Big Data Computing

Master's Degree in Computer Science

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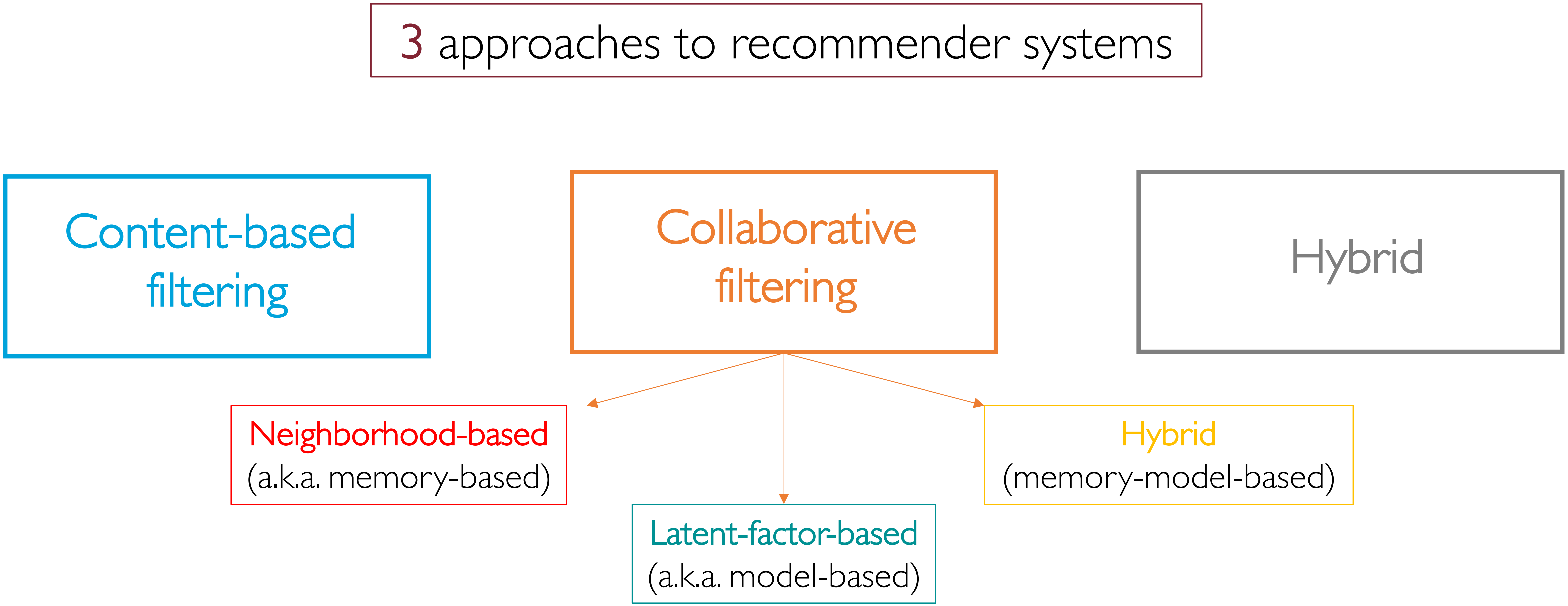
SAPIENZA
UNIVERSITÀ DI ROMA

Recommendation Strategies



COLLABORATIVE FILTERING

Collaborative Filtering (CF)

Idea

Recommend items to user *u* based on preferences of other users similar to *u*

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Core concept:

User-to-User or Item-to-Item similarity

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Core concept:

User-to-User or Item-to-Item similarity

No need for explicit creation of user/item profiles

Collaborative Filtering: Approaches

3 main approaches to collaborative filtering

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Neighborhood-based
(a.k.a. memory-based)

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Latent-factor-based
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3 main approaches to collaborative filtering

Neighborhood-based
(a.k.a. memory-based)

Hybrid
(memory-model-based)

Latent-factor-based
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Neighborhood-based (Memory-based) CF

Compute the relationship between users or items

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User-based

Evaluates a user's preference for an item based on ratings of "neighboring" users for that item

Neighborhood-based (Memory-based) CF

Compute the relationship between users or items

User-based

Evaluates a user's preference for an item based on ratings of "neighboring" users for that item

Item-based

Evaluates a user's preference for an item based on ratings of "neighboring" items by the same user

USER-BASED COLLABORATIVE FILTERING

User-based Neighborhood

Given a user *u* and an item *i* not rated by *u*, we want to estimate *r(u, i)*

User-based Neighborhood

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extract a subset of *k* **neighbours** of *u*

User-based Neighborhood

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k-neighborhood of u is found on the basis of the similarity between user ratings
without the need of explicit user profiles

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k-neighborhood of *u* is found on the basis of the similarity between user ratings
without the need of explicit user profiles

Estimate *r(u, i)* based on the ratings of users in the *k*-neighborhood of *u*

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In theory, rating prediction *r(u,i)* could be defined on any item *i* not rated by *u*

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In practice, we are interested only in estimating $r(u,i)$ for those items i which have been rated by the u's k-neighborhood

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In practice, we are interested only in estimating *r(u,i)* for those items *i* which have been rated by the *u*'s *k*-neighborhood

Intuitively, if a user *v* is not in the *u*'s *k*-neighborhood then very likely *u* will not be interested in any item that only *v* has rated

