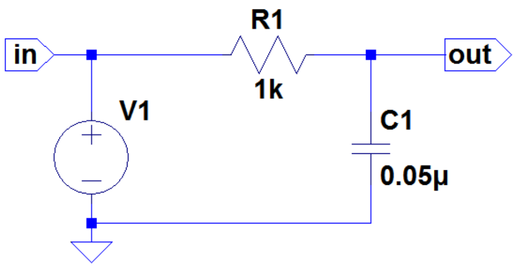
REPORT

Experiment 1: RC Circuit

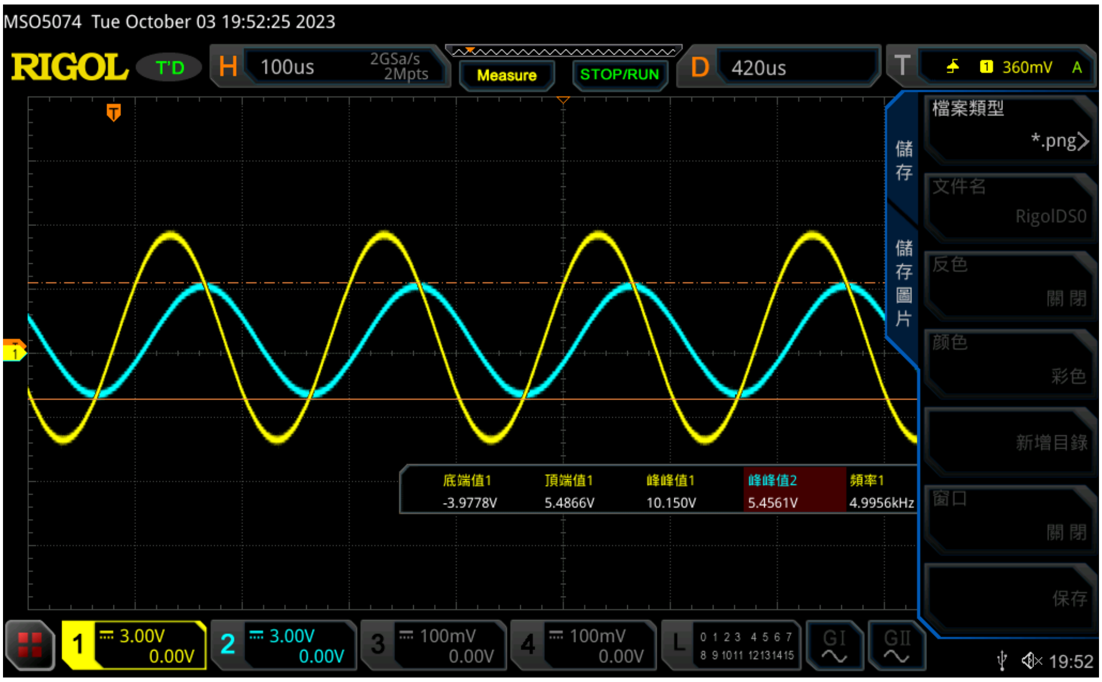


1.

