2.1

111511076
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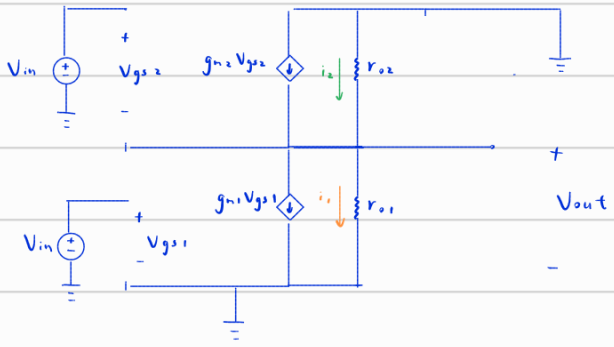
(a)



$V_{gs2} = V_{in} - V_{out}$ $\quad V_{bs2} = -V_{out}$

$V_{gs1} = V_{in}$ $\quad V_{bs1} = 0$

$V_{out} = i_1 r_{o1} = -i_2 r_{o2}$

$\Rightarrow i_1 = \frac{V_{out}}{r_{o1}}$, $i_2 = \frac{-V_{out}}{r_{o2}}$

$g_{m1} V_{gs1} + i_1 = g_{m2} V_{gs2} + i_2 + g_{mb2} V_{gs2}$

$\Rightarrow g_{m1} V_{in} + i_1 = g_{m2} (V_{in} - V_{out}) + i_2$

$g_{m1} V_{in} + \frac{V_{out}}{r_{o1}} = g_{m2} (V_{in} - V_{out}) - \frac{V_{out}}{r_{o2}}$

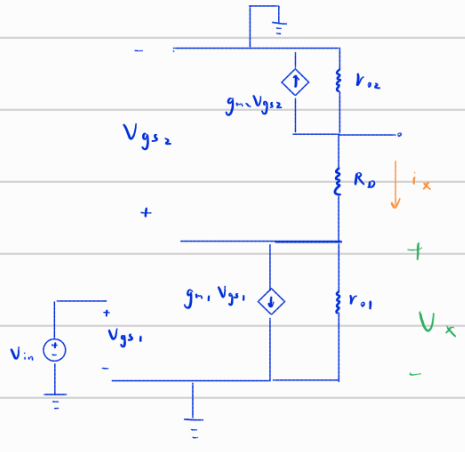
$(g_{m2} - g_{m1}) V_{in} = \left(\frac{1}{r_{o1}} + g_{m2} + \frac{1}{r_{o2}} \right) V_{out}$

$V_{out} = \frac{g_{m2} - g_{m1}}{\frac{1}{r_{o1}} + g_{m2} + \frac{1}{r_{o2}}} V_{in}$

$= \frac{(g_{m2} - g_{m1}) r_{o1} r_{o2}}{g_{m2} r_{o1} r_{o2} + r_{o1} + r_{o2}}$

$A_V = \frac{(g_{m2} - g_{m1}) r_{o1} r_{o2}}{g_{m2} r_{o1} r_{o2} + r_{o1} + r_{o2}}$ V/V #

(b)



Vout

$i_x = g_{m1} V_{gs1} + \frac{V_x}{r_{o1}}$

$V_{gs2} = V_x = V_{out} \times \frac{r_{o1}}{R_D + r_{o1}}$

$g_{m2} V_{gs2} + \frac{V_{out}}{r_{o2}} + i_x = 0$

$i_x = -g_{m2} V_{gs2} - \frac{V_{out}}{r_{o2}}$

$$V_{gs2} = V_{out} - i_x R_D$$

$$V_{gs2} = V_{out} - \left(-g_{m2} V_{gs2} - \frac{V_{out}}{r_{o2}} \right) R_D$$

