

輸电線的勢权(被动元件)

R(电阻五) 能量損失

L(电感H) 有華線 就有电感

C(略F)

$$k = \ell\left(\frac{k}{A}\right) = \frac{k}{MA_{\text{and}}}$$

- ○無限長电線的磁通鏈
- @無限長电線的磁通鏈(多導体)
- ③ a相.b相 c相 互擾(單-導体)
- ® a相.b相 c相 互擾 (每相多導体)
- © 特位(\$摹体互播) : 先求 r'= re<sup>-</sup>华 再求 Rame = ?

$$\lambda = \lambda_{2}^{h(h)} + \lambda_{1}^{h(h)} = \frac{u_{0}i}{2\pi}$$

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## 3.0 简介(Introduction)

A line is characterized by four distributed parameters: series resistance, series inductance, shunt conductance, and shunt capacitance (assuming without mutual inductance and capacitance).

Resistance:  $R=\rho(l/A)$ ,  $\rho$  (resistivity: 電阻率Ωm), length: l, crosssectional area: A. (Conductor, Semiconductor, and Insulator)

Silver:  $\rho = 1.64 \times 10^{-8} \Omega \text{m}$ ,

Copper:  $\rho = 1.72 \times 10^{-8} \Omega \text{m}$ ,  $R = 1.72 \times 10^{-8} \times 1 \text{km} / \pi (0.01 \text{m})^2 = 0.0547 \Omega$ 

Aluminum:  $\rho = 2.8 \times 10^{-8} \Omega \text{m}$ ,

Gold:  $\rho = 2.45 \times 10^{-8} \Omega \text{m}$ ,

Carbon:  $\rho = 4x10^{-5}$  Ωm, Germanium (绪):  $\rho = 4.7x10^{-1}$  Ωm, Silicon:  $\rho = 6.4x10^2$  Ωm,

Paper:  $\rho = 10^{10}$  Ωm, Mica (雲母):  $\rho = 5 \times 10^{11}$  Ωm, Glass:  $\rho = 10^{12}$  Ωm, Teflon:  $\rho = 3 \times 10^{12}$  Ωm, Air:  $\rho = 1.5 \times 10^{14}$  Ωm,

## 集膚效應(Skin Effect)

In high frequency applications the current in a good conductor tends to shift to the surface of the conductor (due to the skin effect).

Resistance:  $R=\rho(l/A)$ ,  $\rho$  (resistivity: 電阻率Ωm),  $\sigma=1/\rho$  (conductivity: 導電率 $1/\Omega$ m = $A/(V\times m)$ )

Skin Depth, $\delta=1/(\pi f\mu\sigma)^{0.5}$  ,  $\mu_o=4\pi\times 10^{-7}$  (H/m)(permeability: 導磁係數)

Silver:  $\sigma = 6.17 \times 10^7$ ,  $\delta = 8.27 \text{mm}$  (at 60Hz),  $\delta = 0.064 \text{mm}$  (at 1MHz)

Copper:  $\sigma = 5.8 \times 10^7$ ,  $\delta = 8.53 \text{mm}$  (at 60Hz),  $\delta = 0.066 \text{mm}$  (at 1MHz)

Aluminum:  $\sigma = 3.54 \times 10^7$ ,  $\delta = 10.92 \text{mm}$  (at 60Hz),  $\delta = 0.084 \text{mm}$  (at 1MHz)

Gold:  $\sigma = 4.1 \times 10^7$ ,  $\delta = 10.14 \text{mm}$  (at 60Hz),  $\delta = 0.079 \text{mm}$  (at 1MHz)

Iron( $\mu_r = 10^3$ ):  $\sigma = 100 \times 10^7$ ,  $\delta = 0.65 \text{mm}$  (at 60Hz),  $\delta = 0.005 \text{mm}$  (at 1MHz)

Seawater:  $\sigma = 4$ ,  $\delta = 32$ m (at 60Hz),  $\delta = 0.25$ m (at 1MHz)

$$ε_0 = (1/36\pi) \times 10^{-9}$$
 (F/m)(permittivity:介電係數)

