Big Data Computing

Master's Degree in Computer Science
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SAPIENZA
UNIVERSITÀ DI ROMA

Recommendation Strategies

3 approaches to recommender systems

Content-based filtering

Collaborative filtering

Hybrid

Neighborhood-based
(a.k.a. memory-based)

Latent-factor-based
(a.k.a. model-based)

Hybrid
(memory-model-based)

LATENT FACTOR MODELS

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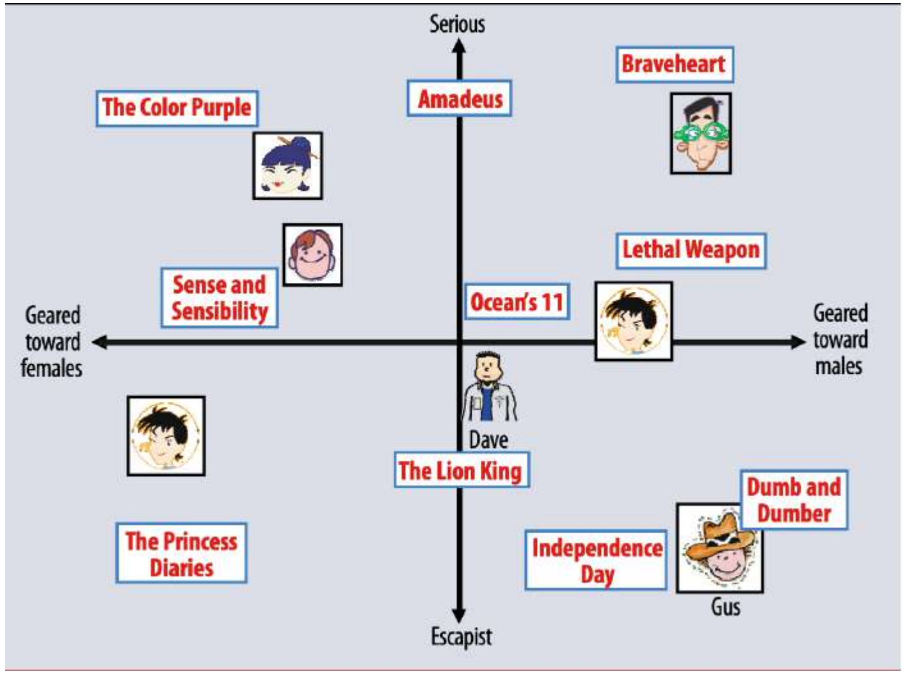

Example: 2 hidden factors

- Dim. 1: Male vs. Female
- Dim. 2: Serious vs. Escapist

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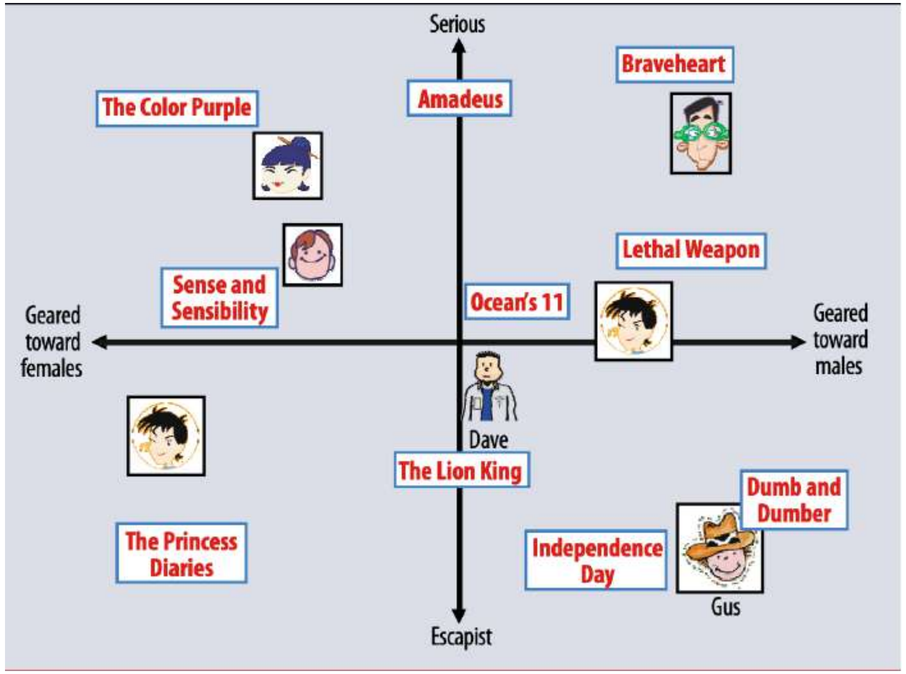

