Power System Analysis
供電=用電

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2.基本原理(Basic Principles)

- 2.0簡介(Introduction)
- 2.1供應單埠的複數功率(Complex Power Supplied to a One-Port)
- 2.2複數功率守恆(Conservation of Complex Power)
- 2.3平衡三相(Balanced Three-Phase)
- 2.4單相分析(Per Phase Analysis)
- 2.5平衡三相功率(Balanced Three-Phase Power)
- 2.6複數功率傳輸(短程)(Complex Power Transmission)
- 2.7複數功率傳輸(輻射線路) (Complex Power Transmission: Radial Line)
- 2.8結論與習題(Summary)

2.0簡介(Introduction)

1直流電:被動元件(R,L,C);V=IR,V=Ldi/dt,I=Cdv/dt

功率P=VI=V2/R=I2R;功J=P*t

2Kirchhoff's current law (KCL): sum of currents entering a node (or a closed boundary) is zero.

3Kirchhoff's voltage law (KVL): sum of all voltages around a closed path (or loop) is zero.

4穩態、暫態、動態(steady state, transient state, dynamic state)

5交流電:單相與三相;相量(phasor)

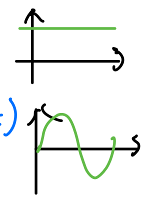
6交流電:被動元件(R,L,C);v=iR,v=Ldi/dt,i=Cdv/dt

功率P=VrmsIrms=Vrms2/R=Irms2R;功J=P*t

7符號:vBE(總瞬間值)=VBE(直流值)+vbe(交流瞬間值)

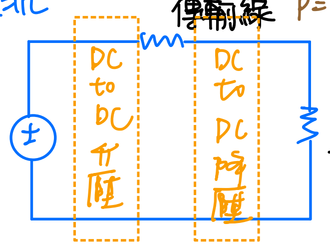
Vbe(相量值)

電
- 直流 (0 Hz)
- 交流 (60 Hz)



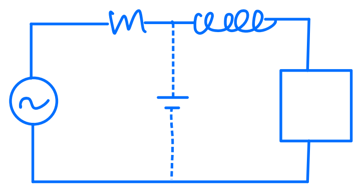
①直流 不能變壓

傳輸線 P=VI



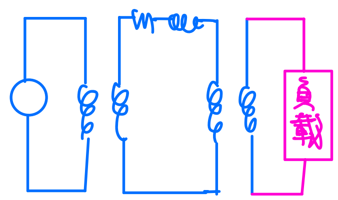
同頻率
相角 0°
电壓

② 交流 (RLC都可當負載)



①被動 RLC
②主動

變壓器 (100 台發电機 / 200 台變壓器 + 許多 負載)



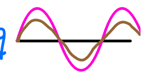
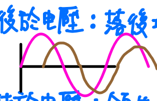
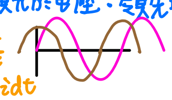
升壓　降壓

同頻率：所有發电機轉一樣快
相角,
电壓

① 直流电：电壓，电流，(實)功率 (有儲能元件)

$$\begin{cases} V = IR \\ V = L\frac{di}{dt} \approx 0 \\ i = C\frac{dv}{dt} \approx 0 \end{cases}$$

$P = VI = \frac{V^2}{R} = I^2R$

② 交流电：电壓，电流
- 同相位：單位功因 
- 电流落後於电壓：落後功因 
- 电流領先於电壓：領先功因 

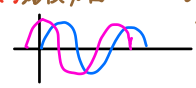
視在功率 = 實功率 + 虛功率
　　　　　　(R)　　(LC)

(L,C 储能)

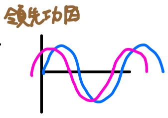
① $V = IR$ 單位功因

② $V = L\frac{di}{dt}$ 电流落後电壓 90°

$\frac{d\sin t}{dt} = \cos t = \sin(t + 90°)$ 落後功因



③ $i = C\frac{dv}{dt}$, $V = \frac{1}{C}\int i\,dt$ 电流領先电壓 90° 領先功因



电的3態

穩態：$t \to \infty$，留下的

暫態：$t \to \infty$，消失不見

動態：

動態

$y'' + 3y' + 2y = 5u(t)$

2階常微分方程式 輸入 $u(t)$，$y(t)$ 輸出

非齊次

y_H：通解，暫態解 $y'' + 3y' + 2y = 0$

$(D^2 + 3D + 2)y = 0$

$(D+1)(D+2)y = 0$

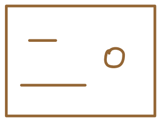
$y_H = C_1e^{-t} + C_2e^{-2t}$

y_p = 特解，穩態解 $y'' + 3y' + 2y = 5u(t)$

$y_p = \frac{5}{2}u(t)$, $y_p' = 0$, $y_p'' = 0$

微分方程
- 偏微分方程 (2個↑變數)
- 常微分方程 (單變數, y)
  - 非線性 (有自乘, 互乘) ex. $y \times y'$
  - 線性
    - 變係數
    - 常係數
      - 齊次
      - 非齊次

