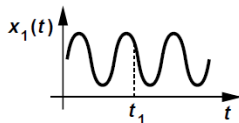
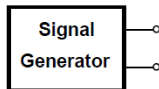


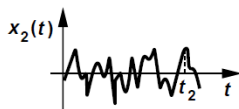
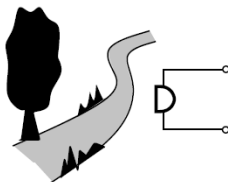
## ***Chapter 7: Noise***

- 7.1 Statistical Characteristics of Noise**
- 7.2 Types of Noise**
- 7.3 Representation of Noise in Circuits**
- 7.4 Noise in Single-Stage Amplifiers**
- 7.5 Noise in Current Mirrors**
- 7.6 Noise in Differential Pairs**
- 7.7 Noise-Power Tradeoff**
- 7.8 Noise Bandwidth**
- 7.9 Problem of Input Noise Integration**

# Statistical Characteristics of Noise

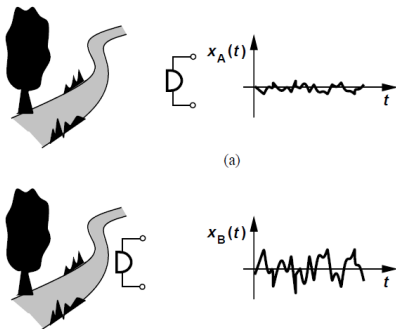


(a)



- Noise is a random process
- Value of  $x_1(t_1)$  can be predicted from observed waveform, that of  $x_2(t_2)$  cannot
- Difference between deterministic and random phenomena
- Instantaneous value of noise in time domain is unpredictable

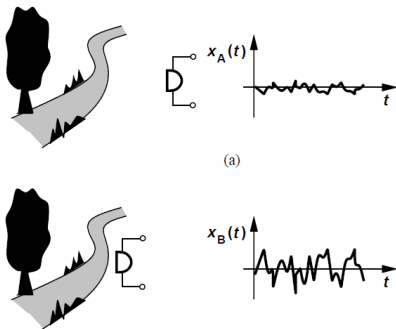
# Statistical Characteristics of Noise



- Need for a “statistical model” for noise
- Average power of noise is predictable
- Applicable to most sources of noise in circuits
- Average power delivered by a periodic voltage  $v(t)$  with period  $T$  to a load resistance  $RL$  is defined as

$$P_{av} = \frac{1}{T} \int_{-T/2}^{+T/2} \frac{v^2(t)}{R_L} dt$$

# Statistical Characteristics of Noise

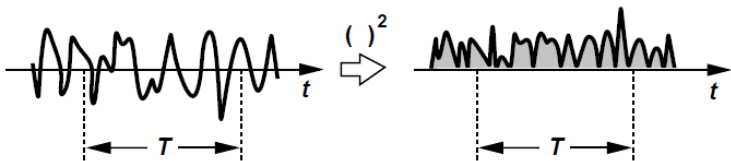


- For a random signal (aperiodic), measurement must be carried out over a long time

$$P_{av} = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{+T/2} \frac{x^2(t)}{R_L} dt$$

- $x(t)$  is a voltage quantity
- $x_A(t)$  delivers more power to a resistive load than  $x_B(t)$

# Statistical Characteristics of Noise



- To calculate average power of (noise) signal  $x(t)$ 
  - Square the signal
  - Find area under resulting waveform for a long period  $T$
  - Normalize area to  $T$
- For simpler calculations,  $P_{av}$  is defined as

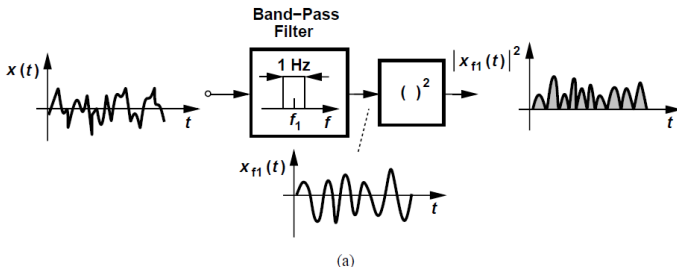
$$P_{av} = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{+T/2} x^2(t) dt$$

- $P_{av}$  is expressed in  $\text{W}$  rather than  $\text{V}^2$
- RMS voltage for noise can be defined as

$$\sqrt{P_{av}}$$

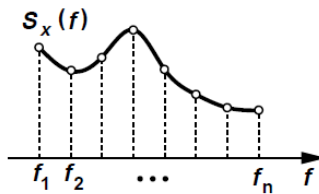
# Noise Spectrum

- Spectrum describes the *frequency content* of noise
- Also called Power Spectral Density (PSD)
- Shows how much power signal carries at each frequency
- PSD  $S_x(f)$  of a noise waveform  $x(t)$  is defined as the average power carried by  $x(t)$  in a 1-Hz bandwidth around  $f$
- Fig. (a) shows calculation of  $S_x(f_1)$ , i.e., power contained in a specific frequency  $f_1$



# Noise Spectrum

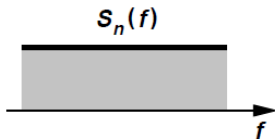
- Repeating previous procedure with bandpass filters with different center frequencies, the overall PSD  $S_x(f)$  can be constructed [Fig. (b)]
- $S_x(f)$  Represents the power carried by signal (or noise) at all frequencies, i.e., total power
- Generally measured in Watts per Hertz
- In signal processing theory, PSD is defined as Fourier transform of Autocorrelation function
- Two definitions are equivalent



(b)

# Noise Spectrum

- As with  $P_{av}$ , it is customary to eliminate  $RL$  from  $S_x(f)$
- $S_x(f)$  is expressed in  $V^2/Hz$  rather than  $W/Hz$
- Also common to take the square root of  $S_x(f)$ , expressing result in  $V/\sqrt{Hz}$
- Common type of noise PSD is “white noise”
  - Displays same value at all frequencies
- White noise does not exist strictly speaking since total power carried by noise cannot be infinite
- Noise spectrum that is flat *in the band of interest* is usually called white





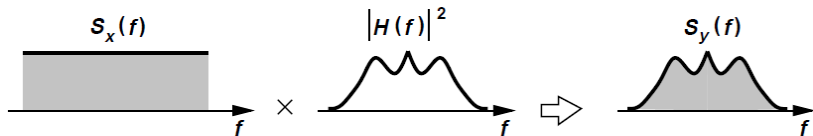
# Theorem

- If a signal with spectrum  $S_x(f)$  is applied to a linear time-invariant (LTI) system with transfer function  $H(s)$ , then the output spectrum  $S_Y(f)$  is given by

$$S_Y(f) = S_x(f)|H(f)|^2$$

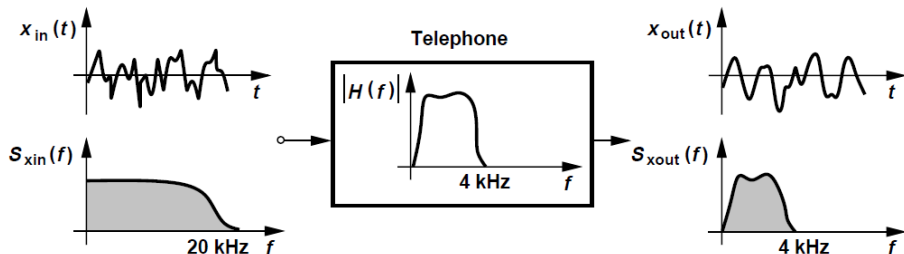
where  $H(f) = H(s = j2\pi f)$

- Spectrum of signal is “shaped” by the transfer function of the system (see Fig. below)