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A user's predicted rating for an item (movie) would equal the dot product of the movie and user vectors on the plot

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- Such vectors are inferred (i.e., learned) from observed item ratings
- High correspondence between item and user factors leads to a recommendation

Matrix Factorization Framework

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- That is why these features are often refer to as **latent features**

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Recommendations for a user are generated by computing the estimated ratings for unseen items, and by taking the top-k highest rated ones

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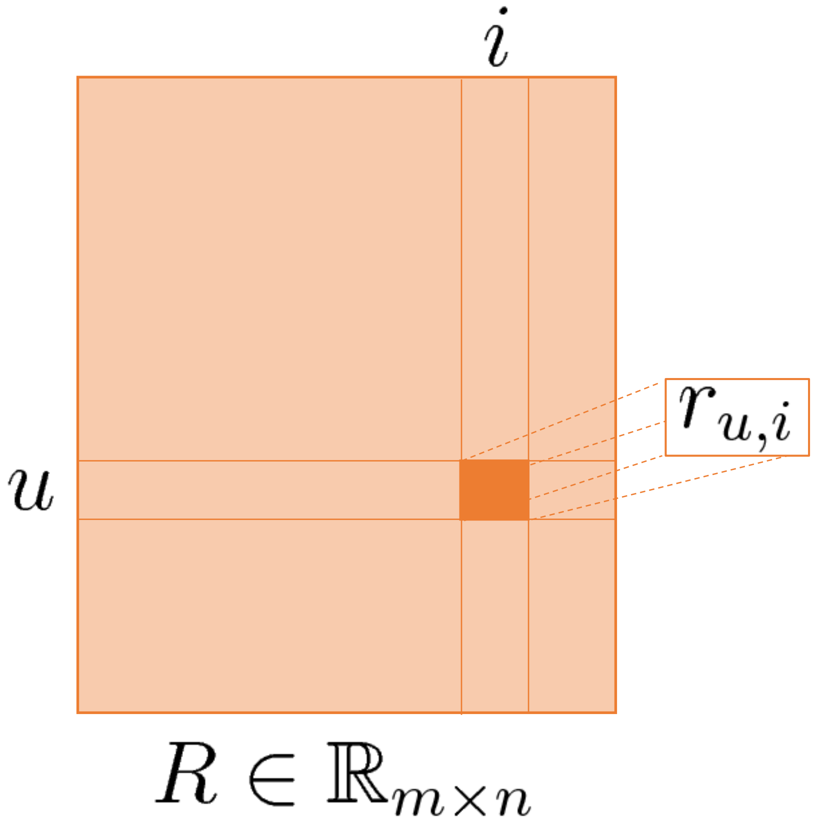