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In practice, we are interested only in estimating *r(u,i)* for those items *i* which have been rated by the *u*'s *k*-neighborhood

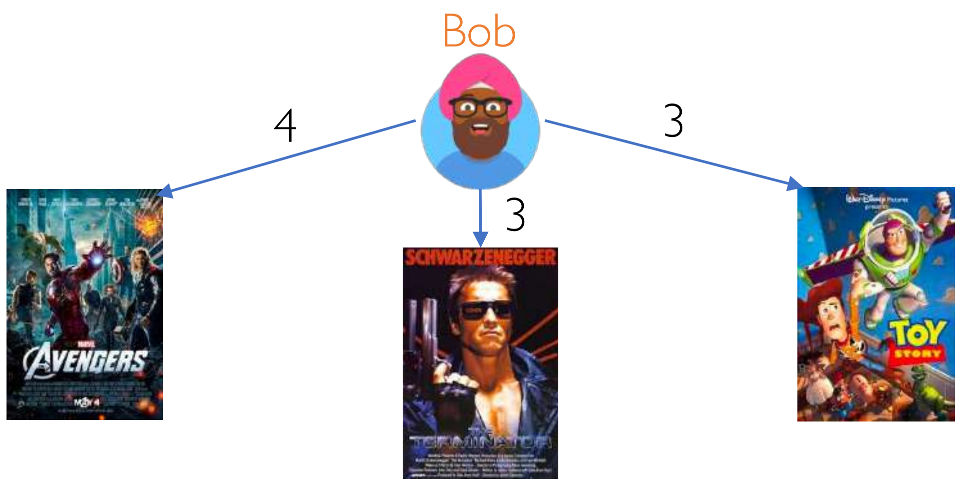
Intuitively, if a user *v* is not in the *u*'s *k*-neighborhood then very likely *u* will not be interested in any item that only *v* has rated

In other words, the *u*'s *k*-neighborhood must be computed first to narrow down the set of items which we must predict the rating of

User-based Neighborhood: Example

		MOVIES							
USERS	Alice	2		5	4	5	4		4
	Bob	4					3		3
	Carl	5	5	3	4	5	4		5
	…	…	…	…	…	…	…	…	…
	Zoe		1	3				5	4

User-based Neighborhood: Example



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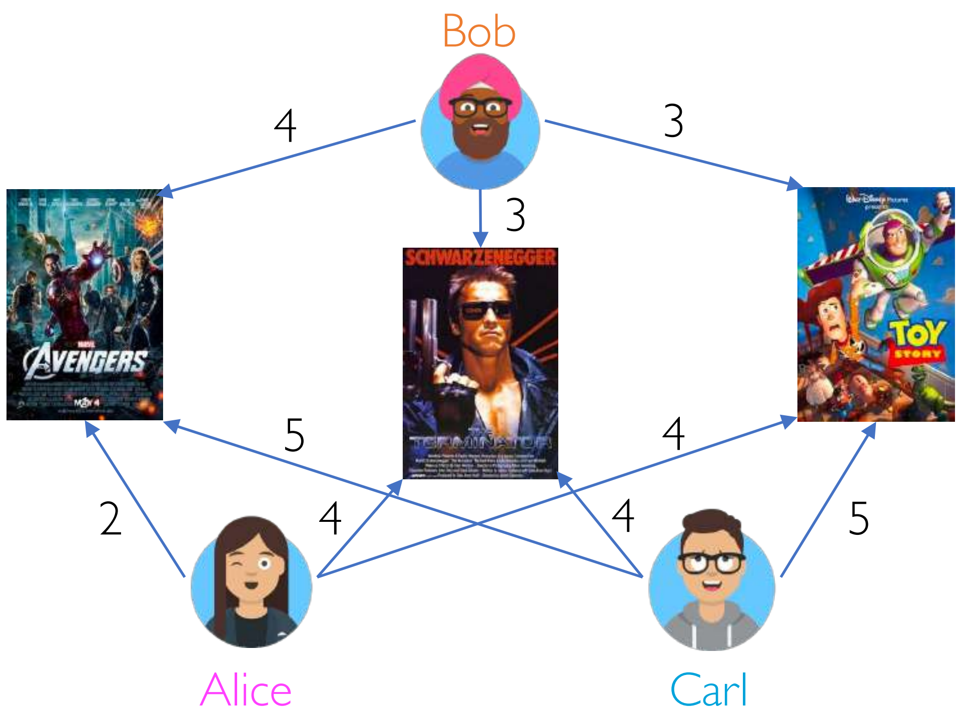
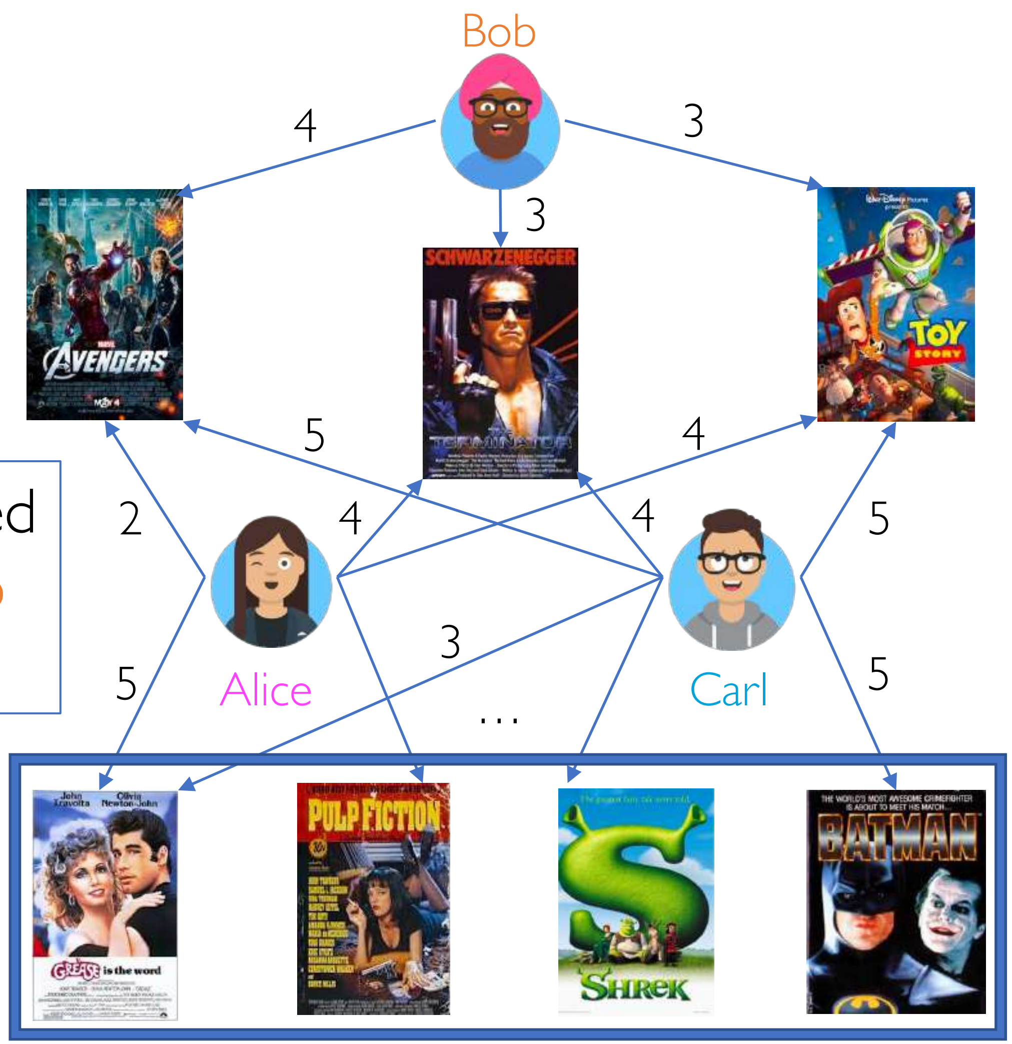


