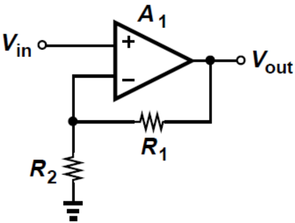
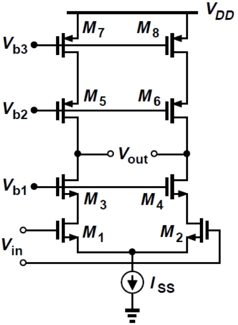
# *Chapter 9: Operational Amplifiers*

# Op Amp Definition

- We loosely define an op amp as a "high-gain differential amplifier."
- Usually employed in a feedback system when precision is a requirement.
- Applications ranging from DC generation, high-speed amplification or filtering.



# Op Amp Design Challenge

Three decades ago

•General-purpose blocks as an “ideal” op amp

•Design effort is to satisfy an ideal op amp

- infinite gain
- infinite input impedance
- zero output impedance

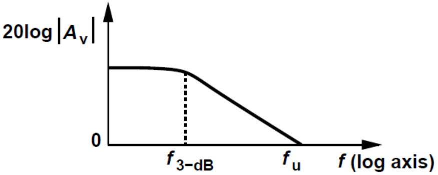
Today

•Design effort is to make trade-offs for a specific application, often sacrificing the unimportant aspects to improve the critical ones.

•E.g., gain error vs speed, open loop gain vs VDD

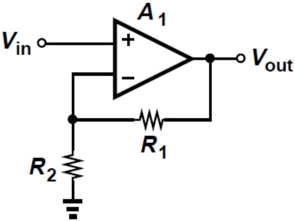
# Performance Parameter

- Gain(Precision), Bandwidth(Speed): 3-dB/fu
  Output Swing, Power dissipation
- Noise, Linearity, Supply Rejection, offset
- Input CM Range, Input/Output Impedance
- Large-Signal behavior (e.g. slew rate)



# Example 9.1

**The circuit has a nominal gain of 10. i.e.,1+R1/R2=10. Determine the minimal value of A1 for a gain error 1%:**




**Solution:**

$$\frac{V_{out}}{V_{in}} = \frac{A_1}{1 + \frac{R_2}{R_1 + R_2} A_1}$$

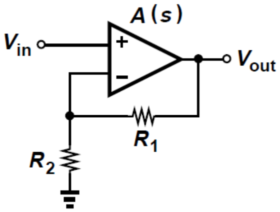
$$= \frac{R_1 + R_2}{R_2} \frac{A_1}{\frac{R_1 + R_2}{R_2} + A_1}$$

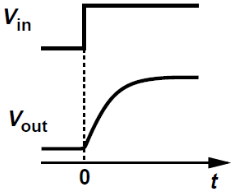
$$\frac{V_{out}}{V_{in}} \approx \left(1 + \frac{R_1}{R_2}\right)\left(1 - \frac{R_1 + R_2}{R_2} \frac{1}{A_1}\right)$$

**Thus, A1>1000. Open-loop gain determines precision.**

# Example 9.2

**Assume the op amp is a single-pole voltage amplifier. For a small step input, calculate the time required for the output to reach within 1% and its unity-gain bandwidth if 1+R1/R2=10 and its settling time is less than 5ns.**




**Solution: Speed vs. Bandwidth**

$$\tau \approx \left(1+\frac{R_1}{R_2}\right)\frac{1}{A_0\omega_0}$$

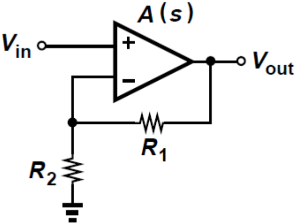
$$\tau \approx 1.09 \text{ ns}$$

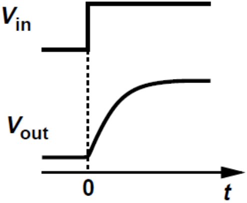
$$1-\exp\frac{-t_{1\%}}{\tau} = 0.99$$

$$A_0\omega_0 \approx (1+R_1/R_2)/\tau = 9.21 \text{ Grad/s } (1.47 \text{ GHz})$$

# Example 9.3

**Explain the circuit behavior if we swap the inverting and non-inverting inputs of the op amp.**



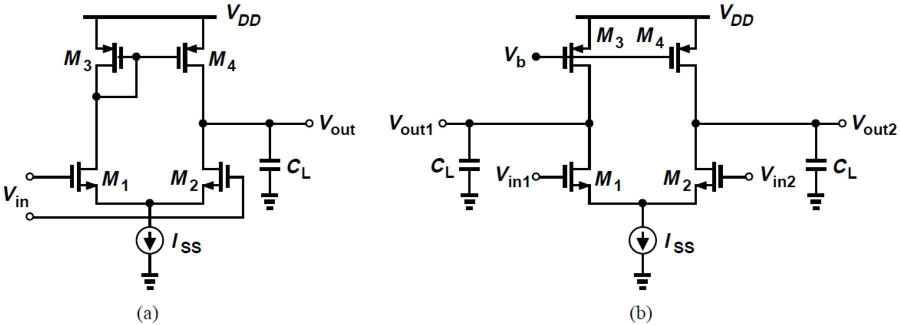

**Solution: Positive feedback destabilizes the circuit. Output grows exponentially to non-linearity range.**

$$\frac{V_{out}}{V_{in}}(s) = \frac{\dfrac{A_0}{1 - \dfrac{R_2}{R_1 + R_2}A_0}}{1 - \dfrac{s}{(1 + \dfrac{R_2}{R_1 + R_2}A_0)\omega_0}}$$

$$V_{out}(t) \approx a\left(1 + \frac{R_1}{R_2}\right)\left(\exp\frac{t}{\tau} - 1\right)u(t)$$

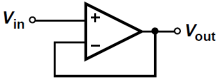
# One-Stage Op Amps

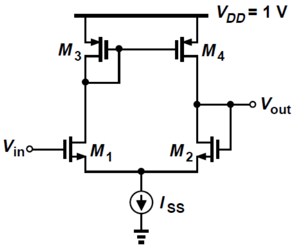
- **Low-frequency gain:** $g_{mN}(r_{ON}\|r_{OP})$
- **Bandwidth: usually proportional to 1/(CL*Rout)**
- **Output Swing (single-side): VDD-3Overdrive**
- **Mirror pole in single-ended circuit**
- **Power and noise: good, with four devices -> input noise**



# Example 9.4

**Calculate the input common-mode voltage range and the closed-loop output impedance of the unity-gain buffer.**




**Solution:**

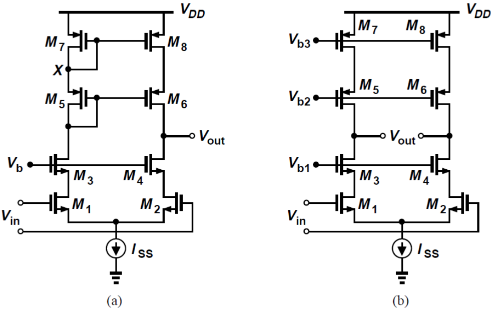
$V_{ISS} + V_{GS1}$ **<Vin<** $V_{in,max} = V_{DD} - |V_{GS3}| + V_{TH1}$

**Output impedance:** $(r_{OP} \| r_{ON})/[g_{mN}(r_{OP} \| r_{ON})] = 1/g_{mN}$

**The closed-loop pole is independent of open-loop output impedance.**

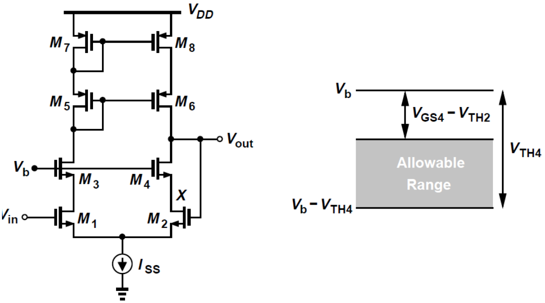
# Telescopic Cascode Op Amps

- **Low-frequency gain:** $g_{mN}[(g_{mN}r_{ON}^2)||(g_{mP}r_{OP}^2)]$
- **Speed: Additional poles**
- **Output Swing (single-side): VDD-5Overdrive**
- **Mirror pole in single-ended**
- **Difficult to short telescopic op amp output to input**
- **Power and noise: good, input noise mainly has four devices contribution**



# Example 9.5

**For this unity-gain buffer configuration, explain in which region each transistor operates as Vin varies from below $V_b - V_{GS4} + V_{TH2}$. to above $V_b - V_{TH4}$**




**Solution: Remedy in switched-capacitor circuit**

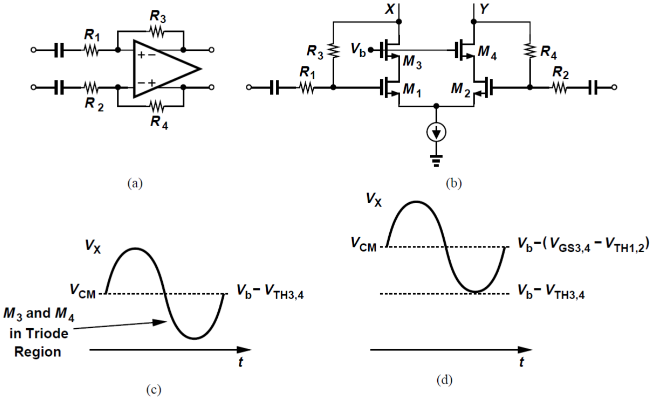
**When** $V_{in} < V_b - V_{TH4}$ **, M4 is in triode, others are saturated;**

