

Big Data Computing

Master's Degree in Computer Science

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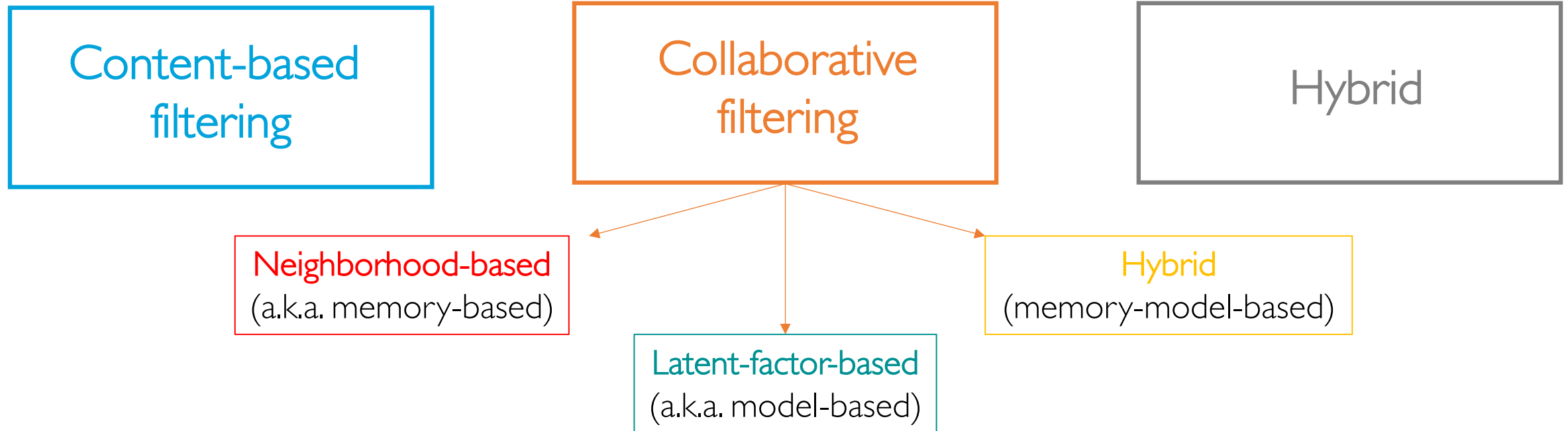
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Recommendation Strategies

3 approaches to recommender systems



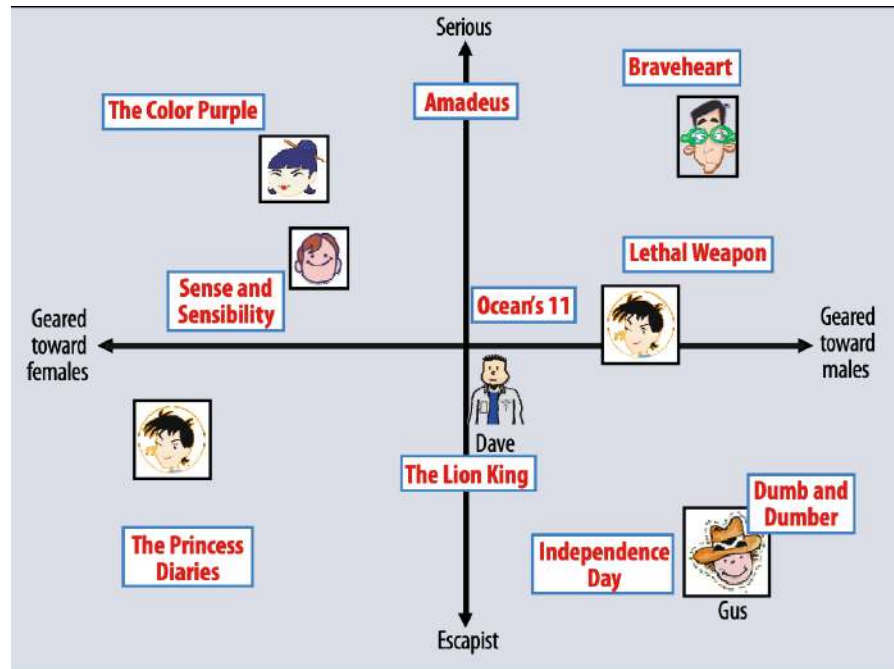
LATENT FACTOR MODELS

Latent Factor (Model-based) CF

Tries to predict ratings by representing both items and users with a number of **hidden factors** inferred from observed ratings