Alice and Carl are the **2-nearest neighbours** of Bob
if we look at their rating behaviours

User-based Neighborhood: Example



Consider all the movies rated by Alice or Carl which Bob hasn't watched yet

User-based Neighborhood: Example

Predict the rating that Bob would give to each of those movies on the basis of Alice's and Carl's ratings

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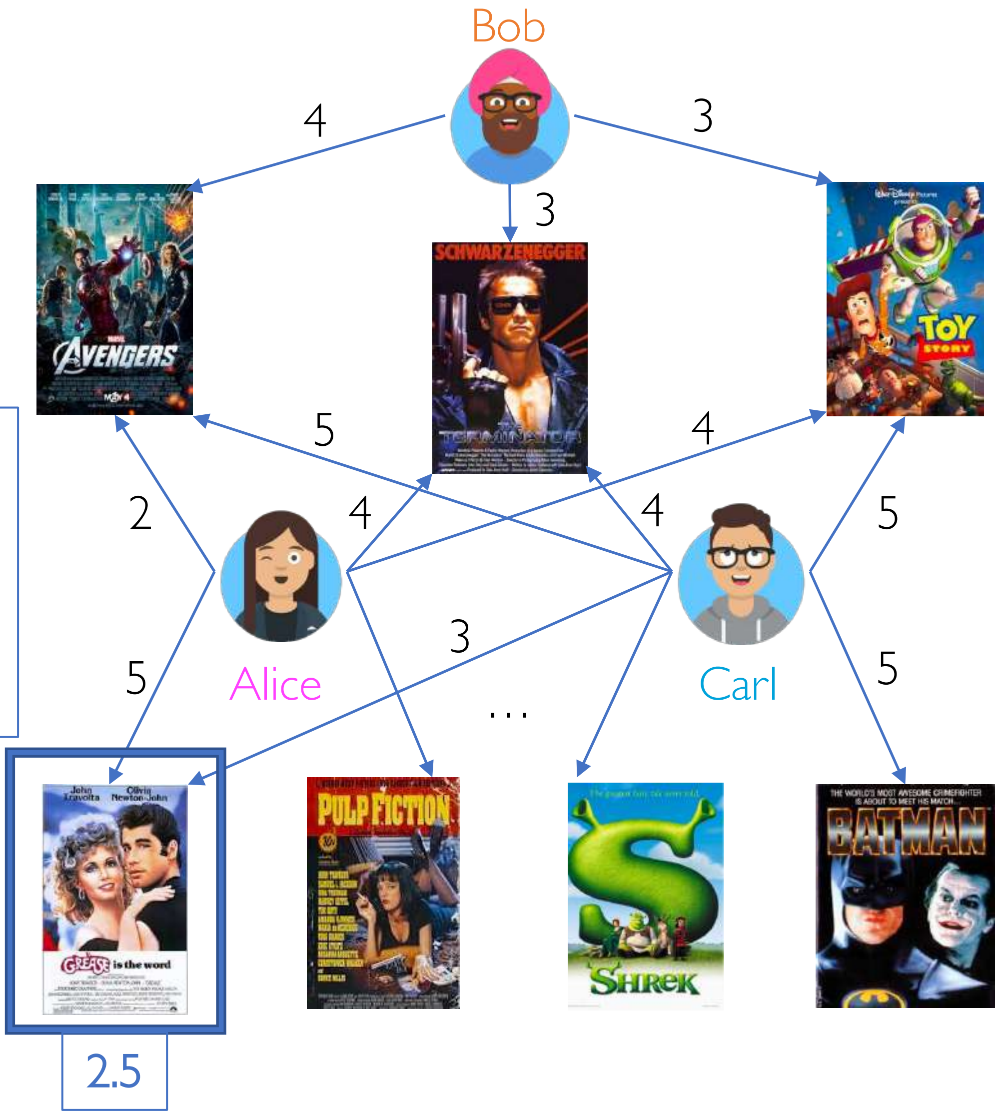