## Theorem: Example

- Regular telephones have a bandwidth of approximately 4kHz and suppress higher frequency components in caller's voice
- Due to limited bandwidth,  $x_{out}(t)$  exhibits slower changes than  $x_{in}(t)$
- Can be difficult to recognize the caller's voice

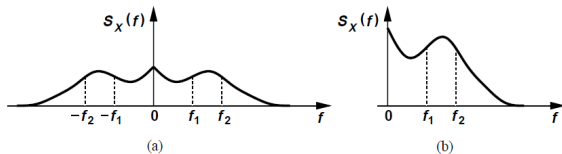


## Two-sided and one-sided spectra

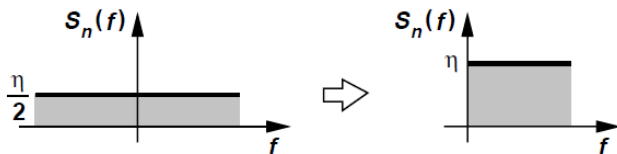
- $S_x(f)$  is an even function of  $f$  for real  $x(t)$
- The total power carried by  $x(t)$  in the frequency range  $[f_1, f_2]$  is equal to

$$\begin{aligned} P_{f_1, f_2} &= \int_{-f_2}^{-f_1} S_x(f) df + \int_{+f_1}^{+f_2} S_x(f) df \\ &= \int_{+f_1}^{+f_2} 2S_x(f) df. \end{aligned}$$

- Negative-frequency part of the spectrum is folded around the vertical axis and added to the positive-frequency part
- Fig. (a) is called “two-sided” spectrum and Fig. (b) is called “one-sided” spectrum



## Two-sided and one-sided spectra



- Two-sided white spectrum can be folded around the vertical axis to give a one-sided white spectrum as shown above
- Spectrum shows the power carried in a small bandwidth at each frequency, revealing how *fast* the waveform is expected to vary in the time domain

# Amplitude Distribution

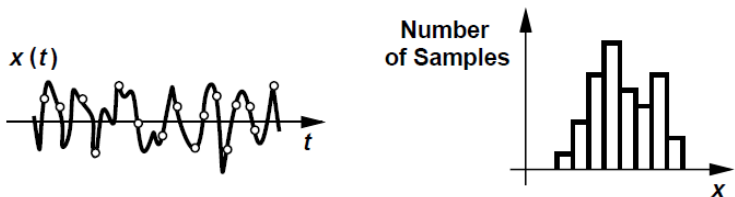
- Although instantaneous value of noise is unpredictable, it is possible to construct a “distribution” of the amplitude by observing the noise waveform for a long time
- The distribution indicates how often each value occurs
- Also called the “Probability Density Function” (PDF), the distribution of  $x(t)$  is defined as

$$p_X(x)dx = \text{probability of } x < X < x + dx$$

where  $X$  is the measured value of  $x(t)$  at some point in time

# Amplitude Distribution

- To estimate distribution, we sample  $x(t)$  at many points, construct bins of small width, choose bin height equal to number of samples whose value falls between two edges of the bin and normalize bin heights to total number of samples
- Fig. below shows an example amplitude distribution



- PDF provides no information as to how fast  $x(t)$  varies in the time domain, i.e., the frequency content of  $x(t)$

# Central Limit Theorem

- The Central Limit Theorem states that if many independent random processes with arbitrary PDFs are added, the PDF of the sum approaches a Gaussian distribution
- Many natural phenomena exhibit Gaussian statistics
  - E.g., noise of a resistor that results from the random “walk” of a very large number of electrons, each with independent statistics
- The Gaussian PDF is defined as

$$p_X(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp \frac{-(x - m)^2}{2\sigma^2}$$

- $\sigma$  and  $m$  are the standard deviation and mean of the distribution respectively
- For a Gaussian distribution,  $\sigma$  is equal to the rms value of the noise

# Correlated and Uncorrelated Sources

- The Superposition Principle is applicable to deterministic voltages and currents, but not very suitable for random noise signals
- Average noise power is of more interest
- Consider the sum of two noise waveforms  $x_1(t)$  and  $x_2(t)$  and take the average of the resulting power

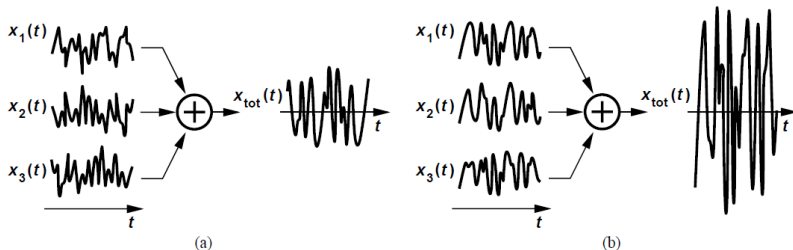
$$\begin{aligned}P_{av} &= \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{+T/2} [x_1(t) + x_2(t)]^2 dt \\&= \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{+T/2} x_1^2(t) dt + \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{+T/2} x_2^2(t) dt \\&\quad + \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{+T/2} 2x_1(t)x_2(t) dt\end{aligned}$$

- $P_{av1}$  and  $P_{av2}$  respectively  $= P_{av1} + P_{av2} + \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{+T/2} 2x_1(t)x_2(t) dt$ ,  $x_1(t)$  and  $x_2(t)$  respectively
- Third term is called “correlation” between  $x_1(t)$  and  $x_2(t)$



# Correlated and Uncorrelated Sources

- Correlation integral indicates how “similar” the two waveforms are
- Noise waveforms generated by independent devices are uncorrelated and integral vanishes to zero
- $P_{av} = P_{av1} + P_{av2}$
- Superposition holds for the *power* of uncorrelated noise sources
- Total power of correlated noise sources [Fig. (a)] is higher than that of uncorrelated sources [Fig. (b)]



# Signal-to-Noise Ratio (SNR)

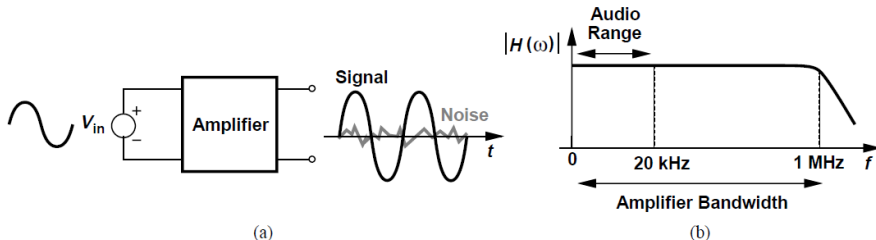
- **Signal-to-noise ratio (SNR) is defined as**

$$\text{SNR} = \frac{P_{sig}}{P_{noise}}$$

- **SNR of a noise-corrupted signal should be high for it to be intelligible**
- **Audio signals require a minimum SNR of 20 dB**
- **For a sinusoid with peak amplitude  $A$ ,  $P_{sig} = A^2/2$**
- **The total average power carried by noise is equal to the area under its spectrum**

- **$P_{noise}$  can be written as  $P_{noise} = \int_{-\infty}^{+\infty} S_{noise}(f)df$ . It spans a wide frequency range**

# Signal-to-Noise Ratio (SNR)



- Above amplifier [Fig. (a)] provides a bandwidth of 1 MHz while sensing an audio signal [Fig. (b)]
- Signal is corrupted by all noise components in the 1-MHz bandwidth
- SNR may be unacceptably low
- Bandwidth of circuit must be limited to the minimum acceptable value to minimize total integrated noise power
- Within the amplifier or by a series low-pass filter