$V_b - V_{TH4} < V_{in} < V_b - (V_{GS4} - V_{TH2})$ **M2, M4 are saturated;**

$V_{in} > V_b - (V_{GS4} - V_{TH2})$ **M2, M1 in triode,M4 is in saturation.**

# Example 9.6

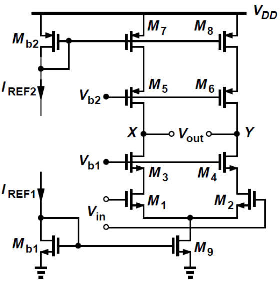
**Assuming that the op amp has a high open-loop gain, determine the maximum allowable output swing.**




**Solution: ±(one threshold-one overdrive)**

# Telescopic Cascode Op Amps Design

- **Generally, power budget determines branch current**
- **Gain and Output voltage swing**
- **Deal with** $I_D, V_{GS}-V_{TH}, W/L, g_m$ and $r_O$




**Design Procedure (Example 9.7)**

**-Define drain current**

**-Distribute overdrive voltage**

**-Calculate the aspect ratio**

$$I_D = (1/2)\mu C_{ox}(W/L)(V_{GS}-V_{TH})^2$$

**-Calculate the gain with Lmin**

**-Iteration by increasing W,L until achieving gain criterion**

**-Finally, DC bias voltage setup and exam residual goals**

**- CMFB is necessary**

$$g_m r_O = \sqrt{2\mu C_{ox}(W/L)I_D}/(\lambda I_D)$$

**PMOS>Cascode_N>Input_N**

# Linear Scaling

- **How to modify design if power budget is different while all the other specifications are the same?**
- **Only scale the widths of all the transistors while keeping the lengths constant.**

## Example 9.6

**Explain what aspects of the performance degrade for a low-power op amp design when we scale down the transistor width.**

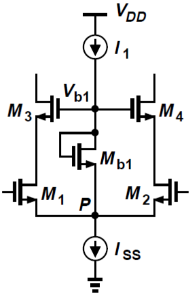
**Solution:**

(1)**The speed of the op amp in driving a capacitive load**

(2)**The input-referred noise voltage rises by a square root factor of scale constant. (for input device)**

# Gate Bias Voltage Generation

- **Ensure bias voltage to track the input CM level**
- **Choose Mb1 to be a narrow, long, “weak” device**



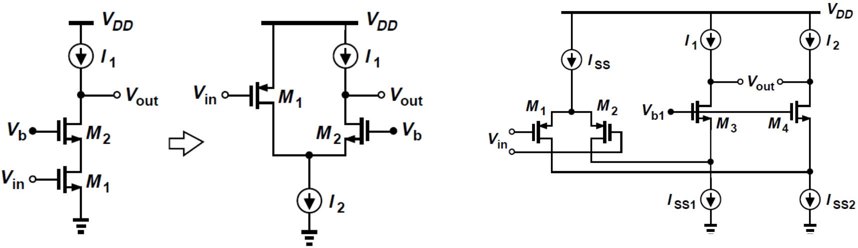

$$V_{b1} = V_P + V_{GS,b1}$$

$$V_{GS,b1} = (V_{GS1,2} - V_{TH1,2}) + V_{GS3,4}$$

# Folded Cascode Op Amps

**Recall Folded Cascode**

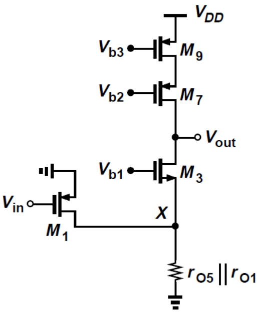
- **Not "stack" the cascode transistor on the input device**
- **Consume higher power**
- **Output Voltage Swing: VDD-4overdrive**
- **Output and input could short together**

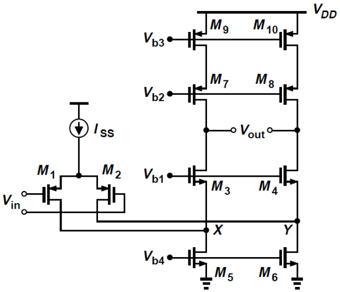


# Folded Cascode Voltage Gain

$$|A_v| = G_m R_{out}$$

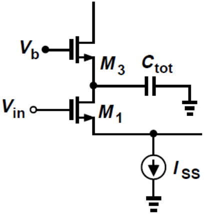
- Since $(g_{m3} + g_{mb3})^{-1} \| r_{O3}$ << $r_{O1} \| r_{O5}$, thus $G_m \approx g_m$
- $|A_v| \approx g_{m1}\{[(g_{m3} + g_{mb3})r_{O3}(r_{O1}\|r_{O5})]\|[(g_{m7} + g_{mb7})r_{O7}r_{O9}]\}$
- Two or three times lower than a telescopic topology

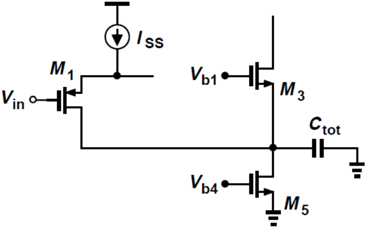


# Effect capacitance on the nondominant pole

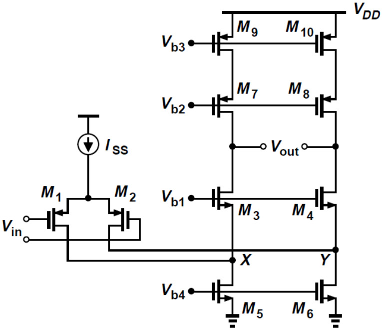
- At “folding point”, a large capacitance due to a large current device M5 would be added to the total capacitance.

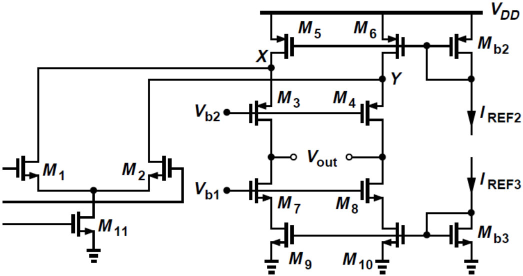


# NMOS vs. PMOS input

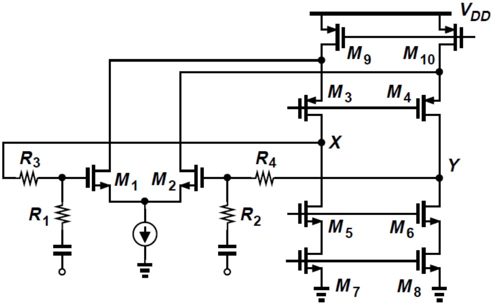
- Greater mobility from NMOS input leads to higher gain
- Lowering the pole at folding point
- PMOS input is less sensitive to flicker noise(wider WL)



# Folded Cascode Properties

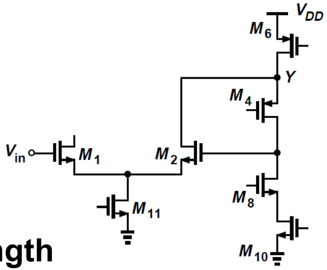
- Slighter Higher Output Swing than telescopic
- Higher Power dissipation, lower voltage gain, lower pole frequency and higher noise
- Input and output can be shorted: 2overdrive from bound
- A better input CM range



# Example 9.9

Design a folded- cascode op amp with an NMOS pair. Specifications: VDD = 3V, differential output swing = 3V, Power dissipation = 10mW, voltage gain = 2000.

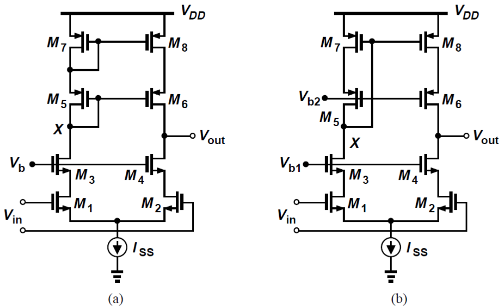
Solution:
(1)Current allocation
(2)Overdrive voltage allocation
(3)Aspect ratio calculation
(4)Small-signal gain with minimal length
(5)Iteration by increase M5/M1/M4 in turns




Note that the folding point capacitance may limit here.

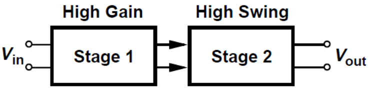
# Low Voltage Single-ended Output

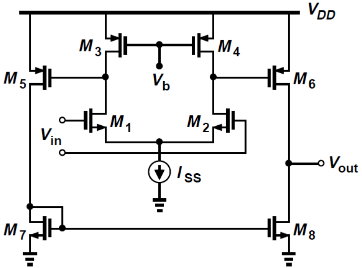
- M7 is biased at the edge of triode
  Could M5 always in saturation?
- Left implementation wastes one threshold voltage
- Still, single-ended output is unfavorable due to half output swing and a mirror pole
- Note that almost all the differential output circuits need a CMFB

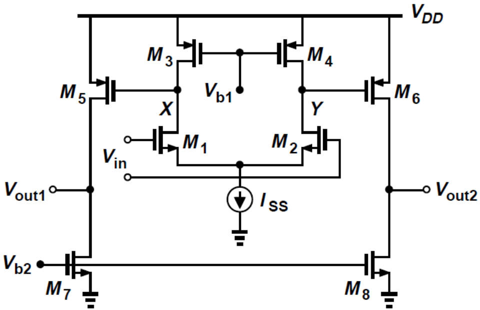


# Two-Stage Op Amps

- **Voltage headroom in today's design is constrained with low supply voltage and large output swing**
- **Gain:** $g_{m1,2}(r_{O1,2}||r_{O3,4})\ g_{m5,6}(r_{O5,6}||r_{O7,8})$
- **Output Swing: Vdd-2Overdrive**