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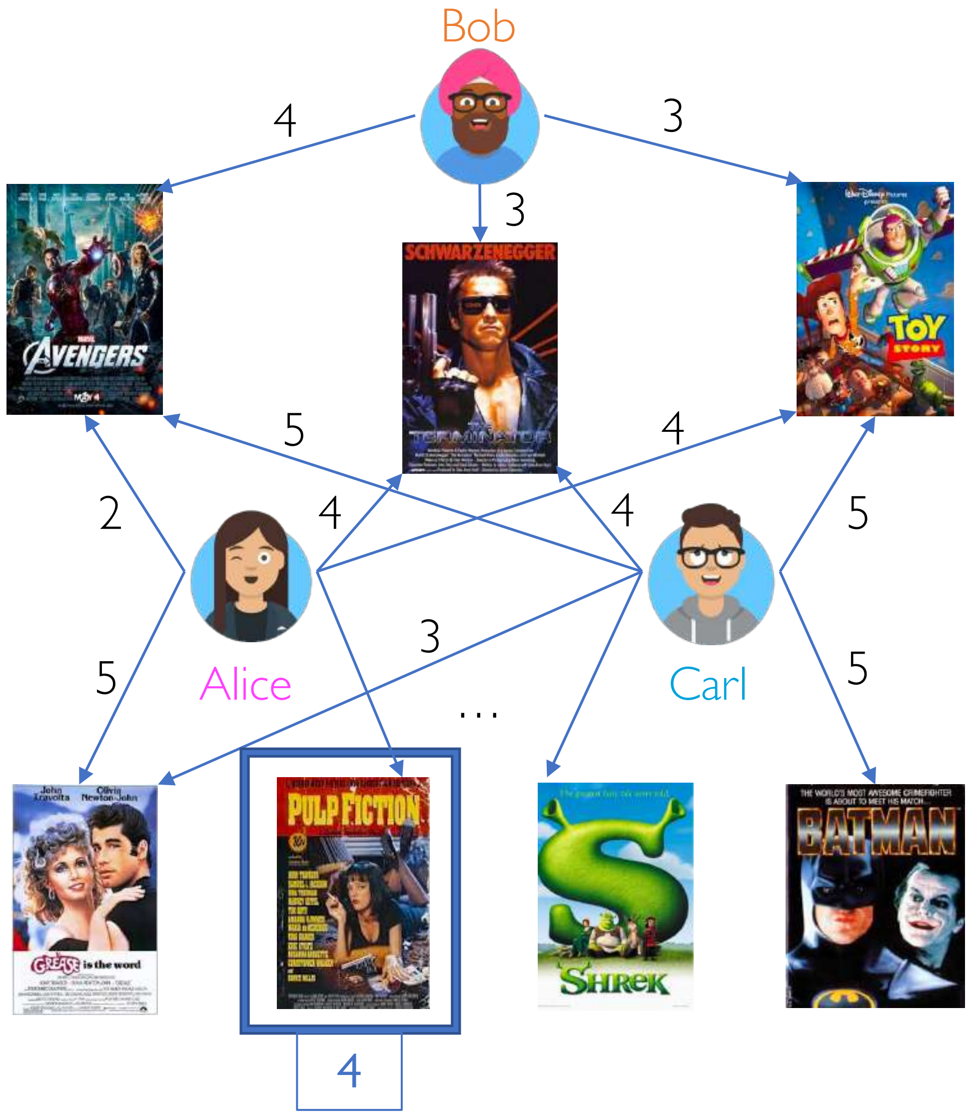

