Big Data Computing

Master's Degree in Computer Science

2021-2022

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SAPIENZA
UNIVERSITÀ DI ROMA

Recommendation Strategies

3 approaches to recommender systems

- Content-based filtering
- Collaborative filtering
  - Neighborhood-based (a.k.a. memory-based)
  - Latent-factor-based (a.k.a. model-based)
  - Hybrid (memory-model-based)
- Hybrid

LATENT FACTOR MODELS

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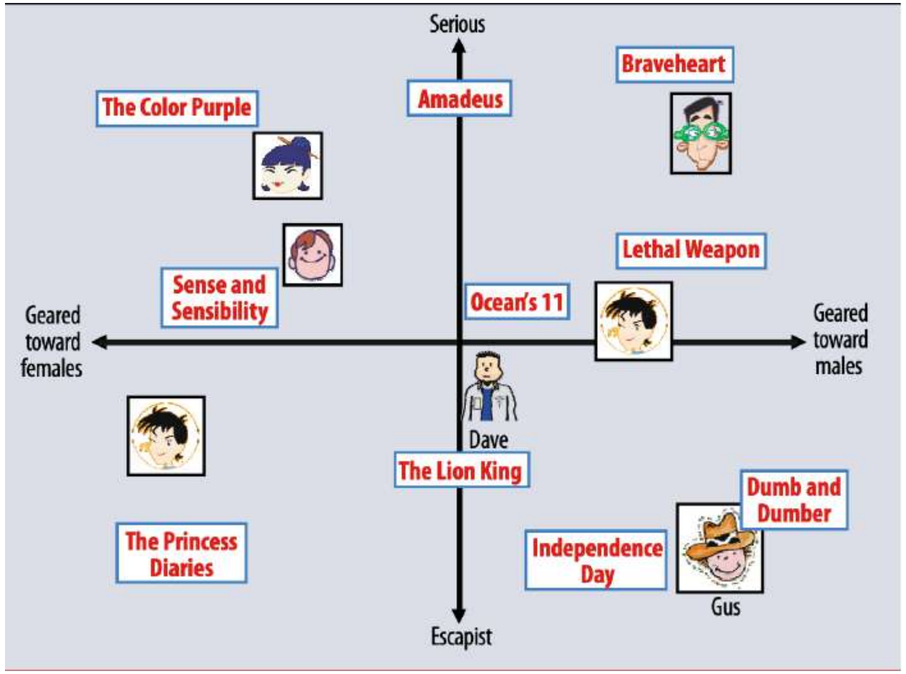

Example: 2 hidden factors

- Dim. 1: Male vs. Female
- Dim. 2: Serious vs. Escapist

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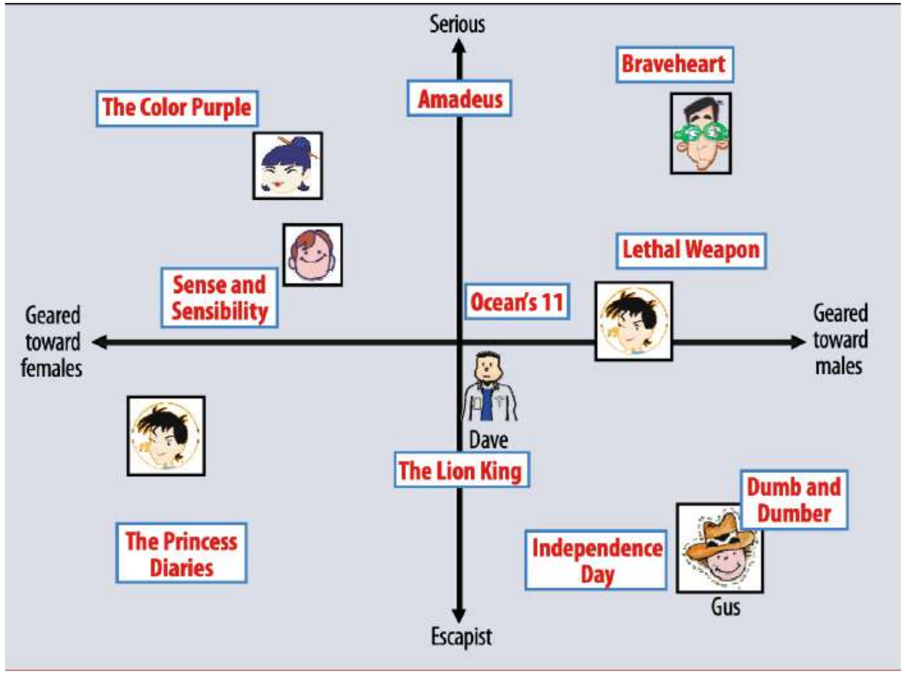