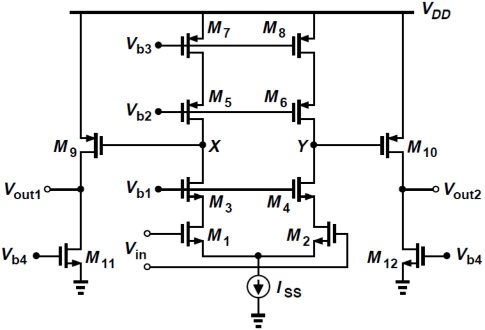


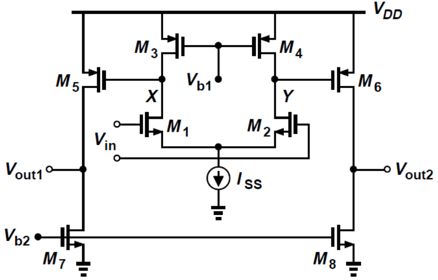
# Two-Stage Op Amps with cascode devices

- Voltage headroom in today's design is constrained with low supply voltage and large output swing
- Gain: $$A_v \approx \{g_{m1,2}[(g_{m3,4}+g_{mb3,4})r_{O3,4}r_{O1,2}] \| [(g_{m5,6}+g_{mb5,6})r_{O5,6}r_{O7,8}]\} \times [g_{m9,10}(r_{O9,10} \| r_{O11,12})].$$
- Can we have more stages? Feedback stability limits



## Example 9.10

**Design the two-stage op amp for VDD = 1V, P = 1mW, a differential output swing of 1 Vpp, and a gain of 100.**




**Solution:**

**(1)Current allocation**

**(2)Voltage allocation:**

**300mV to M7, 200mV to M5, 400mV to M3, 100mV to M1**

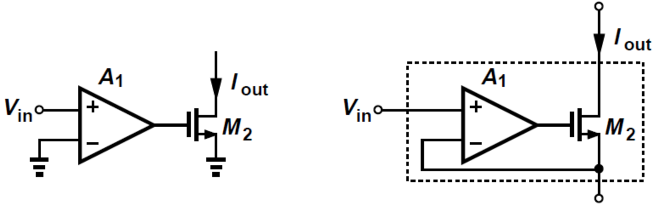
**(noise and gm consideration)**

**(3)Calculate aspect ratio**

**(4)Calculate gain > 2000**

# Gain Boosting

- Increase the output impedance without adding more cascode devices. But how?
- A transistor preceded by an ideal voltage amplifier exhibits a transconductance of gmA1 and a output resistance of ro.



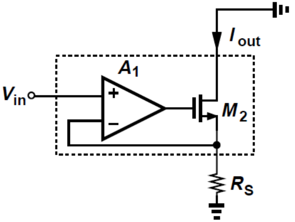
# Gain Boosting

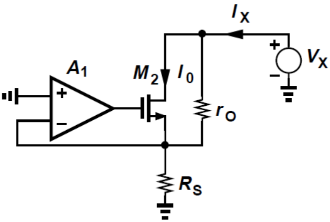
- **Increase the output impedance without adding more cascade devices. But how?**
- **First Perspective:**

**A degenerated transistor preceded by an ideal voltage amplifier**

$$\frac{I_{out}}{V_{in}} = \frac{A_1 g_m}{1 + (A_1 + 1) g_m R_S}. \qquad R_{out} = r_O + (A_1 + 1) g_m r_O R_S + R_S.$$

**In fact, the output resistance is "boosted. The headroom is similar to a simple degenerated transistor**



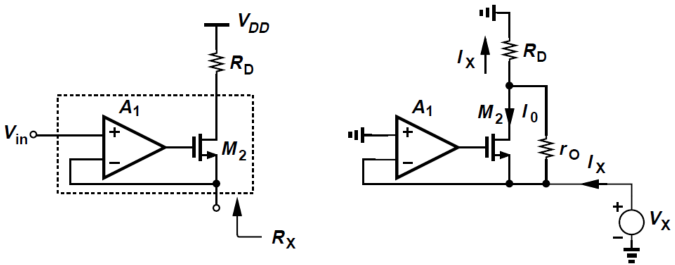
# Example 9.11

**Determine the resistance seen at the source of M2 without considering body effect.**

## Solution:

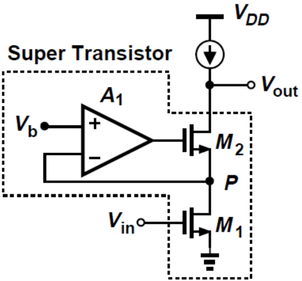
$$\frac{I_X R_D - V_X}{r_O} + (-A_1 V_X - V_X) g_m + I_X = 0$$

$$R_X = \frac{R_D + r_O}{1 + (A_1 + 1) g_m r_O}$$



# Basic gain-boosted stage

• Current-Voltage feedback increase the output impedance by a factor of A1+1, while the real gm raised by A1 is reduced by A1+1 when feedback is applied.




- Rout:

$$R_{out} = r_O + (A_1 + 1)g_m r_O R_S + R_S.$$

- Gm: gm1A1/(A1+1)~gm1
- Rp:(look above P, see example 9.11)

$$r_{O2}/[1 + (A_1 + 1)g_{m2}r_{O2}] \approx [(A_1 + 1)g_{m2}]^{-1}$$

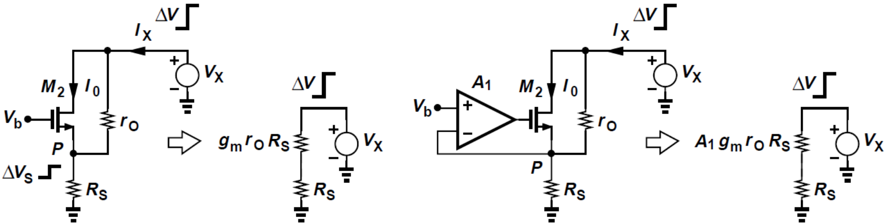
-

$$\begin{aligned} |A_v| &\approx g_{m1}[r_{O2} + (A_1 + 1)g_{m2}r_{O2}r_{O1} + r_{O1}] \\ &\approx g_{m1}g_{m2}r_{O1}r_{O2}(A_1 + 1). \end{aligned}$$

# Regulated Cascode

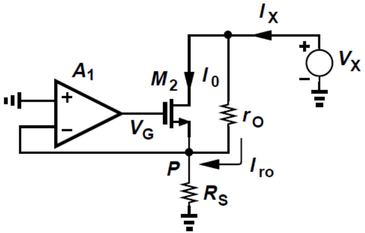
**Second Perspective**

- Loosely view the voltage change divided by Rs and gmroRs.
- Drain current response can be suppressed as
  - Vp is constant
  - Current through Rs is constant
- Vp is "pinned" to Vb by feedback regulation.



# Example 9.12

**Determine the small-signal values of $V_P, V_G, I_0,$ and $I_{ro}$. Assume $(A_1+1)g_m r_O R_S$ is large.**




**Solution: Current circulates M2**

$$V_X = [r_O + (A_1+1)g_m r_O R_S + R_S]I_X$$

$$\begin{aligned} V_P &= I_X R_S \\ &= \frac{R_S}{r_O + (A_1+1)g_m r_O R_S + R_S} V_X. \end{aligned}$$

$$\begin{aligned} V_G &= -A_1 V_P \\ &= \frac{-A_1 R_S}{r_O + (A_1+1)g_m r_O R_S + R_S} V_X \end{aligned}$$

$$V_G - V_P \approx -V_X/(g_m r_O).$$

$$I_0 \approx -V_X/r_O$$

$$\begin{aligned} I_{ro} &= \frac{V_X - V_P}{r_O} \\ &= \frac{r_O + (A_1+1)g_m r_O R_S}{r_O + (A_1+1)g_m r_O R_S + R_S} \frac{V_X}{r_O} \\ &\approx \frac{V_X}{r_O}. \end{aligned}$$

# Gain boosting Key

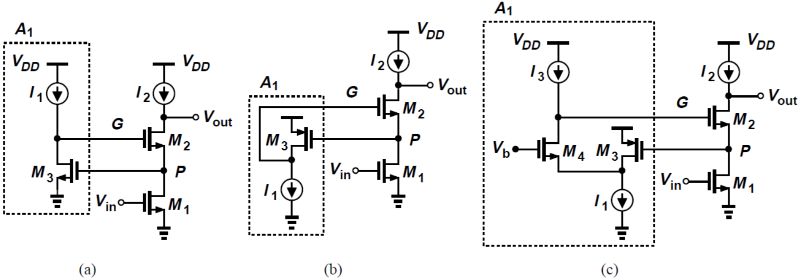
- **The amplifier boosts the gm of the cascode device**
- **The amplifier regulates the output current by monitoring and pinning the source voltage**

# Gain Boosting Circuit Implementation

- **Simplest a common-source stage**

$$|V_{out}/V_{in}| \approx g_{m1}r_{O1}g_{m2}r_{O2}(g_{m3}r_{O3}+1)$$

- **Avoid headroom limitation, PMOS common-source stage is better, but M3 could go in triode**
- **Folded-cascode inserts one more stage**



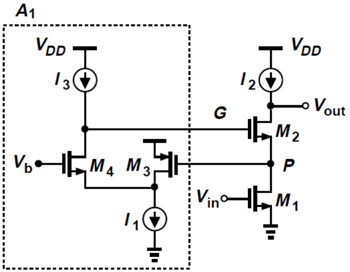
# Example 9.13

**Determine the allowable range for Vb.**

**Solution:**

$$V_{b,min} = V_{GS4} + V_{I1}$$

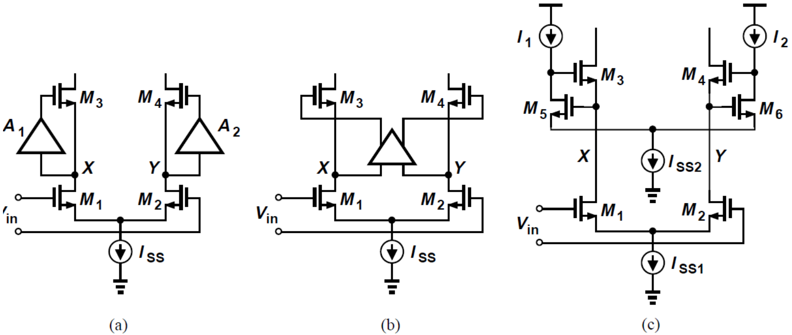
$$V_{b,max} = V_{GS2} + V_P + V_{TH4}$$




(c)

