Power System Analysis
供電＝用電

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4.輸電線模型(Transmission-Line Modeling)

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4.0簡介(Introduction)

In Chapter 3 the distributed per phase inductance and capacitance of a transmission line was related to line geometry.

Some typical values of distributed series reactance, series resistance, and shunt (capacitive) reactance were given.

These elements may be used in the per phase equivalent circuit of a three-phase line operating under balanced conditions.

We now use this per phase model to derive the relationship between voltages and currents at the terminals of the transmission line.

4.1端點處V,I關係的推導(Derivation of Terminal V,I Relations) R=0

We consider the transmission line in the sinusoidal steady state. Thus we may use phasors and impedances. Assume that

$z = r + j\omega l$ = series impedance per meter

$y = g + j\omega c$ = shunt admittance per meter to neutral

The receiving end (right side) is located at $x = 0$; the sending end (left side) is at $x = l$.

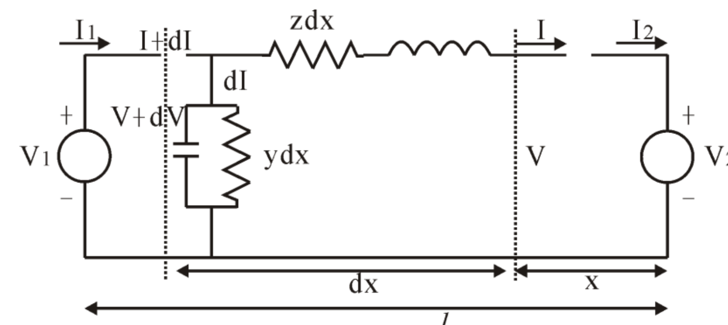
Applying Kirchhoff's voltage law (KVL) and Kirchhoff's current law (KCL) to the section, we get

$dV = Iz\,dx$; $dI = (V + dV)\,y\,dx = V\,y\,dx$ =>

$dV/dx = z\,I$; $dI/dx = y\,V$ =>

$d^2V/dx^2 = y\,z\,V = \gamma^2 V$; $d^2I/dx^2 = y\,z\,I = \gamma^2 I$

Propagation constant : $\gamma = (y\,z)^{0.5} = j\,\omega\sqrt{lc}$ (assume $r = g = 0$).



4.1端點處V,I關係的推導(Derivation of Terminal V,I Relations) $R \neq 0$

$z = r + j\omega l$ = series impedance per meter

$y = g + j\omega c$ = shunt admittance per meter to neutral

$dV/dx = z\,I$; $dI/dx = y\,V$

$d^2V/dx^2 = y\,z\,V = \gamma^2 V$; $d^2I/dx^2 = y\,z\,I = \gamma^2 I$

Propagation constant : $\gamma = (y\,z)^{0.5} = \alpha + j\beta$

$V = k_1 e^{\gamma x} + k_2 e^{-\gamma x} = (k_1 + k_2)(e^{\gamma x} + e^{-\gamma x})/2 + (k_1 - k_2)(e^{\gamma x} - e^{-\gamma x})/2$

$\quad = K_1 \cosh \gamma x + K_2 \sinh \gamma x$; $K_1 = (k_1 + k_2)$, $K_2 = (k_1 - k_2)$

$dV/dx = -K_1 \gamma \sinh \gamma x + K_2 \gamma \cosh \gamma x$

When $x=0$, $V=V_2$ => $V = k_1 + k_2 = K_1 = V_2$,

When $x=0$, $I=I_2$ => $dV(0)/dx = z\,I_2 = K_2 \gamma$ => $K_2 = (z/\gamma)I_2 = (z/y)^{0.5} I_2$

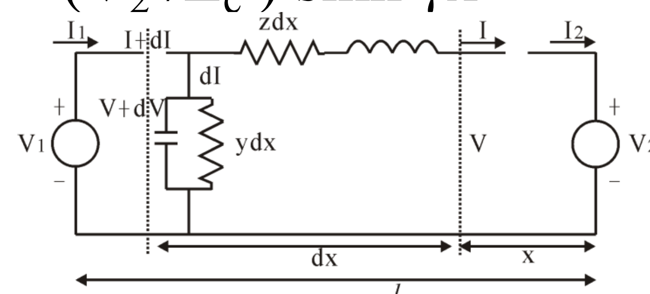
$Z_c = (z/y)^{0.5}$ (characteristic impedance: 線路的特性阻抗)

$V = V_2 \cosh \gamma x + Z_c I_2 \sinh \gamma x$; $I = I_2 \cosh \gamma x + (V_2/Z_c) \sinh \gamma x$

When $x = l$, $V=V_1$, $I = I_1$ =>

$V_1 = V_2 \cosh \gamma l + Z_c I_2 \sinh \gamma l$;

$I_1 = I_2 \cosh \gamma l + (V_2/Z_c) \sinh \gamma l$



Ex4.1 A 60-Hz 138-kV 3Φ transmission line is 225mile long. The distributed line parameters are r =0.169 Ω/mile , l = 2.093 mH/mile , c = 0.01427 μF/mile , g = 0. The transmission line delivers 40 Mwat 132 kV with 95% power factor lagging. Find the sending-end voltage and current. Find the transmission line efficiency.

