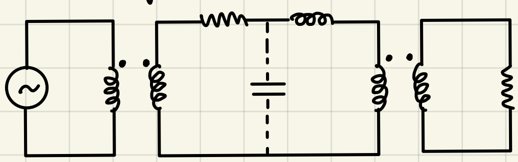
CH1. Introduction
CH2. 基本原理 (複數功率伝輸
CH3. 輸电線參數 (RLC)
CH4. 輸电線模型 (短中長距離. π型)
CH5. 變压器 (標么. per-unit)
CH6. 电力潮流 (很多的發电机)
CH10. 發电机模型
CH13. 不平衡系統操作

AC power system



① 同頻率
② 同相角 (微調)
③ 同电压 (振幅. 微調)
④ 同相序

Power System Analysis
供電=用電

Arthur R. Bergen
授課：侯中權 博士
(Prof. Chung-Chuan Hou)

6.電力潮流分析(Power Flow Analysis)

6.0簡介(Introduction)

6.1電力潮流方程式(Power Flow Equations)

6.2電力潮流問題(The Power Flow Problem)

6.3利用高斯疊代法求解(Solution by Gauss Iteration)

6.4通用疊代法(More General Iteration Scheme)

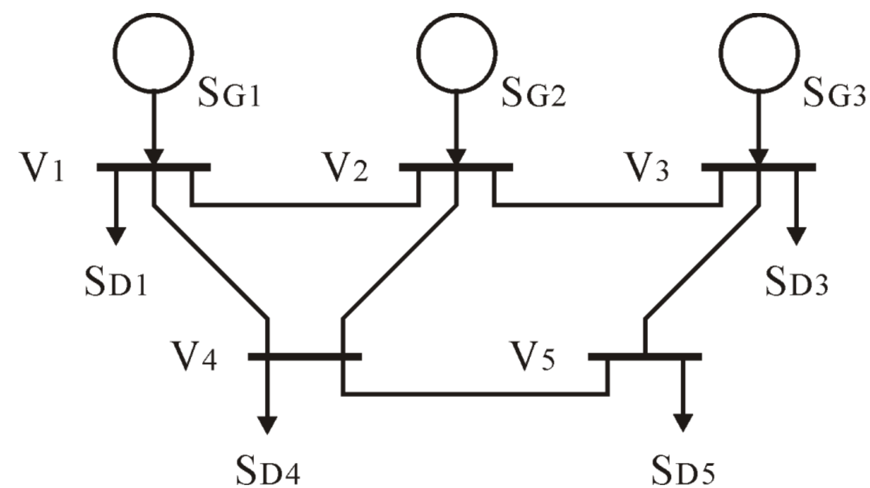
6.5牛頓-拉夫生疊代法(Newton-Raphson Iteration)

6.6應用於電力潮流方程式(Application to Power Flow Equations)

6.7分解電力潮流 (Decoupled Power Flow)

6.8控制上的含意 (Control Implications)

6.9結論與習題(Summary)



6.0 簡 介(Introduction)

In power flow analysis the transmission system is modeled by a set of buses or nodes interconnected by transmission lines. Generators and loads, connected to various nodes of the system, inject and remove power from the transmission system.

The model is appropriate for solving for the steady-state powers and voltages of the transmission system.

The purpose of a power system is to deliver the power the customers require in real time, on demand, within acceptable voltage and frequency limits, and in a reliable and economic manner.

In system operation and planning it is also extremely important to consider the economy of operation.

6.1電力潮流方程式(Power Flow Equations)

Define the complex per phase bus power, S_i , as follows

$S_i = S_{Gi} – S_{Di}$, S_i is what is left of S_{Gi} after stripping away the local load S_{Di} .

Using conservation of complex power, we also have for the *i*th bus,

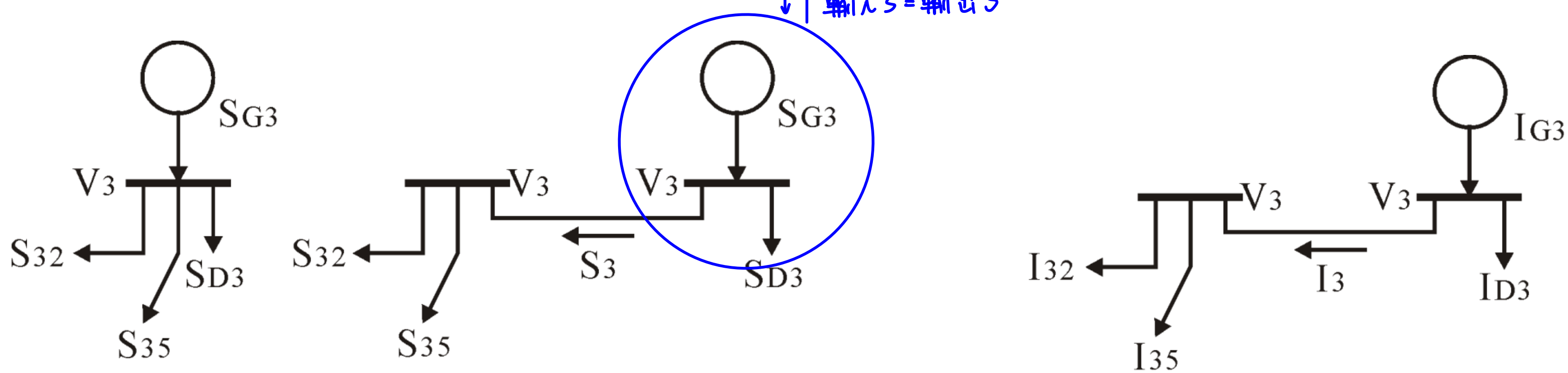
(6.1) $S_i = \Sigma^n{}_{k=1} S_{ik}$, $i = 1,2,,,n$

where we sum S_{ik} over all the transmission links connected to the *i*th bus.

We also define the bus current Ii (the total phase a current entering the transmission system.

$I_i = I_{Gi} – I_{Di} = \Sigma^n{}_{k=1} I_{ik}$, $i = 1,2,,,n$

↓↑ 輸入S=輸出S



Ex6.1 Developed the injected node currents and the node voltages

$I_1 = I_{12} + I_{13} = Y_1V_1 + Y_2(V_1 - V_2) + Y_4V_1 + Y_5(V_1 - V_3)$

$= (Y_1 + Y_2 + Y_4 + Y_5)V_1 - Y_2V_2 - Y_5V_3$,

$I_2 = I_{21} + I_{23} = Y_3V_2 + Y_2(V_2 - V_1) + Y_9V_2 + Y_8(V_2 - V_3)$

$= -Y_2V_1 + (Y_2 + Y_3 + Y_8 + Y_9)V_2 - Y_8V_3$,

$I_3 = I_{31} + I_{32} = Y_6V_3 + Y_5(V_3 - V_1) + Y_7V_3 + Y_8(V_3 - V_2)$

$= -Y_5V_1 - Y_8V_2 + (Y_5 + Y_6 + Y_7 + Y_8)V_3$,

$I = Y_{bus}V$, $I = [I_1\ I_2\ I_3]^T$, $V = [V_1\ V_2\ V_3]^T$,

Y_{bus} is bus admittance matrix

$Y_{bus} = [(Y_1 + Y_2 + Y_4 + Y_5)\ -Y_2\ -Y_5$;

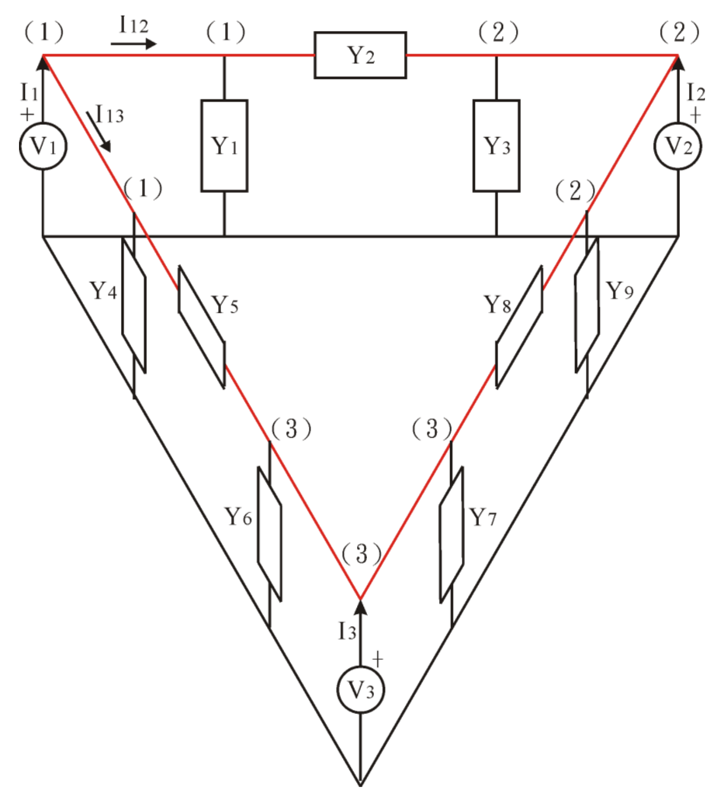
$-Y_2\ \ (Y_2 + Y_3 + Y_8 + Y_9)\ -Y_8$;

$-Y_5 - Y_8\ \ (Y_5 + Y_6 + Y_7 + Y_8)]$

$I_i = \Sigma^n_{k=1} I_{ik} = \Sigma^n_{k=1} Y_{ik} V_k$, $i = 1,2,,,n$

Only V_1 and V_2 without V_3 ,

$Y_{bus} = [(Y_1 + Y_2)\ -Y_2\ ; -Y_2\ \ (Y_2 + Y_3)]$



Calculate the ith bus power ($S_i = V_i I_i^*$, Power Flow Equations)

(6.2) $I_i = \Sigma^n_{k=1} I_{ik} = \Sigma^n_{k=1} Y_{ik} V_k$, $i = 1,2,,,n$

(6.3) $S_i = V_i I_i^* = V_i (\Sigma^n_{k=1} Y_{ik} V_k)^* = V_i \; \Sigma^n_{k=1} Y_{ik}^* V_k^*$, $i = 1,2,,,n$

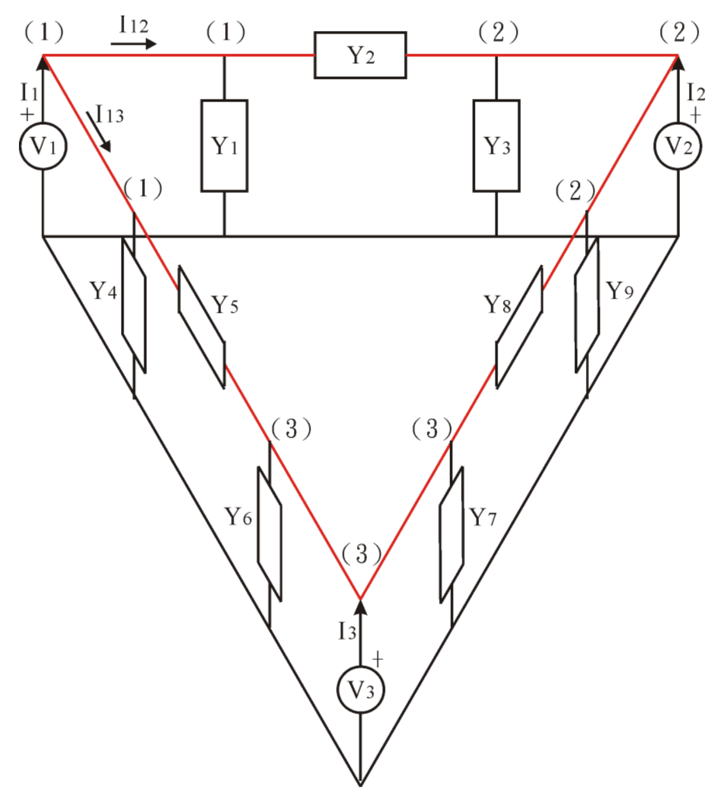
Assume $V_i = |V_i| e^{j\angle Vi} = |V_i| e^{j\theta i}$, $\theta_{ik} = \theta_i - \theta_k$, $Y_{ik} = G_{ik} + jB_{ik}$,