# Gain Boosting with a Differential Pair

- One threshold higher than a simple differential circuit
- Merge two gain boosting blocks to differential one

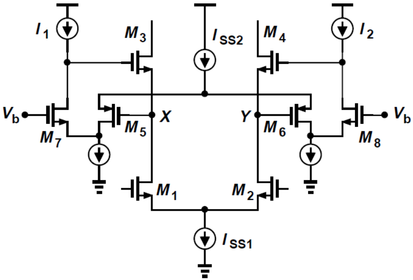


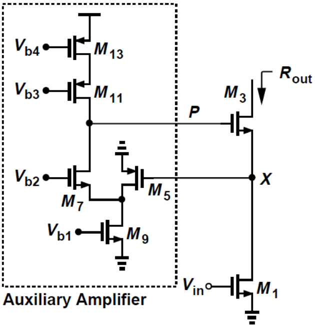
# Differential Folded Cascode Gain Boosting

- **The minimal allowable Vx,Vy is VOD12+VISS1**
- **The output impedance of the circuit (Example 9.14)**

$$R_{out1} \approx [g_{m7}r_{O7}(r_{O9}||r_{O5})]||(g_{m11}r_{O11}r_{O13})$$

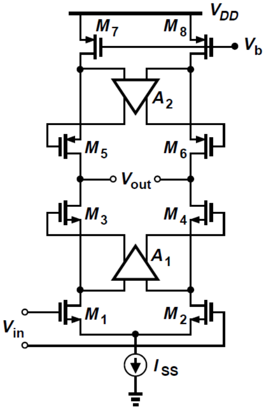
$$R_{out} \approx g_{m3}r_{O3}r_{O1}g_{m5}R_{out1}$$



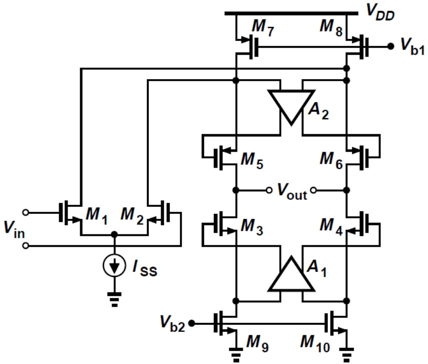



# Gain Boosting in Signal Path and Load

- Gain boosting can be utilized in the load current source
- To allow maximum swings, A2 employs NMOS-input.

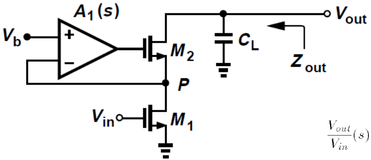



(a)




(b)

# Gain Boosting Frequency Response




$$G_m(s) = g_{m1}\frac{r_{O1}}{r_{O1} + \dfrac{r_{O2}}{1+(A_1+1)g_{m2}r_{O2}}}$$

$$Z_{out} = [r_{O1} + (A_1+1)g_{m2}r_{O2}r_{O1} + r_{O2}]||\frac{1}{C_L s}.$$

$$\frac{V_{out}}{V_{in}}(s) = \frac{-g_{m1}r_{O1}[(1+g_{m2}r_{O2})\frac{s}{\omega_0} + (A_0+1)g_{m2}r_{O2}+1]}{\frac{(r_{O1}+r_{O2})C_L}{\omega_0}[1+g_{m2}(r_{O2}||r_{O1})]s^2 + [(r_{O1}+r_{O2})C_L + (A_0+1)g_{m2}r_{O2}r_{O1}C_L + \frac{1}{\omega_0}]s + 1}.$$

- **Zero:** $|\omega_z| \approx (A_0+1)\omega_0$
- **Dominant pole:**

$$|\omega_{p1}| = \frac{1}{[r_{O1} + (A_0+1)g_{m2}r_{O2}r_{O1} + r_{O2}]C_L + \dfrac{1}{\omega_0}}$$

$$\approx \frac{1}{A_0 g_{m2}r_{O2}r_{O1}C_L}.$$

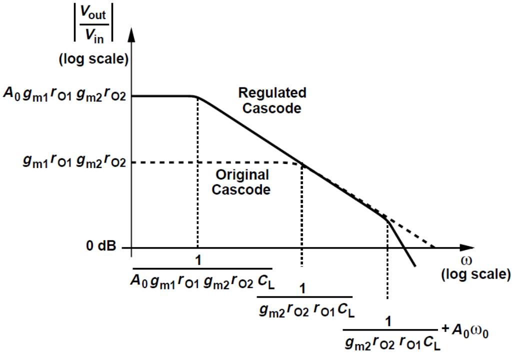
- **Non-dominant pole: Above the original-3dB bandwidth**

$$|\omega_{p2}| = \frac{[r_{O1} + (A_0+1)g_{m2}r_{O2}r_{O1} + r_{O2}]C_L + \dfrac{1}{\omega_0}}{\dfrac{(r_{O1}+r_{O2})C_L}{\omega_0}[1+g_{m2}(r_{O1}||r_{O2})]}$$

$$\approx (A_0+1)\omega_0 + \frac{1}{g_{m2}r_{O2}r_{O1}C_L},$$

# Frequency Response Bode Plot

- Gain boosting frequency response bode plot
- Two poles, non-dominant is below the original 3dB pole



# Example 9.15

**Is the dominant-pole approximation valid here.**

**Solution:**

$$
\begin{aligned}
\frac{\omega_{p2}}{\omega_{p1}} &\approx \left[(A_0+1)\omega_0 + \frac{1}{g_{m2}r_{O2}r_{O1}C_L}\right]\left[(A_0+1)g_{m2}r_{O2}r_{O1}C_L + \frac{1}{\omega_0}\right] \\
&\approx (A_0+1)^2 g_{m2}r_{O2}r_{O1}C_L\omega_0 + 2(A_0+1) + \frac{1}{g_{m2}r_{O2}r_{O1}C_L\omega_0}.
\end{aligned}
$$

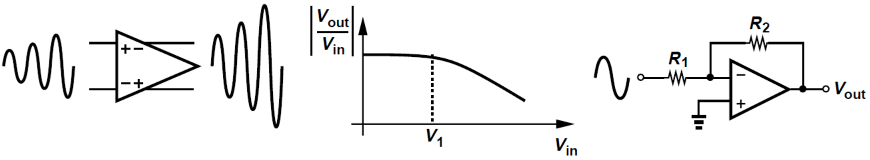
**The s [illegible] an unity and the approximation is valid.**

# Comparison

| | Gain | Output Swing | Speed | Power Dissipation | Noise |
|---|---|---|---|---|---|
| Telescopic | Medium | Medium | Highest | Low | Low |
| Folded−Cascode | Medium | Medium | High | Medium | Medium |
| Two−Stage | High | Highest | Low | Medium | Low |
| Gain−Boosted | High | Medium | Medium | High | Medium |

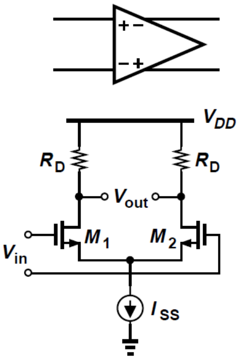
# Output Swing Calculation

- **Be careful about distortion and gain error**
- **The maximum output amplitude that yields an acceptable distortion or gain error**
- **Apply a growing sinusoid wave, monitor the resulting output, and calculate the maximum allowable gain**

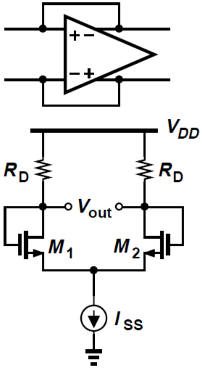



(a) (b)

# Common-Mode Feed Back




(a)

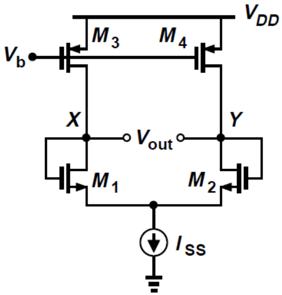



(b)

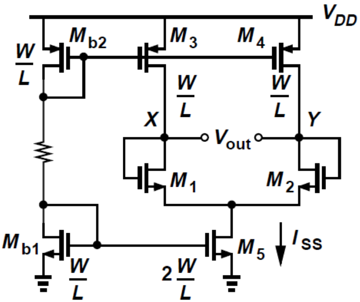
- **Vcm(in) and Vcm(out):** $V_{DD} - I_{SS}R_D/2$

# Basic Concepts

- In fully-differential op amps, the output CM level is usually not well defined.
  - Case 1: $I_{D3,4} < I_{SS}/2$, Vx,Vy decreases, Iss triode;
  - Case 2: In reverse, Vx,Vy increases, M3,M4 triode.



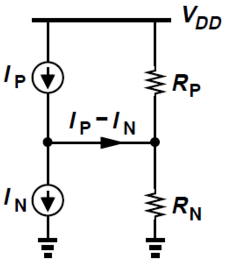

(a)




(b)

# Basic Concepts

- In high-gain amplifiers, CMFB balances the PMOS and NMOS current mismatches, thus avoid driving one of them into triode region.
- Differential feedback cannot define CM level
- In simulation, CM may be well-defined around half VDD, yet in real world, random mismatches and device variations would degrade CM easily without CMFB.



# Example 9.16

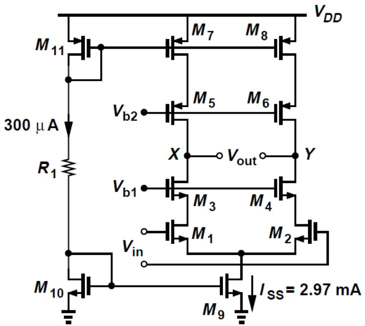
Consider the telescopic op amp below. Suppose M9 suffers from a 1% current mismatch with respect to M10, producing Iss = 2.97 mA rather than 3 mA. Assuming perfect matching for the others. Explain what happens in the circuit.