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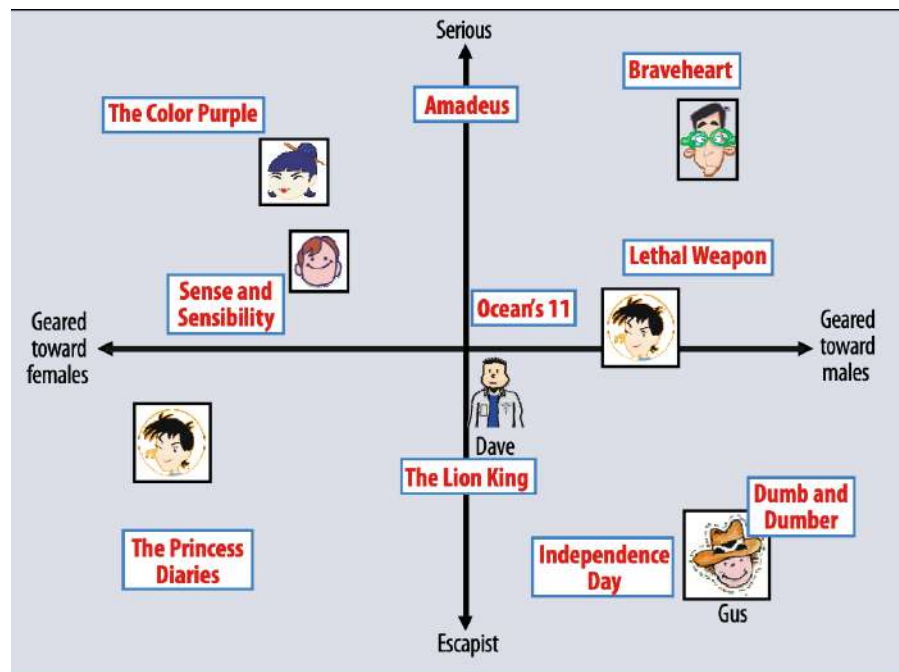


Example: 2 hidden factors

- Dim. 1: Male vs. Female
- Dim. 2: Serious vs. Escapist

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A user's predicted rating for an item (movie) would equal the **dot product** of the movie and user vectors on the plot

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- Such vectors are inferred (i.e., learned) from observed item ratings
- High correspondence between item and user factors leads to a recommendation

Matrix Factorization Framework

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- That is why these features are often refer to as **latent features**

