

Power System Analysis

供電=用電

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鎖相回路(PLL: Phase Lock Loop)

- 1.三相轉靜止座標(abc Reference Frame to Stationary Reference Frame)
- 2.靜止座標轉同步座標(Stationary Reference Frame to Synchronous Reference Frame)
- 3.靜止座標轉非同步座標(Stationary Reference Frame to Asynchronous Reference Frame)
- 4.鎖相回路(PLL: Phase Lock Loop)
- 5.平衡電壓驟降(Balanced voltage sag)
- 6.不平衡電壓驟降(Unbalanced voltage sag)
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三相轉靜止座標(abc Reference Frame to Stationary Reference Frame)

$$[v_{qs} \ v_{ds}] = [2/3 \ -1/3 \ -1/3; 0 \ -1/\sqrt{3} \ 1/\sqrt{3}][v_a \ v_b \ v_c]$$

$$[v_a \ v_b \ v_c] = [1 \ 0; -1/2 \ -\sqrt{3}/2; -1/2 \ \sqrt{3}/2][v_{qs} \ v_{ds}]$$

3-phase 220V_{rms} (60Hz, line-to-line), $\omega = 2\pi f = 120\pi = 377$ rad/sec.

$v_a + v_b + v_c = 0$, balanced system, ($v_a + v_b + v_c \neq 0$, unbalanced system)

$$v_a = 180 \cos \omega t, \ v_b = 180 \cos (\omega t - 120^\circ), \ v_c = 180 \cos (\omega t + 120^\circ),$$

abc Reference Frame to Stationary Reference Frame

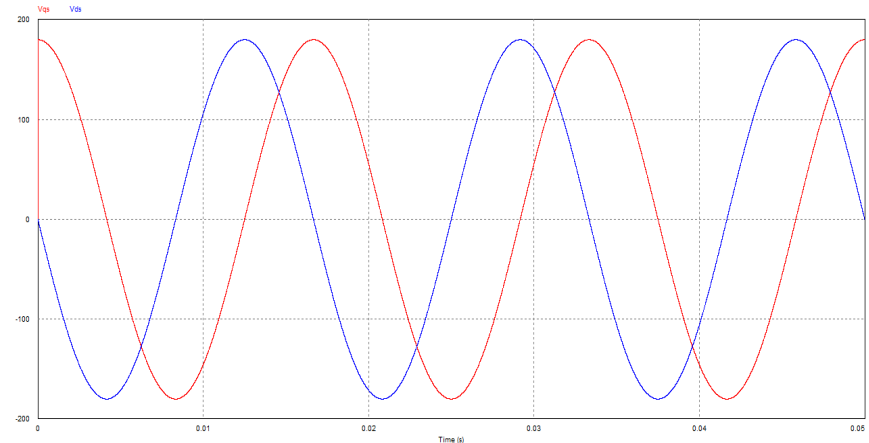
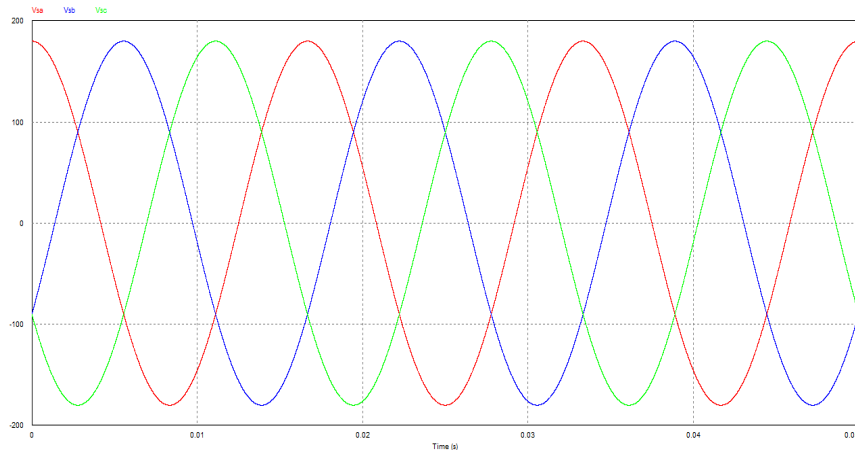
$$v_{qs} = (2/3)v_a + (-1/3)v_b + (-1/3)v_c = (2/3)v_a + (1/3)v_a = v_a = 180 \cos \omega t$$

$$v_{ds} = (-1/\sqrt{3})v_b + (1/\sqrt{3})v_c = (-1/\sqrt{3})[v_a + 2v_b] = -180 \sin \omega t$$

Stationary Reference Frame to abc Reference Frame

$$v_a = v_{qs}, \ v_b = (-1/2)v_{qs} + (-\sqrt{3}/2)v_{ds} = 180 \cos(\omega t - 120^\circ),$$

$$v_c = (-1/2)v_{qs} + (\sqrt{3}/2)v_{ds} = 180 \cos(\omega t + 120^\circ)$$



靜止座標轉同步座標 (Stationary Reference Frame to Synchronous Reference Frame)

$$[v_{qe} \ v_{de}] = [\cos \omega t \ -\sin \omega t; \sin \omega t \ \cos \omega t][v_{qs} \ v_{ds}]$$

$$[v_{qs} \ v_{ds}] = [\cos \omega t \ \sin \omega t; -\sin \omega t \ \cos \omega t][v_{qe} \ v_{de}]$$

3-phase 220V_{rms} (60Hz, line-to-line), $\omega = 2\pi f = 120\pi = 377$ rad/sec.

$$v_{qs} = 180 \cos \omega t, \ v_{ds} = -180 \sin \omega t,$$

Stationary Reference Frame to Synchronous Reference Frame

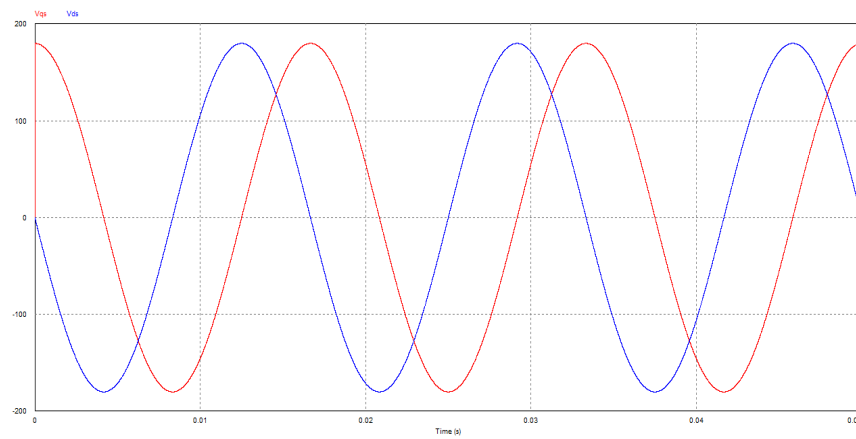
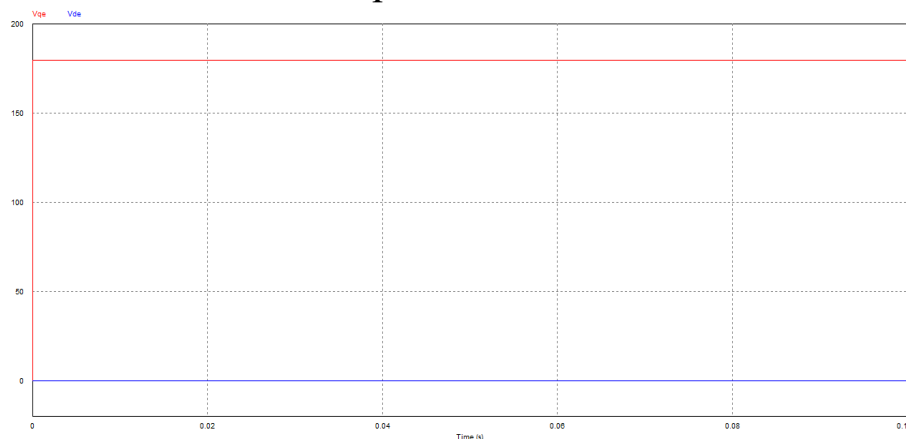
$$v_{qe} = \cos \omega t \ v_{qs} - \sin \omega t \ v_{ds} = 180$$

$$v_{de} = \sin \omega t \ v_{qs} + \cos \omega t \ v_{ds} = 0$$

Synchronous Reference Frame to Stationary Reference Frame

$$v_{qs} = \cos \omega t \ v_{qe} + \sin \omega t \ v_{de} = 180 \cos \omega t,$$

$$v_{ds} = -\sin \omega t \ v_{qe} + \cos \omega t \ v_{de} = -180 \sin \omega t$$