**Solution:**

Output voltage error: $(I_P - I_N)(R_P \| R_N)$ =3.99V

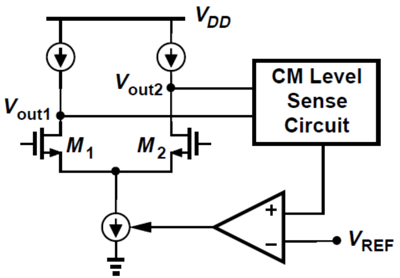
Vx, Vy must rise so much that M5, M6, M7, M8 go to triode, yielding ID7 = 1.485 mA.

Current mismatch is largely depended on different drain-source voltage.



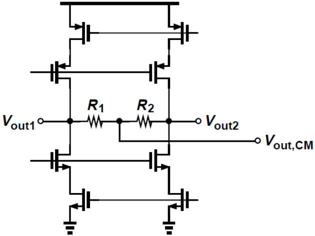
# Conceptual topology

- Measure output CM level;
- Compare with a reference;
- Apply the error to correct the level.

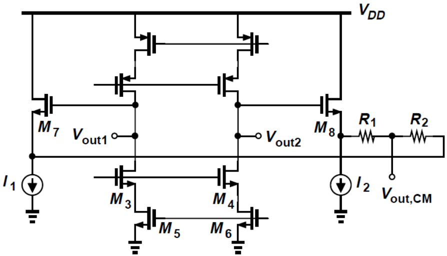


# CM Sensing Techniques

- **Resistive sensing** $V_{out,CM} = (R_1 V_{out2} + R_2 V_{out1})/(R_1 + R_2)$
  - **large R1,R2 to avoid loading effect**
  - **large chip area and parasitic capacitance**
  - **reduce frequency performance**



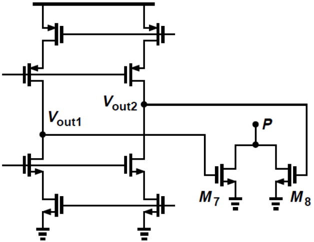

- **Source follower sensing**
  - **lose one Vth in swing**
  - **large R1, I1 to avoid "starved"**



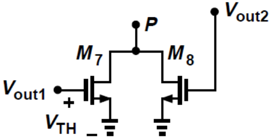
# CM Sensing Techniques

- Capacitive sensing
- Switched-capacitor

- Deep triode sensing
  - Rtot ~ vout1+vout2
  - Rtot has no relationship with differential voltage
  - may go to saturation region




$$
\begin{aligned}
R_{tot} &= R_{on7}||R_{on8} \\
&= \frac{1}{\mu_n C_{ox}\frac{W}{L}(V_{out1}-V_{TH})}\Bigg|\Bigg|\frac{1}{\mu_n C_{ox}\frac{W}{L}(V_{out2}-V_{TH})} \\
&= \frac{1}{\mu_n C_{ox}\frac{W}{L}(V_{out2}+V_{out1}-2V_{TH})},
\end{aligned}
$$

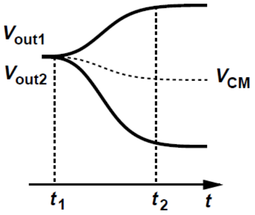


# Example 9.17

A student simulates the step response of a closed-loop op amp circuit and observes the output waveforms shown in below. Explain why Vout1 and Vout2 do not change symmetrically.

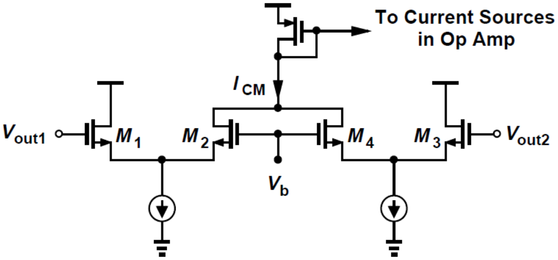
**Solution:**

As evident from waveforms, the output CM levels change from  t1 to t2, indicating CM sensing mechanism is nonlinear. For example, if M7 or M8 in last slide does not in deep triode at t2, the CM level would change because now it is a function of differential output.



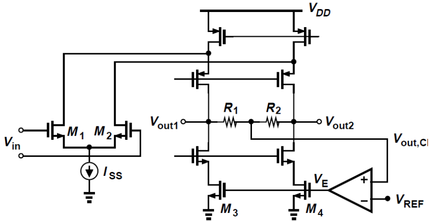
# CM Sensing Techniques

- **Differential pair sensing**
  - $I_{CM} \propto V_{out1} + V_{out2}$ **by small signal analysis**
  - **Under Large swings situation, sensing is not valid due to large non-linearity.**

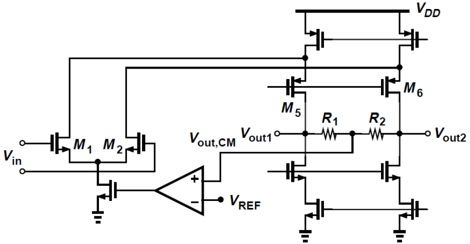


# CM Feedback Techniques

- **Control cascade current source**




- **Control tail current source**

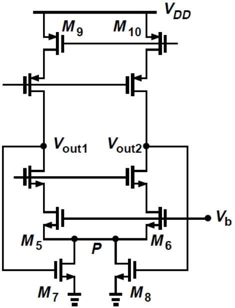


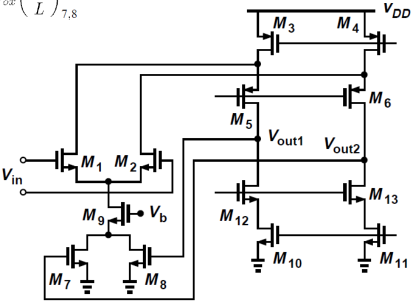
# CM Feedback Techniques

- **Deep triode sensing feedback**
  - **Limited headroom**
  - **Large C**
  - **Device variation**

- **Deep triode folded-cascade sensing feedback**

$$V_{out1} + V_{out2} = \frac{2I_D}{\mu_n C_{ox}\left(\frac{W}{L}\right)_{7,8}} \frac{1}{V_b - V_{GS5}} + 2V_{TH}$$



# Example 9.18

**Determine the sensitivity of Vout,CM to Vb, i.e, dVout,CM/dVb.**

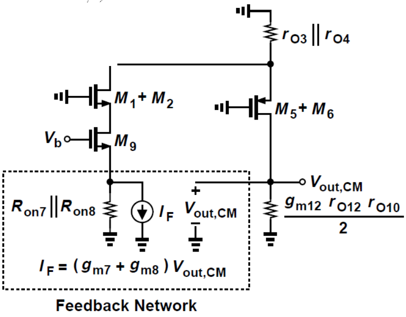
**Solution:**

**CMFB small signal analysis**

$$
\begin{aligned}
\beta &= \frac{V_2}{V_1}\Big|_{I2=0} \\
&= -(g_{m7}+g_{m8})(R_{on7}||R_{on8}) \\
&= -2\mu_n C_{ox}\left(\frac{W}{L}\right)_{7,8} V_{DS7,8} \cdot \frac{1}{2\mu_n C_{ox}(W/L)_{7,8}(V_{GS7,8}-V_{TH7,8})} \\
&= -\frac{V_{DS7,8}}{V_{GS7,8}-V_{TH7,8}},
\end{aligned}
$$

$$\left|\frac{dV_{out,CM}}{dV_b}\right|_{closed} \approx \frac{V_{GS7,8}-V_{TH7,8}}{V_{DS7,8}}$$

**Maximize Vds7,8 for Sensitivity**

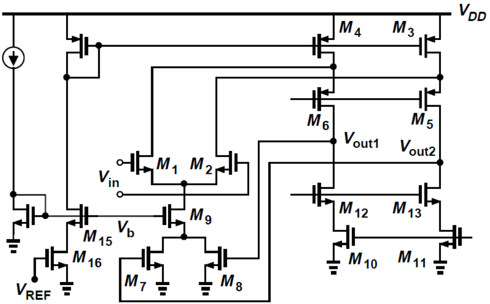


# CM Feedback Techniques

- **Modification of deep triode sensing feedback**

$$(W/L)_{15} = (W/L)_9 \qquad (W/L)_{16} = (W/L)_7 + (W/L)_8$$

- **The output level is relatively independent of device parameters and lowers sensitivity of Vb**

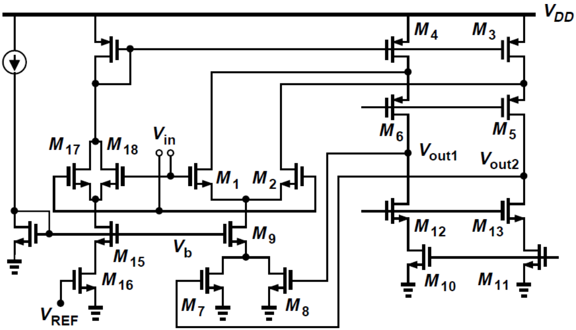


# CM Feedback Techniques

- **Modification of deep triode sensing feedback**

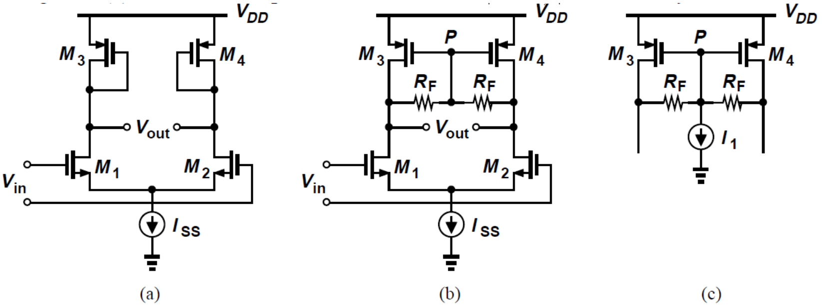
$$(W/L)_{15} = (W/L)_9 \qquad (W/L)_{16} = (W/L)_7 + (W/L)_8$$