Frequency (Hz)	5K	10K	15K
$V_{out,pp}$(V)	5.9184V	3.5510V	2.8408V

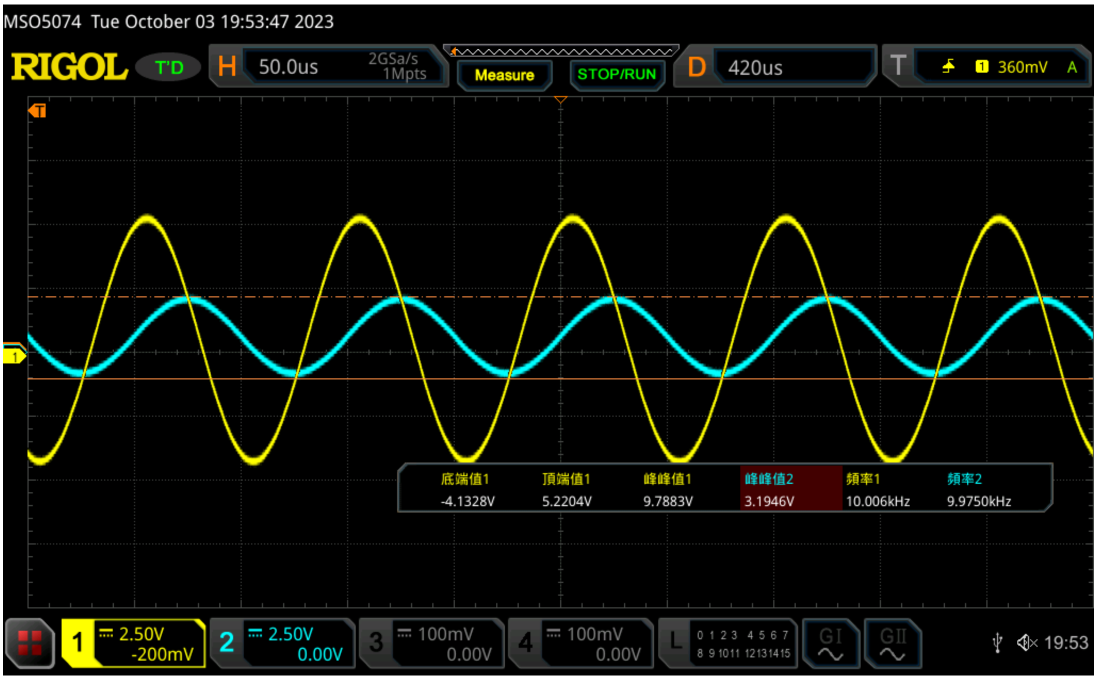
ADJUST THE OSCILLOSCOPE APPROPRIATELY

5k Hz V_{in} and V_{out} waveform

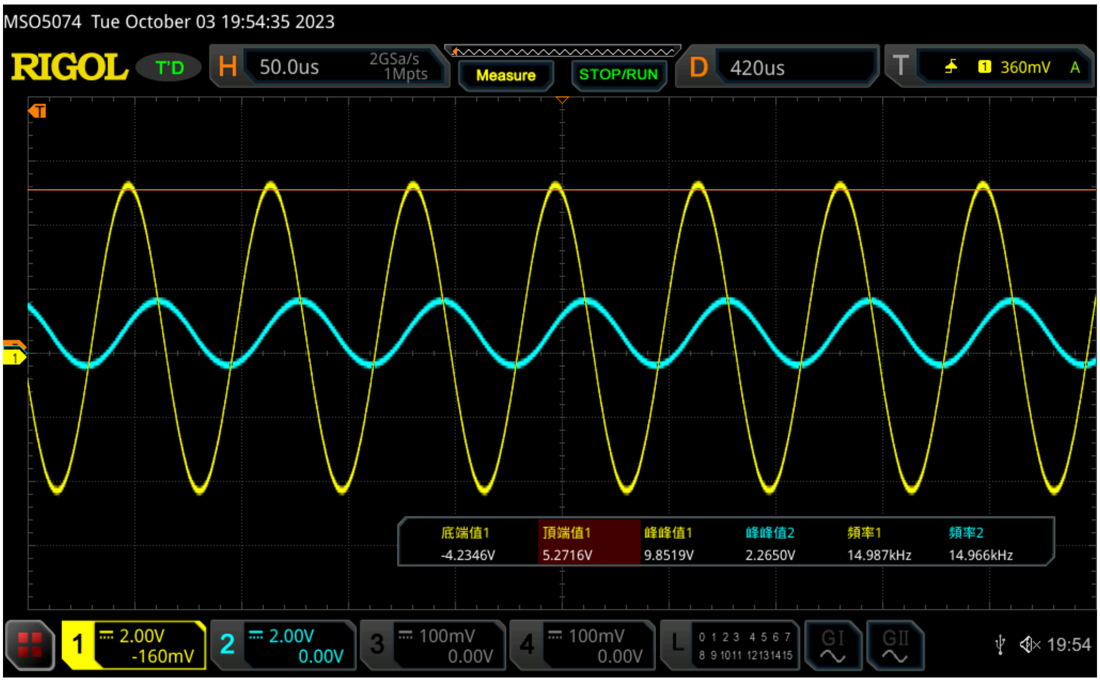


Basic Components

10k Hz V_{in} and V_{out} waveform



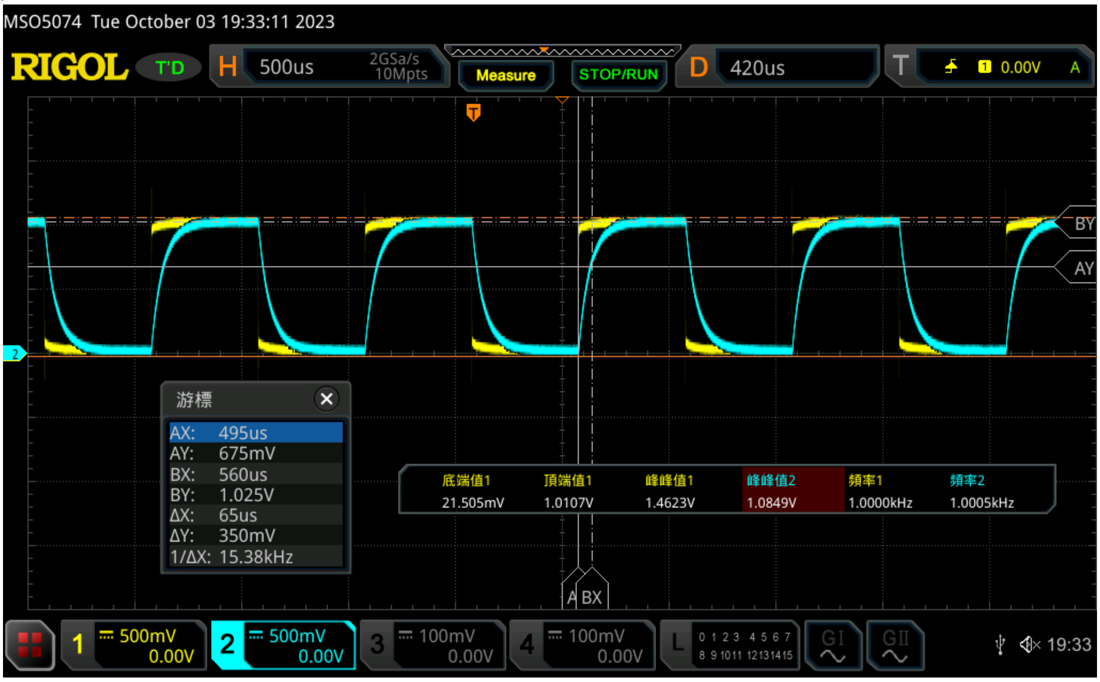
15k Hz V_{in} and V_{out} waveform



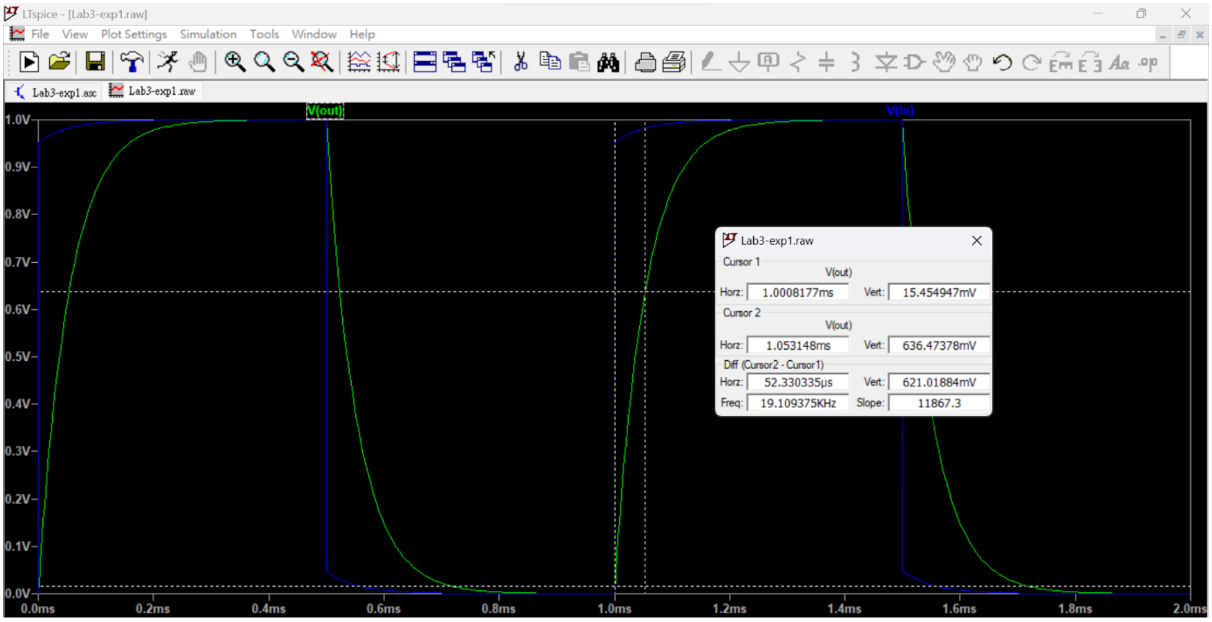
Basic Components

2.

V_{in} and V_{out} waveform



time constant $=\Delta t = \Delta x =$ _65 μ (should be around 50 μ)_ second (i.e. the value you use “cursor” function to measure)(**I have reflected upon the mistakes and learned the correct measuring method. However, I didn’t have access to FGs and Oscilloscopes required to redo this part. I did retry using LTspice though.**)



The simulated result using LTspice gave a time constant of 52 μ s

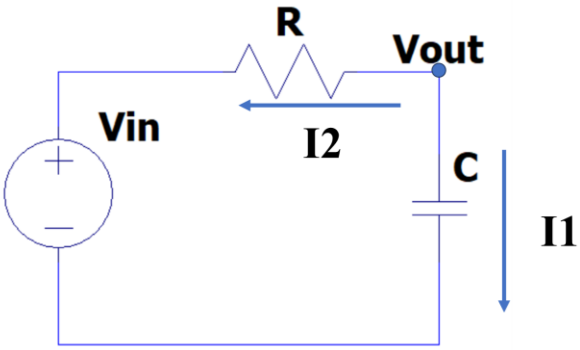
Question:

1. Please use KVL or KCL to derive charging and discharging equations of the capacitor with symbols

(Known: vin: input signal, vout: output signal, R: resistance, C: capacitance)

Charging:

Let V_{in} be a constant DC voltage source. $V_{out} \ = \ V_c$



Applying KCL to the circuit, we acquire: $I1 + I2 = C\frac{dV_{out}}{dt} + \frac{Vout - Vin}{R} = 0$

$$\frac{dV_{out}}{dt} + \frac{Vout}{RC} = \frac{V_{in}}{RC}$$

$$\frac{dV_{out}}{dt} = -\frac{V_{out} - V_{in}}{RC}$$

$$\frac{dV_{out}}{V_{out} - V_{in}} = \frac{-dt}{RC}$$

Integrate both sides about t from 0 to t.

$$\ln(V_{out} - V_{in}) \Big|_{V_{out}(0)}^{V_{out}(t)} = \frac{-t}{RC} \Big|_{0}^{t}$$

$$\ln(V_{out}(t) - V_{in}) - \ln(V_{out}(0) - V_{in}) = \frac{-t}{RC}$$

$$\ln\frac{(V_{out}(t) - V_{in})}{(V_{out}(0) - V_{in})} = \frac{-t}{RC}$$

Take the exponential of both sides.

$$\frac{(V_{out}(t) - V_{in})}{(V_{out}(0) - V_{in})} = e^{\frac{-t}{RC}} = e^{\frac{-t}{\tau}}$$

$$\tau = RC$$

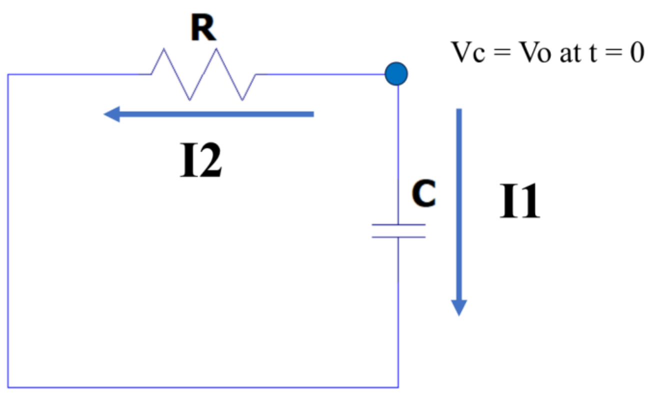
$$V_{out}(t) = V_{in} + (V_{out}(0) - V_{in})e^{\frac{-t}{\tau}}, \qquad t > 0$$

If $V_{out}(0) = 0$, the equation becomes

$$V_{out}(t) = V_c(t) = V_{in}(1 - e^{\frac{-t}{\tau}}), \qquad t > 0$$

Discharging:

Let V_0 be the initial voltage of a charged capacitor, and V_c be the voltage of the capacitor, s.t. $V_c(0) = V_0$



Applying KCL to the circuit, we acquire: $I1 + I2 = C\frac{dV_c}{dt} + \frac{V_c}{R} = 0$

$$\frac{dV_c}{dt} = -\frac{V_c}{RC}$$

$$\frac{dV_c}{V_c} = \frac{-dt}{RC}$$

Integrate both sides.

$$\ln(V_c) = \frac{-t}{RC} + A, \quad \text{A is the integration constant}$$

Take the exponential of both sides

$$V_c = e^{\frac{-t}{RC}+A}$$

$$V_c = Ae^{\frac{-t}{RC}}$$

From the initial condition $V_c(0) = V_0$, we get C = V_0,

$$V_{out}(t) = V_c(t) = V_0 e^{\frac{-t}{RC}} = V_0 e^{\frac{-t}{\tau}}$$

2. Use above answers (charging and discharging equations) to derive time constant.

Let $t = \tau = RC$, then

$$V_{out}(\tau) = V_{in}(1 - e^{-1})$$

$$V_{out}(\tau) = V_{in}(0.63212)$$

Substitute $V_{in} = 1V$, then $V_{out} = 0.63212V$

Cursor AY was originally at 40 mV, so we add 632mV to its original position.

$$40 + 632 = 672\text{mV}$$

We then move AY to around 672mV.

Next, we move cursor BX to the intersection between AY and CH2(blue waveform), while keeping AX at the lowest point of CH2.