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A user's predicted rating for an item (movie) would equal the **dot product** of the movie and user vectors on the plot

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- Such vectors are inferred (i.e., learned) from observed item ratings
- High correspondence between item and user factors leads to a recommendation

Matrix Factorization Framework

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- That is why these features are often refer to as latent features

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Recommendations for a user are generated by computing the estimated ratings for unseen items, and by taking the top-*k* highest rated ones

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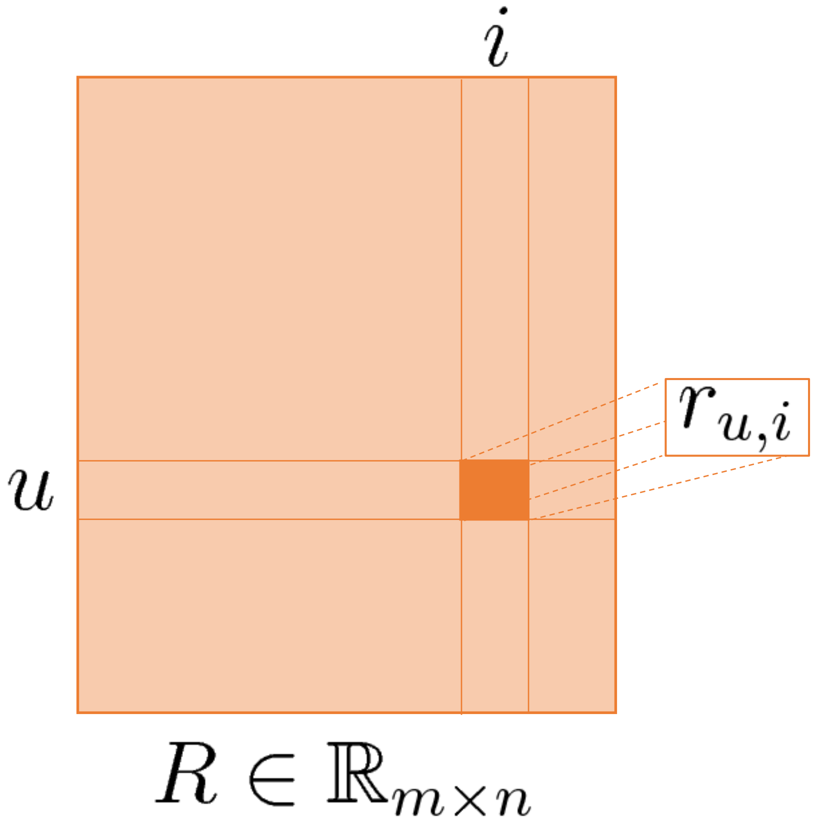