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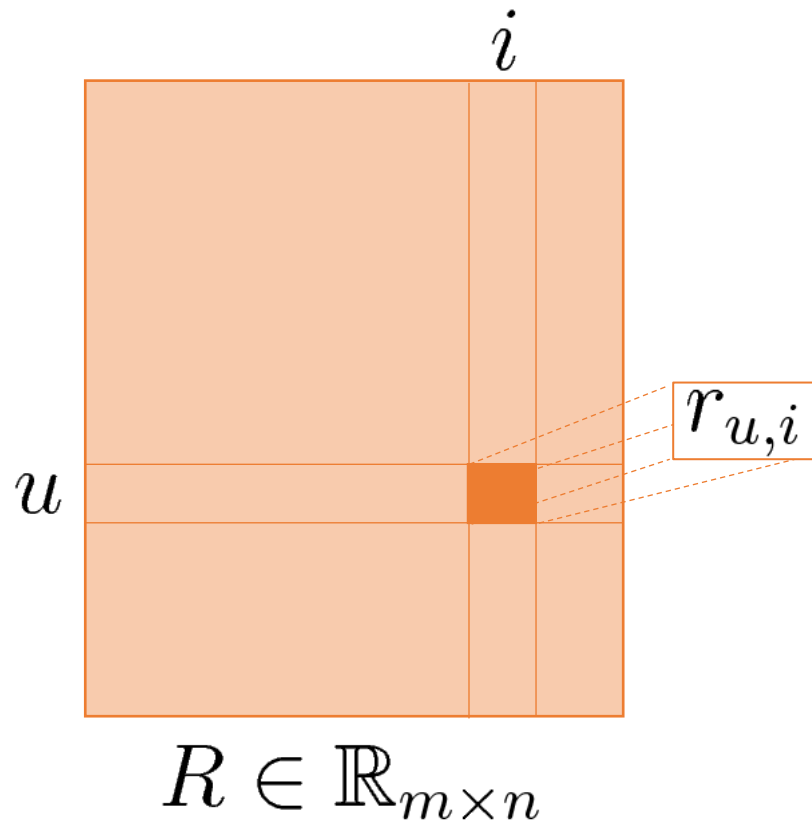
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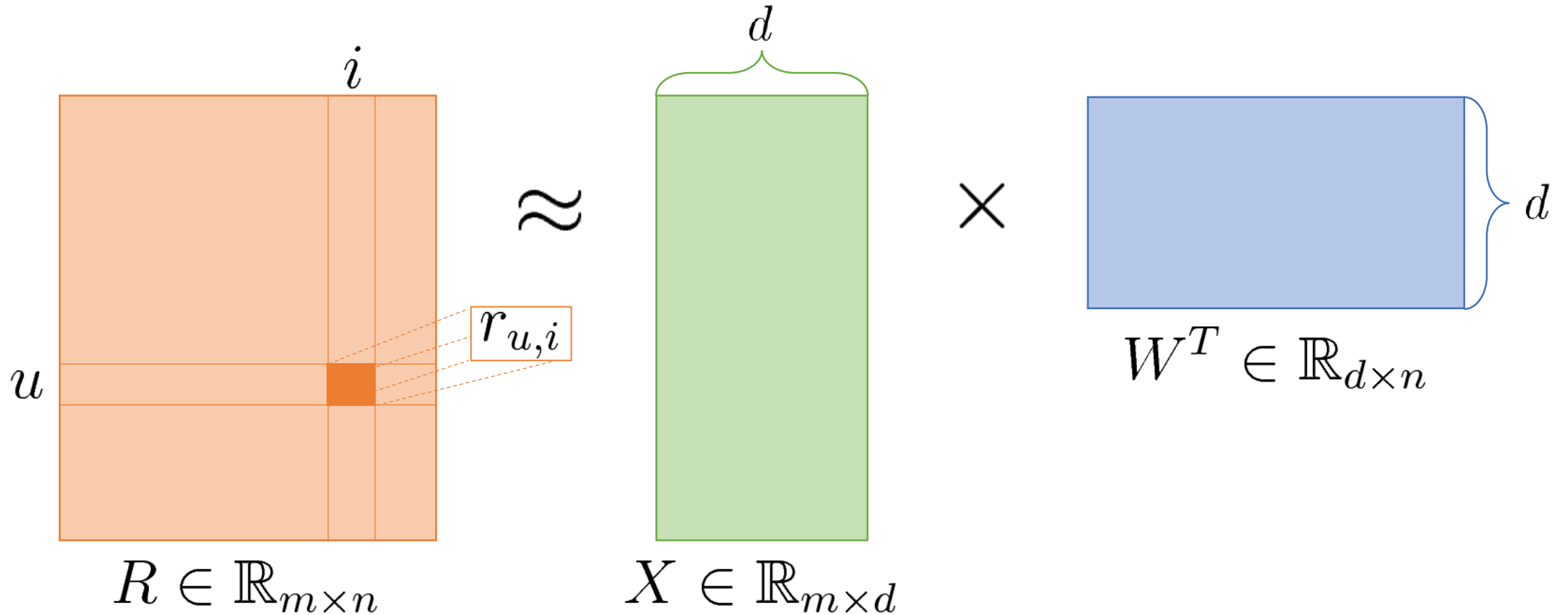
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Recommendations for a user are generated by computing the estimated ratings for unseen items, and by taking the **top- k highest rated** ones

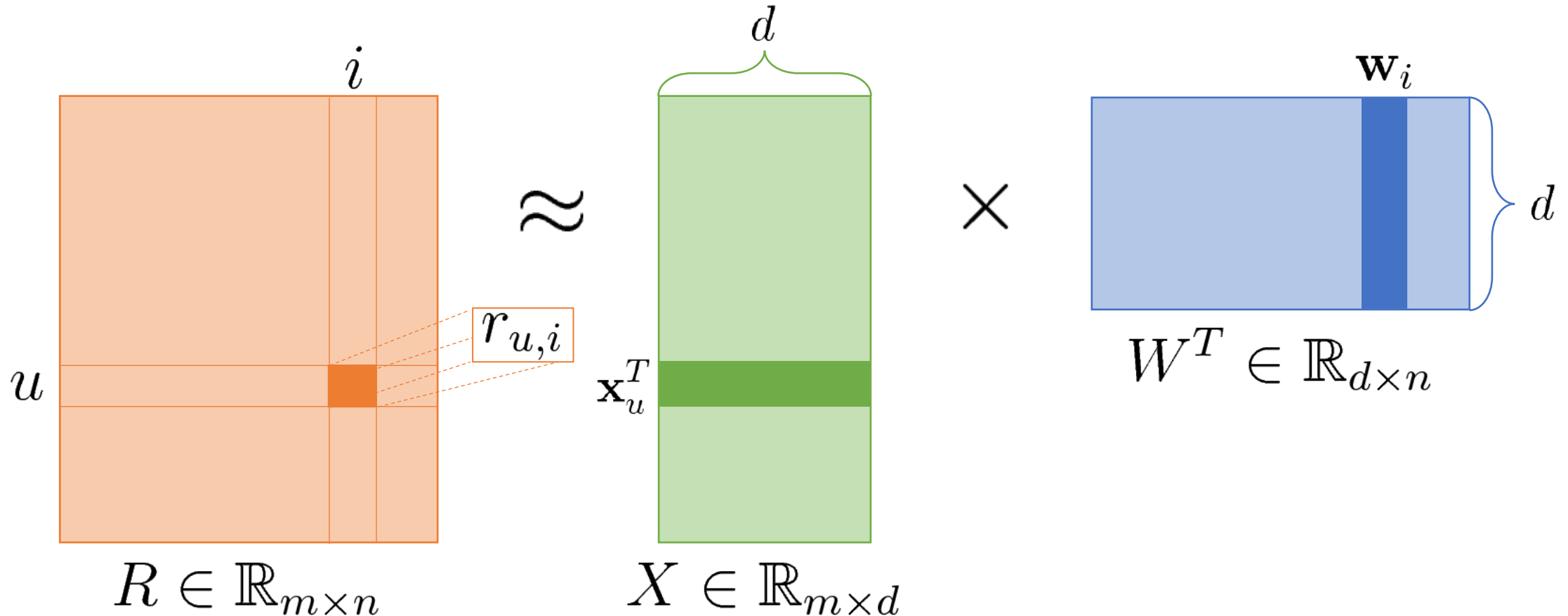
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Matrix Factorization Framework