插座示意



60 ↓

均方根

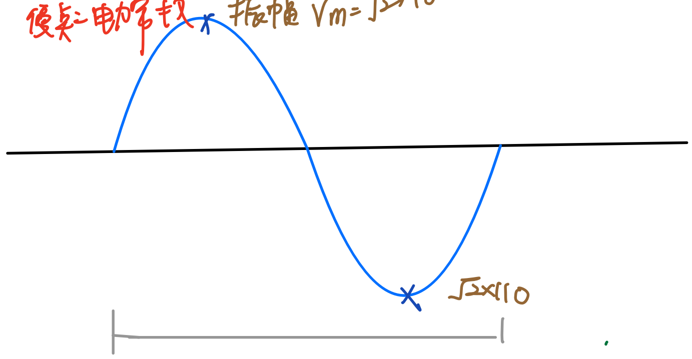
- 單相：110 Vrms / 60Hz
- 缺点：電扇振動
- 三相：3φ 220Vrms / 60Hz
- 優点：電扇平穩

振幅 $V_m = \sqrt{2}\times 110$

ωt

$V(t) = \sqrt{2}\times 110 \cdot \sin 377t$

$V(t) = \sqrt{2}\times 110 \cos 377t$



$\omega = 2\pi f$ 角頻率

一周期 $360^\circ = 2\pi = 6.28$ rad 徑度

$P(t) =$

$f = \frac{1}{T}$　$T = \frac{1}{f} = \frac{1}{60} = 0.0167$秒

(秒)

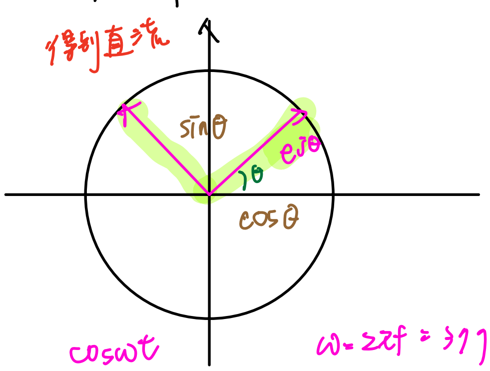
$$\frac{\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{[V_m\cos\theta]^2}{R}\, d\theta}{\pi} = \frac{V_m^2}{\pi R}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} [\cos\theta]^2 d\theta$$

$\theta + \theta$

$$\cos(2\theta) = \cos^2\theta - \sin^2\theta$$
$$= \cos^2\theta - [1 - \cos^2\theta]$$
$$= 2\cos^2\theta - 1$$
$$\cos^2\theta = \frac{1+\cos 2\theta}{2}$$

在同步旋轉標觀察交流

得到直流



在同步旋轉標相

不能達

$\cos\omega t$　　$\omega = 2\pi f = 377$

$$V_m e^{j\theta} = [\cos\theta + j\sin\theta]\, V_m$$

三相

$\pi = 180^\circ$

$\frac{2}{3}\pi = 120^\circ$

$$\begin{cases} V_a(t) = 180 \sin 377t \\ V_b(t) = 180 \sin(377t - 120^\circ) \\ V_c(t) = 180 \sin(377t - 240^\circ) \\ \quad = 180 \sin(377t + 120^\circ) \end{cases}$$