靜止座標轉非同步座標(Stationary Reference Frame to Asynchronous Reference Frame)($\omega_1 \neq \omega$, assume 59Hz, $\omega_1 = 118\pi$ rad/sec.)

$$[v_{qe} \ v_{de}] = [\cos \omega_1 t \ -\sin \omega_1 t; \sin \omega_1 t \ \cos \omega_1 t][v_{qs} \ v_{ds}]$$

$$[v_{qs} \ v_{ds}] = [\cos \omega_1 t \ \sin \omega_1 t; -\sin \omega_1 t \ \cos \omega_1 t][v_{qe} \ v_{de}]$$

3-phase 220V_{rms} (60Hz, line-to-line) , $\omega = 2\pi f = 120\pi = 377$ rad/sec.

$$v_{qs} = 180 \cos \omega t, \ v_{ds} = -180 \sin \omega t,$$

Stationary Reference Frame to Asynchronous Reference Frame

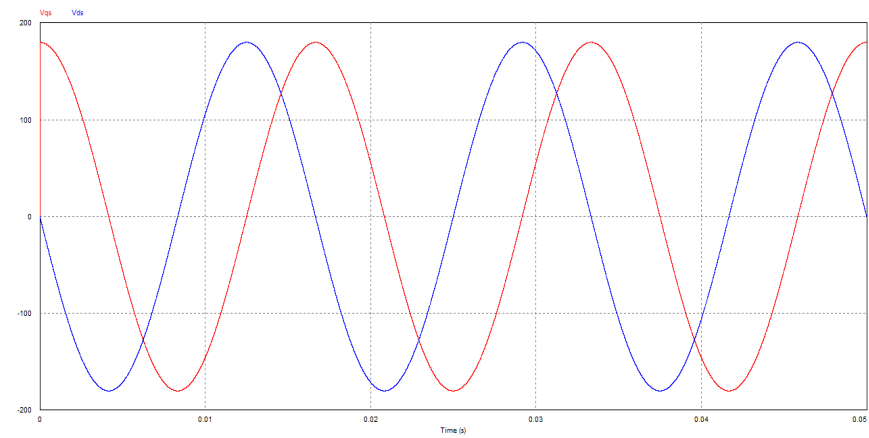
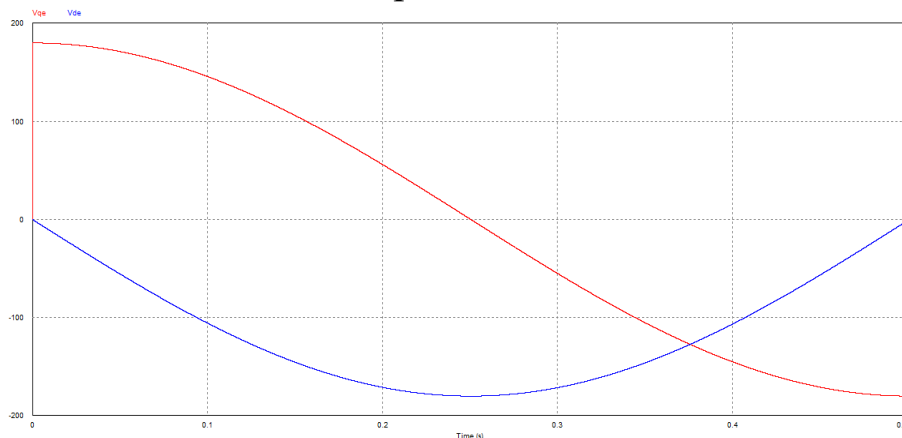
$$v_{qe} = \cos \omega_1 t \ v_{qs} - \sin \omega_1 t \ v_{ds} = 180 \cos (\omega_1 - \omega)t \neq 180$$

$$v_{de} = \sin \omega_1 t \ v_{qs} + \cos \omega_1 t \ v_{ds} = 180 \sin (\omega_1 - \omega)t \neq 0$$

Asynchronous Reference Frame to Stationary Reference Frame

$$v_{qs} = \cos \omega_1 t \ v_{qe} + \sin \omega_1 t \ v_{de} = v_{qs} = 180 \cos \omega t,$$

$$v_{ds} = -\sin \omega_1 t \ v_{qe} + \cos \omega_1 t \ v_{de} = v_{ds} = -180 \sin \omega t$$



鎖相回路(PLL: Phase Lock Loop)($\omega_1 = 314 \text{ rad/sec}$. $\omega_1 \rightarrow \omega$)

$$\begin{bmatrix} v_{qe} & v_{de} \end{bmatrix} = \begin{bmatrix} \cos \omega_1 t & -\sin \omega_1 t \\ \sin \omega_1 t & \cos \omega_1 t \end{bmatrix} \begin{bmatrix} v_{qs} & v_{ds} \end{bmatrix}$$

$$\begin{bmatrix} v_{qs} & v_{ds} \end{bmatrix} = \begin{bmatrix} \cos \omega_1 t & \sin \omega_1 t \\ -\sin \omega_1 t & \cos \omega_1 t \end{bmatrix} \begin{bmatrix} v_{qe} & v_{de} \end{bmatrix}$$

3-phase $220V_{\text{rms}}$ (60Hz, line-to-line), $\omega = 2\pi f = 120\pi = 377 \text{ rad/sec}$.

$$v_{qs} = 180 \cos \omega t, \quad v_{ds} = -180 \sin \omega t,$$

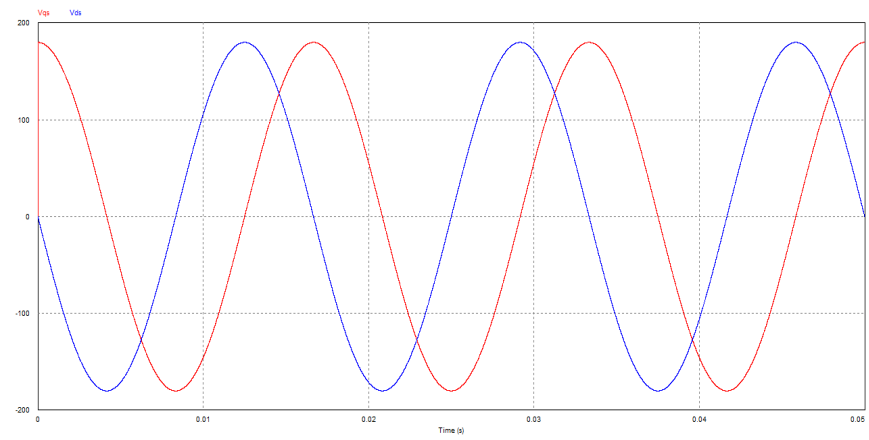
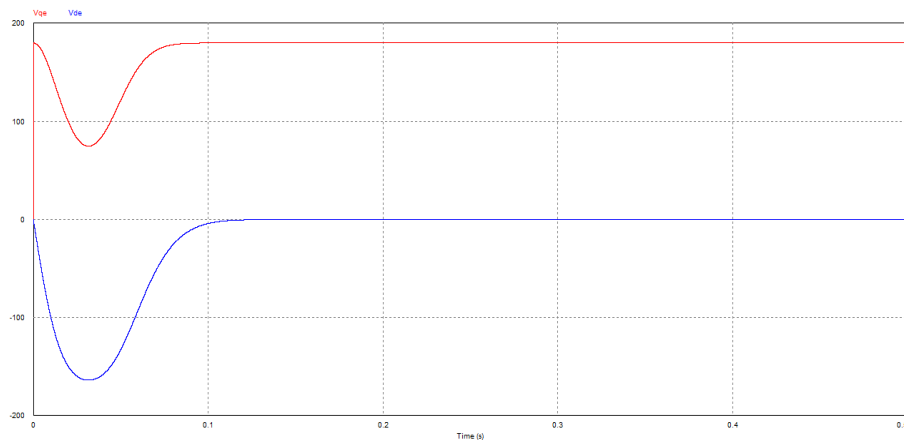
Stationary Reference Frame to Asynchronous Reference Frame

$$v_{qe} = \cos \omega_1 t v_{qs} - \sin \omega_1 t v_{ds} = 180 \cos (\omega_1 - \omega)t \neq 180$$

$$v_{de} = \sin \omega_1 t v_{qs} + \cos \omega_1 t v_{ds} = 180 \sin (\omega_1 - \omega)t \neq 0$$

$$\text{Error} = (0 - v_{de}), \quad \omega_p = P_{\text{gain}} * \text{Error}, \quad \omega_i = \omega_i + I_{\text{gain}} * \text{error}, \quad \omega_1 = \omega_1 + \omega_p + \omega_i$$