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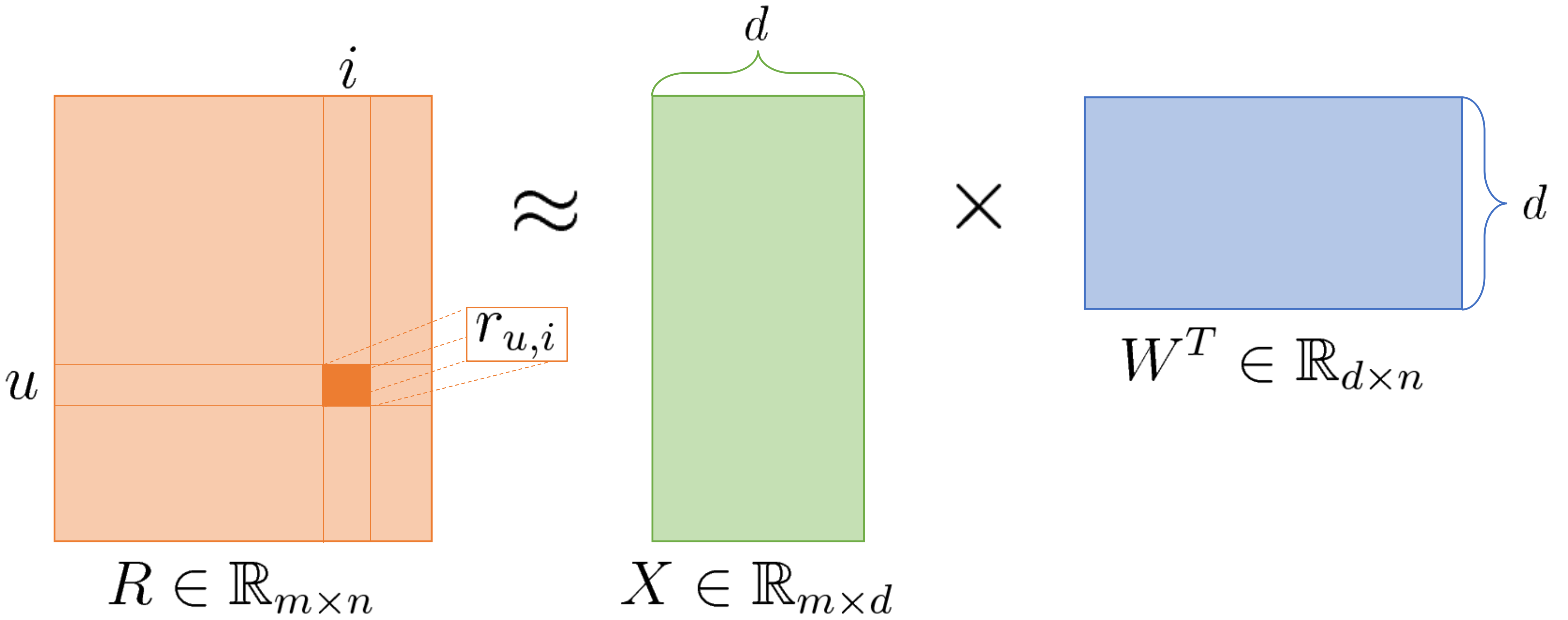