$V_a(t) = 180 \cos 377t$

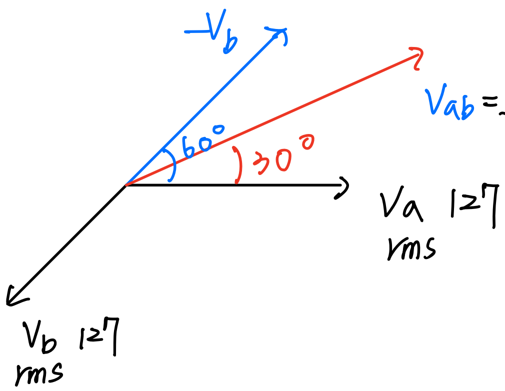
$V_b(t) = 180 \cos(377t - 120^\circ)$

$V_c(t) = 180 \cos(377 - 240^\circ$

$= 180 \cos(377 + 120^\circ)$

$V_m = 180$

$\Rightarrow V_{rms} = \frac{V_m}{\sqrt{2}} = 127$



$V_{ab} = \sqrt{3} V_a \angle 30^\circ = \sqrt{3} \times 127 \angle 30^\circ = 220 \angle 30^\circ$

$V_{ab} = V_a - V_b$

$= V_a + (-V_b)$

$V(t) = \frac{1}{C} \int i\, dt = \frac{1}{Cs} i$ 領先功因

$V(t) = i(t) \times R$ 單位功因

$V(t) = L \frac{di(t)}{dt}$ 落後功因

$\frac{d}{dt} = s = j\omega$

$\frac{d \sin \omega t}{d\omega t} \frac{d\omega t}{dt} = s \sin \omega t$

$= \omega \cos \omega t$

$= \omega \sin(\omega t + 90^\circ)$

$= j\omega \sin \omega t$

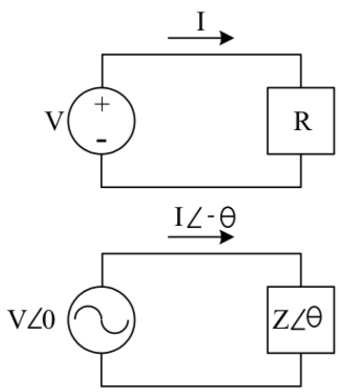
1.9 REAL, REACTIVE, AND APPARENT POWER IN AC CIRCUITS

☆ 必考題!

DC Power

$$P = VI = V^2/R = I^2 R \qquad (1-55)$$

AC Power



$$v(t) = \sqrt{2}\, V \cos \omega t \qquad (1-56)$$

$$i(t) = \sqrt{2}\, I \cos(\omega t - \theta) \qquad (1-57)$$

$$p(t) = v(t)i(t) = 2VI \cos \omega t \cos(\omega t - \theta) \qquad (1-58)$$

$$p(t) = VI[\cos \theta + \cos(2\omega t - \theta)]$$

$$VI \cos \theta \cdot (1 + \cos 2\omega t) + VI \sin \theta \cdot \sin 2\omega t \qquad (1-59)$$

The Power Triangle

$$P = VI \cos \theta \qquad (1-60)$$

$$Q = VI \sin \theta \qquad (1-61)$$

$$S = VI \qquad (1-62)$$

和差化積，積化和差

$\sin(A+B) = \sin A \cos B + \cos A \sin B$

$\sin(A-B) = \sin A \cos B - \cos A \sin B$

$$\begin{cases} \cos(A-B) = \cos A \cos B + \sin A \sin B \\ \cos(A+B) = \cos A \cos B - \sin A \sin B \end{cases}$$

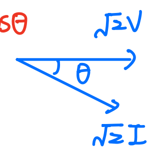
$V(t) = \sqrt{2} V \cos \omega t$

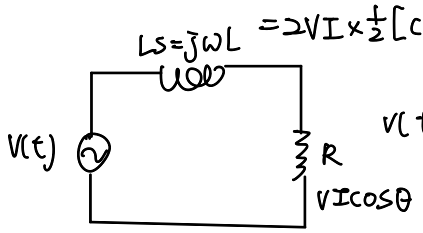
$i(t) = \sqrt{2} I \cos(\omega t - \theta)$

$p(t) = V(t) i(t) = 2VI \cos \omega t \cdot \cos(\omega t - \theta)$

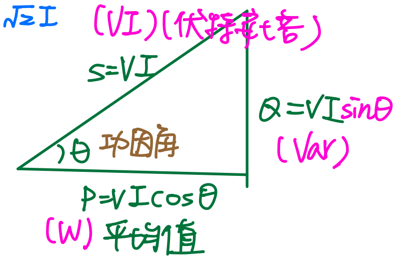
$= 2VI \times \frac{1}{2}[\cos\theta + \cos(2\omega t - \theta)]$

$P(t) = V_{rms} I_{rms}$ 平均值





$v(t) = (j\omega L + R) i(t)$



2.1供應單埠的複數功率

(Complex Power Supplied to a One-Port)