PLL: Phase Lock Loop $\omega_1 \rightarrow \omega$



平衡電壓驟降(Balanced voltage sag)(with PLL)

$$[v_{qs} \ v_{ds}] = [2/3 \ -1/3 \ -1/3; 0 \ -1/\sqrt{3} \ 1/\sqrt{3}][v_a \ v_b \ v_c]$$

$$[v_{qe} \ v_{de}] = [\cos \omega t \ -\sin \omega t; \sin \omega t \ \cos \omega t][v_{qs} \ v_{ds}]$$

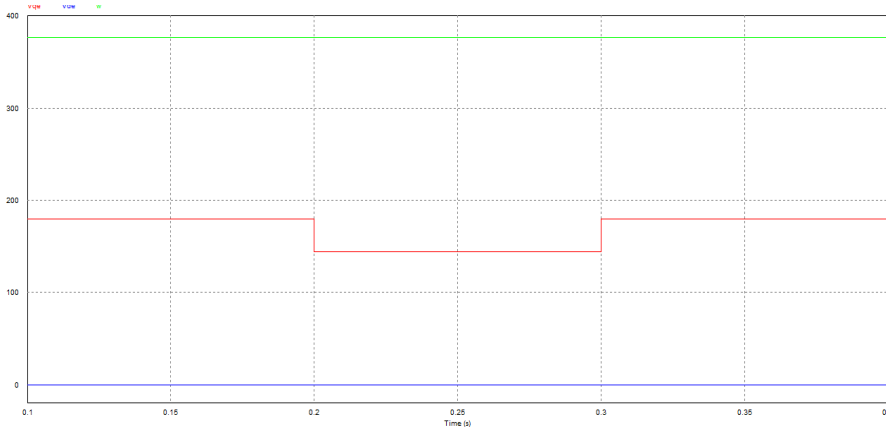
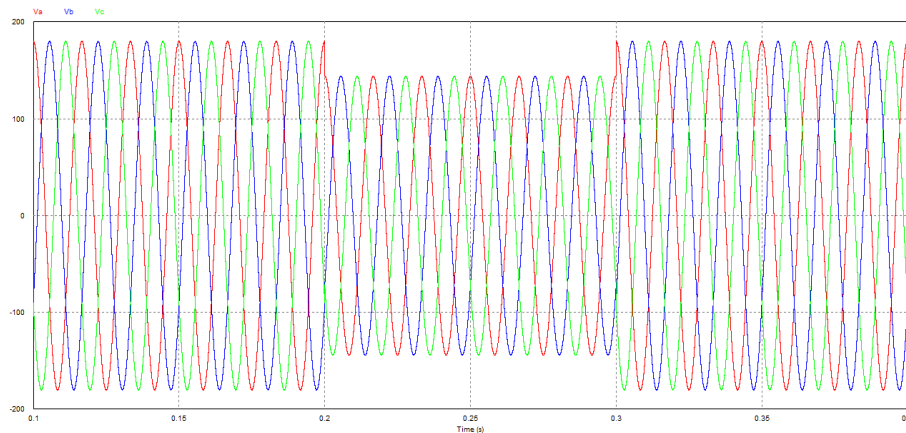
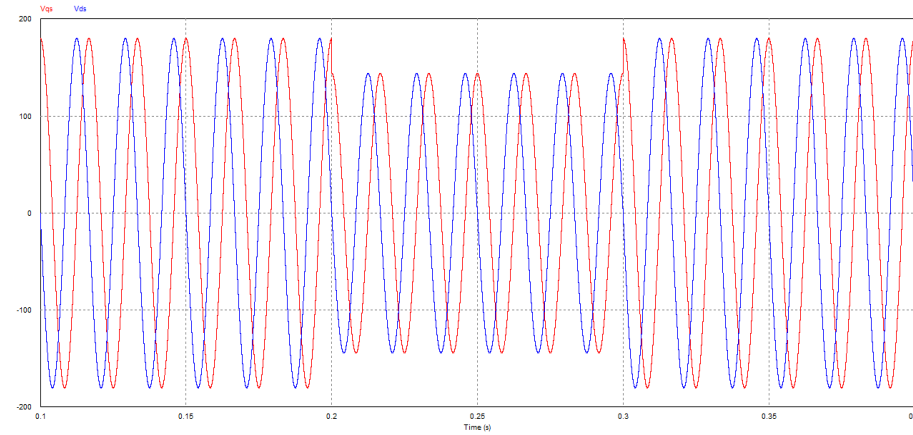
3-phase 220V_{rms} (60Hz, line-to-line) , $\omega = 2\pi f = 120\pi = 377$ rad/sec.

Balanced voltage sag at $t = 0.2 \sim 0.3$, $0.8 * 220V_{rms}$, (180V*0.8)

$$v_a = 180 \cos \omega t ,$$

$$v_b = 180 \cos (\omega t - 120^\circ) ,$$

$$v_c = 180 \cos (\omega t + 120^\circ) ,$$



不平衡電壓驟降1(Unbalanced voltage sag)(with PLL)

$$[v_{qs} \ v_{ds}] = [2/3 \ -1/3 \ -1/3; 0 \ -1/\sqrt{3} \ 1/\sqrt{3}][v_a \ v_b \ v_c]$$

$$[v_{qe} \ v_{de}] = [\cos \omega t \ -\sin \omega t; \sin \omega t \ \cos \omega t][v_{qs} \ v_{ds}]$$

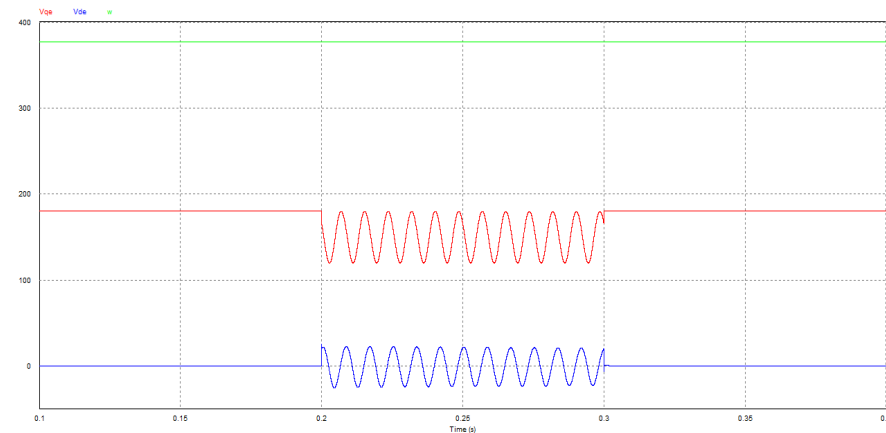
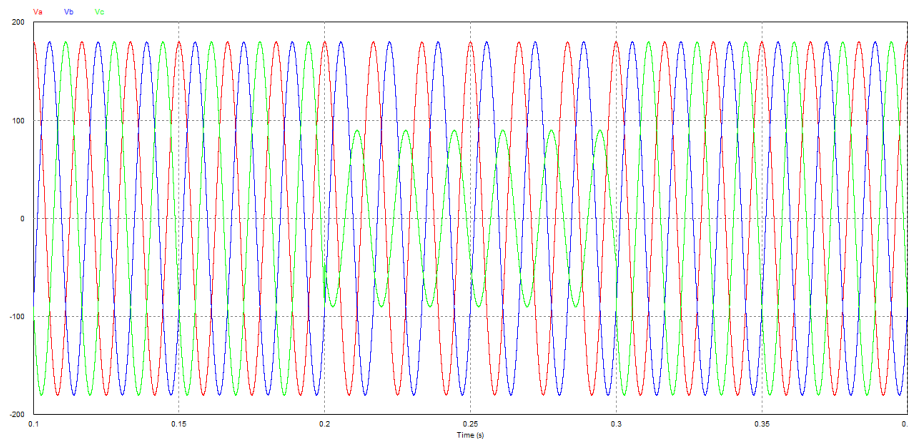
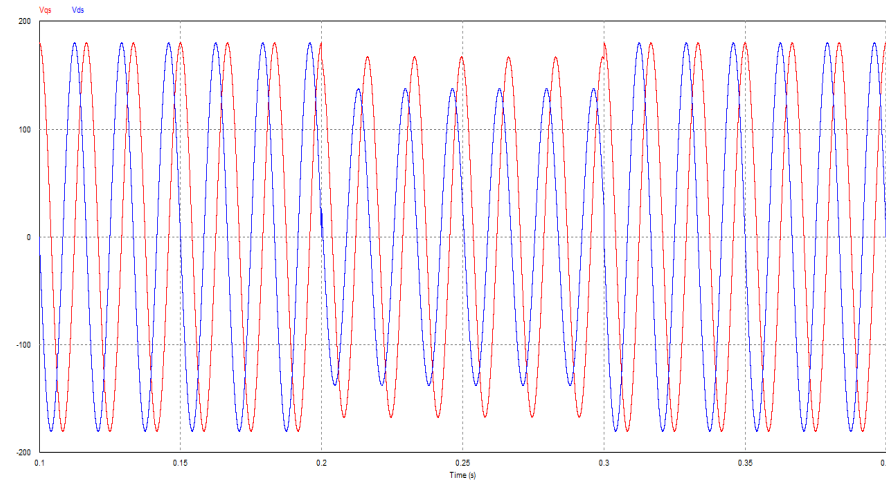
3-phase 220V_{rms} (60Hz, line-to-line) , $\omega = 2\pi f = 120\pi = 377$ rad/sec.

