Домашна работа 3

зад 1 (см): $p=2$

а) $U=\{0;0,25;0,5;0,75;1\}$ $N_{1,2}$

б) $U=\{0;0,0;0,3;0,5;0,5;0,6;1;1;1\}$ $N_{3,2}$

а)

u_0	u_1	u_2	u_3	u_4
0	0,25	0,5	0,75	1

$[u_0,u_1)$ $[0,0,25)$ $N_{0,0}$

$[u_1,u_2)$ $[0,25;0,5)$ $N_{1,0}$ $N_{0,1}$

$[u_2,u_3)$ $[0,5;0,75)$ $N_{2,0}$ $N_{1,1}$ $N_{0,2}$

$[u_3,u_4)$ $[0,75;1)$ $N_{3,0}$ $N_{2,1}$ $N_{1,2}$

$$N_{1,0}(u)=\begin{cases}1, \text{ако } u\in[0,75;0,5)\\ 0, \text{ако } u\notin[0,25;0,5)\end{cases}$$

$$N_{2,0}(u)=\begin{cases}1, \text{ако } u\in[0,5;0,75)\\ 0, \text{ако } u\notin[0,5;0,75)\end{cases}$$

$$N_{3,0}(u)=\begin{cases}1, \text{ако } u\in[0,75;1)\\ 0, \text{ако } u\notin[0,75;1)\end{cases}$$

$$N_{1,1}(u)=\frac{u-u_1}{u_2-u_1}N_{1,0}(u)+\frac{u_3-u}{u_3-u_2}N_{2,0}(u)=$$

$$=\frac{u-0,25}{0,5-0,25}N_{1,0}(u)+\frac{0,75-u}{0,75-0,5}N_{2,0}(u)=$$

$$= \frac{u-0,25}{0,3-0,25} N_{1,0}(u) + \frac{0,75-u}{0,25} N_{2,0}(u) =$$

$$= \frac{u-\frac{1}{4}}{\frac{1}{4}} N_{1,0}(u) + \frac{\frac{3}{4}-u}{\frac{1}{4}} N_{2,0}(u) =$$

$$= 4\left(u-\frac{1}{4}\right) N_{1,0}(u) + 4\left(\frac{3}{4}-u\right) N_{2,0}(u) =$$

$$= (4u-1) N_{1,0}(u) + (3-4u) N_{2,0}(u)$$

$$\Rightarrow N_{1,1}(u) = \begin{cases} 4u-1 & \text{ako } u \in [0,25; 0,5) \\ 3-4u, & \text{ako } u \in [0,5; 0,75) \\ 0, & \text{ako } u \notin [0,25; 0,75) \end{cases}$$

$$N_{2,1}(u) = \frac{u-u_2}{u_3-u_2} \cdot N_{2,0}(u) + \frac{u_4-u}{u_4-u_3} \cdot N_{3,0}(u)$$

$$= \frac{u-0,5}{0,75-0,5} N_{2,0}(u) + \frac{1-u}{1-0,75} \cdot N_{3,0}(u)$$

$$= \frac{u-\frac{1}{2}}{\frac{1}{4}} \cdot N_{2,0}(u) + \frac{1-u}{\frac{1}{4}} \cdot N_{3,0}(u)$$

$$= 4\left(u-\frac{1}{2}\right) \cdot N_{2,0}(u) + 4(1-u) \cdot N_{3,0}(u)$$

$$\Rightarrow (4u-2) N_{2,0}(u) + (4-4u) N_{3,0}(u)$$

$$\Rightarrow N_{2,1}(u) = \begin{cases} 4u-2 & \text{ako } u \in [0,5; 0,75) \\ 4-4u, & \text{ako } u \in [0,75; 1] \\ 0, & \text{ako } u \notin [0,5; 1] \end{cases}$$

$$N_{1,2}(u) = \frac{u-u_1}{u_3-u_1} \cdot N_{1,1}(u) + \frac{u_4-u}{u_4-u_2} \cdot N_{2,1}(u)$$

$$= \frac{u-0{,}25}{0{,}75-0{,}25} \cdot N_{1,1}(u) + \frac{1-u}{1-0{,}5} N_{2,1}(u) =$$

$$= \frac{u-\frac{1}{4}}{\frac{1}{2}} \cdot N_{1,1}(u) + \frac{1-u}{\frac{1}{2}} N_{2,1}(u)$$

$$= 2\left(u-\frac{1}{4}\right) N_{1,1}(u) + 2(1-u) N_{2,1}(u)$$

$$= \left(2u-\frac{1}{2}\right) N_{1,1}(u) + (2-2u) N_{2,1}(u)$$

$\Rightarrow$ 1) за $u \in [0{,}25; 0{,}5)$ $N_{1,1}(u) = 4u-1$ и $N_{2,1}(u) = 0$

$$\Rightarrow N_{1,2}(u) = \left(2u-\frac{1}{2}\right)(4u-1) + (2-2u) \cdot 0 =$$
$$= 8u^2 - 2u - 2u + \frac{1}{2} = 8u^2 - 4u + \frac{1}{2}$$