# Noise Analysis Procedure

- **Output signal of a given circuit is corrupted by noise sources within the circuit**
  - **Interested in noise observed at the output**
- **Four steps:**
  - **Identify the sources of noise and note the spectrum of each**
  - **Find the transfer function from each noise source to the output**
  - **Use the theorem  $S_Y(f) = S_x(f)|H(f)|^2$  to calculate output noise spectrum contributed by each noise source**
  - **Add all the output spectra, accounting for correlated and uncorrelated sources**
- **Integrate the output noise spectrum from  $-\infty$  to  $+\infty$  to get total output noise power**

# Types of Noise

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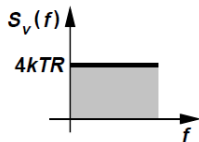
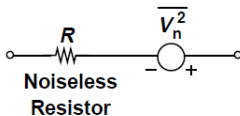
- **Analog signals processed by integrated circuits are corrupted by two types of noise**
  - **Device electronic noise**
  - **“Environmental” noise (Chapter 19)**

# Resistor Thermal Noise

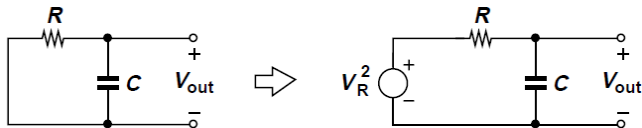
- Random motion of electrons in a conductor induces fluctuations in the voltage measured across it even though the average current is zero
- Thermal noise of a resistor  $R$  can be modeled by a series voltage source, with one-sided spectral density

$$S_v(f) = 4kTR, \quad f \geq 0$$

- Here,  $k = 1.38 \times 10^{-23} \text{ J/K}$  is the Boltzmann constant
- $S_v(f)$  is expressed in  $\text{V}^2/\text{Hz}$ , we also write  $\overline{V_n^2} = 4kTR \Delta f$
- For a 50- $\Omega$  resistor at  $T = 300 \text{ K}$ , thermal noise is  $0.23 \times 10^{-19} \text{ V}^2/\text{Hz}$ , or  $0.91 \text{ nV}/\sqrt{\text{Hz}}$
- $S_v(f)$  is flat up to 100 THz, and is “white” for our purposes



## Resistor Thermal Noise: Example



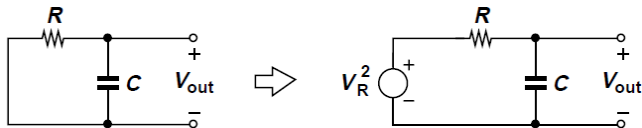
- **To find:** Noise spectrum and total noise power in  $V_{out}$
- **Solution:** Noise spectrum of  $R$  is given by  $S_v(f) = 4kTR$
- **Modeling noise by a series voltage source  $V_R$ , transfer function from  $V_R$  to  $V_{out}$  is**

$$\frac{V_{out}}{V_R}(s) = \frac{1}{RCs + 1}$$

- **Using theorem, noise spectrum at the output  $S_{out}(f)$  is**

$$\begin{aligned} S_{out}(f) &= S_v(f) \left| \frac{V_{out}}{V_R}(j\omega) \right|^2 \\ &= 4kTR \frac{1}{4\pi^2 R^2 C^2 f^2 + 1} \end{aligned}$$

## Resistor Thermal Noise: Example



- White noise spectrum of the resistor is shaped by a low-pass characteristic
- Total noise power at the output is

$$P_{n,out} = \int_0^{\infty} \frac{4kTR}{4\pi^2 R^2 C^2 f^2 + 1} df$$

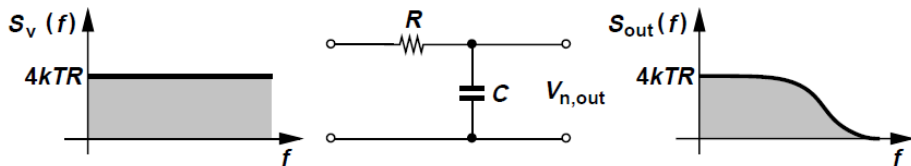
- The integral reduces to

$$\begin{aligned} P_{n,out} &= \frac{2kT}{\pi C} \tan^{-1} u \Big|_{u=0}^{u=\infty} \\ &= \frac{kT}{C} \end{aligned}$$

- The unit of  $P_{n,out}$  is  $\text{V}^2$ , which may be considered as the total rms voltage measured at the output



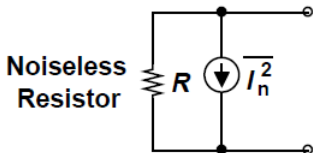
# Resistor Thermal Noise: Example



- The  $RC$  low-pass filter shapes the noise spectrum of the resistor
- Total noise at the output (area under  $S_{out}(f)$ ) is independent of the resistance  $R$
- Intuitively, this is because for larger values of  $R$ , noise per unit bandwidth increases but the overall bandwidth of the circuit decreases
- $kT/C$  noise can only be decreased by increasing  $R$  (if  $T$  is fixed)

# Resistor Thermal Noise

- Thermal noise of a resistor can be represented by a parallel current source too

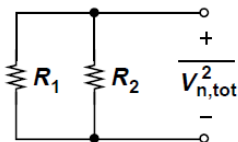


- This representation is equivalent to series voltage source representation with

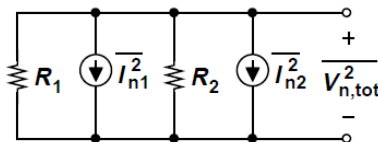
$$\overline{I_n^2} = 4kT/R$$

- $\overline{I_n^2}$  is expressed in  $A^2/Hz$
- $\overline{I_n^2}$  pending on circuit topology, one model may lead to simpler calculations than the other

## Resistor Thermal Noise: Example



(a)

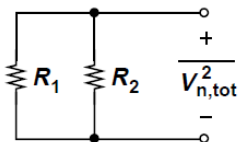


(b)

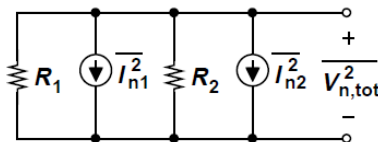
- **To find:** Equivalent noise voltage of two resistors in parallel,  $R_1$  and  $R_2$
- **Solution:**
- Each resistor exhibits equivalent noise current with spectral density  $4kT/R$
- Since the two noise sources are uncorrelated, we add the powers

$$\begin{aligned}\overline{I_{n,tot}^2} &= \overline{I_{n1}^2} + \overline{I_{n2}^2} \\ &= 4kT \left( \frac{1}{R_1} + \frac{1}{R_2} \right)\end{aligned}$$

## Resistor Thermal Noise: Example



(a)



(b)

- Equivalent noise voltage is given by

$$\begin{aligned}\overline{V_{n,tot}^2} &= \overline{I_{n,tot}^2} (R_1 || R_2)^2 \\ &= 4kT (R_1 || R_2),\end{aligned}$$

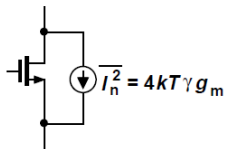
- This notation assumes a 1-Hz bandwidth

# MOSFET Thermal Noise

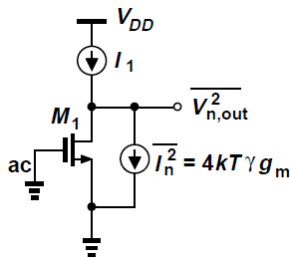
- MOS transistors exhibit thermal noise with the most significant source being the noise generated in the channel
- For long-channel MOS devices operating in saturation, the channel noise can be modeled by a current source connected between the drain and source terminals with a spectral density

$$\overline{I_n^2} = 4kT\gamma g_m$$

- The coefficient ' $\gamma$ ' (not the body effect coefficient) is derived to be 2/3 for long-channel transistors and is higher for submicron MOSFETs
- As a rule of thumb, assume  $\gamma = 1$