ΔX is the time constant. $\Delta X = \tau = 65\mu$

3. Apply experiment rating parameters to calculate theorical time constant is?

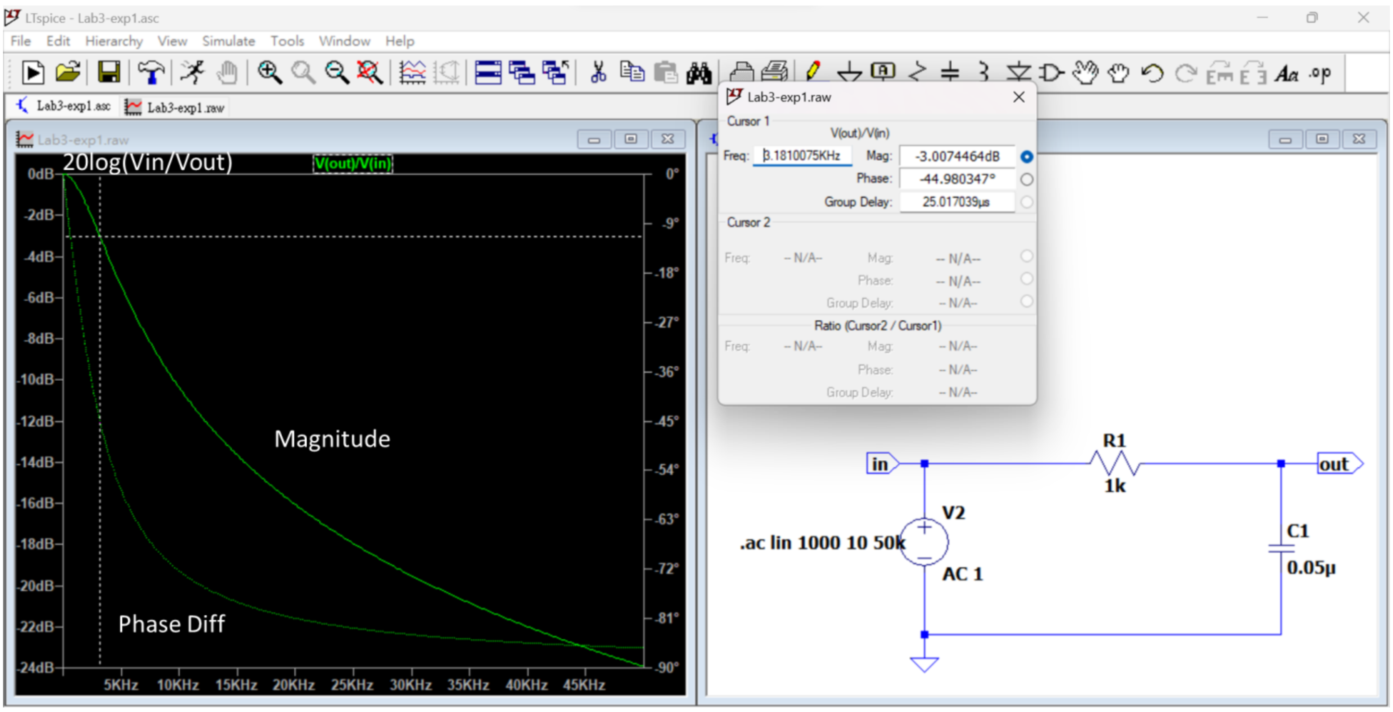
$\tau = RC = 1000 \times 0.05\mu = 0.05m = \mathbf{50\mu}$

The deviation between the theoretical value and the value we measured is $\frac{65-50}{50} = 30\%$

Some mistakes were likely made during measurement. Notice that the waveforms are a little "hairy" and "fat", so we can't rely on the scope's built-in "Measurement". To accurately measure the amplitude, phase difference, etc., we must use cursors. I may have placed the cursor a little too far right. I'll be sure to keep an eye out for this next time.

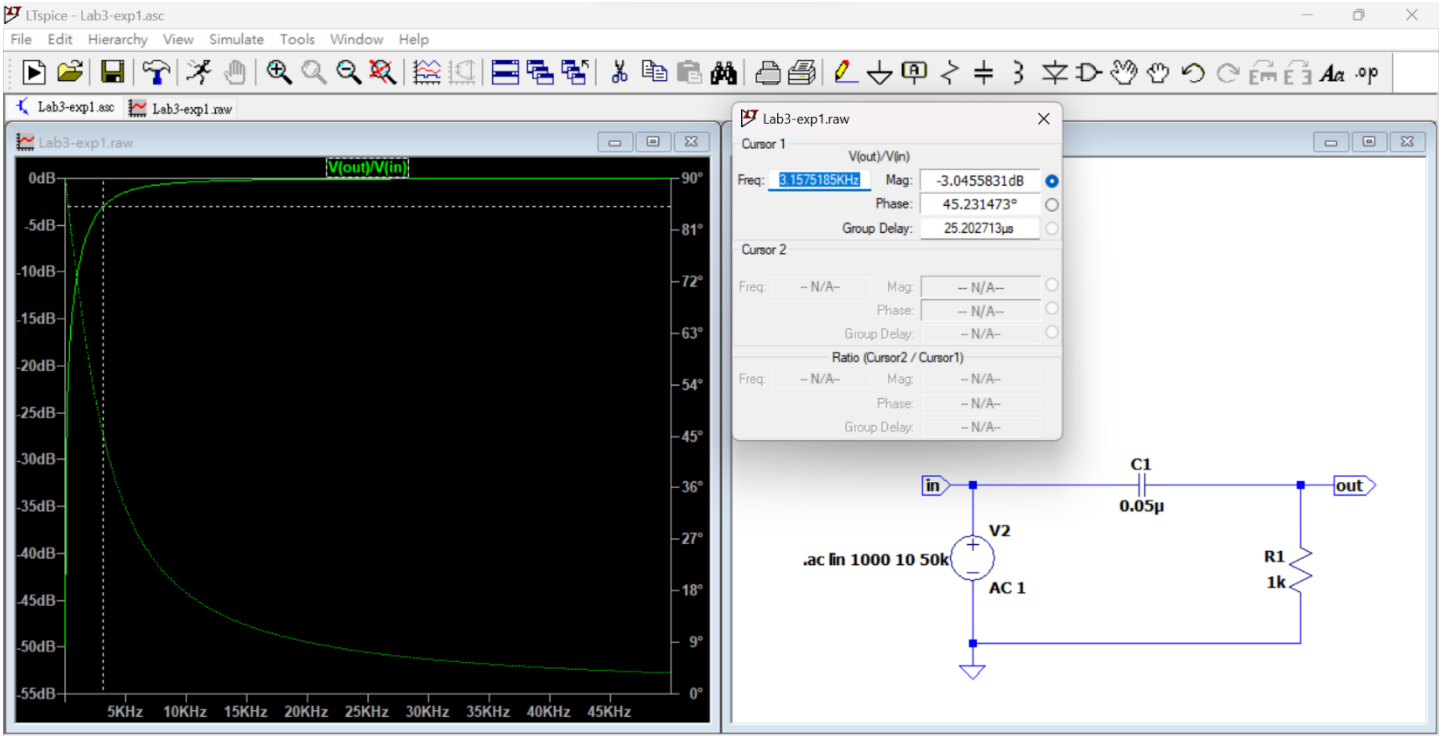
4. Do you find anything about the relationship between output signal and input frequency?

What we have here is essentially a low-pass filter. This circuit allows low-frequency to pass through, while blocking high-frequency signals. I have simulated the filter's frequency response using LTspice. **This filter has a cutoff frequency of $f_c = \frac{1}{2\pi RC} = \frac{1}{2\pi \times 1k \times 0.05\mu} = 3183.09Hz$ according to calculation. This is consistent with the value we obtain with the cursor at 45° phase difference/-3dB, which is 3181.01Hz.**



Input signals with frequencies below 3000 Hz are allowed to reach the output port with minimal attenuation, while signals with frequencies above 3000Hz are progressively attenuated or reduced in amplitude as their frequency increases. **In a low-pass filter, the output lags the input.**

If we **switch the resistor and the capacitor on the circuit with each other**, we **obtain a high-pass** filter with the same cutoff frequency. In a low-pass filter, the output leads the input.



Lastly, I have attached **my own notes of how low-pass and high-pass filters** are made using capacitors on the next page.

Basic Components

Let represent a low-frequency signal

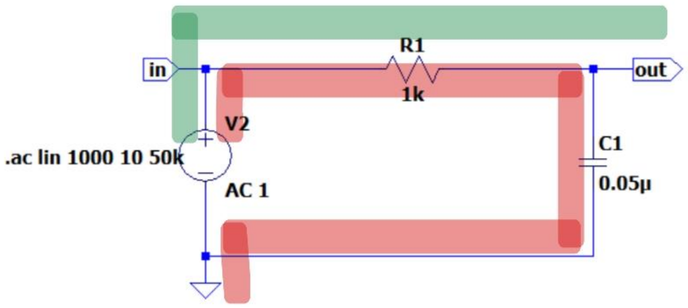
Let represent a high-frequency signal

$$Z_C = \frac{1}{j\omega C} = \frac{1}{j2\pi f C}$$

→ high Z

→ low Z

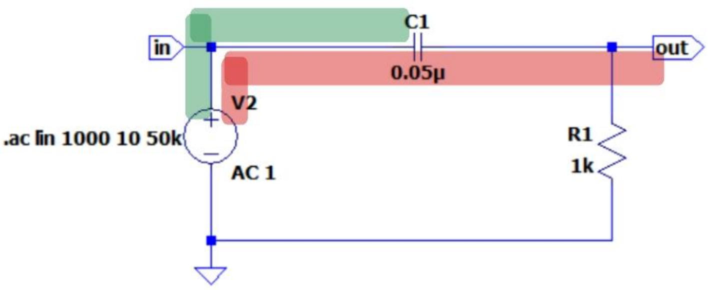
Low-pass



is allowed to pass through because the path to ground is blocked by the capacitor