$z = r + j\omega l = 0.169 + j0.789 = 0.807 \angle 77.9^\circ$ Ω/mile

$y = g + j\omega c = j5.38\times10^{-6} = 5.38\times10^{-6} \angle 90^\circ$ mho/mile

$\Rightarrow Z_c = (z/y)^{0.5} = 387.3 \angle -6.05^\circ$ Ω

$\Rightarrow \gamma\, l = 225\,(y\,z)^{0.5} = 0.4688 \angle 83.95^\circ = 0.0494 + j0.466$

$2 \sinh \gamma l = e^{\gamma l} - e^{-\gamma l} = e^{0.0494}\, e^{j0.466} - e^{-0.0494}\, e^{-j0.466} = 1.051\angle 0.466\text{rad} - 0.952\angle -0.466\text{rad}$

$\sinh \gamma l = 0.452 \angle 84.4^\circ$

$2 \cosh \gamma l = e^{\gamma l} + e^{-\gamma l} = 1.051\angle 0.466\text{rad} + 0.952\angle -0.466\text{rad} = 1.790 \angle 1.42^\circ$

$\cosh \gamma l = 0.895 \angle 1.42^\circ$

$|V_2| = 132 \times 10^3 / \sqrt{3} = 76.2$ kV $\Rightarrow V_2 = 76.2 \angle 0^\circ$ kV ,

1Φ $P_{load} = 40 \times 10^6 / 3 = 13.33$ MW $= 0.95\, |V_2|\, |I_2| \Rightarrow |I_2| = 184.1$, $I_2 = 184.1 \angle -18.195^\circ$

$V_1 = V_2 \cosh \gamma\, l + Z_c\, I_2 \sinh \gamma\, l = 89.28\angle 19.39^\circ$ kV $\Rightarrow V_{LL} = \sqrt{3} \times 89.28 = 154.64$ kV

$I_1 = I_2 \cosh \gamma\, l + (V_2 / Z_c) \sinh \gamma\, l = 162.42\angle 14.76^\circ$ A

$P_{12} = \text{real}(V_1 I_1^*) = 89.28 \times 10^3 \times 162.42 \cos(19.39^\circ - 14.76^\circ) = 14.45$ MW

Efficiency $\eta = 13.33 / 14.45 = 0.92 = 92\ \%$

Ex4.2 Suppose that a radial line is terminated in its characteristic impedance Zc. Find the driving point impedance V_1/I_1 , the voltage gain $|V_2|/|V_1|$, the current gain $|I_2|/|I_1|$, the complex power gain $-S_{21}/S_{12}$, and the real power efficiency $-P_{21}/P_{12}$.

(a) If the line is terminated in Zc, then $V_2 = Zc\, I_2$, $Z_c = (z/y)^{0.5}$

$V_1 = V_2 \cosh \gamma\, l + Z_c\, I_2 \sinh \gamma\, l = V_2 (\cosh \gamma\, l + \sinh \gamma\, l\,) = V_2\, e^{\gamma l} = V_2\, e^{\alpha l}\, e^{j\beta l}$

$I_1 = I_2 \cosh \gamma\, l + (V_2 /Z_c\,) \sinh \gamma\, l = I_2 (\cosh \gamma\, l + \sinh \gamma\, l\,) = I_2\, e^{\gamma l} = I_2\, e^{\alpha l}\, e^{j\beta l}$

So $V_1/I_1 = V_2/I_2 = Zc$,

and we that the driving point impedance is Zc .