# MOSFET Thermal Noise: Example

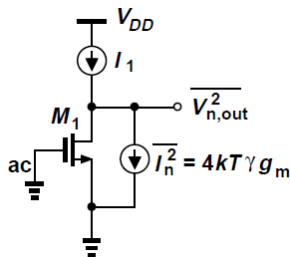


- The maximum output noise occurs if the transistor sees only its own output impedance as the load, i.e., if the external load is an ideal current source
- Output noise voltage spectrum is given by

$$S_{out}(f) = S_{in}(f)|H(f)|^2$$

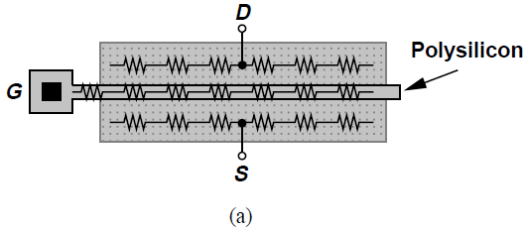
$$\begin{aligned}\overline{V_n^2} &= \overline{I_n^2} r_O^2 \\ &= (4kT\gamma g_m) r_O^2\end{aligned}$$

# MOSFET Thermal Noise: Example



- Noise current of a MOS transistor decreases if the transconductance drops
- Noise measured at the output of the circuit does not depend on where the input terminal is because input is set to zero for noise calculation
- The output resistance  $r_O$  does not produce noise because it is not a physical resistor

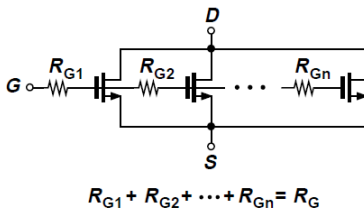
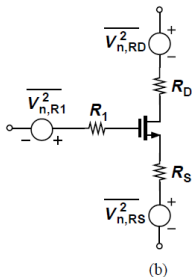
# MOSFET Thermal Noise



- Ohmic sections of a MOSFET have a finite resistivity and exhibit thermal noise [Fig. (a)]
- For a wide transistor, source and drain resistance is negligible whereas the gate distributes resistance may become noticeable

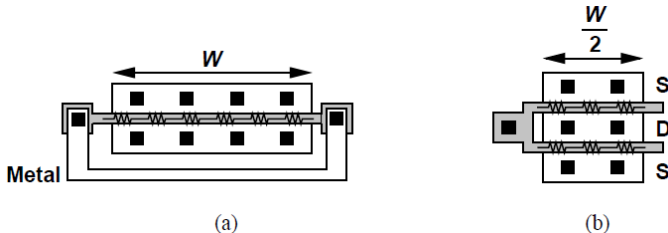


# MOSFET Thermal Noise



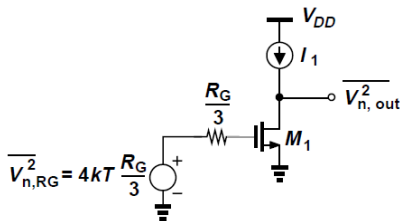
- In the noise model [Fig. (b)], the lumped resistance  $R_1$  represents the distributed gate resistance
- In the distributed structure of Fig. (c), unit transistors near the left end see the noise of only a fraction of  $R_G$  whereas those near the right end see the noise of most of  $R_G$
- It can be proved that  $R_1 = R_G/3$  and hence the noise generated by gate resistance is  $\overline{V_{nRG}^2} = 4kT R_G/3$

# MOSFET Thermal Noise



- Effect of  $R_G$  can be reduced by proper layout
- In Fig. (a), the two ends of the gate are shorted by a metal line, reducing the distributed resistance from  $R_G$  to  $R_G/4$
- Alternatively, the transistor can be folded as in Fig. (b) so that each gate “finger” exhibits a resistance of  $R_G/2$  yielding a total distributed resistance of  $R_G/4$  for the composite transistor

# MOSFET Thermal Noise: Example



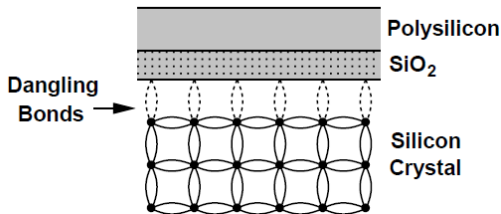
- If the total distributed gate resistance is  $R_G$ , the output noise voltage due to  $R_G$  is given by

$$\overline{V_{n,out}^2} = 4kT \frac{R_G}{3} (g_m r_O)^2$$

- For the gate resistance noise to be negligible, we must ensure

$$\frac{R_G}{3} \ll \frac{\gamma}{g_m}$$

# Flicker Noise



- **At the interface between the gate oxide and silicon substrate, many “dangling” bonds appear, giving rise to extra energy states**
- **Charge carriers moving at the interface are randomly trapped and later released by such energy states, introducing “flicker” noise in the drain current**
- **Other mechanisms in addition to trapping are believed to generate flicker noise**

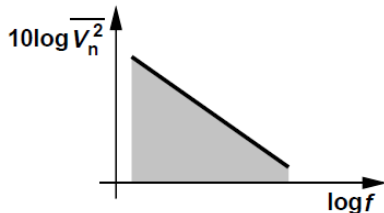
## Flicker Noise

- Average power of flicker noise cannot be predicted easily
- It varies depending on cleanness of oxide-silicon interface and from one CMOS technology to another
- Flicker noise is more easily modeled as a voltage source in series with the gate and in the saturation region, is roughly given by

$$\overline{V_n^2} = \frac{K}{C_{ox}WL} \cdot \frac{1}{f}$$

- $K$  is a process-dependent constant on the order of 10-25  $V^2F$

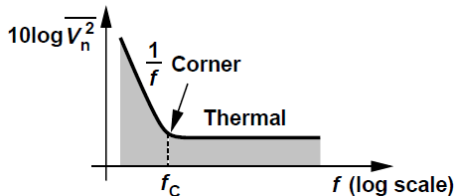
# Flicker Noise



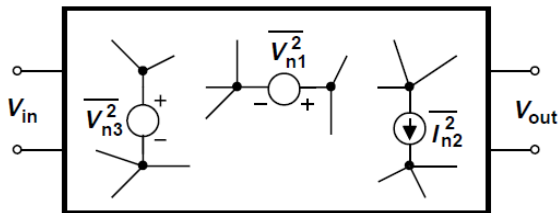
- The noise spectral density is inversely proportional to frequency
- Trap and release phenomenon occurs at low frequencies more often
- Flicker noise is also called “ $1/f$ ” noise
- To reduce  $1/f$  noise, device *area* must be increased
- Generally, PMOS devices exhibit less  $1/f$  noise than NMOS transistors
- Holes are carried in a “buried” channel, at some distance from the oxide-silicon interface

# Flicker Noise Corner Frequency

- At low frequencies, the flicker noise power approaches infinity
- At very slow rates, flicker noise becomes indistinguishable from thermal drift or aging of devices
- Noise component below the lowest frequency in the signal of interest does not corrupt it significantly
- Intersection point of thermal noise and flicker noise spectral densities is called “corner frequency”  $f_C$