Speed of light: C=1/( $\mu_0 \, \epsilon_0$ )<sup>0.5</sup> =3x10<sup>8</sup> m/s, [(H · F)=(s<sup>2</sup>), 亨利 · 法拉= s<sup>2</sup>]

Wave length of 60Hz:  $\lambda = C/f = 5x10^6$  m,

## 3.1磁學回顧(Review of Magnetics)

Ampere's Circuital Law:  $F = \oint H \cdot dl = Ni$ 

 $\ddot{B} = \mu H$ ,  $B = \Phi/A$ ,  $\Phi$ : flux, B: flux density (webers/m<sup>2</sup>), H: magnetic field intensity (A · turn/m)

磁通鏈Flux linkages:  $\lambda = N\Phi = Li = \sum_{i=1}^{N} \Phi_i$ 

### Ex3.1 Calculate the inductance.

$$F = Hl = Ni$$

$$B = \mu H = \mu Ni / l,$$

$$\Phi = BA = (\mu A/l)Ni,$$

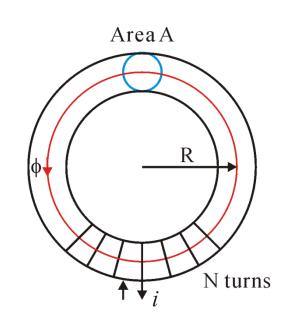
$$L = \lambda/i = N\Phi/i = (\mu AN^2/l) = \mu AN^2/(2\pi R)$$

inductance: relates flux linkage to current

$$e_{ind} = L \frac{di}{dt} = N \frac{d\phi}{dt} \qquad \phi = BA = uHA = u \frac{Ni}{L}A$$

$$\frac{Li}{t} = \frac{N\phi}{t} = Nu \frac{Ni}{Lt}A$$

$$L = \frac{NuA}{L}$$



### 3.2無限長直電線的磁通鏈(Flux Linkages of Infinite Straight Wire)

#### Figure 3.2: wire carrying current

Case 1 (x > r): 
$$F = \oint H \cdot dl = i = H \cdot 2\pi x = H = i/(2\pi x)$$

Case 2 (
$$^{\uparrow 0}$$
 x  $\leq$  r):  $\oint H \cdot dl = i_e = (\pi x^2 / \pi r^2)i = H \cdot 2\pi x => H = (x / 2\pi r^2)i$ 

$$B = \mu_r \ \mu_o H, \ \mu_o = 4\pi \times 10^{-7},$$

 $\mu_r$  of the air, copper, and aluminum is near 1.

Figure 3.3: infinite wire of radius r, find flux crossing rectangle.

Case 1 (x >r, N=1):
$$\lambda_1 = \Phi_1 = \int_A B(x) dx = \mu_0 \int_A H(x) dx = \mu_0 \int_r R i/(2\pi x) dx$$
  
= $(\mu_0 i/2\pi) \ln(R/r)$ . (Caution: R->  $\infty$ )

Case 2 
$$(\mathbf{x} \le \mathbf{r}, \mathbf{N} = \pi \mathbf{x}^2 / \pi \mathbf{r}^2)$$
:  $\lambda_2 = \mu_r \mu_o \int_0^r (\mathbf{x} / 2\pi \mathbf{r}^2) (\pi \mathbf{x}^2 / \pi \mathbf{r}^2) i \, d\mathbf{x} = \mu_r \mu_o i / (8\pi)$ 

Total Flux Linkages per meter of one Infinite Straight Wire:

$$\lambda = \lambda_2 + \lambda_1 = (\mu_0 i/2\pi) [\mu_r/4 + \ln(R/r)]$$

### 3.3多導體情況下的磁通鏈(Flux Linkages of Multi-Conductors)

Total Flux Linkages per meter of one Infinite Straight Wire:

# Ex3.2 Calculate the inductance per meter of each phase of a three-phase transmission line.

Assume that 1. Conductors equally spaced D and have equal radii r.