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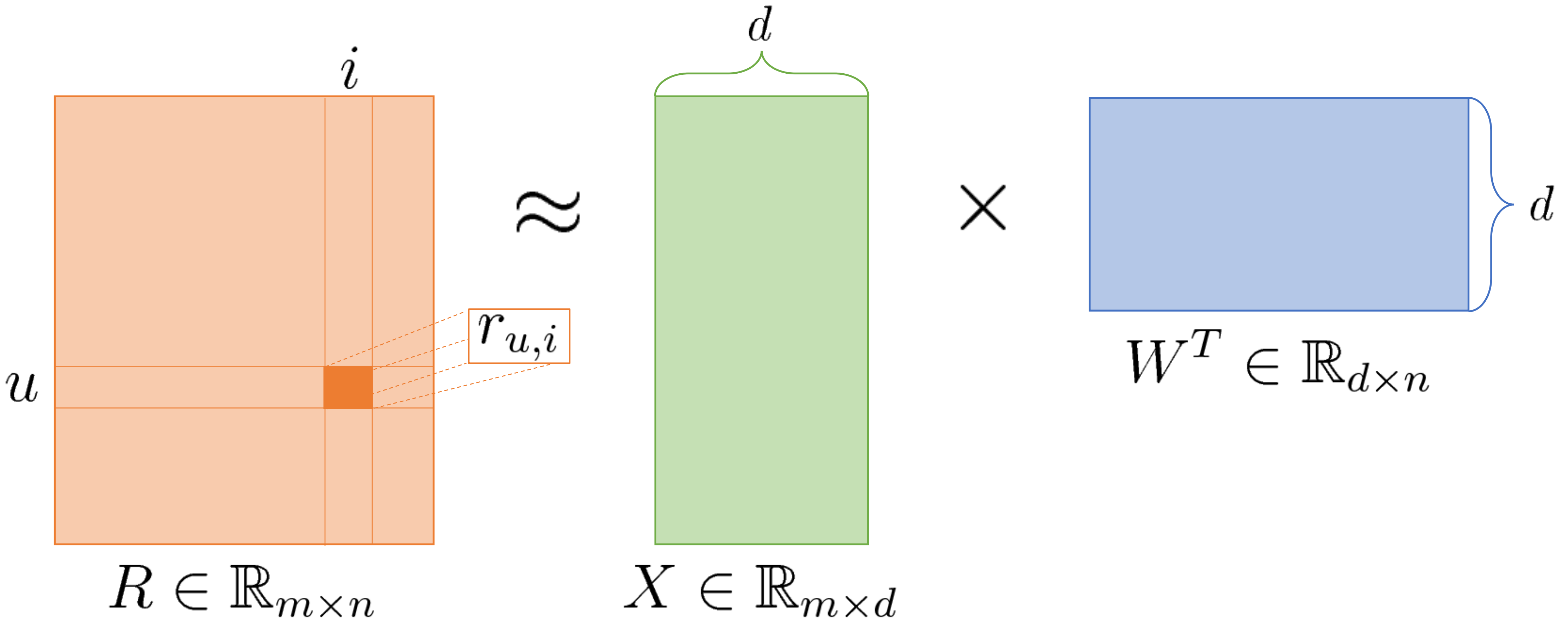

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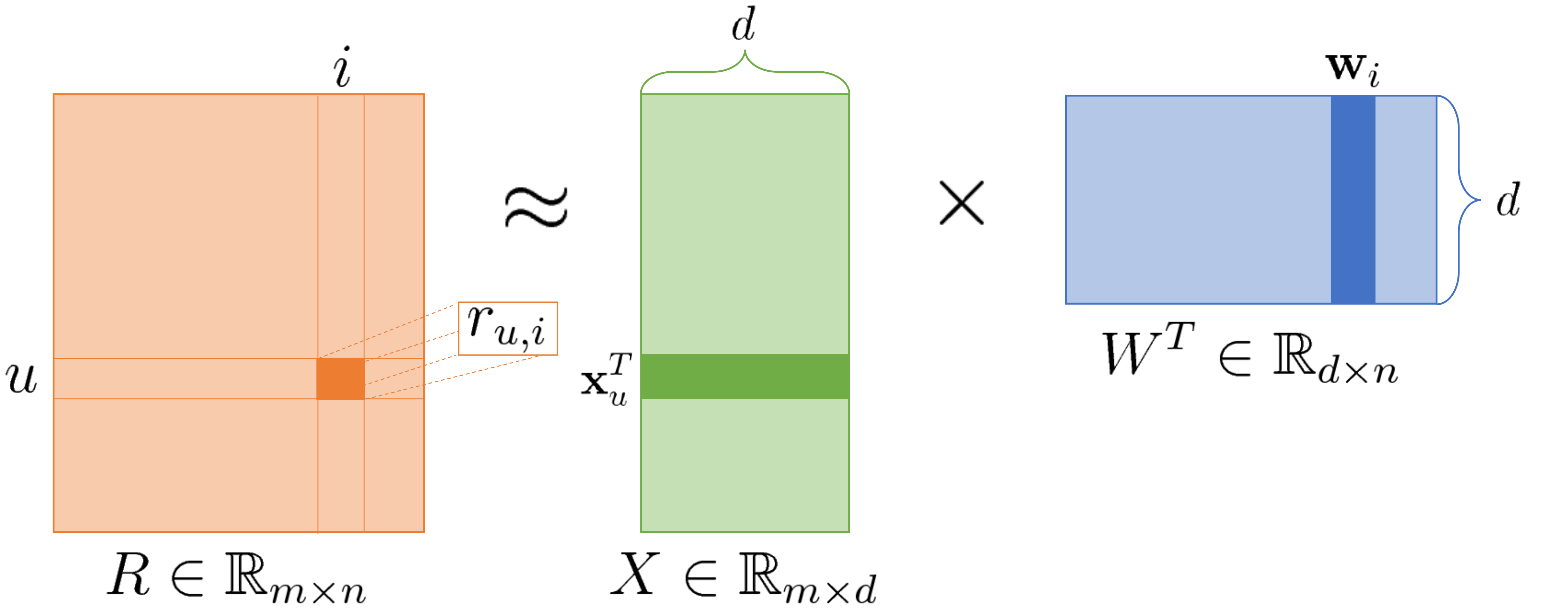

