1 Probability

Use the probability table to calculate the following values:

X_1	X_2	X_3	$P(X_1, X_2, X_3)$
0	0	0	0.05
1	0	0	0.1
0	1	0	0.4
1	1	0	0.1
0	0	1	0.1
1	0	1	0.05
0	1	1	0.2
1	1	1	0.0

1. $P(X_1 = 1, X_2 = 0) = 0.15$

2. $P(X_3 = 0) = 0.65$

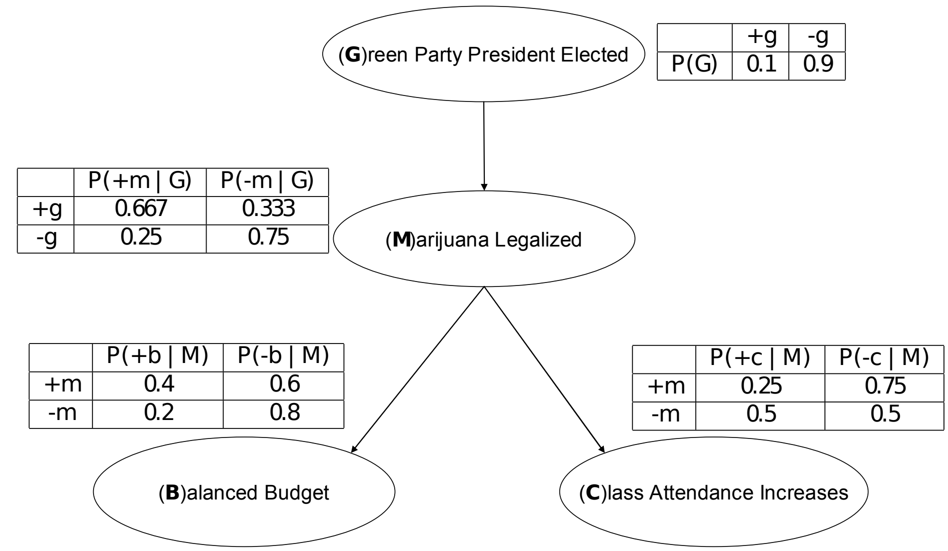
3. $P(X_2 = 1 | X_3 = 1) = 0.2/0.35$

4. $P(X_1 = 0 | X_2 = 1, X_3 = 1) = 1$

5. $P(X_1 = 0, X_2 = 1 | X_3 = 1) = 0.2/0.35$

Q2. Bayes Nets: Green Party President

In a parallel universe the Green Party is running for presidency. Whether a Green Party President is elected (G) will have an effect on whether marijuana is legalized (M), which then influences whether the budget is balanced (B), and whether class attendance increases (C). Armed with the power of probability, the analysts model the situation with the Bayes Net below.



	+g	-g
P(G)	0.1	0.9

	P(+m | G)	P(-m | G)
+g	0.667	0.333
-g	0.25	0.75

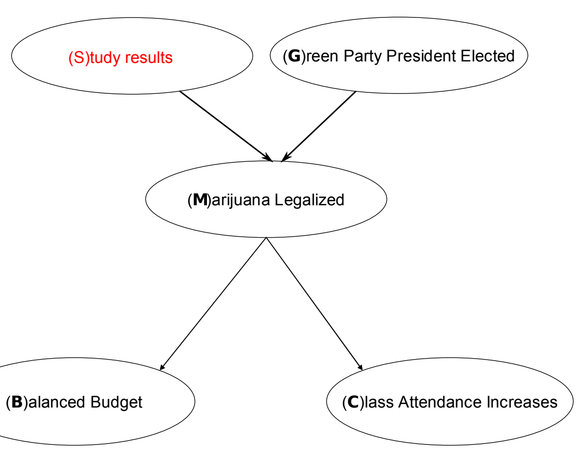
	P(+b | M)	P(-b | M)
+m	0.4	0.6
-m	0.2	0.8

	P(+c | M)	P(-c | M)
+m	0.25	0.75
-m	0.5	0.5

1. The full joint distribution is given below. Fill in the missing values.

G	M	B	C	$P(G,M,B,C)$	G	M	B	C	$P(G,M,B,C)$
+	+	+	+	1/150	-	+	+	+	9/400
+	+	+	-	1/50	-	+	+	-	27/400
+	+	-	+	1/100	-	+	-	+	27/800
+	+	-	-	3/100	-	+	-	-	81/800
+	-	+	+	1/300	-	-	+	+	27/400
+	-	+	-	1/300	-	-	+	-	27/400
+	-	-	+	1/75	-	-	-	+	27/100
+	-	-	-	1/75	-	-	-	-	27/100

2. Now, add a node S to the Bayes net that reflects the possibility that a new scientific study could influence the probability that marijuana is legalized. Assume that the study does not directly influence B or C. Draw the new Bayes net below. Which CPT or CPT's need to be modified?



$P(M|G)$ will become $P(M|G,S)$, and will contain 8 entries instead of 4.

Q3. [Optional Logic Review] In What Worlds?

(a) We wish to come up with hypotheses that entail the following sentences:

- S_1: $X_1 \wedge X_2 \implies Y$
- S_2: $\neg X_1 \vee X_2 \implies Y$

In this problem, we want to come up with a hypothesis H such that $H \vDash S_1 \wedge H \vDash S_2$.

(i) Assume we have the hypothesis H: $Y \iff X_1 \vee X_2$.

Does H entail S_1? ● Yes ○ No

Does H entail S_2? ○ Yes ● No

By looking at the truth table, you see that the for all worlds H is true, S_1 is true. However, H does not entail S_2. One example is $X_1 = false$, $X_2 = false$,$Y = true$.

