

Power System Analysis

供電=用電

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4.0 簡介(Introduction)

In **Chapter 3** the distributed per phase **inductance** and **capacitance** of a transmission line was related to line geometry.

Some typical values of distributed **series reactance**, **series resistance**, and **shunt (capacitive) reactance** were given.

These elements may be used in the per phase equivalent circuit of a three-phase line operating under **balanced conditions**.

We now use this per phase model to derive the relationship between **voltages** and **currents** at the terminals of the transmission line.

4.1 端點處 V, I 關係的推導 (Derivation of Terminal V, I Relations) $R=0$

We consider the transmission line in the sinusoidal steady state. Thus we may use phasors and impedances. Assume that

$z = r + j\omega l =$ series impedance per meter

$y = g + j\omega c =$ shunt admittance per meter to neutral

The receiving end (right side) is located at $x = 0$; the sending end (left side) is at $x = l$.

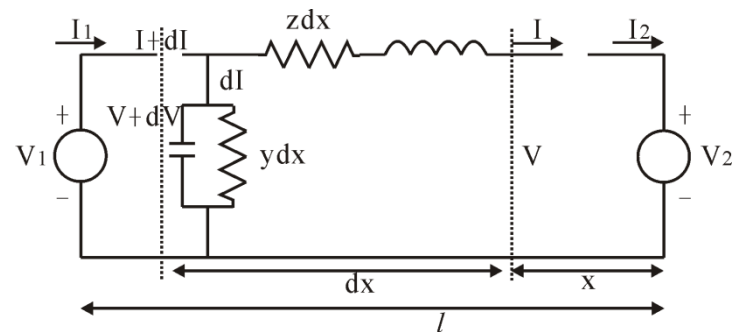
Applying Kirchhoff's voltage law (KVL) and Kirchhoff's current law (KCL) to the section, we get

$$dV = Iz \, dx ; dI = (V + dV) y \, dx = V y \, dx \Rightarrow$$

$$dV / dx = z I ; dI / dx = y V \Rightarrow$$

$$d^2V / dx^2 = y z V = \gamma^2 V ; d^2I / dx^2 = y z I = \gamma^2 I$$

Propagation constant : $\gamma = (y z)^{0.5} = j \omega \sqrt{lc}$ (assume $r = g = 0$).



4.1 端點處 V, I 關係的推導 (Derivation of Terminal V, I Relations) $R \neq 0$

$z = r + j\omega l$ = series impedance per meter

$y = g + j\omega c$ = shunt admittance per meter to neutral

$$dV/dx = z I ; dI/dx = y V$$

$$d^2V/dx^2 = y z V = \gamma^2 V ; d^2I/dx^2 = y z I = \gamma^2 I$$

Propagation constant : $\gamma = (y z)^{0.5} = \alpha + j \beta$

$$V = k_1 e^{\gamma x} + k_2 e^{-\gamma x} = (k_1 + k_2)(e^{\gamma x} + e^{-\gamma x})/2 + (k_1 - k_2)(e^{\gamma x} - e^{-\gamma x})/2 \\ = K_1 \cosh \gamma x + K_2 \sinh \gamma x ; K_1 = (k_1 + k_2) , K_2 = (k_1 - k_2)$$

$$dV/dx = -K_1 \gamma \sinh \gamma x + K_2 \gamma \cosh \gamma x$$

When $x=0$, $V=V_2 \Rightarrow V = k_1 + k_2 = K_1 = V_2$,

When $x=0$, $I=I_2 \Rightarrow dV(0)/dx = z I_2 = K_2 \gamma \Rightarrow K_2 = (z/\gamma) I_2 = (z/y)^{0.5} I_2$

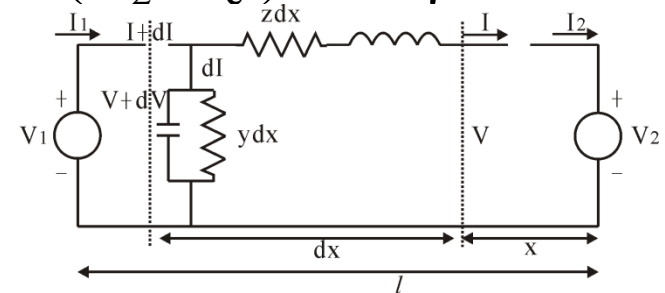
$Z_c = (z/y)^{0.5}$ (characteristic impedance: 線路的特性阻抗)

$$V = V_2 \cosh \gamma x + Z_c I_2 \sinh \gamma x ; I = I_2 \cosh \gamma x + (V_2 / Z_c) \sinh \gamma x$$