is diverted to ground because the path to the ground has low Z

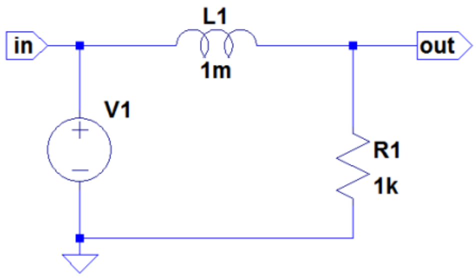
High-pass



is blocked by the high Z of the capacitor

passes through because the capacitor has low Z

Experiment 2: RL Circuit

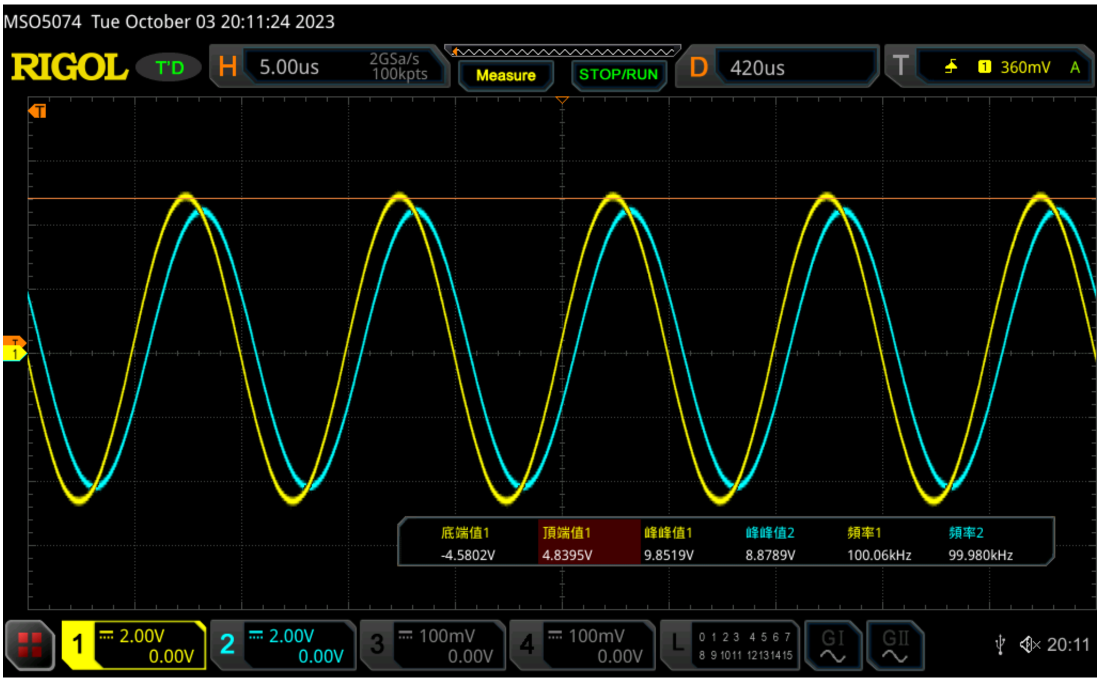


1.

