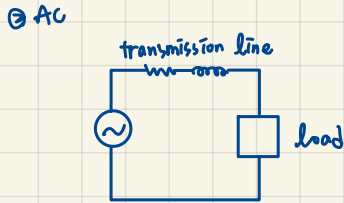
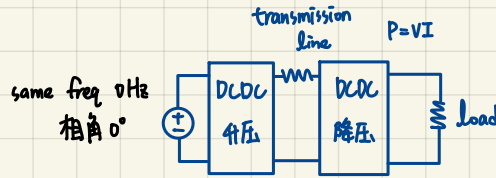
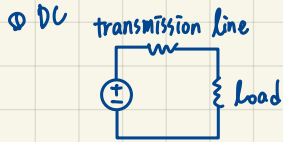


電 { DC (0Hz)
AC (60Hz)



① DC : 电压. 电流. 实功率 (有储能元件)

$$\begin{cases} V=IR \\ V=L\frac{di}{dt} \approx 0 \\ i=C\frac{dV}{dt} \approx 0 \end{cases} \quad P=VI = \frac{V^2}{R} = I^2R$$

② AC : 电压. 电流

- 同相位: 单位功因
- 电流落后于电压: 落后功因
- 电流领先于电压: 领先功因

视在功率 = 实功 + 虚功 (没有储能)
(P) (L+, C-)

① $V=IR$ 单位功因
② $V=L\frac{di}{dt}$ 电流落后电压 90°
落后功因
 $\frac{d\sin t}{dt} = \cos t = \sin(t+90^\circ)$

③ $i=C\frac{dV}{dt}$, $V=\frac{1}{C}\int i dt$
电流领先电压 90°
领先功因

* CH2

直流 DC
0Hz

$$\begin{aligned} V &= iR \text{ 实} \\ \begin{cases} V=L\frac{di}{dt}=0 \\ i=C\frac{dV}{dt}=0 \end{cases} \text{ 虚} \\ P &= VI \\ &= V^2/R \\ &= I^2R \\ Q &= 0 \text{ 虚} \end{aligned}$$

交流 AC
60Hz

$$\begin{aligned} V &= iR \\ V &= L\frac{di}{dt} \neq 0 \\ &= L\omega i \\ i &= C\frac{dV}{dt} \neq 0 \\ &= C\omega V \end{aligned}$$

$S = V_{rms} I_{rms}$
 $Q = S \sin \theta$
 $P = S \cos \theta$
 $\sqrt{2} V_{rms} = V_{in}$

* 功率三角形 (算平均, 非瞬间)

二階常微分方程

穩態: $t \rightarrow \infty$, 留下的 $y'' + 3y' + 2y = 5u(t)$

非齊次
input $u(t)$, output $y(t)$

方程式:

微分方程
偏微分方程 (2個以上變數)
常微分方程 (單變數 y)

非線性
(有自乘、互乘)

線性


變係數

常係數

齊次 非齊次

(輸入項=0) (輸入項≠0)

暫態: $t \rightarrow \infty$, 消失不見

動態: 
ex: switch 改變時突然的變化

y_H : 通解/暫態解 $y'' + 3y' + 2y = 0$

$$(D^2 + 3D + 2)y = 0$$

$$(D+1)(D+2)y = 0$$

$$y_H = C_1 e^{-t} + C_2 e^{-2t}$$

y_P : 特解/穩態解 $y'' + 3y' + 2y = 5u(t)$

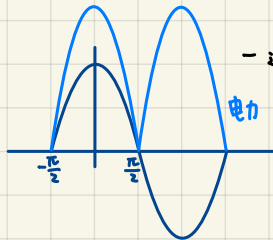
$$y_P = \frac{5}{2}u(t), y_P' = 0, y_P'' = 0$$

