2. Transformers

2.1 Why Transformers Are Important to Modern Life

- A transformer is a device that changes ac electric power at one voltage level to ac electric power at another voltage level through the action of a magnetic field.
- The transformer winding connected to the power source is called the primary winding or input winding, and the winding connected to the loads is called the secondary winding or output winding. If there is a third winding on the transformer, it is called the tertiary winding.
- The first power distribution system in the United States was a 120-V dc system invented by Thomas A. Edison to supply power for incandescent light bulbs in New York City in September 1882.
- In a modern power system, electric power is generated at voltages of 12 to 25kV. Transformers step up the voltage to between 110kV and nearly 1000kV for transmission over long distances at very low losses. Transformers then step down the voltage to the 12kV to 34.5kV range for local distribution and finally permit the power to be used safely in home, offices, and factories at voltages as low as 120V.

2.2 Type and Construction of Transformers

- Transformers are also used for a variety of other purposes (e.g., voltage sampling or potential transformer, current sampling or current transformer, and impedance transformation), but this chapter is primarily devoted to the power transformer.
- The transformer core is constructed of thin laminations electrically isolated from each other in order to minimize eddy currents.
- The primary and secondary windings in a physical transformer are wrapped one on top of the other with the low-voltage winding innermost. Such an arrangement serves two purposes: 1. It simplifies the problem of insulating the high-voltage winding from the core. 2. It results in much less leakage flux than would be the case if the two windings were separated by a distance on the core.
- A transformer connected to the output of a generator and used to step its voltage up to transmission levels (110+ kV) is sometimes called a unit transformer. The transformer at the other end of the transmission line, which steps the voltage down from transmission levels to distribution levels (from 2.3 to 34.5kV), is called a substation transformer. Finally, the transformer that takes the distribution voltage and steps it down to the final voltage at which the power is actually used (110, 208, 220V, etc.) is called a distribution transformer.

2.3 The Ideal Transformer

Power in an Ideal Transformer / Impedance Transformation through a Transformer / Analysis of Circuits Containing Ideal Transformers

$$\frac{v_p(t)}{v_s(t)} = \frac{N_p}{N_s} = a \qquad (2-1)$$

$$turns\ ratio: a = \frac{N_p}{N_s} \qquad (2-2)$$

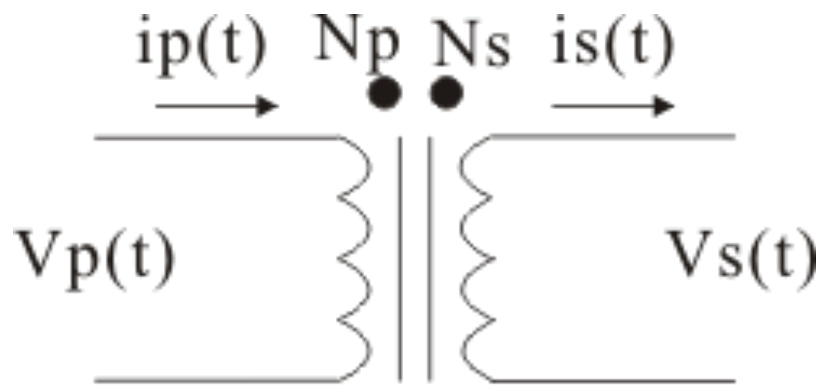
$$N_p i_p(t) = N_s i_s(t) \qquad (2-3a)$$

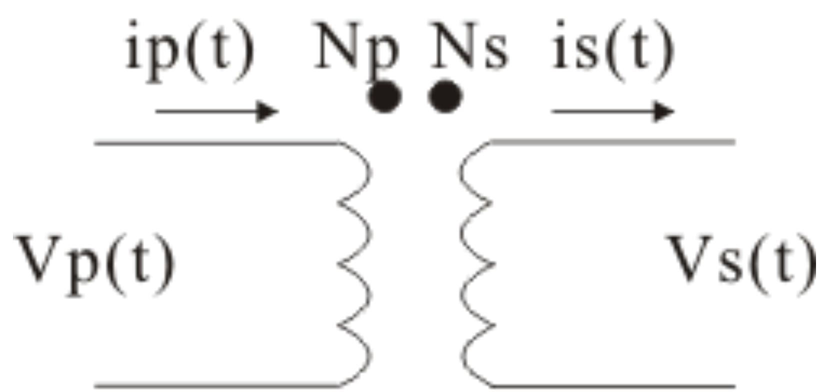
$$\frac{i_p(t)}{i_s(t)} = \frac{1}{a} \qquad (2-3b)$$

Phasor Quantities

$$\frac{V_p}{V_s} = a \qquad (2-4)$$

$$\frac{I_p}{I_s} = \frac{1}{a} \qquad (2-5)$$





Power in an Ideal Transformer

$$P_{in} = V_p I_p \cos\theta_p \qquad (2-6)$$

$$P_{out} = V_s I_s \cos\theta_s \qquad (2-7)$$

$$Power\ Factor : \theta = \theta_p = \theta_s$$

$$P_{out} = V_s I_s \cos\theta = \frac{V_p}{a} \cdot aI_p \cdot \cos\theta = P_{in} \qquad (2-9)$$