Frequency (Hz)	100K	200K	300K
$V_{out,pp}$(V)	8.8789	7.4293	5.9797

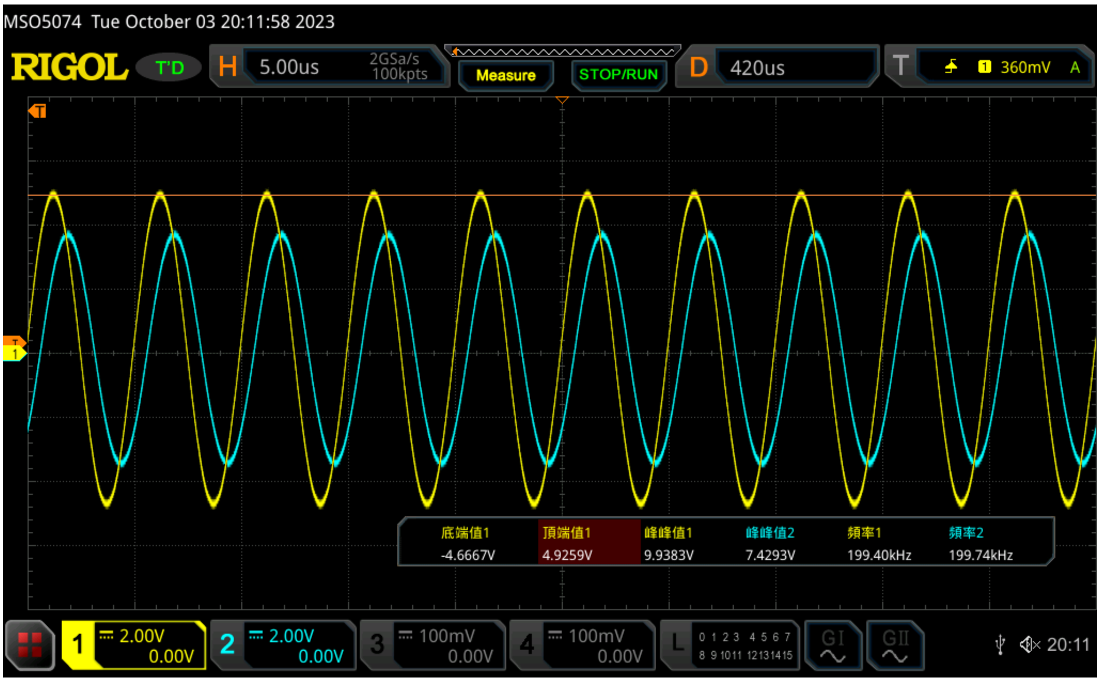
ADJUST THE OSCILLOSCOPE APPROPRIATELY

100k Hz V_{in} and V_{out} waveform

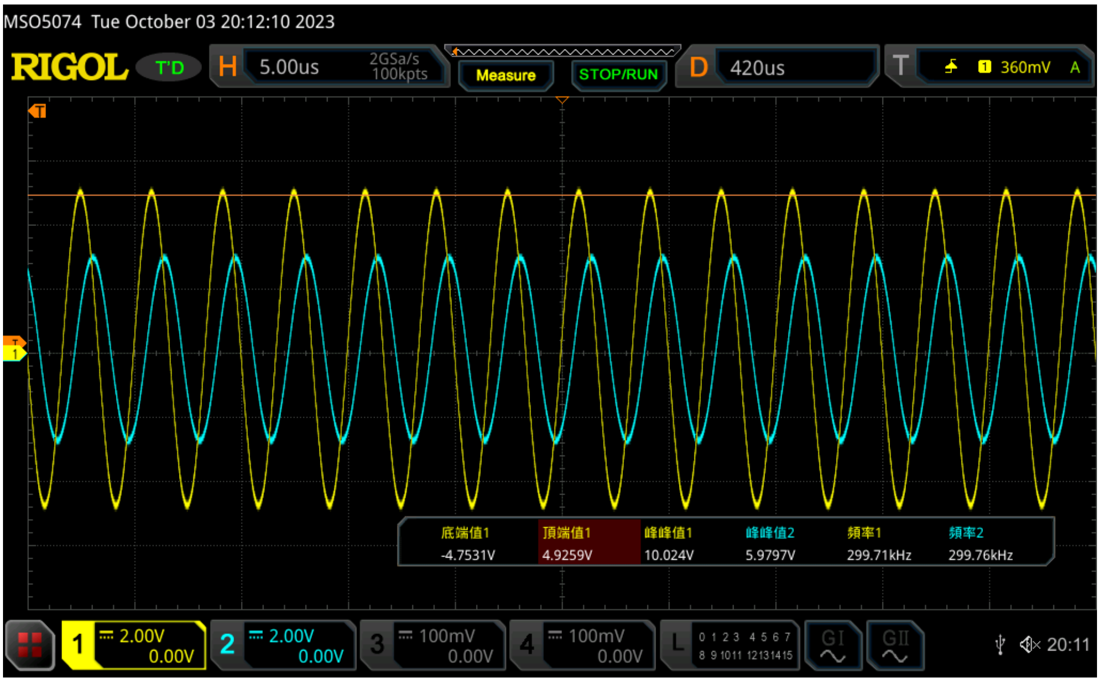


Basic Components

200k Hz V_{in} and V_{out} waveform

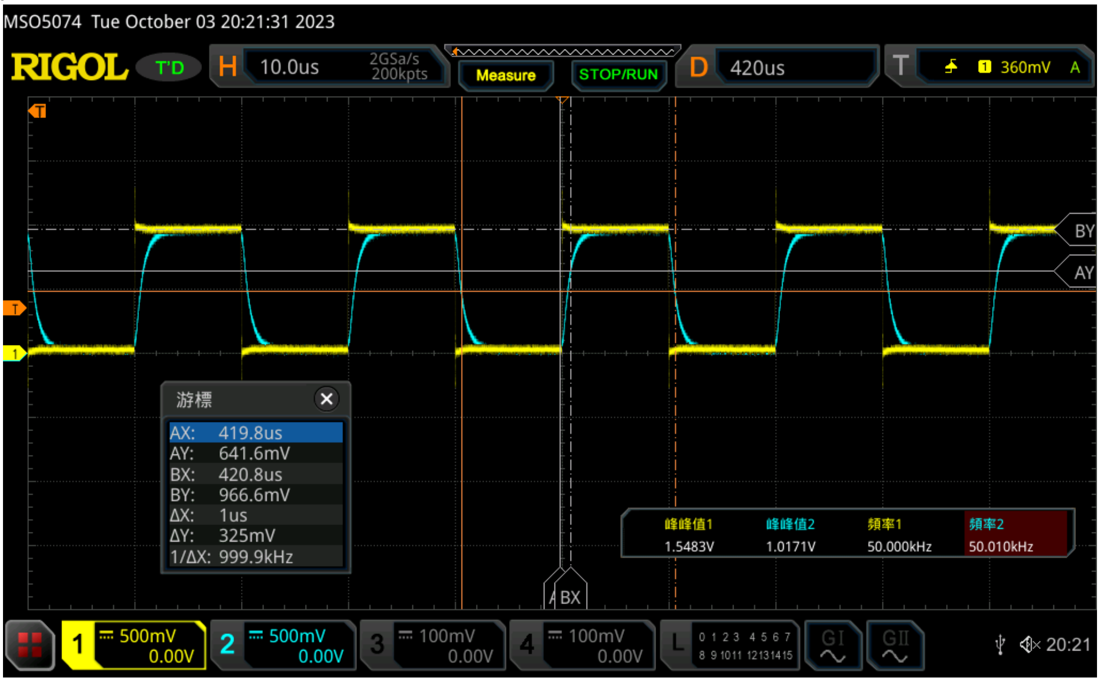


300k Hz V_{in} and V_{out} waveform



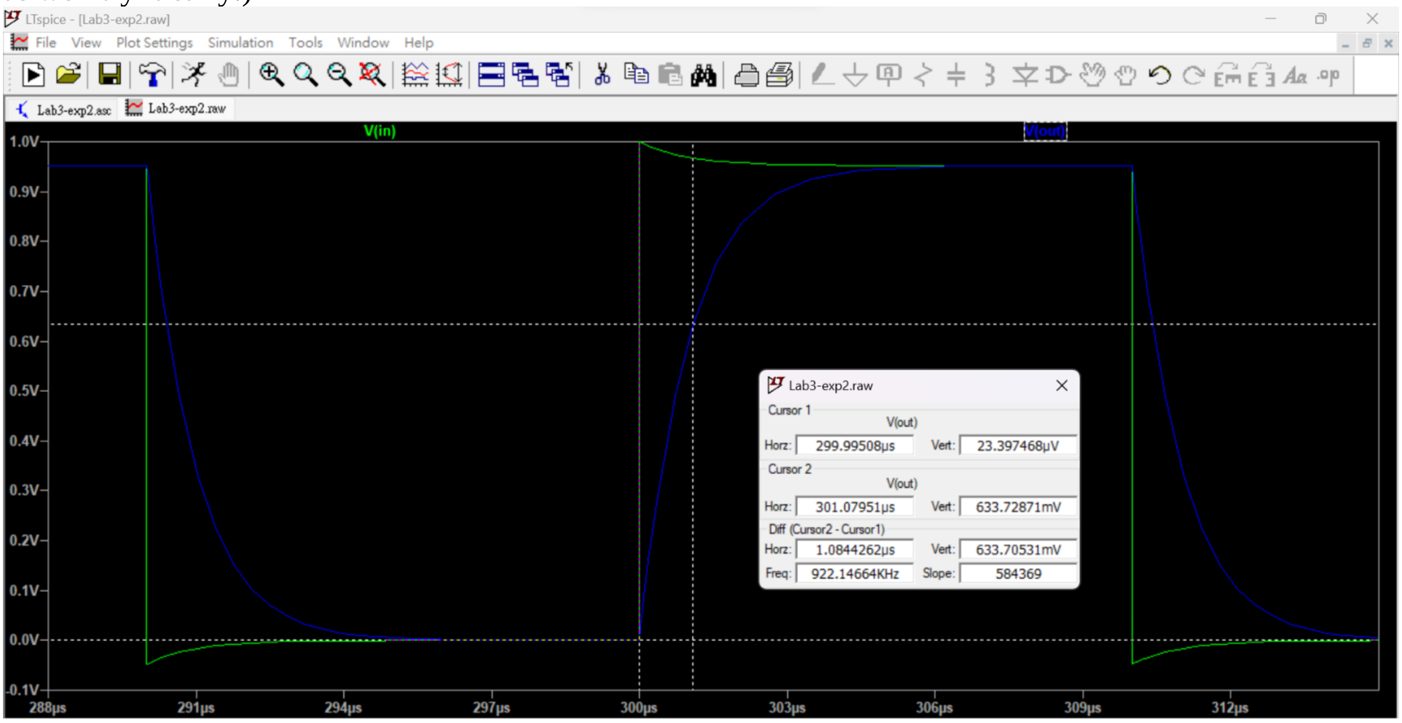
2.

V_{in} and V_{out} waveform



time constant =Δt = Δx = __1 μ___ second (i.e. the value you use "cursor" function to measure)

For good measure, I also **simulated this circuit on LTspice**. (Afterall, it's going to be a part of Lab6. Might as well try it early.)



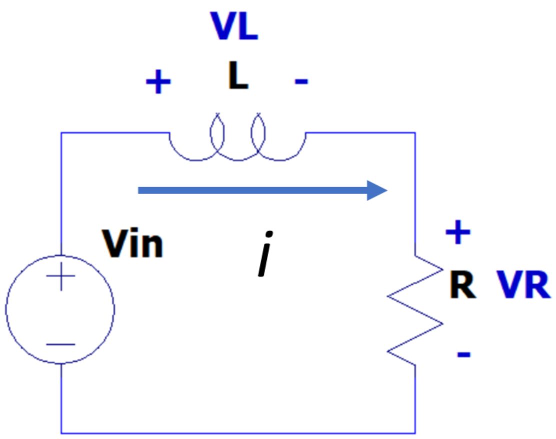
Question:

1. Please use KVL or KCL to derive charging and discharging equations of the capacitor with symbols

(Known: vin: input signal, vout: output signal, R: resistance, L: inductance)

Charging:

Let V_{in} be a constant DC voltage source.



Applying KVL to the circuit, we acquire: $-V_{in} + V_L + V_R = -V_{in} + L\frac{di_L}{dt} + Ri_L = 0$

$$\frac{di_L}{dt} = -\frac{V_{in} - Ri_L}{L} \quad (V_{in} = i_{in} \times R)$$

$$\frac{di_L}{dt} = -\frac{Ri_{in} - Ri_L}{L}$$

$$\frac{di_L}{i_{in} - i_L} = -\frac{Rdt}{L}$$

Integrate both sides about t from 0 to t.

$$\ln(i_L - i_{in}) \mid_{i_L(0)}^{i_L(t)} = \frac{-Rt}{L} \mid_0^t$$

$$\ln(i_L(t) - i_{in}) - \ln(i_L(0) - i_{in}) = \frac{-Rt}{L}$$

$$\ln\frac{(i_L(t) - i_{in})}{(i_L(0) - i_{in})} = \frac{-Rt}{L}$$

Take the exponential of both sides.

$$\frac{(i_L(t) - i_{in})}{(i_L(0) - i_{in})} = e^{\frac{-Rt}{L}} = e^{\frac{-t}{\tau}}$$

$$\tau = RC$$

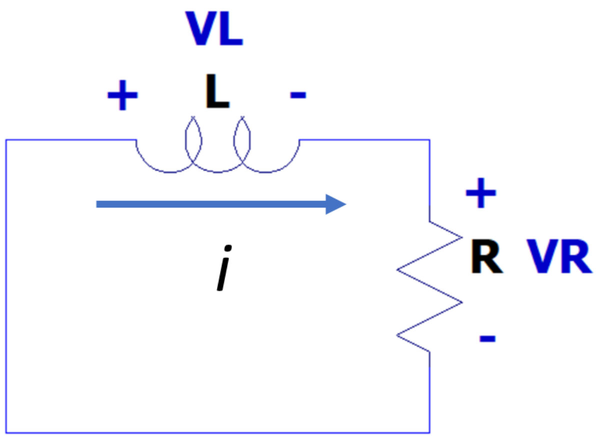
$$i_L(t) = i_{in} + (i_L(0) - i_{in})e^{\frac{-t}{\tau}}, \qquad t > 0$$

If $i_L(0) = 0$, the equation becomes

$$i_{out}(t) = i_L(t) = i_{in}(1 - e^{\frac{-t}{\tau}}), \qquad t > 0$$

Discharging:

Let V_0 be the initial voltage of a charged capacitor, and V_c be the voltage of the capacitor, s.t. $V_c(0) = V_0$



Applying KVL to the circuit, we acquire: $V_L + V_R = L\frac{di_L}{dt} + Ri_L = 0$

$$\frac{di_L}{i_L} = -\frac{R}{L}dt$$

Integrate both sides.

$$\ln(i_L) = \frac{-Rt}{L} + C,\ \text{C is the integration constant}$$

Take the exponential of both sides

$$i_L = e^{\frac{-Rt}{L}+C}$$

$$i_L = Ce^{\frac{-Rt}{L}}$$

From the initial condition $i_L(0) = I_0$, we get C = V_0,

$$i_{out}(t) = i_L(t) = I_0e^{\frac{-Rt}{L}} = I_0e^{\frac{-t}{\tau}}$$

2. Use above answers (charging and discharging equations) to derive time constant.

Let $t = \tau = \frac{L}{R}$, then

$$i_{out}(\tau) = i_{in}(1 - e^{-1})$$

$$i_{out}(\tau) = i_{in}(0.63212)$$

Since $V_{out} = I_{out} \times R;\ V_{in} = I_{in} \times R,$

$$V_{out}(\tau) = V_{in}(0.63212)$$

Substitute $V_{in} = 1V$, then $V_{out} = 0.63212V$

Cursor AY was originally at 8 mV, so we add 632mV to its original position.

8 + 632 = 640mV

We then move AY to around 640mV.

Next, we move cursor BX to the intersection between AY and CH2(blue waveform), while keeping AX at the lowest point of CH2.

ΔX is the time constant. $\boldsymbol{\Delta X = \tau = 1\mu}$

3. Apply experiment rating parameters to calculate theorical time constant is?

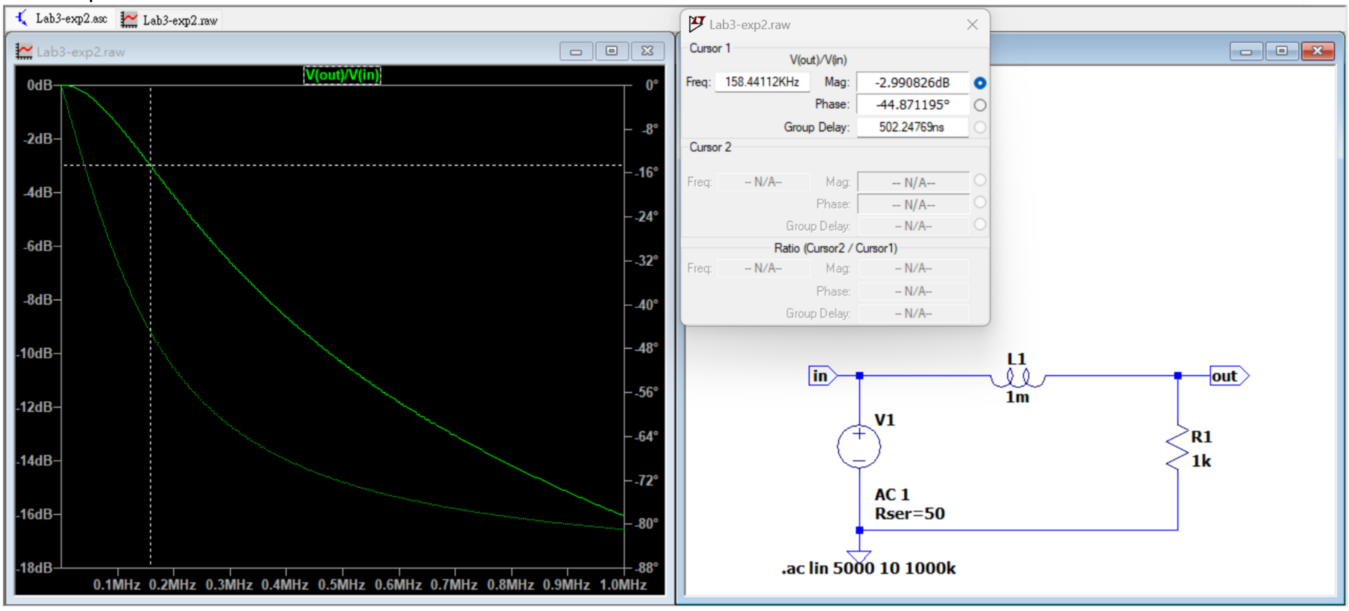
$\tau = \frac{L}{R} = 1m \div 1k = \boldsymbol{1\mu}$