$|V_1| = |V_2\, e^{\gamma l}\,| = |V_2|\, e^{\alpha l} \Rightarrow |V_2|/|V_1| = e^{-\alpha l}$,

$|I_1| = |I_2\, e^{\gamma l}\,| = |I_2|\, e^{\alpha l} \Rightarrow |I_2| / |I_1| = e^{-\alpha l}$,

(b) Noting the reference direction for I_2 , the complex power gain may be calculated

$-S_{21} = V_2\, I_2^* = V_1\, e^{-\alpha l}\, e^{-j\beta l}\, I_1^*\, e^{-\alpha l}\, e^{j\beta l} = (V_1 I_1^*)\, e^{-2\alpha l} = S_{12}\, e^{-2\alpha l}$, thus

$-S_{21} / S_{12} = e^{-2\alpha l}$. Alternatively, we may observe that

$V_1 = V_2\, e^{\gamma l} = Zc\, I_2\, e^{\gamma l} = Zc\, I_1 \Rightarrow V_1 I_1^* = Zc\, |I_1|^2$, while $V_2\, I_2^* = Zc\, I_2\, I_2^* = = Zc\, |I_2|^2$,

$-S_{21}/S_{12} = Zc\, |I_2|^2 / (Zc\, |I_1|^2\,) = |I_2|^2 / |I_1|^2 = e^{-2\alpha l}$.

Finally, since α is real, we have

$\eta = -P_{21} / P_{12} = e^{-2\alpha l}$

Ex4.3 Repeat Ex 4.2 for the case of a lossless line ($r = g = 0$). In addition, find Zc, γ, and P_{12}.

$z = r + j\omega l$ = series impedance per meter

$y = g + j\omega c$ = shunt admittance per meter to neutral

$Zc = (z/y)^{0.5} = (j\omega l / j\omega c)^{0.5} = (l / c)^{0.5} = (L / C)^{0.5}$, Zc is real

$L = \text{length} \times l$ = total inductance of the line

$C = \text{length} \times c$ = total capacitance of the line

$\gamma = (z\,y)^{0.5} = (j\omega l\, j\omega c)^{0.5} = j\omega\,(l\,c)^{0.5} = \alpha + j\beta$, γ is imaginary number

Since $\alpha = 0$, all the ratios calculated in Ex4.2 are unity. Thus

$|V_2|/|V_1| = 1$, $|I_2|/|I_1| = 1$, $-S_{21} / S_{12} = 1$, $\eta = -P_{21} / P_{12} = 1$,

Finally, $P_{12} = \mathrm{Re}(V_1 I_1^*) = \mathrm{Re}(Zc\,|I_1|^2) = Zc\,|I_1|^2$, since Zc in this case is real.

An alternate expression based on $I_1 = V_1 / Zc$ is frequently more useful:

$P_{12} = |V_1|^2 / Zc$, In a lossless line, Zc is surge impedance (突波阻抗). In a loss line, Zc is characteristic impedance (特性阻抗).

A lossless line operating at its normal voltage, terminated in its surge impedance Zc, is said to be surge impedance loaded (SIL).

$P_{SIL} = |V_1|^2 / Zc$ (1 phase), $3P_{SIL} = 3|V_1|^2 / Zc = |V_{LL}|^2 / Zc$ (3 phase),

4.2輸電線上的電波(Waves on Transmission Lines)

Propagation constant : $\gamma = (y\ z)^{0.5} = \alpha + j\beta$,

α is attenuation constant (衰減常數), β is phase constant (相位常數).

$V = k_1\ e^{\gamma x} + k_2\ e^{-\gamma x}$,

$k_1\ e^{\gamma x}$ is a voltage wave traveling to the right (incident wave: 入射波);

$k_2\ e^{-\gamma x}$ is a voltage wave traveling to the left (reflected wave: 反射波).

$v(t, x) = \sqrt{2}\mathrm{Re}\ k_1 e^{\alpha x}\ e^{j(\omega t + \beta x)} + \sqrt{2}\mathrm{Re}\ k_2 e^{-\alpha x}\ e^{j(\omega t - \beta x)} = v_1(t,x) + v_2(t,x)$

Neglecting α and considering $v_2(t,x)$, for fixed x , v_2 is a sinusoidal function of t; for fixed t, v_2 is a sinusoidal function of x .

Assume $\omega t - \beta x$ = constant => v_2 is constant.

So velocity $dx/dt = \omega/\beta = \omega/\ \mathrm{Im}\ (y\ z)^{0.5}$

If an incident lightning voltage pulse (or surge) traveling down a line hits the open-circuited end of the line (or an open switch), there will be a reflected wave generated such that the voltage at the line termination will approximately double . In designing the insulation for transmission lines and equipment connected to it (such as transformers), it is vitally important to take this doubling into account.

4.3傳輸矩陣(Transmission Matrix)

Steady-state terminal voltage and currents.