(6.4) $S_i = \Sigma^n_{k=1} |V_i| |V_k| e^{j\theta ik} (G_{ik} - jB_{ik})$, $i = 1,2,,,n$

$= \Sigma^n_{k=1} |V_i| |V_k| (\cos \theta_{ik} + j\sin \theta_{ik}) (G_{ik} - jB_{ik})$, $i = 1,2,,,n$

$P_i = \Sigma^n_{k=1} |V_i| |V_k| (G_{ik} \cos \theta_{ik} + B_{ik} \sin \theta_{ik})$

$Q_i = \Sigma^n_{k=1} |V_i| |V_k| (G_{ik} \sin \theta_{ik} - B_{ik} \cos \theta_{ik})$



Ex 6.2 assume series impedance $Z_L = j0.1$, parallel admittance $Y_C = j0.01$

$Z_L = j0.1 \Rightarrow Y_L = -j10$,

$I_1 = (Y_1 + Y_2 + Y_4 + Y_5)V_1 - Y_2V_2 - Y_5V_3 = -j19.98V_1 + j10V_2 + j10V_3$,

$I_2 = -Y_2V_1 + (Y_2 + Y_3 + Y_8 + Y_9)V_2 - Y_8V_3 = j10V_1 - j19.98V_2 + j10V_3$,

$I_3 = -Y_5V_1 - Y_8V_2 + (Y_5 + Y_6 + Y_7 + Y_8)V_3 = j10V_1 + j10V_2 - j19.98V_3$,

$I = Y_{bus}V$, $I = [I_1\ I_2\ I_3]^T$, $V = [V_1\ V_2\ V_3]^T$,

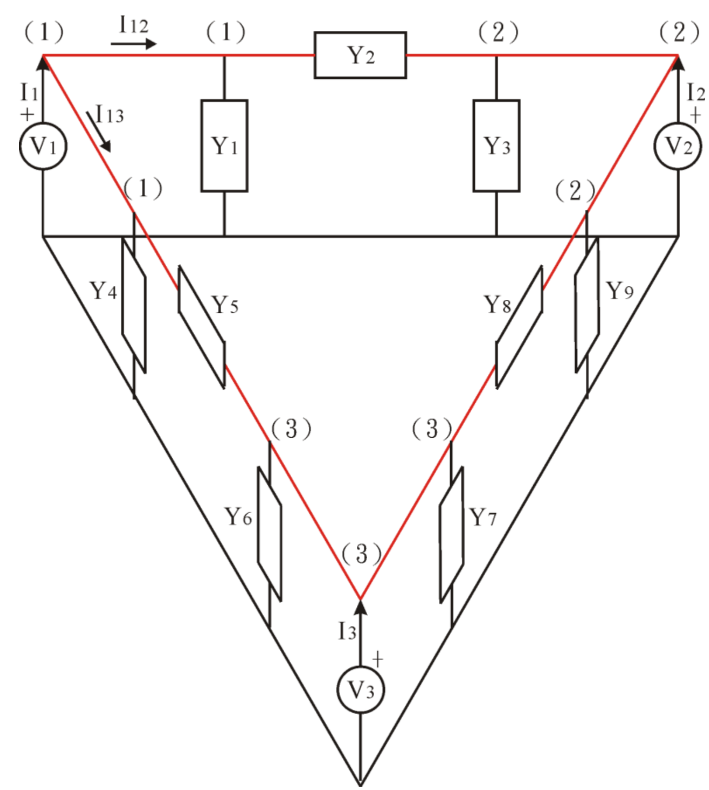
$I_i = \Sigma^n_{k=1} I_{ik} = \Sigma^n_{k=1} Y_{ik} V_k$, $i = 1,2,,,n$

$S_i = V_i I_i^* = V_i (\Sigma^n_{k=1} Y_{ik} V_k)^*$

$S_1 = j19.98|V_1|^2 - j10V_1V_2^* - j10V_1V_3^*$

$S_2 = -j10V_2V_1^* + j19.98|V_2|^2 - j10V_2V_3^*$

$S_3 = -j10V_3V_1^* - j10V_3V_2^* + j19.98|V_3|^2$



6.2電力潮流問題(The Power Flow Problem)

Some buses are supplied by generators. We call these generators buses. Other buses without generators are called load buses.

In summary, there are three types of sources at the different buses:

1. A voltage source. Assume at bus 1 (slack bus or swing bus: 弛放或搖擺滙流排).
2. P, |V| sources (voltage control buses). At the other generator buses.
3. P, Q sources (load buses). At the load buses.

In the case of a load bus with capacitors the bus may be identified as a P, Q bus if the capacitors supply a fixed reactive power, or it may be a P, |V| bus if the capacitor are utilized to maintain a specified P(=0) and |V|.

Sometimes Q rather than |V| is specified at a generator bus. In this case we include it with the load buses. Unless otherwise indicated, we assume voltage control at generator buses.

Two versions of the Power Flow Problem

n個方程式解n個未知數

In both cases we assume that bus 1 is the slack (or swing) bus.

In Case I we assume that all the remaining buses are P, Q buses.

Case I: Given $V_1, S_2, S_3, \ldots, S_n$,

Find $S_1, V_2, V_3, \ldots, V_n$

In Case II we assume both P, |V| and P, Q buses. We number the buses so that buses 2, 3, . . . , m are P, |V| buses and m+1, . . . , n are P, Q buses.

Case II: Given $V_1, (P_2, |V_2|), \ldots, (P_m, |V_m|), S_{m+1}, \ldots, S_n$,

Find $S_1, (Q_2, \angle V_2), \ldots, (Q_m, \angle V_m), V_{m+1} \ldots, V_n$

Ex6.3 $V_1 = 1\angle 0°$, $jQ_{G2} = j1.0$, $Z_L = j0.5$, $S_{D2} = P_{D2} + j1.0$. Find S_1 and V_2. We consider the solution as a function of P_{D2} for $P_{D2} \geqq 0$.

In this case, the capacitor injects a specified power, while the voltage is not controlled. Thus bus2 is a P, Q bus. In fact,

$S_2 = S_{21} = S_{G2} - S_{D2} = j1.0 - (P_{D2} + j1.0) = - P_{D2}$

$S_{12} = |V_1|^2 e^{j\angle Z}/|Z| - |V_1||V_2| e^{j\angle Z} e^{j\theta 12}/|Z|$

$S_{21} = |V_2|^2 e^{j\angle Z}/|Z| - |V_1||V_2| e^{j\angle Z} e^{-j\theta 12}/|Z|$

$R=0$，$Z=jX=j0.5 \Rightarrow \angle Z=90°$，$e^{j\angle Z}=j$, $|V_1|=1$

$S_{21} = j2|V_2|^2 - j2|V_2| e^{-j\theta 12} = - P_{D2}$

We draw a receiving-end circle:

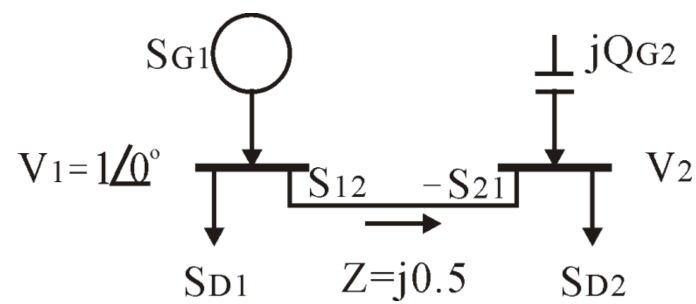
$(2|V_2|^2)^2 + (P_{D2})^2 = (2|V_2|)^2 \Rightarrow 4|V_2|^4 - 4|V_2|^2 + (P_{D2})^2 = 0$

$|V_2|^2 = [1 \pm (1 - (P_{D2})^2)^{0.5}]/2$

If $P_{D2} > 1 \Rightarrow |V_2|$ no solution,

If $P_{D2} = 1 \Rightarrow |V_2| = 0.707$, $\theta_{12} = 45°$

If $0 \leqq P_{D2} < 1 \Rightarrow |V_2|$ has two solution



We can find θ_{12} and $S_1 = S_{12}$

If $P_{D2} = 0.5 \Rightarrow V_2 = 0.97\angle -15°$ (OK) and $V_2 = 0.26\angle -75°$ (not OK)

6.3利用高斯疊代法求解(Solution by Gauss Iteration)

Consider Case I of the power flow problem:

Given $V_1, S_2, S_3, \ldots, S_n$. Find $S_1, V_2, V_3, \ldots, V_n$.

(6.3a) $S_1 = V_1 I_1^* = V_1 (\Sigma^n{}_{k=1} Y_{1k} V_k)^* = V_1 (\Sigma^n{}_{k=1} Y_{1k}^* V_k^*)$

(6.3b) $S_i = V_i I_i^* = V_i (\Sigma^n{}_{k=1} Y_{ik} V_k)^*$, $i = 2, \ldots, n$

利用(6.3b) get $V_2, V_3, \ldots, V_n$.再利用(6.3a) get S_1

(6.5) $S_i^* = V_i^* I_i = V_i^* (\Sigma^n{}_{k=1} Y_{ik} V_k)$, $i = 2, \ldots, n$

(6.6) $S_i^*/V_i^* = (\Sigma^n{}_{k=1} Y_{ik} V_k) = Y_{ii} V_i + (\Sigma^n{}_{k=1,k\neq i} Y_{ik} V_k)$, $i = 2, \ldots, n$

(6.7) $V_i = (1/Y_{ii})[(S_i^*/V_i^*) - (\Sigma^n{}_{k=1,k\neq i} Y_{ik} V_k)]$, $i = 2, \ldots, n$

(6.8) $V_2 = x_1 = h_2 (V_2, V_3, \ldots, V_n), \ldots, V_n = x_{n-1} = h_n (V_2, V_3, \ldots, V_n)$

(6.9) $x = [x_1, \ldots, x_{n-1}] = h(x)$

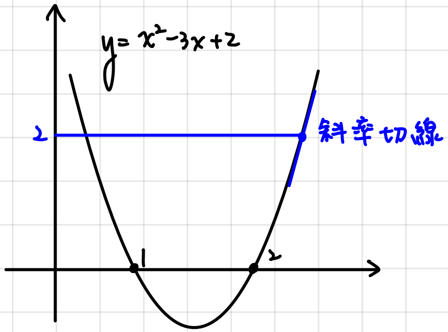
(6.10) $x^{v+1} = h(x^{v})$, iteration $v = 0, 1, 2, \ldots$

Guess an initial value $x^{v=0}$, $x^{v=1} = h(x^{v=0}) \Rightarrow x^{v=2} = h(x^{v=1}) \Rightarrow x^{v=3}, \ldots$

Define error $(\Delta x) = x^{v+1} - x^{v}$, when $\|\Delta x\| \leqq \varepsilon \Rightarrow$ converge

$\|\Delta x\| = \max |(\Delta x)i|$, sup norm , $i = 1, \ldots, n-1$

$\|\Delta x\| = [\Sigma^{n-1}{}_{i=1} |(\Delta x)i|^2]^{1/2}$, Euclidean norm



点斜式(猜)

操作点 $(x=3, y=2)$ $m=3\times2-3=3$

$m=\frac{dy}{dx}=2x-3$

$y=mx+b$

$y=3x-7=0 \Rightarrow x=\frac{7}{3}$

新操作點：$x=\frac{7}{3}$

高斯疊代(Gauss Iteration)高斯-西丹疊代(Gauss-Seidel Iteration)

(6.11)高斯疊代(Gauss Iteration), iteration v = 0, 1, 2, . . .