- **M17, M18 reproduces the drain of M15 a voltage equal to the source voltage of M1 and M2**



# CM Feedback Techniques

- Another type of CM feedback topology
- Diode-connected loads' output CM level is well-defined
- Differential small signal gain $g_{m1,2}(r_{O1,2}||r_{O3,4}||R_F)$
- Common-mode work as a diode-connected $R_F \gg r_{O1,2}||r_{O3,4}$
- Low supply voltage design $I_1 R_F/2 = |V_{TH3,4}|$



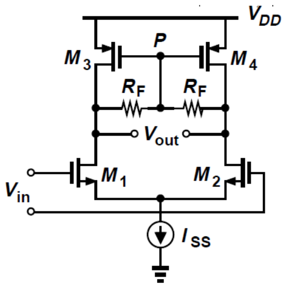
# Example 9.19

**Determine the maximum allowable output swings.**

**Solution:**

**Each output can fall to two overdrive voltages above ground if Vin,CM is chosen to place Iss at the edge of the triode region. The highest level allowed at the output is equal to the output CM level at P plus |Vth3,4| (by selecting suitable RF). Thus, output swing is VDD-3Vov.**

**(RF is small, not like previous setup)**



# Example 9.20

**Facing voltage headroom limitations, a student constructs the circuits below. Determine the small-signal gain from the input CM level to the output CM level.**

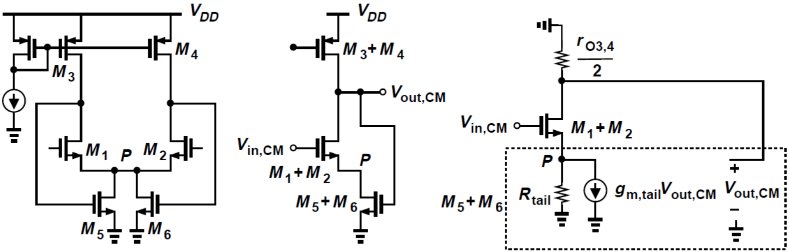
**Solution:**

$$g_{m5}+g_{m6}=2\mu_n C_{ox}(W/L)_{5,6}V_P$$

$$R_{tail}=[2\mu_n C_{ox}(W/L)_{5,6}(V_{out,CM}-V_{TH5,6})]^{-1}$$

$$\frac{V_{out,CM}}{V_{in,CM}}=-\frac{1}{\dfrac{2R_{tail}}{r_{O3,4}}+g_{m,tail}R_{tail}+(g_{m1,2}r_{O3,4})^{-1}}.$$

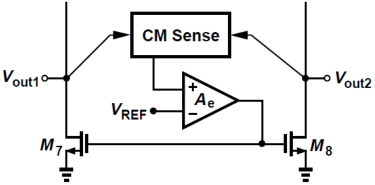
**(A poor CMRR)**

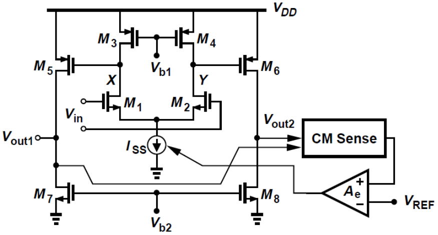



(a) (b) (c)

# CMFB in Two-Stage Op Amps

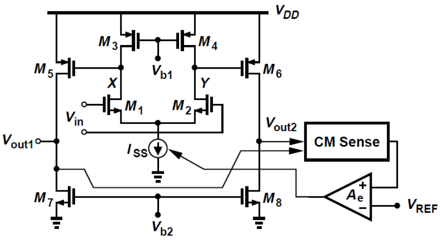
- CMFB around second stage (not good)
  - May establish a current beyond nominal value
- CMFB from second stage to first stage
  - Global loop control of both stages

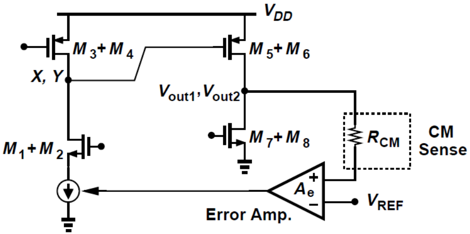


# CMFB from Second to First Stage

- **CMFB from second stage to first stage limitation**
  **- 3 or 4 poles, which makes it difficult for the loop stable**

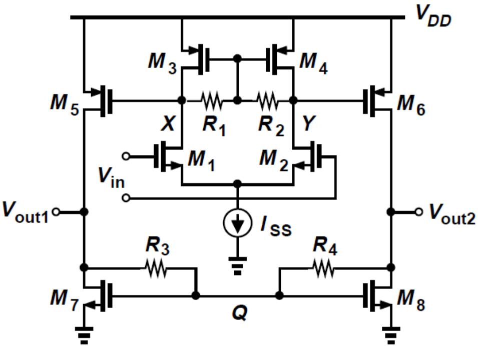


# CMFB at both Stages

- All the drain currents are copied from Iss
- The differential voltage gain is equal to

$$g_{m1}(r_{O1}||r_{O3}||R_1)g_{m5}(r_{O5}||r_{O7}||R_3)$$

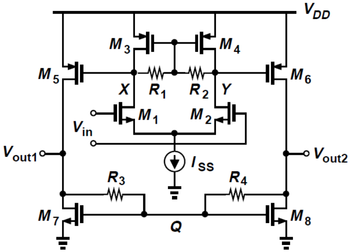


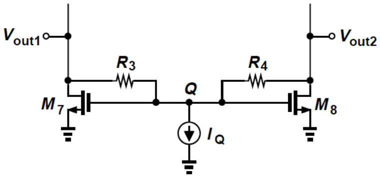
# Example 9.21

For the below design explain why the output CM level is inevitably well below VDD/2 and hence the output swings are limited. Devise a solution.

**Solution:**

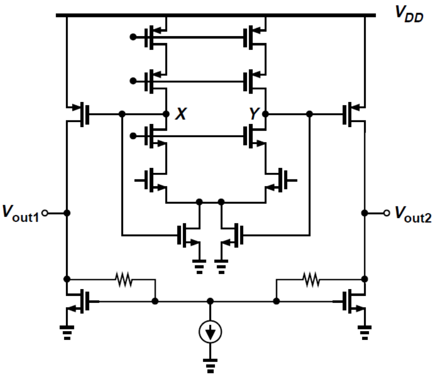
The output CM is equal to VG7,8, which is only slightly greater than one threshold. The issue can be resolved by drawing a small current from node Q. It can be upwards to desired output and the device is still in saturation.



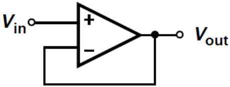
# CMFB for Cascode First Stage

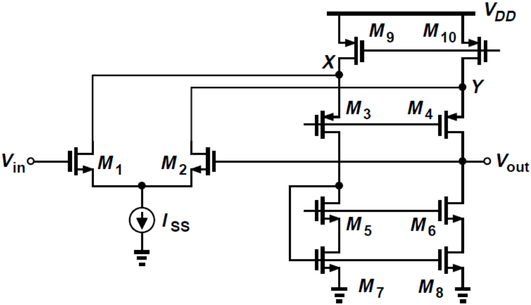
- First stage use deep triode feedback loop to avoid loading.
- Achieving high gain while not precise



# Input Range Limitations

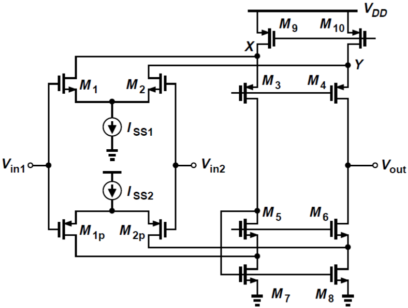
- Input common-mode level may need to vary over a wide range, e.g. ADC input comparator.
- Input swing limits the total range sometimes.
- In the below single-end unity-gain buffer the input CM minimal voltage is VGS1,2+VISS, which is one threshold greater than 2Vov in the output CM.

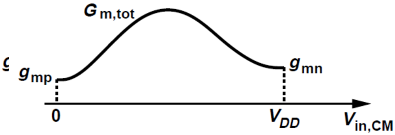


# Extension of Input Range

- Incorporate both NMOS and PMOS differential pair to keep a necessary transconductance
- The transconductance variation should be concerned
- Gain, speed and noise may vary



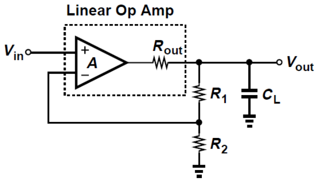
# Slew Rate

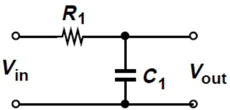
- In a linear circuit, the slope of the step depends on final output value

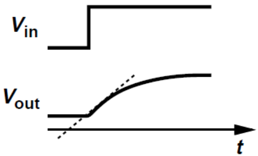
$$\frac{dV_{out}}{dt} = \frac{V_0}{\tau} \exp\frac{-t}{\tau}$$

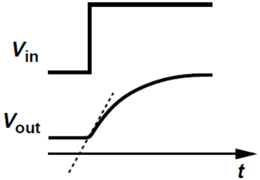
- The observation applies to linear feedback system

$$V_{out} = V_0 \frac{A}{1 + A\frac{R_2}{R_1 + R_2}} \left[ 1 - \exp\frac{-t}{\frac{C_L R_{out}}{1 + AR_2/(R_1 + R_2)}} \right] u(t)$$