This time my measurement was comparable to the theoretical value. **There isn't noticeable deviation.**

4. Do you find anything about the relationship between output signal and input frequency?

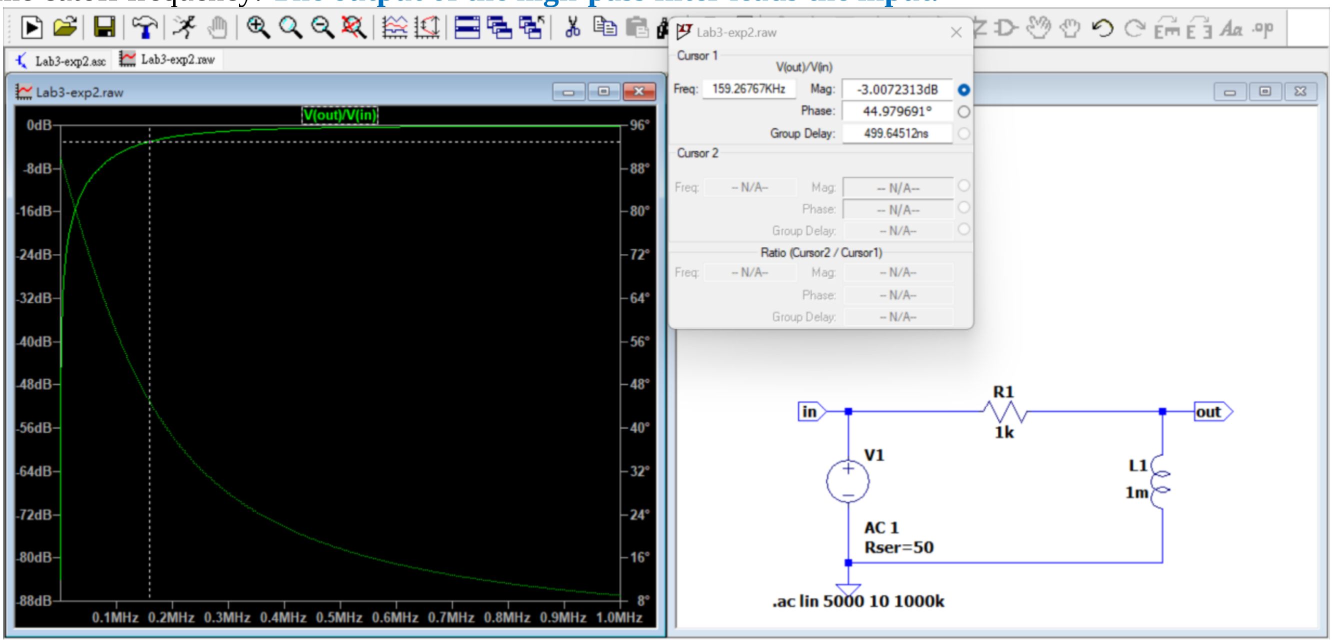
What we have here is yet another **low-pass filter**. This circuit allows low-frequency to pass through, while weakening high-frequency signals. I have simulated the filter's frequency response using LTspice. This filter has a **cutoff frequency of** $\boldsymbol{f_L = \frac{R}{2\pi L} = \frac{1k}{2\pi \times 1m} = 159.155kHz}$ according to calculation. This is consistent with the following simulation result.

Basic Components



Input signals with frequencies **below 159kHz are allowed to reach the output port with minimal attenuation**, while signals with frequencies above 159kHz are progressively attenuated or reduced in amplitude as their frequency increases. **The output lags the input.**

If we **swap the inductor and the resistor with each other**, we obtain a **high-pass filter** with the same cutoff frequency. **The output of the high-pass filter leads the input.**



Basic Components

Let represent a low-frequency signal

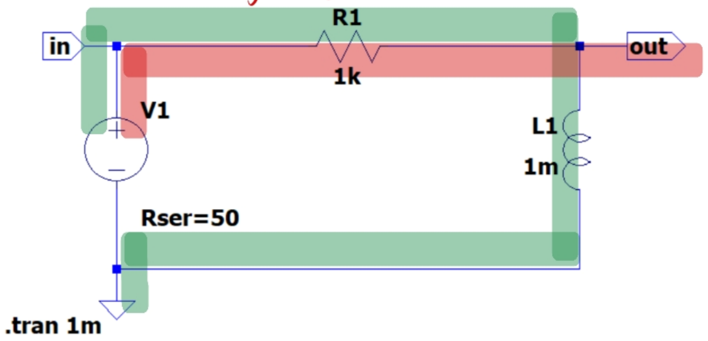
Let represent a high-frequency signal

$Z_L = j\omega L = j2\pi f L$

→ low Z

→ high Z

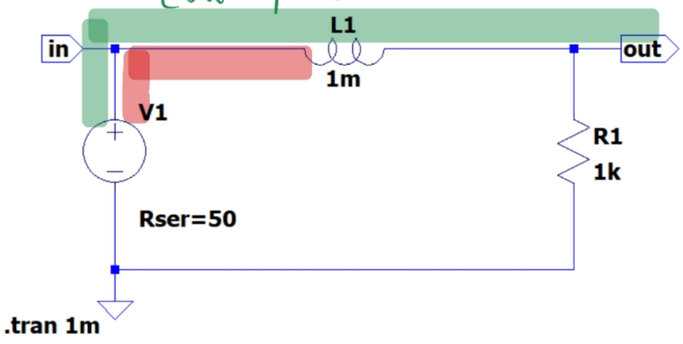
High-pass



is diverted to ground because the path to the ground has low Z

is allowed to pass through because the path to ground is blocked by the inductor

Low-pass



passes through because the inductor has low Z

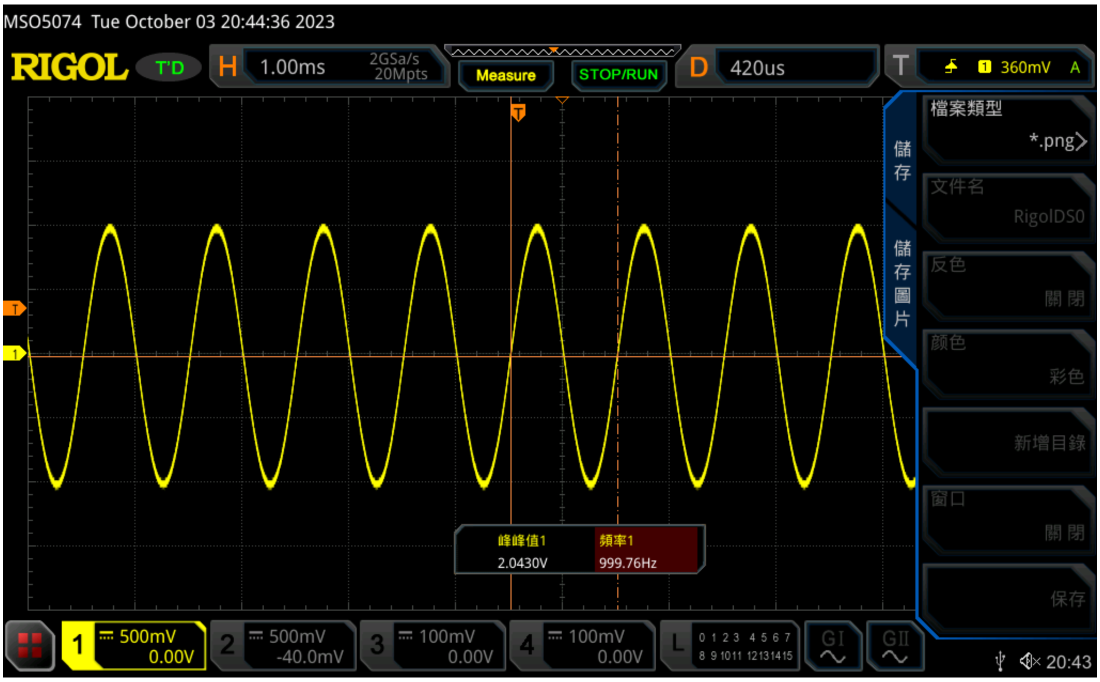
is blocked by the high Z of the inductor