Unbalanced voltage sag at $t = 0.2 \sim 0.3$, $|v_c| = 180V * 0.5$

$$v_a = 180 \cos \omega t ,$$

$$v_b = 180 \cos (\omega t - 120^\circ) ,$$

$$v_c = 180 \cos (\omega t + 120^\circ) , (180 * 0.5)$$



不平衡電壓驟降2(Unbalanced voltage sag)(with PLL)

$$[v_{qs} \ v_{ds}] = [2/3 \ -1/3 \ -1/3; 0 \ -1/\sqrt{3} \ 1/\sqrt{3}][v_a \ v_b \ v_c]$$

$$[v_{qe} \ v_{de}] = [\cos \omega t \ -\sin \omega t; \sin \omega t \ \cos \omega t][v_{qs} \ v_{ds}]$$

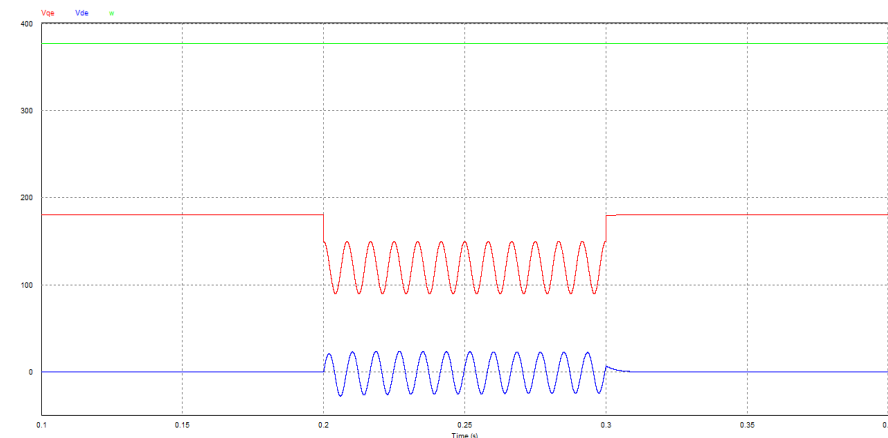
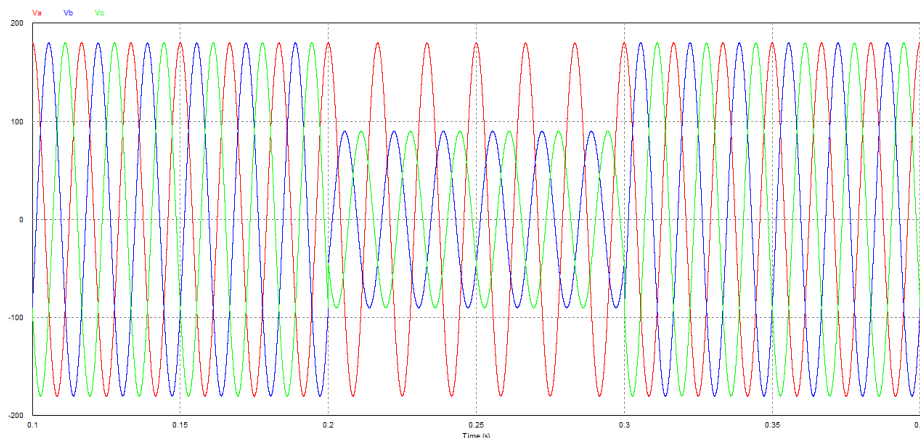
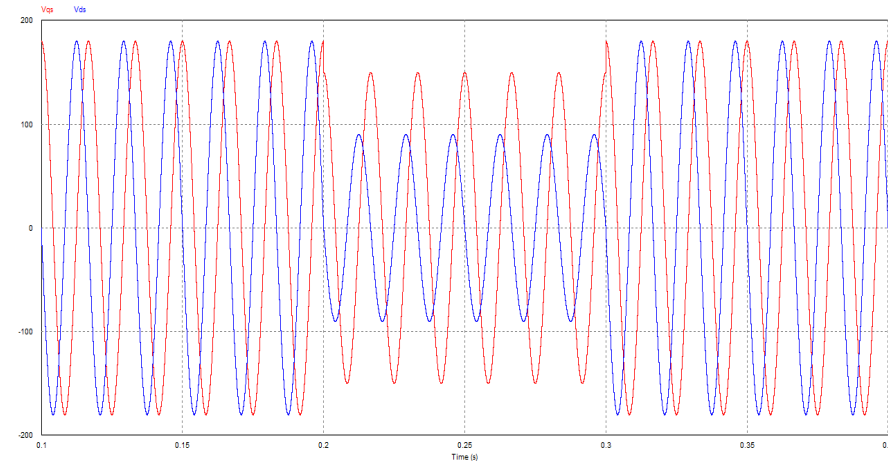
3-phase 220V_{rms} (60Hz, line-to-line), $\omega = 2\pi f = 120\pi = 377 \text{ rad/sec}$.

Unbalanced voltage sag at $t = 0.2 \sim 0.3$, $|v_b| = 180V * 0.5$, $|v_c| = 180V * 0.5$

$$v_a = 180 \cos \omega t,$$

$$v_b = 180 \cos (\omega t - 120^\circ), (180 * 0.5)$$

$$v_c = 180 \cos (\omega t + 120^\circ), (180 * 0.5)$$



不平衡電壓驟降3(Unbalanced voltage sag)(with PLL)

$$[v_{qs} \ v_{ds}] = [2/3 \ -1/3 \ -1/3; 0 \ -1/\sqrt{3} \ 1/\sqrt{3}][v_a \ v_b \ v_c]$$

$$[v_{qe} \ v_{de}] = [\cos \omega t \ -\sin \omega t; \sin \omega t \ \cos \omega t][v_{qs} \ v_{ds}]$$

