Power System Analysis
供電＝用電

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2.0簡介(Introduction)

1直流電:被動元件(R,L,C);V=IR,V=Ldi/dt,I=Cdv/dt

功率$P=VI=V^2/R=I^2R$;功J=P*t

2Kirchhoff's current law (KCL): sum of currents entering a node (or a closed boundary) is zero.

3Kirchhoff's voltage law (KVL): sum of all voltages around a closed path (or loop) is zero.

4穩態、暫態、動態(steady state, transient state, dynamic state)

5交流電:單相與三相;相量(phasor)

6交流電:被動元件(R,L,C);v=iR,v=Ldi/dt,i=Cdv/dt

功率$P=V_{rms}I_{rms}=V_{rms}^2/R=I_{rms}^2R$;功J=P*t

7符號:v_{BE}(總瞬間值)=V_{BE}(直流值)+v_{be}(交流瞬間值)

V_{be}(相量值)

2.1供應單埠的複數功率
(Complex Power Supplied to a One-Port)

$v(t)=V_m\cos(\omega t+\theta_v)= \mathrm{Re}(V_m e^{j(\omega t+\theta v)})$

$i(t)=I_m\cos(\omega t+\theta_i)= \mathrm{Re}(I_m e^{j(\omega t+\theta i)})$

$p(t)=v(t)*i(t)=V_m\cos(\omega t+\theta_v)*I_m\cos(\omega t+\theta_i)$

$\quad =0.5*V_m I_m[\cos(\theta_v-\theta_i)+\cos(2\omega t+\theta_v+\theta_i)]$

Power factor angle: $\theta=\theta_v-\theta_i$

Power factor = $\cos\theta$;(lagging,leading,unity)

Average power$=0.5*V_m I_m\cos\theta=V_{rms}I_{rms}\cos\theta=\mathrm{Re}VI^*$

Power triangle $S=VI^*=P+jQ$;

$|S|$視在功率(VA) apparent power

S複數功率(VA) complex power

P實功(W) real power

Q虛功(無效功率)(Var) reactive power

EX2.1 Inductor L, $Z=j\omega L$, reactive power $Q=\omega L|I|^2$

$i(t)=\sqrt{2}|I|\cos(\omega t+\theta)$

$v(t)=L\,di/dt=-\sqrt{2}\omega L|I|\sin(\omega t+\theta)$

$$p(t)=v(t)*i(t)=-2\omega L|I|^2\sin(\omega t+\theta)\cos(\omega t+\theta)$$
$$=-\omega L|I|^2\sin2(\omega t+\theta)$$

Average Power $P=0$,

瞬時功不為零(Instantaneous power is not zero)

$S=VI^*=ZII^*=Z|I|^2=j\omega L|I|^2=P+jQ$

所以$P=0, Q=\mathrm{Im}S=\omega L|I|^2$

練習1: Capacitor C, $Z=1/j\omega C$, reactive power $Q=-\omega C|V|^2$

$v(t)=\sqrt{2}|V|\cos(\omega t+\theta)$

$i(t)=C\,dv/dt=-\sqrt{2}\omega C|V|\sin(\omega t+\theta)$

$p(t)=v(t)*i(t)=-2\omega C|V|^2\sin(\omega t+\theta)\cos(\omega t+\theta)$

$=-\omega C|V|^2\sin 2(\omega t+\theta)$

Average Power P=0,

瞬時功不為零(Instantaneous power is not zero)

$S=VI^*=V(V/Z)^*=|V|^2/(Z)^*=-j\omega C|V|^2=P+jQ$

So $P=0, Q=\mathrm{Im}S=-\omega C|V|^2$

2.2複數功率守恆
(Conservation of Complex Power)

複數功率守恆($S_{in} = S_{out}$):數個頻率相同的獨立電源供應的網路，

由各個獨立電源供應的複數功率的總和會等於網路會等於網路上所有分支接收到的複數功率

供電=用 電(Power of generators are equal Loads)

EX2.3輸入電源並聯電容C
Input voltage with shunt C

$S_{in}=S_c+S_o$

$S_c=VI^*=V(V/Z)^*=VV^*(1/Z)^*=VV^*(Y)^*=|V|^2(SC)^*$

$=-j\omega C|V|^2$

$S_o=S_{in}-S_c=S_{in}+j\omega C|V|^2$

$P_o=P_{in}$

$Q_o=Q_{in}+\omega C|V|^2$

EX2.4輸入電源串聯電感L(假設$|V_2|=|V_1|$)
Series L between two voltage source

$S_1+S_2=S_L=VI^*=j\omega L|I|^2$

$P_1+P_2=0$

$Q_1+Q_2=Q_L=\omega L|I|^2$

$S_1=V_1I^*$

$S_2=-V_2I^*$

$\because |V_2|=|V_1| => |S_1|=|S_2| => (P_1)^2+(Q_1)^2=(P_2)^2+(Q_2)^2$

$\because |P_2|=|P_1| => |Q_2|=|Q_1| => Q_1=Q_2=0.5\omega L|I|^2$

So $P_1=-P_2$，$Q_1=Q_2 => S_2=-(S_1)^*$

EX2.7三相電源(Three-phase voltages)

