老師說會考題：

已知(2.25)式，$V_1=|V_1|e^{j\theta_1}$，$V_2=|V_2|e^{j\theta_2}$

$$Z=R+j\omega L=|Z|e^{j\angle Z},\quad \theta_{12}=\theta_1-\theta_2$$

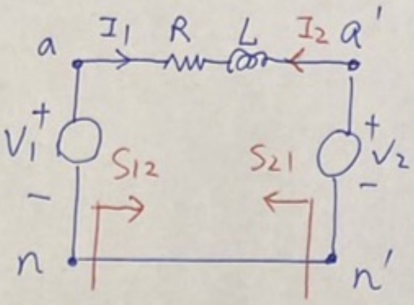

圖 2.14 單相電路

可推導(2.26) $S_{12}=V_1\cdot I_1^*$，因為 $I_1=\dfrac{V_1-V_2}{Z}$

$$S_{12}=V_1\cdot\left(\frac{V_1-V_2}{Z}\right)^*=\frac{V_1\cdot V_1^*-V_1\cdot V_2^*}{Z^*}$$

$$=\frac{|V_1|^2-|V_1|\cdot|V_2|e^{j\theta_{12}}}{|Z|\cdot e^{-j\angle Z}}=\frac{|V_1|^2}{|Z|}\cdot e^{j\angle Z}-\frac{|V_1||V_2|}{|Z|}e^{j\angle Z}\cdot e^{j\theta_{12}}$$

可推導(2.27) $S_{21}=V_2\cdot I_2^*$，因為 $I_2=\dfrac{V_2-V_1}{Z}$

$$S_{21}=V_2\cdot\left(\frac{V_2-V_1}{Z}\right)^*=\frac{|V_2|^2}{|Z|}\cdot e^{j\angle Z}-\frac{|V_2||V_1|}{|Z|}\cdot e^{j\angle Z}\cdot e^{-j\theta_{12}}$$

所以(2.28) $-S_{21}=-\dfrac{|V_2|^2}{|Z|}\cdot e^{j\angle Z}+\dfrac{|V_2||V_1|}{|Z|}\cdot e^{j\angle Z}\cdot e^{-j\theta_{12}}$

(2.29) $S_{12}=C_1-Be^{j\theta_{12}}$，(2.30) $-S_{21}=C_2+Be^{-j\theta_{12}}$

送端圓圓心 $C_1=\dfrac{|V_1|^2}{|Z|}e^{j\angle Z}$，受端圓圓心 $C_2=-\dfrac{|V_2|^2}{|Z|}e^{j\angle Z}$，半徑 $B=\dfrac{|V_1||V_2|}{|Z|}e^{j\angle Z}$

簡化，假設 $R=0$，$Z=j\omega L=jX \Rightarrow |Z|=X$，$\angle Z=90^\circ$，代入(2.26)和(2.27)(2.28)

(2.26) $S_{12}=\dfrac{|V_1|^2}{|Z|}\cdot(\cos 90^\circ+j\sin 90^\circ)-\dfrac{|V_1||V_2|}{|Z|}\cdot(\cos 90^\circ+j\sin 90^\circ)\cdot(\cos\theta_{12}+j\sin\theta_{12})$

$$=j\frac{|V_1|^2}{X}-j\frac{|V_1||V_2|}{X}\cos\theta_{12}+\frac{|V_1||V_2|}{X}\sin\theta_{12}=P_{12}+jQ_{12}$$

所以實功 $P_{12}=\dfrac{|V_1||V_2|}{X}\sin\theta_{12}$ (2.31)，$Q_{12}=\dfrac{|V_1|^2}{X}-\dfrac{|V_1||V_2|}{X}\cos\theta_{12}$ (2.32)

(2.27) $S_{21}=\dfrac{|V_2|^2}{|Z|}\cdot(\cos 90^\circ+j\sin 90^\circ)-\dfrac{|V_2||V_1|}{|Z|}\cdot(\cos 90^\circ+j\sin 90^\circ)\cdot(\cos(-\theta_{12})+j\sin(-\theta_{12}))$

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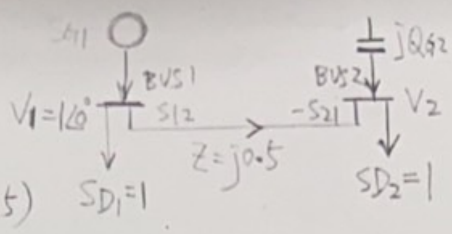
所以實功 $P_{21}=-\dfrac{|V_1||V_2|}{X}\sin\theta_{12}=-P_{12}$

$$Q_{21}=\frac{|V_2|^2}{X}-\frac{|V_2||V_1|}{X}\cos\theta_{12}\qquad(2.33)$$

考古：

例 2.14 如圖系統中所有的值都是單相值



(a) 選取 Q_{G2} 使得 $|V_2|=1$ (b) 此時 $\angle V_2$ 是多少？

當 Bus1 和 Bus2 的傳輸線上只有電抗 $Z=j0.5$ 時，$(X=0.5)$

(2.31) 實功 $P_{12}=-P_{21}=\dfrac{|V_1||V_2|}{X}\sin\theta_{12}$，因為流進 Bus2 的實功等於流出的實功