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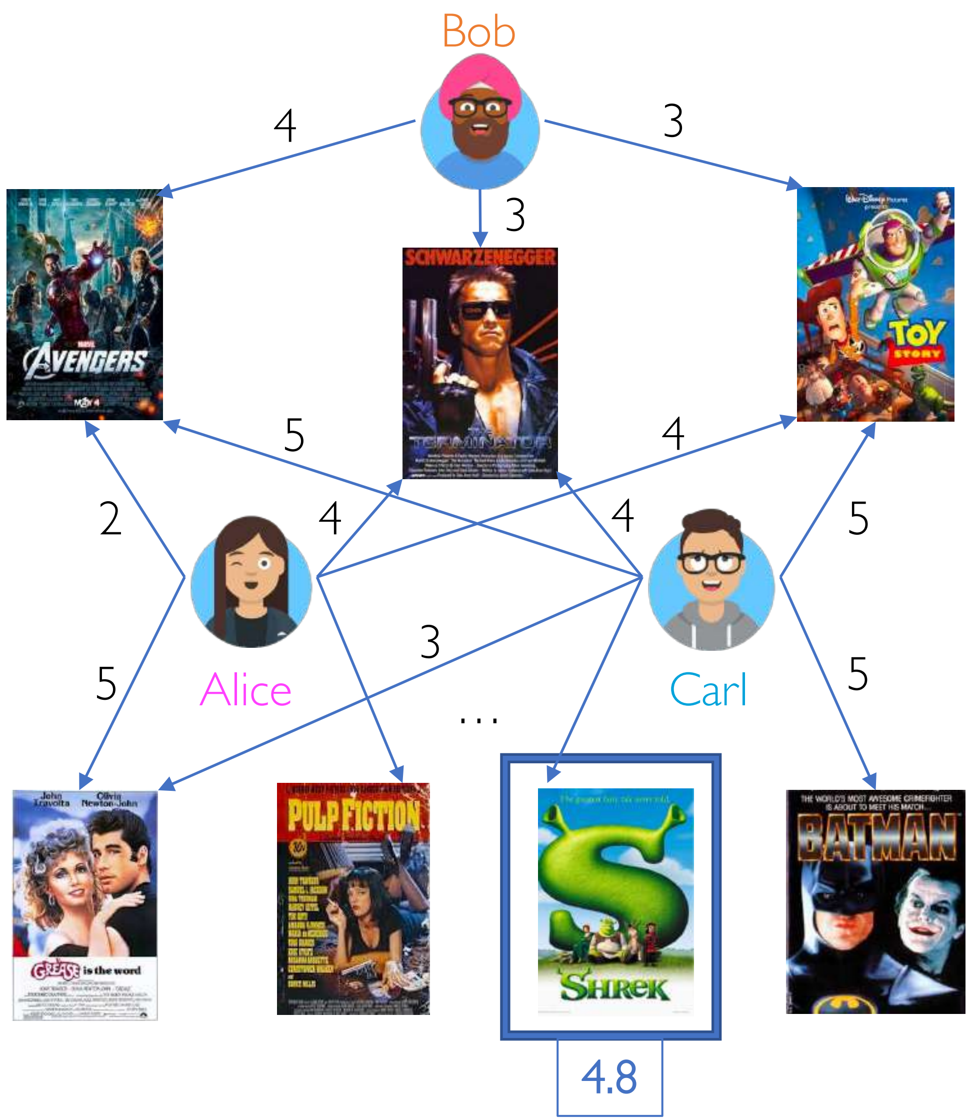