$$Q_{in} = V_p I_p \sin\theta = V_s I_s \sin\theta = Q_{out} \qquad (2-10)$$

$$S_{in} = V_p I_p = V_s I_s = S_{out} \qquad (2-11)$$

Impedance Transformation through a Transformer

$$Z_L = \frac{V_L}{I_L} \qquad (2-12)$$

$$Z_L = \frac{V_s}{I_s} \qquad (2-13)$$

$$Z_L^{'} = \frac{V_p}{I_p} \qquad (2-14)$$

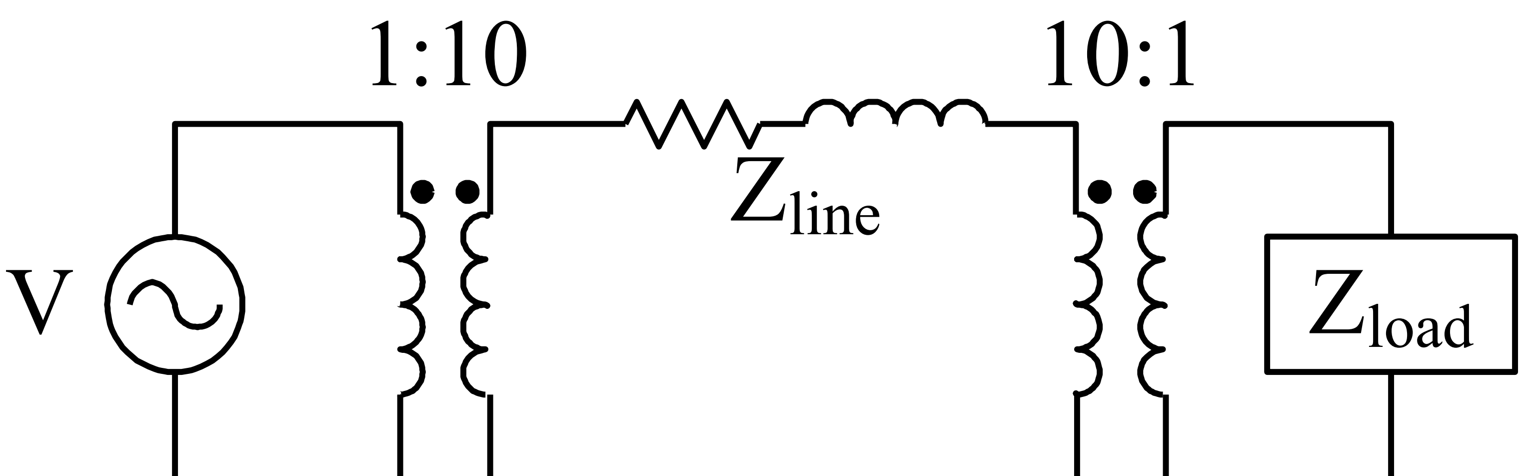
$$V_p = aV_s \quad , \quad I_p = \frac{I_s}{a}$$

$$Z_L^{'} = \frac{V_p}{I_p} = \frac{aV_s}{I_s / a} = a^2 \frac{V_s}{I_s} = a^2 Z_L \quad (2-15)$$

EX2-1. A single-phase power system consists of a 480V/60Hz generator supplying a load $Z_{load} = 4+j3\Omega$ through a transmission line of impedance $Z_{line} = 0.18+j0.24\Omega$. Answer the following questions about this system.

(a) If the power system is exactly as Fig. 2-6 without transformer, what will the voltage at the load be ? What will the transmission line losses be ?

(b) Suppose a 1:10 step-up transformer is placed at the generator end of the transmission line and a 10:1 step-down transformer is placed at the load end of the line (Fig. 2-6 with transformer). What will the load voltage be now? What will the transmission line losses be now?



$$(a)\ I_G = I_{line} = I_{load}$$

$$I_{line} = \frac{V}{Z_{line} + Z_{load}} = \frac{480\angle 0^0 V}{(0.18\Omega + j0.24\Omega) + (4 + j3\Omega)} = 90.8\angle -37.8^0 A$$

$$V_{load} = I_{line} Z_{load} = (90.8\angle -37.8^0 A)(4 + j3\Omega) = 454\angle -0.9^0 V$$

$$P_{loss} = (I_{line})^2 R_{line} = (90.8A)^2 (0.18\Omega) = 1484W$$

$$(b)\ I_G = \frac{10}{1} I_{line} = \frac{1}{10} \cdot \frac{10}{1} I_{load}$$

$$I_G = \frac{V}{Z_{line}^{'} + Z_{load}^{'}} = \frac{480\angle 0^0 V}{(\frac{1}{10})^2 (0.18\Omega + j0.24\Omega) + (4 + j3\Omega)} = 95.94\angle -36.88^0 A$$

$$V_{load} = I_{load} Z_{load} = (95.94\angle -36.88^0 A)(4 + j3\Omega) = 479.7\angle -0.01^0 V$$

$$P_{loss} = (I_{line})^2 R_{line} = (\frac{1}{10} \cdot 95.94A)^2 (0.18\Omega) = 16.7W$$

2.4 Theory of Operation of Real Single-Phase Transformers

The Voltage Ratio across a Transformer / The Magnetization Current in a Real Transformer / The Current Ratio on a Transformer and the Dot Convention

Faraday's Law

$$e_{ind} = \frac{d\lambda}{dt} \qquad (1-41)$$

$$\lambda = \sum_{i=1}^{N} \phi_i \qquad (1-42)$$

average flux per turn

$$\bar{\phi} = \frac{\lambda}{N} \qquad (2-16)$$

$$e_{ind} = N\frac{d\bar{\phi}}{dt} \qquad (2-17)$$

$$\bar{\phi} = \frac{1}{N_p}\int v_p(t)dt \qquad (2-18)$$

$$\bar{\phi}_p = \phi_M + \phi_{Lp} \qquad (2-19)$$

$$\bar{\phi}_s = \phi_M + \phi_{Ls} \qquad (2-20)$$

$$v_p(t) = N_p\frac{d\bar{\phi}_p}{dt} = N_p\frac{d\phi_M}{dt} + N_p\frac{d\phi_{Lp}}{dt} \quad (2-21)$$

$$v_p(t) = e_p(t) + e_{Lp}(t) \qquad (2-22)$$

$$v_s(t) = N_s\frac{d\bar{\phi}_s}{dt} = N_s\frac{d\phi_M}{dt} + N_p\frac{d\phi_{Ls}}{dt} \quad (2-23)$$

$$v_s(t) = e_s(t) + e_{Ls}(t) \qquad (2-24)$$

$$e_p(t) = N_p\frac{d\phi_M}{dt} \qquad (2-25)$$

$$e_s(t) = N_s\frac{d\phi_M}{dt} \qquad (2-26)$$

$$\frac{e_p(t)}{e_s(t)} = \frac{N_p}{N_s} = a \qquad (2-27)$$

$$\frac{v_p(t)}{v_s(t)} = \frac{N_p}{N_s} = a \quad , \ \phi_M >> \phi_{Lp} \quad (2-28)$$

$$\bar{\phi} = \frac{1}{N_p}\int V_M \cos\omega t dt = \frac{V_M}{\omega N_p}\sin\omega t \quad Wb \quad (2-29)$$

The Magnetization Current in a Real Transformer

- The magnetization current i_M , which is the current required to produce flux in the transformer core
- The core-loss current i_{h+e} , which is the current required to make up for hysteresis and eddy current losses
- The magnetization current in the transformer is not sinusoidal. The higher frequency components in the magnetization current are due to magnetic saturation in the transformer core
- Once the peak flux reaches the saturation point in the core, a small increase in peak flux requires a very large increase in the peak magnetization current
- The fundamental component of the magnetization current lags the voltage applied to the core by 90^0
- The higher frequency components in the magnetization current can be quite large compared to the fundamental component. In general, the further a transformer core is driven into saturation, the larger the harmonic components will become.
- The core-loss current is nonlinear because of the nonlinear effects of hysteresis
- The fundamental component of the core-loss current is in phase with the voltage applied to the core
- $i_{ex} = i_m + i_{h+e}$ (2-30)