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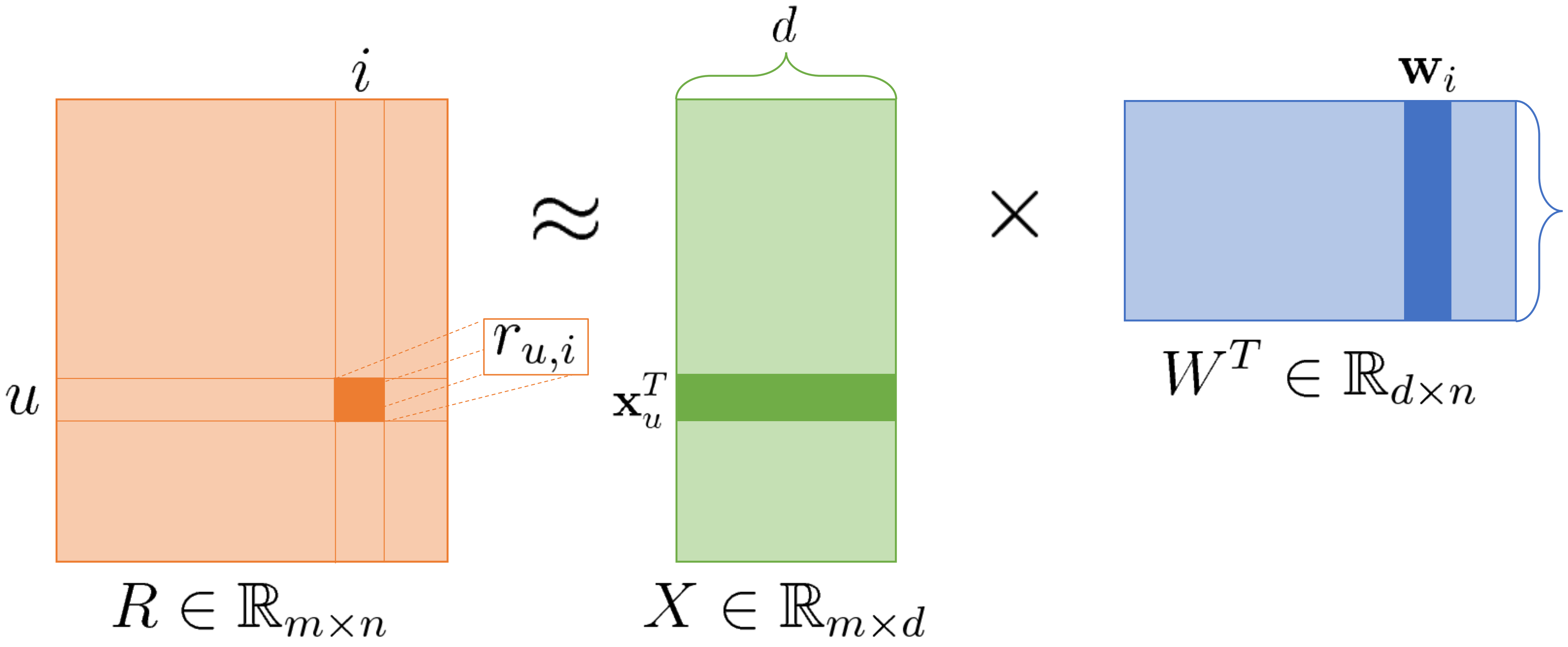

Approximate the user-item rating matrix R with the product of X x W^T

How Do We Learn X and W?

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To actually learn the latent factor representations x_u and w_i we minimize the following loss function

$$L(X, W) = \sum_{(u,i) \in \mathcal{D}} \left(r_{u,i} - \mathbf{x}_u^T \cdot \mathbf{w}_i \right)^2 + \lambda \Big(\sum_{u \in \mathcal{D}} ||\mathbf{x}_u||^2 + \sum_{i \in \mathcal{D}} ||\mathbf{w}_i||^2 \Big)$$

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Mathematically convenient

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Still, how do we solve this?

Learning Algorithms

$\boxed{\text{2 main optimization methods}}$

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$$\nabla L(\mathbf{x}_u; \mathbf{w}_i) = \frac{1}{2}\Big[-2(r_{u,i} - \mathbf{x}_u^T \cdot \mathbf{w}_i)\mathbf{w}_i + 2\lambda\mathbf{x}_u\Big] = -(r_{u,i} - \mathbf{x}_u^T \cdot \mathbf{w}_i)\mathbf{w}_i + \lambda\mathbf{x}_u$$

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We know that the updating strategy for SGD is as follows:

$$\mathbf{x}_u^{(t+1)} \leftarrow \mathbf{x}_u^{(t)} - \eta \nabla L(\mathbf{x}_u^{(t)}; \mathbf{w}_i^{(t)})$$

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At each iteration, both user and item latent vectors are updated by a magnitude proportional to η in the opposite direction of the gradient

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- In real life problems, this number can get very large quite often, requiring both a parallelization mechanism or an alternative optimizer

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- Each alternating iteration reduces to traditional least squares and can be solved using OLS or its regularized variant (e.g., pseudo-inverse)

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We want to set this to $\boxed{-\sum_{i\in\mathcal{D}}(r_{u,i} - \mathbf{x}_u^T \cdot \mathbf{w}_i)\mathbf{w}_i + \lambda\mathbf{x}_u = 0}$