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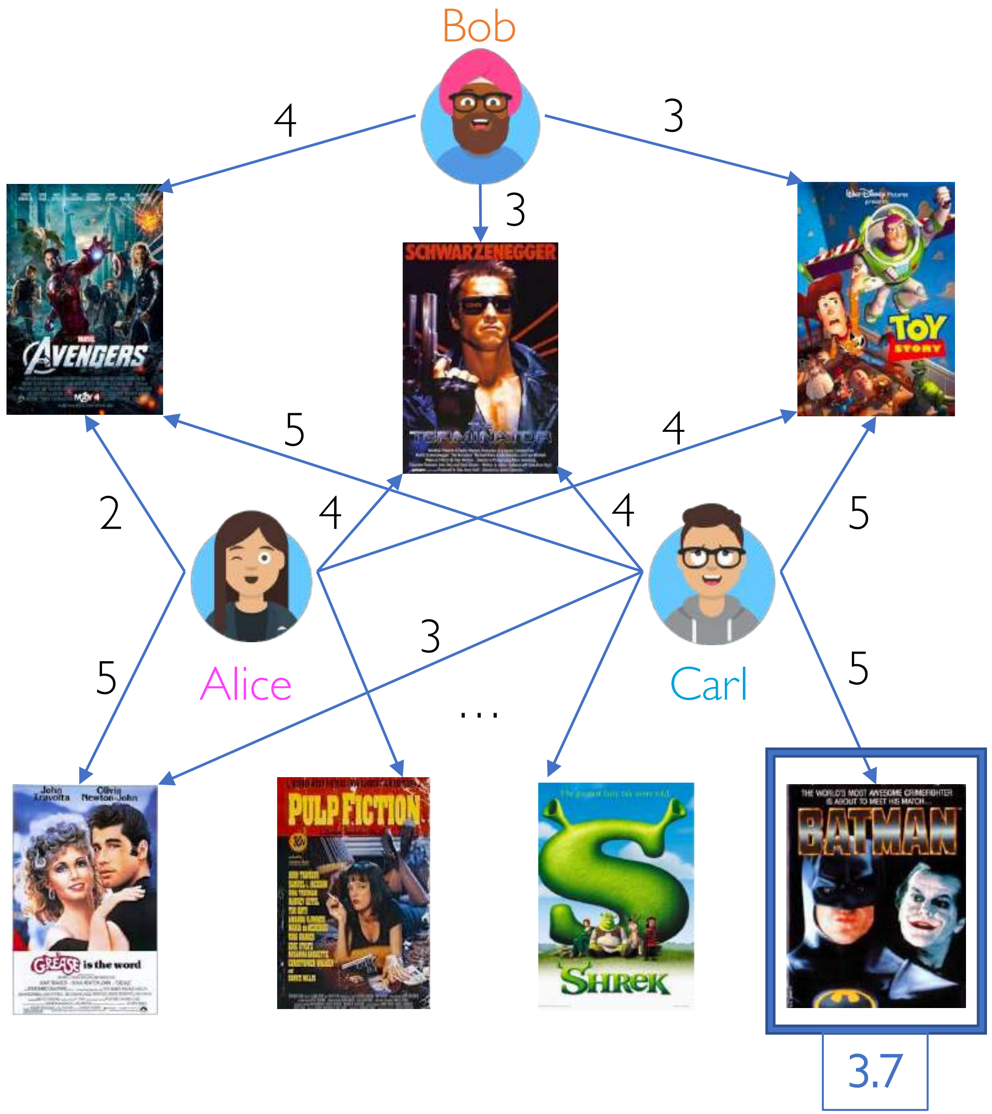
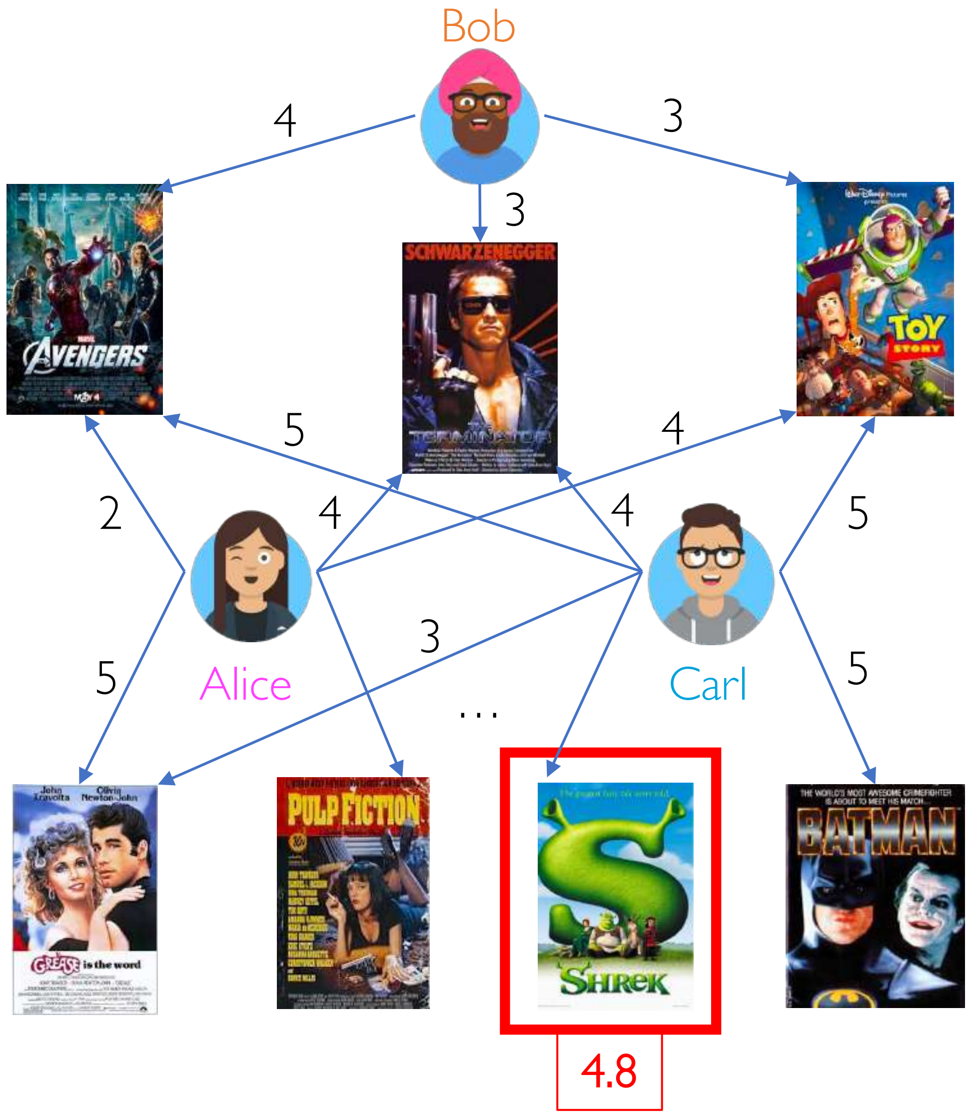


User-based Neighborhood: Example

Recommend the highest rated movie(s) to Bob!