The Current Ratio on a Transformer and the Dot Convention

- What assumptions are required to convert a real transformer into the ideal transformer described previously? They are as follows:
- 1. The core must have no hysteresis or eddy.
- 2. The magnetization curve must have the shape shown in Fig. 2-15. Notice that for an unsaturated core the net magnetomotive force is zero, implying that $N_p i_p = N_s i_s$.
- 3. The leakage flux in the core must be zero, implying that all the flux in the core couples both windings.
- 4. The resistance of the transformer windings must be zero.

$$F_{net} = N_p i_p - N_s i_s \qquad (2-31)$$

$$F_{net} = N_p i_p - N_s i_s = \phi R \qquad (2-32)$$

ideal transformer

$$F_{net} = N_p i_p - N_s i_s \approx 0 \qquad (2-33)$$

$$N_p i_p \approx N_s i_s \qquad (2-34)$$

$$\frac{i_p}{i_s} \approx \frac{N_s}{N_p} = \frac{1}{a} \qquad (2-35)$$

2.5 The Equivalent Circuit of a Transformer

The Exact Equivalent Circuit of a Real Transformer / Approximate Equivalent Circuits of a Transformer / Determining the Values of Components in the Transformer Model

- Copper (I^2R) losses : Copper losses are the resistive heating losses in the primary and secondary windings of the transformer. They are proportional to the square of the current in the windings.
- Eddy current losses : Eddy current losses are resistive heating losses in the core of the transformer. They are proportional to the square of the voltage applied to the transformer.
- Hysteresis losses : Hysteresis losses are associated with the rearrangement of the magnetic domains in the core during each half-cycle, as explained in Chapter 1. They are a complex, nonlinear function of the voltage applied to the transformer.
- Leakage flux : The leakage fluxes which escape the core and pass through only one of the transformer windings are leakage fluxes. These escaped fluxes produce a self-inductance in the primary and secondary coils, and the effects of this inductance must be accounted for.

The Exact Equivalent Circuit of a Real Transformer

$$e_{Lp}(t) = N_p \frac{d\phi_{Lp}}{dt} \qquad (2-36a)$$

$$e_{Ls}(t) = N_s \frac{d\phi_{Ls}}{dt} \qquad (2-36b)$$

$$\phi_{Lp} = \frac{N_p i_p}{R} \qquad (2-37a)$$

$$\phi_{Ls} = \frac{N_s i_s}{R} \qquad (2-37a)$$

$$e_{Lp}(t) = N_p \frac{d\phi_{Lp}}{dt} = \frac{N_p^2}{R} \frac{di_p}{dt} = L_p \frac{di_p}{dt} \qquad (2-38.39a)$$

$$e_{Ls}(t) = N_s \frac{d\phi_{Ls}}{dt} = \frac{N_s^2}{R} \frac{di_s}{dt} = L_s \frac{di_s}{dt} \qquad (2-38.39b)$$

Figure 2-16: The model of a real transformer

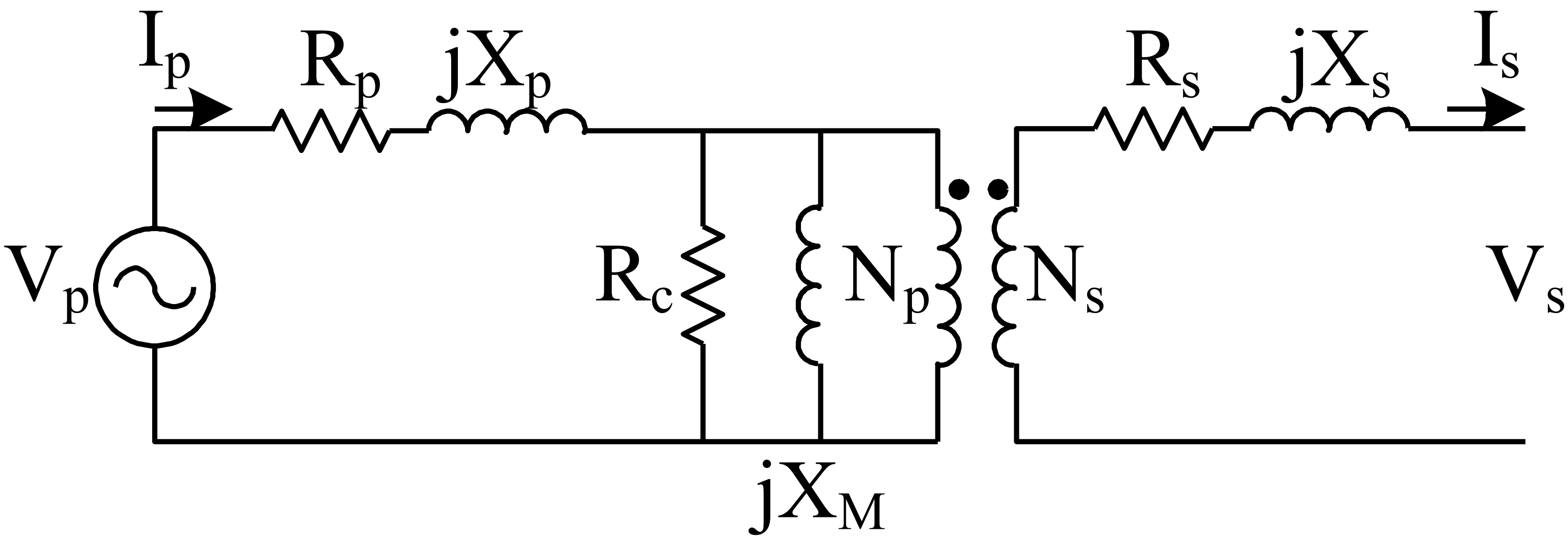


Figure 2-17: (a) The transformer model referred to its primary voltage level. (b) The transformer model referred to its secondary voltage level.

