Chapter 10: Stability and Frequency Compensation

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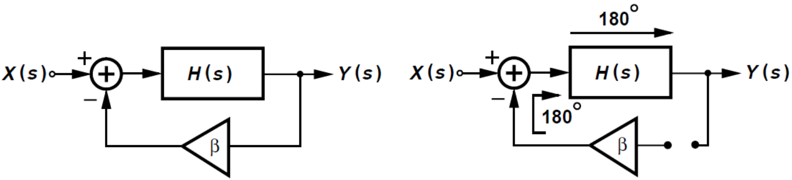
General Considerations



(a) (b)

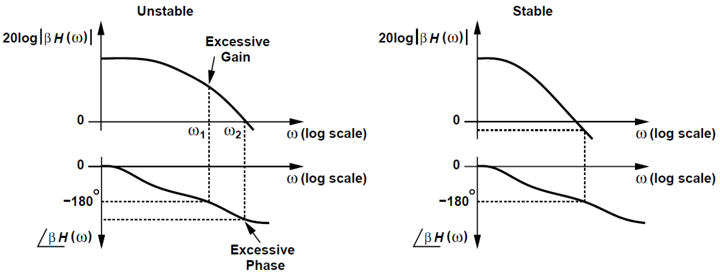
- Feedback systems suffer from potential instability and they may oscillate.
- Closed-loop transfer function: $\frac{Y}{X}(s) = \frac{H(s)}{1+\beta H(s)}$
- What happens if the denominator goes to infinity

"Barkhausen's criteria"

$$|\beta H(j\omega_1)| = 1$$
$$\angle \beta H(j\omega_1) = -180^\circ$$

- Negative feedback itself provide 180 phase shift
- Loop transmission determines the stability issue

MOSFET as a Switch



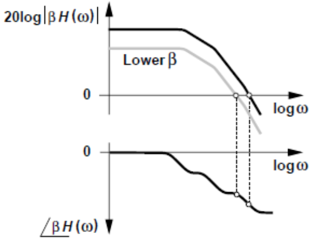
- **Phase shift changes the negative feedback to positive**
- **Gain crossover and phase crossover when gain is unity and phase is – 180 degrees**
- **PX must be behind GX, and GX is equal to unity-gain bandwidth in the open-loop system**

Example 10.1

Explain whether the system depicted before becomes more or less stable if the feedback is weakened.

Solution:

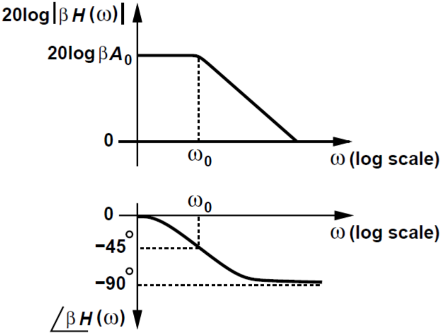
If β is reduced, a lower β shifts the plot of 20log $\angle\beta H(\omega)$ down and GX to the left. Since the phase does not change, the system becomes more stable. If there is no feed back, the circuit will not oscillate. The worst case is unity gain feedback, when $\beta H = H$.



Review of Bode Approximation

- The slope of the magnitude plot changes by +20dB/dec at every zero frequency and by -20dB/dec at every pole frequency.
- The change begins to change at one-tenth of the pole (left zero)frequency, change by -45 degrees (+45 degrees) at the pole (left zero), and approaches -90 degrees (+90 degrees) at 10 times the pole (left zero)
- The key point is that phase changes faster than magnitude.

One-Pole System



$$H(s) = A_0/(1 + s/\omega_0)$$

$$\frac{Y}{X}(s) = \frac{\frac{A_0}{1+\beta A_0}}{1 + \frac{s}{\omega_0(1+\beta A_0)}}$$

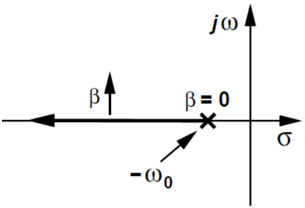
- **A phase shift less than 90 degrees**
- **A one-pole system is unconditionally stable**

Example 10.2

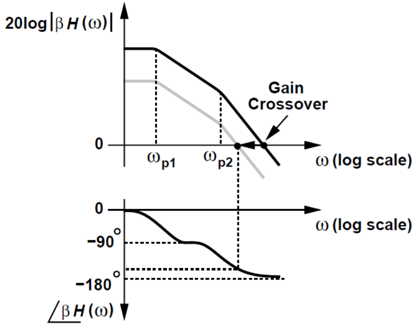
Construct the root locus for a one-pole system.

Solution:

The closed-loop transfer function implies that the system has a pole $s_p = -\omega_0(1 + \beta A_0)$. So this is a real-valued pole in the left half plane that moves away from the origin as the loop gain increases.



Multipole Systems



- The system is stable when below gain crossover, phase is less than -180 degrees
- If the feedback becomes weaker, the system is more stable

Example 10.3

Construct the root locus for a two-pole system.

Solution:

The closed-loop transfer function implies that the system has a pole . So this is a real-valued pole in the left half plane that moves away from the origin as the loop gain increases.

Open loop gain

$$H(s) = \frac{A_0}{\left(1+\frac{s}{\omega_{p1}}\right)\left(1+\frac{s}{\omega_{p2}}\right)}$$

Close-loop transfer function

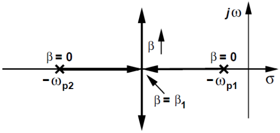
$$\frac{Y}{X}(s) = \frac{A_0}{\left(1+\frac{s}{\omega_{p1}}\right)\left(1+\frac{s}{\omega_{p2}}\right)+\beta A_0}$$

$$= \frac{A_0\omega_{p1}\omega_{p2}}{s^2+(\omega_{p1}+\omega_{p2})s+(1+\beta A_0)\omega_{p1}\omega_{p2}}$$

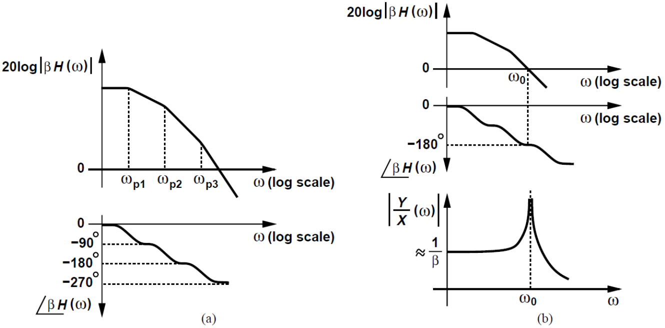
$$s_{1,2} = \frac{-(\omega_{p1}+\omega_{p2}) \pm \sqrt{(\omega_{p1}+\omega_{p2})^2 - 4(1+\beta A_0)\omega_{p1}\omega_{p2}}}{2}$$

$$\beta = 0,\ s_{1,2} = -\omega_{p1}, -\omega_{p2}$$

$$\beta_1 = \frac{1}{A_0}\frac{(\omega_{p1}-\omega_{p2})^2}{4\omega_{p1}\omega_{p2}}$$



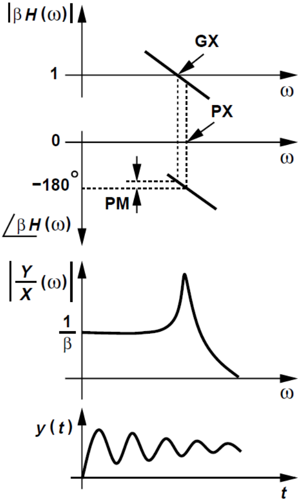
Three poles systems



- The system is no long stable
- If the feedback becomes weaker, the system is more stable
- The close-loop response has infinite gain when gain and phase crossover frequency coincide, which leads to oscillation

Phase Margin

How far should PX be from GX?