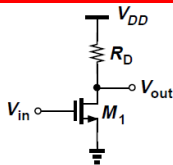
# Representation of Noise in Circuits



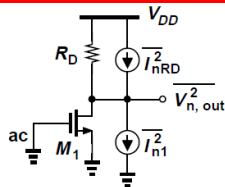
- To find the output noise, the input is set to zero and total noise is calculated at the output due to all the noise sources in the circuit
- This is how noise is measured in laboratories and in simulations



# Representation of Noise in Circuits: Example



(a)



(b)

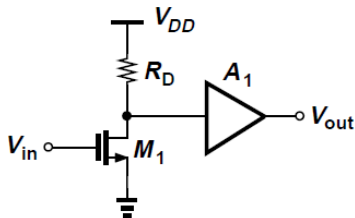
- **To find:** Total output noise voltage of the common-source stage [Fig. (a)]
- Follow noise analysis procedure described earlier
- Thermal and flicker noise of M1 and thermal noise of  $R_D$  are modeled as current sources [Fig. (b)]

- **Output noise voltage per unit bandwidth, added as power quantities is**

$$\overline{I_{n,th}^2} = 4kT\gamma g_m \quad \overline{I_{n,1/f}^2} = K g_m^2 / (C_{ox} W L f) \quad \overline{I_{n,RD}^2} = 4kT / R_D$$

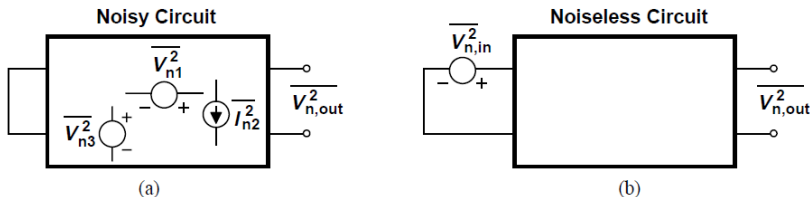
$$\overline{V_{n,out}^2} = \left( 4kT\gamma g_m + \frac{K}{C_{ox} W L} \cdot \frac{1}{f} \cdot g_m^2 + \frac{4kT}{R_D} \right) R_D^2$$

# Input-Referred Noise



- Output-referred noise does not allow a fair comparison of noise in different circuits since it depends on the gain
- In above figure, a CS stage is succeeded by a noiseless amplifier with voltage gain  $A_1$ , then the net output noise is now multiplied by  $A_1$
- This may indicate that circuit becomes noisier as  $A_1$  increases, which is incorrect since the signal level also increases proportionally, and net SNR at the output does not depend on  $A_1$

# Input-Referred Noise

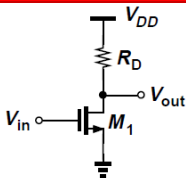


- Input-referred noise represents the effect of all noise sources in the circuit by a single source  $\overline{V_{n,in}^2}$ , at the input such that the output noise in Fig. (b) is equal to that in Fig. (a)
- If the voltage gain is  $A_v$ , then we must have

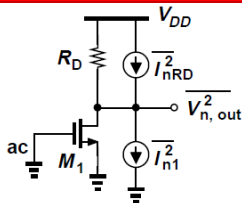
$$\overline{V_{n,out}^2} = A_v^2 \overline{V_{n,in}^2}$$

- The input-referred noise voltage in this simple case is simply the output noise divided by the gain squared.

# Input-Referred Noise: Example



(a)



(b)

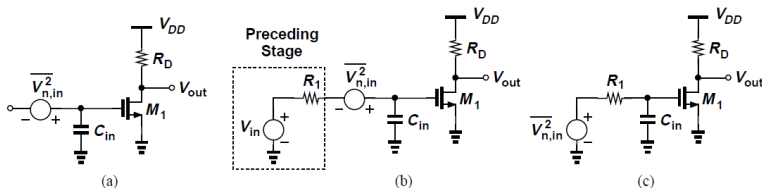
- For the simple CS stage, the input-referred noise voltage is given by

$$\begin{aligned}\overline{V_{n,in}^2} &= \frac{\overline{V_{n,out}^2}}{A_v^2} \\ &= \left( 4kT\gamma g_m + \frac{K}{C_{ox}WL} \cdot \frac{1}{f} \cdot g_m^2 + \frac{4kT}{R_D} \right) R_D^2 \frac{1}{g_m^2 R_D^2} \\ &= 4kT \frac{\gamma}{g_m} + \frac{K}{C_{ox}WL} \cdot \frac{1}{f} + \frac{4kT}{g_m^2 R_D}.\end{aligned}$$

- First and third terms combined can be viewed as thermal noise of an equivalent resistance  $RT$ , so that the equivalent input-referred thermal noise is  $4kTRT$

# Input-Referred Noise

- Single voltage source in series with the input is an incomplete representation of the input-referred noise for a circuit with a finite input impedance and driven by a finite source impedance
- For the CS stage, the input-referred noise voltage is independent of the preceding stage

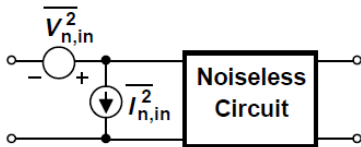


- If the preceding stage is equivalent with an output impedance of  $R_1$ , the output noise due to voltage division is

$$\begin{aligned}\overline{V_{n,out}^2} &= \overline{V_{n,in}^2} \left| \frac{1}{R_1 C_{in} j\omega + 1} \right|^2 (g_m R_D)^2 \\ &= \frac{4kT\gamma g_m R_D^2}{R_1^2 C_{in}^2 \omega^2 + 1}.\end{aligned}$$

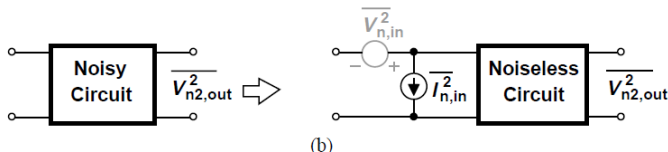
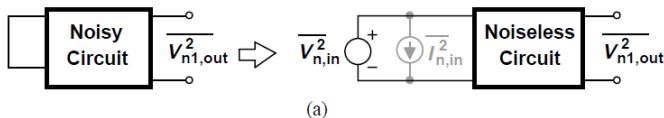
## Input-Referred Noise

- The previous result is incorrect since the output noise due to  $M1$  does not diminish as  $R1$  increases
- To solve this issue, we model the input-referred noise by both a series voltage source and a parallel current source, so that if the output impedance of the previous stage assumes large values, thereby reducing the effect of  $\overline{V_{n,in}^2}$ , the noise current still flows through the finite impedance, producing noise at the input
- It can be proved that  $\overline{V_{n,in}^2}$  and  $\overline{I_{n,in}^2}$  are necessary and sufficient to represent the noise of any linear two-port circuit



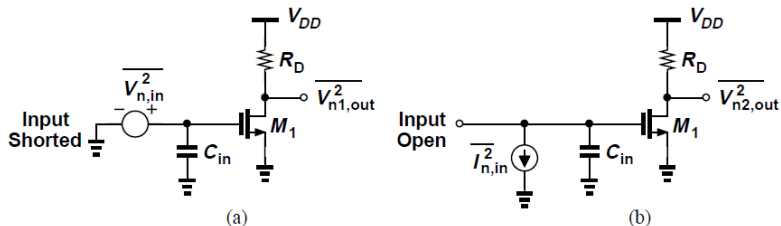
# Input-Referred Noise

- To calculate  $\overline{V_{n,in}^2}$  and  $\overline{I_{n,in}^2}$ , two extreme cases are considered: zero and infinite source impedances



- If the source impedance is zero [Fig. (a)],  $\overline{V_{n,in}^2}$  flows through  $\overline{I_{n,in}^2}$  and has no effect at the output. The output noise  $\overline{V_{n1,out}^2}$  measured arises solely from  $\overline{I_{n,in}^2}$ .
- If the input is left open [Fig. (b)], then  $\overline{I_{n,in}^2}$  has no effect and the output noise is only due to  $\overline{V_{n,in}^2}$ .