$x_1^{v+1} = h_1(x_1^{v}, x_2^{v}, \ldots, x_{n-1}^{v})$

$x_2^{v+1} = h_2(x_1^{v}, x_2^{v}, \ldots, x_{n-1}^{v})$

.

$x_{n-1}^{v+1} = h_{n-1}(x_1^{v}, x_2^{v}, \ldots, x_{n-1}^{v})$

(6.12)高斯-西丹疊代(Gauss-Seidel Iteration), iteration v = 0, 1, 2, . . .

$x_1^{v+1} = h_1(x_1^{v}, x_2^{v}, \ldots, x_{n-1}^{v})$

$x_2^{v+1} = h_2(x_1^{v+1}, x_2^{v}, \ldots, x_{n-1}^{v})$

.

$x_{n-1}^{v+1} = h_{n-1}(x_1^{v+1}, x_2^{v+1}, \ldots, x_{n-2}^{v+1}, x_{n-1}^{v})$

Ex6.4 $V_1 = 1\angle 0^\circ$, $jQ_{G2} = j1.0$, $Z_L = j0.5$, $S_{D2} = 0.5 + j1.0$. Find S_1 and V_2 by Gauss iteration. If $P_{D2} = 0.5 \Rightarrow V_2 = 0.97\angle -15^\circ$ (O) and $V_2 = 0.26\angle -75^\circ$ (X)

Bus2 is a P, Q bus. $S_{21} = V_2 I_2^* = V_2[(V_2 - V_1)/Z]^* = |V_2|^2 e^{j\angle Z}/|Z| - |V_1||V_2| e^{j\angle Z} e^{-j\theta 12}/|Z|$

$S_2 = S_{21} = S_{G2} - S_{D2} = j1.0 - (0.5 + j1.0) = -0.5$

$Z_L = j0.5$, $Y = 1/Z_L = -j2$, Ybus = [Y -Y ; -Y Y] = [-j2 j2; j2 –j2]

(6.7) $V_i = (1/Y_{ii})[(S_i^*/V_i^*) - (\Sigma^n_{k=1, k\neq i} Y_{ik} V_k)]$, $i = 2, \ldots, n$

$V_2 = (1/Y_{22})[S_2^*/V_2^* - Y_{21}V_1] = (1/-j2)[(-0.5)/(V_2^*) - (j2)1]$

$= -j(0.25)/(V_2^*) + 1$

Guess an initial value $V_2^{\ 0} = 1\angle 0^\circ \Rightarrow$

$V_2^{\ 1} = 1 - j0.25 = 1.030776\angle -14.036243^\circ$

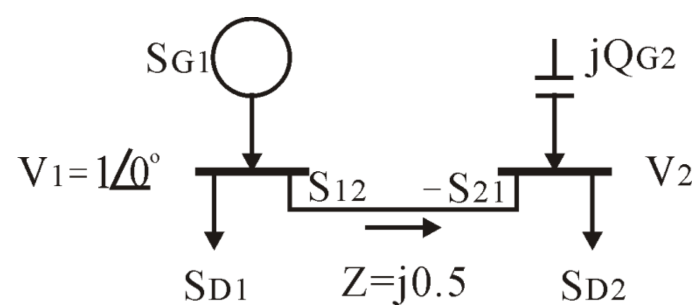
$V_2^{\ 2} = -j(0.25)/(1-j0.25)^* + 1 = 0.970143\angle -14.036249^\circ$

$V_2^{\ 3} = 0.970261\angle -14.931409^\circ$

$V_2^{\ 4} = 0.966235\angle -14.931416^\circ$

$V_2^{\ 5} = 0.966236\angle -14.995078^\circ$

$V_2^{\ 6} = 0.965948\angle -14.995072^\circ$



Guess an initial value $V_2^{\ 0} = 0.1\angle 0^\circ \Rightarrow V_2^{\ 8} = 0.965918\angle -14.995310^\circ$

Guess an initial value $V_2^{\ 0} = 0.25\angle -75^\circ \Rightarrow V_2^{\ ?} = 0.97\angle -15^\circ$ (why ?)

Case II of the Power Flow Problem

Given V_1, $(P_2, |V_2|)$, . . . , $(P_m, |V_m|)$, S_{m+1}, . . . , S_n,

Find S_1, $(Q_2, \angle V_2)$, . . . ,$(Q_m, \angle V_m)$, V_{m+1} . . . , V_n

(6.7) $V_i = (1/Y_{ii})[(S_i^*/V_i^*) - (\Sigma^n_{k=1,k\neq i} Y_{ik} V_k)]$, $i = 2, \ldots, n$

(6.13) $V_i^{v+1} = (1/Y_{ii})[(P_i - jQ_i^v)/(V_i^v)^* - (\Sigma^n_{k=1,k\neq i} Y_{ik} V_k)]$, $i = 2, .., n$

(6.14) $Q_i^v = \text{Im}[(V_i^v)(\Sigma^n_{k=1} Y_{ik}^*(V_k^v)^*]$, $i = 2, \ldots, m$

(6.3a) $S_1 = V_1 I_1^* = V_1 (\Sigma^n_{k=1} Y_{1k} V_k)^* = V_1 (\Sigma^n_{k=1} Y_{1k}^* V_k^*)$

(6.3b) $S_i = V_i I_i^* = V_i (\Sigma^n_{k=1} Y_{ik} V_k)^*$, $i = 2, \ldots, n$

(2.26) $S_{12} = V_1 I_1^* = V_1[(V_1 - V_2)/Z]^* = |V_1|^2/(Z)^* - V_1 V_2^*/(Z)^*$

$= |V_1|^2 e^{j\angle Z}/|Z| - |V_1||V_2| e^{j\angle Z} e^{j\theta 12}/|Z|$

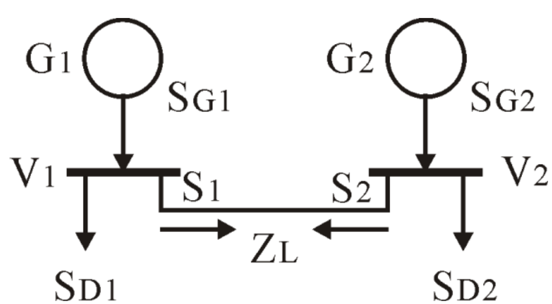
(2.27) $S_{21} = |V_2|^2 e^{j\angle Z}/|Z| - |V_1||V_2| e^{j\angle Z} e^{-j\theta 12}/|Z|$

Assume $R=0$，$Z=jX \Rightarrow \angle Z = 90^\circ$，$e^{j\angle Z} = j$

So (2.31) $P_{12} = -P_{21} = (|V_1||V_2|/X)\sin\theta_{12}$

(2.32) $Q_{12} = |V_1|^2/X - (|V_1||V_2|/X)\cos\theta_{12}$

(2.33) $Q_{21} = |V_2|^2/X - (|V_1||V_2|/X)\cos\theta_{12}$



Ex6.5 Gauss iteration for Case II. The power flow problem can be stated as follows: given $V_1=1\angle 0^\circ$ and $P_2= -0.75$, $V_2=1\angle V_2$, $Z_L=j0.5$, $S_{G2}=0.25+jQ_{G2}$, $S_{D2}=1.0+j0.5$. Find S_1, Q_2, $\angle V_2$

$S_2 = S_{21} = S_{G2} - S_{D2} = (0.25+jQ_{G2}) - (1.0 +j0.5)$

$Z_L =j0.5$, $Y=1/ Z_L = -j2$, Ybus = [Y -Y ; -Y Y] = [-j2 j2; j2 –j2]

(6.13) $V_i^{\,v+1} = (1/Y_{ii})[(P_i -jQ_i^{v})/(V_i^{v})^* - (\Sigma^n_{k=1,k\neq i} Y_{ik} V_k)]$, $i = 2, . . , n$

(6.14) $Q_i^{v} = \mathrm{Im}\,[(V_i^{v}) (\Sigma^n_{k=1} Y_{ik}^*(V_k^{v})^*]$, $i = 2, ... , m$

$V_2^{\,v+1} =(-1/j2)[(-0.75 - jQ_{G2}^{v})/(V_2^{v})^* - j2 \times 1]$

$Q_2^{v} = \mathrm{Im}\,[(V_2^{v}) (\Sigma^n_{k=1} Y_{ik}^*(V_k^{v})^*]= \mathrm{Im}\,\{V_2^{v} [Y_{21}^*V_1^*+Y_{22}^*(V_2^{v})^*]\}$

$= \mathrm{Im}\{V_2^{v} [-j2 \times 1 +j2(V_2^{v})^*]\} = \mathrm{Im}\,(-j2V_2^{v} +j2|V_2^{v}|^2)= -2\mathrm{Re}(V_2^{v})+2$

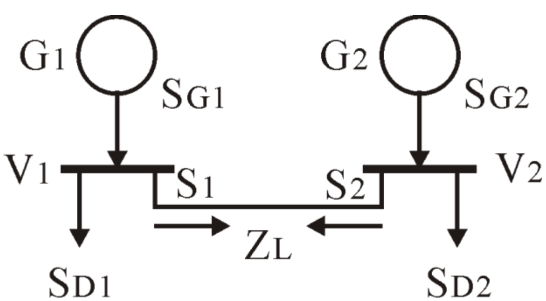
$V_2^0 =1\angle 0^\circ \Rightarrow Q_2^0 =0 \Rightarrow V_2^1 =(-1/j2)[(-0.75-j0)-j2]=1-j0.375=1.068\angle -20.556^\circ$

$V_2^1 =1\angle -20.556^\circ \Rightarrow Q_2^1 =0.12734 \Rightarrow V_2^2 =(-1/j2)[(-0.75-j0.12734)/(1\angle -20.556^\circ)^* -j2]$
$=0.927965-j0.3734706=1.0003\angle -21.9229^\circ$

$V_2^2 =1\angle -21.9229^\circ$, $Q_2^2 =0.1446$, $V_2^3 =1.0000\angle -22.0169^\circ$

$V_2^3 =1\angle -22.0169^\circ$, $Q_2^3 =0.1459$, $V_2^4 =1.0000\angle -22.0238^\circ$

$V_2^4 =1\angle -22.0238^\circ$, $Q_2^4 =0.1459$,



$P_{12}= -P_{21}=(|V_1||V_2|/X)\sin\theta_{12} =0.75 \Rightarrow \theta_{12} = 22.0238^\circ$

$Q_2 =Q_{21}=|V_2|^2/X - (|V_1||V_2|/X)\cos\theta_{12} =0.1459$

$S_1=V_1 I_1^*=V_1(\Sigma^n_{k=1} Y_{1k}^*V_k^*)=V_1(Y_{11}^*V_1^*+Y_{12}^*V_2^*)= j2-j2\angle 22.0238^\circ=0.75+j0.1459$

6.4通用疊代法(More General Iteration Scheme) y=f(x)=mx+b

If we use Gauss or Gauss-Seidel, sometimes we get convergence, sometimes not. It is known that convergence (and existence and uniqueness of solutions) is assumed if the map x->h(x) is so called contraction mapping (縮型映射).

In general, the conditions required are hard to check and in practice we just try the iterative scheme and hope for convergence.

Still, we would like some control over convergence. This is not available using the basic Gauss or Gauss-Seidel scheme. We need and will now derive a more general formula.