Example : Phase Margin

GX is only slightly below PX. At GX the phase equals -175 degrees.

$$\beta H(j\omega_1) = 1 \times \exp(-j175^\circ)$$

$$\begin{aligned}\frac{Y}{X}(j\omega_1) &= \frac{H(j\omega_1)}{1+\beta H(j\omega_1)} \\ &= \frac{\frac{1}{\beta}\exp(-j175^\circ)}{1+\exp(-j175^\circ)} \\ &= \frac{1}{\beta}\cdot\frac{-0.9962-j0.0872}{0.0038-j0.0872}\end{aligned}$$

$$\begin{aligned}\left|\frac{Y}{X}(j\omega_1)\right| &= \frac{1}{\beta}\cdot\frac{1}{0.0872} \\ &\approx \frac{11.5}{\beta}.\end{aligned}$$

- **The closed-loop frequency response exhibits a sharp peak (frequency domain)**
- **The closed-loop system is near oscillation and exhibits a very underdamped behavior.**
- **It would suffer from ringing (time domain)**

Definition

- **Phase Margin(PM), defined as**

$$\textbf{PM} = 180^\circ + \angle \beta H(\omega = \omega_1)$$

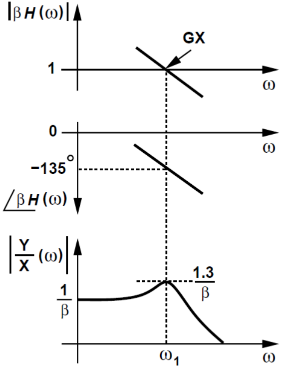
- **A "well-behaved" closed-loop response concludes a greater spacing between GX and PX**
- **The unity-gain bandwidth cannot exceed the second pole frequency**
- **For large-signal application, time-domain simulation of closed-loop system more relevant and useful than small-signal as computations**

Phase Margin Comparison

- **How much phase margin is adequate?**

PM = 45 degrees

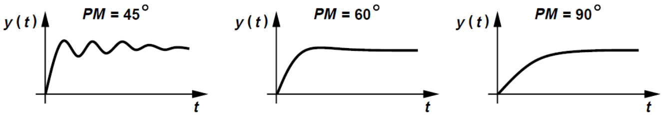
$$\frac{Y}{X} = \frac{H(j\omega_1)}{1 + 1 \times \exp(-j135^\circ)}$$
$$= \frac{H(j\omega_1)}{0.29 - 0.71j}. \approx \frac{1.3}{\beta}.$$



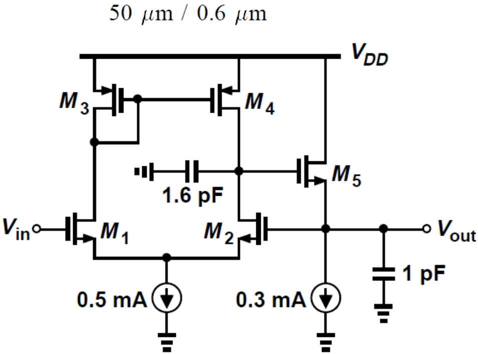
PM = 60 degrees

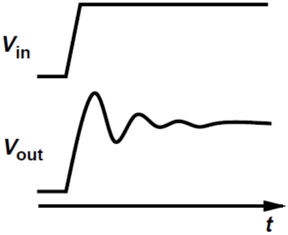
$$Y(j\omega_1)/X(j\omega_1) = 1/\beta$$

Well-suited for small signal



Example: good PM but poor settling

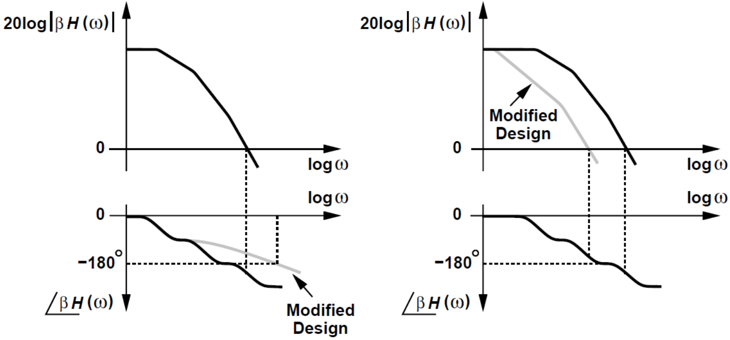




- SPICE yields a phase margin of 65 degrees and a unity frequency of 150 MHz
- The large-signal step response suffers from ringing

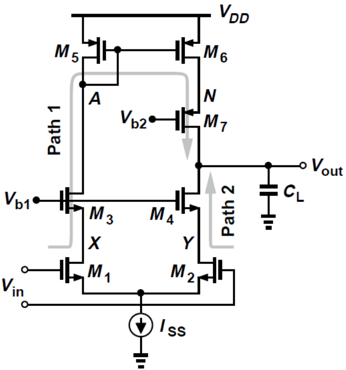
Basic Frequency Compensation

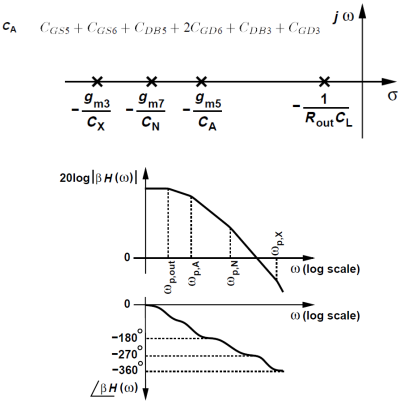
- Be "compensated", the circuit open-loop transfer function must be modified such that closed-loop circuit is stable
  - minimizing the overall phase shift (low gain/swing)
  - dropping the gain with frequency (low bandwidth)



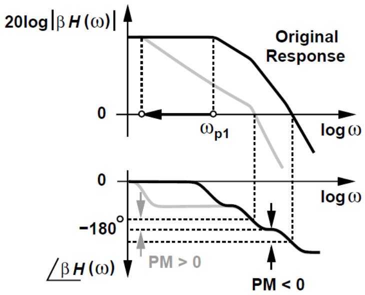
Telescopic Op Amp Stability

- Path 1: HF pole at source of M3, a mirror pole at A, and another HF pole at source of M7
- Path 2: HF pole at source of M4, shared pole at output
- Dominant pole at output due to high output impedance
- N is the next dominant pole due to wider device
- A zero at the twice of mirror pole



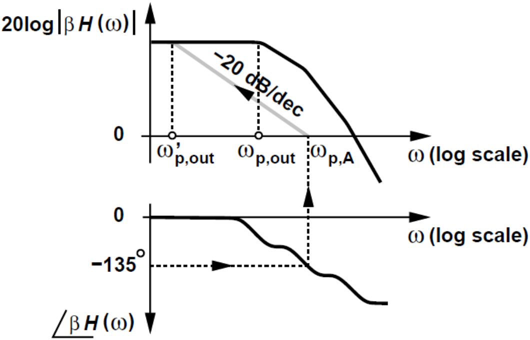


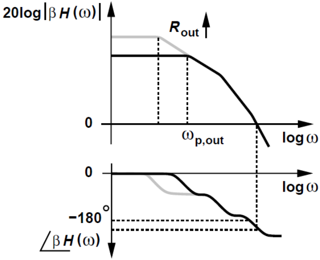
Compensation Procedure



- Remember that our mission is to minimize the number of poles while maintaining one dominant pole enough far away from next pole
- Lower the dominant pole frequency by a large load capacitance
- Phase plot is affected by not the critical part