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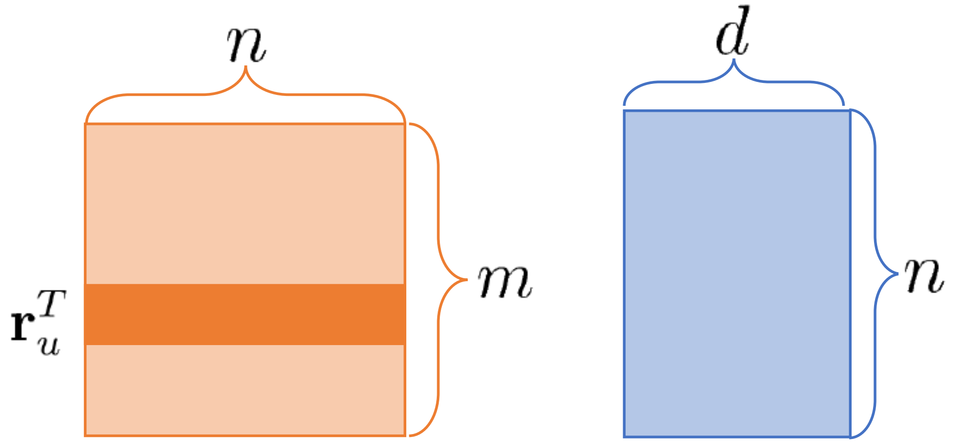

user-item rating matrix R item-matrix W

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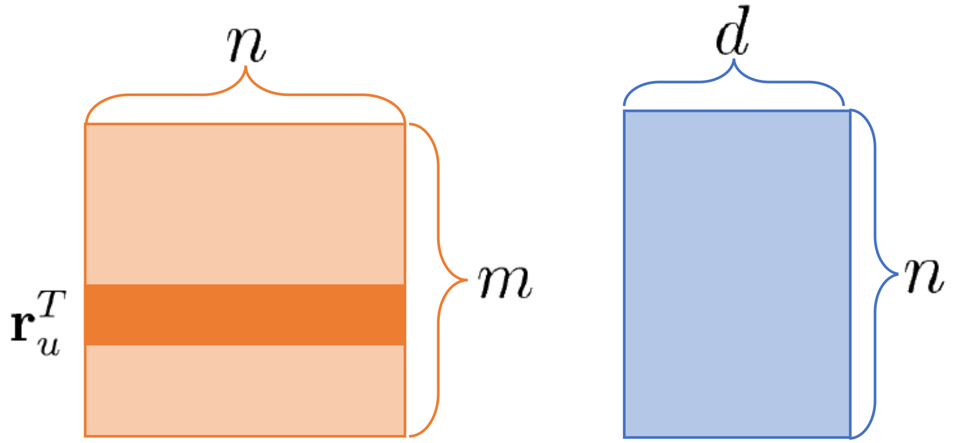

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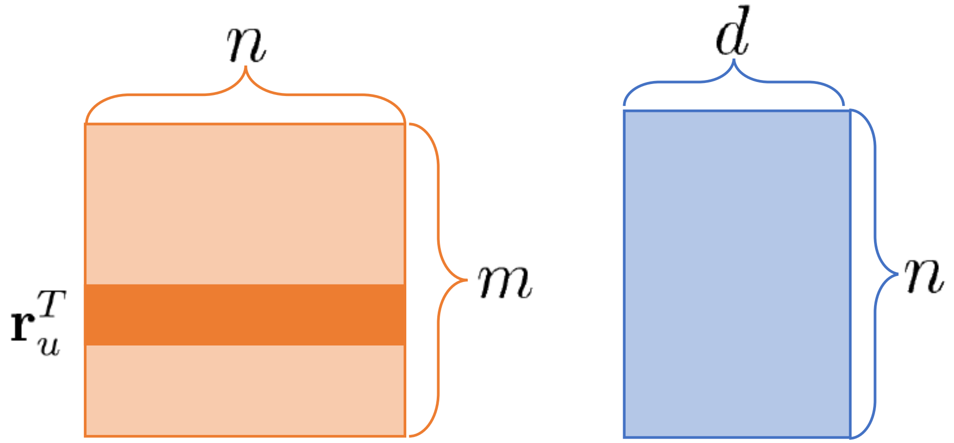