單相: $110 \text{ V}_{rms} / 60 \text{ Hz}$
cons: 電力脈動
三相: 3ϕ $220 \text{ V}_{rms} / 60 \text{ Hz}$
pros: 電力常規

$$v(t) = \sqrt{2} \times 110 \sin(\omega t)$$

$$v(t) = \sqrt{2} \times 110 \cos(\omega t)$$

振幅 $V_m = \sqrt{2} V_{rms} = \sqrt{2} \times 110$



一週期 $360^\circ = 2\pi = 6.28 \text{ rad}$ 弧度

單相

$$\cos(\theta + \theta) = \cos^2 \theta - \sin^2 \theta$$

$$= \cos^2 \theta - [1 - \cos^2 \theta]$$

$$= 2\cos^2 \theta - 1$$

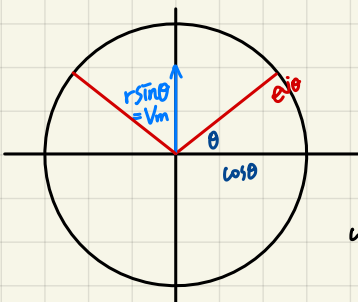
$$\cos^2 \theta = \frac{1 + \cos 2\theta}{2}$$

$$\int_{-\pi/2}^{\pi/2} (\cos \theta)^2 d\theta = \int_{-\pi/2}^{\pi/2} \frac{1 + \cos 2\theta}{2} d\theta = \frac{\pi}{2}$$

$$\frac{\int_{-\pi/2}^{\pi/2} \frac{V_m \cos \theta}{R} d\theta}{\pi} = \frac{V_m^2}{2R} = \frac{V_{rms}^2}{R}$$

在 60Hz 同步座標觀察交流得到直流
→ 只能量一個 freq.

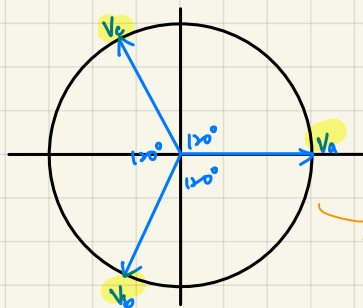
60Hz → 0Hz
0Hz → -60Hz



$$\cos(\omega t) \quad \omega = 2\pi f = 377$$

$$V_m e^{j\theta} = [\cos \theta + j \sin \theta] V_m$$

三相



$$\pi = 180^\circ$$

$$\frac{2\pi}{3} = 120^\circ$$

$$V_a(t) = 180 \sin 377t$$

$$V_b(t) = 180 \sin(377t - 120^\circ)$$

$$V_c(t) = 180 \sin(377t - 240^\circ) = 180 \sin(377t + 120^\circ)$$

$$V_a(t) = 180 \cos 377t$$

$$V_b(t) = 180 \cos(377t - 120^\circ)$$

$$V_c(t) = 180 \cos(377t - 240^\circ) = 180 \cos(377t + 120^\circ)$$

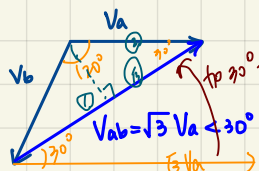
$$V_a + V_b + V_c = 0$$

$$i_a + i_b + i_c = 0$$

$$P = V_a i_a + V_b i_b + V_c i_c = 3VI \cos \theta \neq 0$$

$$V_m = 180 \Rightarrow V_{rms} = \frac{V_m}{\sqrt{2}} = 127$$

$$V_{ab} = V_a - V_b = \sqrt{3} V_{arm} \angle 30^\circ = \sqrt{3} \times 127 = 220$$



$$V(t) = \frac{1}{C} \int i dt = \frac{i}{\omega C} \quad \text{領先功因}$$

$$V(t) = i(t) \times R \quad \text{單位功因}$$

$$V(t) = L \frac{di(t)}{dt} \quad \text{落後功因}$$

$$\frac{d \sin(\omega t)}{dt}$$

$$= \frac{d \sin(\omega t)}{d \omega t} \times \frac{d \omega t}{dt}$$

$$= \cos(\omega t) \times \omega = \sin(\omega t + 90^\circ) \times \omega = j \sin(\omega t) \times \omega \Rightarrow \frac{d}{dt} = j\omega$$

和差化積

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$

$$\sin(A-B) = \sin A \cos B - \cos A \sin B$$

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$\cos(A-B) = \cos A \cos B + \sin A \sin B$$