Determine the Dominant Pole





Realize a phase margin with 45 degrees

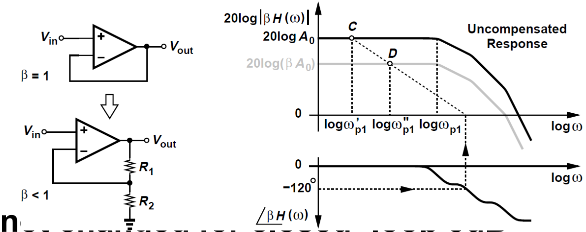
- draw a 20dB/dec line from gain crossover(A)
- determine the new dominant pole
- load capacitance increased by $\omega_{p,out}/\omega'_{p,out}$

- Increasing Rout or moving non-dominant pole?
  - undesired

Example 10.5

An Op amp has a phase margin of 60 degrees with unity-gain feedback. By what factor can the compensation relaxed if the circuit is to operate with a feedback factor of $\beta < 1$?

Solution:



$$\frac{-20\log\beta}{\log\omega''_{p1} - \log\omega'_{p1}} = 20,$$

C can scale down by $1/\beta$

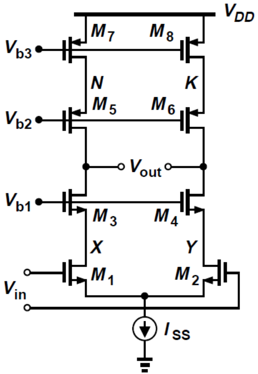
Notice that the speed is not changed for closed-loop 3dB bandwidth calc, which means time constant is not changed.

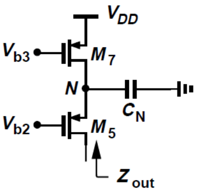
$(1+\beta A_0)\omega''_{p1} \approx \beta A_0\omega''_{p1} \approx A_0\omega'_{p1}$ Consider the PM improvement?

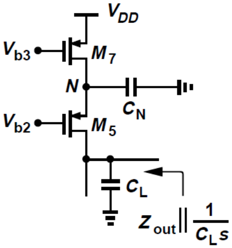
$(1+A_0)\omega'_{p1} \approx A_0\omega'_{p1}$

Note that unity-gain feedback fastest but worst stability

Differential Telescopic Cascode







$$Z_{out} \| \frac{1}{C_L s} = \frac{(1+g_{m5}r_{O5})\dfrac{r_{O7}}{r_{O7}C_N s+1} \cdot \dfrac{1}{C_L s}}{(1+g_{m5}r_{O5})\dfrac{r_{O7}}{r_{O7}C_N s+1} + \dfrac{1}{C_L s}}$$

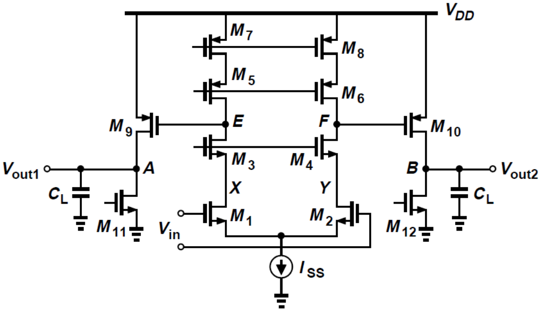
$$= \frac{(1+g_{m5}r_{O5})r_{O7}}{[(1+g_{m5}r_{O5})r_{O7}C_L + r_{O7}C_N]s+1}$$

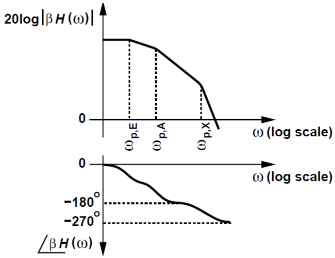
- **No mirror pole**
- **Lower the dominant pole by a slight amount as if there is no pole in cascade current source**

$(1+g_{m5}r_{O5})r_{O7}C_L + r_{O7}C_N$

- **Low gain but fast path can create zero**

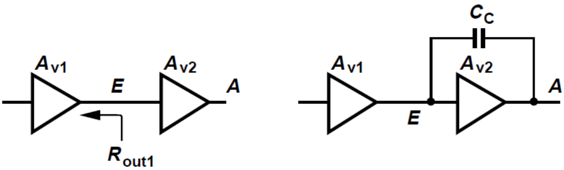
Compensation of Two-Stage Op Amp



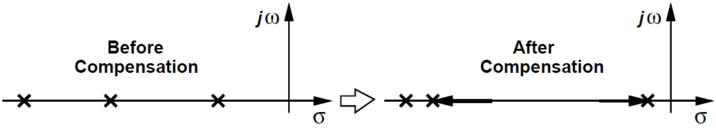


- Three poles but two dominant at A and E
- If only moving one dominant pole,
  - limited bandwidth
  - a large compensation capacitor

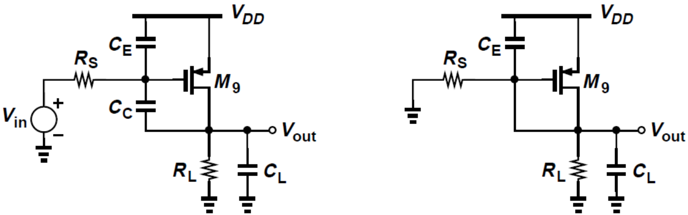
Miller Compensation(1)



- **A larger C creating a dominating pole** $R_{out1}^{-1}[C_E + (1 + A_{v2})C_C]^{-1}$
- **Pole splitting**



Miller Compensation(2)



$$\omega'_{p1} \approx \frac{1}{R_S[(1+g_{m9}R_L)(C_C+C_{GD9})+C_E]+R_L(C_C+C_{GD9}+C_L)}$$

$$\omega'_{p2} \approx \frac{R_S[(1+g_{m9}R_L)(C_C+C_{GD9})+C_E]+R_L(C_C+C_{GD9}+C_L)}{R_SR_L[(C_C+C_{GD9})C_E+(C_C+C_{GD9})C_L+C_EC_L)]}$$

- **Hand calculation. Miller effect provides a low impedance path at high frequency**

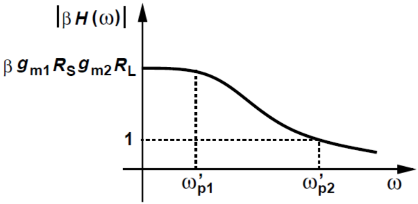
$$\omega'_{p1} \approx \frac{1}{R_S[(1+g_{m9}R_L)(C_C+C_{GD9})}$$

$$\omega'_{p2} \approx g_{m9}/(C_E+C_L).$$

- **Require iteration calculation**

Example 10.6

The two-stage op amp incorporates Miller compensation to reach a phase margin of 45 degrees. Estimate the compensation capacitor value.