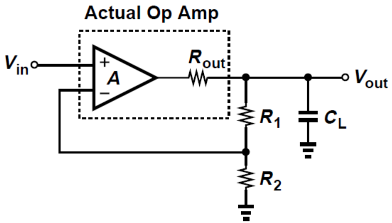



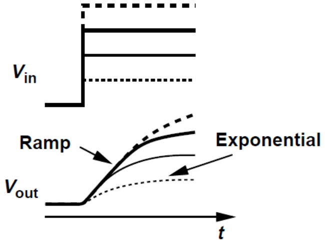



# Slew Rate

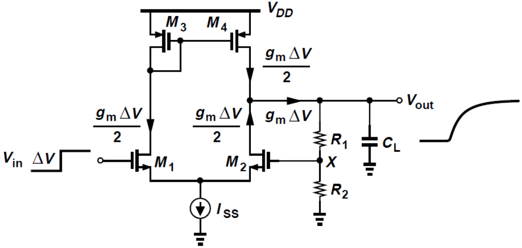
- In a realistic case, with large input steps, the output displays a linear ramp having a constant slope. The slope of ramp is the “slew rate”.
- It seems that the maximum current to charge the load capacitance is limited.
- Nonlinear behavior. Reduce speed and increase distortion.
- Increase SR would consume power and wider device

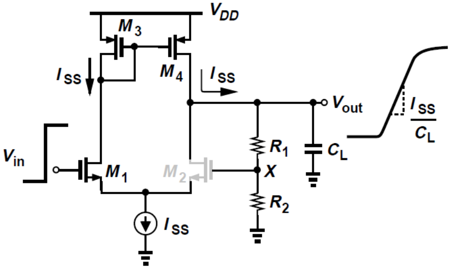


# Slew Rate Example

- A small step rises Vout by $g_m \Delta V$ and hence adjusts through R1, R2 negative feedback circuit.
- When M1 experiences a large step, M2 turns off. Thus, CL is charged by a constant current ISS.
- Feedback is broken but after M2 turns on, the circuit returns to a linear operation.




(a)

# Example 9.22

**(a)Determine the small-signal step response of the circuit.**
**(b)Calculate the positive and negative slew rates.**

## Solution:

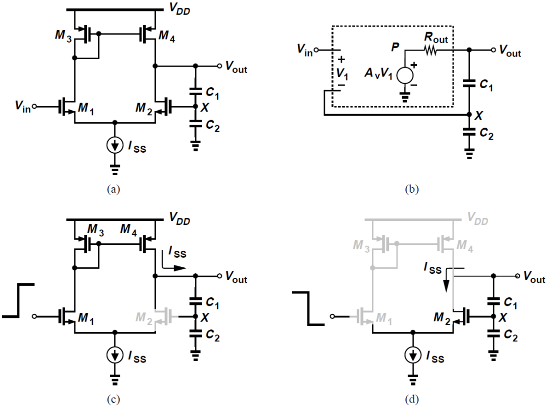
$$\frac{V_{out}}{V_{in}}(s) = \frac{A_v}{1 + A_v\dfrac{C_1}{C_1+C_2} + \dfrac{C_1C_2}{C_1+C_2}R_{out}s}$$

$$= \frac{A_v/\left(1 + A_v\dfrac{C_1}{C_1+C_2}\right)}{1 + \dfrac{C_1C_2}{C_1+C_2}R_{out}s/\left(1 + A_v\dfrac{C_1}{C_1+C_2}\right)}$$

$$V_{out}(t) = \frac{A_v}{1 + A_v\dfrac{C_1}{C_1+C_2}}V_0\left(1 - \exp\frac{-t}{\tau}\right)u(t)$$

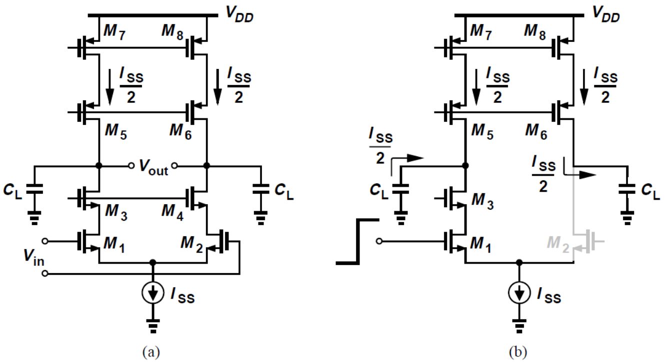
$$V_{out}(t) = I_{SS}/[C_1C_2/(C_1+C_2)]t$$

$$V_{out} = -I_{SS}/[C_1C_2/(C_1+C_2)]t$$



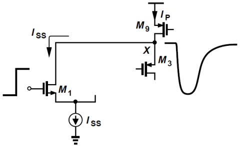
# Slew Rate of Telescopic Op Amp

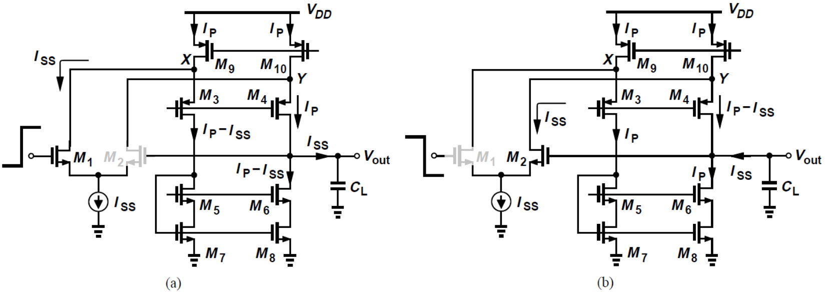
- Each side appears a ramp with slope equal to $\pm I_{SS}/(2C_L)$
- The total slew rate for Vout1- Vout2 equal to $I_{SS}/C_L$



# Slew Rate of Folded-cascode Op Amp

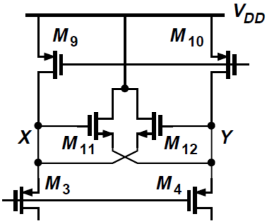
- Yield a slew rate of $I_{SS}/C_L$ if $I_P \geq I_{SS}$
- Otherwise, M3 turns off and tail current source enters the triode region. The settling time increases.



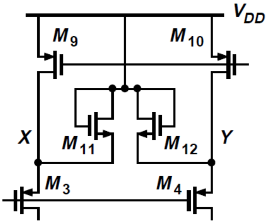



# Clamp transistor

- Limit Vx,Vy to produce large different voltage
- More aggressive design, Vx,Vy higher than VDD-VTHN




(a)

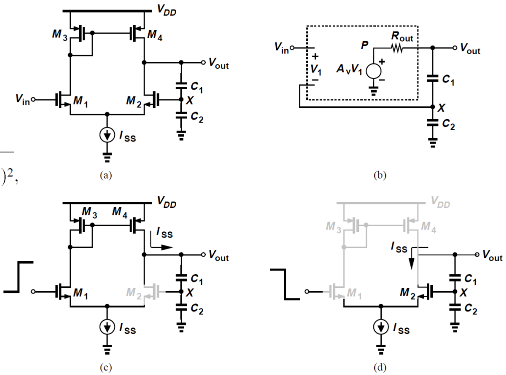



(b)

# Example 9.23

**As Vout rises, so does Vx, eventually turning M2 on. As ID2 increases from zero, the plot becomes linear. Considering M1 and M2 becomes linear if difference between their drain current is less than** $\alpha I_{SS}$ (e.g., $\alpha = 0.1$).

**Calculate the time to linear.**




**Solution:**

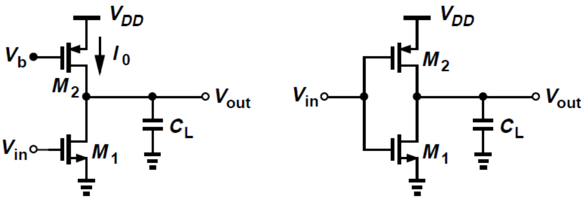
$$\alpha I_{SS} = \frac{1}{2}\mu_n C_{ox}\frac{W}{L}(V_{in1} - V_{in2})\sqrt{\frac{4I_{SS}}{\mu_n C_{ox}\frac{W}{L}} - (V_{in1} - V_{in2})^2},$$

$$\Delta V_G \approx \alpha\sqrt{\frac{I_{SS}}{\mu_n C_{ox}\frac{W}{L}}}$$

$$t = \frac{C_2}{I_{SS}}\left(V_0 - \alpha\sqrt{\frac{I_{SS}}{\mu_n C_{ox}\frac{W}{L}}}\right)$$

# High-Slew-Rate Op Amp

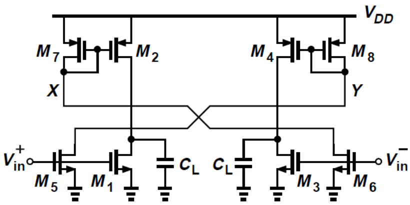
- Slew rate is limited by power consumption
- The trade-off could be mitigated if the capacitor could be charged to a desired value quickly. And the voltage falls back to original value.
- Complementary topology jumps fast but suffer from poor power supply rejection



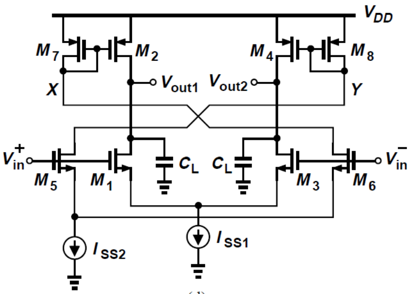

(a) (b)

# Push-Pull Stages

- Use current-mirror. E.g. If Vin+ jumps down, and Vin- jumps up then
  - M5 draws less current, lowering ID4;
  - M3, M6 draws more current;
  - M7 draws more current, raising ID2;
  - M1 draws less current, charging CL.