$V_1 = AV_2 + BI_2$; $I_1 = CV_2 + DI_2$,

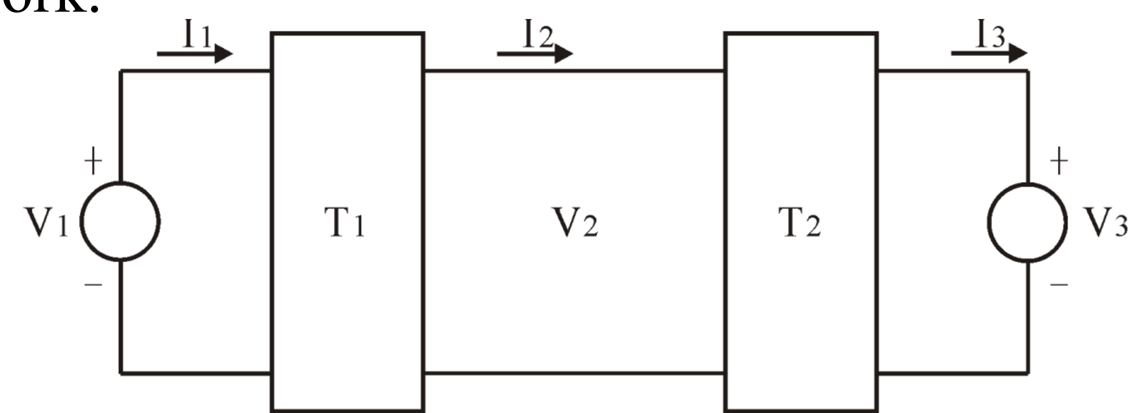
where $A= \cosh \gamma l$, $B= Z_c \sinh \gamma l$, $C= (1/Z_c) \sinh \gamma l$, $D= \cosh \gamma l$,

Transmission matrix (chain matrix) $T = [A\ B;C\ D]$, $T^{-1} =[D\ –B;-C\ A]$

$\det T = AD – BC = (\cosh \gamma l)^2 – (\sinh \gamma l)^2 = 1$

$[V_1\ I_1] = T_1\ [V_2\ I_2] = T_1T_2\ [V_3\ I_3] = T\ [V_3\ I_3]$

That det T = 1, holds in general for two-port networks composed of (linear time-invariant) resistors, capacitors, inductors, coupled inductors, and transformers. This provides a useful check of analytical or numerical work.



4.4等效集總電路(Lumped-Circuit Equivalent), $\gamma = (y\,z)^{0.5}$

$V_1 = AV_2 + BI_2$; $I_1 = CV_2 + DI_2$,

Find a Π equivalent circuit that has the same A, B, C, D parameters as the transmission line. We note that a T equivalent circuit may also be derived.

$V_1 = V_2 \cosh \gamma l + Z_c I_2 \sinh \gamma l$; $I_1 = I_2 \cosh \gamma l + (V_2/Z_c) \sinh \gamma l$

$V_1 = V_2 + Z'[I_2 + (Y'/2)V_2] = (1 + Z'Y'/2)V_2 + Z'I_2$

$I_1 = (Y'/2)V_1 + (Y'/2)V_2 + I_2 = Y'(1 + Z'Y'/4)V_2 + (1 + Z'Y'/2)I_2$

A= $\cosh \gamma l = 1 + Z'Y'/2$, B= $Z_c \sinh \gamma l$ =Z' ,

C= $(1/Z_c) \sinh \gamma l = Y'(1 + Z'Y'/4)$, D= $\cosh \gamma l = 1 + Z'Y'/2$,

$Z' = Z_c \sinh \gamma l = (z/y)^{0.5} \sinh \gamma l = zl \sinh \gamma l / l(zy)^{0.5} = Z \sinh \gamma l / \gamma l = Z$

Assume $|\gamma l| \ll 1 \Rightarrow \sinh \gamma l / \gamma l = 1$; $Z = zl$ is the total series impedance of line

$1 + Z'Y'/2 = \cosh \gamma l \Rightarrow$ $Y'/2$ $= (\cosh \gamma l - 1)/Z' = (\cosh \gamma l - 1)/(Z_c \sinh \gamma l)$

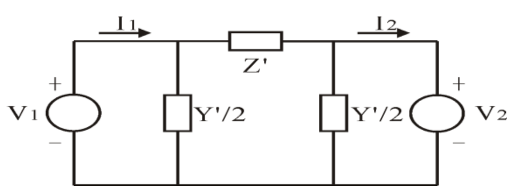
$=(1/Z_c)(e^{\gamma l} + e^{-\gamma l} - 2)/(e^{\gamma l} - e^{-\gamma l}) = (1/Z_c)(e^{\gamma l/2} - e^{-\gamma l/2})^2/(e^{\gamma l/2} + e^{-\gamma l/2})(e^{\gamma l/2} - e^{-\gamma l/2})$