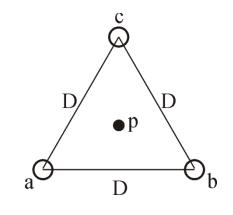
2. 
$$i_a + i_b + i_c = 0$$
.  
 $\lambda = (\mu / 2\pi) \{i \mid \mu \}$ 

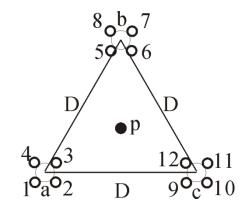
$$\begin{split} \lambda_a &= (\mu_o/2\pi) \{ i_a [\mu_r/4 + \ln(1/r)] + i_b \ln(1/D) + i_c \ln(1/D) \} \\ &= (\mu_o/2\pi) \{ i_a [\mu_r/4 + \ln(1/r)] - i_a \ln(1/D) \} \\ &= (\mu_o/2\pi) [\mu_r/4 + \ln(1/r) - \ln(1/D)] \times i_a \\ &= (\mu_o/2\pi) [\ln e^{(\mu r/4)} + \ln(1/r) - \ln(1/D)] \times i_a \\ &= (\mu_o/2\pi) [\ln(1/r e^{-(\mu r/4)}) - \ln(1/D)] \times i_a \\ &= (\mu_o/2\pi) [\ln(1/r e^{-(\mu r/4)}) - \ln(1/D)] \times i_a \end{split}$$

$$= (\mu_o/2\pi)[\ln(D/\underline{r'})] \times i_a$$
$$= l_a \times i_a$$

Assume that 1. D = 1m, r = 0.01m.

$$\mu_{\rm r} = 1, \, \mu_{\rm o} = 4\pi \times 10^{-7},$$
 $l_{\rm a} = ?$ 





## 3.4捆束導體(Conductor Bundling)

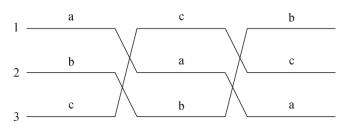
$$\lambda_{l} = (\mu_{o}/2\pi) \{ (i_{a}/4)[\mu_{r}/4 + \ln(1/r) + \ln(1/d_{12}) + \ln(1/d_{13}) + \ln(1/d_{14})] \\ + (i_{b}/4)[\ln(1/d_{15}) + \ln(1/d_{16}) + \ln(1/d_{17}) + \ln(1/d_{18})] \\ + (i_{c}/4)[\ln(1/d_{19}) + \ln(1/d_{1,10}) + \ln(1/d_{1,11}) + \ln(1/d_{1,12})] \} \\ = (\mu_{o}/2\pi)(i_{a} \ln 1/R_{GMR} + i_{b} \ln 1/D_{1b} + i_{c} \ln 1/D_{1c} \\ \mathbf{r'} = \mathbf{r} e^{-(\mu r/4)}, R_{GMR}^{\text{sath}} = (\mathbf{r'} d_{12} d_{13} d_{14})^{1/4} \text{ (geometric mean radius )} \\ D_{1b} = (d_{15} d_{16} d_{17} d_{18})^{1/4}, D_{1c} = (d_{19} d_{1,10} d_{1,11} d_{1,12})^{1/4} \\ \text{Assuming } D_{1b} = D_{1c} = D \text{ (geometric mean distance), } i_{a} + i_{b} + i_{c} = 0 \text{ ,} \\ \lambda_{l} = (\mu_{o}/2\pi) i_{a} \ln D/R_{GMR} \\ \text{Inductance } l_{1} = \lambda_{l}/(i_{a}/4) = 4 (\mu_{o}/2\pi) \ln D/R_{GMR} \\ \text{For phase a: } l_{1} = l_{2} = l_{3} = l_{4} \text{ , and four parallel lines,} \\ l_{a} = l_{1}/4 = (\mu_{o}/2\pi) \ln D/R_{GMR} \text{ , and } l_{a} = l_{b} = l_{c} \\ \text{Ex3.3 Find the geometric mean radius (GMR) of three} \\ \text{symmetrically spaced conductors. Assume that } r = 2cm \text{ and} \\ \mathbf{r'} = \mathbf{r} e^{-(\mu r/4)} = 2e^{-1/4} = 1.56 \text{ cm} \text{ .}$$