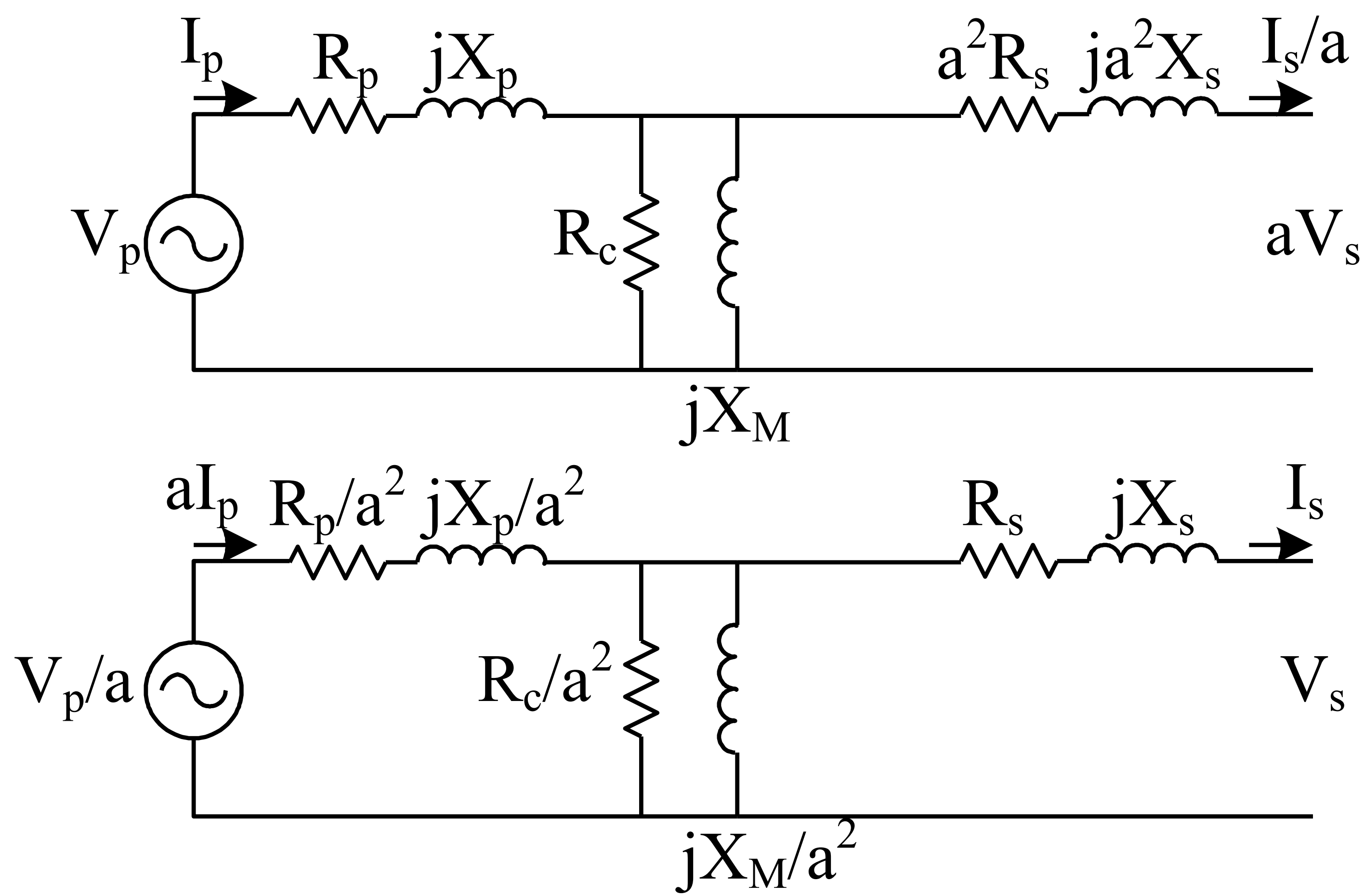
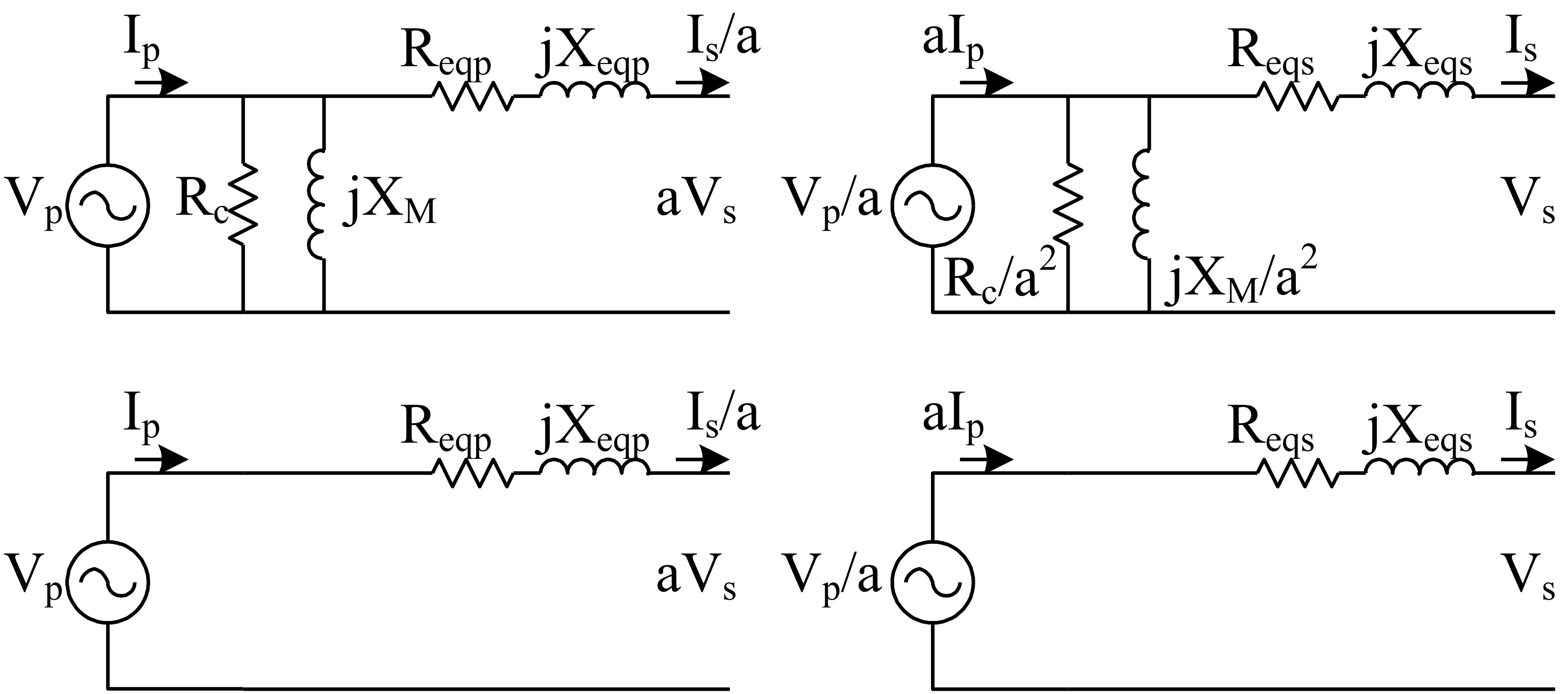


Figure 2-18: Approximate transformer models. (a) Referred to the primary side; (b) referred to the secondary side; (c) with no excitation branch, referred to the primary side; (d) with no excitation branch, referred to the secondary side.