Approximate the user-item rating matrix R with the product of $X \times W^T$

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Assuming we have access to a **dataset** of **observed ratings**

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$$L(X, W) = \sum_{(u,i) \in \mathcal{D}} \left(r_{u,i} - \mathbf{x}_u^T \cdot \mathbf{w}_i \right)^2 + \lambda \left(\sum_{u \in \mathcal{D}} ||\mathbf{x}_u||^2 + \sum_{i \in \mathcal{D}} ||\mathbf{w}_i||^2 \right)$$

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Mathematically convenient

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Still, how do we solve this?

Learning Algorithms

2 main optimization methods

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Stochastic Gradient Descent (SGD)

Learning Algorithms

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graph TD; A[2 main optimization methods] -- blue --> B[Stochastic Gradient Descent (SGD)]; A -- green --> C[Alternating Least Squares (ALS)];
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We know that the updating strategy for SGD is as follows:

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At each iteration, both user and item latent vectors are updated by a magnitude proportional to η in the **opposite direction** of the gradient

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- In real life problems, this number can get very large quite often, requiring both a parallelization mechanism or an alternative optimizer

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- Each alternating iteration reduces to traditional least squares and can be solved using OLS or its regularized variant (e.g., pseudo-inverse)

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We want to set this to 0

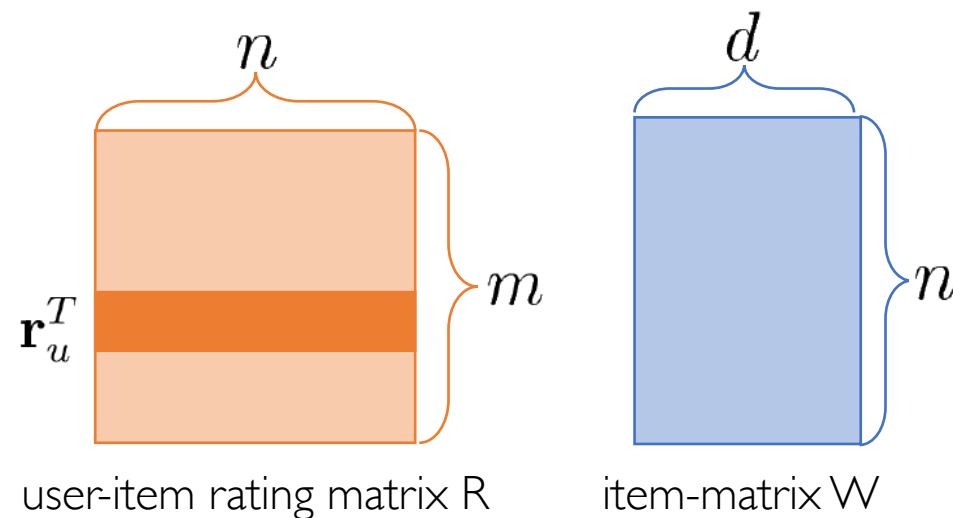
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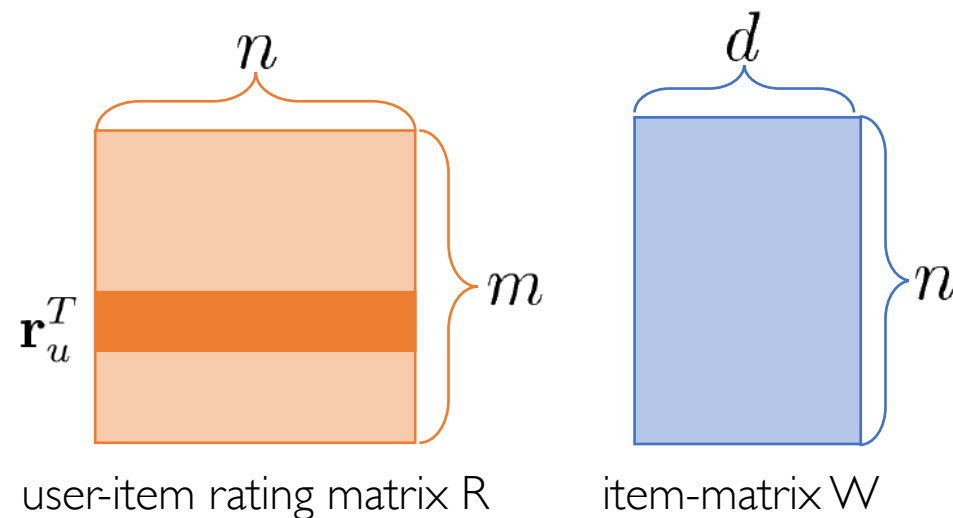