$v(t)=V_m\cos(\omega t+\theta_v)= \mathrm{Re}(V_m e^{j(\omega t+\theta_v)})$

$i(t)=I_m\cos(\omega t+\theta_i)= \mathrm{Re}(I_m e^{j(\omega t+\theta_i)})$

$p(t)=v(t)*i(t)=V_m\cos(\omega t+\theta_v)*I_m\cos(\omega t+\theta_i)$

$=0.5*V_mI_m[\cos(\theta_v-\theta_i)+\cos(2\omega t+\theta_v+\theta_i)]$

Power factor angle: $\theta=\theta_v-\theta_i$

Power factor = $\cos\theta$;(lagging,leading,unity)

Average power=$0.5*V_mI_m\cos\theta=V_{rms}I_{rms}\cos\theta=\mathrm{Re}\,VI^*$

Power triangle $S=VI^*=P+jQ$;

|S|視在功率(VA) apparent power

S複數功率(VA) complex power

P實功(W) real power

Q虛功(無效功率)(Var) reactive power

EX2.1 Inductor L,$Z=j\omega L$,reactive power $Q=\omega L|I|2$

$i(t)=\sqrt{2}|I|\cos(\omega t+\theta)$

$v(t)=L\,di/dt=-\sqrt{2}\omega L|I|\sin(\omega t+\theta)$

$p(t)=v(t)*i(t)=-2\omega L|I|2\sin(\omega t+\theta)\cos(\omega t+\theta)$

$=-\omega L|I|2\sin 2(\omega t+\theta)$

Average Power P=0,

瞬時功不為零(Instantaneous power is not zero)

但平均值是0

$S=VI^*=ZII^*=Z|I|2=j\omega L|I|2=P+jQ$

所以$P=0,Q=\mathrm{Im}S=\omega L|I|2$

練習1: Capacitor C,$Z=1/j\omega C$,reactive power $Q=-\omega C|V|2$

$v(t)=\sqrt{2}|V|\cos(\omega t+\theta)$

$i(t)=Cdv/dt=-\sqrt{2}\omega C|V|\sin(\omega t+\theta)$

$p(t)=v(t)*i(t)=-2\omega C|V|2\sin(\omega t+\theta)\cos(\omega t+\theta)$

$=-\omega C|V|2\sin 2(\omega t+\theta)$

Average Power P=0,

瞬時功不為零(Instantaneous power is not zero)

$S=VI*=V(V/Z)*=|V|2/(Z)*=-j\omega C|V|2=P+jQ$

So $P=0,Q=ImS=-\omega C|V|2$

2.2複數功率守恆
(Conservation of Complex Power)

複數功率守恆(Sin = Sout):數個頻率相同的獨立電源供應的網路，

由各個獨立電源供應的複數功率的總和會等於網路會等於網路上所有分支接收到的複數功率

供電=用電(Power of generators are equal Loads)

EX2.3輸入電源並聯電容C

Input voltage with shunt C

Sin=Sc+So

Sc=VI*=V(V/Z)*=VV*(1/Z)*=VV*(Y)*=|V|2(SC)*

=-jωC|V|2

So=Sin-Sc=Sin+jωC|V|2

Po=Pin

Qo=Qin+ωC|V|2

EX2.4輸入電源串聯電感L(假設|V2|=|V1|)

Series L between two voltage source

S1+S2=SL=VI*=jωL|I|2

P1+P2=0

Q1+Q2=QL=ωL|I|2

S1=V1I*

S2=-V2I*

∵ |V2|=|V1|=>|S1|=|S2|=>(P1)2+(Q1)2=(P2)2+(Q2)2

∵ |P2|=|P1|=>|Q2|=|Q1|=>Q1=Q2=0.5ωL|I|2

So P1=-P2，Q1=Q2=>S2=-(S1)*

EX2.7三相電源(Three-phase voltages)

以n1為基點(n1 is basis point)

S=Van1Ia*+Vbn1Ib*+Vcn1Ic*(三瓦特計法)

以b為基點(b is basis point)

S=VabIa*+VbbIb*+VcbIc*=VabIa*+VcbIc*(二瓦特計法)

2.3平衡三相(Balanced Three-Phase) 重要!