積化和差

$$2 \cos A \cos B = \cos(A+B) + \cos(A-B)$$

1.9 REAL, REACTIVE, AND APPARENT POWER IN AC CIRCUITS

DC Power

$$P = VI = V^2 / R = I^2 R \quad (1-55)$$

AC Power

$$v(t) = \sqrt{2} V \cos \omega t \quad (1-56)$$

$$i(t) = \sqrt{2} I \cos(\omega t - \theta) \quad (1-57)$$

$$p(t) = v(t)i(t) = 2VI \cos \omega t \cos(\omega t - \theta) \quad (1-58)$$

$$p(t) = VI[\cos \theta + \cos(2\omega t - \theta)]$$

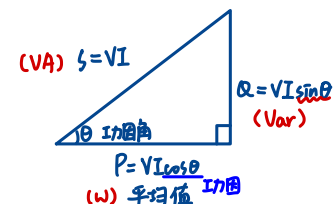
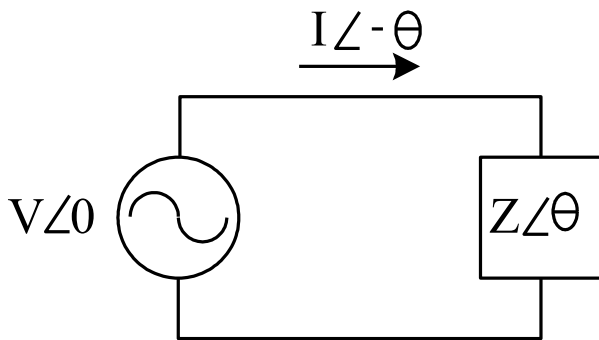
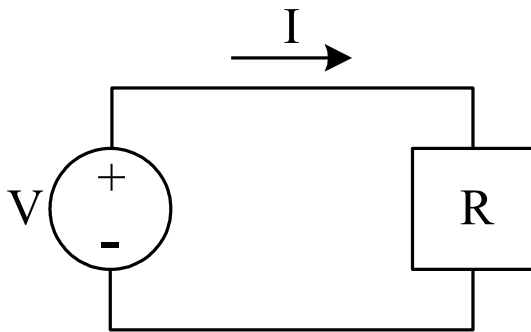
$$= VI \cos \theta \cdot (1 + \cos 2\omega t) + VI \sin \theta \cdot \sin 2\omega t \quad (1-59)$$

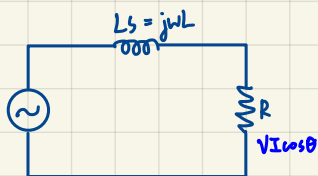
The Power Triangle

$$P = VI \cos \theta \quad (1-60)$$

$$Q = VI \sin \theta \quad (1-61)$$

$$S = VI \quad (1-62)$$





$$v(t) = (j\omega L + R) i(t)$$

7 符號: v_{BE} (總瞬間值) = V_{BE} (直流值) + v_{be} (交流瞬間值)

V_{be} (相量值)

2.1 供應單埠的複數功率

(Complex Power Supplied to a One-Port)

$$v(t) = V_m \cos(\omega t + \theta_v) = \text{Re}(V_m e^{j(\omega t + \theta_v)})$$

$$i(t) = I_m \cos(\omega t + \theta_i) = \text{Re}(I_m e^{j(\omega t + \theta_i)})$$

$$\begin{aligned} p(t) &= v(t) * i(t) = V_m \cos(\omega t + \theta_v) * I_m \cos(\omega t + \theta_i) \\ &= 0.5 * V_m I_m [\cos(\theta_v - \theta_i) + \cos(2\omega t + \theta_v + \theta_i)] \end{aligned}$$

Power factor angle: $\theta = \theta_v - \theta_i$

Power factor = $\cos\theta$; (lagging, leading, unity)

$$\text{Average power} = 0.5 * V_m I_m \cos\theta = V_{\text{rms}} I_{\text{rms}} \cos\theta = \text{Re} VI^*$$

Power triangle $S = VI^* = P + jQ$;