$$V_{gs2} = V_{out} + g_{m2} V_{gs2} R_D + \frac{V_{out}}{r_{o2}} R_D$$

$$(1 - g_{m2} R_D) V_{gs2} = \left(1 + \frac{R_D}{r_{o2}} \right) V_{out}$$

$$V_{gs2} = \frac{r_{o2} + R_D}{r_{o2} - g_{m2} r_{o2} R_D} V_{out}$$

$$g_{m1} V_{gs1} + \frac{V_x}{r_{o1}} = -g_{m2} V_{gs2} - \frac{V_{out}}{r_{o2}}$$

$$g_{m1} V_{in} + \frac{V_{gs2}}{r_{o1}} = -g_{m2} V_{gs2} - \frac{V_{out}}{r_{o2}}$$

$$g_{m1} V_{in} + \frac{1}{r_{o1}} \frac{r_{o2} + R_D}{r_{o2} - g_{m2} r_{o2} R_D} V_{out} = -g_{m2} \frac{r_{o2} + R_D}{r_{o2} - g_{m2} r_{o2} R_D} V_{out} - \frac{V_{out}}{r_{o2}}$$

$$g_{m1} V_{in} = \left(- \frac{r_{o2} + R_D}{r_{o1} r_{o2} - g_{m2} r_{o1} r_{o2} R_D} - \frac{g_{m2} r_{o2} + g_{m2} R_D}{r_{o2} - g_{m2} r_{o2} R_D} - \frac{1}{r_{o2}} \right) V_{out}$$

$$g_{m1} V_{in} = \left(- \frac{r_{o2} + R_D}{r_{o1} r_{o2} - g_{m2} r_{o1} r_{o2} R_D} - \frac{g_{m2} r_{o1} r_{o2} + g_{m2} r_{o1} R_D}{r_{o1} r_{o2} - g_{m2} r_{o1} r_{o2} R_D} - \frac{1}{r_{o2}} \right) V_{out}$$

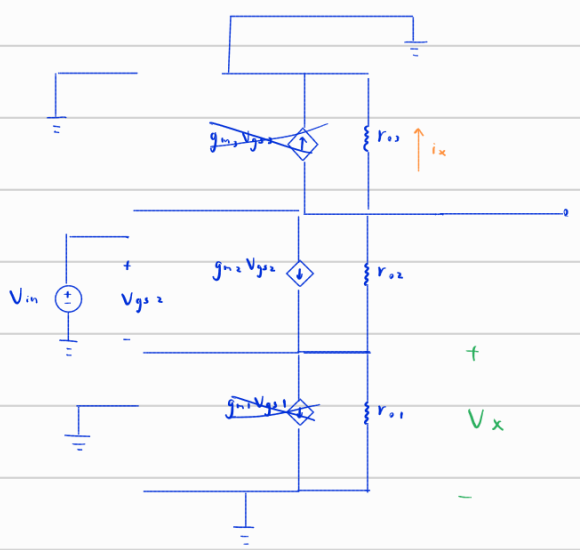
$$= \left(- \frac{r_{o2} + R_D + g_{m2} r_{o1} r_{o2} + g_{m2} r_{o1} R_D}{r_{o1} r_{o2} - g_{m2} r_{o1} r_{o2} R_D} - \frac{1}{r_{o2}} \right) V_{out}$$

$$(r_{o1} r_{o2} - g_{m2} r_{o1} r_{o2} R_D) g_{m1} V_{in} = \left(- r_{o2} - R_D - g_{m2} r_{o1} r_{o2} - \cancel{g_{m2} r_{o1} R_D} - r_{o1} + \cancel{g_{m2} r_{o1} R_D} \right) V_{out}$$

$$V_{out} = - \frac{g_{m1} r_{o1} r_{o2} - g_{m1} g_{m2} r_{o1} r_{o2} R_D}{R_D + r_{o1} + r_{o2} + g_{m2} r_{o1} r_{o2}} V_{in}$$

$$A_v = - \frac{g_{m1} r_{o1} r_{o2} - g_{m1} g_{m2} r_{o1} r_{o2} R_D}{R_D + r_{o1} + r_{o2} + g_{m2} r_{o1} r_{o2}} \; V/V \quad \#$$