When $x=l$, $V=V_1$, $I=I_1 \Rightarrow$

$$V_1 = V_2 \cosh \gamma l + Z_c I_2 \sinh \gamma l ;$$

$$I_1 = I_2 \cosh \gamma l + (V_2 / Z_c) \sinh \gamma l$$



Ex4.1 A **60-Hz** 138-kV 3 Φ transmission line is 225mile long. The distributed line parameters are $r = 0.169 \Omega/\text{mile}$, $l = 2.093 \text{ mH}/\text{mile}$, $c = 0.01427 \mu\text{F}/\text{mile}$, $g = 0$. The transmission line delivers 40 Mwat 132 kV with **95% power factor lagging**. Find the sending-end voltage and current. Find the transmission line **efficiency**.

$$z = r + j\omega l = 0.169 + j0.789 = 0.807 \angle 77.9^\circ \Omega/\text{mile}$$

$$y = g + j\omega c = j5.38 \times 10^{-6} = 5.38 \times 10^{-6} \angle 90^\circ \text{ mho}/\text{mile}$$

$$\Rightarrow Z_c = (z/y)^{0.5} = 387.3 \angle -6.05^\circ \Omega$$

$$\Rightarrow \gamma l = 225 (y z)^{0.5} = 0.4688 \angle 83.95^\circ = 0.0494 + j0.466$$

$$2 \sinh \gamma l = e^{\gamma l} - e^{-\gamma l} = e^{0.0494} e^{j0.466} - e^{-0.0494} e^{-j0.466} = 1.051 \angle 0.466 \text{ rad} - 0.952 \angle -0.466 \text{ rad}$$

$$\sinh \gamma l = 0.452 \angle 84.4^\circ$$

$$2 \cosh \gamma l = e^{\gamma l} + e^{-\gamma l} = 1.051 \angle 0.466 \text{ rad} + 0.952 \angle -0.466 \text{ rad} = 1.790 \angle 1.42^\circ$$

$$\cosh \gamma l = 0.895 \angle 1.42^\circ$$

$$|V_2| = 132 \times 10^3 / \sqrt{3} = 76.2 \text{ kV} \Rightarrow V_2 = 76.2 \angle 0^\circ \text{ kV},$$

$$1\Phi P_{\text{load}} = 40 \times 10^6 / 3 = 13.33 \text{ MW} = 0.95 |V_2| |I_2| \Rightarrow |I_2| = 184.1, I_2 = 184.1 \angle -18.195^\circ$$

$$V_1 = V_2 \cosh \gamma l + Z_c I_2 \sinh \gamma l = 89.28 \angle 19.39^\circ \text{ kV} \Rightarrow V_{\text{LL}} = \sqrt{3} \times 89.28 = 154.64 \text{ kV}$$

$$I_1 = I_2 \cosh \gamma l + (V_2 / Z_c) \sinh \gamma l = 162.42 \angle 14.76^\circ \text{ A}$$

$$P_{12} = \text{real}(V_1 I_1^*) = 89.28 \times 10^3 \times 162.42 \cos(19.39^\circ - 14.76^\circ) = 14.45 \text{ MW}$$

$$\text{Efficiency } \eta = 13.33 / 14.45 = 0.92 = 92 \%$$

Ex4.2 Suppose that a radial line is terminated in its characteristic impedance Z_c . Find the driving point impedance V_1/I_1 , the voltage gain $|V_2|/|V_1|$, the current gain $|I_2|/|I_1|$, the complex power gain $-S_{21}/S_{12}$, and the real power efficiency $-P_{21}/P_{12}$.

(a) If the line is terminated in Z_c , then $V_2 = Z_c I_2$, $Z_c = (z/y)^{0.5}$

$$V_1 = V_2 \cosh \gamma l + Z_c I_2 \sinh \gamma l = V_2 (\cosh \gamma l + \sinh \gamma l) = V_2 e^{\gamma l} = V_2 e^{\alpha l} e^{j\beta l}$$

$$I_1 = I_2 \cosh \gamma l + (V_2 / Z_c) \sinh \gamma l = I_2 (\cosh \gamma l + \sinh \gamma l) = I_2 e^{\gamma l} = I_2 e^{\alpha l} e^{j\beta l}$$

$$\text{So } V_1/I_1 = V_2/I_2 = Z_c ,$$

and we that the driving point impedance is Z_c .

$$|V_1| = |V_2 e^{\gamma l}| = |V_2| e^{\alpha l} \Rightarrow |V_2|/|V_1| = e^{-\alpha l} ,$$

$$|I_1| = |I_2 e^{\gamma l}| = |I_2| e^{\alpha l} \Rightarrow |I_2| / |I_1| = e^{-\alpha l} ,$$