Experiment 3: Speaker properties and signal sound

ADJUST THE OSCILLOSCOPE APPROPRIATELY

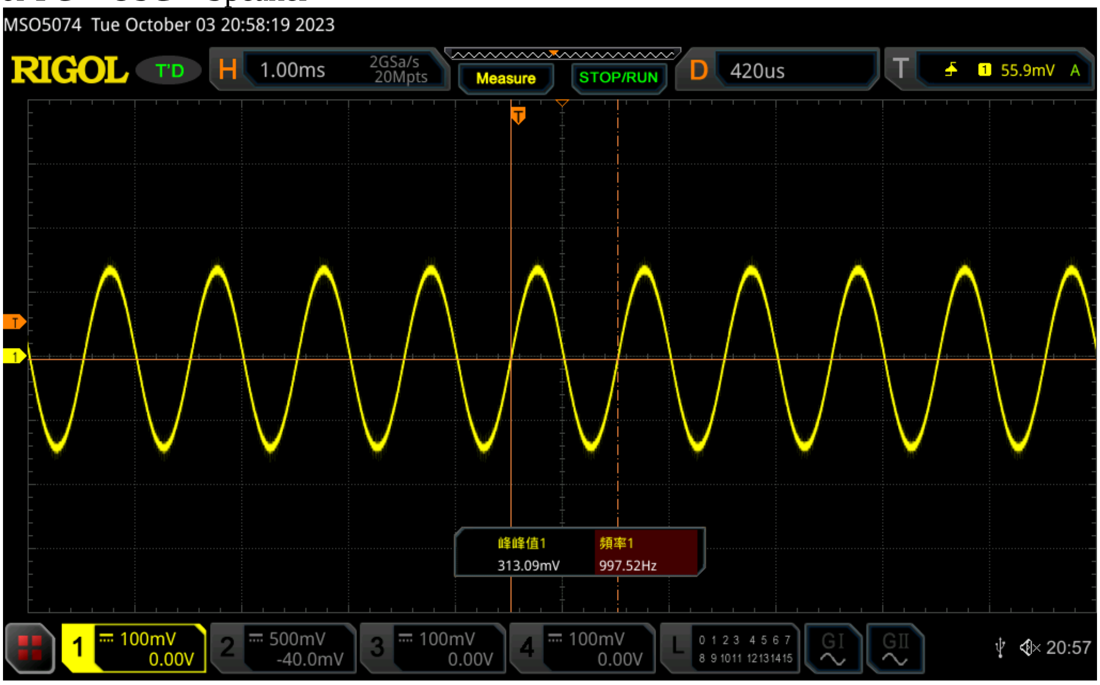
1.

waveform for FG + OSC



2.

waveform for FG + OSC + Speaker



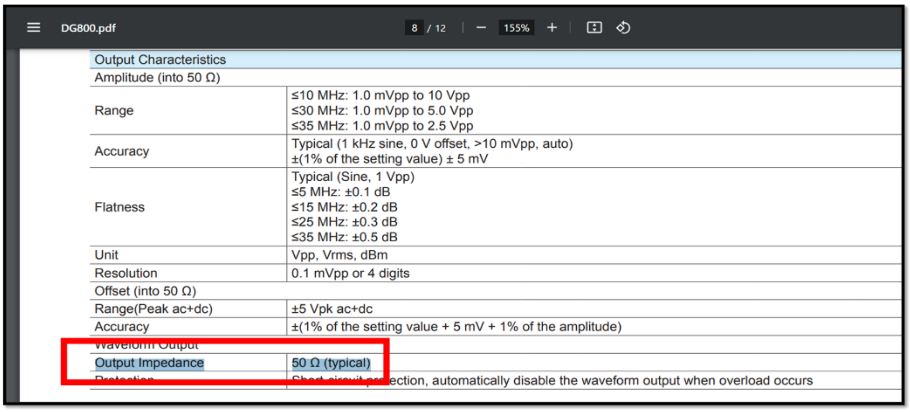
Configuration	V_{pp} of OSC CH1 (V)
FG + OSC	2.043V
FG + OSC + Speaker	313.09mV

Question:

Are there any differences between these two connections?

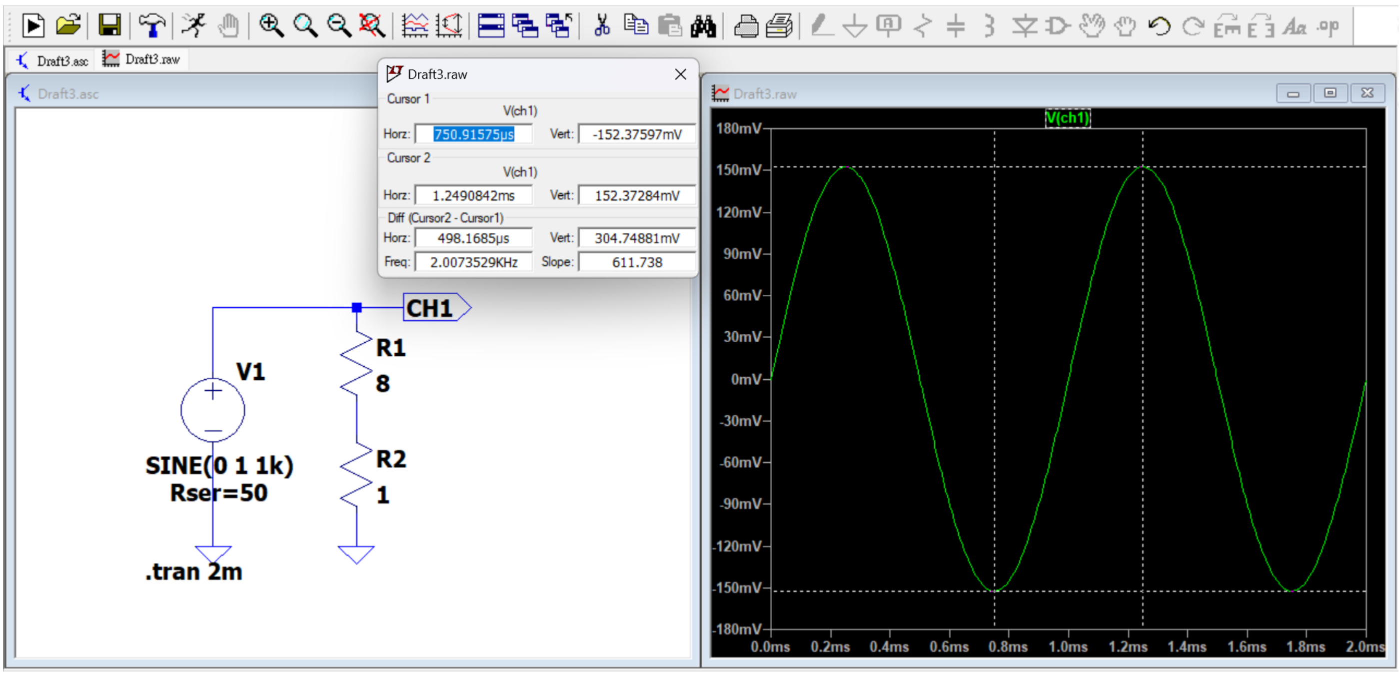
It's common sense that the Vpp measured from "FG + OSC" should be around the same value as what was outputted from the function generator, since there were no other components in the circuit. However, the **Vpp measured from "FG + OSC+Speaker" is considerably reduced**. This likely has something to do with our components having low resistance.

Can you explain the phenomena? Hint: voltage divider


DG800.pdf 8 / 12 155%

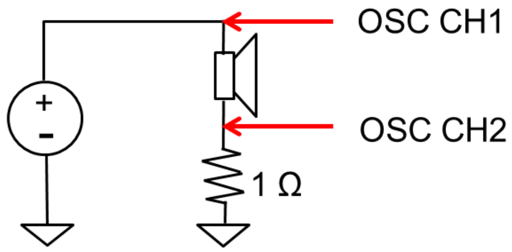
Output Characteristics	
Amplitude (into 50 Ω)	
Range	≤10 MHz: 1.0 mVpp to 10 Vpp ≤30 MHz: 1.0 mVpp to 5.0 Vpp ≤35 MHz: 1.0 mVpp to 2.5 Vpp
Accuracy	Typical (1 kHz sine, 0 V offset, >10 mVpp, auto) ±(1% of the setting value) ± 5 mV
Flatness	Typical (Sine, 1 Vpp) ≤5 MHz: ±0.1 dB ≤15 MHz: ±0.2 dB ≤25 MHz: ±0.3 dB ≤35 MHz: ±0.5 dB
Unit	Vpp, Vrms, dBm
Resolution	0.1 mVpp or 4 digits
Offset (into 50 Ω)	
Range(Peak ac+dc)	±5 Vpk ac+dc
Accuracy	±(1% of the setting value + 5 mV + 1% of the amplitude)
Waveform Output	
Output Impedance	50 Ω (typical)
Protection	Short-circuit protection, automatically disable the waveform output when overload occurs

According to Rigol's official **"DG1022 Calibration Guide"**, the function generator we were using has an internal resistance of 50Ω. This was not directly revealed in the circuit diagram provided to us, so I redrew one myself on LTspice with the 50Ω parasitic resistance included. The **simulation resulted in a Vpp of 304.749mV**



Additionally, using the voltage divider formula, we acquire $2V \times \frac{(8+1)}{50+(8+1)} = 305.085mV$. **This is comparable to both what we measured and simulated proving that our experiment results were accurate**. I then started wondering why we previously didn't have to account for the function generator's parasitic resistance during Exp1 and Exp2. It occurred to me that we were using 1000Ω in those two circuits. The effects of **the internal resistance(50Ω) can be ignored when connected to a much larger resistance.** It is only impacting this circuit due to the speaker and the resistor we used having much lower resistance than 50Ω.