We will also formulate the problem in a slightly different way. For the general discussion we will use the notation f(x), reserving h(x) for the equations defined in

(6.9) $x = [x_1, . . ,x_{n-1}] = h(x)$

(6.10) $x^{v+1} = h(x^{v})$, iteration $v = 0, 1, 2, . . .$

Problem: Solve $f(x) = 0$.

Method: Use the iteration formula $x^{v+1} = \Phi(x^{v})$ with the function Φ still to be determined. Starting with an initial value x^0 .

More General Iteration Scheme:
Solve $f(x) = 0$. Use the iteration formula $x^{v+1} = \Phi(x^{v})$

(6.15) $x^* = \Phi(x^*) \Longleftrightarrow f(x^*)=0$

For any x, it satisfies

(6.16) $A(x)[\, x - \Phi(x)] = f(x)$, where $A(x)$ is a nonsingular matrix.

(6.17) $\Phi(x) = x - A(x)^{-1}f(x)$, $x^{v+1} = x^{v} - [A(x^{v})]^{-1} f(x^{v})$, until $f(x^{v}) =0$.

As a special case we consider the solution of $f(x)=0$ with $A(x)^{-1} = \alpha I$, where α is a real scalar and I is the identity matrix.

(6.18) $x^{v+1} = x^{v} - \alpha f(x^{v})$, solution $f(x) = x - h(x)=0$

When $f(x^{v+1}) - f(x^{v}) = m(x^{v+1} - x^{v}) = (1/\alpha)(x^{v+1} - x^{v})$, $f(x^{v+1}) =0$

(6.19) $x^{v+1} = x^{v} - \alpha\, [x^{v} - h(x^{v})]$

If $\alpha =1$, $x^{v+1} = h(x^{v})$, Gauss (or Gauss-Seidel) scheme.

Choosing $\alpha \neq 1$, accelerated Gauss (or Gauss-Seidel) , α is the accelerated factor, α can be either positive or negative.

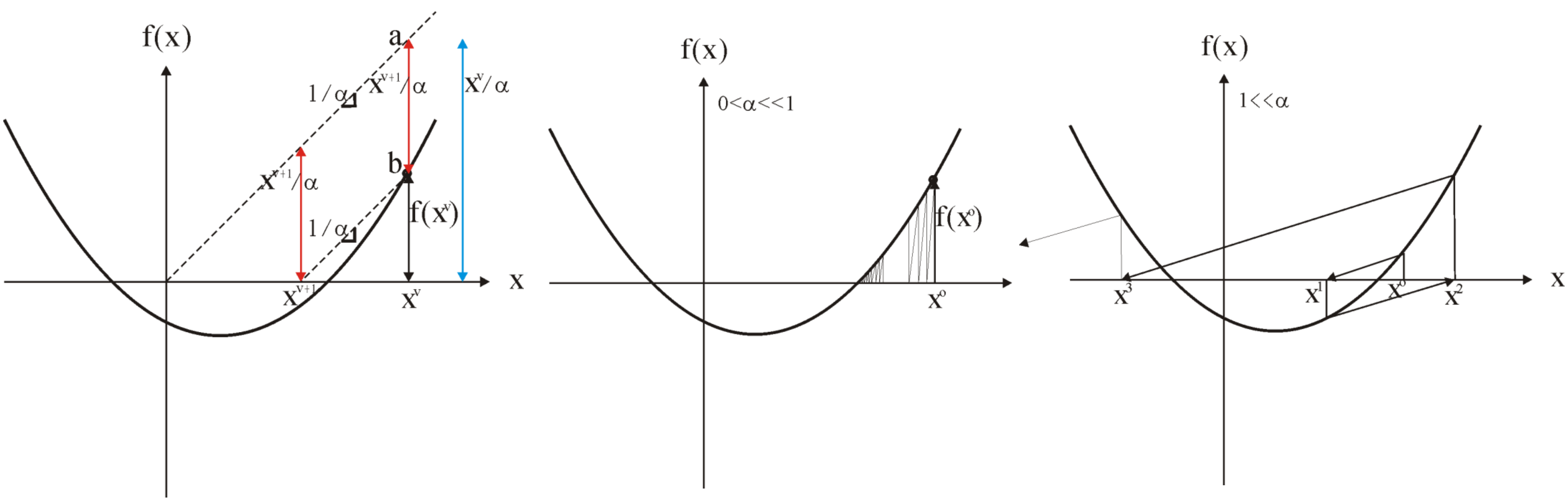
Ex6.6 Solve f(x) = 0, where f(x) is shown as follow.

(6.18) $x^{v+1} = x^{v} - \alpha f(x^{v})$, solution $f(x) = x - h(x)=0$

$(1/\alpha)x^{v+1} = (1/\alpha)x^{v} - f(x^{v})$

1. Given x^v, move vertically to $f(x^v)$. 2. Return to horizontal axis along a line of slope $(1/\alpha)$. 3. The horizontal-axis intercept of the line is x^{v+1} .

$(0<\alpha<<1)$ Sluggish (遲鈍) , $(1<<\alpha)$ Unstable (不穩定)



Exercise 2. $f(x) = x^2 - x - 2, 0$, (a) Find a good value of α and a range of value for x^0 such that the sequence of iterations converges to the zero at $x = 0$. (b) Repeat part (a) to converge to the zero at $x = -1$. Exercise 3. $f(x) = x - 1$. Find a value of α such that the sequence of iterations (a) is monotonically convergent (單調收斂). (b) converges in one step (一步收斂). (c) is oscillatory and convergent (振盪與收斂). (d) is oscillatory and divergent (振盪與發散). (e) is monotonically divergent (單調發散).

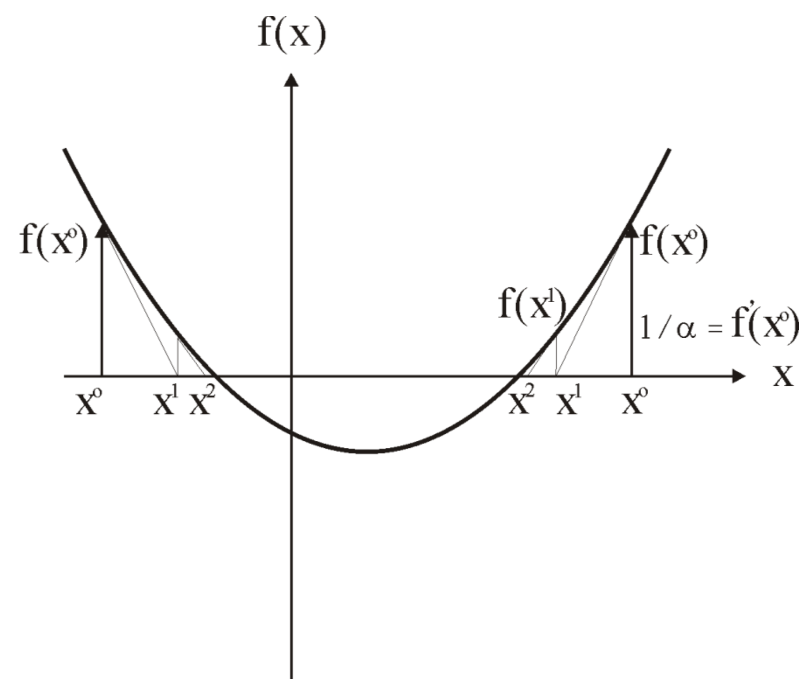
Picking $(1/\alpha) = f'(x^v)$, where f' is the derivative of f. Iterations under this scheme are shown in figure solving $f(x) = 0$.

(6.20) $x^{v+1} = x^v - [f'(x^v)]^{-1} f(x^v)$

Slope $= [f(x^{v+1}) - f(x^v)]/[(x^{v+1}) - (x^v)]$

$= [f'(x^v)]$

操作桌: $f_1(x_1 \ldots x_n)$
$f_2(x_1 \ldots x_n)$
$f_3(x_1 \ldots x_n)$
$f_4(x_1 \ldots x_n)$



6.5牛頓-拉夫生疊代法(Newton-Raphson Iteration) $x^{v+1} = x^v - [f'(x^v)]^{-1} f(x^v)$

(6.20) $x^{v+1} = x^v - [f'(x^v)]^{-1} f(x^v)$

(6.21) $f(x+\Delta x) = f(x) + f'(x)\,\Delta x + \text{h.o.t.}$ (higher-order terms) (Taylor Series)

(6.22) $f_1(x+\Delta x) = f_1(x)+[\partial f_1(x)/\partial x_1]\,\Delta x_1 +\ldots+[\partial f_1(x)/\partial x_n]\,\Delta x_n + \text{h.o.t}$

$f_2(x+\Delta x) = f_2(x)+[\partial f_2(x)/\partial x_1]\,\Delta x_1 +\ldots+[\partial f_2(x)/\partial x_n]\,\Delta x_n + \text{h.o.t}$

. . .

$f_n(x+\Delta x) = f_n(x)+[\partial f_n(x)/\partial x_1]\,\Delta x_1 +\ldots+[\partial f_n(x)/\partial x_n]\,\Delta x_n + \text{h.o.t}$

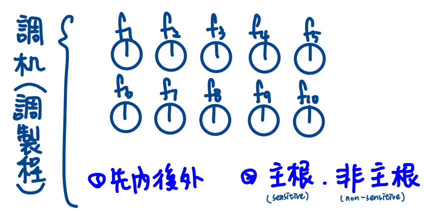
(6.23) $f(x+\Delta x) = f(x) + J(x)\,\Delta x + \text{h.o.t.}$

斜率矩陣

(6.24) $J(x)=[\partial f_1(x)/\partial x_1 \;. . .\; \partial f_1(x)/\partial x_n \;; . . . ;\; \partial f_n(x)/\partial x_1 \;. . .\; \partial f_n(x)/\partial x_n\,]$

$\Delta x = [\Delta x_1\ \Delta x_2\ . . .\ \Delta x_n\,]$, $J(x)$ is Jacobian Matrix

General Newton-Raphson Iteration



(6.25) $x^{v+1} = x^v - [J(x^v)]^{-1} f(x^v)$

(6.26) $\Delta x^v = x^{v+1} - x^v = -[J(x^v)]^{-1} f(x^v)$

(6.27) $J(x^v)\,\Delta x^v = -f(x^v)$

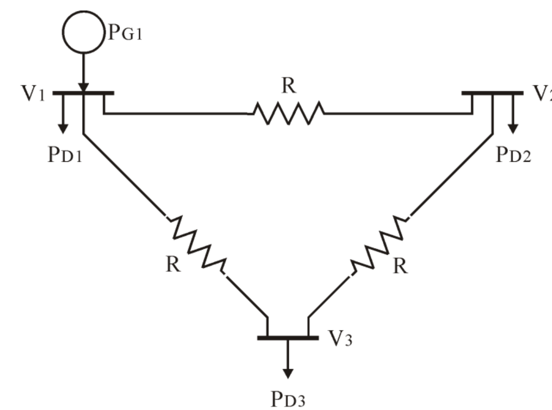
Calculating inverse matrix is computationally expensive and not really needed.

Ex 6.7 Given the DC system , use the Newton-Raphson method to find the DC bus voltages V_2, V_3, and P_{G1} .