直流電與交流電的優缺點

Advantages and disadvantages of DC and AC voltages

單相交流電與三相交流電的優缺點

Advantages and disadvantages of single-phase voltage and three-phase voltages

正序與負序(產生旋轉磁場)，零序

Positive sequence, negative sequence, zero sequence

平衡與不平衡電壓和負載(線性與非線性負載)

Balanced and unbalanced voltages and loads

中性點電壓與電流(voltage and current of neutral point)

Δ-Y

$\angle 0^\circ \quad \angle -120^\circ \quad \angle -240$

$a \longrightarrow b \longrightarrow c$

$a \longrightarrow c \longrightarrow b$

$\angle 0^\circ \quad \angle -120^\circ \quad \angle -240^\circ$

EX2.8三相電源與負載中性點電壓

Three-phase voltages and neutral point voltage

以n1為基點(n1 is basis point)

Ia=Van1/Z=(Van-Vn1n)/Z=(Van-Vn1n)Y

Ib=Vbn1/Z=(Vbn-Vn1n)/Z=(Vbn-Vn1n)Y

Ic=Vcn1/Z=(Vcn-Vn1n)/Z=(Vcn-Vn1n)Y

So Ia+Ib+Ic=(Van+Vbn+Vcn)Y-3Vn1nY=0

If (Van+Vbn+Vcn)=0 =>Vn1n=0

EX2.9中性點阻抗不為零時?

Δ-Y

Y 人

Δ型負載無法分析，要變成Y。

阻抗(Impedance)

ZY=ZΔ/3

EX2.10線對線電壓與相電壓?

Line-to-line voltages and Phase voltages?

2.4單相分析(平衡三相)Per Phase Analysis

平衡三相(Balanced three-phase)

假設:平衡三相系統；負載與電源是星形連接；電路模型中，相之間無互感存在

所以:所有的中性點電位相同；各相是完全去耦合；所有對應的網路變數和平衡電源系統具有相同相序

EX2.11 Balanced three-phase?

2.5平衡三相功率(瞬時功率為常數)

Power of the balanced three-phase is constant

S3=VaIa*+VbIb*+VcIc*

Balanced three-phase and positive sequency 用exponential去消他

S3=VaIa*+Vae -j2 π/3(Iae -j2π/3)*+ Vae j2 π/3(Iae j2π/3)*=3VaIa*

Instantaneous Power:p3(t)=pa(t)+pb(t)+pc(t)

p3(t)=va(t)ia(t)+vb(t)ib(t)+vc(t)ic(t)

va(t)*ia(t)=Vmcos(ωt+θv)*Imcos(ωt+θi)

=0.5*VmIm[cos(θv-θi)+cos(2ωt+θv+θi)]

vb(t)*ib(t)=Vmcos(ωt+θv-2π/3)*Imcos(ωt+θi-2π/3)

=0.5*VmIm[cos(θv-θi)+cos(2ωt+θv+θi-4π/3)]

vc(t)*ic(t)=Vmcos(ωt+θv+2π/3)*Imcos(ωt+θi+2π/3)

=0.5*VmIm[cos(θv-θi)+cos(2ωt+θv+θi+4π/3)]

p3(t)=3*0.5*VmIm[cos(θv-θi)]=3|V||I|cos(θv-θi)

加电容,加电感

2.6複數功率傳輸(短程)Complex power Transmission

100km

短程輸電線，用串聯的RL電路來表示電線$Z=R+j\omega L$

$V1=|V1|ej\theta 1$，$V2=|V2|ej\theta 2$，$Z=|Z|\ ej\angle Z$，$\theta 12=\theta 1-\theta 2$

$S12=V1I1^*=V1[(V1-V2)/Z]^*= |V1|2/(Z)^*-V1V2^*/(Z)^*$

$= |V1|2\ ej\angle Z\ /|Z|-|V1||V2|\ ej\angle Zej\theta 12/|Z|$

$S21=|V2|2\ ej\angle Z\ /|Z|-|V1||V2|\ ej\angle Ze\text{-}j\theta 12/|Z|$

Assume $R=0$，$Z=jX \Rightarrow \angle Z=90°$，$ej\angle Z=j$

So $P12= -P21=(|V1||V2|/X)\sin\theta 12$

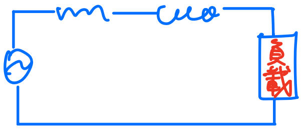
$Q12=|V1|2/X - (|V1||V2|/X)\cos\theta 12$

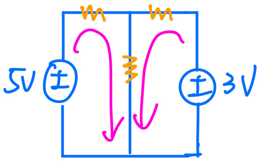
$Q21=|V2|2/X - (|V1||V2|/X)\cos\theta 12$

EX2.12兩個發電機失去同步? Two generators without synchronous?