$=(1/Z_c)\tanh(\gamma l/2) = (y/z)^{0.5} \tanh(\gamma l/2) = (yl/\gamma l)\tanh(\gamma l/2) = (Y/2) \tanh(\gamma l/2)/(\gamma l/2)$

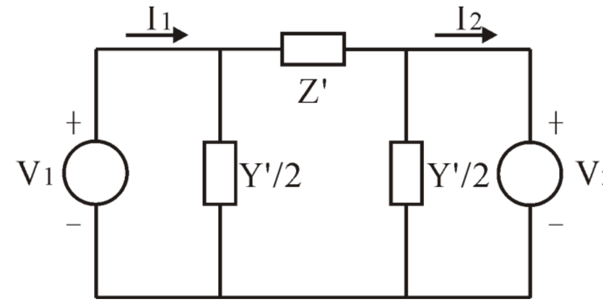
Assume $|\gamma l| \ll 1 \Rightarrow \tanh(\gamma l/2)/(\gamma l/2) = 1$; $Y = yl$ is the total line-neutral admittance of line

$Z_c = (z/y)^{0.5} = (zl/yl)^{0.5} = (Z/Y)^{0.5}$

$\gamma l = (zy)^{0.5}\, l = (zl\, yl)^{0.5} = (Z\,Y)^{0.5}$



Ex4.4 Find the Π equivalent circuit in Ex4.1 A 60-Hz 138-kV 3Φ transmission line is 225mile long. The distributed line parameters are r =0.169 Ω/mile , l = 2.093 mH/mile , c = 0.01427 μF/mile , g = 0.



$z = r + j\omega l = 0.169 + j0.789 = 0.807\ \angle 77.9^\circ\quad \Omega/\text{mile}$

$y = g + j\omega c = j5.38\times 10^{-6} = 5.38\times 10^{-6}\ \angle 90^\circ\quad \text{mho/mile}$

$\Rightarrow Z_c = (z/y)^{0.5} = 387.3\ \angle -6.05^\circ\quad \Omega$

$\Rightarrow \gamma\, l = 225\ (y\ z)^{0.5} = 0.4688\ \angle 83.95^\circ\quad = 0.0494 + j0.466$

$2\sinh \gamma l = e^{\gamma l} - e^{-\gamma l} = e^{0.0494}\, e^{j0.466} - e^{-0.0494}\, e^{-j0.466} = 1.051\angle 0.466\text{rad} - 0.952\angle -0.466\text{rad}$

$\sinh \gamma l = 0.452\ \angle 84.4^\circ$

$Z' = Zc \sinh \gamma l = 387.3\ \angle -6.05^\circ\quad \Omega \times 0.452\ \angle 84.4^\circ\quad = 175.06\ \angle 78.35^\circ$

$Z' = Z \sinh \gamma l / \gamma l = 225 \times 0.807\angle 77.9 \times 0.452\angle 84.4^\circ\ / (0.4688\angle 83.95^\circ\ )$

$= 181.57\angle 77.9 \times 0.9642\angle 0.45^\circ\quad = 175.07\ \angle 78.35^\circ$

$2\cosh \gamma l = e^{\gamma l} + e^{-\gamma l} = 1.051\angle 0.466\text{rad} + 0.952\angle -0.466\text{rad}\quad = 1.790\ \angle 1.42^\circ$

$\cosh \gamma l = 0.895\ \angle 1.42^\circ$

$Y'/2 = (\cosh \gamma l - 1)/Z' = (0.895\angle 1.42^\circ\ - 1)\ /\ 175.06\angle 78.35^\circ\quad = 614.57 \times 10^{-6}\angle 89.8^\circ\quad \text{mho}$

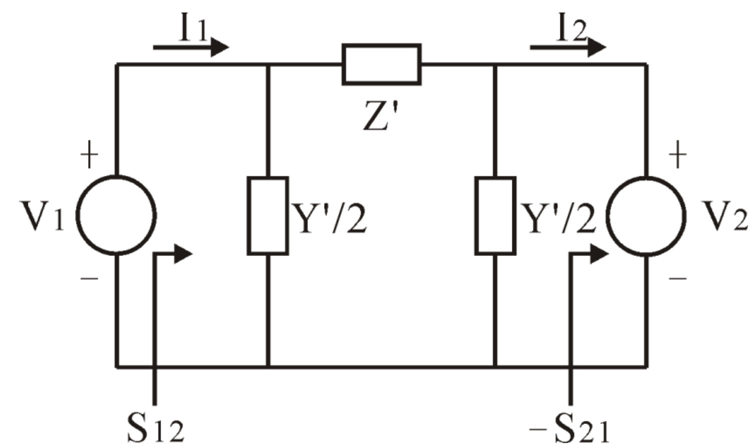
$Y/2 = (yl)/2 = (225 \times 5.38\times 10^{-6}\ \angle 90^\circ\ )/2\ = 605.25\ \times 10^{-6}\ \angle 90^\circ\quad \text{mho}$

4.5簡化模型(Simplified Models)

Long line (l > 150mile = 241.4 km): Use the Π equivalent circuit model with Z' and Y'/2.