 $d_{12} = d_{13} = d_{23} = 50 \text{cm}$ ,  $R_{GMR} = (r'd_{12}d_{13})^{1/3} = ?$ ,

### 3.5移位(Transposition)

It is usually more convenient to arrange the phases in a horizontal or vertical configuration, therefore the symmetry is lost. One way to regain the symmetry and restore balanced conditions is to use the method of transposition of lines.

$$\begin{split} \lambda_{a} & = (\lambda_{a1} + \lambda_{a2} + \lambda_{a3})/3 \;, \; D_{m} = (d_{12} \; d_{23} \; d_{13})^{1/3} \;, \\ \lambda_{a} & = (1/3)(\mu_{o}/2\pi) \{ \; i_{a} \; \ln(1/r' \;) + i_{b} \; \ln(1/d_{12}) + i_{c} \; \ln(1/d_{13}) \\ & \quad + i_{a} \; \ln(1/r' \;) + i_{b} \; \ln(1/d_{23}) + i_{c} \; \ln(1/d_{12}) \\ & \quad + i_{a} \; \ln(1/r' \;) + i_{b} \; \ln(1/d_{13}) + i_{c} \; \ln(1/d_{23}) \; \} \\ \lambda_{a} & = (\mu_{o}/2\pi) \{ \; i_{a} \; \ln(1/r' \;) + i_{b} \; \ln(1/D_{m}) + i_{c} \; \ln(1/D_{m}) \; \} \\ & = (\mu_{o}/2\pi) \{ \; i_{a} \; \ln(1/r' \;) - i_{a} \; \ln(1/D_{m}) \; \} = (\mu_{o}/2\pi) \; i_{a} \; \ln(D_{m}/r' \;) \\ l_{a} & = l_{b} & = l_{c} & = (\mu_{o}/2\pi) \; \ln(D_{m}/r' \;) \; \text{for one line transposition} \\ l_{a} & = l_{b} & = l_{c} & = (\mu_{o}/2\pi) \; \ln(D_{m}/R_{GMR} \;) \; \text{for conductor bundling transposition} \end{split}$$



Ex 3.5 Find the inductance per meter of the 3-phase line shown in figure E3.5. The conductors are aluminum ( $\mu_r = 1$ ), with radius r = 0.5 inch,  $d_{12} = d_{23} = 20$  ft,  $d_{13} = 40$  ft, each phase has two conductors and distance is 18 inch.

(a) r'=r e 
$$^{-(\mu r/4)} = 0.5 \times 0.78$$
,  $R_{GMR} = (r' \times 18)^{1/2} = 2.65$  inch = 0.22 ft

(b) Dm = 
$$(20 \text{ ft} \times 20 \text{ ft} \times 40 \text{ ft})^{1/3} = 25.2 \text{ ft}$$

(c) 
$$l_a = l_b = l_c = (\mu_o/2\pi) \ln(D_m/R_{GMR}) = 2 \times 10^{-7} \ln(25.2/0.22)$$
  