$R = 0.01$ ($Y=1/R=100$), $V_1 = 1.0$, $P_{D1} = 0.5$, $P_{D2} = 1.0$, $P_{D3} = 0.5$

$Ybus = [(Y_2 +Y_5) -Y_2 -Y_5 ; -Y_2 \ (Y_2 +Y_8) -Y_8 \ ;- Y_5 - Y_8 \ (Y_5 +Y_8)]$

$= 100[2 \ -1 \ -1 \ ; -1 \ 2 \ -1 \ ; -1 \ -1 \ 2]$



Using $S_i = V_i I_i^* = V_i (\Sigma^n_{k=1} Y_{ik} V_k)^*$

$P_1 = 200(V_1)^2 - 100V_1V_2 - 100V_1V_3 =P_{G1} -P_{D1}$

$P_2 = -100V_2V_1 + 200(V_2)^2 -100V_2V_3 = -P_{D2} = -1.0$

$P_3 = -100V_3V_1 -100V_3V_2 +200(V_3)^2 = -P_{D3} = -0.5$

$f_1(x) = -P_2 -100V_2V_1 + 200(V_2)^2 -100V_2V_3 = 1.0 -100V_2 + 200(V_2)^2 -100V_2V_3 =0$

$f_2(x) = -P_3 -100V_3V_1 -100V_3V_2 +200(V_3)^2 = 0.5 -100V_3 -100V_3V_2 +200(V_3)^2 =0$

$V_2 = x_1$, $V_3 = x_2$, $J(x)=[\partial f_1(x)/\partial x_1 \ \partial f_1(x)/\partial x_2 \ ; \partial f_2(x)/\partial x_1 \ \ \partial f_2(x)/\partial x_2]$

$J(x) =100[(-1+4V_2 -V_3) \ \ (-V_2);(-V_3) \ \ (-1-V_2+4V_3)]$, assume $V_2=1,V_3=1$

$J^0 =100[2 \ -1;-1 \ 2]$, $(J^0)^{-1} = (1/300)[2 \ 1;1 \ 2]$, $f(x^0) =[f_1(x^0) \ f_2(x^0)]=[1 \ 0.5]$

$x^{v+1} =x^v -[J(x^v)]^{-1} f(x^v)$, $x^1 =[1 \ 1]- (1/300)[2 \ 1;1 \ 2][1 \ 0.5]^T =[0.991667 \ \ 0.993333]$

$J_1 =100[1.973333 \ -0.991667;-0.993333 \ 1.981667]$, $f(x^1) =[0.00843 \ 0.00323] =[0 \ 0]$

$x^2 = [0.991599 \ 0.993283]$, $f(x^2)=[0.000053 \ 0.000040]$

=> $P_1 = 1.5118$, line loss = $P_1 - P_{D2} - P_{D3} = 0.0118$ (?)

6.6應用於電力潮流方程式(Application to Power Flow Equations)

(6.3) $S_i = V_i I_i^* = V_i (\Sigma^n_{k=1} Y_{ik} V_k)^* = V_i \Sigma^n_{k=1} Y_{ik}^* V_k^*$, $i = 1,2,,,n$

(6.4) $S_i = \Sigma^n_{k=1} |V_i| |V_k| e^{j\theta ik} (G_{ik} - jB_{ik})$, $i = 1,2,,,n$

$= \Sigma^n_{k=1} |V_i| |V_k| (\cos\theta_{ik} + j\sin\theta_{ik}) (G_{ik} - jB_{ik})$, $i = 1,2,,,n$

(6.29) $P_i = \Sigma^n_{k=1} |V_i| |V_k| (G_{ik} \cos\theta_{ik} + B_{ik} \sin\theta_{ik})$, $i = 1,2,,,n$

$Q_i = \Sigma^n_{k=1} |V_i| |V_k| (G_{ik} \sin\theta_{ik} - B_{ik} \cos\theta_{ik})$, $\theta_{ik} = \theta_i - \theta_k$

(6.30) Define $\theta=[\theta_2 \,.\,.\, \theta_n]$, $|V|=[|V_2| \,.\,.\, |V_n|]$, $x=[\theta\ |V|]$

(6.31) $P_i(x) = \Sigma^n_{k=1} |V_i| |V_k| (G_{ik} \cos\theta_{ik} + B_{ik} \sin\theta_{ik})$, $i = 1,2,,,n$

$Q_i(x) = \Sigma^n_{k=1} |V_i| |V_k| (G_{ik} \sin\theta_{ik} - B_{ik} \cos\theta_{ik})$, $\theta_{ik} = \theta_i - \theta_k$

(6.32) $P_i = P_i(x)$, $Q_i = Q_i(x)$, $i = 2,,,n$

(6.33) $P_i(x) - P_i = 0$, $Q_i(x) - Q_i = 0$, $i = 2,,,n$

$f_1(x) = P_2(x) - P_2$, $f_2(x) = P_3(x) - P_3$, . . . , $f_{2n-2}(x) = Q_n(x) - Q_n$,

(6.34) $f(x) = [(P_2(x) - P_2) \,.\,.\, (P_n(x) - P_n)\ (Q_2(x) - Q_2) \,.\,.\, (Q_n(x) - Q_n)] = 0$

(6.35) $J=[J_{11}\ J_{12}; J_{21}\ J_{22}]$, $J_{11} = \partial P_i(x)/\partial\theta_k$, $J_{12} = \partial P_i(x)/\partial|V_k|$, $J_{21} = \partial Q_i(x)/\partial\theta_k$, $J_{22} = \partial Q_i(x)/\partial|V_k|$

(6.27) $J(x^v)\,\Delta x^v = - f(x^v) = J(x^v)(x^{v+1} - x^v)$

(6.36) $\Delta P(x)=[(P_2 - P_2(x))..(P_n - P_n(x))]$, $\Delta Q(x)=[(Q_2 - Q_2(x))..(Q_n - Q_n(x))]$

(6.37) $f(x) = -[\Delta P(x)\ \Delta Q(x)]$,

(6.38) $[J^v_{11}\ \ J^v_{12} ;\ J^v_{21}\ \ J^v_{22}]\ [\Delta\theta^v\ \ \Delta|V|^v] = [\Delta P(x^v)\ \ \Delta Q(x^v)]$

Application to Power Flow Equations

(6.31) $P_i(x) = \Sigma^n_{k=1} |V_i| |V_k| (G_{ik} \cos \theta_{ik} + B_{ik} \sin \theta_{ik})$, $i = 1,2,,,n$

$Q_i(x) = \Sigma^n_{k=1} |V_i| |V_k| (G_{ik} \sin \theta_{ik} - B_{ik} \cos \theta_{ik})$, $\theta_{ik} = \theta_i - \theta_k$

(6.39) $\partial P_2(x)/\partial\theta_2 = (\partial/\partial\theta_2) \Sigma^n_{k=1} |V_2| |V_k| (G_{2k} \cos \theta_{2k} + B_{2k} \sin \theta_{2k})$

$= \Sigma^n_{k=1, k \neq 2} |V_2| |V_k| (-G_{2k} \sin \theta_{2k} + B_{2k} \cos \theta_{2k})$

(6.40) $\partial P_2(x)/\partial\theta_3 = |V_2| |V_3| (G_{23} \sin \theta_{23} - B_{23} \cos \theta_{23})$

$\partial P_2(x)/\partial|V_2| = \Sigma^n_{k=1} |V_k|(G_{2k} \cos \theta_{2k} + B_{2k} \sin \theta_{2k}) + |V_2|G_{22}$

$\partial P_2(x)/\partial|V_3| = |V_2| (G_{23} \cos \theta_{23} + B_{23} \sin \theta_{23})$

$\partial Q_2(x)/\partial|V_2| = \Sigma^n_{k=1} |V_k|(G_{2k} \sin \theta_{2k} - B_{2k} \cos \theta_{2k}) - |V_2|B_{22}$

$\partial Q_2(x)/\partial|V_3| = |V_2| (G_{23} \sin \theta_{23} - B_{23} \cos \theta_{23})$

$\partial Q_2(x)/\partial\theta_2 = \Sigma^n_{k=1, k \neq 2} |V_2| |V_k| (G_{2k} \cos \theta_{2k} + B_{2k} \sin \theta_{2k})$

$\partial Q_2(x)/\partial\theta_3 = - |V_2| |V_3| (G_{23} \cos \theta_{23} + B_{23} \sin \theta_{23})$

Power Flow Equations

For indices $p \neq q$

$$J^{11}_{pq} = \partial P_p(x)/\partial\theta_q = |V_p|\,|V_q|\,(G_{pq} \sin\theta_{pq} - B_{pq} \cos\theta_{pq}),$$

$$J^{21}_{pq} = \partial Q_p(x)/\partial\theta_q = -|V_p|\,|V_q|\,(G_{pq} \cos\theta_{pq} + B_{pq} \sin\theta_{pq}),$$

$$J^{12}_{pq} = \partial P_p(x)/\partial|V_q| = |V_p|\,(G_{pq} \cos\theta_{pq} + B_{pq} \sin\theta_{pq}),$$

$$J^{22}_{pq} = \partial Q_p(x)/\partial|V_q| = |V_p|\,(G_{pq} \sin\theta_{pq} - B_{pq} \cos\theta_{pq}),$$

For indices $p=q$

$$J^{11}_{pp} = \partial P_p(x)/\partial\theta_p = -Q_p - B_{pp}\,|V_p|^2,$$

$$J^{21}_{pp} = \partial Q_p(x)/\partial\theta_p = P_p - G_{pp}\,|V_p|^2,$$

$$J^{12}_{pp} = \partial P_p(x)/\partial|V_p| = (P_p/|V_p|) + G_{pp}\,|V_p|,$$

$$J^{22}_{pp} = \partial Q_p(x)/\partial|V_p| = (Q_p/|V_p|) - B_{pq}\,|V_p|,$$

Ex 6.8 Find θ_2 , $|V_3|$, θ_3 , S_{G1} , and Q_{G2} for the system. In the transmission system all the shunt elements are capacitor with an admittance Yc =j0.01, while all the series elements are inductors with an impedance of Z_L=j0.1 ($1/Z_L$ = -j10).

$V_1=1\angle 0^\circ$, $|V_2|=1.05$, $P_{G2}=0.6661$, $S_{D3}=2.8653+j1.2244$, $x=[\theta_2\ \ \theta_3\ \ |V_3|]$

$Y_{bus}=[(Y_1+Y_2+Y_4+Y_5)\ \ -Y_2\ \ -Y_5;\ -Y_2\ \ (Y_2+Y_3+Y_8+Y_9)\ -Y_8;\ -Y_5\ -Y_8\ \ (Y_5+Y_6+Y_7+Y_8)]$
$=[-j19.98\ \ j10\ \ j10;\ j10\ \ -j19.98\ \ j10;\ j10\ \ j10\ \ -j19.98]$

(6.31) $P_i(x) = \Sigma^n_{k=1} |V_i|\,|V_k|\,(G_{ik} \cos\theta_{ik} + B_{ik}\sin\theta_{ik})$, $i = 1,2,,,n$

$Q_i(x) = \Sigma^n_{k=1} |V_i|\,|V_k|\,(G_{ik} \sin\theta_{ik} - B_{ik}\cos\theta_{ik})$, $\theta_{ik} = \theta_i - \theta_k$

(6.41a) $P_2(x)= |V_2|\,|V_1|\,B_{21}\sin\theta_{21} + |V_2|\,|V_3|\,B_{23}\sin\theta_{23} = 10.5\sin\theta_2 + 10.5\,|V_3|\sin\theta_{23}$

(6.41b) $P_3(x)=|V_3|\,|V_1|\,B_{31}\sin\theta_{31} + |V_3|\,|V_2|\,B_{32}\sin\theta_{32} = 10|V_3|\sin\theta_3 + 10.5\,|V_3|\sin\theta_{32}$

(6.41c) $Q_3(x)=|V_3|\,|V_1|\,B_{31}\cos\theta_{31} + |V_3|\,|V_2|\,B_{32}\cos\theta_{32} + |V_3|^2 B_{33}$

$= -[10|V_3|\cos\theta_3 + 10.5\,|V_3|\cos\theta_{32} - 19.98\,|V_3|^2]$

$J(x)=[\partial P_2(x)/\partial\theta_2\ \ \partial P_2(x)/\partial\theta_3\ \ \partial P_2(x)/\partial|V_3|$;