$P12= -P21=(|V1||V2|/X)\sin[(\omega 1- \omega 2)t+\theta 12]$

電力系統 ① 只有一部發电機(少數)





② 兩台發电機

③ 多個發电機

EX2.13(短程 short distance) $Z=1\angle 85°$，$\theta 12=10°$

(a)$|V1|=|V2|=1$

$S12=V1I1^*=V1[(V1-V2)/Z]^*= |V1|2/(Z)^*-V1V2^*/(Z)^*$

$= |V1|2\ ej\angle Z\ /|Z|-|V1||V2|\ ej\angle Zej\theta 12/|Z|$

$S12=1\angle 85°-1\angle 95°$

$S21=|V2|2\ ej\angle Z\ /|Z|-|V1||V2|\ ej\angle Ze-j\theta 12/|Z|$

$S21=1\angle 85°-1\angle 75°$

$P12= -P21=0.1743$

$Q12=0$

$Q21=0.0303$

(b)$|V1|=1.1$ ，$|V2|=0.9$ ，$Z=1\angle 85°$，$\theta 12=10°$

EX2.14 SG1:$V1=1\angle 0°$，SD1=1，jQG2 :V2=? ，SD2=1，Z=j0.5

(a)Find QG2 for $|V2|=1$ (b) and $\angle V2$?(c)If QG2 =0,could be supplied load SD2 ?(d) and $\angle V2$?

∵ SD2=1 real，and jQG2 imaginary number，So P12= -P21=1

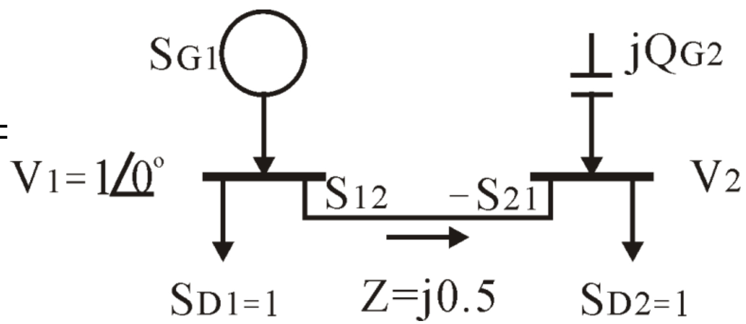
So $P12= -P21=(|V1||V2|/X)\sin\theta12=2\sin\theta12=1$

So $\theta12=30°$，and $\angle V2=-30°$

$QG2 =Q21=|V2|2/X - (|V1||V2|/X)\cos\theta12=2-2\cos30°=0.268$

(c) and (d)

If QG2 =0,-S21= SD2 =1



$S21=|V2|2\ ej\angle Z\ /|Z|-|V1||V2|\ ej\angle Ze-j\theta12/|Z|=$

Find $2|V2|2=1,\theta12=45° \Rightarrow V2=0.707\angle -45°$

(c) and (d)

If QG2 =0,so $QG2 =Q21=|V2|2/X - (|V1||V2|/X)\cos\theta12=0$

so $|V2|=|V1|\cos\theta12= \cos\theta12$

If $P12= -P21=(|V1||V2|/X)\sin\theta12=2|V2|\sin\theta12=1$

so $\theta12=45°,|V2|=0.707$

2.7複數功率傳輸(輻射線路)

Complex Power Transmission: Radial Line

較遠的一端有複數功率負載，沒有發電機或電容器組來維持電壓，求遠端電壓受負載變化的影響?

$S_D=V_2I^*=|V_2||I|e^{j\psi}$

$=|V_2||I|(\cos\psi+j\sin\psi)=P_D(1+j\beta)$

$P_D=P_{12}= -P_{21}=(|V_1||V_2|/X)\sin\theta_{12}$

$Q_D= -Q_{21}= -|V_2|^2/X + (|V_1||V_2|/X)\cos\theta_{12}$

2.8結論與習題(Summary)

瞬時功率(Instantaneous Power)
複數功率(Complex Power)
有效功率(Real Power)
無效功率(Reactive Power)
相量(Phasor)
平衡三相(Balanced Three-Phase)
單相分析(Per Phase Analysis)