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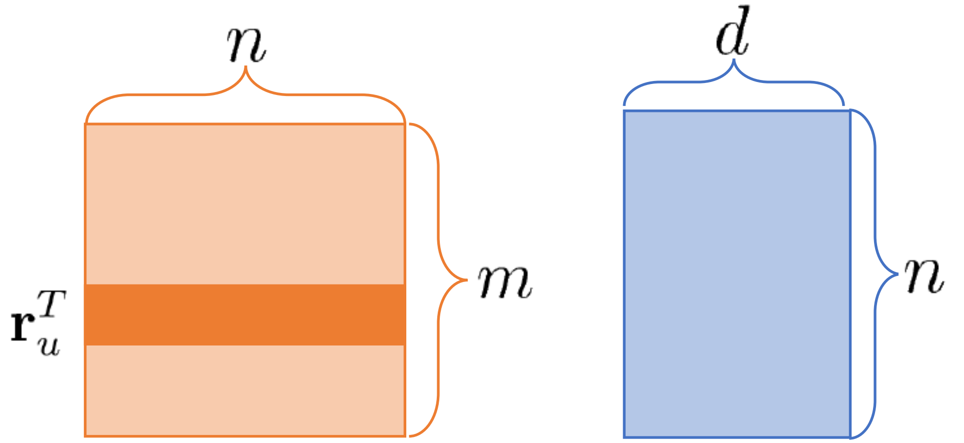
$$= W^T \cdot \mathbf{r}_u = \mathbf{x}_u (W^T W + \lambda I)$$ $I \in \mathbb{R}_{d \times d}$ identity matrix

$$= (W^T W + \lambda I)^{-1} \cdot W^T \cdot \mathbf{r}_u = \mathbf{x}_u (W^T W + \lambda I) \cdot (W^T W + \lambda I)^{-1}$$



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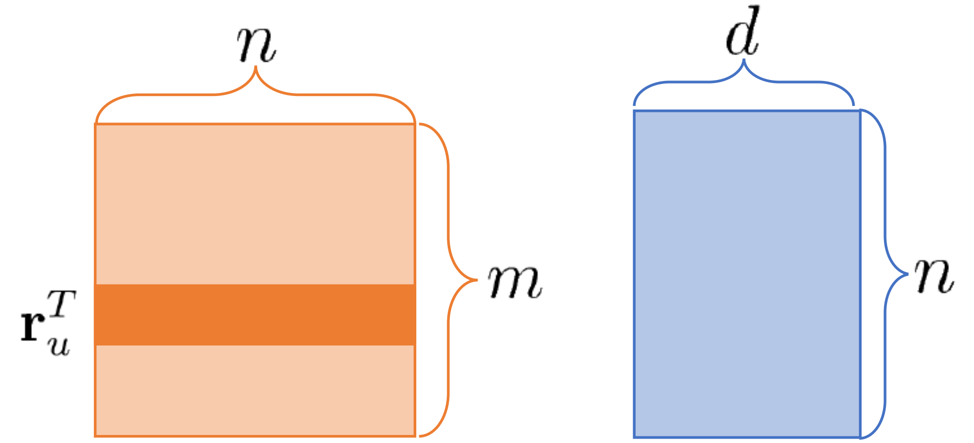

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$$=W^T\cdot\mathbf{r}_u=W^TW\cdot\mathbf{x}_u+\lambda\mathbf{x}_u$$

$$=W^T\cdot\mathbf{r}_u=\mathbf{x}_u(W^TW+\lambda I)$$ $I\in\mathbb{R}_{d\times d}$ identity matrix

$$=(W^TW+\lambda I)^{-1}\cdot W^T\cdot\mathbf{r}_u=\mathbf{x}_u\cancel{(W^TW+\lambda I)}\cdot\cancel{(W^TW+\lambda I)^{-1}}$$

$$\boxed{\mathbf{x}_u=(W^TW+\lambda I)^{-1}\cdot W^T\cdot\mathbf{r}_u}$$

ALS: User Vector Fixed

$$-\sum_{u\in\mathcal{D}}(r_{u,i} - \mathbf{x}_u^T \cdot \mathbf{w}_i)\mathbf{x}_u + \lambda \mathbf{w}_i = 0$$