Determining the Values of Components in the Transformer Model

(a) open-circuit test: a transformer's secondary winding is open-circuited, and its primary winding is connected to a full-rated line voltage.

(b) short-circuit test: the secondary terminals of the transformer are short-circuited, and the primary terminals are connected to a fairly low-voltage source.

$$open-circuit\ test$$

$$V_{OC} = I_{OC}\left(\frac{R_C jX_M}{R_C + jX_M}\right) \qquad (2-44)$$

$$\frac{I_{OC}}{V_{OC}}\angle -\theta = \frac{R_C + jX_M}{R_C jX_M} = \frac{1}{R_C} - j\frac{1}{X_M}$$

$$PF = \cos\theta = \frac{P_{OC}}{V_{OC} I_{OC}} \qquad (2-45)$$

$$short-circuit\ test$$

$$|Z_{SE}| = \frac{V_{SC}}{I_{SC}} \qquad (2-48)$$

$$PF = \cos\theta = \frac{P_{SC}}{V_{OC} I_{OC}} \qquad (2-49)$$

$$Z_{SE} = \frac{V_{SC}\angle 0^0}{I_{SC}\angle -\theta^0} = \frac{V_{SC}}{I_{SC}}\angle\theta^0 \qquad (2-51)$$

$$Z_{SE} = R_{eq} + jX_{eq} = (R_P + a^2 R_S) + j(X_P + a^2 X_S) \qquad (2-52)$$

EX2-2: The equivalent impedances of a 20-kVA, 8000/240V, 60Hz transformer are to be determined. The open-circuit test and the short-circuit test were performed on the primary side of the transformer, and the following data were taken (Voc=8000V, Ioc=0.214A, Poc=400W ; Vsc=489V, Isc=2.5A, Psc=240W). Find the impedances of the approximate equivalent circuit referred to the primary side, and sketch the circuit.

$$open-circuit\ test$$

$$PF = \cos\theta = \frac{Poc}{Voc \cdot Ioc} = \frac{400W}{(8000V)(0.214A)} = 0.234\ lagging$$

$$\theta = \cos^{-1} PF = 76.5^0$$

$$\frac{Ioc}{Voc}\angle -\theta = \frac{0.214A}{8000V}\angle -76.5^0 = 0.0000063 - j0.0000261 = \frac{1}{Rc} - j\frac{1}{X_M}$$

$$Rc = 159k\Omega\ ,\ X_M = 38.4k\Omega$$

$$short-circuit\ test$$

$$PF = \cos\theta = \frac{Psc}{Vsc \cdot Isc} = \frac{240W}{(489V)(2.5A)} = 0.196\ lagging$$

$$\theta = \cos^{-1} PF = 78.7^0$$

$$Z_{SE} = \frac{Vsc}{Isc}\angle\theta = \frac{489V}{2.5A}\angle 78.7^0 = 38.4 + j192\Omega$$

$$R_{eq} = 38.4\Omega\ ,\ X_{eq} = 192\Omega$$

2.6 The Per-Unit System of Measurements

$$Quantity\ \ per\ unit = \frac{Actual\ value}{base\ value\ of\ \ quantity} \qquad (2-53)$$

$$P_{base}\ , Q_{base}\ , or\ S_{base} = V_{base} I_{base} \qquad (2-54)$$

$$Z_{base} = \frac{V_{base}}{I_{base}} \qquad (2-55)$$

$$Y_{base} = \frac{I_{base}}{V_{base}} \qquad (2-56)$$

$$Z_{base} = \frac{(V_{base})^2}{S_{base}} \qquad (2-57)$$

$$(P, Q, S)_{pu\ on\ base2} = (P, Q, S)_{pu\ on\ base1} \frac{S_{base1}}{S_{base2}} \qquad (2-58)$$

$$V_{pu\ on\ base2} = V_{pu\ on\ base1} \frac{V_{base1}}{V_{base2}} \qquad (2-59)$$

$$(R, X, Z)_{pu\ on\ base2} = (R, X, Z)_{pu\ on\ base1} \frac{(V_{base1})^2 S_{base2}}{(V_{base2})^2 S_{base1}} \qquad (2-60)$$

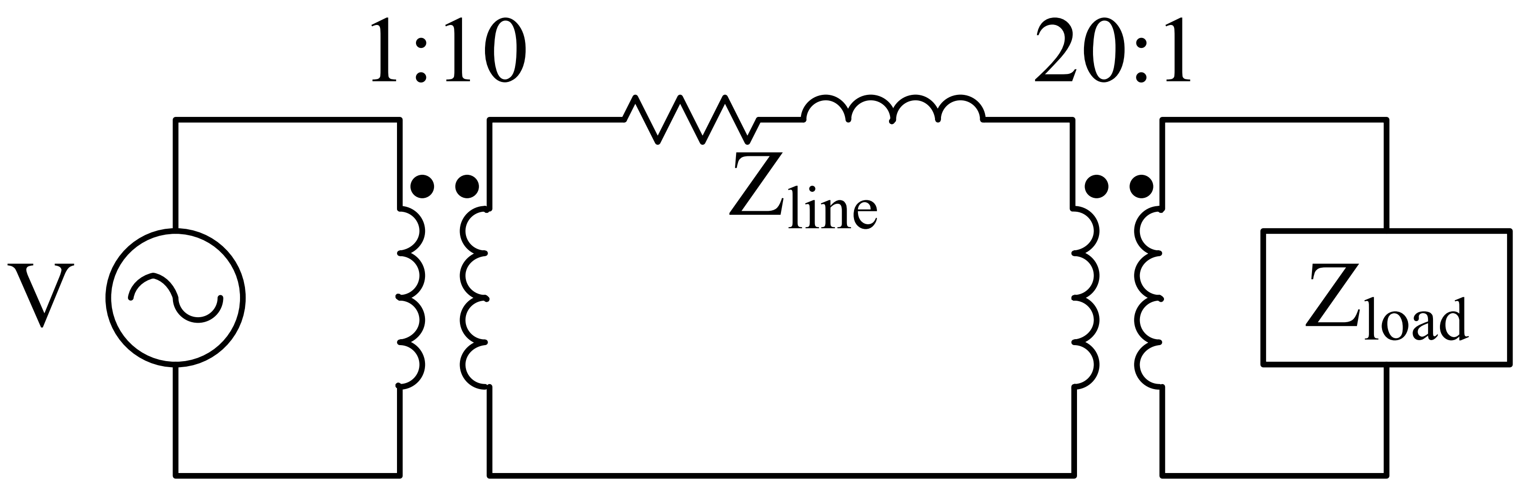
EX2-3. A simple power system is shown in figure. This system contains a $480V(0^0)/60Hz$ generator connected to an ideal 1:10 step-up transformer, a transmission line, an ideal 20:1 step-down transformer, and a load. The impedance of the transmission line is $Z_{line} = 20+j60\Omega$, and $Z_{load} = 8.66+j5\Omega$. The base values for this system are chosen to be 480V and 10kVA at the generator.