2) за $u \in [0{,}5; 0{,}75)$ $N_{1,1}(u) = 3-4u$ и $N_{2,1}(u) = 4u-2$

$$\Rightarrow N_{1,2}(u) = \left(2u-\frac{1}{2}\right)(3-4u) + (2-2u)(4u-2) =$$
$$= 6u - 8u^2 - \frac{3}{2} + 2u + 8u - 4 - 8u^2 + 4u =$$
$$= -16u^2 + 20u - \frac{11}{2}$$

3) за $u \in [0{,}75; 1]$ $N_{1,1}(u) = 0$ и $N_{2,1}(u) = 4-4u$

$$\Rightarrow N_{1,2}(u) = \left(2u-\frac{1}{2}\right) \cdot 0 + (2-2u)(4-4u) =$$
$$= 8 - 8u - 8u + 8u^2 =$$
$$= 8u^2 - 16u + 8$$

$$\Rightarrow N_{1,2}(u) = \begin{cases} 8u^2 - 4u + \frac{1}{2}, & \text{ako } u \in [0{,}25; 0{,}5) \\ -16u^2 + 20u - \frac{11}{2}, & \text{ako } u \in [0{,}5; 0{,}75) \\ 8u^2 - 16u + 8, & \text{ako } u \in [0{,}75; 1] \\ 0, & \text{ako } u \notin [0{,}25; 1] \end{cases}$$

$$N_{1,2}(0{,}4) = 8 \cdot (0{,}4)^2 - 4 \cdot 0{,}4 + \frac{1}{2} = 8\left(\frac{2}{5}\right)^2 - 4 \cdot \frac{2}{5} + \frac{1}{2} = \frac{32}{25} - \frac{8}{5} + \frac{1}{2}$$

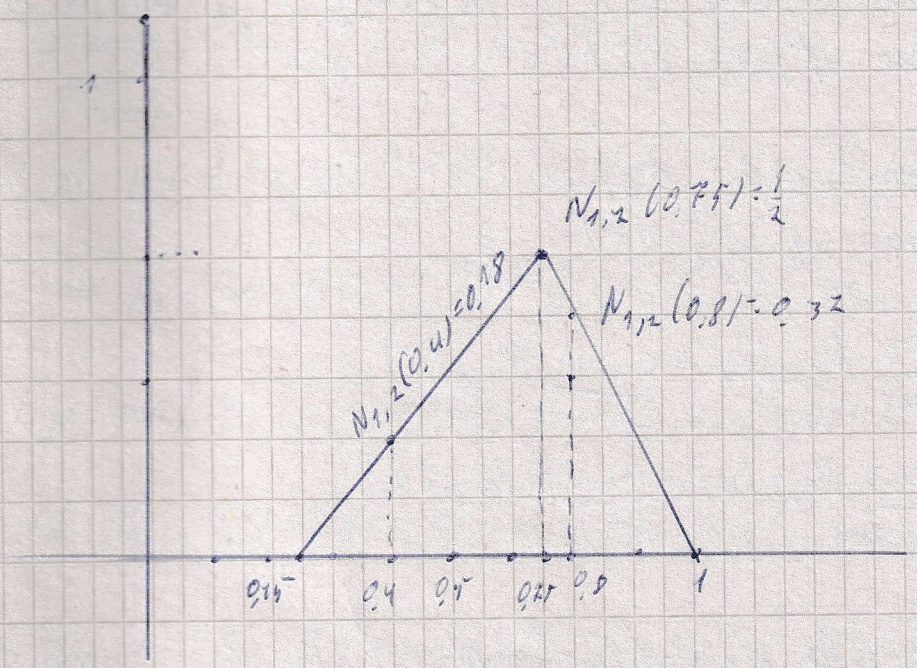
$$= \frac{64}{50} - \frac{80}{50} + \frac{25}{50} = \frac{89 - 80}{50} = \frac{9}{50} = 0{,}18 \in [0{,}25; 0{,}5]$$