(b) Noting the reference direction for I_2 , the complex power gain may be calculated

$$-S_{21} = V_2 I_2^* = V_1 e^{-\alpha l} e^{-j\beta l} I_1^* e^{-\alpha l} e^{j\beta l} = (V_1 I_1^*) e^{-2\alpha l} = S_{12} e^{-2\alpha l} , \text{ thus}$$

$$-S_{21} / S_{12} = e^{-2\alpha l} . \text{ Alternatively, we may observe that}$$

$$V_1 = V_2 e^{\gamma l} = Z_c I_2 e^{\gamma l} = Z_c I_1 \Rightarrow V_1 I_1^* = Z_c |I_1|^2 , \text{ while } V_2 I_2^* = Z_c I_2 I_2^* = Z_c |I_2|^2 ,$$

$$-S_{21}/S_{12} = Z_c |I_2|^2 / (Z_c |I_1|^2) = |I_2|^2 / |I_1|^2 = e^{-2\alpha l} .$$

Finally, since α is real, we have

$$\eta = -P_{21} / P_{12} = e^{-2\alpha l}$$

Ex4.3 Repeat Ex 4.2 for the case of a lossless line ($r = g = 0$). In addition, find Z_c , γ , and P_{12} .

$z = r + j\omega l$ = series impedance per meter

$y = g + j\omega c$ = shunt admittance per meter to neutral

$Z_c = (z/y)^{0.5} = (j\omega l / j\omega c)^{0.5} = (l / c)^{0.5} = (L / C)^{0.5}$, Z_c is real

$L = \text{length} \times l$ = total inductance of the line

$C = \text{length} \times c$ = total capacitance of the line

$\gamma = (z y)^{0.5} = (j\omega l j\omega c)^{0.5} = j\omega (l c)^{0.5} = \alpha + j\beta$, γ is imaginary number

Since $\alpha = 0$, all the ratios calculated in Ex4.2 are unity. Thus

$|V_2|/|V_1| = 1$, $|I_2|/|I_1| = 1$, $-S_{21} / S_{12} = 1$, $\eta = -P_{21} / P_{12} = 1$,

Finally, $P_{12} = \text{Re}(V_1 I_1^*) = \text{Re}(Z_c |I_1|^2) = Z_c |I_1|^2$, since Z_c in this case is real.

An alternate expression based on $I_1 = V_1 / Z_c$ is frequently more useful:

$P_{12} = |V_1|^2 / Z_c$, In a lossless line, Z_c is surge impedance (突波阻抗). In a loss line, Z_c is characteristic impedance (特性阻抗).

A lossless line operating at its normal voltage, terminated in its surge impedance Z_c , is said to be surge impedance loaded (SIL).

$P_{\text{SIL}} = |V_1|^2 / Z_c$ (1 phase), $3P_{\text{SIL}} = 3|V_1|^2 / Z_c = |V_{\text{LL}}|^2 / Z_c$ (3 phase),

4.2 輸電線上的電波(Waves on Transmission Lines)

Propagation constant : $\gamma = (y z)^{0.5} = \alpha + j\beta$,

α is attenuation constant (衰減常數), β is phase constant (相位常數).

$$V = k_1 e^{\gamma x} + k_2 e^{-\gamma x} ,$$

$k_1 e^{\gamma x}$ is a voltage wave traveling to the right (incident wave: 入射波);

$k_2 e^{-\gamma x}$ is a voltage wave traveling to the left (reflected wave: 反射波).

$$v(t, x) = \sqrt{2} \operatorname{Re} k_1 e^{\alpha x} e^{j(\omega t + \beta x)} + \sqrt{2} \operatorname{Re} k_2 e^{-\alpha x} e^{j(\omega t - \beta x)} = v_1(t, x) + v_2(t, x)$$

Neglecting α and considering $v_2(t, x)$, for fixed x , v_2 is a sinusoidal function of t ; for fixed t , v_2 is a sinusoidal function of x .

Assume $\omega t - \beta x = \text{constant} \Rightarrow v_2$ is constant.

So velocity $dx/dt = \omega/\beta = \omega / \operatorname{Im} (y z)^{0.5}$

If an incident **lightning** voltage pulse (or surge) traveling down a line hits the open-circuited end of the line (or an open switch), there will be a reflected wave generated such that the **voltage** at the line termination will approximately **double** . In designing the insulation for transmission lines and equipment connected to it (such as transformers), it is vitally important to take this doubling into account.