以n1為基點(n1 is basis point)

$S=V_{an1}I_a^*+V_{bn1}I_b^*+V_{cn1}I_c^*$(三瓦特計法)

以b為基點(b is basis point)

$S=V_{ab}I_a^*+V_{bb}I_b^*+V_{cb}I_c^*=V_{ab}I_a^*+V_{cb}I_c^*$(二瓦特計法)

2.3平衡三相(Balanced Three-Phase)

直流電與交流電的優缺點

Advantages and disadvantages of DC and AC voltages

單相交流電與三相交流電的優缺點

Advantages and disadvantages of single-phase voltage and three-phase voltages

正序與負序(產生旋轉磁場)，零序

Positive sequence, negative sequence, zero sequence

平衡與不平衡電壓和負載(線性與非線性負載)

Balanced and unbalanced voltages and loads

中性點電壓與電流(voltage and current of neutral point)

Δ-Y

EX2.8三相電源與負載中性點電壓

Three-phase voltages and neutral point voltage

以n1為基點(n1 is basis point)

$I_a=V_{an1}/Z=(V_{an}-V_{n1n})/Z=(V_{an}-V_{n1n})Y$

$I_b=V_{bn1}/Z=(V_{bn}-V_{n1n})/Z=(V_{bn}-V_{n1n})Y$

$I_c=V_{cn1}/Z=(V_{cn}-V_{n1n})/Z=(V_{cn}-V_{n1n})Y$

So $I_a+I_b+I_c=(V_{an}+V_{bn}+V_{cn})Y-3V_{n1n}Y=0$

If $(V_{an}+V_{bn}+V_{cn})=0 \Rightarrow V_{n1n}=0$

EX2.9中性點阻抗不為零時?

Δ-Y

阻抗(Impedance)

$Z_Y = Z_\Delta / 3$

EX2.10線對線電壓與相電壓?

Line-to-line voltages and Phase voltages?

2.4單相分析(平衡三相)Per Phase Analysis

平衡三相(Balanced three-phase)

假設:平衡三相系統；負載與電源是星形連接；電路模型中，相之間無互感存在

所以:所有的中性點電位相同；各相是完全去耦合；所有對應的網路變數和平衡電源系統具有相同相序

EX2.11 Balanced three-phase?

2.5平衡三相功率(瞬時功率為常數)

Power of the balanced three-phase is constant

$S_3 = V_a I_a^* + V_b I_b^* + V_c I_c^*$

Balanced three-phase and positive sequency

$S_3 = V_a I_a^* + V_a e^{-j2\pi/3}(I_a e^{-j2\pi/3})^* + V_a e^{j2\pi/3}(I_a e^{j2\pi/3})^* = 3V_a I_a^*$

Instantaneous Power: $p_3(t) = p_a(t) + p_b(t) + p_c(t)$

$p_3(t) = v_a(t) i_a(t) + v_b(t) i_b(t) + v_c(t) i_c(t)$

$v_a(t) * i_a(t) = V_m \cos(\omega t + \theta_v) * I_m \cos(\omega t + \theta_i)$

$= 0.5 * V_m I_m [\cos(\theta_v - \theta_i) + \cos(2\omega t + \theta_v + \theta_i)]$

$v_b(t) * i_b(t) = V_m \cos(\omega t + \theta_v - 2\pi/3) * I_m \cos(\omega t + \theta_i - 2\pi/3)$

$= 0.5 * V_m I_m [\cos(\theta_v - \theta_i) + \cos(2\omega t + \theta_v + \theta_i - 4\pi/3)]$

$v_c(t) * i_c(t) = V_m \cos(\omega t + \theta_v + 2\pi/3) * I_m \cos(\omega t + \theta_i + 2\pi/3)$

$= 0.5 * V_m I_m [\cos(\theta_v - \theta_i) + \cos(2\omega t + \theta_v + \theta_i + 4\pi/3)]$

$p_3(t) = 3 * 0.5 * V_m I_m [\cos(\theta_v - \theta_i)] = 3|V||I|\cos(\theta_v - \theta_i)$

2.6複數功率傳輸(短程)Complex power Transmission

短程輸電線，用串聯的RL電路來表示電線$Z=R+j\omega L$

$V_1=|V_1|e^{j\theta 1}$，$V_2=|V_2|e^{j\theta 2}$，$Z=|Z|\ e^{j\angle Z}$，$\theta_{12}=\theta_1-\theta_2$

$S_{12}=V_1 I_1^{*}=V_1[(V_1-V_2)/Z]^{*}=|V_1|^2/(Z)^{*}-V_1 V_2^{*}/(Z)^{*}$

$=|V_1|^2\ e^{j\angle Z}/|Z|-|V_1||V_2|\ e^{j\angle Z}e^{j\theta 12}/|Z|$

$S_{21}=|V_2|^2\ e^{j\angle Z}/|Z|-|V_1||V_2|\ e^{j\angle Z}e^{-j\theta 12}/|Z|$

Assume $R=0$，$Z=jX \Rightarrow \angle Z=90^\circ$，$e^{j\angle Z}=j$

So $P_{12}=-P_{21}=(|V_1||V_2|/X)\sin\theta_{12}$

$Q_{12}=|V_1|^2/X-(|V_1||V_2|/X)\cos\theta_{12}$

$Q_{21}=|V_2|^2/X-(|V_1||V_2|/X)\cos\theta_{12}$

EX2.12兩個發電機失去同步? Two generators without synchronous?