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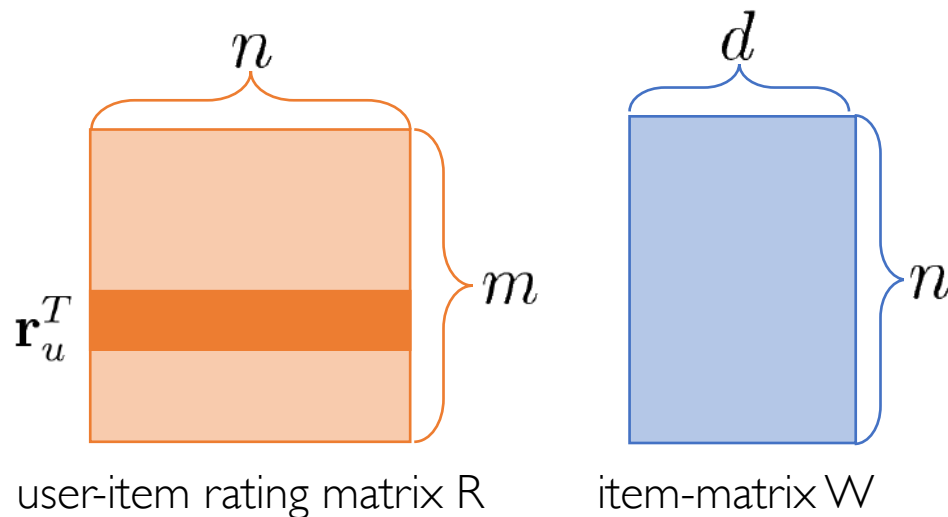
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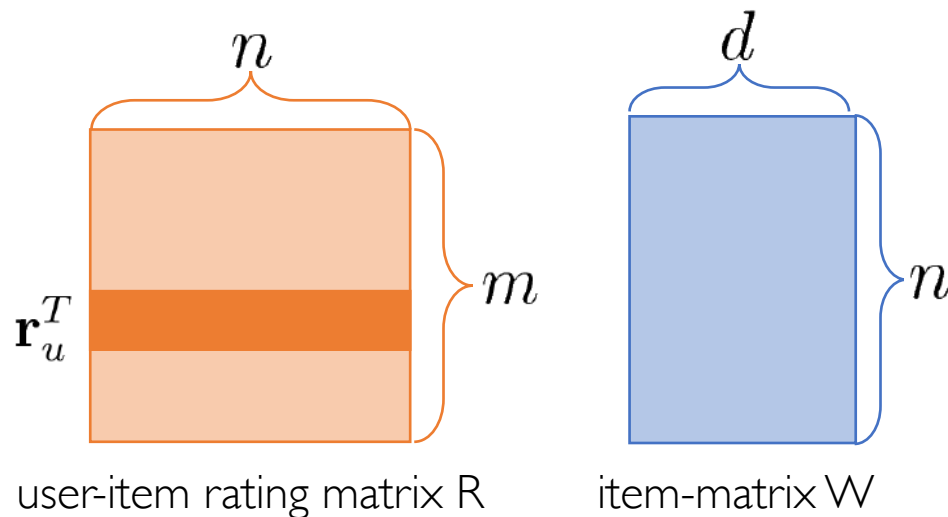
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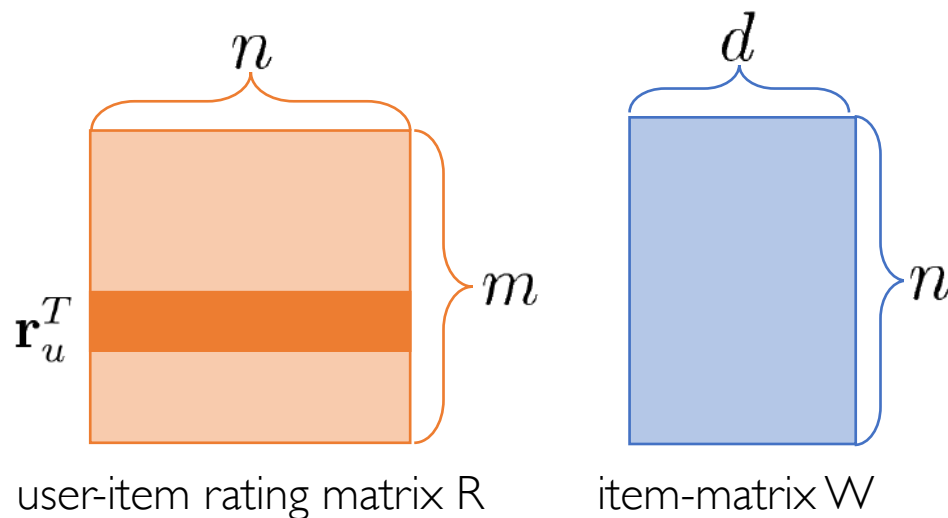
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$$= W^T \cdot \mathbf{r}_u = \mathbf{x}_u (W^T W + \lambda I) \quad I \in \mathbb{R}_{d \times d} \text{ identity matrix}$$