$$N_{1,2}(0{,}8) = 8\left(\frac{4}{5}\right)^2 - 16 \cdot \frac{4}{5} + 8 = 8 \cdot \frac{16}{25} - \frac{64}{5} + 8 = \frac{128}{25} - \frac{64}{5} + \frac{200}{25}$$

$$= \frac{128 - 320 + 200}{25} = \frac{8}{25} = 0{,}32 \in [0{,}75; 1]$$

$$N_{1,2}(0{,}5) = -16 \cdot \left(\frac{1}{2}\right)^2 + 20 \cdot \frac{1}{2} - \frac{11}{2} = -\frac{8}{2} + \frac{20}{2} - \frac{11}{2} = \frac{1}{2}$$

$$N_{1,2}(0{,}75) = 8 \cdot \left(\frac{3}{4}\right)^2 - 16 \cdot \left(\frac{3}{4}\right)^2 - 16 \cdot \frac{3}{4} + 8 = \frac{18}{4} - \frac{48}{4} + \frac{32}{4} = \frac{2}{4} = \frac{1}{2}$$



б) $u = \{0; 0; 0; 0{,}3; 0{,}5; 0{,}5; 0{,}6; 1; 1; 1\}$ №3.2

u_0	u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8	u_9
0	0	0	0,3	0,5	0,5	0,6	1	1	1

$[u_0, u_1)$ $[0; 0)$ $N_{0,0}$

$[u_1, u_2)$ $[0, 0)$ $N_{1,0}$ — $N_{0,1}$

$[u_2, u_3)$ $[0; 0{,}3)$ $N_{2,0}$ — $N_{1,1}$ — $N_{0,2}$

$[u_3; u_4)$ $[0{,}3; 0{,}5)$ $N_{3,0}$ — $N_{2,1}$ — $N_{1,2}$

$[u_4, u_5)$ $[0{,}5; 0{,}5)$ $N_{4,0}$ — $N_{3,1}$ — $N_{2,2}$

$[u_5; u_6)$ $[0{,}5; 0{,}6)$ $N_{5,0}$ — $N_{4,1}$ — $N_{3,2}$

$[u_6; u_7)$ $[0{,}6; 1)$ $N_{6,0}$ — $N_{5,1}$ — $N_{4,2}$

$[u_7; u_8)$ $[1; 1)$ $N_{7,0}$ — $N_{6,1}$ — $N_{5,2}$

$[u_8; u_9)$ $[1; 1)$ $N_{8,0}$ — $N_{7,1}$ — $N_{6,2}$

$$N_{3,0}(u) = \begin{cases} 1, \text{ako } u \in [0{,}3; 0{,}5) \\ 0, \text{ako } u \notin [0{,}3; 0{,}5) \end{cases}$$

$$N_{4,0}(u) = \begin{cases} 1, \text{ako } u \in [0{,}5; 0{,}5) \\ 0, \text{ako } u \notin [0{,}5; 0{,}5) \end{cases} > N_{4,0}(u) = 0 \text{ за } \forall u \in [0; 1)$$

$$N_{5,0}(u) = \begin{cases} 1, \text{ako } u \in [0{,}5; 0.6) \\ 0, \text{ako } u \notin [0{,}5; 0{,}6) \end{cases}$$

$$N_{3,1}(u) = \frac{u - u_3}{u_4 - u_3} N_{3,0}(u) + \frac{u_5 - u}{u_5 - u_4} N_{4,0}(u) =$$

i p

$$= \frac{u - 0{,}3}{0{,}5 - 0{,}3} N_{3,0}(u) = \frac{u - 0{,}3}{0{,}2} N_{3,0}(u)$$

$$= 5(u - 0{,}3) N_{3,0}(u) = \left(5u - \frac{3}{2}\right) N_{3,0}(u)$$

$$= N_{3,1}(u) = \begin{cases} 5u - \frac{3}{2}, & \text{ако } u \in [0{,}3; 0{,}5) \\ 0, & \text{ако } u \notin [0{,}3; 0{,}5) \end{cases}$$