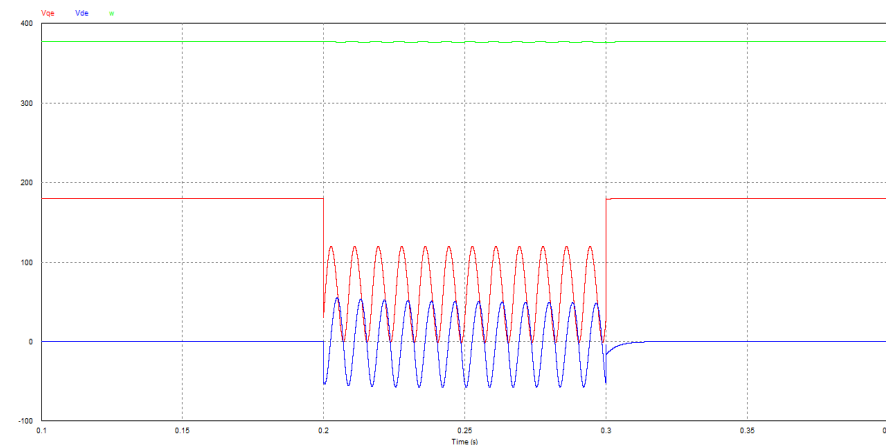
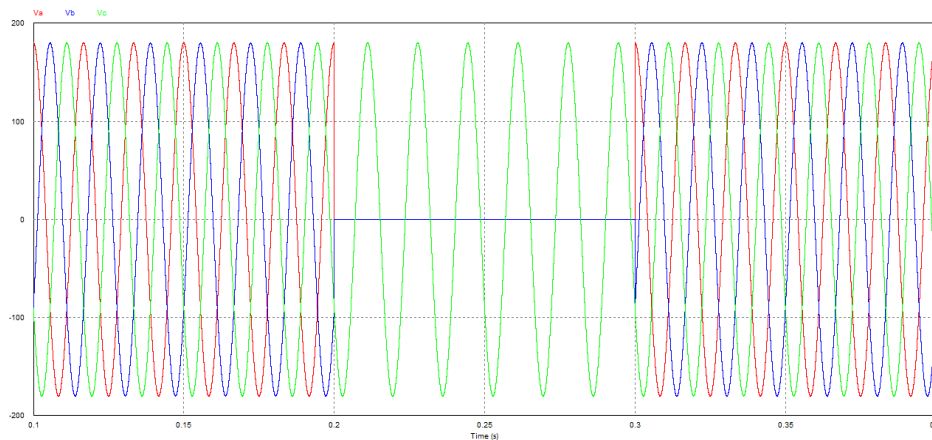
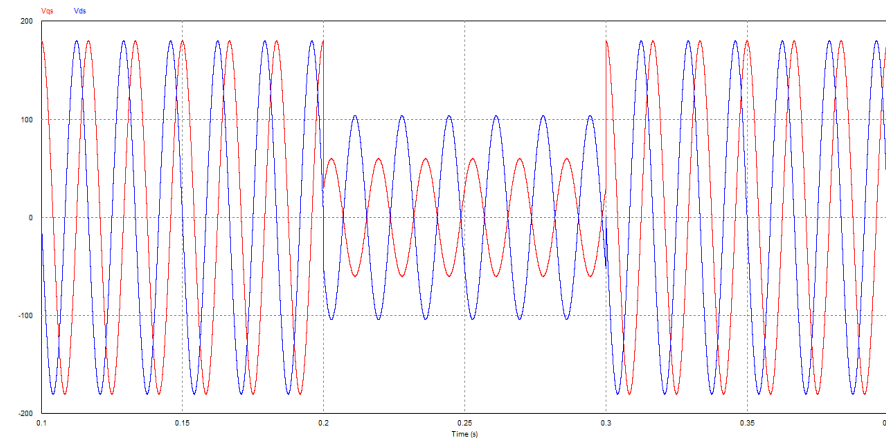
3-phase 220V_{rms} (60Hz, line-to-line), $\omega = 2\pi f = 120\pi = 377 \text{ rad/sec}$.

Unbalanced voltage sag at $t = 0.2 \sim 0.3$, $|v_a| = 0$, $|v_b| = 0$

$$v_a = 180 \cos \omega t, \ v_a = 0$$

$$v_b = 180 \cos (\omega t - 120^\circ), \ v_b = 0$$

$$v_c = 180 \cos (\omega t + 120^\circ),$$



停電(Power Failure)(PLL is fail and fixed at 377 rad/sec.)

$$[v_{qs} \ v_{ds}] = [2/3 \ -1/3 \ -1/3; 0 \ -1/\sqrt{3} \ 1/\sqrt{3}][v_a \ v_b \ v_c]$$

$$[v_{qe} \ v_{de}] = [\cos \omega t \ -\sin \omega t; \sin \omega t \ \cos \omega t][v_{qs} \ v_{ds}]$$

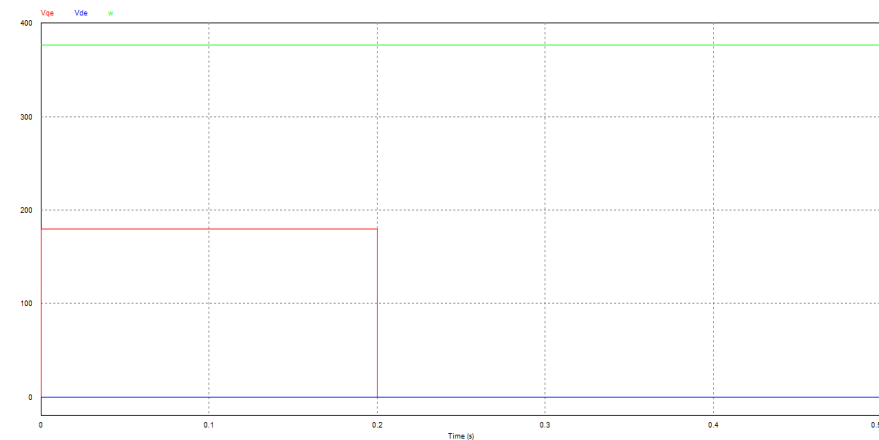
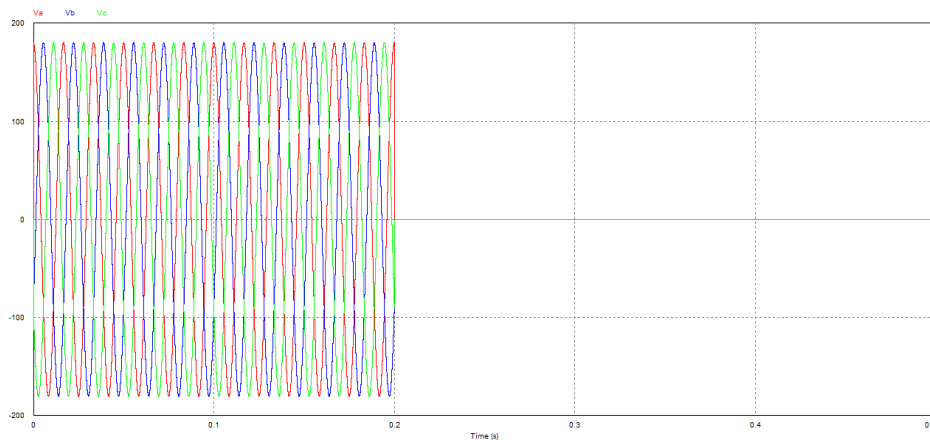
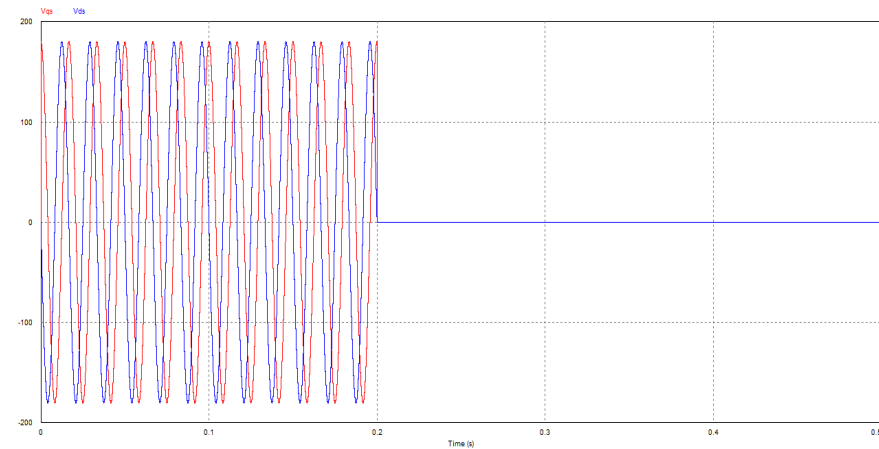
3-phase 220V_{rms} (60Hz, line-to-line), $\omega = 2\pi f = 120\pi = 377 \text{ rad/sec.}$

Unbalanced voltage sag at $t = 0.2 \sim 0.5$, $|v_a| = 0$, $|v_b| = 0$, $|v_c| = 0$

$$v_a = 180 \cos \omega t, \ v_a = 0$$

$$v_b = 180 \cos (\omega t - 120^\circ), \ v_b = 0$$

$$v_c = 180 \cos (\omega t + 120^\circ), \ v_c = 0$$



相序錯誤(phase abc -> acb)(PLL is fail)

$$[v_{qs} \ v_{ds}] = [2/3 \ -1/3 \ -1/3; 0 \ -1/\sqrt{3} \ 1/\sqrt{3}][v_a \ v_b \ v_c]$$