4.3 傳輸矩陣(Transmission Matrix)

Steady-state terminal voltage and currents.

$$V_1 = AV_2 + BI_2 \ ; \ I_1 = CV_2 + DI_2 \ ,$$

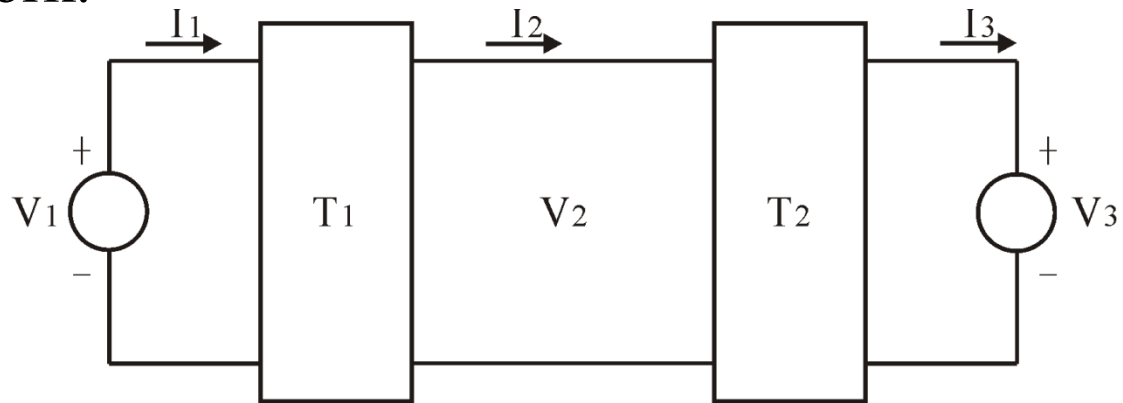
where $A = \cosh \gamma l$, $B = Z_c \sinh \gamma l$, $C = (1/Z_c) \sinh \gamma l$, $D = \cosh \gamma l$,

Transmission matrix (chain matrix) $T = [A \ B; C \ D]$, $T^{-1} = [D \ -B; -C \ A]$

$$\det T = AD - BC = (\cosh \gamma l)^2 - (\sinh \gamma l)^2 = 1$$

$$[V_1 \ I_1] = T_1 [V_2 \ I_2] = T_1 T_2 [V_3 \ I_3] = T [V_3 \ I_3]$$

That $\det T = 1$, holds in general for two-port networks composed of (linear time-invariant) resistors, capacitors, inductors, coupled inductors, and transformers. This provides a useful check of analytical or numerical work.



4.4 等效集總電路(Lumped-Circuit Equivalent), $\gamma = (y z)^{0.5}$

$$V_1 = AV_2 + BI_2 ; I_1 = CV_2 + DI_2 ,$$

Find a **Π equivalent circuit** that has the same A, B, C, D parameters as the transmission line. We note that a T equivalent circuit may also be derived.

$$V_1 = V_2 \cosh \gamma l + Z_c I_2 \sinh \gamma l ; I_1 = I_2 \cosh \gamma l + (V_2 / Z_c) \sinh \gamma l$$

$$V_1 = V_2 + Z' [I_2 + (Y'/2)V_2] = (1 + Z'Y'/2)V_2 + Z'I_2$$

$$I_1 = (Y'/2)V_1 + (Y'/2)V_2 + I_2 = Y'(1 + Z'Y'/4)V_2 + (1 + Z'Y'/2)I_2$$

$$\mathbf{A} = \cosh \gamma l = 1 + Z'Y'/2 , \mathbf{B} = Z_c \sinh \gamma l = \mathbf{Z}' ,$$

$$\mathbf{C} = (1/Z_c) \sinh \gamma l = Y'(1 + Z'Y'/4), \mathbf{D} = \cosh \gamma l = 1 + Z'Y'/2 ,$$

$$\mathbf{Z}' = Z_c \sinh \gamma l = (z/y)^{0.5} \sinh \gamma l = zl \sinh \gamma l / l(zy)^{0.5} = Z \sinh \gamma l / \gamma l = \mathbf{Z}$$

Assume $|\gamma l| \ll 1 \Rightarrow \sinh \gamma l / \gamma l = 1 ; Z = zl$ is the total series impedance of line

$$1 + Z'Y'/2 = \cosh \gamma l \Rightarrow \mathbf{Y}'/2 = (\cosh \gamma l - 1)/Z' = (\cosh \gamma l - 1)/(Z_c \sinh \gamma l)$$

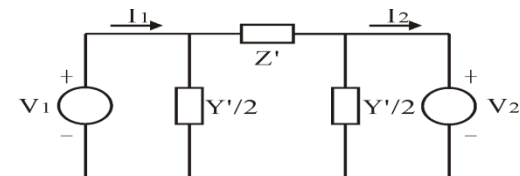
$$= (1/Z_c)(e^{\gamma l} + e^{-\gamma l} - 2)/(e^{\gamma l} - e^{-\gamma l}) = (1/Z_c)(e^{\gamma l/2} - e^{-\gamma l/2})^2 / (e^{\gamma l/2} + e^{-\gamma l/2})(e^{\gamma l/2} - e^{-\gamma l/2})$$

$$= (1/Z_c) \tanh(\gamma l/2) = (y/z)^{0.5} \tanh(\gamma l/2) = (yl/\gamma l) \tanh(\gamma l/2) = (\mathbf{Y}/2) \tanh(\gamma l/2) / (\gamma l/2)$$