# Input-Referred Noise: Example



- For the circuit in Fig. (a), the input-referred noise voltage is simply

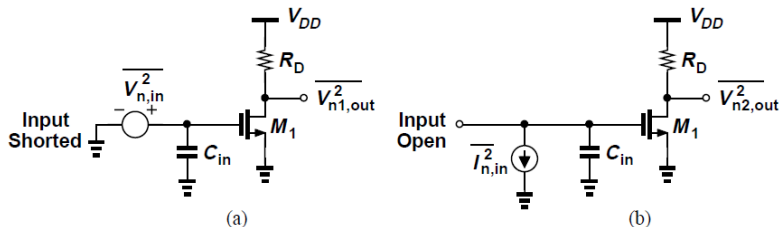
$$\overline{V_{n,in}^2} = 4kT \frac{\gamma}{g_m} + \frac{4kT}{g_m^2 R_D}$$

- To obtain the input-referred noise current, the input is left open and we find the output noise in terms of  $\overline{I_{n,in}^2}$
- The noise current flows through  $C_{in}$ , generating at the output [Fig. (b)]

$$\overline{V_{n2,out}^2} = \overline{I_{n,in}^2} \left( \frac{1}{C_{in}\omega} \right)^2 g_m^2 R_D^2$$



# Input-Referred Noise: Example



- This value must be equal to the output of the noisy circuit when the input is left open [Fig. (b)]

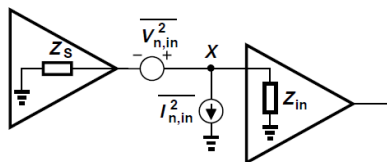
$$\overline{V_{n2,out}^2} = \left( 4kT\gamma g_m + \frac{4kT}{R_D} \right) R_D^2$$

- Thus,

$$\overline{I_{n,in}^2} = (C_{in}\omega)^2 \frac{4kT}{g_m^2} \left( \gamma g_m + \frac{1}{R_D} \right)$$

# Input-Referred Noise

- The input noise current  $I_{n,in}$  becomes significant for low enough values of the input impedance  $Z_{in}$



- In above figure,  $Z_S$  denotes the output impedance of the preceding circuit; total noise voltage sensed by the second stage at node X is

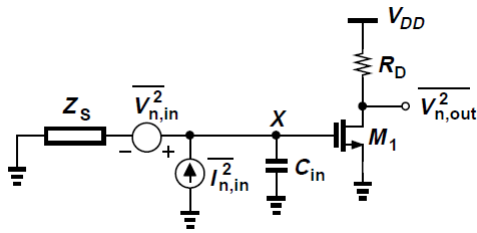
$$V_{n,X} = \frac{Z_{in}}{Z_{in} + Z_S} V_{n,in} + \frac{Z_{in} Z_S}{Z_{in} + Z_S} I_{n,in}$$

- If  $\frac{Z_{in}}{Z_{in} + Z_S} \approx 1$ , the effect of  $I_{n,in}$  is negligible
- If  $|I_{n,in}|^2 |Z_S|^2 \ll \overline{V_{n,in}^2}$ , the effect of  $I_{n,in}$  can be neglected if

$$|Z_S|^2 \ll \frac{\overline{V_{n,in}^2}}{\overline{I_{n,in}^2}}$$

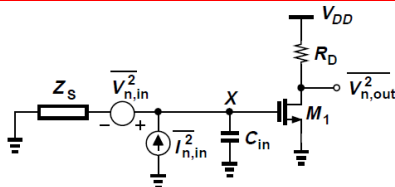
# Input-Referred Noise: Correlation

- Input-referred noise voltages and currents may be correlated
- Noise calculations must include correlations between the two
- Use of both a voltage source and a current source to represent the input-referred noise does not “count the noise twice”



- It can be p...irect for  
any source impedance  $Z_S$ , with both  $\overline{V_{n,i}^2}$  and  $\overline{I_{n,i}^2}$   
included

# Input-Referred Noise: Correlation

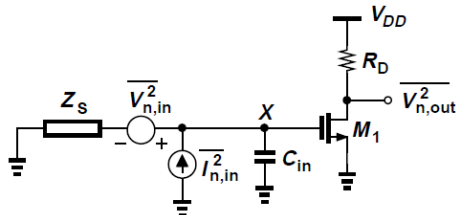


- Assuming  $Z_S$  is noiseless for simplicity, we first calculate the total noise voltage at the gate of  $M_1$  due to  $\overline{V_{n,in}^2}$  and  $\overline{I_{n,in}^2}$
- Cannot apply superposition of powers since they are correlated, but can be applied to voltages and currents since the circuit is linear and time-invariant
- Therefore, if  $V_{n,M1}$  denotes the gate-referred noise voltage of  $M_1$  and  $V_{n,RD}$  the noise voltage of  $R_D$ , then

$$V_{n,in} = V_{n,M1} + \frac{1}{g_m R_D} V_{n,RD}$$

$$I_{n,in} = C_{in} s V_{n,M1} + \frac{C_{in} s}{g_m R_D} V_{n,RD}$$

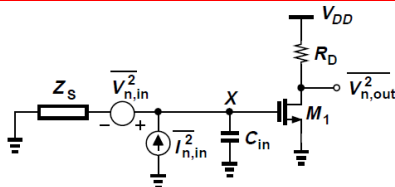
# Input-Referred Noise: Correlation



- $V_{n,M1}$  and  $V_{n,RD}$  appear in both  $V_{n,in}$  and  $I_{n,in}$ , creating a strong correlation between the two
- Adding the contributions of  $V_{n,in}$  and  $I_{n,in}$  at node  $X$ , as if they were deterministic quantities, we have

$$\begin{aligned}
 V_{n,X} &= V_{n,in} \frac{\frac{1}{C_{in}s}}{\frac{1}{C_{in}s} + Z_S} + I_{n,in} \frac{\frac{Z_S}{C_{in}s}}{\frac{1}{C_{in}s} + Z_S} \\
 &= \frac{V_{n,in} + I_{n,in} Z_S}{Z_S C_{in}s + 1}.
 \end{aligned}$$

# Input-Referred Noise: Correlation



- Substituting for  $V_{n,in}$  and  $I_{n,in}$

$$\begin{aligned} V_{n,X} &= \frac{1}{Z_S C_{in} s + 1} \left[ V_{n,M1} + \frac{1}{g_m R_D} V_{n,RD} + C_{in} s Z_S \left( V_{n,M1} + \frac{1}{g_m R_D} V_{n,RD} \right) \right] \\ &= V_{n,M1} + \frac{1}{g_m R_D} V_{n,RD}. \end{aligned}$$

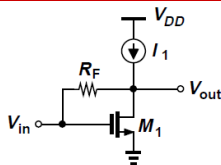
- $V_{n,X}$  is independent of  $Z_S$  and  $C_{in}$
- It follows that  $\overline{V_{n,out}^2} = g_m^2 R_D^2 \overline{V_{n,X}^2}$   

$$= 4kT \left( \gamma g_m + \frac{1}{R_D} \right) R_D^2$$
- $V_{n,in}$  and  $I_{n,in}$  do not double count the noise

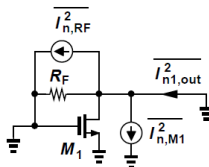
## Input-Referred Noise: Correlation

- In some cases, it is simpler to consider the output short-circuit noise *current* rather than the output noise voltage
- This current is then multiplied by the circuit's output resistance to yield the output noise voltage or simply divided by a proper gain to give input-referred quantities