(c)



$$i_x = \frac{V_{out}}{r_{o3}}$$

$$g_{m3} V_{gs3} + i_x + g_{m2} V_{gs2} + \frac{V_{out} - V_x}{r_{o2}} = 0$$

$$g_{m2} V_{gs2} + \frac{V_{out} - V_x}{r_{o2}} = \frac{V_x}{r_{o1}}$$

$$\frac{V_x}{r_{o1}} = -i_x \qquad V_x = -r_{o1} i_x$$

$$V_{gs2} = V_{in} - V_x$$

$$= V_{in} - V_{out} \frac{r_{o1}}{r_{o1} + r_{o2}}$$

$$V_{gs3} = 0\,V$$

$$V_{gs1} = 0\,V$$

$$-g_{m3}\cancel{V_{gs3}} + \frac{0 - V_{out}}{r_{o3}} = g_{m2}V_{gs2} + \frac{V_{out} - V_x}{r_{o2}}$$

$$g_{m2}V_{gs2} + \frac{V_{out} - V_x}{r_{o2}} = g_{m1}\cancel{V_{gs1}} + \frac{V_x}{r_{o1}}$$

$$V_x = -\frac{r_{o1}}{r_{o3}} V_{out}$$

$$-\frac{V_{out}}{r_{o3}} = g_{m2}(V_{in} - V_x) + \frac{V_{out} - V_x}{r_{o2}}$$

$$-g_{m2}V_{in} = \frac{V_{out}}{r_{o3}} - g_{m2}\left(-\frac{r_{o1}}{r_{o3}} V_{out}\right) + \frac{V_{out}}{r_{o2}} - \frac{1}{r_{o2}}\left(-\frac{r_{o1}}{r_{o3}} V_{out}\right)$$

$$-g_{m2}V_{in} = \left(\frac{1}{r_{o2}} + \frac{1}{r_{o3}} + g_{m2}\frac{r_{o1}}{r_{o3}} + \frac{r_{o3}}{r_{o2}r_{o3}}\right) V_{out}$$

$$\frac{V_{out}}{V_{in}} = -\frac{g_{m2}r_{o2}r_{o3}}{r_{o2} + g_{m2}r_{o2}r_{o1} + r_{o3} + r_{o1}}$$

$$A_v = -\frac{g_{m2}r_{o2}r_{o3}}{r_{o1} + r_{o2} + r_{o3} + g_{m2}r_{o2}r_{o1}} \quad V/V \quad \#$$

2.2

$V_{DD} = 3V$

$C_{ox} = 3.9 \frac{\epsilon_0}{t_{ox}} = 3.83693 \times 10^{-7}\ F/cm^2$

$\mu_n = 350\ cm^2/V/s$ $\quad V_{t_n} = 0.7$

$\mu_p = 100\ cm^2/V/s$ $\quad V_{t_p} = -0.8$

$k_n' = 0.13428\ mA/V^2$ $\quad k_p' = 0.038337\ mA/V^2$

$k_{1,2} = 20.142\ mA/V^2$ $\quad \lambda_n = 0.1$

$k_{3,4} = 1.279\ mA/V^2$ $\quad \lambda_p = 0.2$

$k_5 = 10.071\ mA/V^2$

(a)