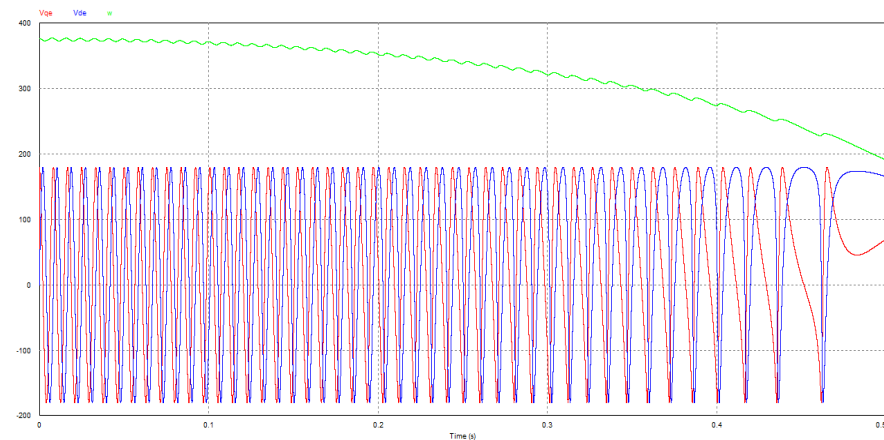
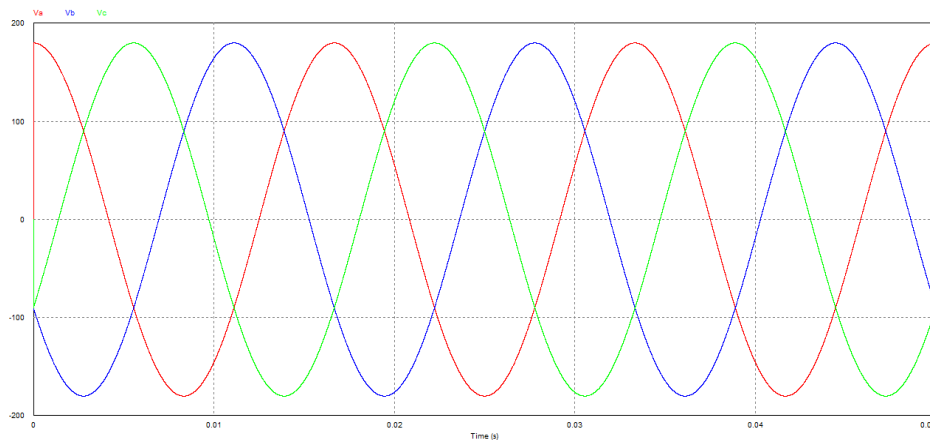
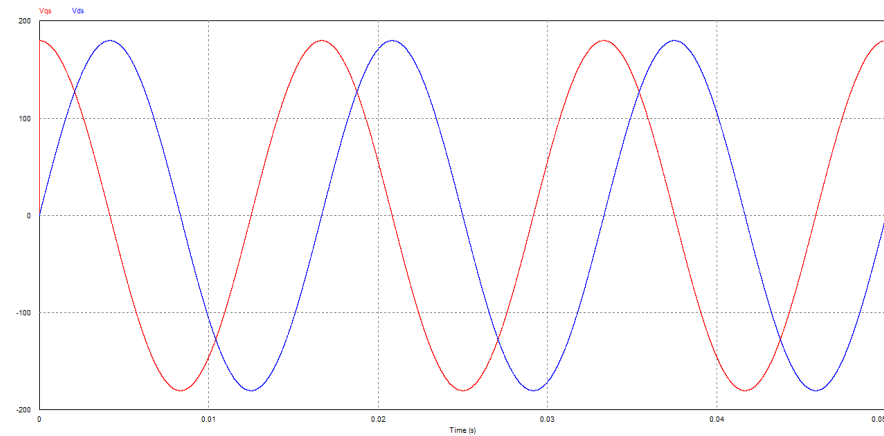
$$[v_{qe} \ v_{de}] = [\cos \omega t \ -\sin \omega t; \sin \omega t \ \cos \omega t][v_{qs} \ v_{ds}]$$

3-phase 220V_{rms} (60Hz, line-to-line), $\omega = 2\pi f = 120\pi = 377 \text{ rad/sec}$.

$$v_a = 180 \cos \omega t,$$

$$v_c = 180 \cos (\omega t - 120^\circ),$$

$$v_b = 180 \cos (\omega t + 120^\circ),$$



帕克變換(Park Transformation) $\theta = \omega t$, $[v_0 \ v_d \ v_q] = P[v_a \ v_b \ v_c]$

$$[v_{qs} \ v_{ds}] = [2/3 \ -1/3 \ -1/3; 0 \ -1/\sqrt{3} \ 1/\sqrt{3}][v_a \ v_b \ v_c]$$

$$[v_{qe} \ v_{de}] = [\cos\theta \ -\sin\theta; \sin\theta \ \cos\theta][v_{qs} \ v_{ds}]$$

$$[v_{qe} \ v_{de}] = (2/3) [\cos\theta \ \cos(\theta-2\pi/3) \ \cos(\theta+2\pi/3); \sin\theta \ \sin(\theta-2\pi/3) \ \sin(\theta+2\pi/3)] [v_a \ v_b \ v_c]$$

$$v_0 = (1/3)(v_a + v_b + v_c), \quad v_0 = (1/\sqrt{3})(v_a + v_b + v_c)$$

$$[v_0 \ v_d \ v_q] = (2/3)^{0.5} \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2}; \\ \cos\theta & \cos(\theta-2\pi/3) & \cos(\theta+2\pi/3); \\ \sin\theta & \sin(\theta-2\pi/3) & \sin(\theta+2\pi/3) \end{bmatrix} [v_a \ v_b \ v_c]$$

$$P^{-1} = P^T = (2/3)^{0.5} \begin{bmatrix} 1/\sqrt{2} & \cos\theta & \sin\theta; \\ 1/\sqrt{2} & \cos(\theta-2\pi/3) & \sin(\theta-2\pi/3); \\ 1/\sqrt{2} & \cos(\theta+2\pi/3) & \sin(\theta+2\pi/3) \end{bmatrix}$$

To simplify the equations and in some important cases obtain **linear time-invariant equations**, we use a **Park** transformation (also called a **Blondel** transformation, or **0dq** transformation to **rotor** coordinates) of the **stator** abc quantities. We transform abc voltages, current, and flux linkages.

對稱分量(Symmetrical Components)

Represent V_a , V_b , and V_c in terms of nine symmetrical components , zero sequence set ($V_a^0 = V_b^0 = V_c^0$), positive (abc) sequence set, and negative (acb) sequence set. ($V_a^0, V_a^+, V_a^-, V_b^0, V_b^+, V_b^-, V_c^0, V_c^+, V_c^-$)

$$V_a = V_a^0 + V_a^+ + V_a^-, \quad V_b = V_b^0 + V_b^+ + V_b^-, \quad V_c = V_c^0 + V_c^+ + V_c^-,$$

Define $\alpha = e^{j2\pi/3} = 1 \angle 120^\circ$, $\alpha^2 = e^{j4\pi/3} = 1 \angle 240^\circ = 1 \angle -120^\circ$, $\alpha^* = \alpha^2$, $1 + \alpha + \alpha^2 = 0$

$$[V_a \ V_b \ V_c] = A [V_a^0 \ V_a^+ \ V_a^-], \quad A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha^2 & \alpha \\ 1 & \alpha & \alpha^2 \end{bmatrix}$$

A is symmetrical components transformation matrix.

$$A^{-1} = (1/3) \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix}, \quad [V_a^0 \ V_a^+ \ V_a^-] = A^{-1} [V_a \ V_b \ V_c],$$

Balanced Positive Sequence $V_a = 1 \angle 0^\circ$, $V_b = 1 \angle -120^\circ$, $V_c = 1 \angle 120^\circ$,

$$[V_a^0 \ V_a^+ \ V_a^-] = A^{-1} [V_a \ V_b \ V_c] = \begin{bmatrix} 0 & 1 \angle 0^\circ & 0 \end{bmatrix}$$

Balanced Negative Sequence $V_a = 1 \angle 0^\circ$, $V_b = 1 \angle 120^\circ$, $V_c = 1 \angle -120^\circ$,

$$[V_a^0 \ V_a^+ \ V_a^-] = A^{-1} [V_a \ V_b \ V_c] = \begin{bmatrix} 0 & 0 & 1 \angle 0^\circ \end{bmatrix}$$