$|S|$ 視在功率 (VA) apparent power

S 複數功率 (VA) complex power

P 實功 (W) real power

Q 虛功 (無效功率) (Var) reactive power

EX2.1 Inductor L , $Z=j\omega L$, reactive power $Q=\omega L|I|^2$

$$i(t)=\sqrt{2}|I|\cos(\omega t+\theta)$$

$$v(t)=Ldi/dt=-\sqrt{2}\omega L|I|\sin(\omega t+\theta)$$

$$\begin{aligned} p(t)=v(t)*i(t) &= -2\omega L|I|^2\sin(\omega t+\theta)\cos(\omega t+\theta) \\ &= -\omega L|I|^2\sin 2(\omega t+\theta) \end{aligned}$$

Average Power $P=0$,

瞬時功不為零 (Instantaneous power is not zero)

$$S=VI^*=ZII^*=Z|I|^2=j\omega L|I|^2=P+jQ$$

$$\text{所以 } P=0, Q=\text{Im}S=\omega L|I|^2$$

練習1: Capacitor C , $Z=1/j\omega C$, reactive power $Q=-\omega C|V|^2$

$$v(t)=\sqrt{2}|V|\cos(\omega t+\theta)$$

$$i(t)=Cdv/dt=-\sqrt{2}\omega C|V|\sin(\omega t+\theta)$$

$$\begin{aligned} p(t)=v(t)*i(t) &= -2\omega C|V|^2\sin(\omega t+\theta)\cos(\omega t+\theta) \\ &= -\omega C|V|^2\sin 2(\omega t+\theta) \end{aligned}$$

Average Power $P=0$,

瞬時功不為零(Instantaneous power is not zero)

$$S=VI^*=V(V/Z)^*=|V|^2/(Z)^*=-j\omega C|V|^2=P+jQ$$

$$\text{So } P=0, Q=\text{Im}S=-\omega C|V|^2$$

2.2 複數功率守恆 (Conservation of Complex Power)

複數功率守恆($S_{in} = S_{out}$): 數個頻率相同的獨立電源供應的網路，

由各個獨立電源供應的複數功率的總和會等於網路會等於網路上所有分支接收到的複數功率

供電=用電(Power of generators are equal Loads)

EX2.3 輸入電源並聯電容C

Input voltage with shunt C

$$S_{in} = S_c + S_o$$

$$\begin{aligned} S_c &= VI^* = V(V/Z)^* = VV^*(1/Z)^* = VV^*(Y)^* = |V|^2(SC)^* \\ &= -j\omega C|V|^2 \end{aligned}$$

$$S_o = S_{in} - S_c = S_{in} + j\omega C|V|^2$$

$$P_o = P_{in}$$

$$Q_o = Q_{in} + \omega C|V|^2$$

EX2.4 輸入電源串聯電感L(假設 $|V_2|=|V_1|$)

Series L between two voltage source

$$S_1 + S_2 = S_L = VI^* = j\omega L |I|^2$$

$$P_1 + P_2 = 0$$

$$Q_1 + Q_2 = Q_L = \omega L |I|^2$$

$$S_1 = V_1 I^*$$

$$S_2 = -V_2 I^*$$

$$\because |V_2| = |V_1| \Rightarrow |S_1| = |S_2| \Rightarrow (P_1)^2 + (Q_1)^2 = (P_2)^2 + (Q_2)^2$$

$$\because |P_2| = |P_1| \Rightarrow |Q_2| = |Q_1| \Rightarrow Q_1 = Q_2 = 0.5\omega L |I|^2$$

$$\text{So } P_1 = -P_2, \quad Q_1 = Q_2 \Rightarrow S_2 = -(S_1)^*$$

EX2.7 三相電源(Three-phase voltages)

以n1為基點(n1 is basis point)

$$S = V_{an1} I_a^* + V_{bn1} I_b^* + V_{cn1} I_c^* \text{ (三瓦特計法)}$$

以b為基點(b is basis point)

$$S = V_{ab} I_a^* + V_{bb} I_b^* + V_{cb} I_c^* = V_{ab} I_a^* + V_{cb} I_c^* \text{ (二瓦特計法)}$$