Solution:

Dominant pole $(g_{m9} R_L C_C R_S)^{-1}$

Next pole g_{m9}/C_L

Loop gain transfer function

$$|\beta H(\omega)| \approx \frac{\beta g_{m1} R_S g_{m9} R_L}{\sqrt{1 + \omega^2/\omega'^2_{p1}}}$$

$C_C = \dfrac{g_{m1}}{g_{m9}} C_L$ **smaller for a weaker feedback**

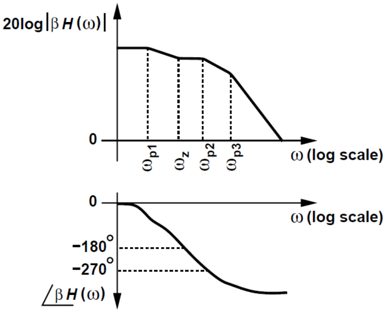
Right-Half-Plane Zero

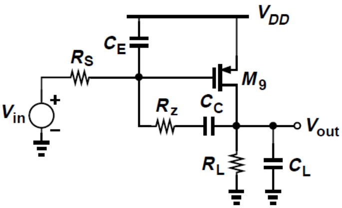
- Zero $g_{m9}/(C_C + C_{GD9})$. from a "feedforward" path
- Right-half-plane zero yields a negative phase as well as slows down the drop of magnitude
- In series with one resistor to move to the left plane

$$\omega_z \approx \frac{1}{C_C\left(g_{m9}^{-1} - R_z\right)}$$

$$\frac{1}{C_C\left(g_{m9}^{-1} - R_z\right)} = \frac{-g_{m9}}{C_L + C_E},$$

$$R_z = \frac{C_L + C_E + C_C}{g_{m9}C_C} \approx \frac{C_L + C_C}{g_{m9}C_C},$$





Example 10.7

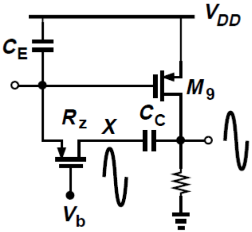
Noting that the Miller compensation has a right half plane zero. By making it equals to the non-dominant pole. Explain what happens.

Solution:

Because of the different sign, both of them still take effect.

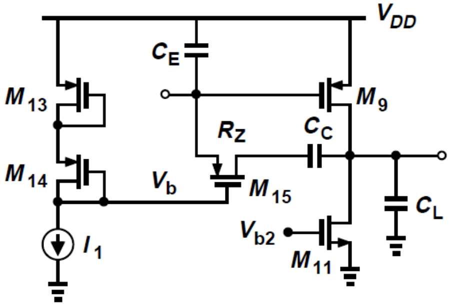
$$\beta H(s) = \frac{\beta A_0(1 - \frac{s}{\omega_z})}{(1 + \frac{s}{\omega_{p1}})(1 + \frac{s}{\omega_{p2}})}.$$

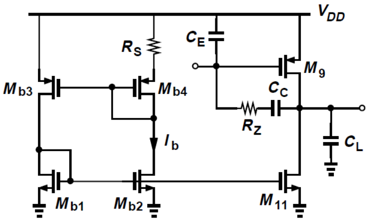
Disadvantage of Canceling Pole



- The load capacitance is variable
- Actual implementation of Rz due to output voltage excursion is complex

Rz Implementation





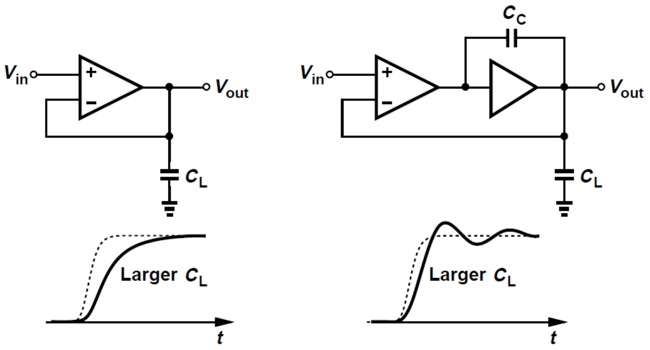
- VGS13 = VGS9, VGS14 = VGS15

$$g_{m14}^{-1}\frac{(W/L)_{14}}{(W/L)_{15}} = g_{m9}^{-1}\left(1+\frac{C_L}{C_C}\right)$$

$$(W/L)_{15} = \sqrt{(W/L)_{14}(W/L)_9}\sqrt{\frac{I_{D9}}{I_{D14}}\frac{C_C}{C_C+C_L}}$$

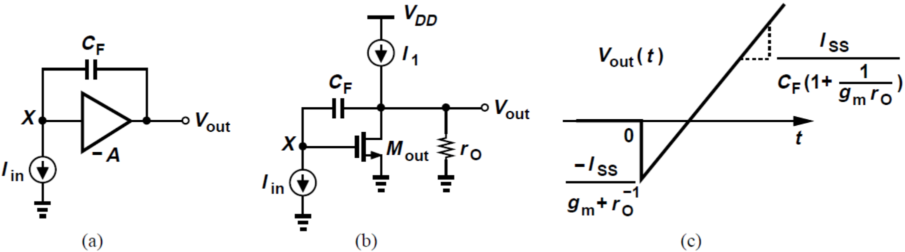
- Use constant-gm circuit

Effect of Load Capacitance



- One-stage op amp improves the phase margin
- Two-stage op amp degrades the phase margin

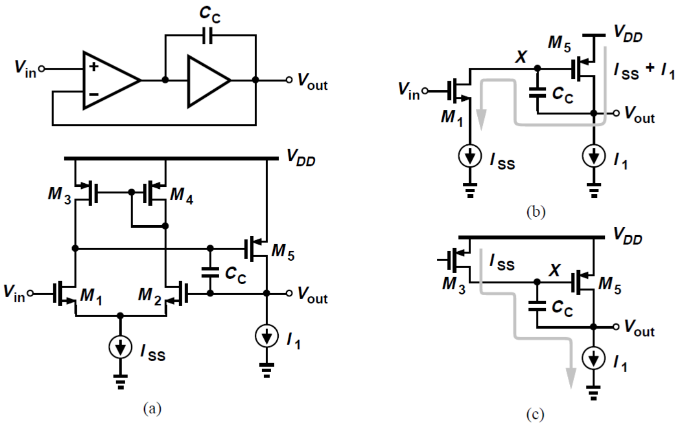
Slewing Study



- **For large A** $V_{out}(t) \approx \frac{I_{SS}}{C_F} t.$
- **For circuit in (b)** $V_{out}(t) = \frac{I_{SS}}{C_F(1+\frac{1}{g_m r_O})} t u(t) - \frac{I_{SS}}{g_m + \frac{1}{r_O}} u(t).$

- CF acts as a short circuit
- The slope reveals Miller Effect at the output
- I1 only serves as the bias current

Slewing in Two-Stage Op Amp



- **Positive slew rate** I_{SS}/C_C

  if M5 is not wide enough, M1 goes to triode
- **Negative slew rate** I_{SS}/C_C
  - $I_1 = I_{SS}$ **Vx rises to turn off M5**
  - $I_1 < I_{SS}$ **M3 goes to triode** I_{D3}/C_C

Example 10.8

Op amps typically drive a heavy load capacitance. Repeat the slew rate analysis.