$\partial P_3(x)/\partial\theta_2\ \ \partial P_3(x)/\partial\theta_3\ \ \partial P_3(x)/\partial|V_3|$;

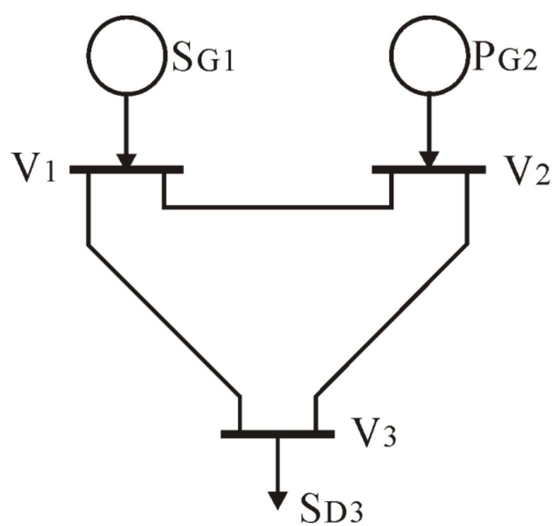
$\partial Q_3(x)/\partial\theta_2\ \ \partial Q_3(x)/\partial\theta_3\ \ \partial Q_3(x)/\partial|V_3|\ ]$

$\partial P_2(x)/\partial\theta_2 = |V_2|\,|V_1|\,B_{21}\cos\theta_{21} + |V_2|\,|V_3|\,B_{23}\cos\theta_{23}$

$= 10.5\cos\theta_2 + 10.5\,|V_3|\cos\theta_{23}$

$\partial P_2(x)/\partial\theta_3 = -\,|V_2|\,|V_3|\,B_{23}\cos\theta_{23} = -10.5\,|V_3|\cos\theta_{23}$

$\partial P_2(x)/\partial|V_3| = |V_2|\,B_{23}\sin\theta_{23} = 10.5\sin\theta_{23}$



Ex6.8 $V_1=1\angle 0^\circ$, $|V_2|=1.05$, $P_{G2}=0.6661$, $S_{D3}=2.8653+j1.2244$, $x=[\theta_2\ \ \theta_3\ \ |V_3|]$

(6.41a) $P_2(x)=|V_2|\,|V_1|\,B_{21}\sin\theta_{21}+|V_2|\,|V_3|\,B_{23}\sin\theta_{23}=10.5\sin\theta_2+10.5\,|V_3|\sin\theta_{23}$

(6.41b) $P_3(x)=|V_3|\,|V_1|\,B_{31}\sin\theta_{31}+|V_3|\,|V_2|\,B_{32}\sin\theta_{32}=10|V_3|\sin\theta_3+10.5\,|V_3|\sin\theta_{32}$

(6.41c) $Q_3(x)=-[|V_3|\,|V_1|\,B_{31}\cos\theta_{31}+|V_3|\,|V_2|\,B_{32}\cos\theta_{32}+|V_3|^2\,B_{33}]$

$=-[10|V_3|\cos\theta_3+10.5\,|V_3|\cos\theta_{32}-19.98\,|V_3|^2]$

$\partial P_3(x)/\partial\theta_2=|V_3|\,|V_2|\,B_{32}\cos\theta_{32}=-10.5|V_3|\cos\theta_{32}$

$\partial P_3(x)/\partial\theta_3=|V_3|\,|V_1|\,B_{31}\cos\theta_{31}+|V_3|\,|V_2|\,B_{32}\cos\theta_{32}=10|V_3|\cos\theta_3+10.5\,|V_3|\cos\theta_{32}$

$\partial P_3(x)/\partial|V_3|=|V_1|\,B_{31}\sin\theta_{31}+|V_2|\,B_{32}\sin\theta_{32}=10\sin\theta_3+10.5\sin\theta_{32}$

$\partial Q_3(x)/\partial\theta_2=-|V_3|\,|V_2|\,B_{32}\sin\theta_{32}=-10.5|V_3|\sin\theta_{32}$

$\partial Q_3(x)/\partial\theta_3=|V_3|\,|V_1|\,B_{31}\sin\theta_{31}+|V_3|\,|V_2|\,B_{32}\sin\theta_{32}=10|V_3|\sin\theta_3+10.5|V_3|\sin\theta_{32}$

$\partial Q_3(x)/\partial|V_3|=-[|V_1|\,B_{31}\cos\theta_{31}+|V_2|B_{32}\cos\theta_{32}+2|V_3|\,B_{33}]=-[10\cos\theta_3+10.5\cos\theta_{32}-39.96|V_3|]$

$P_2=P_{G2}=0.6661$, $P_3=-P_{D3}=-2.8653$, $Q_3=-Q_{D3}=-1.2244$

Guess $\theta^0_2=\theta^0_3=0$, $|V_3|=1.0$,

$[\Delta P_2\ \Delta P_3\ \Delta Q_3]^0=[P_2\ P_3\ Q_3]-[P_2(x^0)\ P_3(x^0)\ Q_3(x^0)]$

$=[0.6661\ -2.8653\ -1.2244]-[0\ \ 0\ \ -0.52]=[0.6661\ -2.8653\ -0.7044]$

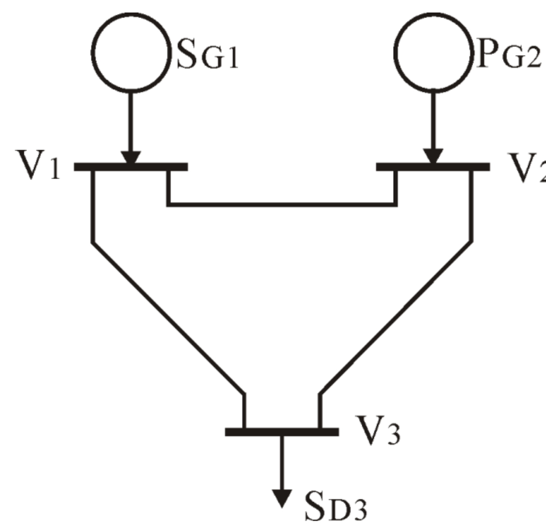
$J^0=[21\ \ -10.5\ \ 0;-10.5\ \ 20.5\ \ 0;0\ \ 0\ \ 19.46]=[J_{11}\ 0;0\ J_{22}]$

$(J^0)^{-1}=[J^{-1}_{11}\ 0;0\ J^{-1}_{22}]=[0.064\ \ 0.0328\ \ 0;0.0328\ \ 0.0656\ \ 0;0\ 0\ 0.0514]$

$\Delta x^0=[\Delta\theta_2\ \Delta\theta_3\ \Delta|V_3|]^0=(J^0)^{-1}[\Delta P_2\ \Delta P_3\ \Delta Q_3]$

$=[-0.0513\text{rad}\ -0.166\text{rad}\ -0.0362]=[-2.9395^\circ\ -9.5111^\circ\ -0.0362]$

$x^1=x^0+\Delta x^0=[0\ 0\ 1]+[-2.93965^\circ\ -9.5111^\circ\ -0.0362]=[-2.9395^\circ\ -9.5111^\circ\ 0.9638]$



Ex6.8 $x = [\theta_2 \;\; \theta_3 \;\; |V_3|] = [-3^\circ \;\; -10^\circ \;\; 0.95]$

(6.41a) $P_2(x) = |V_2|\,|V_1|\,B_{21}\sin\theta_{21} + |V_2|\,|V_3|\,B_{23}\sin\theta_{23} = 10.5\sin\theta_2 + 10.5\,|V_3|\sin\theta_{23}$

(6.41b) $P_3(x) = |V_3|\,|V_1|\,B_{31}\sin\theta_{31} + |V_3|\,|V_2|\,B_{32}\sin\theta_{32} = 10|V_3|\sin\theta_3 + 10.5\,|V_3|\sin\theta_{32}$

(6.41c) $Q_3(x) = -[|V_3|\,|V_1|\,B_{31}\cos\theta_{31} + |V_3|\,|V_2|\,B_{32}\cos\theta_{32} + |V_3|^2 B_{33}]$

$= -[10|V_3|\cos\theta_3 + 10.5\,|V_3|\cos\theta_{32} - 19.98\,|V_3|^2]$

$\partial P_2(x)/\partial\theta_2 = 10.5\cos\theta_2 + 10.5\,|V_3|\cos\theta_{23}$, $\partial P_2(x)/\partial\theta_3 = -10.5\,|V_3|\cos\theta_{23}$, $\partial P_2(x)/\partial|V_3| = 10.5\sin\theta_{23}$

$\partial P_3(x)/\partial\theta_2 = -10.5|V_3|\cos\theta_{32}$, $\partial P_3(x)/\partial\theta_3 = 10|V_3|\cos\theta_3 + 10.5|V_3|\cos\theta_{32}$, $\partial P_3(x)/\partial|V_3| = 10\sin\theta_3 + 10.5\sin\theta_{32}$

$\partial Q_3(x)/\partial\theta_2 = -10.5|V_3|\sin\theta_{32}$, $\partial Q_3(x)/\partial\theta_3 = 10|V_3|\sin\theta_3 + 10.5|V_3|\sin\theta_{32}$

$\partial Q_3(x)/\partial|V_3| = -[10\cos\theta_3 + 10.5\cos\theta_{32} - 39.96|V_3|]$

$P_2 = P_{G2} = 0.6661$, $P_3 = -P_{D3} = -2.8653$, $Q_3 = -Q_{D3} = -1.2244$

$x^1 = [-2.9395^\circ \;\; -9.5111^\circ \;\; 0.9638]$, $P_2(x^1) = 0.6198$, $P_3(x^1) = -2.7508$, $Q_3(x^1) = -0.9993$

$[\Delta P_2 \; \Delta P_3 \; \Delta Q_3]^1 = [P_2 \; P_3 \; Q_3] - [P_2(x^1) \; P_3(x^1) \; Q_3(x^1)] = [0.0463 \;\; -0.1145 \;\; -0.2251]$

$J^1 = [20.5396 \;\; -10.0534 \;\; 1.2017; -10.0534 \;\; 19.5589 \;\; -2.8541; 1.1582 \;\; -2.7508 \;\; 18.2199]$

$(J^1)^{-1} = [0.0651 \;\; 0.0336 \;\; 0.001; 0.0336 \;\; 0.0696 \;\; 0.0087; 0.0009 \;\; 0.0084 \;\; 0.0561]$

$\Delta x^1 = [\Delta\theta_2 \; \Delta\theta_3 \; \Delta|V_3|]^1 = (J^1)^{-1}[\Delta P_2 \; \Delta P_3 \; \Delta Q_3]$

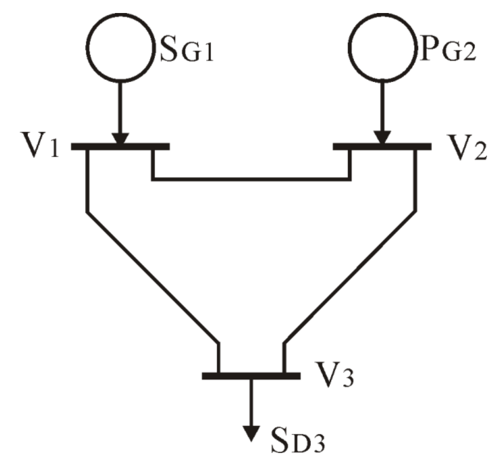
$x^2 = [-3.0023^\circ \;\; -9.9924^\circ \;\; 0.9502]$

$[\Delta P_2 \; \Delta P_3 \; \Delta Q_3]^1 = [P_2 \; P_3 \; Q_3] - [P_2(x^2) \; P_3(x^2) \; Q_3(x^2)] = [0.0019 \;\; -0.0023 \;\; -0.0031]$