# Input-Referred Noise: Correlation (Example)



(a)



(b)

- **To find:** Input-referred noise voltage and current. Assume  $I_1$  is noiseless and  $\lambda=0$
- To compute the input-referred noise voltage, we short the input port [Fig. (b)]. Here, it is also possible to short the output port and hence

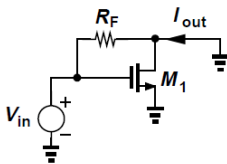
$$\overline{I_{n1,out}^2} = \frac{4kT}{R_F} + 4kT\gamma g_m$$

- The output impedance of the circuit with the input shorted is simply  $R_F$ , therefore,

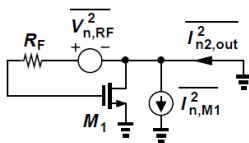
$$\overline{V_{n1,out}^2} = \left( \frac{4kT}{R_F} + 4kT\gamma g_m \right) R_F^2$$



# Input-Referred Noise: Correlation (Example)



(c)



(d)

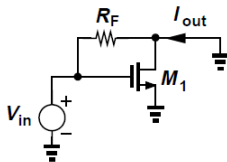
- Input-referred noise voltage can be found by dividing previous equation by voltage gain or  $\overline{I_{n1,out}^2}$  by  $G_m$
- As shown in Fig. (c),

$$\begin{aligned} G_m &= \frac{I_{out}}{V_{in}} \\ &= g_m - \frac{1}{R_F} \end{aligned}$$

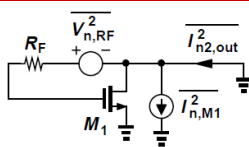
- Therefore,

$$\overline{V_{n,in}^2} = \frac{\frac{4kT}{R_F} + 4kT\gamma g_m}{\left(g_m - \frac{1}{R_F}\right)^2}$$

# Input-Referred Noise: Correlation (Example)



(c)



(d)

- To find the input-referred noise current [Fig. (d)], we find the output noise current with the input left open

$$\overline{I_{n2,out}^2} = 4kT R_F g_m^2 + 4kT \gamma g_m$$

- Next, we need to find the current gain of the circuit according to the arrangement in Fig. (c)
- Since,  $V_{GS} = I_{in} R_F$  and  $I_D = g_m I_{in} R_F$ ,

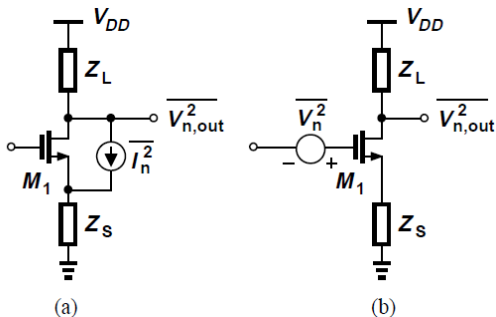
$$I_{out} = g_m R_F I_{in} - I_{in}$$

- Thus,

$$= (g_m R_F - 1) I_{in}$$

$$\overline{I_{n,in}^2} = \frac{4kT R_F g_m^2 + 4kT \gamma g_m}{(g_m R_F - 1)^2}$$

# Noise in Single-Stage Amplifiers: Lemma



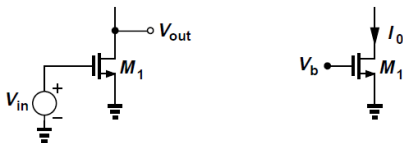
- **Lemma:** The circuits in Fig. (a) and (b) are equivalent at low frequencies if  $\overline{V_n^2} = \overline{I_n^2}/g_m^2$  and the circuits are driven by a finite impedance
- The noise source can be transformed from a drain-source current to a gate series voltage for arbitrary  $Z_S$

## Common-Source Stage

- From a previous example, the input-referred noise voltage of a simple CS stage was found to be

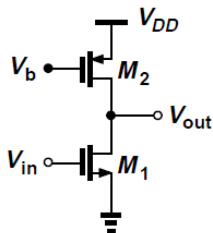
$$\overline{V_{n,in}^2} = 4kT \left( \frac{\gamma}{g_m} + \frac{1}{g_m^2 R_D} \right) + \frac{K}{C_{ox} W L f}$$

- From above expression,  $4kTy/gm$  is the thermal noise current expressed as a voltage in series with the gate

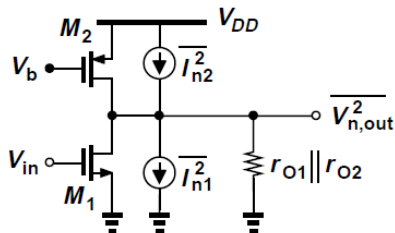


- To reduce input-referred noise voltage, transconductance must be maximized if the transistor is to amplify a voltage signal applied to its gate [Fig. (a)] whereas it must be minimized if operating as a current source [Fig. (b)]

# Common-Source Stage: Example



(a)

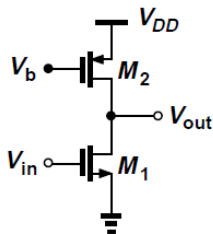


(b)

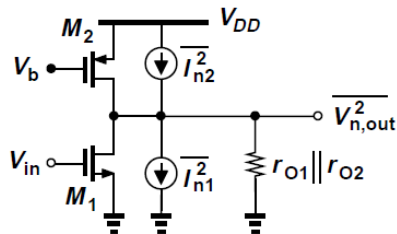
- **To find:** Input-referred thermal noise, total output thermal noise with a load capacitance  $CL$
- **Representing the thermal noise of  $M1$  and  $M2$  by current sources and noting that they are uncorrelated,**

$$\overline{V_{n,out}^2} = 4kT(\gamma g_{m1} + \gamma g_{m2})(r_{O1} \parallel r_{O2})^2$$

# Common-Source Stage: Example



(a)



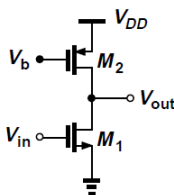
(b)

- Since the voltage gain is equal to  $g_{m1}(r_{O1} \parallel r_{O2})$ , total noise voltage referred to the gate of **M1** is

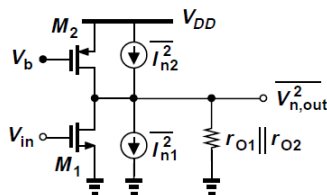
$$\begin{aligned} \overline{V_{n,in}^2} &= 4kT(\gamma g_{m1} + \gamma g_{m2}) \frac{1}{g_{m1}^2} \\ &= 4kT\gamma \left( \frac{1}{g_{m1}} + \frac{g_{m2}}{g_{m1}^2} \right). \end{aligned}$$

- Thus,  $g_{m2}$  must be minimized because **M2** serves as a current source rather than a transconductor

# Common-Source Stage: Example



(a)



(b)

- **Total output noise is**

$$\overline{V_{n,out,tot}^2} = \int_0^\infty 4kT\gamma(g_{m1} + g_{m2})(r_{O1}||r_{O2})^2 \frac{df}{1 + (r_{O1}||r_{O2})^2 C_L^2 (2\pi f)^2}$$

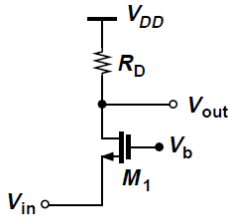
$$\overline{V_{n,out,tot}^2} = \gamma(g_{m1} + g_{m2})(r_{O1}||r_{O2}) \frac{kT}{C_L}$$