$$= (W^T W + \lambda I)^{-1} \cdot W^T \cdot \mathbf{r}_u = \mathbf{x}_u (W^T W + \lambda I) \cdot (W^T W + \lambda I)^{-1}$$

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ALS: User Vector Fixed

$$-\sum_{u \in \mathcal{D}} (r_{u,i} - \mathbf{x}_u^T \cdot \mathbf{w}_i) \mathbf{x}_u + \lambda \mathbf{w}_i = 0$$

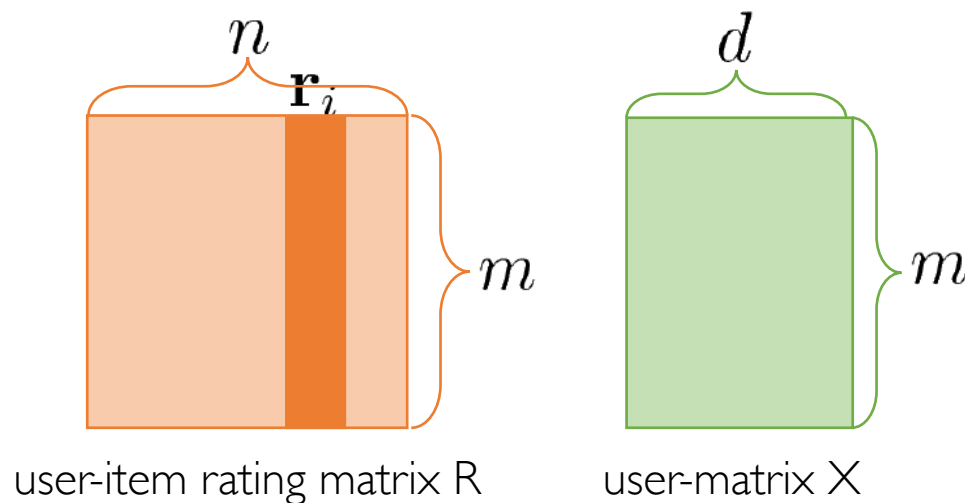
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Convergence is guaranteed because in each step the loss function can either decrease or stay unchanged, but never increase

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- However, ALS is favorable in at least **2** cases:
 - **Parallelization:** each \mathbf{x}_u and \mathbf{w}_i is computed independently of user/item factors
 - **Implicit Data:** the training set is dense and looping over each single instance - as SGD does - would be unfeasible

Singular Value Decomposition (SVD)

A well-known technique to decompose a matrix into the product of **3 matrices**

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SVD solution is unique

Applying SVD to Collaborative Filtering

- If we let the matrix A be the user-item ratings R
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Each factor k is the result of the **similarity** between user i and item j and its overall effect on ratings across all users and items

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- Possible workaround to apply SVD: use **imputation** to fill missing values in the matrix R

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- For example, some users systematically tend to give higher ratings than others, and some items receive higher ratings than others

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Overall avg. rating

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$$b_{\text{Joe,Titanic}} = 3.7 - 0.3 + 0.5 = 3.9$$