$P_{G1} = P_1 = |V_1|\,|V_2|\,B_{12}\sin\theta_{12} + |V_1|\,|V_3|\,B_{13}\sin\theta_{13} = 2.1987$

$Q_{G1} = Q_1 = |V_1|\,|V_2|\,B_{12}\cos\theta_{12} + |V_1|\,|V_3|\,B_{13}\cos\theta_{13} + |V_1|^2 B_{11} = 0.1365$

$Q_{G2} = Q_2 = |V_2|\,|V_1|\,B_{21}\cos\theta_{21} + |V_2|\,|V_3|\,B_{23}\cos\theta_{23} + |V_2|^2 B_{22} = -1.6395$



6.7分解電力潮流(Decoupled Power Flow)

$\partial P_2(x)/\partial|V_3| = |V_2|[G_{23} \cos \theta_{23} + B_{23} \sin \theta_{23}]$, G_{23} is quite small

$\partial Q_2(x)/\partial\theta_3 = -|V_2||V_3|[G_{23} \cos \theta_{23} + B_{23} \sin \theta_{23}]$

Not choose $A(x^v)=[J^v_{11}\ J^v_{12};\ J^v_{21}\ J^v_{22}]$, choose $A(x^v)=[J^v_{11}\ \ 0;\ 0\ \ J^v_{22}]$,

(6.47a) $J^v_{11}\ \Delta\theta^v = \Delta P(x^v)$

(6.47b) $J^v_{22}\ \Delta|V|^v = \Delta Q(x^v)$

(6.48) $\partial P_2(x)/\partial\theta_2 = -Q_2 - B_{22}|V_2|^2 = \Sigma^n_{k=1,k\neq 2} |V_2|\,|V_k|B_{2k}$

$$= (\Sigma^n_{k=1} |V_2|\,|V_k|B_{2k}) - |V_2|^2 B_{22} = -|V_2|^2 B_{22}$$

1. With all the $|V_k|$ approximately equal, $(\Sigma^n_{k=1}|V_2||V_k|B_{2k}) = |V_2|^2\, \Sigma^n_{k=1} B_{2k}$
2. B_{22} = sum of susceptances (電納為電抗的倒數) of all the elements of the Π-equivalent circuit incident to bus 2.
3. For $k\neq 2$, B_{2k} = -susceptance of the bridging element from bus 2 to bus k.
4. Because of observations 2 and 3, in $\Sigma^n_{k=1} B_{2k}$, all the bridging elements cancel, leaving only the sum of (small) shunt element susceptances (capacitive). Thus we have $|\Sigma^n_{k=1} B_{2k}| \ll |B_{22}|$

Note: If all the (per unit) $|V_i|$ are equal, we have a so-called flat profile (扁平圖形); under normal operating conditions this is a reasonable approximation.

Decoupled Power Flow

(6.48) $\partial P_2(x)/\partial\theta_2 = -Q_2 - B_{22}|V_2|^2 = \Sigma^n{}_{k=1,k\neq 2} |V_2|\,|V_k|B_{2k}$

$= (\Sigma^n{}_{k=1} |V_2|\,|V_k|B_{2k}) - |V_2|^2 B_{22} = -|V_2|^2 B_{22}$

(6.49) $\partial Q_2(x)/\partial|V_2| = (-\Sigma^n{}_{k=1} |V_k|B_{2k}) - |V_2|B_{22}$

Equations (6.48) and (6.49) give the pattern for the diagonal terms. For the off-diagonal terms, we obtain the following approximations:

(6.50) $\partial P_2(x)/\partial\theta_3 = -|V_2||V_3|B_{23}$, $\partial Q_2(x)/\partial|V_3| = -|V_2|B_{23}$,

(6.51) $B=[B_{22}\ B_{23}\ \ldots\ B_{2n}\ ;\ .\ ;\ .\ ;\ B_{n2}\ B_{n3}\ \ldots\ B_{nn}]$, B is a constant matrix

$[V]=[|V_2|\ 0\ldots\ 0;\ 0\ |V_3|\ 0\ldots\ 0;\ .\ ;\ 0\ 0\ldots\ |V_n|]$,

B may be obtained from Y_{bus} by stripping away the first row and column and then taking the imaginary part.

(6.52a) $J_{11} = -[V]B[V]$, (6.52b) $J_{22} = -[V]B$,

(6.53a) $-[V^v]B[V^v]\ \Delta\theta^v = \Delta P(x^v)$, (6.53b) $-[V^v]B\ \Delta|V|^v = \Delta Q(x^v)$,

Assume the second $[V^v]$ is identity matrix.

(6.54a) $-[V^v]B\ \Delta\theta^v = \Delta P(x^v) \Rightarrow -B\ \Delta\theta^v = [V^v]^{-1}\ \Delta P(x^v) = \Delta P\backslash(x^v)$,

(6.54b) $-[V^v]B\Delta|V|^v = \Delta Q(x^v) \Rightarrow -B\ \Delta|V|^v = [V^v]^{-1}\ \Delta Q(x^v) = \Delta Q\backslash(x^v)$,

B is a constant matrix, independent of iteration count.

Ex 6.9 Use fast-decoupled power flow to find θ_2 , $|V_3|$, θ_3 , S_{G1} , and Q_{G2} for the system. In the transmission system all the shunt elements are capacitor with an admittance Yc =j0.01, while all the series elements are inductors with an impedance of Z_L=j0.1 ($1/Z_L$ = -j10).

$V_1=1\angle 0^\circ$, $|V_2|=1.05$, $P_{G2}=0.6661$, $S_{D3}=2.8653+j1.2244$, $x=[\theta_2\ \ \theta_3\ \ |V_3|]$

(6.36) $\Delta P(x)=[(P_2-P_2(x))..(P_n-P_n(x))]$, $\Delta Q(x)=[(Q_2-Q_2(x))..(Q_n-Q_n(x))]$

(6.41a) $P_2(x)=|V_2|\,|V_1|\,B_{21}\sin\theta_{21}+|V_2|\,|V_3|\,B_{23}\sin\theta_{23}=10.5\sin\theta_2+10.5\,|V_3|\sin\theta_{23}$

(6.41b) $P_3(x)=|V_3|\,|V_1|\,B_{31}\sin\theta_{31}+|V_3|\,|V_2|\,B_{32}\sin\theta_{32}=10|V_3|\sin\theta_3+10.5\,|V_3|\sin\theta_{32}$

(6.41c) $Q_3(x)=-[|V_3|\,|V_1|\,B_{31}\cos\theta_{31}+|V_3|\,|V_2|\,B_{32}\cos\theta_{32}+|V_3|^2 B_{33}]$

$= -[10|V_3|\cos\theta_3+10.5\,|V_3|\cos\theta_{32}-19.98\,|V_3|^2]$

$P_2=P_{G2}=0.6661$, $P_3=-P_{D3}=-2.8653$, $Q_3=-Q_{D3}=-1.2244$

$Y_{bus}=[(Y_1+Y_2+Y_4+Y_5)\ \ -Y_2\ \ -Y_5;\ -Y_2\ \ (Y_2+Y_3+Y_8+Y_9)\ -Y_8;\ -Y_5\ -Y_8\ \ (Y_5+Y_6+Y_7+Y_8)]$

$=[-j19.98\ \ j10\ \ j10;\ j10\ \ -j19.98\ \ j10;\ j10\ \ j10\ \ -j19.98]$

$B=[B_{22}\ B_{23}\ ;B_{32}\ B_{33}]=[-19.98\ \ 10;10\ \ -19.98]$, $-B^{-1}=[0.0668\ 0.0334;\ 0.0334\ 0.0668]$

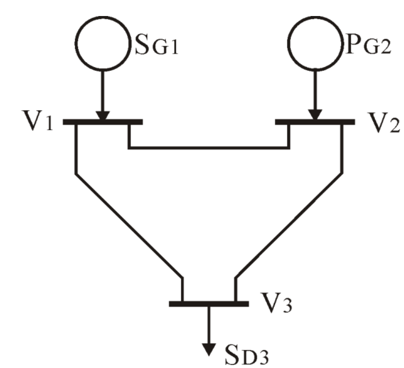
$-B[\Delta\theta_2\ \ \Delta\theta_3\ ]^v=[\Delta P_2/|V_2|\ \ \ \Delta P_3/|V_3|]^v=[\Delta P_2/1.05\ \ \ \Delta P_3/|V_3|]^v$

$-B[\Delta|V_2|\ \ \Delta|V_3|]^v=-B[0\ \ \Delta|V_3|]^v=[\Delta Q_2/|V_2|\ \ \Delta Q_3/|V_3|]^v \Rightarrow -(-19.98)\Delta|V_3|^v=(\Delta Q_3/|V_3|)^v$

(6.55a) $[\Delta\theta_2\ \ \Delta\theta_3\ ]^v=-B^{-1}[\Delta P_2/1.05\ \ \ \Delta P_3/|V_3|]^v$

(6.55b) $\Delta|V_3|^v=0.0501\,(\Delta Q_3/|V_3|)^v$

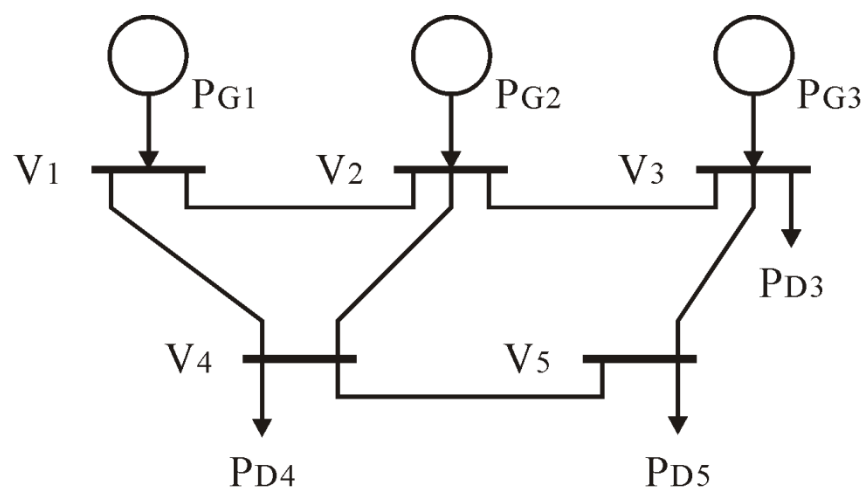
No:	θ_2 ,	θ_3 ,	$\|V_3\|$,	$\Delta P_2/1.05$,	$\Delta P_3/\|V_3\|$,	$\Delta Q_3/\|V_3\|$
0:	0	0	1	0.6344	-2.8653	-0.7044
1:	-3.0539	-9.7517	0.9647	0.0420	-0.0517	-0.2601
2:	-2.9908	-9.8721	0.9517	0.0159	0.0382	-0.0252
3:	-3.0023	-9.9867	0.9504	0.0025	-0.0039	-0.0067



6.8控制上的含意(Control Implications)

The Jacobian matrix comes up in another connection related to system control.

Suppose that the system is in a particular operating state or condition x^0 with corresponding bus powers $P(x^0)$ and $Q(x^0)$. Suppose we now wish to make a small change in the bus powers by exercising control at the generator buses (i.e., by changing some of the components of x). We then need to consider how changes in x affect changes in P(x) and Q(x); for small increments the relationship is linear and is given by ((6.38) $[J^v_{11}\ \ J^v_{12}\ ;\ J^v_{21}\ \ J^v_{22}]\ [\Delta\theta^v\ \ \Delta|V|^v] = [\Delta P(x^v)\ \ \Delta Q(x^v)]$), where the Jacobian matrix is evaluated at the operating state x^0 .



Ex 6.10 Assume that the series line impedances are $z_L=r_L+jx_L=0.0099+j0.099=0.0995\angle 84.2894^\circ$. Neglect the capacitive (shunt) impedances.