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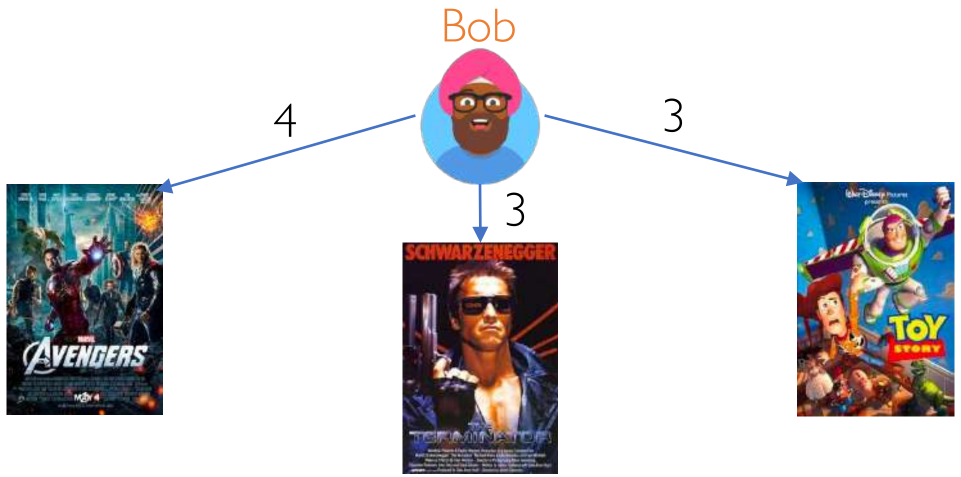
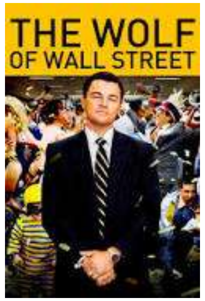


There is no point in predicting the rating of a movie which has only been rated by a user (Zoe) who is **not** in the Bob's neighborhood



User-to-User Similarity

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- Each user represented by her/his rating vector and similarity between them is measured in the item (rating) space

User-to-User Similarity

$\text{sim}(u, v)$ Similarity metric between any pair of users

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		MOVIES							
		AVENGERS	BATMAN	Grease	PULP FICTION	SHREK	SCHWARZENEGGER	THE WOLF OF WALL STREET	TOY STORY
USERS	Alice	2		5	4	5	4		4
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	…	…	…	…	…	…	…	…	…
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Must capture the intuition that $\mathrm{sim}(\mathrm{Alice}, \mathrm{Carl}) > \mathrm{sim}(\mathrm{Alice}, \mathrm{Bob})$

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$\mathbf{r}_u$ *n*-dimensional vector of ratings provided by user *u* (*n* = #movies)

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$\mathbf{r}_{\text{Bob}}$

User-to-User Similarity: Jaccard Similarity

$$\text{sim}(u, v) = J(\mathbf{r}_u, \mathbf{r}_v) = \frac{|\mathbf{r}_u \cap \mathbf{r}_v|}{|\mathbf{r}_u \cup \mathbf{r}_v|}$$

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$$= \frac{3}{6} = 0.5$$

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$$\mathrm{sim}(\mathrm{Alice}, \mathrm{Carl}) = \frac{|\mathbf{r}_{\mathrm{Alice}} \cap \mathbf{r}_{\mathrm{Carl}}|}{|\mathbf{r}_{\mathrm{Alice}} \cup \mathbf{r}_{\mathrm{Carl}}|}$$

$$= \frac{6}{7} \approx 0.86$$

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Problem!
Jaccard ignores rating values

User-to-User Similarity: Cosine Similarity

$$\mathrm{sim}(u, v) = \mathrm{cosine}(\mathbf{r}_u, \mathbf{r}_v) = \frac{\mathbf{r}_u \cdot \mathbf{r}_v}{||\mathbf{r}_u||\,||\mathbf{r}_v||}$$

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$$\mathrm{sim}(\mathrm{Alice}, \mathrm{Bob}) = \frac{\mathbf{r}_{\mathrm{Alice}} \cdot \mathbf{r}_{\mathrm{Bob}}}{||\mathbf{r}_{\mathrm{Alice}}||\,||\mathbf{r}_{\mathrm{Bob}}||}$$

$$= \frac{32}{\sqrt{102}\sqrt{44}} \approx 0.48$$

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$$= \frac{102}{\sqrt{102}\sqrt{141}} \approx 0.85$$

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Problem!

Missing rating values are treated as 0s and have a negative effect

User-to-User Similarity: Pearson Correlation

$$\mathrm{sim}(u, v) = \mathrm{Pearson}(\mathbf{r}_u, \mathbf{r}_v) = \frac{(\mathbf{r}_u - \bar{\mathbf{r}}_u) \cdot (\mathbf{r}_v - \bar{\mathbf{r}}_v)}{\sqrt{(\mathbf{r}_u - \bar{\mathbf{r}}_u)^T \cdot (\mathbf{r}_u - \bar{\mathbf{r}}_u)} \times \sqrt{(\mathbf{r}_v - \bar{\mathbf{r}}_v)^T \cdot (\mathbf{r}_v - \bar{\mathbf{r}}_v)}}$$

		MOVIES							
USERS	Alice	-2		1	0	1	0		0
	Bob	2/3					-1/3		-1/3
	Carl	4/7	4/7	-10/7	-3/7	4/7	-3/7		4/7
	...	...	...	...	...	...	...	...	...
	Zoe		-9/4	-1/4				7/4	-1/4

Solution:
Normalize ratings by subtracting the mean rating

Now 0 means neutral, and if we treat missing ratings as 0, it doesn't mean it's negative