Bias term

Including Bias into the Optimization

$$\hat{r}_{u,i} = \underbrace{\mathbf{x}_u^T \cdot \mathbf{w}_i}_{\text{latent factors}} + \underbrace{\mu + b_u + b_i}_{\text{bias}}$$

The estimated rating of an item i for the user u is now made of **2 components**

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models global average, user and item bias

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Overall, the original optimization problem becomes as follows

$$X^*, W^* = \operatorname{argmin}_{X, W} \left\{ \frac{1}{2} \sum_{(u, i) \in \mathcal{D}} \left[r_{u, i} - (\mathbf{x}_u^T \cdot \mathbf{w}_i + \mu + b_u + b_i) \right]^2 + \lambda \left(\sum_{u \in \mathcal{D}} \|\mathbf{x}_u\|^2 + \sum_{i \in \mathcal{D}} \|\mathbf{w}_i\|^2 + \sum_{u \in \mathcal{D}} b_u^2 + \sum_{i \in \mathcal{D}} b_i^2 \right) \right\}$$

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Can still be solved using ALS

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the vast majority of items are not rated by users

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- **Netflix** is a good example of hybrid recommender systems

Netflix's Hybrid Recommender System

Recommendations are generated

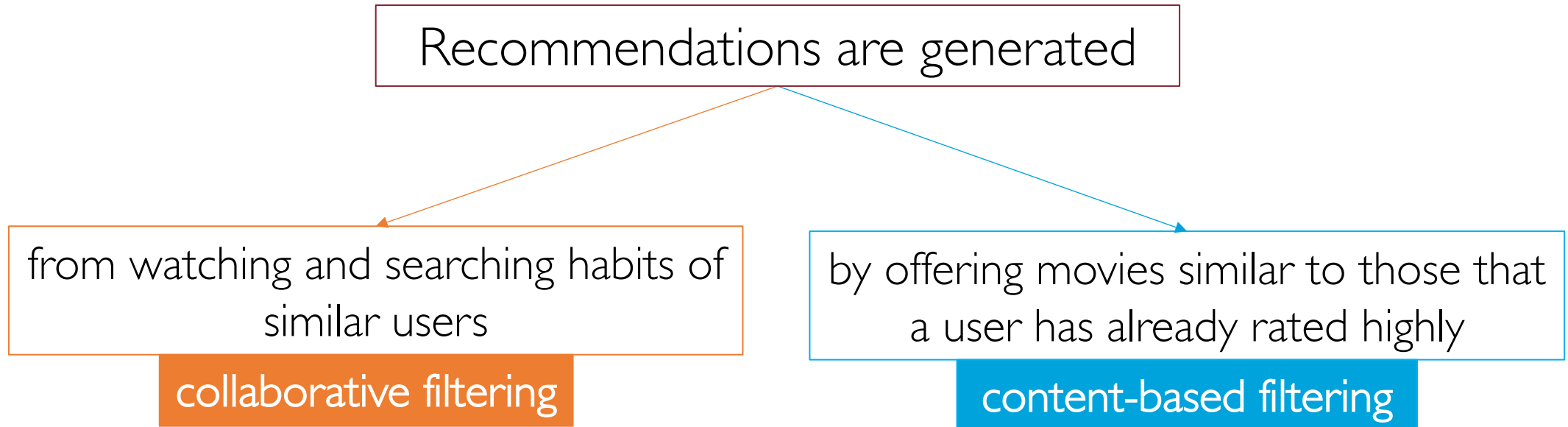
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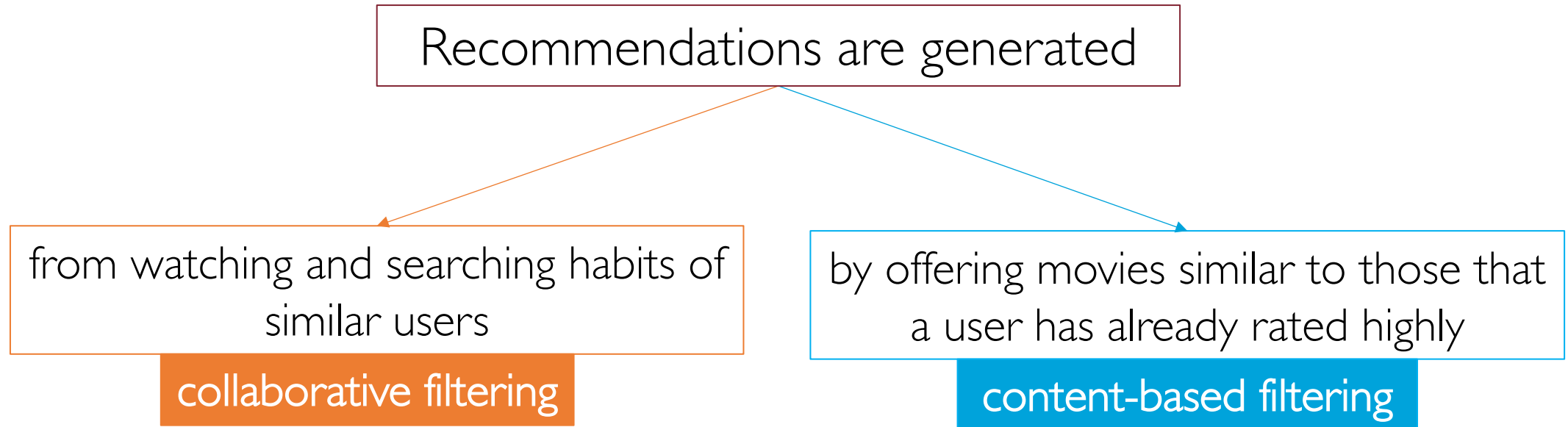
from watching and searching habits of
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collaborative filtering