Medium length line (50mile < l < 150mile = 241.4 km): Use the Π equivalent circuit model with Z and Y/2 instead of Z' and Y'/2.

Short line (l < 50mile = 80.5 km): Same as the medium length line except that we neglect Y/2.



$z = r + j\omega l \Rightarrow Z = zl$,

$y = g + j\omega c \Rightarrow Y = yl$,

$Z' = Z_c \sinh \gamma l$

$Y'/2 = (\cosh \gamma l - 1)/Z'$

$\gamma = (yz)^{0.5}$, $Z_c = (z/y)^{0.5}$

$\sinh \gamma l = (e^{\gamma l} - e^{-\gamma l})/2$, $\cosh \gamma l = (e^{\gamma l} + e^{-\gamma l})/2$

Ex 4.5 Consider the receiving-end voltage of a lossless open-circuited line and compare the results by use of the three models. V1 is fixed.

Open circuited => $I2 = 0$, lossless => $\alpha = 0$, $\gamma = j\,\beta$, assume $\beta = 0.002$ rad/mile

M1: Long line (l > 150mile):

$V_1 = V_2 \cosh \gamma l + Z_c I_2 \sinh \gamma l = V_2 \cosh \gamma l = V_2 (e^{\gamma l} + e^{-\gamma l})/2 = V_2 \cos \beta l$

$I_1 = I_2 \cosh \gamma l + (V_2 / Z_c) \sinh \gamma l = (V_2 / Z_c) \sinh \gamma l$

M2: Medium length line: Use the Π model with Z and Y/2 instead of Z' and Y'/2.

$V_1 = V_2 + Z' [I_2 + (Y'/2)V_2] = (1 + Z'Y'/2)V_2 + Z'I_2$

$= (1 + ZY/2)V_2 = [1 + (\gamma l)^2 /2]\, V_2 = [1 - (\beta l)^2 /2]\, V_2$

This is the first two terms in the Taylor series expansion of $\cos \beta l$.

M3: Short line (l < 50mile = 80.5 km): Same as the medium length line (Y/2 = 0).

$V_1 = V_2 + Z' [I_2 + (Y'/2)V_2] = V_2$

This is the first term in the Taylor series expansion of $\cos \beta l$.

(1) 50 miles => $\beta l = 0.1$ rad ,

M1: $V_1 = 0.995004\, V_2$, M2: $V_1 = 0.995000\, V_2$, M3: $V_1 = V_2$,

(2) 200 miles => $\beta l = 0.4$ rad ,

M1: $V_1 = 0.921\, V_2$, M2: $V_1 = 0.920\, V_2$, M3: $V_1 = V_2$,

(1) 600 miles => $\beta l = 1.2$ rad ,

M1: $V_1 = 0.362\, V_2$, M2: $V_1 = 0.280\, V_2$ (error ?), M3: $V_1 = V_2$ (inaccurate),

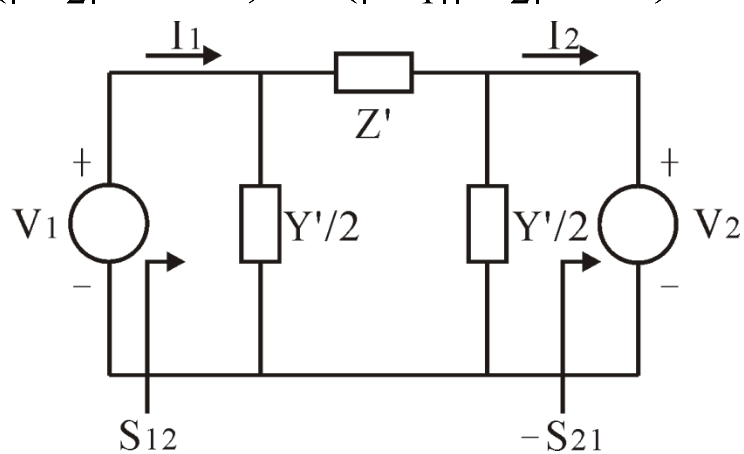
4.6複數功率傳輸(長程或中程線路)(Complex Power Transmission: Long or Medium Lines)