(i) $V_{D_1} = 3 - 0.25 \cdot 5 = 1.75$

$V_{in,cm\ max} = 1.75 + V_{t_n} = 2.45\ V$

$I_{D_5} = \frac{1}{2} \cdot 10.071m \cdot V_{ov_5}^2 = 0.5m$

$V_{ov5} = 0.315 = V_{DS\ min}$

$I_{D_1} = \frac{1}{2} \cdot 20.142m \cdot V_{ov_1}^2 = 0.25m$

$V_{ov1} = 0.1576\ V$

$V_{gs_1} = 0.8576\ V$

$V_{in,cm\ min} = V_{DS_5\ min} + V_{gs_1} = 0.315 + 0.8576$

$= 1.1726$

$1.173\ V \leq V_{in,cm} \leq 2.450\ V$ #

(ii) $V_{out\ max} = V_{DD} = 3$

$V_{out\ min} = V_{in,cm} - V_t = 1.2 - 0.7 = 0.5$

$2(3 - 0.5) = 5\ V$ #

(iii) $\lambda = 0.1$

$r_{o_1} = \frac{1}{\lambda I_D} = \frac{1}{0.1 \cdot 0.25m}$

$= 40k$

$V_{s1} = 1.2 - V_{gs_1} = 0.3424$

$g_{m_1} = \sqrt{2 k_1 I_D} = \sqrt{2 \cdot 20.142m \cdot 0.25m} = 3.1735m$

$$A_v = -g_{m_1}(R_o \parallel r_o)$$

$$= -3.1735m \cdot 4.4444k$$

$$= -14.104\ V/V \quad \#$$

(b)

(i) $g_{m3,4} = \sqrt{2 \cdot 1.279m \cdot 0.25m} = 0.79969\ m$

$$\frac{1}{g_{3,4}} = 1.25098k$$

$$I_{D3} = 0.25m = \frac{1}{2} \cdot 1.279m \cdot V_{ov3}^2$$

$$V_{ov3} = 0.62524V$$

$$V_{sg3} = 0.62524 + 0.8 = 1.42524V \qquad V_{g3} = 3 - 1.42524 = 1.57476V$$

$$V_{in,cm\,max} = V_{out} + V_{t1} = 1.57476V + 0.7V = 2.27476$$

$$V_{in,cm\,min} = V_{DS5\,min} + V_{gs1} = 0.315 + 0.8576$$

$$= 1.1726$$

$$1.173 \le V_{in,cm} \le 2.275V \quad \#$$

(ii) $V_{out,max} = V_{DD} - |V_{tp}| = 3 - 0.8 = 2.2V$

$$V_{d,sat} = V_{in} - V_t$$

$$= 1.2 - 0.7 = 0.5$$

$$0.5m = \frac{1}{2} \cdot 1.279m \cdot V_{ov3}^2 \qquad V_{ov3} = 0.88423V$$

$$|V_{gs3}|_{I_D = I_{SS}} = 1.68423V \qquad V_{g3} = 1.31577$$

$$V_{out,min} = \max\{1.31577, 0.5\}$$

$$2(2.2 - 1.31577) = 1.76846$$

$$= 1.768V \quad \#$$

(iii) $r_{o3} = \frac{1}{\lambda_n I_D} = \frac{1}{0.1 \cdot 0.25m} = 40k$

$$r_{o1} = \frac{1}{\lambda_p I_D} = \frac{1}{0.2 \cdot 0.25m} = 20k$$

$$g_{m3,4} = \sqrt{2 \cdot 1.279m \cdot 0.25m} = 0.79969\ m \qquad \frac{1}{0.79969m} = 1.25098k$$

$$g_{m1,2} = \sqrt{2 \cdot 20.142n \cdot 0.25m} = 3.17398m$$

$$A_v = -3.17398m \cdot \left(r_{o1} \parallel r_{o3} \parallel \frac{1}{g_{m3,4}}\right)$$

$$= -3.62812\ V/V$$

$$= -3.628\ V/V \quad \#$$

(c)

(i) $0.25\,m = \frac{1}{2} \cdot 1.279\,m \cdot V_{ov3}^2 \qquad V_{ov3} = 0.62524\,V$