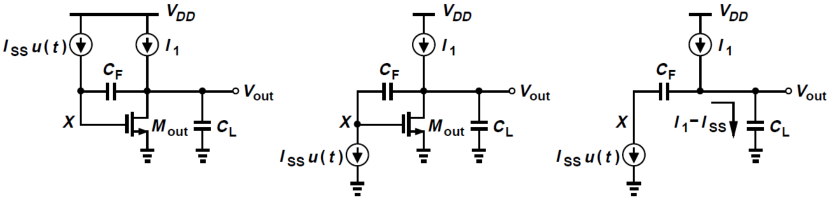
Solution:

Case(a) I_{SS}/C_F

Case(b)

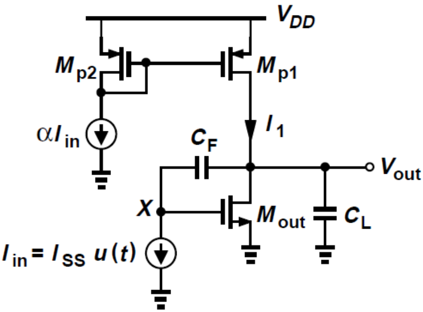
$I_1 > I_{SS}(C_L/C_F) + I_{SS}$ I_{SS}/C_F

$I_1 < (1 + C_L/C_F)I_{SS}$ $(I_1 - I_{SS})/C_L$



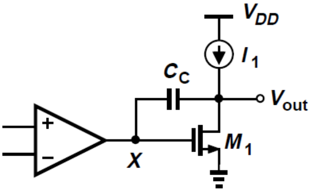
(a) (b) (c)

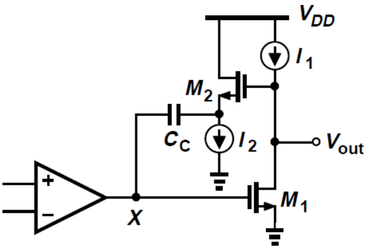
Slewing in Two-Stage Class-AB Op Amp

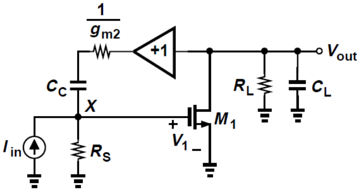


- Recall class-AB op amp incorporate Miller compensation, degrading the phase margin.
- Slower than class-A Op Amp
- Current mirror yields $I_1 = (W_{p1}/W_{p2})\alpha I_{in}$
- Slewing rate $[\alpha(W_{p1}/W_{p2}) - 1]I_{SS}/C_L$

Source Follower to Remove Zero







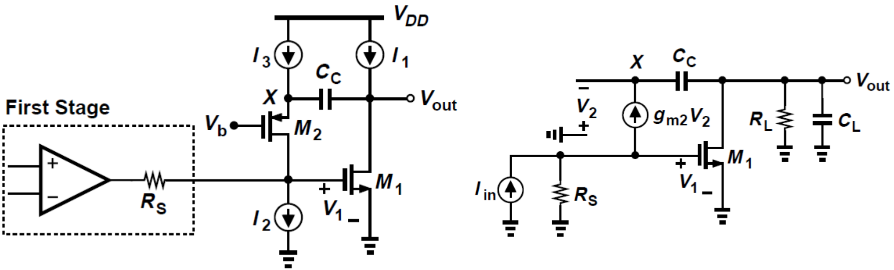
$$\omega_{p1} \approx \frac{g_{m2}}{g_{m1}g_{m2}R_L R_S C_C} \approx \frac{1}{g_{m1}R_L R_S C_C},$$

$$\omega_{p2} \approx \frac{g_{m1}g_{m2}R_L R_S C_C}{R_L C_L C_C g_{m2} R_S} \approx \frac{g_{m1}}{C_L}.$$

$$\frac{V_{out}}{I_{in}} = \frac{-g_{m1}R_L R_S(g_{m2}+C_C s)}{R_L C_L C_C(1+g_{m2}R_S)s^2 + [(1+g_{m1}g_{m2}R_L R_S)C_C + g_{m2}R_L C_L]s + g_{m2}}$$

- **Expect left-plane-zero at higher frequency**

Common-gate to Remove Zero

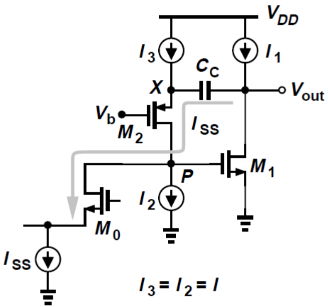


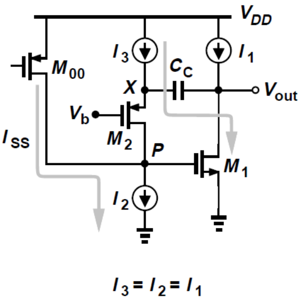
$$\frac{V_{out}}{I_{in}} = \frac{-g_{m1}R_S R_L(g_{m2} + C_C s)}{R_L C_L C_C s^2 + [(1 + g_{m1}R_S)g_{m2}R_L C_C + C_C + g_{m2}R_L C_L]s + g_{m2}}.$$

$$\omega_{p1} \approx \frac{g_{m2}}{g_{m1}g_{m2}R_L R_S C_C} \approx \frac{1}{g_{m1}R_L R_S C_C}, \qquad \omega_{p2} \approx \frac{g_{m2}R_s g_{m1}}{C_L}$$

- **Source follower limits lower end output**
- **Further splitting provides more bandwidth**

Common-gate to Remove Zero Slew Rate

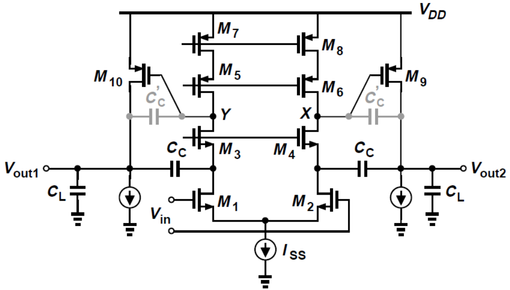


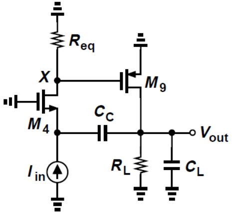


- **Positive slewing** $I_1 \geq I_{SS} + I_{D1}$ $\quad I_{SS}/C_C$

  $I_1 < I_{SS}$, $\quad I_1/C_C$
- **Negative slewing I3, I2 must as large as Iss**

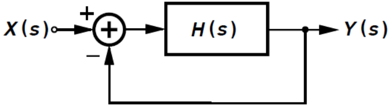
Two-Stage Cascode Op Amp

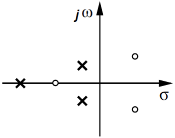


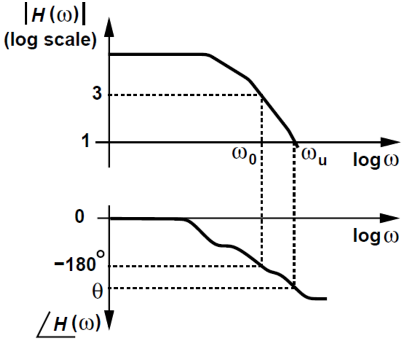


- **Zero** $(g_{m4}R_{eq})(g_{m9}/C_C)$
- **Dominant pole** $(R_{eq}g_{m9}R_LC_C)^{-1}$,
- **Non-dominant pole** $g_{m4}g_{m9}R_{eq}/C_L$
- **It is plausible to combine two methods and the capacitance at X is quite large, which should also be taken into consideration**

Nyquist's Stability Criterion



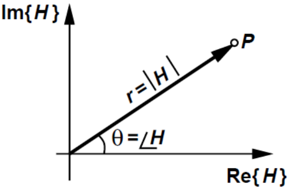




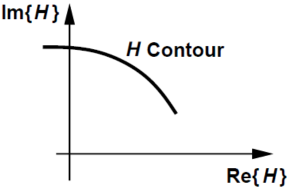
- Bode plot only consider simple sinusoid waveform
- Nyquist's stability criterion exams RHP poles.
- $\sigma_1 + j\omega_1$ is a growing sinusoid could lead to unstable state
- Nyquist stability analysis predicts how many zeros $1 + \beta H(s)$ has in the right half plane(RHP) or on the jw axis.