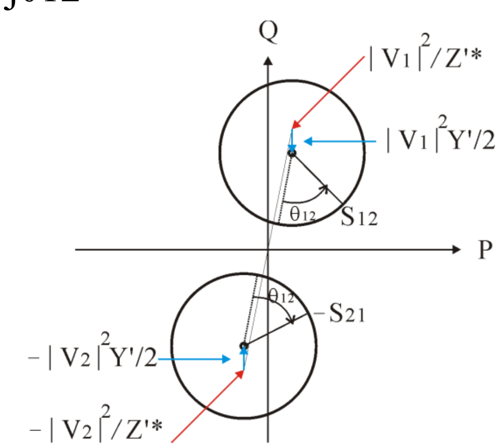
Z between voltages V_1 and V_2

$$S_{12} = V_1 I_1^* = V_1[(V_1 - V_2)/Z]^* = |V_1|^2/Z^* - V_1 V_2^*/Z^*$$
$$= (|V_1|^2/|Z|)\, e^{j\angle Z} - (|V_1||V_2|/|Z|)\, e^{j\angle Z}\, e^{j\theta 12}$$
$$S_{21} = V_2 I_2^* = V_2((V_2 - V_1)/Z)^* = |V_2|^2/Z^* - V_2 V_1^*/Z^*$$
$$= (|V_2|^2/|Z|)\, e^{j\angle Z} - (|V_2||V_1|/|Z|)\, e^{j\angle Z}\, e^{-j\theta 12}$$
$$-S_{21} = -(|V_2|^2/|Z|)\, e^{j\angle Z} + (|V_2||V_1|/|Z|)\, e^{j\angle Z}\, e^{-j\theta 12}$$

Z' and Y'/2 between voltages V_1 and V_2

$$S_{12} = (Y'^*/2)|V_1|^2 + (|V_1|^2/Z'^*) - (|V_1||V_2|/Z'^*)\, e^{j\theta 12}$$
$$-S_{21} = -(Y'^*/2)|V_2|^2 - (|V_2|^2/Z'^*) + (|V_1||V_2|/Z'^*)\, e^{-j\theta 12}$$





4.7線路的功率處理容量(Power-Handling Capability of Lines)

Power line are limited in their ability to deliver power. The two most important limits can be understood by considering thermal effects and stability.

When current flows in conductors, there are I^2R losses and heat is generated (T increased, T is limited 100° C). This loss of power reduces transmission efficiency.

If the cable gets too hot, the insulation will begin to deteriorate and may fail in time.

With limitations on maximum current and voltage there is a corresponding limitation on the MVA that can safely be transmitted.

The maximum angle θ_{12} is 90° , for stability, the maximum angle θ_{12} is 40° ~50° , A lossless line with equal voltage magnitudes at each end as Ex 4.3.

$z = r + j\omega l = j\omega l$ series impedance per meter

$y = g + j\omega c = j\omega c$ shunt admittance per meter to neutral

$Z_c = (z/y)^{0.5} = (j\omega l / j\omega c)^{0.5} = (l / c)^{0.5} = (L/C)^{0.5}$,

$\gamma = (z\, y)^{0.5} = \alpha + j\beta = j\beta = (j\omega l\, j\omega c)^{0.5} = j\omega\, (l\, c)^{0.5}$

$\sinh \gamma l = (e^{\gamma l} - e^{-\gamma l})/2 = (e^{j\beta l} - e^{-j\beta l})/2 = j \sin \beta l$

$\cosh \gamma l = (e^{\gamma l} + e^{-\gamma l})/2 = (e^{j\beta l} + e^{-j\beta l})/2 = \cos \beta l$

$Z' = Z_c \sinh \gamma l = jZ_c \sin \beta l$

$Y'/2 = (\cosh \gamma l - 1)/Z' = (Y/2) \tanh(\gamma l/2)/(\gamma l/2) = (j\omega C/2) \tan(\beta l/2)/(\beta l/2)$

Assume $|V_1| = |V_2|$, $P_{SIL} = |V_1|^2 / Z_c$

$S_{12} = (Y'^*/2)|V_1|^2 + (|V_1|^2 / Z'^*) - (|V_1||V_2|/Z'^*)\, e^{j\theta 12}$

$P_{12} = -\, [|V_1||V_2| / (-\, jZ_c \sin \beta l)]\, (j\sin \theta_{12}) = (|V_1|^2 / Z_c)\, (\sin \theta_{12} / \sin \beta l)$

$P_{12} = P_{SIL}\, (\sin \theta_{12} / \sin \beta l)$

$-S_{21} = -(Y'^*/2)|V_2|^2 - (|V_2|^2 / Z'^*) + (|V_1||V_2|/Z'^*)\, e^{-j\theta 12}$

Ex 4.6 Assume that β =0.002 rad/mile and $\theta_{12} = 45°$. Find P12/PSIL as a function of line length.