Approximate the user-item rating matrix R with the product of X x W^T

How Do We Learn X and W?

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To actually learn the latent factor representations $\mathbf{x}_u$ and $\mathbf{w}_i$ we minimize the following loss function

$$L(X,W) = \sum_{(u,i)\in\mathcal{D}} \left(r_{u,i} - \mathbf{x}_u^T \cdot \mathbf{w}_i\right)^2 + \lambda\Big(\sum_{u\in\mathcal{D}} ||\mathbf{x}_u||^2 + \sum_{i\in\mathcal{D}} ||\mathbf{w}_i||^2\Big)$$

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Mathematically convenient

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Still, how do we solve this?

Learning Algorithms

$\boxed{\text{2 main optimization methods}}$

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Stochastic Gradient Descent (SGD)

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- Stochastic Gradient Descent (SGD)
- Alternating Least Squares (ALS)

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We know that the updating strategy for SGD is as follows:

$$\mathbf{x}_u^{(t+1)} \leftarrow \mathbf{x}_u^{(t)} - \eta \nabla L(\mathbf{x}_u^{(t)}; \mathbf{w}_i^{(t)})$$

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At each iteration, both user and item latent vectors are updated by a magnitude proportional to η in the opposite direction of the gradient

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- Indeed, there are $d(m+n)$ parameters to optimize
- In real life problems, this number can get very large quite often, requiring both a parallelization mechanism or an alternative optimizer

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- Each alternating iteration reduces to traditional least squares and can be solved using OLS or its regularized variant (e.g., pseudo-inverse)

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We want to set this to 0 $\boxed{-\sum_{i\in\mathcal{D}}(r_{u,i} - \mathbf{x}_u^T \cdot \mathbf{w}_i)\mathbf{w}_i + \lambda\mathbf{x}_u = 0}$

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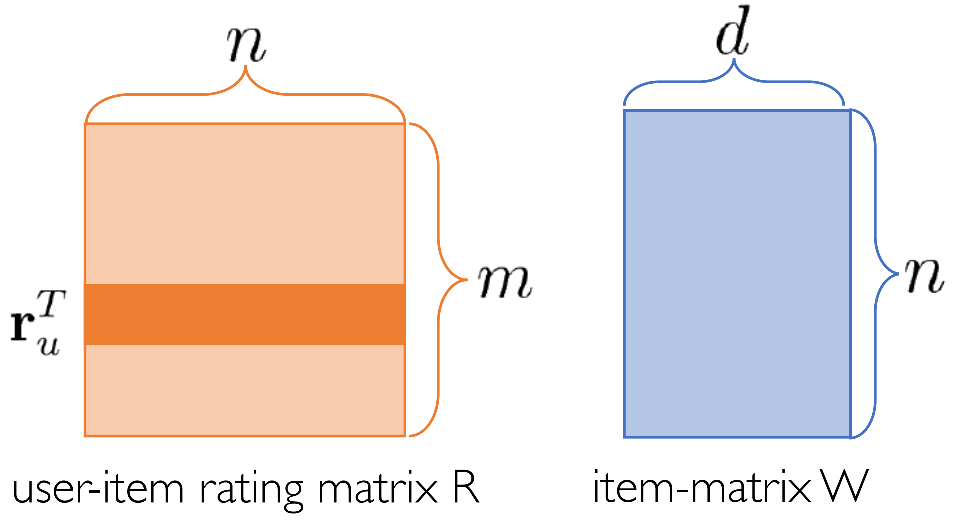