$V_{sg3} = 1.42524\,V \qquad V_b = 1.57476\,V \qquad V_{d3,max} = 1.57476 + 0.8$

$= 2.37476\,V$

$V_{in,max} = 3.07476\,V$ (can't be larger than V_{DD}) ✗

$V_{in,cm\,max} = \min\{V_{DD} - V_{ov3} + V_{th1}, V_{DD}\} = 3$

$V_{in,cm\,min} = V_{DSS,min} + V_{gs1} = 0.315 + 0.8576$

$= 1.1726$

$1.173\,V \le V_{in,cm} \le 3\,V$ #

(ii) $V_{out,max} = V_{d3,max}$

$= 2.37476$

$V_{out,min} = 1.2 - 0.7$

$= 0.5$

$2(2.37476 - 0.5) = 3.74952$

$= 3.750\,V$ #

(iii) $r_{o3} = \frac{1}{\lambda_n I_b} = \frac{1}{0.1 \cdot 0.25m} = 40k$

$r_{o1} = \frac{1}{\lambda_p I_b} = \frac{1}{0.2 \cdot 0.25m} = 20k$

$g_{m1,2} = \sqrt{2 \cdot 20.142m \cdot 0.25m} = 3.17348m$

$A_v = -3.17348m\,(40k \parallel 20k)$

$= -42.313\,V/V$ #

2.3

(a) $I_{D5} = I_{D6} = 0.9mA \qquad k_n' = 0.13428\,mA/V^2 \qquad k_p' = 0.03837\,mA/V^2$

$I_{D3} = I_{D4} = 0.1mA \qquad k_n = 16.1136\,mA/V^2 \qquad k_p = 4.6044\,mA/V^2$

$g_{m1} = \sqrt{2k_n \cdot 0.5m} = 4.01417m$

$g_{m3} = \sqrt{2k_p \cdot 0.1m} = 0.95962m \qquad \frac{1}{g_{m3}} = 1.04208k$

$r_{o5} = \frac{1}{0.2 \cdot 0.9m} = 12.5k$

$r_{o3} = \frac{1}{0.2 \cdot 0.1m} = 50k$

$r_{o1} = \frac{1}{0.1 \cdot 0.5m} = 20k$

$A_v = -g_{m1} \cdot (\frac{1}{g_{m3}} \parallel r_{o1} \parallel r_{o3} \parallel r_{o5})$

$= -3.6176$

$= -3.618\,V/V$ #

(b)

$$I_{DS} = 0.4\,mA = \frac{1}{2} \cdot 4.6044\,m \cdot V_{ovs}^2$$

$$V_{ovs} = 0.41683$$

$$V_{sgs} = 1.21683$$

$$3 - 1.21683 = 1.78317$$

$$V_b = 1.783 \quad \#$$

(c)

$$V_{out\,1,2\,max} = \min\{ V_b + |V_{thsp}|,\ V_{DD} - |V_{th3p}| \}$$

$$= \min\{ 1.783 + 0.8,\ 3 - 0.8 \}$$

$$= 2.2$$

$$0.6\,mA = \frac{1}{2} \cdot 16.1136\,m \cdot (V_{gs1} - 0.7)^2$$

$$V_{gs1}\big|_{I_D = 0.6mA} = 0.97289$$

$$0.2\,mA = \frac{1}{2} \cdot 4.6044\,m \cdot (V_{sg3} - 0.8)^2$$

$$V_{sg3}\big|_{I_D = 0.2mA} = 1.09474 \quad (\text{No current goes through } M_4)$$

$$V_{out\,1,2\,min} = \max\{ V_{in\,min} + V_{gs1}\big|_{I_D = 0.6mA} - V_{th1n},\ V_{DD} - |V_{gs3}|_{I_D = 0.2mA} \}$$

$$= \max\{ 0.9 + 0.97289 - 0.7,\ 3 - 1.09474 \}$$

$$= \max\{ 0.97289,\ 1.90529 \}$$

$$= 1.90529$$

$$2(2.2 - 1.90529) = 0.58942$$

$$= 0.589\,V \quad \#$$