Assume $|V_1| = |V_2|$, $P_{SIL} = |V_1|^2 / Z_c$

$S_{12} = (Y'^*/2)|V_1|^2 + (|V_1|^2 / Z'^*) - (|V_1||V_2|/Z'^*)\, e^{j\theta 12}$

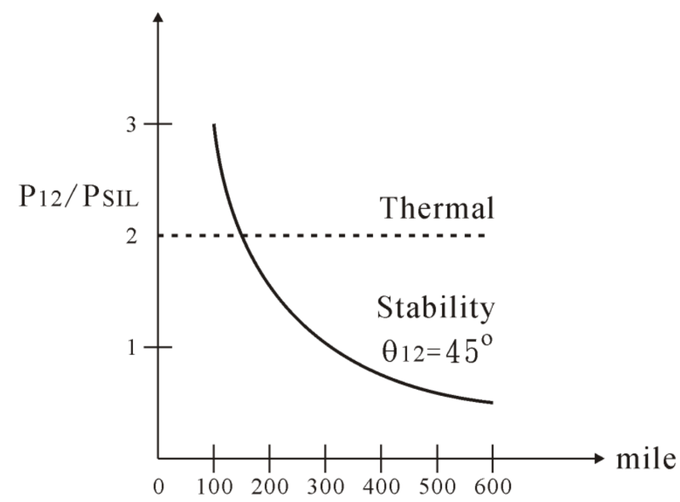
$P_{12} = -\, [|V_1||V_2| / (-jZ_c \sin \beta l)]\, (j\sin \theta_{12}) = (|V_1|^2 / Z_c)\, (\sin \theta_{12} / \sin \beta l)$

$P_{12} = P_{SIL}\, (\sin \theta_{12} / \sin \beta l) \Rightarrow P_{12} / P_{SIL} = (\sin \theta_{12} / \sin \beta l)$

$\theta_{12} = 45°$, $\beta l = 0.002$ rad/mile $\times\, l = 0.1146$ °/mile $\times\, l$

$P_{12} / P_{SIL} = 0.707 / \sin\,(0.1146$ °/mile $\times\, l\,)$

We see that for short lines the thermal limit governs, where as for long lines the stability limit prevails.

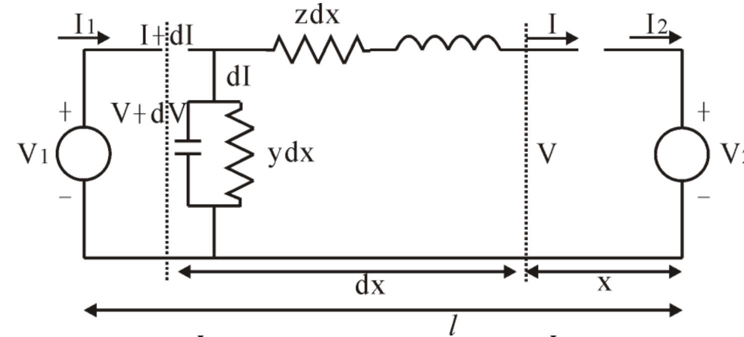


4.8結論與習題(Summary)

The long line equation is frequently used.

$$V_1 = V_2 \cosh \gamma l + Z_c I_2 \sinh \gamma l$$

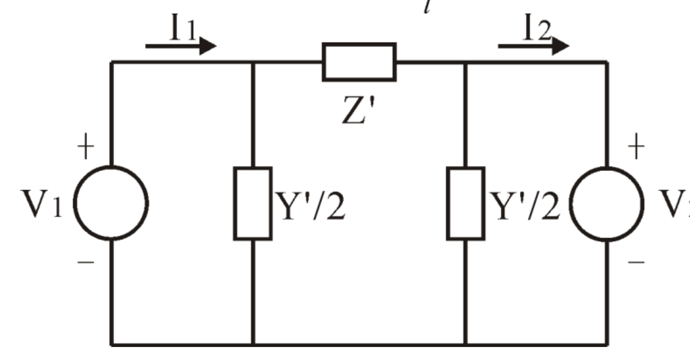
$$I_1 = I_2 \cosh \gamma l + (V_2 / Z_c) \sinh \gamma l$$



The Π equivalent circuit

$Z' = Zc \sinh \gamma l$

$Y'/2 = (\cosh \gamma l - 1)/Z'$



Finally, we note that thermal effects limit the power handling capability of short and medium length lines, where as stability requirements impose the limitations on long lines.