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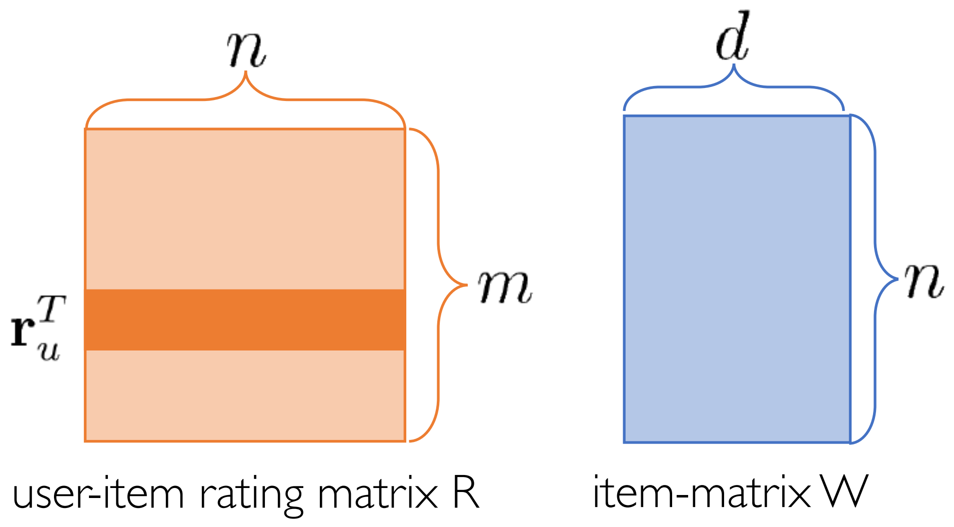

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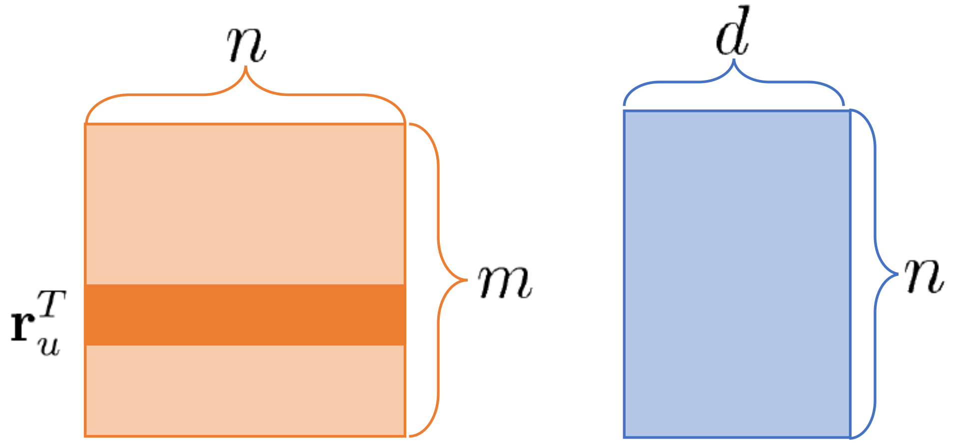

user-item rating matrix R

item-matrix W

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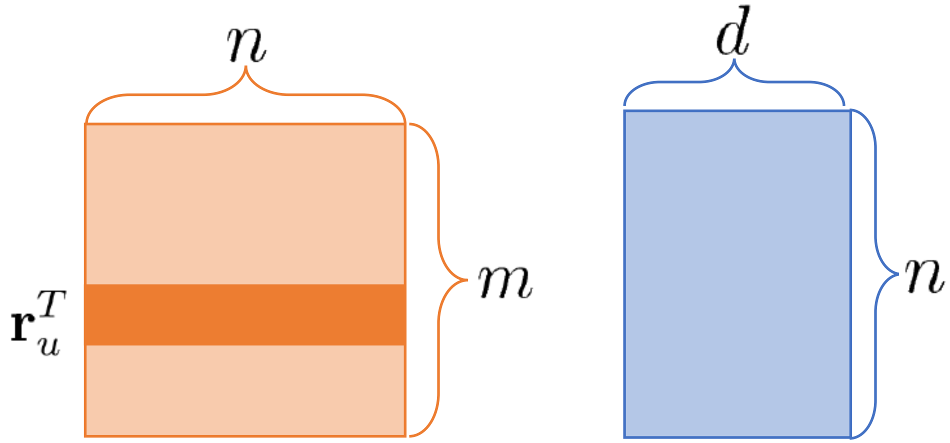
$$=W^T\cdot\mathbf{r}_u=\mathbf{x}_u(W^TW+\lambda I)$$ $I\in\mathbb{R}_{d\times d}$ identity matrix