Basic Components

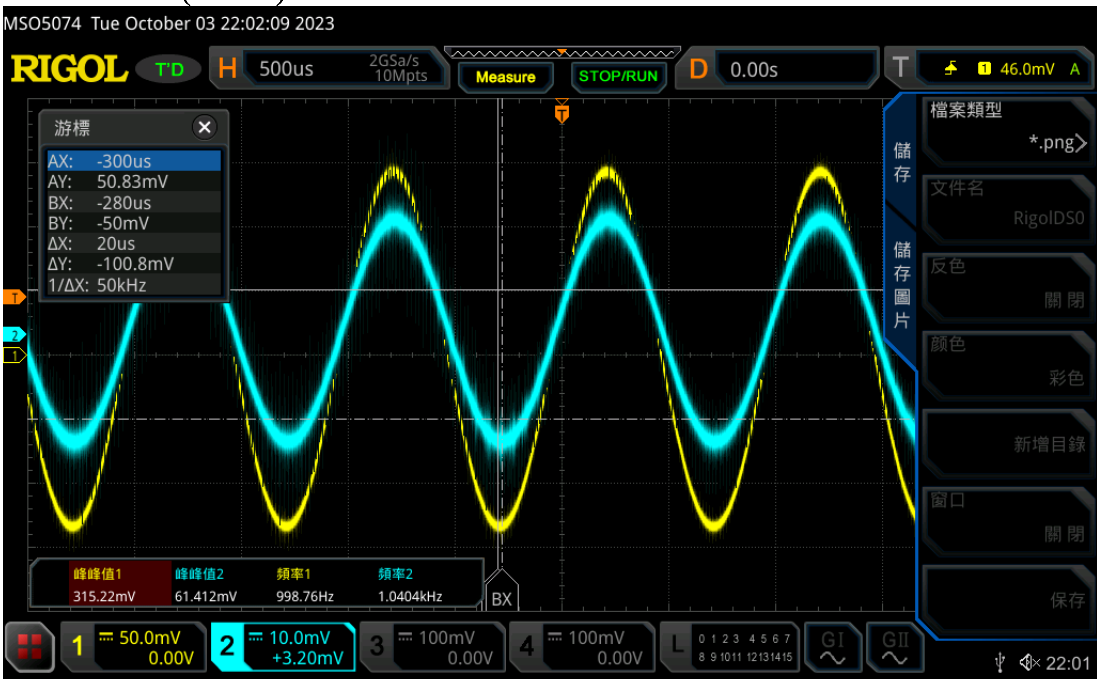


3.

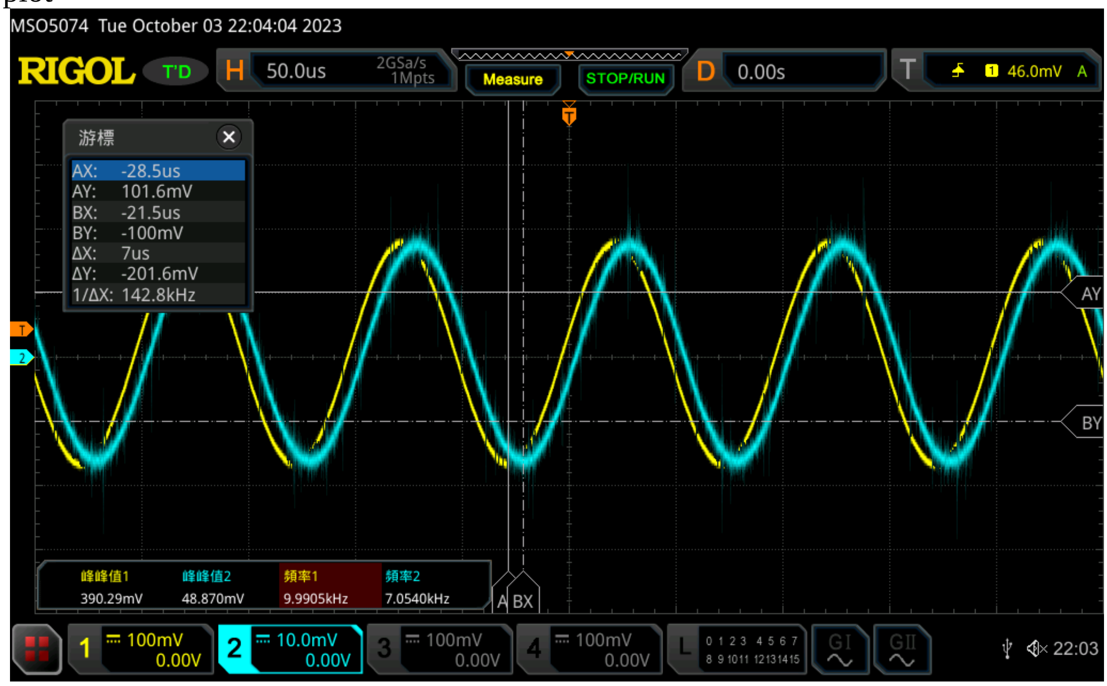
①

CH_1_ leads CH_2_ by _7.2_ degree. Calculated using $\frac{360° \times time\ diff}{wave\ period} = \frac{360 \times 20\mu}{0.001} = 7.2$

CH1 and CH2 waveform (1 KHz)



X-Y mode plot

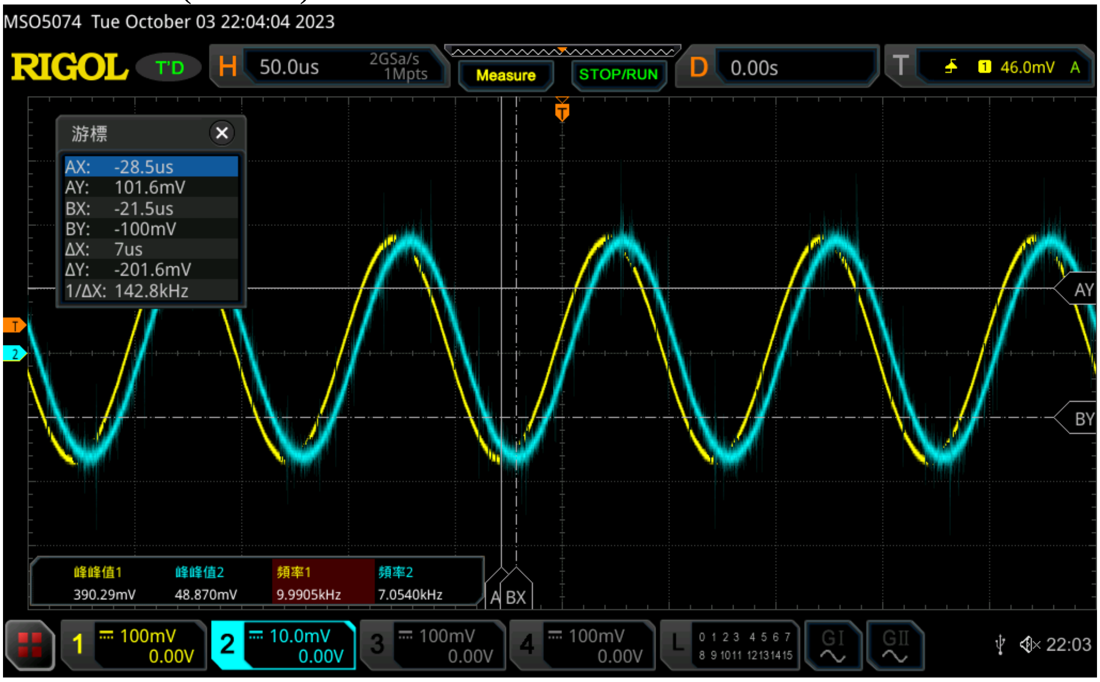


②

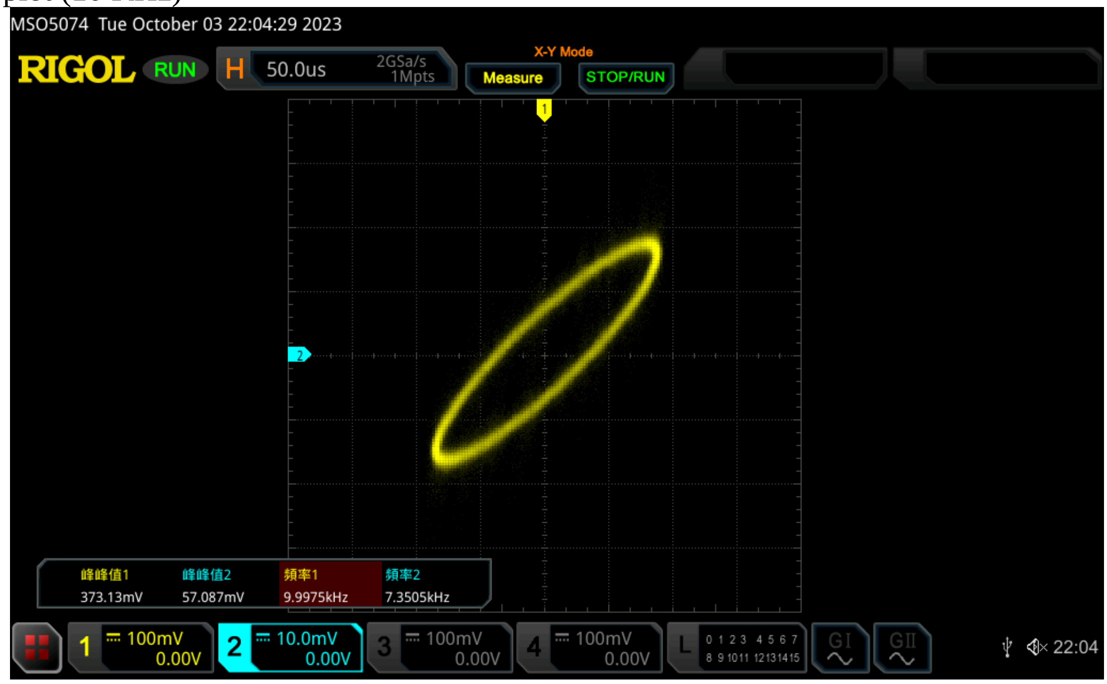
Basic Components

CH_1_ leads CH_2_ by _25.2_ degree. Calculated using $\frac{360^\circ \times time\ diff}{wave\ period} = \frac{360\times 7\mu}{0.0001} = 25.2$

CH1 and CH2 waveform (10 KHz)



X-Y mode plot (10 KHz)



Question:

Please describe the sound produced by different shape.

Sine: The sine wave sounds **smooth and rounded**. It's comfortable to listen to.

Square: The square wave sounds **harsh and sharp**. Extensive exposure causes discomfort

Ramp: An **in-between** of the square and the sine wave. Not so smooth as a sine wave, but it's also not as sharp as a square wave.

Audio: https://youtu.be/v4XG_NHLWcs?si=tc472wZlsve8oEu6&t=199 (VOLUME WARNING)

References:

1. Stack Exchange- High Pass vs Low Pass simple Circuit (RC vs CR)

2. YouTube- Can you hear the difference between a sine wave and a square wave?

3. AnalogDialogue- Phase Response in Active Filters Part 2, the Low-Pass and High-Pass Response