Basic Concepts

- **Plot H function in polar coordinates by angle and radius;**
- **With frequencies vary, they create a contour. The horizontal and vertical axes are two projections** $|H|\cos(\angle H)$ **and** $|H|\sin(\angle H)$, **- called H contour**

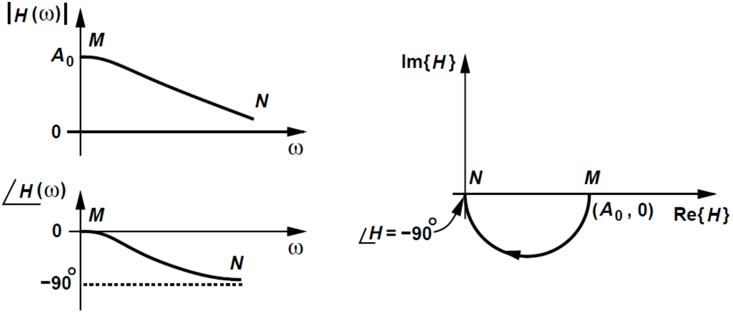


(a)



(b)

Example: One Pole System

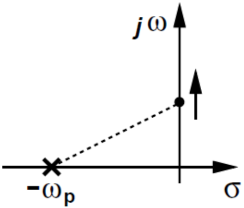


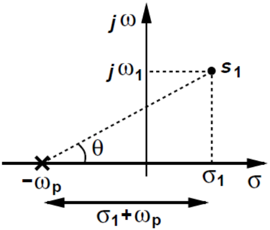
- **Begin at (A0, 0) for w=0 travels to the left until origin for w=infinite**
- **Falls below the horizontal axis because angle is negative**
- **At the origin the angle is -90 degrees**

$-\tan^{-1}(\omega/\omega_p)$, $\quad A_0/\sqrt{1+\omega^2/\omega_p^2}$

Calculate Phase from s-Plane

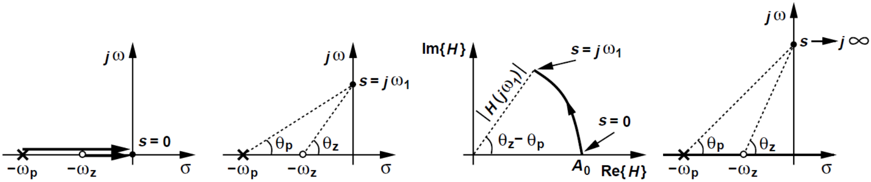
- Draw a vector from the pole to s1, measure the angle of this vector with respect to the positive horizontal axis, and multiply by -1
- Draw a vector from the zero to s1, repeat the measurement and multiply by 1
- The total phase contribution is the algebraically sum
- H magnitude is available to analyze by distance calculation, not necessary for Nyquist's approach



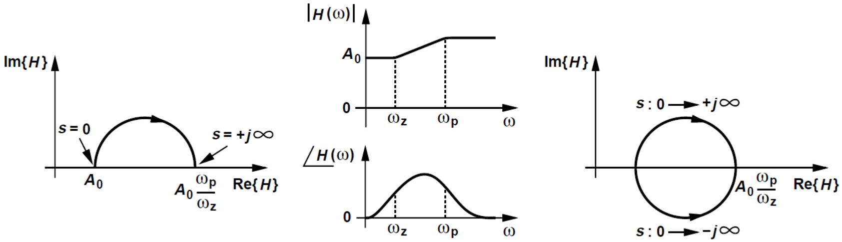


General First-Order System:1 pole, 1 zero

$H(s) = A_0(1 + s/\omega_z)/(1 + s/\omega_p)$ $\qquad\qquad$ $\omega_p > \omega_z$.



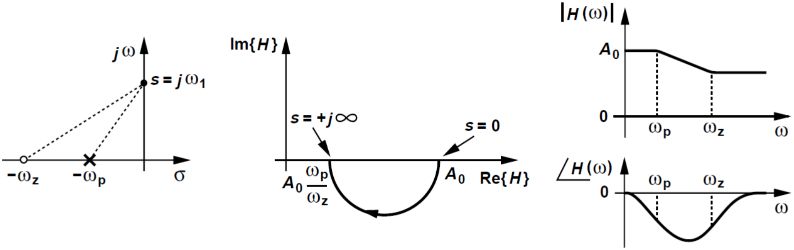
(a) (b) (c) (d)



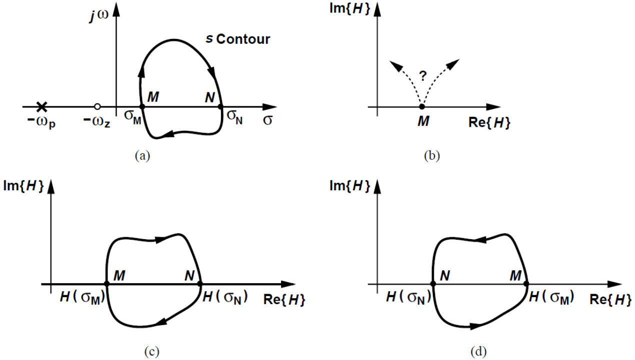
(e) (f) (g)

General First-Order System:1 pole, 1zero

$$H(s) = A_0(1 + s/\omega_z)/(1 + s/\omega_p) \qquad \omega_p < \omega_z$$



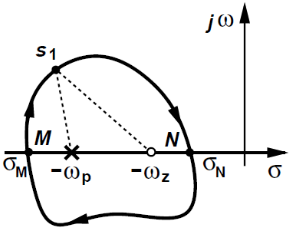
Travel on an Arbitrary Path(1)



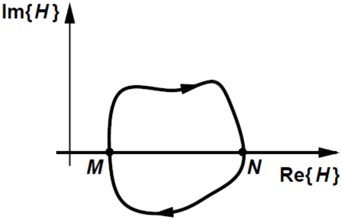
Abs. $\omega_p > \omega_z$. $\omega_p < \omega_z$

- **Analyze M and N points**
- **Determine the rotation direction by observing the net angle value**

Travel on an Arbitrary Path(2)



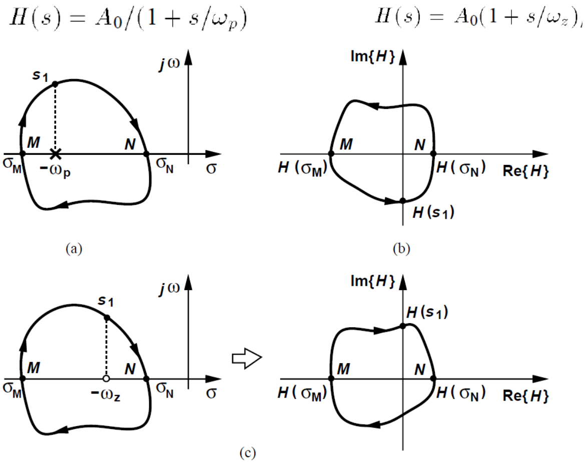
(a)