Assume $|\gamma l| \ll 1 \Rightarrow \tanh(\gamma l/2) / (\gamma l/2) = 1 ; Y = yl$ is the total line-neutral admittance of line

$$Z_c = (z/y)^{0.5} = (zl/yl)^{0.5} = (Z/Y)^{0.5}$$

$$\gamma l = (zy)^{0.5} l = (zl/yl)^{0.5} = (Z/Y)^{0.5}$$



Ex4.4 Find the Π equivalent circuit in Ex4.1 A **60-Hz** 138-kV 3 Φ transmission line is 225mile long. The distributed line parameters are $r = 0.169 \Omega/\text{mile}$, $l = 2.093 \text{ mH}/\text{mile}$, $c = 0.01427 \mu\text{F}/\text{mile}$, $g = 0$.

$$z = r + j\omega l = 0.169 + j0.789 = 0.807 \angle 77.9^\circ \Omega/\text{mile}$$

$$y = g + j\omega c = j5.38 \times 10^{-6} = 5.38 \times 10^{-6} \angle 90^\circ \text{ mho}/\text{mile}$$

$$\Rightarrow Z_c = (z/y)^{0.5} = 387.3 \angle -6.05^\circ \Omega$$

$$\Rightarrow \gamma l = 225 (y z)^{0.5} = 0.4688 \angle 83.95^\circ = 0.0494 + j0.466$$

$$2 \sinh \gamma l = e^{\gamma l} - e^{-\gamma l} = e^{0.0494} e^{j0.466} - e^{-0.0494} e^{-j0.466} = 1.051 \angle 0.466 \text{ rad} - 0.952 \angle -0.466 \text{ rad}$$

$$\sinh \gamma l = 0.452 \angle 84.4^\circ$$

$$Z' = Z_c \sinh \gamma l = 387.3 \angle -6.05^\circ \Omega \times 0.452 \angle 84.4^\circ = 175.06 \angle 78.35^\circ$$

$$Z' = Z \sinh \gamma l / \gamma l = 225 \times 0.807 \angle 77.9^\circ \times 0.452 \angle 84.4^\circ / (0.4688 \angle 83.95^\circ)$$

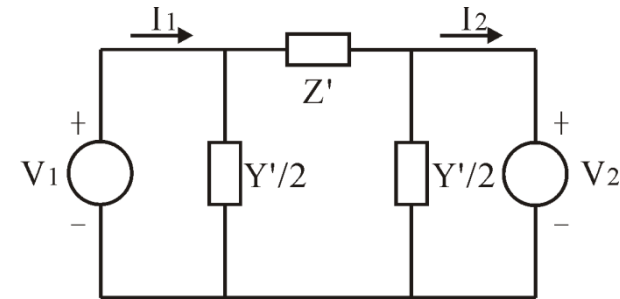
$$= 181.57 \angle 77.9^\circ \times 0.9642 \angle 0.45^\circ = 175.07 \angle 78.35^\circ$$

$$2 \cosh \gamma l = e^{\gamma l} + e^{-\gamma l} = 1.051 \angle 0.466 \text{ rad} + 0.952 \angle -0.466 \text{ rad} = 1.790 \angle 1.42^\circ$$

$$\cosh \gamma l = 0.895 \angle 1.42^\circ$$

$$Y'/2 = (\cosh \gamma l - 1)/Z' = (0.895 \angle 1.42^\circ - 1) / 175.06 \angle 78.35^\circ = 614.57 \times 10^{-6} \angle 89.8^\circ \text{ mho}$$

$$Y/2 = (yl)/2 = (225 \times 5.38 \times 10^{-6} \angle 90^\circ) / 2 = 605.25 \times 10^{-6} \angle 90^\circ \text{ mho}$$



4.5 簡化模型(Simplified Models)

Long line ($l > 150\text{mile} = 241.4 \text{ km}$): Use the Π equivalent circuit model with Z' and $Y'/2$.

Medium length line ($50\text{mile} < l < 150\text{mile} = 241.4 \text{ km}$): Use the Π equivalent circuit model with Z and $Y/2$ instead of Z' and $Y'/2$.