(ii) Pretend that we have obtained a magical solver, $SAT(s)$ which takes in a sentence s and returns $true$ if s is satisfiable and $false$ otherwise. We wishes to use this solver to determine whether a hypothesis H' entails the two sentences S_1 and S_2. Mark all of the following expressions that correctly return $true$ if and only if $H' \vDash S_1 \wedge H' \vDash S_2$. If none of the expressions are correct, select "None of the above".

- ☐ $SAT(H' \wedge \neg(S_1 \wedge S_2))$
- ☐ $SAT(\neg H' \vee (S_1 \wedge S_2))$
- ■ $\neg SAT(H' \wedge \neg(S_1 \wedge S_2))$
- ☐ $\neg SAT(\neg H' \vee (S_1 \wedge S_2))$
- ☐ None of the above

Recall for H' to entail both S_1 and S_2, it must hold that $H' \wedge \neg(S_1 \wedge S_2)$ is **not** satisfiable.

Four people, Alex, Betty, Cathy, and Dan are going to a famliy gathering. They can bring dishes or games. They have the following predicates in their vocabulary:

[topsep=-10pt]$Brought(p, i)$: Person p brought a dish or game i. $Cooked(p, d)$: Person p cooked dish d. $Played(p, g)$: Person p played game g.

(b) Select which first-order logic sentences are syntactically correct translations for the following English sentences. You must use the syntax shown in class (eg. ∀, ∃, ∧, ⇒, ⇔). **Please select all that apply**.

(i) At least one dish cooked by Alex was brought by Betty.

- ■ $\exists d\ Cooked(A, d) \wedge Brought(B, d)$
- ☐ $[\exists d\ Cooked(A, d)] \wedge [\forall d' \wedge (d' = d)\ Brought(B, d')]$
- ☐ $\neg[\forall d\ Cooked(A, d) \vee Brought(B, d)]$
- ■ $\exists d_1, d_2\ Cooked(A, d_1) \wedge (d_2 = d_1)\ \wedge Brought(B, d_2)$

(ii) At least one game played by Cathy is only played by people who brought dishes.

- ☐ $\neg[\forall g\ Played(C, g) \vee [\exists p\ Played(p, g) \implies \forall d\ Brought(p, d)]]$
- ☐ $\forall p\ \exists g\ Played(C, g) \wedge Played(p, g) \implies \exists d\ Brought(p, d)$
- ☐ $\exists g\ Played(C, g) \implies \forall p \exists d\ Played(p, g) \wedge Brought(p, d)$
- ■ $\exists g\ Played(C, g) \wedge [\forall p\ Played(p, g) \Rightarrow \exists d,\ Brought(p, d)]$

(c) Assume we have the following sentence with variables A, B, C, and D, where each variable takes Boolean values:

$$S3 : (A \vee B \vee \neg C) \wedge (A \vee \neg B \vee D) \wedge (\neg B \vee \neg D)$$

(i) For the above sentence $S3$, state how many worlds make the sentence true. [Hint: you can do this and the next part without constructing a truth table!]

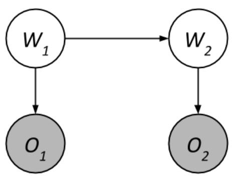
$\boxed{8}$ (1) Clauses disjoint (2) clauses with k literals remove 2^{n-k} models.

(ii) Does $S3 \vDash (A \wedge B \wedge D)$? ○ Yes ● No

1 HMMs

No solutions provied yet

Consider the following Hidden Markov Model. O_1 and O_2 are supposed to be shaded.



W_1	$P(W_1)$
0	0.3
1	0.7

W_t	W_{t+1}	$P(W_{t+1}\|W_t)$
0	0	0.4
0	1	0.6
1	0	0.8
1	1	0.2

W_t	O_t	$P(O_t\|W_t)$
0	a	0.9
0	b	0.1
1	a	0.5
1	b	0.5

Suppose that we observe $O_1 = a$ and $O_2 = b$.
Using the forward algorithm, compute the probability distribution $P(W_2|O_1 = a, O_2 = b)$ one step at a time.

(a) Compute $P(W_1, O_1 = a)$.

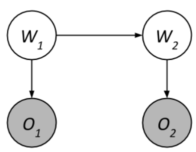
(b) Using the previous calculation, compute $P(W_2, O_1 = a)$.

(c) Using the previous calculation, compute $P(W_2, O_1 = a, O_2 = b)$.

(d) Finally, compute $P(W_2|O_1 = a, O_2 = b)$.

2 Particle Filtering

Let's use Particle Filtering to estimate the distribution of $P(W_2|O_1 = a, O_2 = b)$. Here's the HMM again. O_1 and O_2 are supposed to be shaded.



W_1	$P(W_1)$
0	0.3
1	0.7

W_t	W_{t+1}	$P(W_{t+1}|W_t)$
0	0	0.4
0	1	0.6
1	0	0.8
1	1	0.2

W_t	O_t	$P(O_t|W_t)$
0	a	0.9
0	b	0.1
1	a	0.5
1	b	0.5

We start with two particles representing our distribution for W_1.
$P_1 : W_1 = 0$
$P_2 : W_1 = 1$
Use the following random numbers to run particle filtering:

$$[0.22, 0.05, 0.33, 0.20, 0.84, 0.54, 0.79, 0.66, 0.14, 0.96]$$

(a) Observe: Compute the weight of the two particles after evidence $O_1 = a$.

(b) Resample: Using the random numbers, resample P_1 and P_2 based on the weights.

(c) Predict: Sample P_1 and P_2 from applying the time update.

(d) Update: Compute the weight of the two particles after evidence $O_2 = b$.

(e) Resample: Using the random numbers, resample P_1 and P_2 based on the weights.

(f) What is our estimated distribution for $P(W_2|O_1 = a, O_2 = b)$?