1.

$$\overline{I^2_{n,out}} = 4kT\frac{1}{R_D} + 4kT\gamma g_m$$

$$\overline{V^2_{n,in}} = \frac{\overline{I^2_{n,out}}}{G_m^2}$$

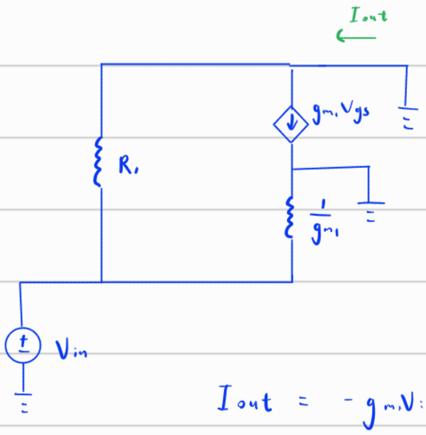
$$= 4kT\frac{1}{R_D}\frac{1}{g_m^2} + 4kT\gamma\frac{1}{g_m} \quad \#$$

$$\overline{I^2_{n,in}}\left(\frac{1}{C_{in}\,\omega}\right)^2 = \overline{V^2_{n,in}}$$

$$\overline{I^2_{n,in}} = \left[\left(4kT\frac{1}{R_D}\frac{1}{g_m^2} + 4kT\gamma\frac{1}{g_m}\right)(C_{in}\,\omega)^2\right] \left(A^2/Hz\right) \quad \#$$

2.

(a)



$$V_{gs} = 0 - V_{in}$$

$$I_{out} = -g_{m1}V_{in} + \frac{0 - V_{in}}{R_1}$$

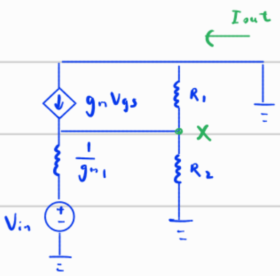
$$= -\left(g_{m1} + \frac{1}{R_1}\right)V_{in}$$

$$\frac{I_{out}}{V_{in}} = G_m = -\left(g_{m1} + \frac{1}{R_1}\right)$$

$$\overline{I^2_{n,out}} = 4kT\cdot\frac{1}{R_D} + 4kT\frac{1}{R_1} + 4kT\gamma g_{m1}$$

$$\overline{V^2_{n,in}} = \frac{\overline{I^2_{n,out}}}{G_m^2} = \left[\left(4kT\frac{1}{R_D} + 4kT\frac{1}{R_1} + 4kT\gamma g_{m1}\right)\left(\frac{1}{g_{m1} + \frac{1}{R_1}}\right)^2\right] \left(V^2/Hz\right) \quad \#$$

(b)



$$V_{gs} = V_X - V_{in}$$

$$\text{KCL @ } X: \quad \frac{0 - V_X}{R_1} + g_{m1}(V_X - V_{in}) = (V_X - V_{in})\,g_{m1} + \frac{V_X}{R_2}$$

$$V_X = \left[I_{out} - g_{m1}(V_X - V_{in})\right]R_2$$

$$V_X = I_{out}R_2 - g_{m1}R_2V_X + g_{m1}R_2V_{in}$$

$$(1 + g_{m1}R_2)V_X = I_{out}R_2 + g_{m1}R_2V_{in}$$

$$V_x = \frac{I_{out} R_2 + g_{m1} R_2 V_{in}}{1 + g_{m1} R_2}$$

$$\frac{-V_x}{R_1} = \frac{V_x}{R_2}$$

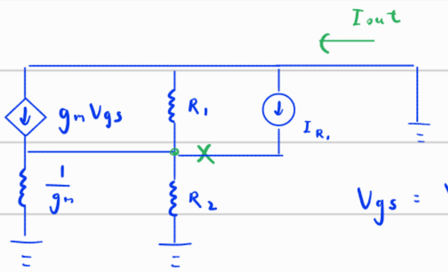
$$\Rightarrow \; - \frac{I_{out} R_2 + g_{m1} R_2 V_{in}}{R_1 \left(\cancel{1+g_{m1}R_2}\right)} = \frac{I_{out} + g_{m1} V_{in}}{\cancel{1+g_{m1}R_2}}$$

$$- \frac{I_{out} R_2 + g_{m1} R_2 V_{in}}{R_1} = I_{out} + g_{m1} V_{in}$$

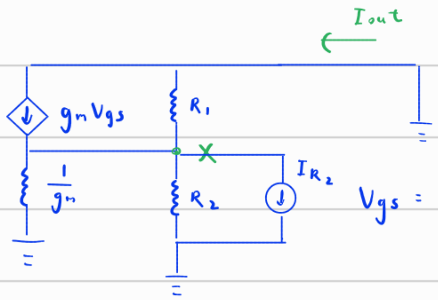
$$- I_{out} R_2 - g_{m1} R_2 V_{in} = R_1 I_{out} + g_{m1} R_1 V_{in}$$

$$\cancel{(R_1 + R_2)}\, I_{out} = - g_{m1} \cancel{(R_1 + R_2)}\, V_{in}$$

$$G_m = \frac{I_{out}}{V_{in}} = - g_{m1}$$



$V_{gs} = V_x$



$V_{gs} = V_x$

$$I_{out} = g_{m1} V_x + \frac{0 - V_x}{R_1} + I_{R_1}$$

$$= g_{m1} V_x + \frac{V_x}{R_2}$$

$$\left(g_m + \frac{1}{R_2}\right) V_x = I_{out}$$

$$V_x = \frac{I_{out}}{g_m + \frac{1}{R_2}}$$

$$I_{out} = g_m \frac{I_{out}}{g_m + \frac{1}{R_2}} - \frac{1}{R_1} \frac{I_{out}}{g_m + \frac{1}{R_2}} + I_{R_1}$$