Short line ($l < 50\text{mile} = 80.5 \text{ km}$): Same as the medium length line except that we neglect $Y/2$.

$$z = r + j\omega l \Rightarrow Z = zl ,$$

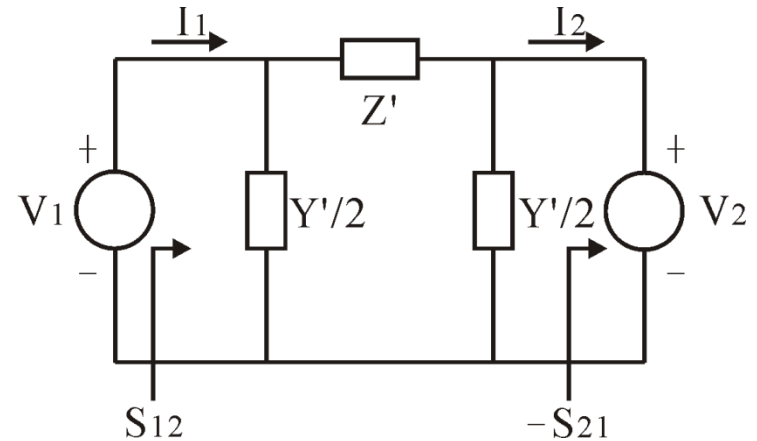
$$y = g + j\omega c \Rightarrow Y = yl ,$$

$$Z' = Z_c \sinh \gamma l$$

$$Y'/2 = (\cosh \gamma l - 1)/Z'$$

$$\gamma = (yz)^{0.5} , Z_c = (z/y)^{0.5}$$

$$\sinh \gamma l = (e^{\gamma l} - e^{-\gamma l})/2 , \cosh \gamma l = (e^{\gamma l} + e^{-\gamma l})/2$$



Ex 4.5 Consider the receiving-end voltage of a lossless open-circuited line and compare the results by use of the three models. V_1 is fixed.

Open circuited $\Rightarrow I_2 = 0$, lossless $\Rightarrow \alpha = 0$, $\gamma = j\beta$, assume $\beta = 0.002$ rad/mile

M1: Long line ($l > 150$ mile):

$$V_1 = V_2 \cosh \gamma l + Z_c I_2 \sinh \gamma l = V_2 \cosh \gamma l = V_2 (e^{\gamma l} + e^{-\gamma l})/2 = V_2 \cos \beta l$$

$$I_1 = I_2 \cosh \gamma l + (V_2 / Z_c) \sinh \gamma l = (V_2 / Z_c) \sinh \gamma l$$

M2: Medium length line: Use the Π model with Z and $Y/2$ instead of Z' and $Y'/2$.

$$\begin{aligned} V_1 &= V_2 + Z' [I_2 + (Y'/2)V_2] = (1 + Z'Y'/2)V_2 + Z'I_2 \\ &= (1 + ZY/2)V_2 = [1 + (\gamma l)^2 / 2] V_2 = [1 - (\beta l)^2 / 2] V_2 \end{aligned}$$

This is the first two terms in the Taylor series expansion of $\cos \beta l$.

M3: Short line ($l < 50$ mile = 80.5 km): Same as the medium length line ($Y/2 = 0$).

$$V_1 = V_2 + Z' [I_2 + (Y'/2)V_2] = \mathbf{V_2}$$

This is the first term in the Taylor series expansion of $\cos \beta l$.

(1) 50 miles $\Rightarrow \beta l = 0.1$ rad ,

$$\text{M1: } V_1 = 0.995004 V_2, \text{ M2: } V_1 = 0.995000 V_2, \text{ M3: } V_1 = V_2,$$

(2) 200 miles $\Rightarrow \beta l = 0.4$ rad ,

$$\text{M1: } V_1 = 0.921 V_2, \text{ M2: } V_1 = 0.920 V_2, \text{ M3: } V_1 = V_2,$$

(1) 600 miles $\Rightarrow \beta l = 1.2$ rad ,

$$\text{M1: } V_1 = 0.362 V_2, \text{ M2: } V_1 = 0.280 V_2 \text{ (error ?), M3: } V_1 = V_2 \text{ (inaccurate),}$$

4.6 複數功率傳輸(長程或中程線路)(Complex Power Transmission: Long or Medium Lines)

Z between voltages V_1 and V_2

$$\begin{aligned} S_{12} &= V_1 I_1^* = V_1 [(V_1 - V_2)/Z]^* = |V_1|^2 / Z^* - V_1 V_2^* / Z^* \\ &= (|V_1|^2 / |Z|) e^{j\angle Z} - (|V_1| |V_2| / |Z|) e^{j\angle Z} e^{j\theta_{12}} \end{aligned}$$