$$= \frac{0{,}6 - u}{0{,}6 - 0{,}5} N_{5,0}(u) = (6 - 10u) \cdot N_{5,0}(u)$$

$$= \frac{0{,}6 - u}{0{,}6 - 0{,}5} N_{5,0}(u) = (6 - 10u) N_{5,0}(u) \Rightarrow N_{3,1}(u) = \begin{cases} 5u - \frac{3}{2}, & u \in [0{,}3; 0{,}5) \\ 0, & u \notin [0{,}3; 0{,}5) \end{cases}$$

$$N_{4,1}(u) = \frac{u - u_4}{u_5 - u_4} N_{4,0}(u) + \frac{u_6 - u}{u_6 - u_5} N_{5,0}(u)$$

$$= \frac{0{,}6 - u}{0{,}6 - 0{,}5} \cdot N_{5,0}(u) = (6 - 10u) \cdot N_{5,0}(u)$$

$$\Rightarrow N_{4,1}(u) = \begin{cases} 6 - 10u, & \text{ако } u \in [0{,}5; 0{,}6) \\ 0, & \text{ако } u \notin [0{,}5; 0{,}6) \end{cases}$$

$$N_{3,2}(u) = \frac{u - u_3}{u_6 - u_3} N_{3,1}(u) + \frac{u_6 - u}{u_6 - u_4} \cdot N_{4,1}(u) =$$

$$= \left(5u - \frac{3}{2}\right) \cdot N_{3,1}(u) + (6 - 10u) N_{4,1}(u)$$

$\Rightarrow$ 1) за $u \in [0{,}3; 0{,}5) \Rightarrow N_{3,1}(u) = 5u - \frac{3}{2}$ и $N_{4,1}(u) = 0$

$\Rightarrow N_{3,2}(u) = \left(5u - \frac{3}{2}\right)\left(5u - \frac{3}{2}\right) = \left(5u - 3/2\right)^2$ при $u \in [0{,}3; 0{,}5)$

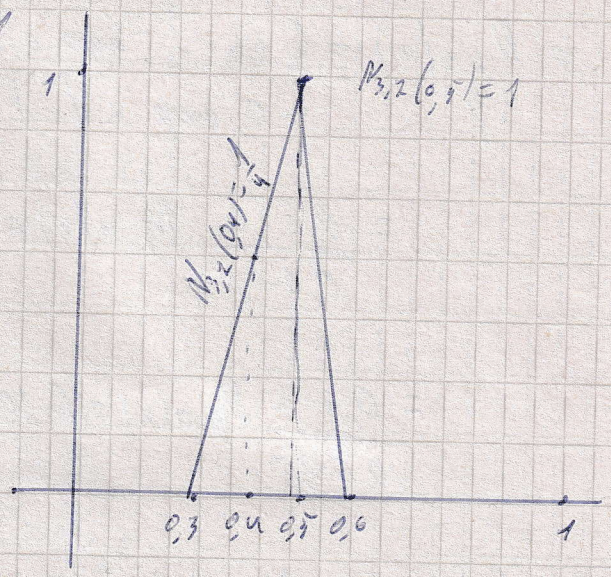
2) за $u \in [0{,}5; 0{,}6) \Rightarrow N_{3,1}(u) = 0$ и $N_{4,1}(u) = 6 - 10u$

$\Rightarrow N_{3,2}(u) = (6 - 10u)(6 - 10u) = (6 - 10u)^2$ при $u \in [0{,}5; 0{,}6)$

$$\Rightarrow N_{3,2}(u) = \begin{cases} \left(5u - \frac{3}{2}\right)^2, & \text{ако } u \in [0{,}3; 0{,}5) \\ (6 - 10u)^2 & \text{ако } u \in [0{,}5; 0{,}6) \\ 0, & \text{ако } u \notin [0{,}3; 0{,}6) \end{cases}$$

$$N_{3,2}(0,4) = \left(5 - \frac{7}{5} - \frac{7}{2}\right)^2 = \frac{1}{4}$$

$$N_{3,2}(0,5) = \left(6 - 10 \cdot \frac{1}{2}\right)^2 = 1$$



węzły

u_0	u_1	u_2	u_3	u_4	u_5	u_6	u_7	u_8	u_9	u_{10}	u_{11}	u_{12}
0	0	0	0	$\frac{1}{6}$	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{5}{6}$	1	1	1	1

I^bo bowiem:

$t = 0,4 \in [u_5, u_6) = [0,33; 0,5) \rightarrow P_5, P_4, P_3, P_2$ - zaczerpniemy punkty

$$a_{3,i} = \frac{t - u}{u_{i+p} - u_i} \qquad t = 0,4 = \frac{2}{5}$$

$$a_5 = \frac{\frac{2}{5} - \frac{1}{3}}{\frac{5}{6} - \frac{1}{3}} = \frac{1}{15} \cdot 2 = \frac{2}{15}$$

$$a_4 = \frac{\frac{2}{5} - \frac{1}{6}}{\frac{5}{6} - \frac{1}{6}} = \frac{14}{20} = \frac{7}{15}$$

$$a_3 = \frac{\frac{2}{5} - 0}{\frac{1}{2} - 0} = \frac{4}{5}$$

$$Q_i = (1 - a_i) P_{i-1} + a_i \cdot P_i$$

$$Q_5 = \left(1 - \frac{2}{15}\right) \cdot P_4 + \frac{2}{5} \cdot P_5 = \frac{13}{15}(0,0) + \frac{2}{15}(1,1) = \left(\frac{2}{15}, \frac{2}{15}\right) = Q_5$$

$$Q_4 = \left(1-\frac{7}{10}\right)P_3 + \frac{7}{10}P_4 = \frac{8}{10}(-1;1) + \frac{7}{10}(0,0) = \left(-\frac{8}{10}, \frac{8}{10}\right) = Q_4$$

$$Q_3 = \left(1-\frac{4}{5}\right)P_2 + \frac{4}{5}P_3 = \frac{1}{5}(-1,2) + \frac{4}{5}(-1,1) = \left(-1, \frac{6}{5}\right) = Q_3$$

II^ро внутване

$t = 0{,}4 \in [0{,}6; 0{,}7) = \left[\frac{2}{5}; \frac{1}{2}\right) \to P_6', P_5', P_4', P_3'$ - засегнати точки

$$a_{1,2} = \frac{t - v_i}{v_{i+p} - v_i}$$

$$a_{6,2} = \frac{0{,}4 - \frac{2}{5}}{\frac{5}{6} - \frac{2}{5}} = 0$$

$$a_{5,2} = \frac{2}{5} \cdot \frac{1}{3} = \frac{1}{15} \cdot 3 = \frac{1}{5}$$

$$a_{4,2} = \frac{\frac{2}{5} - \frac{1}{6}}{\frac{1}{2} - \frac{1}{6}} = \frac{\frac{7}{30}}{\frac{1}{3}} = \frac{7}{30} \cdot 3 = \frac{7}{10}$$

$$R_i = (1-a_{i,2})P_{i-1}' + a_{i,2}P_i'$$

$$R_6 = (1-0)P_5' + 0 \cdot P_6' = P_5' = Q_5 = \left(\frac{2}{15}, \frac{2}{15}\right) R_6$$

$$R_5 = (1-a_{5,2})R_4' + a_{5,2}R_5' = \frac{4}{5}\left(-\frac{8}{10}, \frac{8}{10}\right) + \frac{1}{5}\left(\frac{2}{15}, \frac{2}{15}\right) = \left(-\frac{302}{5 \cdot 15}, \frac{34}{5 \cdot 15}\right) = \left(-\frac{2}{5}, \frac{34}{75}\right) = R_5$$

$$R_4 = \left(1-\frac{7}{10}\right)P_3' + \frac{7}{10}P_4' = \frac{3}{10}\left(-1, \frac{6}{5}\right) + \frac{7}{10}\left(-\frac{8}{10}, \frac{8}{15}\right) = \left(-\frac{101}{100}, \frac{11}{15}\right) = R_4$$

w_0	w_1	w_2	w_3	w_4	w_5	w_6	w_7	w_8	w_9	w_{10}	w_{11}	w_{12}	w_{13}	w_{14}
0	0	0	0	$\frac{1}{10}$	$\frac{1}{3}$	$\frac{2}{5}$	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{5}{6}$	1	1	1	1	1

P_0''	P_1''	P_2''	P_3''	P_4''	P_5''	P_6''	P_7''	P_8''	P_9''	P_{10}''
P_0	P_1	P_2	Q_3	R_4	R_5	R_6	P_5	P_6	P_7	P_8

$Q_3\left(-1, \frac{6}{9}\right) \to$

$Q_4\left(-\frac{8}{15}, \frac{8}{15}\right) \to$

$Q_5\left(\frac{2}{15}; \frac{2}{15}\right) \to$

$\equiv R_6$

$R_4\left(-\frac{101}{130}, \frac{11}{15}\right)$

$R_5\left(-\frac{2}{5}; \frac{34}{15}\right)$