= 9.47 × 10<sup>-7</sup> H/m

$$\approx 10^{-6} \, \text{H/m} = 1 \, \text{mH/m} = 1 \, \text{mH/km}$$

## 3 8 曲 刑 象 數 值 (Typical Parameter Values)

3.8 典型 参數值(Typical Parameter Values)		
Conductors per phase (18-in. spacing): 1(138kV), 2(345kV), 4(765kV)		
54/7,	45/7,	54/19
0.977,	1.165,	1.424
0.0329,	0.0386,	0.0479
or(A): 770,	1010,	1250
0.0329,	0.2406,	0.6916
17.5,	26.0,	45.0
22.05,	32.76,	56.70
13.02,	9.83,	8.81
0.789,	0.596,	0.535
8.84,	11.59,	12.78
0.186,	0.142,	0.129
0.1618,	0.0539,	0.0190
0.1688,	0.0564,	0.0201
50,	415,	2268
	: 1(138kV), 54/7, 0.977, 0.0329, or(A): 770, 0.0329, 17.5, 22.05, 13.02, 0.789, 8.84, 0.186, 0.1618, 0.1688,	: 1(138kV), 2(345kV) 54/7, 45/7, 0.977, 1.165, 0.0329, 0.0386, 0r(A): 770, 1010, 0.0329, 0.2406, 17.5, 26.0, 22.05, 32.76, 13.02, 9.83, 0.789, 0.596, 8.84, 11.59, 0.186, 0.142, 0.1618, 0.0539, 0.1688, 0.0564,

### Three phase line-to-line 138 kVrms / 60Hz

Conductors per phase (18-in. spacing): 1(138kV)

Number of strands aluminum/steel:

Diameter (in.):  $0.977 \div \overset{71}{2} \div 1\overset{72}{2} = 0.0407 \text{ ft,}$ Conductor GMR (ft):  $0.0329 = 0.0407 \times 0.8? = 0.0317 = 0.0407 \times e^{-\mu r/4} = 0.0407 \times 0.7788$ Diameter (in.):

Current-carrying capacity per conductor(A): 770, 1 mm \* 74

Bundle GMR- $R_{GMR}$  (ft): r' =0.0329, r =0.0407=0.977 ÷ 2÷12

Flat phase spacing (ft): 17.5,

GMD phase spacing (ft):  $22.05 = (17.5 \times 17.5 \times 2 \times 17.5)^{1/3} = 17.5 \times 1.26$ 

Inductance (H/m  $\times 10^{-7}$ ):  $13.02 = 2 \times 10^{-7} \times \ln (22.05 / 0.0329)$ 

 $X_L (\Omega/mile) : 0.789 = 2\pi \times 60 \times 13.02 \times 10^{-7} \times 1609.34$ 

Capacitance (F/m  $\times 10^{-12}$ ):  $8.84 = 2\pi \times 8.854 \times 10^{-12} / \ln (22.05 / 0.407)$ 

 $|X_C|$  (M $\Omega$ -mile to neutral):  $0.186 = 1/(2\pi \times 60 \times 8.84 \times 10^{-12} \times 1609.34)$ 

Resistance ( $\Omega$ /mile), dc, 50 °C: 0.1618=  $\rho \times 1609.34/[\pi(0.977 \times 2.54 \times 10^{-2}/2)^2]$ ,  $\rho$ =4.863 × 10<sup>-8</sup>

Resistance ( $\Omega$ /mile), 60Hz, 50 °C : 0.1688 = 0.1618 × 1.0433

Surge impedance loading (MVA): 50 ?=  $61.4=1\times\sqrt{3}\times138\times0.77/3$  (MVA)

## Three phase line-to-line 345 kVrms / 60Hz

Conductors per phase (18-in. spacing) : 2(345kV)

Number of strands aluminum/steel: 45/7,

Diameter (in.):  $1.165 \div 2 \div 12 = 0.0485 \text{ ft},$ 

Conductor GMR (ft): $0.0386 = 0.0485 \times 0.8? = 0.0378 = 0.0485 \times e^{-\mu r/4} = 0.0485 \times 0.7788$ Current-carrying capacity per conductor(A): 1010,

Bundle GMR-R<sub>GMR</sub> (ft):  $0.2406 = [0.0386 \times (18/12)]^{1/2}$ ,  $0.2697 = [0.0485 \times (18/12)]^{1/2}$ 

Flat phase spacing (ft): 26.0,

GMD phase spacing (ft):  $32.76 = (26.0 \times 26.0 \times 2 \times 26.0)^{1/3} = 26.0 \times 1.26$ Inductance (H/m ×10<sup>-7</sup>): 9.83 = 2× 10<sup>-7</sup> × ln (32.76 / 0.2406)

Inductance (H/m  $\times 10^{-7}$ ):  $9.83 = 2 \times 10^{-7} \times \ln (32.76 / 0.2406)$ 