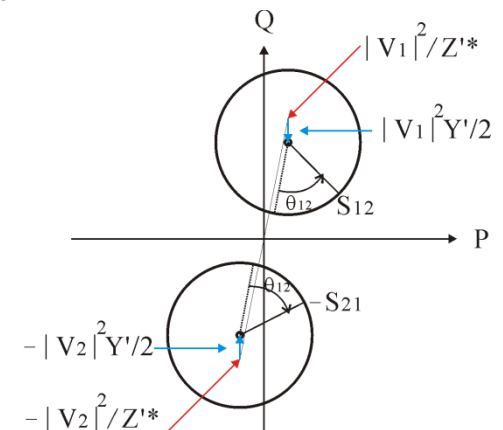
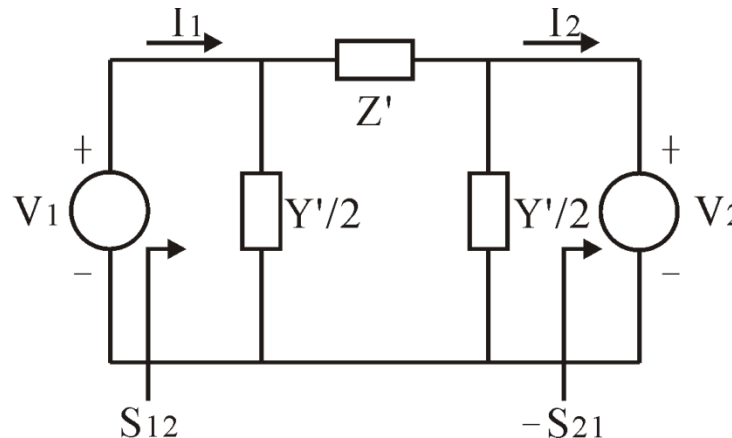
$$\begin{aligned} S_{21} &= V_2 I_2^* = V_2 ((V_2 - V_1)/Z)^* = |V_2|^2 / Z^* - V_2 V_1^* / Z^* \\ &= (|V_2|^2 / |Z|) e^{j\angle Z} - (|V_2| |V_1| / |Z|) e^{j\angle Z} e^{-j\theta_{12}} \end{aligned}$$

$$-S_{21} = -(|V_2|^2 / |Z|) e^{j\angle Z} + (|V_2| |V_1| / |Z|) e^{j\angle Z} e^{-j\theta_{12}}$$

Z' and $Y'/2$ between voltages V_1 and V_2

$$S_{12} = (Y'^*/2) |V_1|^2 + (|V_1|^2 / Z'^*) - (|V_1| |V_2| / Z'^*) e^{j\theta_{12}}$$

$$-S_{21} = -(Y'^*/2) |V_2|^2 - (|V_2|^2 / Z'^*) + (|V_1| |V_2| / Z'^*) e^{-j\theta_{12}}$$



4.7 線路的功率處理容量(Power-Handling Capability of Lines)

Power line are limited in their ability to deliver power. The two most important limits can be understood by considering thermal effects and stability.

When current flows in conductors, there are I^2R losses and heat is generated (T increased, T is limited 100° C). This loss of power reduces transmission efficiency.

If the cable gets too hot, the insulation will begin to deteriorate and may fail in time.

With limitations on maximum current and voltage there is a corresponding limitation on the MVA that can safely be transmitted.

The maximum angle θ_{12} is 90° , for stability, the maximum angle θ_{12} is $40^\circ \sim 50^\circ$,

A lossless line with equal voltage magnitudes at each end as Ex 4.3.

$$z = r + j\omega l = j\omega l \text{ series impedance per meter}$$

$$y = g + j\omega c = j\omega c \text{ shunt admittance per meter to neutral}$$

$$Z_c = (z/y)^{0.5} = (j\omega l / j\omega c)^{0.5} = (l / c)^{0.5} = (L/C)^{0.5} ,$$

$$\gamma = (z y)^{0.5} = \alpha + j\beta = j\beta = (j\omega l j\omega c)^{0.5} = j\omega (l c)^{0.5}$$

$$\sinh \gamma l = (e^{\gamma l} - e^{-\gamma l})/2 = (e^{j\beta l} - e^{-j\beta l})/2 = j \sin \beta l$$

$$\cosh \gamma l = (e^{\gamma l} + e^{-\gamma l})/2 = (e^{j\beta l} + e^{-j\beta l})/2 = \cos \beta l$$

$$Z' = Z_c \sinh \gamma l = jZ_c \sin \beta l$$

$$Y'/2 = (\cosh \gamma l - 1)/Z' = (\text{Y/2}) \tanh(\gamma l/2)/(\gamma l/2) = (j\omega C/2) \tan(\beta l/2)/(\beta l/2)$$

$$\text{Assume } |V_1| = |V_2| , P_{\text{SIL}} = |V_1|^2 / Z_c$$

$$S_{12} = (Y'^*/2)|V_1|^2 + (|V_1|^2 / Z'^*) - (|V_1||V_2|/Z'^*) e^{j\theta_{12}}$$

$$P_{12} = - [|V_1||V_2| / (-jZ_c \sin \beta l)] (j \sin \theta_{12}) = (|V_1|^2 / Z_c) (\sin \theta_{12} / \sin \beta l)$$

$$P_{12} = P_{\text{SIL}} (\sin \theta_{12} / \sin \beta l)$$

$$-S_{21} = -(Y'^*/2)|V_2|^2 - (|V_2|^2 / Z'^*) + (|V_1||V_2|/Z'^*) e^{-j\theta_{12}}$$