$P_{G2}=0.883$, $P_{G3}=0.2076$, $V_1=1\angle 0^\circ$, $|V_2|=1$, $|V_3|=1$, $S_{D3}=0.2+j0.1$, $S_{D4}=1.7137+j0.5983$, $S_{D5}=1.7355+j0.5496$,

(a)Verify that a solution of the power flow equations is given by

$\theta=[\theta_2\ \theta_3\ \theta_4\ \theta_5]=[-5^\circ\ -10^\circ\ -10^\circ\ -15^\circ]$, $|V|=[|V_4|\ \ |V_5|]=[1\ \ 1]$

(b)Calculate the slack bus power $S_1=S_{G1}$.

(c)Calculate the total line losses.

(d)Show that the (complex) load demand may be met with lower line losses by shifting generation to generator 3.

(6.5) $S_i^*=V_i^*I_i=V_i^*(\Sigma^n_{k=1}Y_{ik}V_k)$, $i=2,\ldots,n$

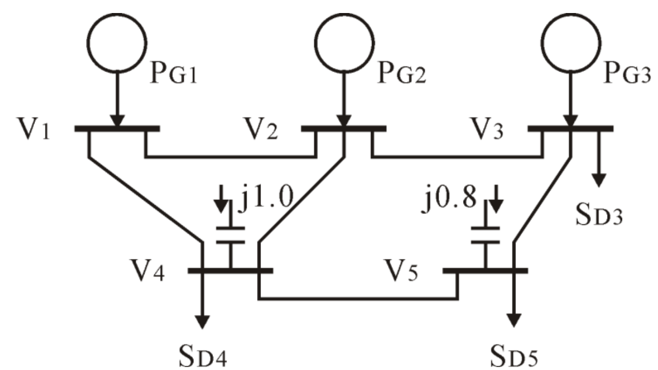
(6.29) $P_i=\Sigma^n_{k=1}|V_i|\,|V_k|\,(G_{ik}\cos\theta_{ik}+B_{ik}\sin\theta_{ik})$, $i=1,2,,,n$

$Q_i=\Sigma^n_{k=1}|V_i|\,|V_k|\,(G_{ik}\sin\theta_{ik}-B_{ik}\cos\theta_{ik})$, $\theta_{ik}=\theta_i-\theta_k$

$Y_L=(z_L)^{-1}=10.503\angle -84.2894^\circ=1-j10$,

$Y_{bus}=[(Y_1+Y_2+Y_4+Y_5)\ \ -Y_2\ \ -Y_5;\ -Y_2\ \ (Y_2+Y_3+Y_8+Y_9)\ -Y_8;\ -Y_5\ -Y_8\ \ (Y_5+Y_6+Y_7+Y_8)]$

$Y_{bus}=[2(1-j10)\ -(1-j10)\quad 0\quad -(1-j10)\quad 0;$
$-(1-j10)\quad 3(1-j10)\ -(1-j10)\ -(1-j10)\quad 0;$
$0\quad -(1-j10)\ 2(1-j10)\quad 0\quad -(1-j10);$
$-(1-j10)\ -(1-j10)\quad 0\quad 3(1-j10)\ -(1-j10);$
$0\quad 0\quad -(1-j10)\ -(1-j10)\ 2(1-j10)]$



Ex 6.10(a)(b)(c) $z_L=r_L+jx_L = 0.0099 + j0.099 = 0.0995 \angle 84.2894^\circ$. $Y_L = (z_L)^{-1} = 1-j10$,

$P_{G2} = 0.883$, $P_{G3} = 0.2076$, $V_1 = 1\angle 0^\circ$, $|V_2|=1$, $|V_3|=1$, $S_{D3} = 0.2+j0.1$, $S_{D4} = 1.7137+j0.5983$, $S_{D5} = 1.7355+j0.5496$,

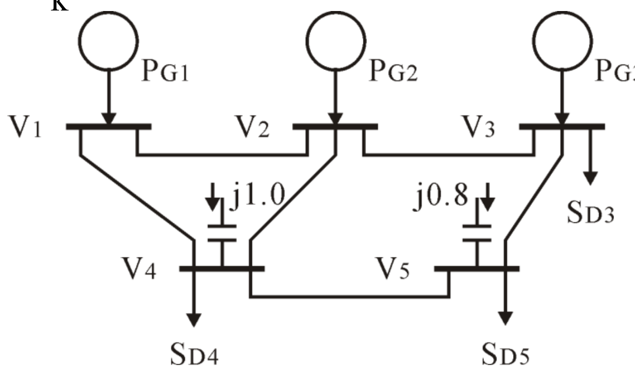
(a)Verify that a solution of the power flow equations is given by

$\theta = [\theta_2\ \theta_3\ \theta_4\ \theta_5] = [-5^\circ\ -10^\circ\ -10^\circ\ -15^\circ]$, $|V| = [|V_4|\ \ |V_5|] = [1\ \ 1]$

(6.29) $P_i = \Sigma^n_{k=1} |V_i|\,|V_k|\,(G_{ik} \cos\theta_{ik} + B_{ik} \sin\theta_{ik})$, $i = 1,2,,,n$

$Q_i = \Sigma^n_{k=1} |V_i|\,|V_k|\,(G_{ik} \sin\theta_{ik} - B_{ik} \cos\theta_{ik})$, $\theta_{ik} = \theta_i - \theta_k$

$Y_{bus} = [2(1-j10)\ \ -(1-j10)\ \ 0\ \ -(1-j10)\ \ 0;$
$-(1-j10)\ \ 3(1-j10)\ \ -(1-j10)\ \ -(1-j10)\ \ 0;$
$0\ \ -(1-j10)\ \ 2(1-j10)\ \ 0\ \ -(1-j10);$
$-(1-j10)\ \ -(1-j10)\ \ 0\ \ 3(1-j10)\ \ -(1-j10);$
$0\ \ 0\ \ -(1-j10)\ \ -(1-j10)\ \ 2(1-j10)]$



(6.5) $S_4^* = V_4^*(\Sigma^5_{k=1} Y_{4k}V_k) = Y_{44}|V_4|^2 + V_4^*(Y_{41}V_1 + Y_{42}V_2 + Y_{43}V_3 + Y_{45}V_5)$

$S_4^* = (3-j30) + 1\angle 10^\circ(-1+j10)(1\angle 0^\circ + 1\angle -5^\circ + 1\angle -15^\circ) = -1.7137 - j0.4017$

$S_4 = -1.7137 + j0.4017 = S_{G4} - S_{D4} = (j1.0) - (1.7137+j0.5983) = -1.7137 + j0.4017$

(b)$S_1 = V_1(\Sigma^5_{k=1} Y^*_{1k}V_k^*) = Y^*_{11}|V_1|^2 + V_1(Y^*_{12}V^*_2 + Y^*_{13}V^*_3 + Y^*_{14}V^*_4 + Y^*_{15}V^*_5)$

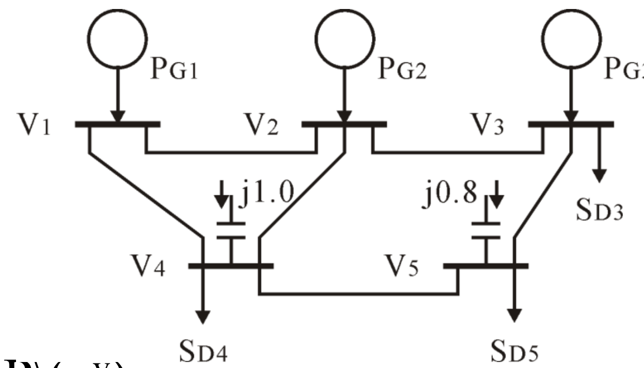
$= (2+j20) + 1\angle 10^\circ(-1-j10)(1\angle 5^\circ + 1\angle 10^\circ) = 2.627 - j0.0709$

(c)In a lossless transmission system, $\Sigma^n_{i=1} P_i = 0$. In loss system, power is conservation.

$P_L = \Sigma^5_{i=1} P_i = P_{G1} + P_{G2} + P_{G3} - S_{D3} - S_{D4} - S_{D5} = 2.627+0.883+0.0076 - 1.7137 - 1.7355 = 0.0684$

Ex 6.10(d) $z_L=r_L+jx_L=0.0099+j0.099=0.0995\ \angle 84.2894^\circ$. $Y_L=(z_L)^{-1}=1-j10$,

Y_{bus} =[2(1-j10) -(1-j10) 0 -(1-j10) 0;
-(1-j10) 3(1-j10) -(1-j10) -(1-j10) 0;
0 -(1-j10) 2(1-j10) 0 -(1-j10);
-(1-j10) -(1-j10) 0 3(1-j10) -(1-j10);
0 0 -(1-j10) -(1-j10) 2(1-j10)]



B =[30 10 10 0;10 -20 0 10;10 0 -30 10; 0 10 10 -20]

(6.54a) $-[V^v]B\ \Delta\theta^v=\Delta P(x^v) \Rightarrow -B\ \Delta\theta^v=[V^v]^{-1}\ \Delta P(x^v)=\Delta P\backslash(x^v)$,

(6.54b) $-[V^v]B\Delta|V|^v=\Delta Q(x^v) \Rightarrow -B\ \Delta|V|^v=[V^v]^{-1}\ \Delta Q(x^v)=\Delta Q\backslash(x^v)$,

(6.57) $-B[\Delta\theta_2\ \Delta\theta_3\ \Delta\theta_4\ \Delta\theta_5]=[\Delta P_2\ \Delta P_3\ \Delta P_4\ \Delta P_5]=[0\ \ 0.1\ \ 0\ \ 0]$, only P_{G3} is increased.

(6.58) $-[-30\ \ 10;\ 10\ \ -30][\Delta|V_4|\ \ \Delta|V_5|]=[\Delta Q_4\ \Delta Q_5]=[0\ \ 0]$, $|V_2|=1$ and $|V_3|=1$ are fixed.

Reactive power injections at buses 4 and 5 are not change, therefore $\Delta|V_4|=\Delta|V_5|=0$

Solve (6.57) $[\Delta\theta_2\ \Delta\theta_3\ \Delta\theta_4\ \Delta\theta_5]=[0.00545\ \ 0.01182\ \ 0.00455\ \ 0.00818]$rad $=[0.312^\circ\ \ 0.677^\circ\ \ 0.261^\circ\ \ 0.469^\circ]$

$\theta=\theta^0+\Delta\theta=[-5\ \ -10\ \ -10\ \ -15]+\Delta\theta=[-4.688^\circ\ \ -9.323^\circ\ \ -9.739^\circ\ \ -14.531^\circ]$

Losses between buses 1 and 2

$P_{loss}=\mathrm{Re}(S_{12}+S_{21})=(R_L/|Z_L|^2)\ [|V_1|^2+|V_2|^2-2|V_1||V_2|\cos\theta_{12}]=2(1-\cos\theta_{12})=2(1-\cos 4.688)$

Losses between bus 1-2 , 2-3 , 1-4 , 2-4, 4-5 , 3-5

Ploss = 2(6-cos4.688 -cos4.635 –cos9.739 –cos5.051 -cos4.792 –cos5.208)=0.0651 < 0.0684

As P_{G3} is increased, the transmission losses are decreased

6.9結論與習題(Summary)

In summary, there are three types of sources at the different buses:

1. A voltage source. Assume at bus 1 (slack bus or swing bus: 弛放或搖擺滙流排).
2. P, |V| sources (voltage control buses). At the other generator buses.
3. P, Q sources (load buses). At the load buses.

高斯疊代(Gauss Iteration)高斯-西丹疊代(Gauss-Seidel Iteration)
牛頓-拉夫生疊代法(Newton-Raphson Iteration)