所以 $P_{12}=SD_2=1=\dfrac{1\times 1}{0.5}\sin\theta_{12} \Rightarrow \sin\theta_{12}=0.5 \Rightarrow \theta_{12}=30° \Rightarrow \angle V_2=-30°$

(2.32) $Q_{12}=\dfrac{|V_1|^2}{X}-\dfrac{|V_1||V_2|}{X}\cos\theta_{12}$

(2.33) $Q_{21}=\dfrac{|V_2|^2}{X}-\dfrac{|V_1||V_2|}{X}\cos\theta_{12}$

因為流進 Bus2 的虛功等於流出的虛功

所以 $Q_{G2}=Q_{21}=\dfrac{|V_2|^2}{X}-\dfrac{|V_1||V_2|}{X}\cos\theta_{12}$ （已知 $|V_1|=1$, $|V_2|=1$, $X=0.5$, $\theta_{12}=30°$）

$=2-2\cos 30°=0.268$，$Q_{G2}>0$ 相當於電容器電源

(c) 如果 $Q_{G2}=0$，能供應負載 SD_2 嗎？(d) 如果能，V_2 又是多少？

已知 $|V_1|=1$，$X=0.5$，未知 $V_2=?$，$\theta_{12}=?$

由 (2.31) $P_{12}=-P_{21}=\dfrac{|V_1||V_2|}{X}\sin\theta_{12}=2|V_2|\sin\theta_{12}=SD_2=1$ —— ①

由 (2.33) $Q_{G2}=0=Q_{21}=\dfrac{|V_2|^2}{X}-\dfrac{|V_1||V_2|}{X}\cos\theta_{12}=2|V_2|^2-2|V_2|\cos\theta_{12}$ —②

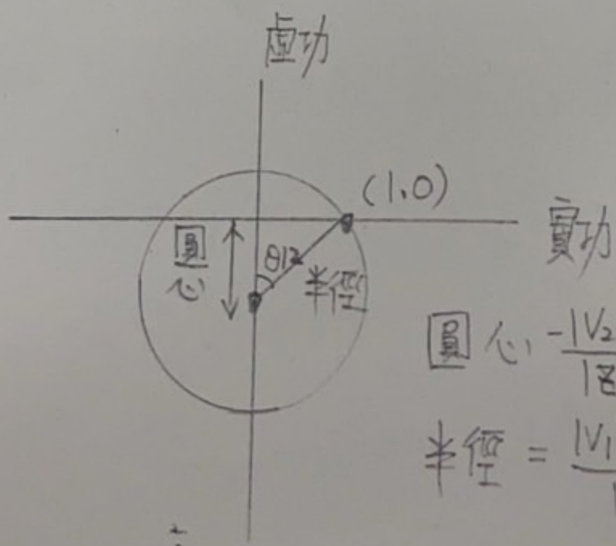
由①可得 $\sin\theta_{12}=\dfrac{1}{2|V_2|}$，由②可得 $\cos\theta_{12}=|V_2|$

所以 $\sin\theta_{12}=\dfrac{1}{2\cos\theta_{12}} \Rightarrow 2\sin\theta_{12}\cos\theta_{12}=1 \Rightarrow \sin 2\theta_{12}=1$

$\Rightarrow 2\theta_{12}=90° \Rightarrow \theta_{12}=45°$

$\Rightarrow |V_2|=\cos\theta_{12}=\dfrac{1}{\sqrt{2}}$

$\Rightarrow V_2=0.707\angle -45°$



圓心 $\dfrac{-|V_2|^2}{|Z|}=-2|V_2|^2$，絕對值 $2|V_2|^2$

半徑 $=\dfrac{|V_1||V_2|}{|Z|}=2|V_2|$

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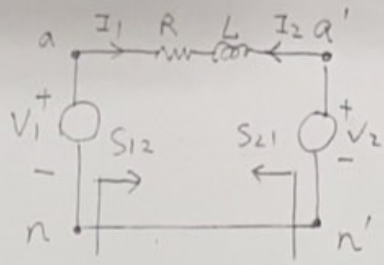

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