Ex 4.6 Assume that $\beta = 0.002$ rad/mile and $\theta_{12} = 45^\circ$. Find P_{12}/P_{SIL} as a function of line length.

$$\text{Assume } |V_1| = |V_2|, P_{SIL} = |V_1|^2 / Z_c$$

$$S_{12} = (Y'^*/2)|V_1|^2 + (|V_1|^2 / Z'^*) - (|V_1||V_2|/Z'^*) e^{j\theta_{12}}$$

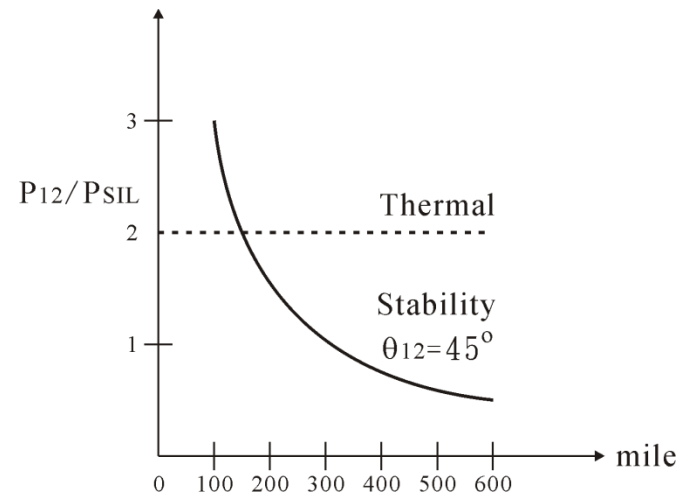
$$P_{12} = - [|V_1||V_2| / (-jZ_c \sin \beta l)] (j \sin \theta_{12}) = (|V_1|^2 / Z_c) (\sin \theta_{12} / \sin \beta l)$$

$$P_{12} = P_{SIL} (\sin \theta_{12} / \sin \beta l) \Rightarrow P_{12} / P_{SIL} = (\sin \theta_{12} / \sin \beta l)$$

$$\theta_{12} = 45^\circ, \beta l = 0.002 \text{ rad/mile} \times l = 0.1146^\circ/\text{mile} \times l$$

$$P_{12} / P_{SIL} = 0.707 / \sin (0.1146^\circ/\text{mile} \times l)$$

We see that for short lines the **thermal** limit governs, where as for long lines the **stability** limit prevails.



4.8 結論與習題(Summary)

The long line equation is frequently used.

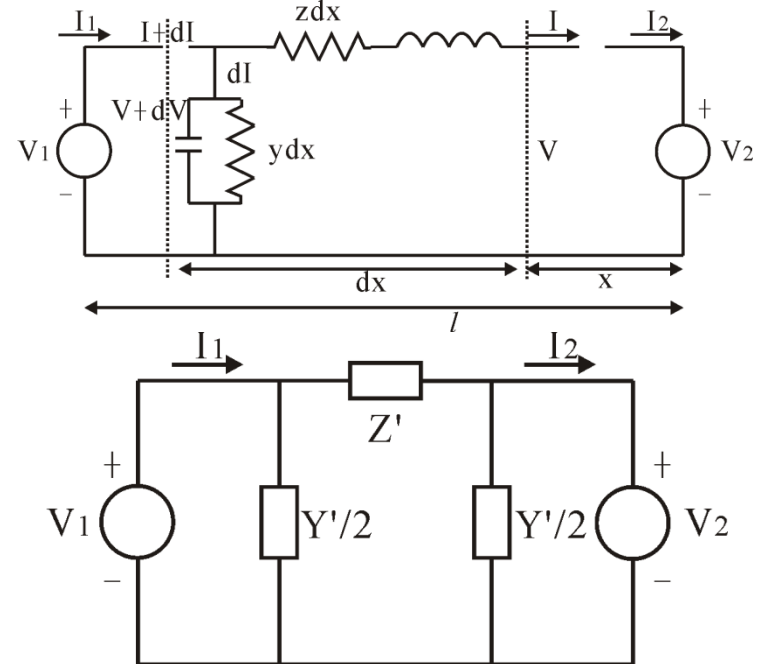
$$V_1 = V_2 \cosh \gamma l + Z_c I_2 \sinh \gamma l$$

$$I_1 = I_2 \cosh \gamma l + (V_2 / Z_c) \sinh \gamma l$$

The Π equivalent circuit

$$Z' = Z_c \sinh \gamma l$$

$$Y'/2 = (\cosh \gamma l - 1)/Z'$$



Finally, we note that **thermal** effects limit the power handling capability of short and medium length lines, whereas **stability** requirements impose the limitations on long lines.