$$=(W^TW+\lambda I)^{-1}\cdot W^T\cdot\mathbf{r}_u=\mathbf{x}_u(W^TW+\lambda I)\cdot(W^TW+\lambda I)^{-1}$$



user-item rating matrix R

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$$= W^T\cdot\mathbf{r}_u = \mathbf{x}_u(W^TW + \lambda I)$$ $I \in \mathbb{R}_{d\times d}$ identity matrix

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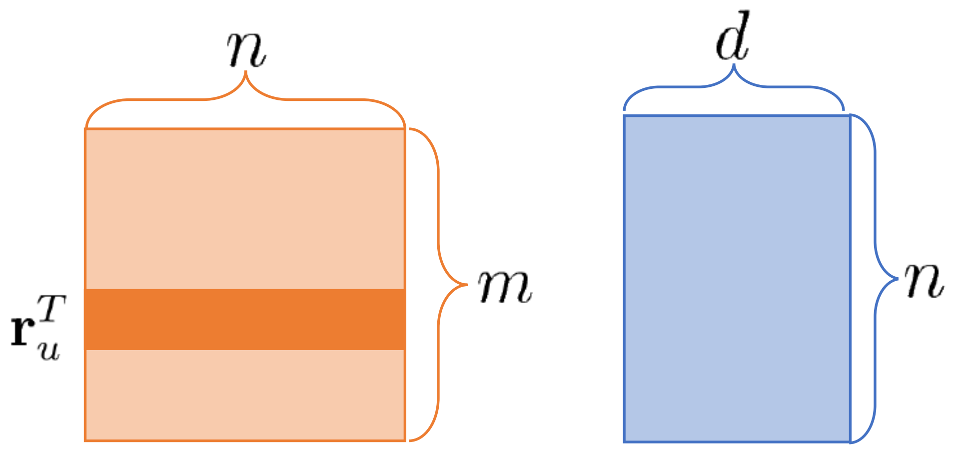

user-item rating matrix R   item-matrix W

ALS: User Vector Fixed

$$-\sum_{u\in\mathcal{D}}(r_{u,i} - \mathbf{x}_u^T \cdot \mathbf{w}_i)\mathbf{x}_u + \lambda \mathbf{w}_i = 0$$

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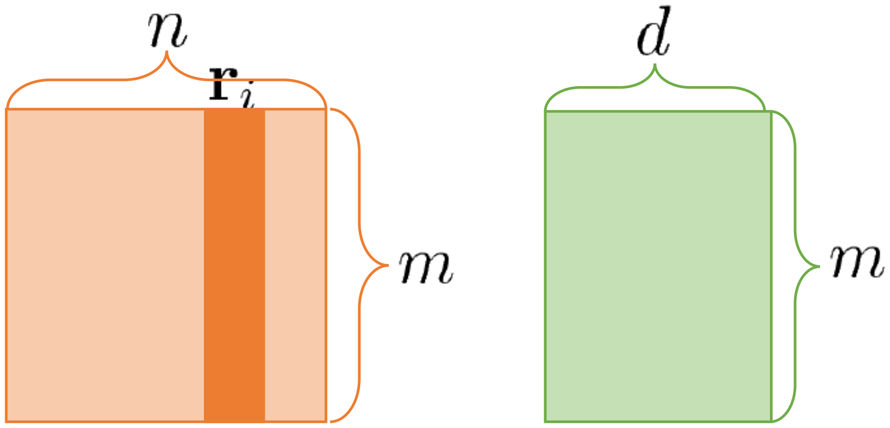

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Convergence is guaranteed because in each step the loss function can either decrease or stay unchanged, but never increase

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  - Implicit Data: the training set is dense and looping over each single instance - as SGD does - would be unfeasible

Singular Value Decomposition (SVD)

A well-known technique to decompose a matrix
into the product of 3 matrices

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$\boxed{\text{SVD solution is unique}}$

Applying SVD to Collaborative Filtering

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Each factor k is the result of the similarity between user i and item j and its overall effect on ratings across all users and items

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- Possible workaround to apply SVD: use imputation to fill missing values in the matrix R