(a) Find the base voltage, current, impedance, and apparent power at every point in the power system.

(b) Convert this system to its per-unit equivalent circuit.

(c) Find the power supplied to the load in this system.

(d) Find the power lost in the transmission line



(a) *In the generator region,* $V_{base1} = 480V$ *and* $S_{base} = 10kVA$

$$I_{base1} = \frac{S_{base}}{V_{base1}} = \frac{10000VA}{480V} = 20.83A$$

$$Z_{base1} = \frac{V_{base1}}{I_{base1}} = \frac{480V}{20.83A} = 23.04\Omega$$

In the transmission line region, $V_{base2} = 4800V$ *and* $S_{base} = 10kVA$

$$I_{base2} = \frac{S_{base}}{V_{base2}} = \frac{10000VA}{4800V} = 2.083A$$

$$Z_{base2} = \frac{V_{base2}}{I_{base2}} = \frac{4800V}{2.083A} = 2304\Omega$$

In the load region, $V_{base3} = 240V$ *and* $S_{base} = 10kVA$

$$I_{base3} = \frac{S_{base}}{V_{base3}} = \frac{10000VA}{240V} = 41.67A$$

$$Z_{base3} = \frac{V_{base3}}{I_{base3}} = \frac{4800V}{41.67A} = 5.76\Omega$$

(b) *In the generator region*

$$V_{G,pu} = \frac{V_G}{V_{base1}} = \frac{480\angle 0^0 V}{480V} = 1.0\angle 0^0\ pu$$

In the transmission line region

$$Z_{line,pu} = \frac{Z_{line}}{Z_{base2}} = \frac{20 + j60\Omega}{2304\Omega} = 0.0087 + j0.026\,pu$$

In the load region

$$Z_{load,pu} = \frac{Z_{load}}{Z_{base3}} = \frac{10\angle 30^0\Omega}{5.76\Omega} = 1.736\angle 30^0\ pu = (1.503 + j0.868)\,pu$$

$$(c)\ I_{pu} = \frac{V_{pu}}{Z_{tot,pu}} = \frac{1\angle 0^0}{(0.0087 + j0.026) + (1.736\angle 30^0)} = 0.569\angle -30.6^0\ pu$$

$$P_{load,pu} = I_{pu}^2 R_{load,pu} = (0.569)^2(1.503) = 0.487$$

$$P_{load} = P_{load,pu} S_{base} = 0.487 \times 10000VA = 4870W$$

$$(d)\ P_{line,pu} = I_{pu}^2 R_{line,pu} = (0.569)^2(0.0087) = 0.00282$$

$$P_{line} = P_{line,pu} S_{base} = 0.00282 \times 10000VA = 28.2W$$

2.7 Transformer Voltage Regulation and Efficiency

The Transformer Phasor Diagram / Transformer Efficiency

$$VR = \frac{V_{S,nl} - V_{S,fl}}{V_{S,fl}} \times 100\% \qquad (2-61)$$

$$VR = \frac{V_P / a - V_{S,fl}}{V_{S,fl}} \times 100\% \qquad (2-62)$$

$$VR = \frac{V_{P,pu} - V_{S,fl,pu}}{V_{S,fl,pu}} \times 100\% \qquad (2-63)$$

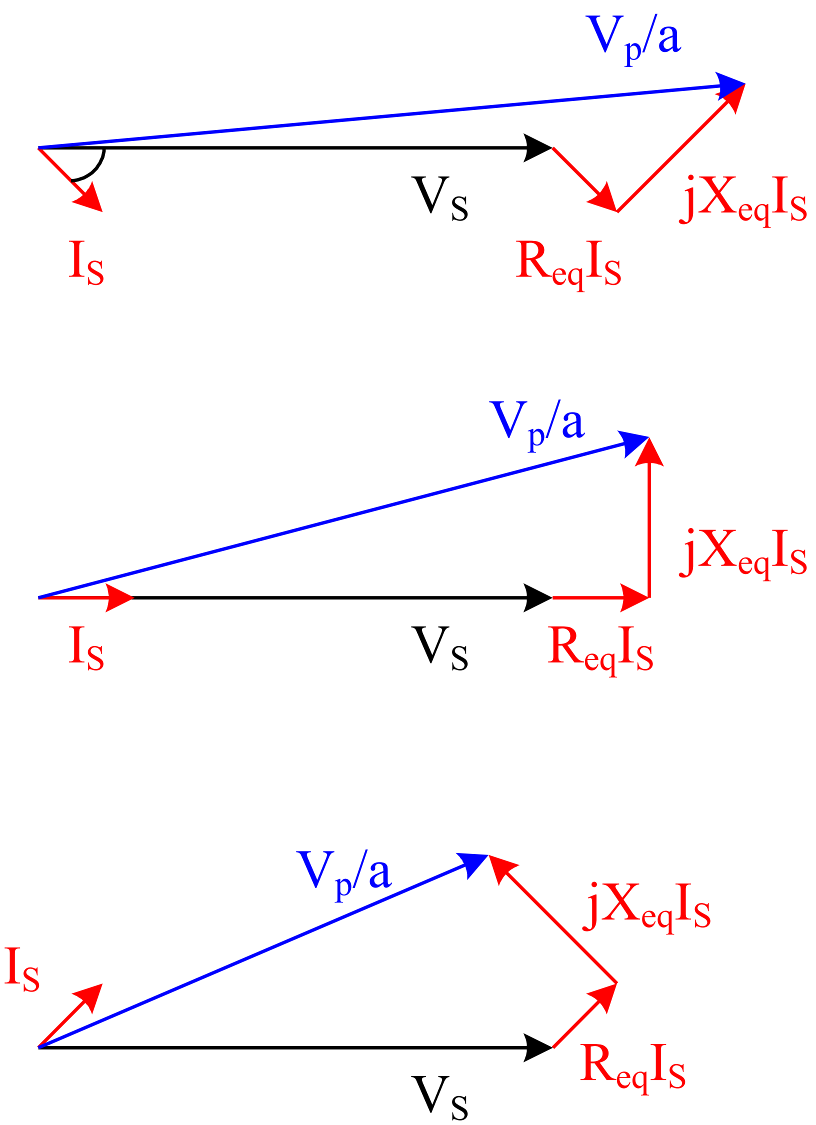
$$\frac{V_P}{a} = V_S + R_{eq} I_S + jX_{eq} I_S \qquad (2-64)$$