Ex13.1 Finding Symmetrical Components ($\theta = \omega t$)

$$V_a = 1 \angle 0^\circ, V_b = 1 \angle -90^\circ, V_c = 2 \angle 135^\circ,$$

$$\text{Define } \alpha = e^{j2\pi/3} = 1 \angle 120^\circ, \alpha^2 = e^{j4\pi/3} = 1 \angle 240^\circ = 1 \angle -120^\circ$$

$$A^{-1} = (1/3) [1 \ 1 \ 1; 1 \ \alpha \ \alpha^2; 1 \ \alpha^2 \ \alpha], [V_a^0 \ V_a^+ \ V_a^-] = A^{-1} [V_a \ V_b \ V_c],$$

$$[V_a^0 \ V_a^+ \ V_a^-] = [0.195 \angle 135^\circ \quad 1.311 \angle 15^\circ \quad 0.494 \angle -105^\circ]$$

$$V_a^0 = 0.195 \angle 135^\circ = 0.195 \cos(\omega t + 135^\circ)$$

$$\text{Zero Sequence: } V_a^0 = V_b^0 = V_c^0 = 0.195 \angle 135^\circ$$

$$V_a^+ = 1.311 \angle 15^\circ = 1.311 \cos(\omega t + 15^\circ)$$

$$\text{Positive Sequence: } 1.311 \angle 15^\circ, 1.311 \angle (15^\circ - 120^\circ), 1.311 \angle (15^\circ + 120^\circ)$$

$$V_a^- = 0.494 \angle -105^\circ = 0.494 \cos(\omega t - 105^\circ)$$

$$\text{Negative Sequence: } 0.494 \angle -105^\circ, 0.494 \angle (-105^\circ + 120^\circ), 0.494 \angle (-105^\circ - 120^\circ)$$

$$V_a = 1 \angle 0^\circ = V_a^0 + V_a^+ + V_a^- = 0.195 \angle 135^\circ + 1.311 \angle 15^\circ + 0.494 \angle -105^\circ$$

$$\begin{aligned} V_b &= 1 \angle -90^\circ = V_b^0 + V_b^+ + V_b^- = V_a^0 + \alpha^2 V_a^+ + \alpha V_a^- \\ &= 0.195 \angle 135^\circ + 1.311 \angle (15^\circ - 120^\circ) + 0.494 \angle (-105^\circ + 120^\circ) \end{aligned}$$

$$V_c = 2 \angle 135^\circ = V_c^0 + V_c^+ + V_c^- = V_a^0 + \alpha V_a^+ + \alpha^2 V_a^-$$

Ex13.1 $V_a = 1\angle 0^\circ$, $V_b = 1\angle -90^\circ$, $V_c = 2\angle 135^\circ$, (PLL is fail)

$$[v_{qs} \ v_{ds}] = [2/3 \ -1/3 \ -1/3; 0 \ -1/\sqrt{3} \ 1/\sqrt{3}][v_a \ v_b \ v_c]$$

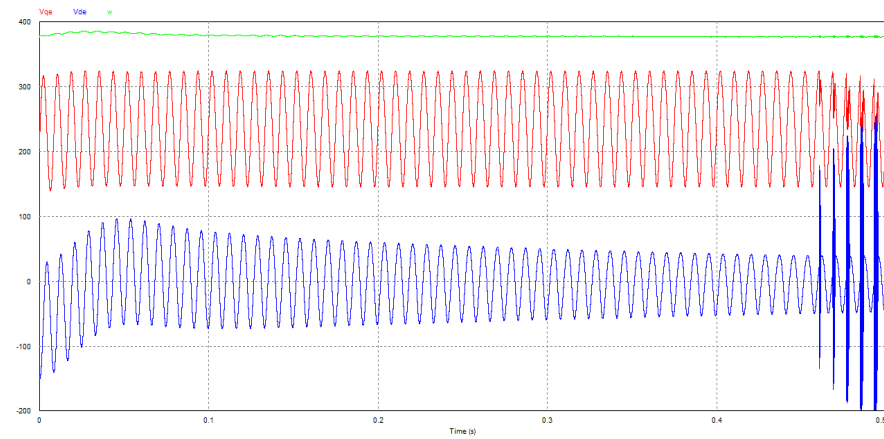
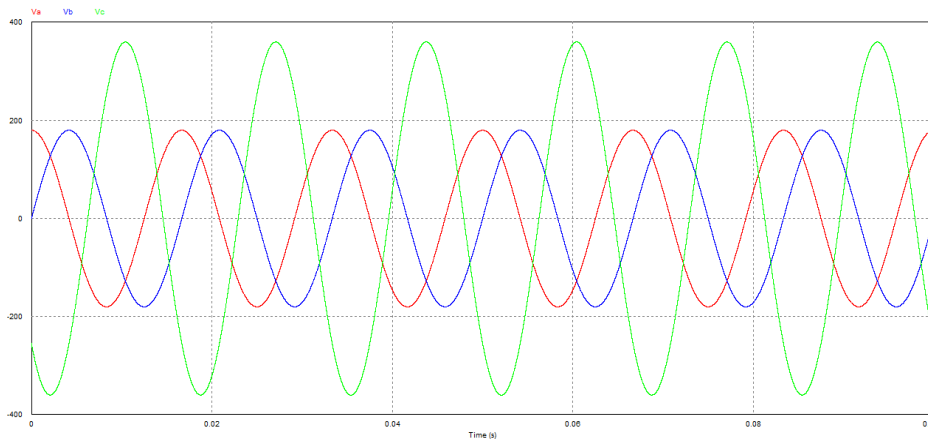
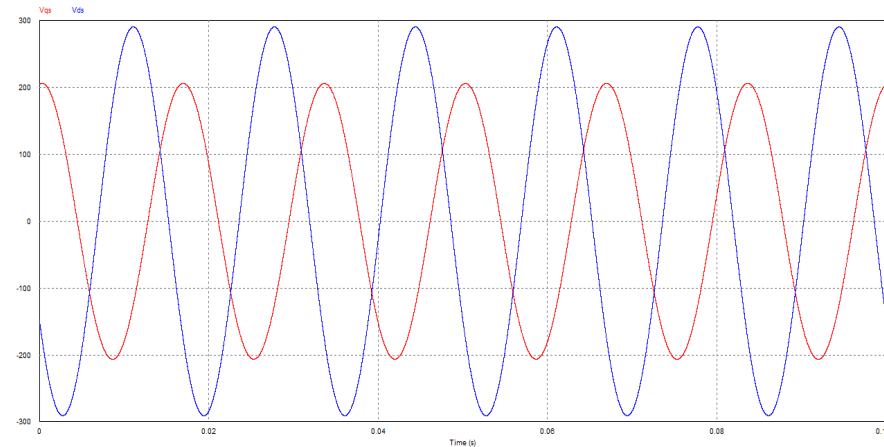
$$[v_{qe} \ v_{de}] = [\cos \omega t \ -\sin \omega t; \sin \omega t \ \cos \omega t][v_{qs} \ v_{ds}]$$

3-phase 220V_{rms} (60Hz, line-to-line) , $\omega = 2\pi f = 120\pi = 377 \text{ rad/sec}$.