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- For example, some users systematically tend to give higher ratings than others, and some items receive higher ratings than others

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$$\boxed{b_{\text{Joe,Titanic}} = 3.7 - 0.3 + 0.5 = 3.9}$$

Bias term

Including Bias into the Optimization

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The estimated rating of an item *i* for the user *u* is now made of 2 components

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models global average, user and item bias

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Overall, the original optimization problem becomes as follows

$$X^*, W^* = \text{argmin}_{X,W} \left\{ \frac{1}{2} \sum_{(u,i) \in \mathcal{D}} \left[r_{u,i} - (\mathbf{x}_u^T \cdot \mathbf{w}_i + \mu + b_u + b_i) \right]^2 + \lambda \Big(\sum_{u \in \mathcal{D}} ||\mathbf{x}_u||^2 + \sum_{i \in \mathcal{D}} ||\mathbf{w}_i||^2 + \sum_{u \in \mathcal{D}} b_u^2 + \sum_{i \in \mathcal{D}} b_i^2 \Big) \right\}$$

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Can still be solved using ALS

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the vast majority of items are not rated by users

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- Netflix is a good example of hybrid recommender systems

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[Netflix: What Happens When You Press Play?]

For more details about how Netflix actually works

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- According to the contest website, more than 48,000 teams from 182 different countries have downloaded the data

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A combination of 100 different predictor sets, mostly factorization models

Evaluation Metrics

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Online

A/B testing measuring CTR, ROI, and other "live" metrics

Evaluation Metrics: RMSE

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The RMSE might penalize a method that does well for high ratings and badly for others

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Mapping of binary classification terminology to recommender systems

binary classifier	recommender system
# with condition (y = 1)	# of all possible relevant items for a user
# predicted positive (TP + FP)	# of recommended items
# correct positives (TP)	# of recommended items that are relavant

Evaluation Metrics: Precision & Recall

For a recommender system, we can therefore define

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A recommender system generates k=5 items to recommend

There are only 3 relevant items

The success/failure of our recommendations: [0, 1, 1, 0, 0] 0=not relevant/1=relevant

$$P = \frac{2}{5} \qquad R = \frac{2}{3}$$

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- Imagine taking our list of N recommendations and considering only the first element, then only the first two, then only the first three, and so on
- P@k and R@k are simply the precision and recall calculated only from the subset of the first k recommendations

P@k: Example

$$k = 3$$

$$P@3 = \frac{1}{3}$$

Rank	Product Recommended	Result
1	Credit card	Correct positive
2	Christmas Fund	False positive
3	Debit Card	False positive
4	Auto loan	False positive
5	HELOC	Correct Positive
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$$AP@N = \frac{1}{|\mathrm{Rel}|} \sum_{k=1}^{N} P@k \times \mathbf{1}_{\mathrm{Rel}}(k)$$

indicator function

$$\mathbf{1}_{\mathrm{Rel}}(k) = \begin{cases} 1 & \text{if item } k \in \mathrm{Rel} \\ 0 & \text{otherwise} \end{cases}$$

Mean Average Precision (MAP)

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Personalization

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Intuitively, a high personalization score indicates the recommender system is able to provide a highly personalized experience to the users

Personalization

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u_1 = [A, B, C, D] u_2 = [A, B, C, E] u_3 = [A, B, F, G]

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	A	B	C	D	E	F	G
u_1	1	1	1	1	0	0	0
u_2	1	1	1	0	1	0	0
u_3	1	1	0	0	0	1	1

Personalization

Compute the 3-by-3 triangular matrix containing the cosine similarity between each pair of user's recommendation binary vector

$$M_{i,j} = \text{cosine}(\mathbf{u}_i, \mathbf{u}_j)$$

	u_1	u_2	u_3
u_1	1	0.75	0.58
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Take the average of the upper triangle of the matrix M above

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Personalization = 1 – 0.64 = 0.36

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- Evaluation metrics must capture the accuracy, personalization and serendipity of recommendations

Recommended Readings and Information :)

- A huge body of work on recommender systems is available out there!
- Surveys:
  - Adomavicius & Tuzhilin [2005]
  - Koren & Volinsky [2009]
  - Bobadilla *et al*. [2013]
  - Zhang *et al*. [2019]
- Well-renowed series of Conferences: RecSys, KDD, SIGIR, TheWebConf