2.3 平衡三相(Balanced Three-Phase)

pros: ① 无磁功

pros: ① 可升降压 ② 伝送距離遠

直流電與交流電的優缺點

cons: ① 无法升降压 ② 伝送距離短

Advantages and disadvantages of DC and AC voltages

pros: 電力系統

單相交流電與三相交流電的優缺點

cons: ① 電力系統

Advantages and disadvantages of single-phase voltage and three-phase voltages

$V_a(0^\circ) \rightarrow V_b(-120^\circ) \rightarrow V_c(-240^\circ)$

$V_a + V_b + V_c \neq 0$

正序與負序(產生旋轉磁場)，零序

$V_a(0^\circ) \rightarrow V_c(-240^\circ) \rightarrow V_b(-120^\circ)$

Positive sequence, negative sequence, zero sequence

平衡與不平衡電壓和負載(線性與非線性負載)

Balanced and unbalanced voltages and loads

在發電機才是real 中性點其它的摸法會變

中性點電壓與電流(voltage and current of neutral point)

Δ -Y

EX2.8 三相電源與負載中性點電壓

Three-phase voltages and neutral point voltage

以n1為基點(n1 is basis point)

$$I_a = V_{an1}/Z = (V_{an} - V_{n1n})/Z = (V_{an} - V_{n1n})Y$$

$$I_b = V_{bn1}/Z = (V_{bn} - V_{n1n})/Z = (V_{bn} - V_{n1n})Y$$

$$I_c = V_{cn1}/Z = (V_{cn} - V_{n1n})/Z = (V_{cn} - V_{n1n})Y$$

$$\text{So } I_a + I_b + I_c = (V_{an} + V_{bn} + V_{cn})Y - 3V_{n1n}Y = 0$$

$$\text{If } (V_{an} + V_{bn} + V_{cn}) = 0 \Rightarrow V_{n1n} = 0 \quad \text{theoretically, but not likely}$$

EX2.9 中性點阻抗不為零時？

Δ -Y

阻抗(Impedance)

$$Z_Y = Z_{\Delta} / 3$$


EX2.10 線對線電壓與相電壓？

Line-to-line voltages and Phase voltages?

2.4 單相分析(平衡三相)Per Phase Analysis

平衡三相(Balanced three-phase)

假設:平衡三相系統；負載與電源是星形連接；電路模型中，相之間無互感存在

所以:所有的中性點電位相同；各相是完全去耦合；所有對應的網路變數和平衡電源系統具有相同相序

EX2.11 Balanced three-phase?

2.5 平衡三相功率(瞬時功率為常數)

Power of the balanced three-phase is constant

$$S_3 = V_a I_a^* + V_b I_b^* + V_c I_c^*$$

Balanced three-phase and positive sequency

$$S_3 = V_a I_a^* + V_a e^{-j2\pi/3} (I_a e^{-j2\pi/3})^* + V_a e^{j2\pi/3} (I_a e^{j2\pi/3})^* = 3 V_a I_a^*$$

Instantaneous Power: $p_3(t) = p_a(t) + p_b(t) + p_c(t)$

$$p_3(t) = v_a(t) i_a(t) + v_b(t) i_b(t) + v_c(t) i_c(t)$$

$$\begin{aligned} v_a(t) i_a(t) &= V_m \cos(\omega t + \theta_v) * I_m \cos(\omega t + \theta_i) \\ &= 0.5 * V_m I_m [\cos(\theta_v - \theta_i) + \cos(2\omega t + \theta_v + \theta_i)] \end{aligned}$$

$$\begin{aligned} v_b(t) i_b(t) &= V_m \cos(\omega t + \theta_v - 2\pi/3) * I_m \cos(\omega t + \theta_i - 2\pi/3) \\ &= 0.5 * V_m I_m [\cos(\theta_v - \theta_i) + \cos(2\omega t + \theta_v + \theta_i - 4\pi/3)] \end{aligned}$$

$$\begin{aligned} v_c(t) i_c(t) &= V_m \cos(\omega t + \theta_v + 2\pi/3) * I_m \cos(\omega t + \theta_i + 2\pi/3) \\ &= 0.5 * V_m I_m [\cos(\theta_v - \theta_i) + \cos(2\omega t + \theta_v + \theta_i + 4\pi/3)] \end{aligned}$$

$$p_3(t) = 3 * 0.5 * V_m I_m [\cos(\theta_v - \theta_i)] = 3 |V| |I| \cos(\theta_v - \theta_i)$$