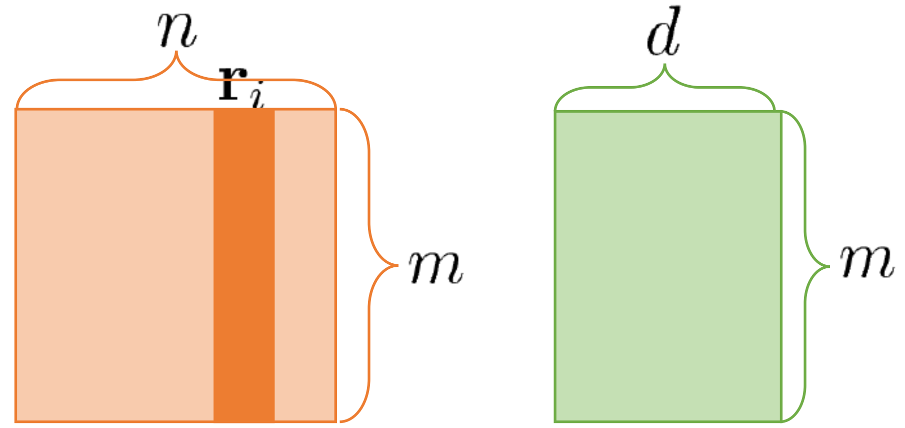
$$= -X^T(\mathbf{r}_i - X \cdot \mathbf{w}_i) - \lambda \mathbf{w}_i = 0$$

$$= X^T \cdot \mathbf{r}_i = X^T X \cdot \mathbf{w}_i + \lambda \mathbf{w}_i$$

$$= X^T \cdot \mathbf{r}_i = \mathbf{w}_i(X^T X + \lambda I)$$ $I \in \mathbb{R}_{d\times d}$ identity matrix

$$= (X^T X + \lambda I)^{-1} \cdot X^T \cdot \mathbf{r}_i = \mathbf{w}_i(\cancel{X^T X + \lambda I}) \cdot (\cancel{X^T X + \lambda I})^{-1}$$

$$\boxed{\mathbf{w}_i = (X^T X + \lambda I)^{-1} \cdot X^T \cdot \mathbf{r}_i}$$



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Convergence is guaranteed because in each step the loss function either decreases or stays unchanged, never increases

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- However, ALS is favorable in at least 2 cases:
  - **Parallelization:** each $\mathbf{x}_u$ and $\mathbf{w}_i$ is computed independently of user/item factors
  - **Implicit Data:** the training set is dense and looping over each single instance - as SGD does - would be unfeasible

Including Biases

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- The basic learning framework tries to capture the interactions between users and items that produce the different rating values

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- For example, some users systematically tend to give higher ratings than others, and some items receive higher ratings than others

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$$b_{\text{Joe,Titanic}} = 3.7 - 0.3 + 0.5 = 3.9$$

Bias term

Including Bias into the Optimization

$$\hat{r}_{u,i} = \underbrace{\mathbf{x}_u^T \cdot \mathbf{w}_i}_{\text{latent factors}} + \underbrace{\mu + b_u + b_i}_{\text{bias}}$$

The estimated rating of an item i for the user u is now made of 2 components

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models global average, user and item bias

Including Bias into the Optimization

Overall, the original optimization problem becomes as follows

$$X^*, W^* = \text{argmin}_{X,W} \left\{ \frac{1}{2} \sum_{(u,i) \in \mathcal{D}} \left[r_{u,i} - (\mathbf{x}_u^T \cdot \mathbf{w}_i + \mu + b_u + b_i) \right]^2 + \lambda \left(\sum_{u \in \mathcal{D}} ||\mathbf{x}_u||^2 + \sum_{i \in \mathcal{D}} ||\mathbf{w}_i||^2 + \sum_{u \in \mathcal{D}} b_u^2 + \sum_{i \in \mathcal{D}} b_i^2 \right) \right\}$$

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Can still be solved using ALS

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the vast majority of items are not rated by users

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- They can also be used to overcome common problems in recommender systems such as cold start and the sparseness of user-item matrix
- Netflix is a good example of hybrid recommender systems

Netflix's Hybrid Recommender System

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[Netflix: What Happens When You Press Play?]