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[Netflix: What Happens When You Press Play?](#)

For more details about how Netflix actually works

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- According to the [contest website](#), more than 48,000 teams from 182 different countries have downloaded the data

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A combination of 100 different predictor sets, mostly factorization models

Evaluation Metrics

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Online

A/B testing measuring CTR,
ROI, and other "live" metrics

Evaluation Metrics: RMSE

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The RMSE might penalize a method that does well for high ratings and
badly for others

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$$P = \frac{TP}{TP + FP} \quad R = \frac{TP}{TP + FN}$$

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Mapping of binary classification terminology to recommender systems

binary classifier	recommender system
# with condition ($y = 1$)	# of all possible relevant items for a user
# predicted positive ($TP + FP$)	# of recommended items
# correct positives (TP)	# of recommended items that are relevant

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A recommender system generates $k=5$ items to recommend

There are only 3 relevant items

The success/failure of our recommendations: $[0, 1, 1, 0, 0]$ 0=not relevant/1=relevant

$$P = \frac{2}{5} \quad R = \frac{2}{3}$$

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- Imagine taking our list of N recommendations and considering only the first element, then only the first two, then only the first three, and so on
- $P@k$ and $R@k$ are simply the precision and recall calculated only from the subset of the first k recommendations

P@k: Example

$k = 3$

$P@3 = \frac{1}{3}$

Rank	Product Recommended	Result
1	Credit card	Correct positive
2	Christmas Fund	False positive
3	Debit Card	False positive
4	Auto loan	False positive
5	HELOC	Correct Positive
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$k = 6$

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indicator function

$$\mathbf{1}_{\text{Rel}}(k) = \begin{cases} 1 & \text{if item } k \in \text{Rel} \\ 0 & \text{otherwise} \end{cases}$$

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$AP@N$ is computed for a single data point (i.e., user)

We define the Mean Average Precision (MAP) as follows:

$$MAP@N = \frac{1}{|\mathcal{U}|} \sum_{u=1}^{|\mathcal{U}|} AP@N(u) = \frac{1}{|\mathcal{U}|} \sum_{u=1}^{|\mathcal{U}|} \frac{1}{|\text{Rel}|} \sum_{k=1}^N P@k(u) \times \mathbf{1}_{\text{Rel}}(k, u)$$

Personalization

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Intuitively, a high personalization score indicates the recommender system is able to provide a **highly personalized** experience to the users

Personalization

Suppose 3 users are recommended the following lists of items

$$u_1 = [A, B, C, D]$$

$$u_2 = [A, B, C, E]$$

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	A	B	C	D	E	F	G
u_1	1	1	1	1	0	0	0
u_2	1	1	1	0	1	0	0
u_3	1	1	0	0	0	1	1

Personalization

Compute the 3-by-3 triangular matrix containing the cosine similarity between each pair of user's recommendation binary vector

$$M_{i,j} = \text{cosine}(\mathbf{u}_i, \mathbf{u}_j)$$

	u_1	u_2	u_3
u_1	1	0.75	0.58
u_2	0.75	1	0.58
u_3	0.58	0.58	1

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Take the average of the upper triangle of the matrix M above

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$$\text{Personalization} = 1 - 0.64 = 0.36$$

Take-Home Message of Today

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- Evaluation metrics must capture the accuracy, personalization and serendipity of recommendations

Recommended Readings and Information :)

- A huge body of work on recommender systems is available out there!
- Surveys:
 - [Adomavicius & Tuzhilin](#) [2005]
 - [Koren & Volinsky](#) [2009]
 - [Bobadilla et al.](#) [2013]
 - [Zhang et al.](#) [2019]
- Well-renowed series of Conferences: [RecSys](#), [KDD](#), [SIGIR](#), [TheWebConf](#)