$$v_a = 180 \cos \omega t ,$$

$$v_b = 180 \cos (\omega t - 90^\circ) ,$$

$$v_c = 2 * 180 \cos (\omega t + 135^\circ) ,$$

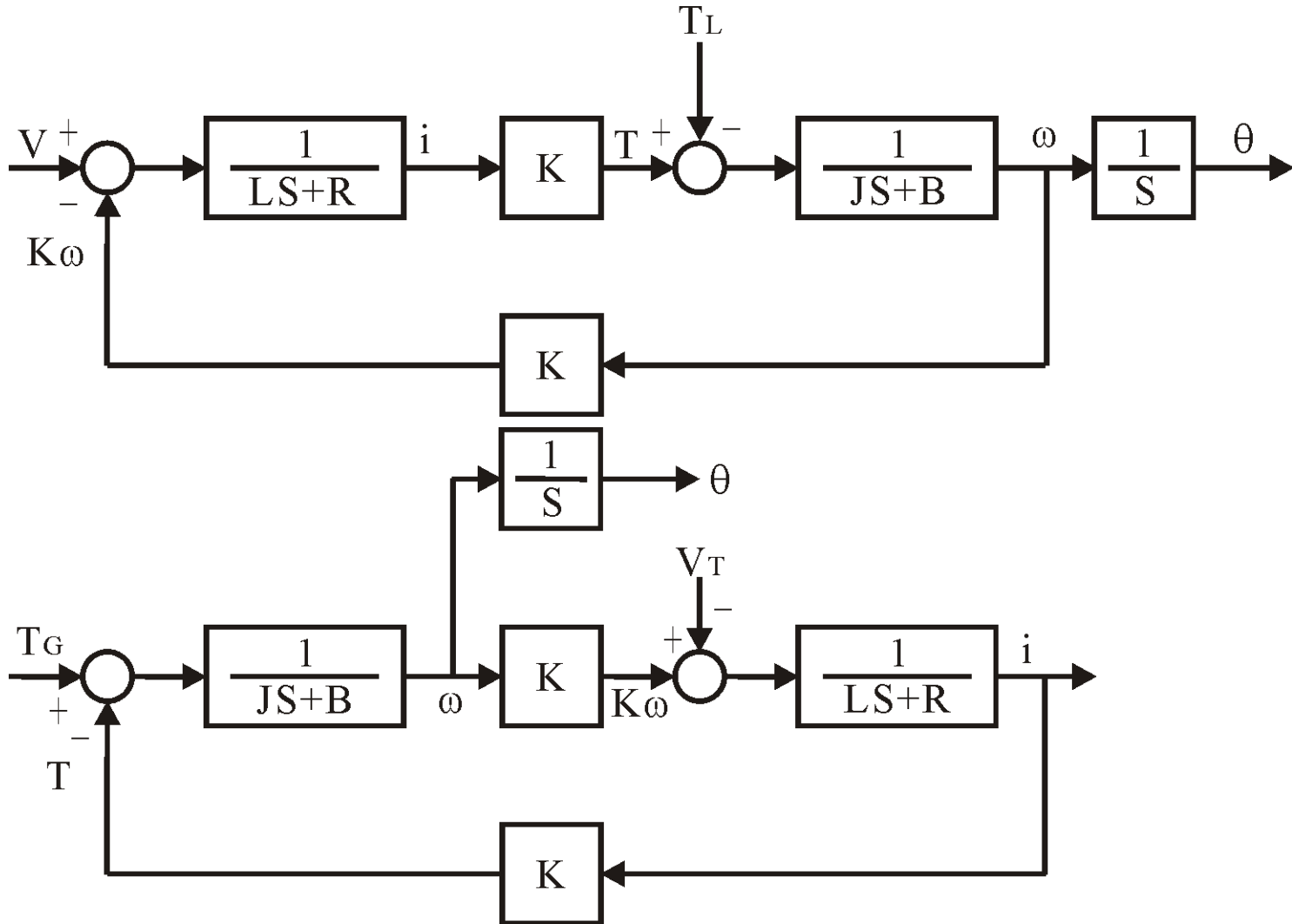


馬達與發電機(Motor and generator)

$$V = L \frac{di}{dt} + iR + K\omega = (LS + R)i + K\omega; K\omega = V_T + L \frac{di}{dt} + iR = V_T + (LS + R)i$$

Induced Torque: $T = Ki$; Induced Voltage: $K\omega$

$$T - T_L = J \frac{d\omega}{dt} + B\omega = (JS + B)\omega ; T_G - T = J \frac{d\omega}{dt} + B\omega = (JS + B)\omega$$

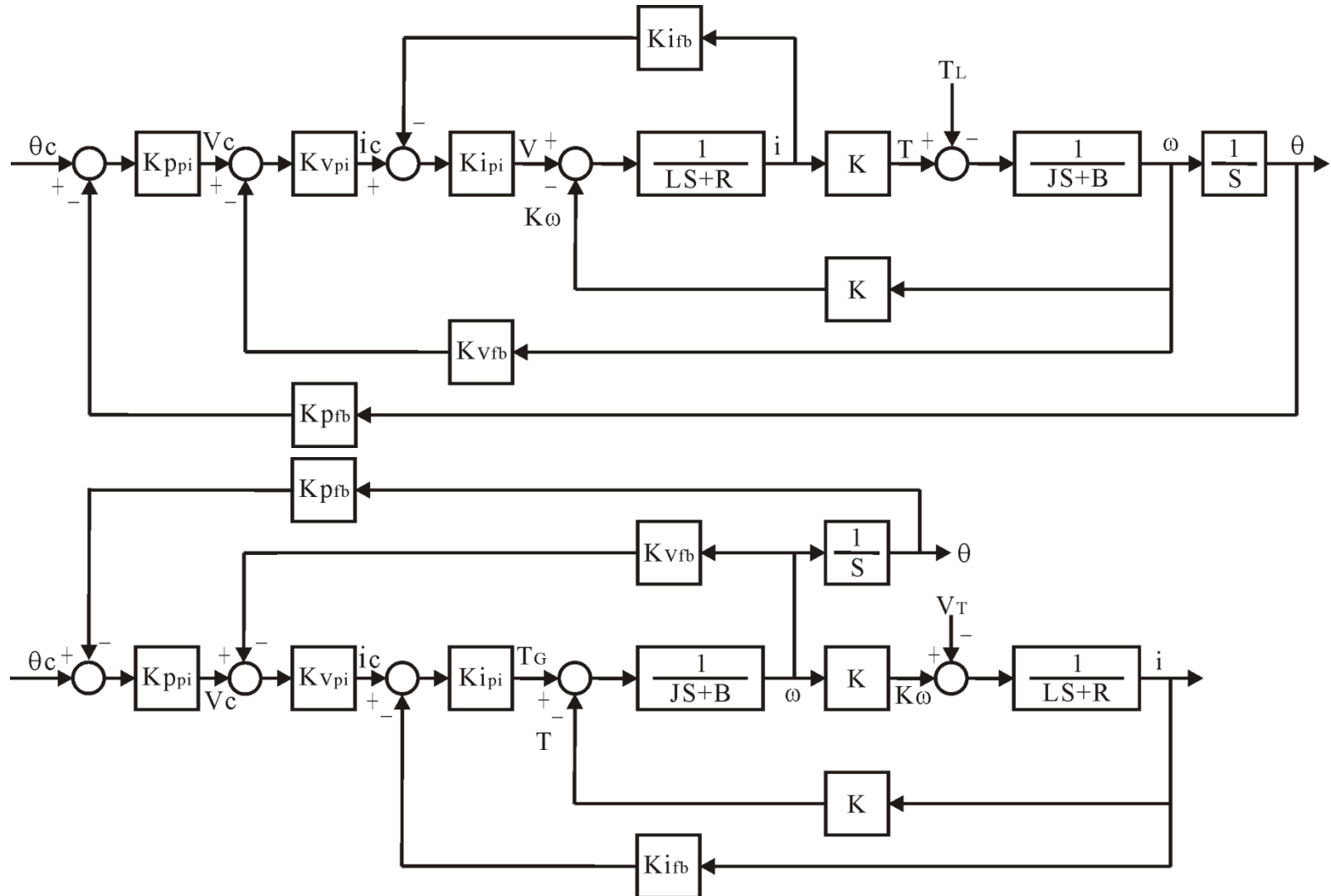


馬達與發電機控制(Control of motor and generator)

$$V = L \frac{di}{dt} + iR + K\omega = (LS + R)i + K\omega; K\omega = V_T + L \frac{di}{dt} + iR = V_T + (LS + R)i$$

Torque equation: $T = Ki$; Induced Voltage: $K\omega$

$$T - T_L = J \frac{d\omega}{dt} + B\omega = (JS + B)\omega ; T_G - T = J \frac{d\omega}{dt} + B\omega = (JS + B)\omega$$



重疊定理(Superposition Principle)

The **superposition principle** states that the voltage across (or current through) an element in a **linear circuit** is the algebraic **sum** of the voltage across (or current through) that element due to each **independent source** acting alone.

We consider **one independent source at a time** while all other independent sources are turned off. This implies that we replace every voltage source by 0V (or a **short circuit**), and every current source by 0A (or an **open circuit**). This way we obtain a simpler and more manageable circuit.

Dependent sources are left intact because they are controlled by circuit variables.

Step1: Turn off all independent sources except one source. Find the output (voltage or current) due to that active source using the Kirchhoff's Voltage and Current Laws (KVL and KCL).

Step2: Repeat step 1 for each of the other independent sources.

Step3: Find the total contribution by adding algebraically all the contributions due to the independent sources.