 $X_L (\Omega/\text{mile}) : 0.596 = 2\pi \times 60 \times 9.83 \times 10^{-7} \times 1609.34$ 

Capacitance (F/m ×10<sup>-12</sup>):  $11.59 = 2\pi \times 8.854 \times 10^{-12} / \ln (32.76 / 0.2697)$ 

 $|X_C|$  (M $\Omega$ -mile to neutral) :  $0.142 = 1/(2\pi \times 60 \times 11.59 \times 10^{-12} \times 1609.34)$ 

Resistance ( $\Omega/\text{mile}$ ),dc,50°C: 0.0539= $\rho \times 1609.34/[2 \times \pi (1.165 \times 2.54 \times 10^{-2}/2)^2]$ ,  $\rho = 4.6066 \times 10^{-8}$ Resistance ( $\Omega/\text{mile}$ ), 60Hz, 50°C: 0.0564 = 0.0539  $\times$  1.0464

Resistance (12/mile), 60Hz, 50°C: 0.0564 = 0.0539  $\times$  1.0464 Surge impedance loading (MVA):415?=402.3= $2\times\sqrt{3}\times345\times1.010/3$  (MVA) Three phase line-to-line 765 kVrms / 60Hz

Conductors per phase (18-in. = 1.5-ft spacing) : 4(765kV)

Number of strands aluminum/steel: 54/19,

Diameter (in.):  $1.424 \div 2 \div 12 = 0.0593 \text{ ft},$ 

Conductor GMR (ft): $0.0479 = 0.0593 \times 0.8? = 0.0462 = 0.0593 \times e^{-\mu r/4} = 0.0593 \times 0.7788$ 

Current-carrying capacity per conductor(A): 1250,

Bundle GMR-R<sub>GMR</sub> (ft):  $0.6916 = [0.0479 \times 1.5 \times 1.5 \times 1.5 \times \sqrt{2}]^{1/4}$ ,  $0.7294 = [0.0593 \times 1.5 \times 1.5 \times 1.5 \times \sqrt{2}]^{1/4}$ ,

Flat phase spacing (ft): 45.0, GMD phase spacing (ft):  $56.7 = (45.0 \times 45.0 \times 2 \times 45.0)^{1/3} = 45.0 \times 1.26$ 

Inductance (H/m ×10<sup>-7</sup>):  $8.81 = 2 \times 10^{-7} \times \ln (56.7 / 0.6916)$ 

 $X_{I}$  ( $\Omega$ /mile):  $0.535 = 2\pi \times 60 \times 8.81 \times 10^{-7} \times 1609.34$ 

Capacitance (F/m  $\times 10^{-12}$ ):  $12.78 = 2\pi \times 8.854 \times 10^{-12} / \ln (56.7 / 0.7294)$ 

 $|X_C|$  (M $\Omega$ -mile to neutral) :  $0.129 = 1/(2\pi \times 60 \times 12.78 \times 10^{-12} \times 1609.34)$ 

Resistance ( $\Omega/\text{mile}$ ),dc,50°C: 0.0190= $\rho \times 1609.34/[4 \times \pi (1.424 \times 2.54 \times 10^{-2}/2)^2]$ ,  $\rho = 4.854 \times 10^{-8}$ 

Resistance ( $\Omega$ /mile), 60Hz, 50 °C : 0.0201 = 0.0190 × 1.0579

Surge impedance loading (MVA) : 2268 ?= 2208.3= $4 \times \sqrt{3} \times 765 \times 1.250/3$  (MVA)

### 3.9結論與習題(Summary)

For a three-phase line with transposition and bundling, the average per phase inductance (H/m) is given by

The formula for average capacitance (F/m) to neutral is  $c = (2\pi\epsilon) \ln(D_m/R_{GMR}) \text{ for conductor bundling transposition}$   $D_m = (D_{12} D_{23} D_{13})^{1/3}$   $R^c_b = R_{GMR} = (r d_{12} d_{13} \cdot d_{1b})^{1/b} , b > 1 ; R^c_b = R_{GMR} = r , \text{ when b=1}$