(b)

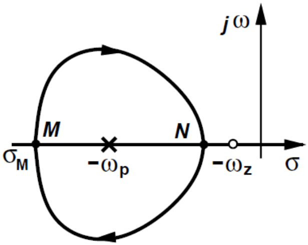
- **S contour encloses pole and zero**
- **H contour does not encloses zero**

General First-Order System:1 pole or 1 zero

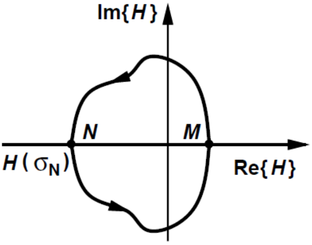


- **S contour and H contour has opposite direction for one pole**
- **They are symmetric for one zero**

General First-Order System:1 pole and 1 zero



(a)

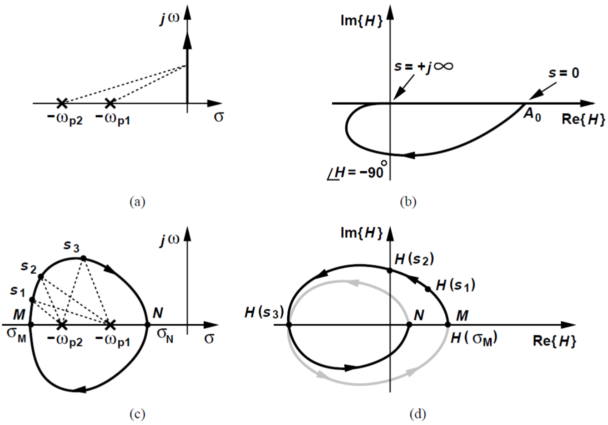


(b)

- **S contour encloses only one pole or one zero**
- **H contour encloses the origin**

System with two poles

$$H(s) = A_0[(1+s/\omega_{p1})(1+s/\omega_{p2})]^{-1}$$



- $|H(s)|$ falls from 0 to infinity
- $\angle H(s)$ from 0 to -90 degrees and then to -180 degrees
- S encloses both poles, H contour encloses origin twice

Cauchy's Principle

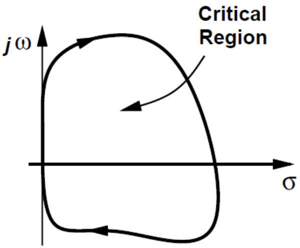
- If s contour encircles P poles and Z zeros of H(s) in the clockwise direction, then the polar plot of H(s) encircles the origin Z-P times in the same direction.
- If it has a negative number, then H(s) encircles in the counter direction.

  - Cauchy's Principle of Argument
- If we manage to draw the polar plot of the transfer function and find it encircles the origin clockwise N times. Thus, in s contour Z = N + P.

Nyquist's Method

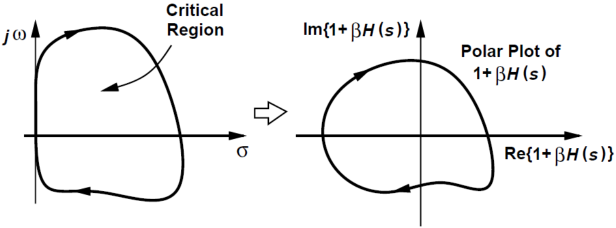
- **Nyquist's stability analysis** $\frac{Y}{X}(s) = \frac{H(s)}{1+\beta H(s)}$.
  - A negative-feedback system becomes unstable if it has any poles on the jw axis or in the RHP

  (critical region)
  - The same as if $1+\beta H(s)$ **has any zeros**



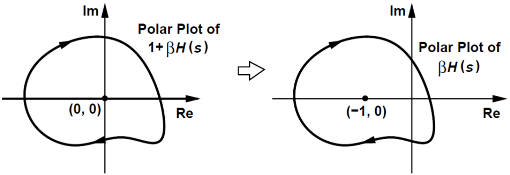
Nyquist's Theorem(1)

- How to determine whether $1+\beta H(s)$ has any zeros?
- (1)independently determine P
- (2)draw the polar plot of $1+\beta H(s)$ as s traverses the contour shown below
- (3)determine the number of times N. that encircles the origin clockwise
- (4)find P+N as the number of zeros in critical region
- Poles of H(s) or $1+\beta H(s)$ are usually none. Since the open-loop system is stable

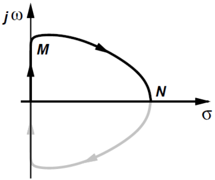


Nyquist's Theorem(2)

- For a closed-looped system $H(s)/[1+\beta H(s)]$, the polar plot of $\beta H(s)$ must not encircle the point (-1,0) clockwise as s traverses a contour around the critical region clockwise.

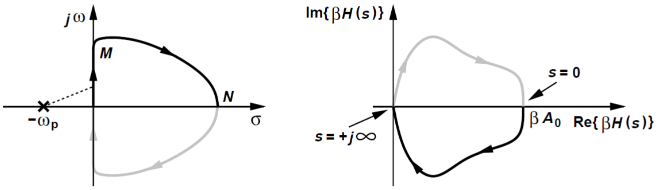


- Choose t[illegible]ematical labor
  - may go through (-1,0) when zeros are on this axis
  - the result from M to N is good enough due to symmetry



Example 10.9

Study the closed-loop stability if $H(s) = A_0/(1 + s/\omega_{p1})$



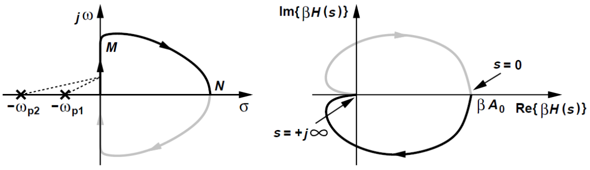
(a) (b)

Solution:

For the s contour shown in the left. $\beta H(s)$ begins at βA_0 for s=0. As s=jw moves upwards, the phase becomes more negative. At $+j\infty$, the phase goes to -90 degrees and the magnitude drops to zero. From M to N, only the phase of $\beta H(s)$ changes. We reflect this plot around real axis. Since the H contour does not encircle (-1,0), the closed-loop system is stable.

Example 10.10

Study the closed-loop stability if $H(s) = A_0/[(1 + s/\omega_{p1})(1 + s/\omega_{p2})]$



(a) (b)

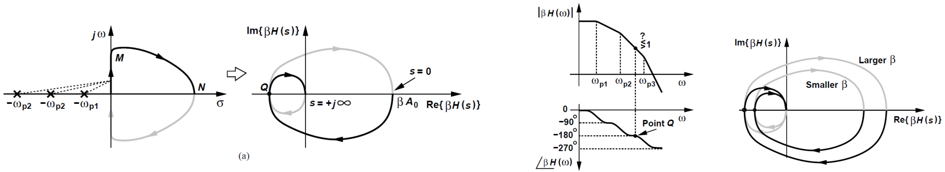
Solution:

As s=0, $\beta H(s) = \beta A_0$**. As s=jw moves upward, the phase is negative. At** $s = +j\infty$**, phase is -180 degrees and** βA_0 **magnitude falls to zero. From M to N, H(s) remains at zero. Since it does not encloses (-1,0), the system is stable.**

Example 10.11

Study the closed-loop stability if

$H(s) = A_0/[(1+s/\omega_{p1})(1+s/\omega_{p2})(1+s/\omega_{p3})]$.