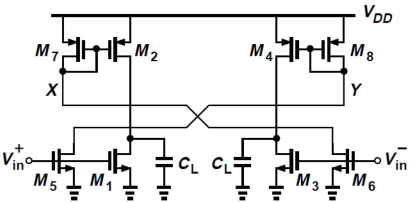

(c)



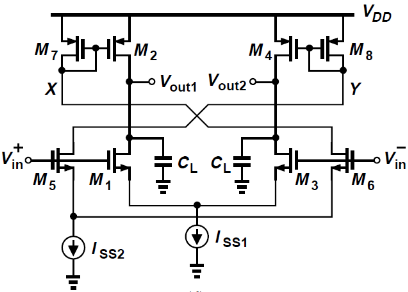

(d)

# Push-Pull Stages

- Improve the input common-mode rejection by adding tail current sources to build differential circuits
- The differential slew rate is $[I_{SS1} + I_{SS2}(W_4/W_8)]/C_L$
- SR increases with a around twofold power penalty




(c)



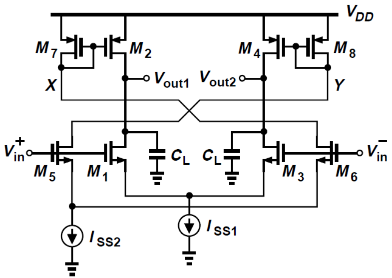

(d)

# Example 9.24

**Calculate the small-signal voltage gain of the class-AB op amp shown below.**

**Solution:**

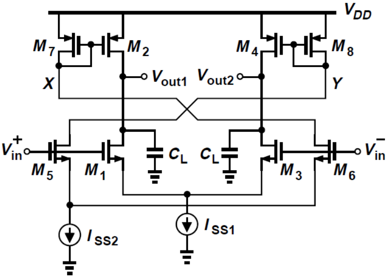
$$\begin{aligned}|A_v| &\approx g_{m1}(r_{O3}||r_{O4}) + (W_4/W_8)g_{m5}(r_{O3}||r_{O4}) \\ &\approx [g_{m1} + (W_4/W_8)g_{m5}](r_{O3}||r_{O4}).\end{aligned}$$



# Frequency Response and Mirror Pole

- **It is not possible to equate zero and pole**
- **Raise SR would decrease mirror pole frequency**

$$g_{m8}/C_Y \qquad C_Y \approx 2(W_4 + W_8)LC_{ox}$$




$$H_{mirr}(s) = \frac{W_4}{W_8} g_{m5}(r_{O3}||r_{O4}) \frac{1}{1+\frac{s}{\omega_{p,X}}} \frac{1}{1+\frac{s}{\omega_{out}}}$$

$\omega_{p,X} \approx g_{m8}/C_Y$ and $\omega_{out} = [(r_{O3}||r_{O4})C_L]^{-1}$

$$\begin{aligned} H_{tot}(s) &= H_{main}(s) + H_{mirr}(s) \\ &= \frac{r_{O3}||r_{O4}}{1+\frac{s}{\omega_{out}}} \left[ \frac{W_4}{W_8} \frac{g_{m5}}{1+\frac{s}{\omega_{p,X}}} + g_{m1} \right] \\ &= \frac{r_{O3}||r_{O4}}{1+\frac{s}{\omega_{out}}} \cdot \frac{(W_4/W_8)g_{m5} + g_{m1} + g_{m1}s/\omega_{p,X}}{1+\frac{s}{\omega_{p,X}}} \end{aligned}$$

$$|\omega_z| = \left( \frac{W_4}{W_8} \frac{g_{m5}}{g_{m1}} + 1 \right) \omega_{p,X}.$$

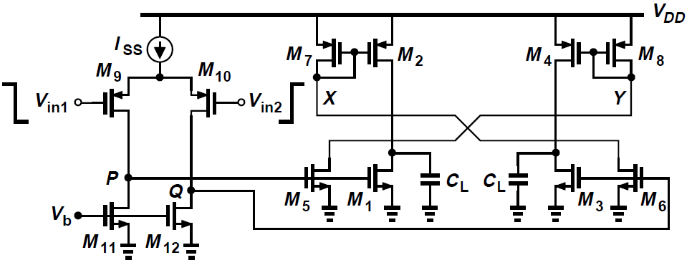
# Two-Stage Op Amp with High SR

- **Voltage Gain:**

$$|A_v| = g_{m9}(r_{O9}||r_{O11})[g_{m1} + (W_4/W_8)g_{m5}](r_{O1}||r_{O2})$$

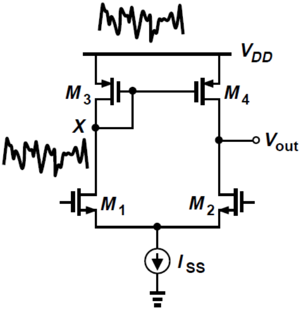
- **Slew Rate**

  **- If P is low capacitance, it is "agile" enough to go upon to VDD, thus providing a large SR.**



# Power Supply Rejection

- **Power line contains noise**
- **PSRR(power supply rejection ration):**
  **- Gain from input to output divided by the gain from supply to the output**
- **At low frequency** $PSRR \approx g_{mN}(r_{OP} \| r_{ON})$



# Example 9.25

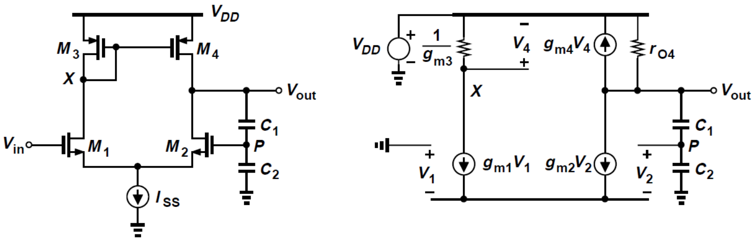
**Calculate the low-frequency PSRR of the feedback circuit shown below**

**Solution:**

$$\frac{V_{out}}{V_{DD}} = \frac{1}{g_{m2}r_{O4}\dfrac{C_1}{C_1+C_2}+1}.$$

$$PSRR \approx \left(1+\frac{C_2}{C_1}\right)\left(g_{m2}r_{O4}\frac{C_1}{C_1+C_2}+1\right)$$
$$\approx g_{m2}r_{O4}.$$

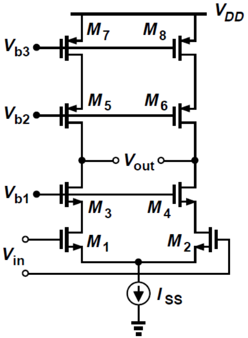
**Feedback reduce** $\partial V_{out}/\partial V_{DD}$ and $\partial V_{out}/\partial V_{in}$ **the same and PSRR is relatively constant.**



# Noise in Telescopic Op Amp

- **At low frequency the cascode devices contribute negligible noise**

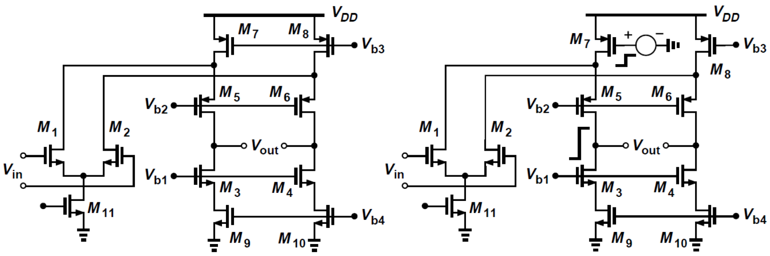
$$\overline{V_n^2} = 4kT\left(2\frac{\gamma}{g_{m1,2}} + 2\frac{\gamma g_{m7,8}}{g_{m1,2}^2}\right) + 2\frac{K_N}{(WL)_{1,2}C_{ox}f} + 2\frac{K_P}{(WL)_{7,8}C_{ox}f}\frac{g_{m7,8}^2}{g_{m1,2}^2}$$



# Noise in Folded-Cascode Op Amp

- **At low frequency the cascode devices contribute negligible noise**

$$\overline{V_{n,int}^2} = 8kT\left(\frac{\gamma}{g_{m1,2}} + \gamma\frac{g_{m7,8}}{g_{m1,2}^2} + \gamma\frac{g_{m9,10}}{g_{m1,2}^2}\right)$$



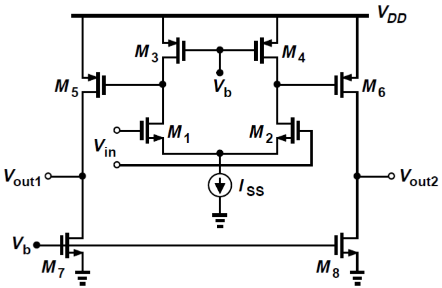

(a)

(b)

# Noise in two-stage Op Amp

- **The noise in the second stage contributes negligible noise**

$$\overline{V_{n,tot}^2} = 8kT\gamma \frac{1}{g_{m1}^2}\left[g_{m1} + g_{m3} + \frac{g_{m5} + g_{m7}}{g_{m5}^2(r_{O1}||r_{O3})^2}\right]$$



# Example 9.26

**A simple amplifier is constructed below. Note that the first stage incorporates diode-connected – rather than current-source loads. Assuming all the transistors in saturation. (W/L)1=50/0.6 (W/L)3=10/0.6 (W/L)5=20/0.6 (W/L)7=56/0.6**

**Solution:**

$$A_{v1} \approx \frac{g_{m1}}{g_{m3}} = \sqrt{\frac{50 \times 75}{10 \times 30}} \approx 3.54.$$

$$(8kT/3)(g_{m3} + g_{m1})/g_{m1}^2 = 1.10 \times 10^{-17}\ \mathrm{V^2/Hz}.$$

$$4kT(2/3)(g_{m5} + g_{m7})/g_{m5}^2\, A_{v1}^2 = 2.29 \times 10^{-18}\ \mathrm{V^2/Hz}$$

$$\overline{V_{n,in}^2} = 2(2.29 \times 10^{-18} + 1.10 \times 10^{-17}) = 2.66 \times 10^{-17}\ \mathrm{V^2/Hz},$$