$$\eta = \frac{P_{out}}{P_{in}} \times 100\% \qquad (2-65)$$

$$\eta = \frac{P_{out}}{P_{out} + P_{loss}} \times 100\% \qquad (2-66)$$

$$\eta = \frac{V_S I_S \cos\theta}{P_{Cu} + P_{core} + V_S I_S \cos\theta} \times 100\% \qquad (2-67)$$

The Transformer Phasor Diagram



2.8 Transformer Taps Voltage Regulation

EX2-6: A 500-kVA, 13200/480V distribution transformer has four 2.5 percent taps on its primary winding. What are the voltage ratios of this transformer at each tap setting?

Solution:

The five possible voltage ratings of this transformer are

+5.0% tap	13860/480V
+2.5% tap	13530/480V
Nominal rating	13200/480V
-2.5% tap	12870/480V
-5.0% tap	12540/480V

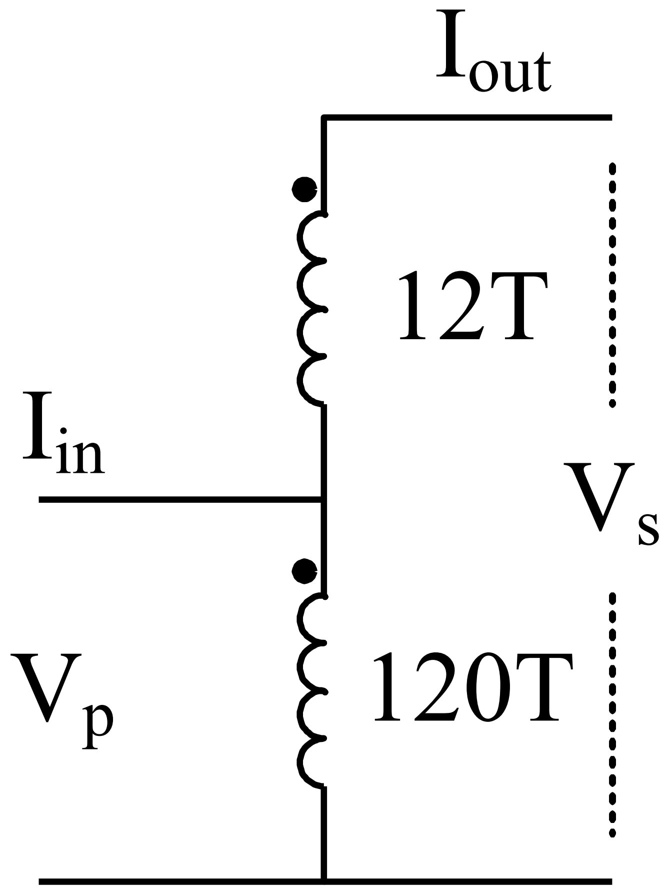
2.9 The Autotransformer

Voltage and Current Relationships in an Autotransformer / The Apparent Power Rating Advantage of Autotransformers / The Internal Impedance of an Autotransformer

$$\frac{V_P}{V_S} = \frac{N_P}{N_S}$$

$$I_P N_P = I_S N_S$$

$$V_P I_P = V_S I_S$$



EX2-7: A 100VA 120/12-V transformer is to be connected so as to form a step-up autotransformer. A primary voltage of 120V is applied to the transformer.

(a) What is the secondary voltage of the transformer?

(b) What is its maximum VA rating in this mode of operation?

(c) Calculate the rating advantage of this autotransformer connection over the transformer's rating in conventional 120/12-V operation.

$$(a)\ V_H = \frac{N_{SE}+N_C}{N_C} V_L = \frac{12+120}{120}\cdot 120V = 132V$$

$$(b)\ I_{SE,\max} = \frac{S_{\max}}{V_{SE}} = \frac{100VA}{12V} = 8.33A$$

$$S_{out} = V_S I_S = V_H I_H = (132V)(8.33A) = 1100VA = S_{in}$$

$$(c)\ \frac{S_{IO}}{S_W} = \frac{1100VA}{100VA} = 11$$

$$\frac{S_{IO}}{S_W} = \frac{N_{SE}+N_C}{N_{SE}} = \frac{12+120}{12} = 11$$

2.10 Three-Phase Transformers

Three-Phase Transformer Connections / The Per-Unit System for Three-Phase Transformers

$$WYE - WYE \ (Y - Y)$$

$$\frac{V_{LP}}{V_{LS}} = \frac{\sqrt{3}\, V_{\phi P}}{\sqrt{3}\, V_{\phi S}} = a \qquad (Y - Y) \qquad (2-87)$$

$$WYE - DELTA \ (Y - \Delta)$$

$$\frac{V_{LP}}{V_{LS}} = \frac{\sqrt{3}\, V_{\phi P}}{V_{\phi S}} = \sqrt{3}\, a \qquad (Y - \Delta) \qquad (2-88)$$

$$DELTA - WYE \ (\Delta - Y)$$

$$\frac{V_{LP}}{V_{LS}} = \frac{V_{\phi P}}{\sqrt{3}\, V_{\phi S}} = \frac{a}{\sqrt{3}} \qquad (\Delta - Y) \qquad (2-89)$$

$$DELTA - DELTA \ (\Delta - \Delta)$$

$$\frac{V_{LP}}{V_{LS}} = \frac{V_{\phi P}}{V_{\phi S}} = a \qquad (\Delta - \Delta) \qquad (2-90)$$

The Per-Unit System for Three-Phase Transformers

$$S_{base} = 3S_{1\phi,base} \qquad (2-91)$$

$$I_{\phi,base} = \frac{S_{1\phi,base}}{V_{\phi,base}} \qquad (2-92a)$$

$$I_{\phi,base} = \frac{S_{base}}{3V_{\phi,base}} \qquad (2-92b)$$

$$Z_{base} = \frac{(V_{\phi,base})^2}{S_{1\phi,base}} \qquad (2-93a)$$

$$Z_{base} = \frac{3(V_{\phi,base})^2}{S_{base}} \qquad (2-93b)$$

$$I_{L,base} = \frac{S_{base}}{\sqrt{3}\, V_{L,base}} \qquad (2-94)$$

EX2-9: A 50-kVA 13800/208 V Δ-Y distribution transformer has a resistance of 1 percent and a reactance of 7 percent per unit.