For more details about how Netflix actually works

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- Participating teams submit predicted ratings for a test set of approximately 3M ratings

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- According to the contest website, more than 48,000 teams from 182 different countries have downloaded the data

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A combination of 100 different predictor sets, mostly factorization models

Evaluation Metrics

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Online

A/B testing measuring CTR, ROI, and other "live" metrics

Evaluation Metrics: RMSE

$$\mathrm{RMSE} = \frac{1}{|\mathcal{D}_{\text{test}}|} \sqrt{\sum_{(u,i) \in \mathcal{D}_{\text{test}}} (r_{u,i} - \hat{r}_{u,i})^2}$$

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The RMSE might penalize a method that does well for high ratings and badly for others

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For a binary classifier predicting a condition (y = 1) or not, we define

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Mapping of binary classification terminology to recommender systems

binary classifier	recommender system
# with condition (y = 1)	# of all possible relevant items for a user
# predicted positive (TP + FP)	# of recommended items
# correct positives (TP)	# of recommended items that are relavant

Evaluation Metrics: Precision & Recall

For a recommender system, we can therefore define

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A recommender system generates k=5 items to recommend

There are only 3 relevant items

The success/failure of our recommendations: [0, 1, 1, 0, 0] 0=not relevant/1=relevant

$$P = \frac{2}{5} \qquad R = \frac{2}{3}$$

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- Imagine taking our list of N recommendations and considering only the first element, then only the first two, then only the first three, and so on
- P@k and R@k are simply the precision and recall calculated only from the subset of the first k recommendations

P@k: Example

$$k = 3$$

$$P@3 = \frac{1}{3}$$

Rank	Product Recommended	Result
1	Credit card	Correct positive
2	Christmas Fund	False positive
3	Debit Card	False positive
4	Auto loan	False positive
5	HELOC	Correct Positive
6	College Fund	Correct positive
7	Personal loan	False positive

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$$k = 6$$

$$P@6 = \frac{3}{6}$$

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indicator function $\mathbf{1}_{\mathrm{Rel}}(k) = \begin{cases} 1 & \text{if item } k \in \mathrm{Rel} \\ 0 & \text{otherwise} \end{cases}$

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Intuitively, a high personalization score indicates the recommender system is able to provide a highly personalized experience to the users

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u_1 = [A, B, C, D] u_2 = [A, B, C, E] u_3 = [A, B, F, G]

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	A	B	C	D	E	F	G
u_1	1	1	1	1	0	0	0
u_2	1	1	1	0	1	0	0
u_3	1	1	0	0	0	1	1

Personalization

Compute the 3-by-3 triangular matrix containing the cosine similarity between each pair of user's recommendation binary vector

$$M_{i,j} = \text{cosine}(\mathbf{u}_i, \mathbf{u}_j)$$

	u_1	u_2	u_3
u_1	1	0.75	0.58
u_2	0.75	1	0.58
u_3	0.58	0.58	1

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Take the average of the upper triangle of the matrix M above

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Personalization = 1 – 0.64 = 0.36

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  - Content-based (explicitly creating user and item profiles)
  - Collaborative-filtering (extract patterns from past observed ratings)

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  - Collaborative-filtering (extract patterns from past observed ratings)
- Hybrid approaches combining both usually work better in practice
- New Neural-Network-based approaches have been proposed recently

Recommended Readings and Information :)

- A huge body of work on recommender systems is available out there!
- Surveys:
  - Adomavicius & Tuzhilin [2005]
  - Koren & Volinsky [2009]
  - Bobadilla *et al.* [2013]
  - Zhang *et al.* [2019]
- Well-renowned series of Conferences: RecSys, KDD, SIGIR, TheWebConf