$$\left(g_m + \frac{1}{R_2}\right) I_{out} = \left(g_m - \frac{1}{R_1}\right) I_{out} + \left(g_m + \frac{1}{R_2}\right) I_{R_1}$$

$$\left(\frac{1}{R_1} + \frac{1}{R_2}\right) I_{out} = \left(g_m + \frac{1}{R_2}\right) I_{R_1}$$

$$\frac{I_{out}}{I_{R_1}} = \frac{g_m + \frac{1}{R_2}}{\frac{1}{R_1} + \frac{1}{R_2}} = \frac{g_m R_1 R_2 + R_1}{R_1 + R_2}$$

$$I_{out} = g_m V_x + \frac{0 - V_x}{R_1}$$

$$= g_m V_x + \frac{V_x}{R_2} + I_{R_2}$$

$$\left(g_m - \frac{1}{R_1}\right) V_x = I_{out}$$

$$V_x = \frac{I_{out}}{g_m - \frac{1}{R_1}}$$

$$I_{out} = g_{m1} \frac{I_{out}}{g_m - \frac{1}{R_1}} + \frac{1}{R_2} \frac{I_{out}}{g_m - \frac{1}{R_1}} + I_{R_2}$$

$$\left(g_{m1} - \frac{1}{R_1}\right) I_{out} = \left(g_{m1} + \frac{1}{R_2}\right) I_{out} + \left(g_m - \frac{1}{R_1}\right) I_{R_2}$$

$$\left(-\frac{1}{R_1} - \frac{1}{R_2}\right) I_{out} = \left(g_{m1} - \frac{1}{R_1}\right) I_{R_2}$$

$$\frac{I_{out}}{I_{R_2}} = - \frac{g_{m1} - \frac{1}{R_1}}{\frac{1}{R_1} + \frac{1}{R_2}}$$

$$= - \frac{g_m R_1 R_2 - R_2}{R_1 + R_2}$$

$$\overline{I^2_{n,out}} = 4kT \frac{1}{R_0} + 4kT \gamma g_{m1} + 4kT \frac{1}{R_1} \left(\frac{g_{m1} R_1^2 + R_1}{2R_1}\right)^2 + 4kT \frac{1}{R_1} \left(\frac{g_{m1} R_1^2 - R_1}{2R_1}\right)^2$$

$$\overline{V^2_{n,in}} = \frac{\overline{I^2_{n,out}}}{G_m^2} = \frac{\overline{I^2_{n,out}}}{(-g_{m1})^2}$$

$$= \Big[\left(\frac{1}{g_{m1}}\right)^2 \Big[4kT \frac{1}{R_0} + 4kT \gamma g_{m1} + 4kT \frac{1}{R_1} \left(\frac{g_{m1} R_1 + 1}{2}\right)^2 + 4kT \frac{1}{R_1} \left(\frac{g_{m1} R_1 - 1}{2}\right)^2 \Big] \; (V^2/Hz)$$

#

3.

$G_m = g_{m1}$

$$\overline{I^2_{n,out}} = \frac{K}{C_{ox} W_1 L_1} \frac{1}{f} g_{m1}^2 + \frac{K}{C_{ox} W_2 L_2} \frac{1}{f} g_{m2}^2$$

$$\overline{V^2_{n,in}} = \frac{\overline{I^2_{n,out}}}{G_m^2}$$

$$= \left[\frac{K}{C_{ox} W_1 L_1} \frac{1}{f} + \frac{K}{C_{ox} W_2 L_2} \frac{1}{f} \left(\frac{g_{m2}}{g_{m1}} \right)^2 \right] (V^2/Hz) \ \#$$

4.

$$\overline{V^2_{n,in}} = 4kTR_s$$

$$H(s) = \frac{A_0}{1 + \frac{s}{w_0}} = \frac{A_0}{1 + \frac{s}{f_0}}$$

$$\overline{V^2_{n,out}} = |H(s)|^2 \, 4kTR_s$$

$$= \left(\frac{A_0^2}{\frac{s^2}{f_0^2} + \frac{s}{f_0} + 1} \right) 4kTR_s$$

$$\approx \left(\frac{A_0^2}{\frac{s^2}{f_0^2} + 1} \right) 4kTR_s$$

$$P_{noise} = \int_0^\infty \left(\frac{A_0^2}{\frac{s^2}{f_0^2} + 1} \right) 4kTR_s \, df$$

$$= A_0^2 \, 4kTR_s \, \frac{\pi}{2} f_0 \, (V^2)$$

$$= A_0^2 \, 2kTR_s \, \pi f_0 \, (V^2)$$

$$= A_0^2 \, kTR_s \, w_0 \, (V^2) \ \#$$

Let $x = \frac{f}{f_0}$ $\quad dx = \frac{1}{f_0} df$

$$\int_0^\infty \frac{1}{1 + \left(\frac{f}{f_0}\right)^2} df \qquad df = f_0 \, dx$$

$$= \int_0^\infty \frac{1}{1+x^2} f_0 \, dx$$

$$= \left[\tan^{-1} x \right]_0^\infty f_0$$

$$= \frac{\pi}{2} f_0$$