$P_{12}=-P_{21}=(|V_1||V_2|/X)\sin[(\omega_1-\omega_2)t+\theta_{12}]$

EX2.13(短程 short distance) $Z=1\angle 85^\circ$ ，$\theta_{12}=10^\circ$

(a)$|V_1|=|V_2|=1$

$S_{12}=V_1I_1^*=V_1[(V_1-V_2)/Z]^*=|V_1|^2/(Z)^*-V_1V_2^*/(Z)^*$

$=|V_1|^2\, e^{j\angle Z}/|Z|-|V_1||V_2|\, e^{j\angle Z}e^{j\theta 12}/|Z|$

$S_{12}=1\angle 85^\circ\ \ -1\angle 95^\circ$

$S_{21}=|V_2|^2\, e^{j\angle Z}/|Z|-|V_1||V_2|\, e^{j\angle Z}e^{-j\theta 12}/|Z|$

$S_{21}=1\angle 85^\circ\ \ -1\angle 75^\circ$

$P_{12}=-P_{21}=0.1743$

$Q_{12}=0$

$Q_{21}=0.0303$

(b)$|V_1|=1.1$ ，$|V_2|=0.9$ ，$Z=1\angle 85^\circ$ ，$\theta_{12}=10^\circ$

EX2.14 S_{G1}:$V_1=1\angle 0^\circ$，$S_{D1}=1$，jQ_{G2} :$V_2=?$ ，$S_{D2}=1$，$Z=j0.5$

(a)Find Q_{G2} for $|V_2|=1$ (b) and $\angle V_2$?(c)If $Q_{G2}=0$,could be supplied load S_{D2} ?(d) and $\angle V_2$?

$\because$ $S_{D2}=1$ real，and jQ_{G2} imaginary number，So $P_{12}= -P_{21}=1$

So $P_{12}= -P_{21}=(|V_1||V_2|/X)\sin\theta_{12}=2\sin\theta_{12}=1$

So $\theta_{12}=30^\circ$ ，and $\angle V_2=-30^\circ$

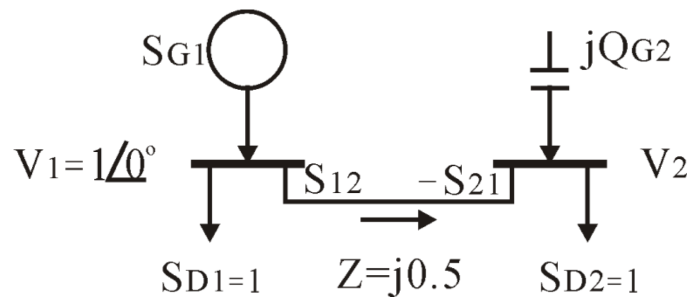
$Q_{G2}=Q_{21}=|V_2|^2/X - (|V_1||V_2|/X)\cos\theta_{12}=2-2\cos 30^\circ =0.268$

(c) and (d)

If $Q_{G2}=0$,$-S_{21}= S_{D2}=1$

$S_{21}=|V_2|^2 e^{j\angle Z}/|Z|-|V_1||V_2| e^{j\angle Z}e^{-j\theta 12}/|Z|=-1$

Find $2|V_2|^2=1,\theta_{12}=45^\circ$ => $V_2=0.707\angle -45^\circ$



(c) and (d)

If $Q_{G2}=0$,so $Q_{G2}=Q_{21}=|V_2|^2/X - (|V_1||V_2|/X)\cos\theta_{12}=0$

so $|V_2|=|V_1|\cos\theta_{12}= \cos\theta_{12}$

If $P_{12}= -P_{21}=(|V_1||V_2|/X)\sin\theta_{12}=2|V_2|\sin\theta_{12}=1$

so $\theta_{12}=45^\circ$,$|V_2|=0.707$

2.7複數功率傳輸(輻射線路) Complex Power Transmission: Radial Line

較遠的一端有複數功率負載，沒有發電機或電容器組來維持電壓，求遠端電壓受負載變化的影響?

$S_D=V_2I^*=|V_2||I|e^{j\psi}$

$=|V_2||I|(\cos\psi+j\sin\psi)=P_D(1+j\beta)$

$P_D=P_{12}= -P_{21}=(|V_1||V_2|/X)\sin\theta_{12}$

$Q_D= -Q_{21}= -|V_2|^2/X + (|V_1||V_2|/X)\cos\theta_{12}$

2.8結論與習題(Summary)

瞬時功率(Instantaneous Power)

複數功率(Complex Power)

有效功率(Real Power)

無效功率(Reactive Power)

相量(Phasor)

平衡三相(Balanced Three-Phase)

單相分析(Per Phase Analysis)