User-to-User Similarity: Pearson Correlation

$\mathbf{r}'_u = \mathbf{r}_u - \bar{\mathbf{r}}_u$ mean-scaled rating vector of *u*

$\mathbf{r}'_v = \mathbf{r}_v - \bar{\mathbf{r}}_v$ mean-scaled rating vector of *v*

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$$= \frac{(\mathbf{r}_u - \bar{\mathbf{r}}_u) \cdot (\mathbf{r}_v - \bar{\mathbf{r}}_v)}{\sqrt{(\mathbf{r}_u - \bar{\mathbf{r}}_u)^T \cdot (\mathbf{r}_u - \bar{\mathbf{r}}_u)} \times \sqrt{(\mathbf{r}_v - \bar{\mathbf{r}}_v)^T \cdot (\mathbf{r}_v - \bar{\mathbf{r}}_v)}} = \mathrm{Pearson}(\mathbf{r}_u, \mathbf{r}_v)$$

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Top-*k* most "similar" users to *u*

u's *k*-neighborhood

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Set of items rated by *u*'s neighbors

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Predicted rating given by user *u* to item *i*

$$\mathbf{r}_u[i] = r(u, i) = r_{u,i}$$

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plain average

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weighted average

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Aging

user profiles changed quickly and the entire system model had to be recomputed

ITEM-BASED COLLABORATIVE FILTERING

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- The model doesn't suffer from aging and therefore it does not need to be recomputed frequently

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Estimate $r(u, i)$ based on the ratings of items in the k-neighborhood of i

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Item-to-Item Similarity

- The key "trick" to discover the *k*-neighborhood of a given item *i* is the ability of finding items that are "similar" to *i*
- Remember that we are not building any content-based user/item profile
- Intuitively, 2 items i_1 and i_2 are similar to each other if users who rated them are similar
- Each item represented by the user ratings vector and similarity between them is measured in the user (rating) space

Item-to-Item Similarity

$\mathbf{r}_i$ $\quad$ *m*-dimensional vector of ratings provided for item *i* (m = #users)

		MOVIES							
USERS	Alice	2		5	4	5	4		4
	Bob	4					3		3
	Carl	5	5	3	4	5	4		5
	…	…	…	…	…	…	…	…	…
	Zoe		1	3				5	4

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$\mathbf{r}_{\text{Shrek}}$

Item-based Neighborhood: Example

Let's consider again Bob!

		MOVIES							
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	Bob	4					3		3
	Carl	5	5	3	4	5	4		5
	…	…	…	…	…	…	…	…	…
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Item-based Neighborhood: Example

Suppose we want to predict the rating Bob would give to Shrek

		MOVIES							
USERS	Alice	2		5	4	5	4		4
	Bob	4				?	3		3
	Carl	5	5	3	4	5	4		5
	...	...	...	...	...	...	...	...	...
	Zoe		1	3				5	4

Item-based Neighborhood: Example

We first extract the subset of *k* most similar items to Shrek which have been rated by Bob

		MOVIES							
		AVENGERS	BATMAN	Grease	PULP FICTION	SHREK	SCHWARZENEGGER	THE WOLF OF WALL STREET	TOY STORY
USERS	Alice	2		5	4	5	4		4
	Bob	4					3		3
	Carl	5	5	3	4	5	4		5
	…	…	…	…	…	…	…	…	…
	Zoe		1	3				5	4

$\mathbf{r}_{\text{Shrek}}$

Item-based Neighborhood: Example

Suppose those are: The Avengers and The Terminator

		MOVIES							
USERS	Alice	2		5	4	5	4		4
	Bob	4					3		3
	Carl	5	5	3	4	5	4		5
	…	…	…	…	…	…	…	…	…
	Zoe		1	3				5	4

For example, item similarity is measured using Pearson's correlation

Item-based Neighborhood: Example

The predicted rating is computed as an aggregating function of the ratings that Bob gave to the *k* most similar movies to Shrek

		MOVIES							
USERS	Alice	2		5	4	5	4		4
	Bob	4				?	3		3
	Carl	5	5	3	4	5	4		5
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$$\mathcal{I}_u^k = \operatorname{argmax}_{\mathcal{I}'_u \subseteq \mathcal{I}_u, |\mathcal{I}'_u| = k} \sum_{i' \in \mathcal{I}'_u} \operatorname{sim}(i, i')$$

Top-*k* most "similar" items to *i* among those rated by *u*

i's *k*-neighborhood

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Predicted rating given by user *u* to item *i*

$$\mathbf{r}_u[i] = r(u, i) = r_{u,i}$$

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$$\forall i' \in \mathcal{I}_u^k$$

$$r_{u,i} = \frac{1}{k} \sum_{i' \in \mathcal{I}_u^k} r_{u,i'}$$

plain average

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$$r_{u,i} = \frac{1}{k} \sum_{i' \in \mathcal{I}_u^k} \mathrm{sim}(i, i') \cdot r_{u,i'}$$

weighted average

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In general, item-based works better than user-based CF

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  - Locality-Sensitive Hashing (LSH) approximation

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  - In large scale systems, this must be pre-computed offline
  - At inference time, make use of ad hoc data structures (e.g., k-d trees) to efficiently compute the set of (approximated) nearest neighbors for a query user/item