(a) What is the transformer's phase impedance referred to the high-voltage side?

(b) Calculate this transformer's voltage regulation at full load and 0.8 PF lagging, using the calculated high-side impedance.

(c) Calculate this transformer's voltage regulation under the same conditions, using the per-unit system.

$$Z_{base} = \frac{3(V_{\phi,base})^2}{S_{base}} = \frac{3(13800V)^2}{50000VA} = 11426\Omega \ , \ Z_{eq,pu} = 0.01 + j0.07\,pu$$

$$Z_{eq} = Z_{eq,pu} Z_{base} = (0.01 + j0.07\,pu)(11426\Omega) = 114.2 + j800\Omega$$

$$(b)\ VR = \frac{V_{\phi P} - aV_{\phi S}}{aV_{\phi S}} \times 100\% \, , \ PF = 0.8\ lagging \Rightarrow \theta = 36.87^0, I_\phi = \frac{S}{3V_\phi} = \frac{50000VA}{3(13800V)} = 1.208A$$

$$V_{\phi P} = aV_{\phi S} + R_{eq} I_\phi + jX_{eq} I_\phi$$

$$= 13800\angle 0^0 + (114.2\Omega)(1.208\angle -36.87^0 A) + (j800\Omega)(1.208\angle -36.87^0 A)$$

$$= 14490 + j690.3 = 14506\angle 2.73^0 V$$

$$VR = \frac{V_{\phi P} - aV_{\phi S}}{aV_{\phi S}} \times 100\% = \frac{14506 - 13800}{13800} \times 100\% = 5.1\%$$

$$(c)\ In\ the\ per-unit\ system, V_{o,pu} = 1\angle 0^0, \ I_{o,pu} = 1\angle -36.87^0$$

$$V_P = V_{o,pu} + Z_{eq,pu} I_{o,pu} = 1\angle 0^0 + (0.01 + j0.07\,pu)1\angle -36.87^0 = 1.05 + j0.05 = 1.051\angle 2.73^0$$

$$VR = \frac{1.051 - 1.0}{1.0} \times 100\% = 5.1\%$$

2.11 Three-Phase Transformation Using Two Transformers

The Open-Δ (or V-V) Connection / The Open-Wye-Open-Delta Connection / The Scott-T Connection / The Three-Phase T Connection

$$V_C = -V_A - V_B = -V\angle 0^0 - V\angle -120^0$$
$$= -V - (-0.5V - j0.866V) = -0.5V + j0.866V$$
$$= V\angle 120^0$$

$$P_1 = 3V_\phi I_\phi \cos(150^0 - 120^0) = 3V_\phi I_\phi \cos 30^0$$
$$= \frac{\sqrt{3}}{2} V_\phi I_\phi \qquad (2-96)$$

$$P_2 = 3V_\phi I_\phi \cos(30^0 - 60^0) = 3V_\phi I_\phi \cos(-30^0)$$
$$= \frac{\sqrt{3}}{2} V_\phi I_\phi \qquad (2-97)$$

$$P = P_1 + P_2 = \sqrt{3}\, V_\phi I_\phi \qquad (2-98)$$

$$\frac{P_{open\Delta}}{P_{3\,phase}} = \frac{\sqrt{3}\, V_\phi I_\phi}{3V_\phi I_\phi} = \frac{1}{\sqrt{3}} = 0.577 \qquad (2-99)$$

2.12 Transformer Ratings and Related Problems

The Voltage and Frequency Ratings of a Transformer / The apparent Power Ratings of a Transformer / The Problem of Current Inrush / The Transformer Nameplate

$$v(t) = V_M \sin \omega t \ V$$

$$\phi(t) = \frac{1}{N_P} \int v(t)dt = -\frac{V_M}{\omega N_P} \cos \omega t \qquad (2-100)$$

$$\phi_{\max} = \frac{V_{\max}}{\omega N_P} \qquad (2-101)$$

$$i = \frac{\mathcal{F}}{N_P} \qquad (2-102)$$

$$I_{rms} = \sqrt{\frac{1}{T}\int_0^T i^2 dt} \qquad (2-103)$$

2.13 Instrument Transformers

Two special transformers are used with power systems for taking measurements. One is the potential transformer, and the other is the current transformer.

A potential transformer is a specially wound transformer with a high voltage primary and a low voltage secondary.

Current transformers sample the current in a line and reduce it to a safe and measurable level.

2.14 Summary

- A transformer is a device for converting electric energy at one voltage level to electric energy at another voltage level through the action of a magnetic field. It plays an extremely important role in modern life by making possible the economical long-distance transmission of electric power.
- When a voltage is applied to the primary of a transformer, a flux is produced in the core as given by Faraday's law. The changing flux in the core then induces a voltage in the secondary winding of the transformer. Because transformer cores have very high permeability, the net magnetomotive force required in the core to produce its flux is very small. Since the net magnetomotive force is very small, the primary circuit's magnetomotive force. This fact yields the transformer current ratio.
- A real transformer has leakage fluxes that pass through either the primary or the secondary winding, but not both. In addition there are hysteresis, eddy current, and copper losses. These effects are accounted for in the equivalent circuit of the transformer. Transformer imperfections are measured in a real transformed by its voltage regulation and its efficiency.
- The per-unit system of measurement is a convenient way to study systems containing transformers, because in this system the different system voltage levels disappear. In addition, the per-unit impedances of a transformer expressed to its own ratings base fall within a relatively narrow range, providing a convenient check for reasonableness in problem solutions.
- An autotransformer differs from a regular transformer that the two windings of the autotransformer are connected. The voltage on one side of the transformer is the voltage across a single winding, while the voltage on the other sided of the transformer is the sum of the voltage across both windings. Because only a portion of the power in an autotransformer actually passes through the windings, an autotransformer has a poer rating advantage compared to a regular transformer of equal size. However, the connection destroys the electrical isolation between a transformeer's primary and secondary sides.
- The voltage levels of three-phase circuits can be transformed by a proper combinations of two or three transformers. Potential transformers and current transformers can sample the voltages and current present in a circuit. Both devices are very common in large distributions systems.