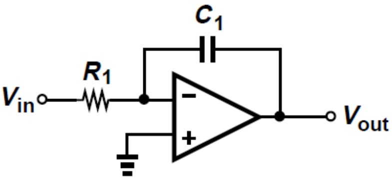


(a) (b) (c)

Solution:

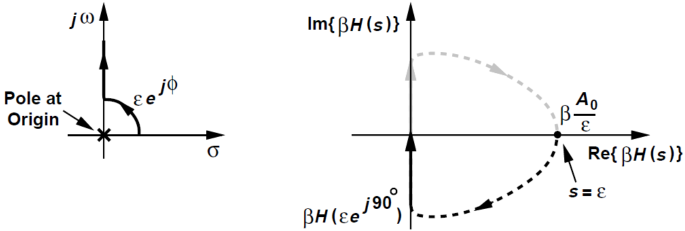
As s=0, $\beta H(s) = \beta A_0$. As s=jw moves upward, the phase is negative. At $s = +j\infty$, phase is +90 degrees and magnitude falls to zero. From M to N, H(s) remains at zero. It could encircle(-1,0) if at the point of Q $\tan^{-1}(\omega_Q/\omega_{p1}) + \tan^{-1}(\omega_Q/\omega_{p2}) + \tan^{-1}(\omega_Q/\omega_{p3}) = 180°$ and $|\beta H(s = j\omega_Q)|$ on the left of (-1,0). Higher β leads to unstable.

Systems with Poles at Origin(1)



- Some open-loop system contain one or more poles at the origin. i.e. an integrator $H(s) = \frac{-1}{R_1 C_1 s}$
- Nyquist stability analysis choose a different s contour that does not go through origin
- Travel on an infinitesimally small circle around zero

Systems with Poles at Origin(2)



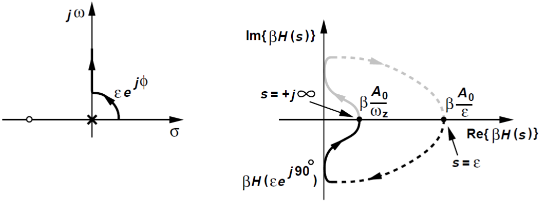
- **If $H(s) = A_0/s$ and we choose $s = \epsilon \exp(j\phi)$, then**
- **$\beta H(s) = \beta(A_0/\epsilon)\exp(-j\phi)$. At $\phi = 0,\ s = \epsilon$, and βH is real and very large . As ϕ rotates to +90 degree , the H phase towards -90 degree while radius is large. Then s travels upwards on the jw axis and BH falls. Then it remains at (0,0). Note that the system is stable since the polar plot does not encircle (-1,0).**

Example 10.12

Analyze the closed-loop stability of $H(s) = A_0(1 + s/\omega_z)/s$
The zero can be created, for example, by inserting a Resistor in series with C1.

Solution:

With $s = \epsilon \exp(j\phi)$, $\beta H(s) = \beta(A_0/\epsilon)\exp(-j\phi)$ because a small ϵ. At $\phi = 90^\circ$, the zero contributes negligible phase. As we travel upwards to $s = +j\infty$, the function approaches to $\beta A_0/\omega_z$. It deflects away from the origin.

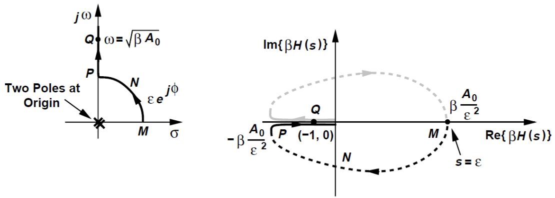


Example 10.13

A negative–feedback loop employs two ideal integrators, i.e., $H(s) = A_0/s^2$. Study the closed-loop stability of the system.

Solution:

With $s = \epsilon \exp(j\phi)$, $\phi = 0$, and hence $\beta H = \beta A_0/\epsilon^2$. As ϕ goes to +45 degrees, $\beta H(s) = \beta(A_0/\epsilon^2)\exp(-2j\phi)$ still at a very large radius. For $\phi = +90^\circ$, it returns to the real axis. As s travels at the imaginary axis, $\beta|H(j\omega)| = \beta A_0/\omega^2$. The contour passes (-1,0) at $\omega = \sqrt{\beta A_0}$. The closed-loop system therefore contains poles on the jw axis



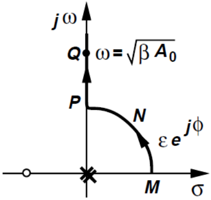
Example 10.14

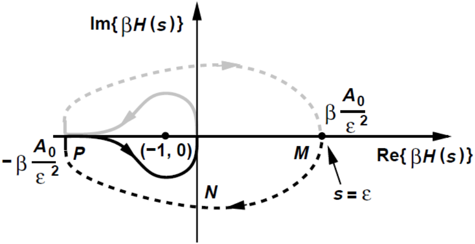
Repeat the previous example if a zero is added to one of the integrators , i.e., H(s) = $A_0(1+s/\omega_z)/s^2$

Solution:

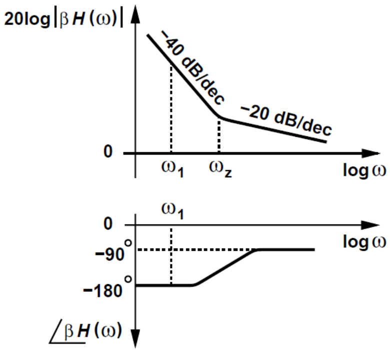
The behavior of BH is similar to that in the previous example. But as s=+jw travels upward, the zero begins to contribute appreciable phase and $|\beta H(j\omega)| = \beta A_0\sqrt{1+\omega^2/\omega_z^2}/\omega^2$

The zero ensures that BH does not cross or encircle (-1,0), stabilizing the closed-loop system.



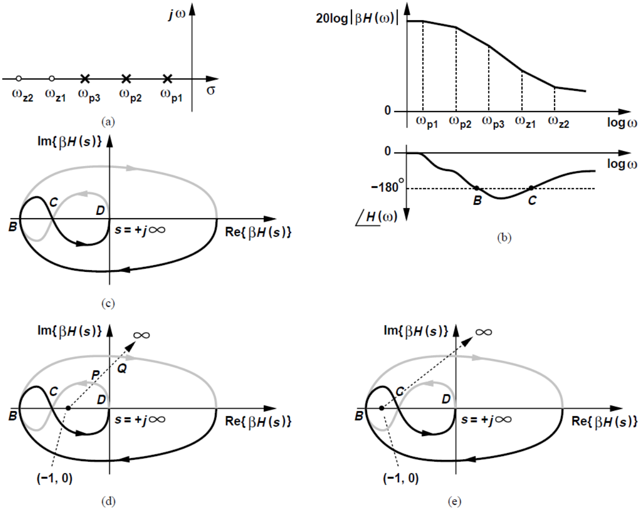


Paradox from Bode Plots



- It appears that at frequency ω_1, at which $|\beta H|$ is greater than unity and $\angle\beta H = -180°$ the circuit is capable of oscillation
- From Nyquist plot, $\beta H(j\omega_1) = -\beta A_0(1 + j\omega_1/\omega_z)/\omega_1^2$ owing to zero, the phase of BH never reach 180 degree.

Systems with Multiple 180 degree Crossings



- **If $\angle\beta H$ crosses 180 degree an even(odd) number of times while $|H|>1$ then the system is stable(unstable).**