- **A low-frequency input sinusoid of amplitude  $V_m$  yields an output amplitude equal to  $g_{m1}(r_{O1}||r_{O2})V_m$  and output SNR of**

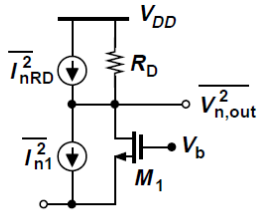
$$SNR_{out} = \left[ \frac{g_{m1}(r_{O1}||r_{O2})V_m}{\sqrt{2}} \right]^2 \cdot \frac{1}{\gamma(g_{m1} + g_{m2})(r_{O1}||r_{O2})(kT/C_L)}$$

$$= \frac{C_L}{2\gamma kT} \cdot \frac{g_{m1}^2(r_{O1}||r_{O2})}{g_{m1} + g_{m2}} V_m^2$$

# Common-Gate Stage: Thermal Noise



(a)

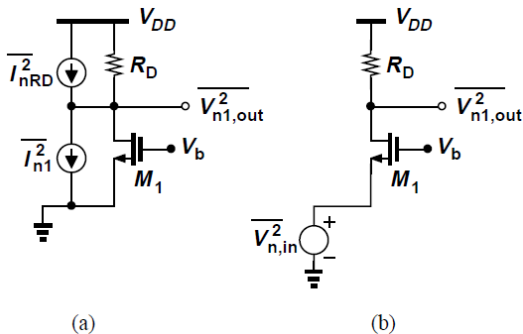


(b)

- Neglecting channel-length modulation, we represent the thermal noise of  $M_1$  and  $R_D$  by two current sources [Fig. (b)]
- Due to low input impedance of the circuit, the input-referred noise current is not negligible even at low frequencies



# Common-Gate Stage: Thermal Noise

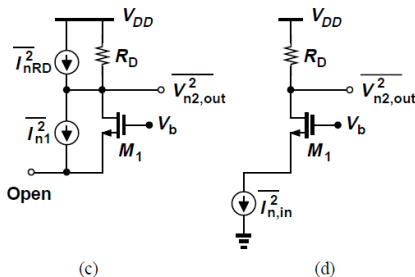


- To calculate the input-referred noise voltage, we short the input to ground and equate the output noises of the circuits in Figs. (a) and (b)

$$\left(4kT\gamma g_m + \frac{4kT}{R_D}\right) R_D^2 = \overline{V_{n,in}^2} (g_m + g_{mb})^2 R_D^2$$

$$\overline{V_{n,in}^2} = \frac{4kT(\gamma g_m + 1/R_D)}{(g_m + g_{mb})^2}$$

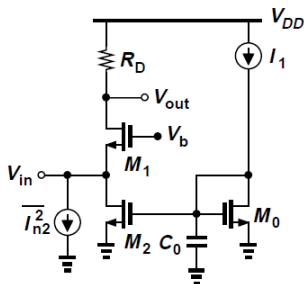
# Common-Gate Stage: Thermal Noise



- To calculate the input-referred noise current, we equate the output noises of the circuits in Figs. (c) and (d)
- $I_{n1}$  produces no noise at the output since the sum of the currents at the source of  $M1$  is zero
- The output noise voltage is therefore  $4kTR_D$  and hence  $\overline{I_{n,in}^2} R_D^2 = 4kTR_D$
- Thus,

$$\overline{I_{n,in}^2} = \frac{4kT}{R_D}$$

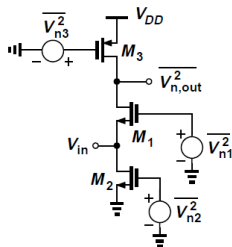
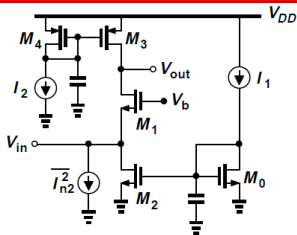
# Common-Gate Stage: Thermal Noise



- Bias current source in the common-gate stage also contributes thermal noise
- Current mirror arrangement establishes bias current of  $M1$  as a multiple of  $I1$

- If input is shorted to ground, drain noise current of  $M2$  does not contribute to input-referred noise voltage
- If input is open, all of  $\overline{I_{n2}^2}$  flows from  $M1$  and  $RD$ , producing an output noise of  $\overline{I_{n2}^2} R_D^2$  and hence an input-referred noise current of  $\overline{I_{n2}^2}$ .
- It is desirable to minimize transconductance of  $M2$ , at the cost of reduced output swing

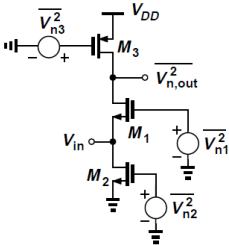
# Common-Gate Stage: Flicker Noise



- To compute input-referred  $1/f$  noise voltage and current of above circuit, each  $1/f$  noise generator is modeled by a voltage source in series with the gate of the corresponding transistor ( $1/f$  noise of  $M0$  and  $M4$  is neglected)
- With the input shorted to ground, we have

$$\overline{V_{n1,out}^2} = \frac{1}{C_{owf}} \left[ \frac{g_{m1}^2 K_N}{(WL)_1} + \frac{g_{m3}^2 K_P}{(WL)_3} \right] (r_{O1} || r_{O3})^2$$

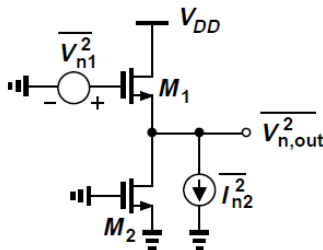
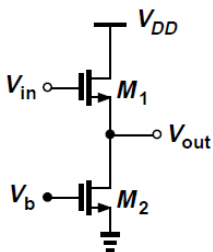
- $K_P$  and  $K_N$  denote flicker noise coefficients of NMOS and PMOS devices respectively



- **It follows that**

$$\overline{I_{n,in}^2} = \frac{1}{C_{oxf}} \left[ \frac{g_{m2}^2 K_N}{(WL)_2} + \frac{g_{m3}^2 K_P}{(WL)_3} \right]$$

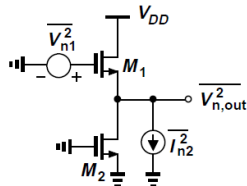
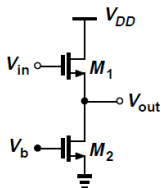
# Source Followers: Thermal Noise



- Since the input impedance of the source follower is quite high, the input-referred noise current can be neglected for moderate driving source impedances
- To compute the input-referred noise voltage, the output noise of  $M2$  can be expressed as

$$\overline{V_{n,out}^2}|_{M2} = \overline{I_{n2}^2} \left( \frac{1}{g_{m1}} \parallel \frac{1}{g_{mb1}} \parallel r_{O1} \parallel r_{O2} \right)^2$$

# Source Followers: Thermal Noise



- The voltage gain is (from Chapter 3),

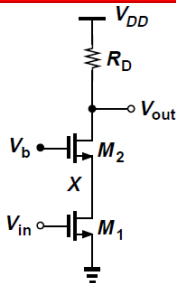
$$A_v = \frac{\frac{1}{g_{mb1}} \parallel r_{O1} \parallel r_{O2}}{\frac{1}{g_{mb1}} \parallel r_{O1} \parallel r_{O2} + \frac{1}{g_{m1}}}$$

- Total input-referred noise voltage is

$$\begin{aligned} \overline{V_{n,in}^2} &= \overline{V_{n1}^2} + \frac{\overline{V_{n,out}^2}|_{M2}}{A_v^2} \\ &= 4kT\gamma \left( \frac{1}{g_{m1}} + \frac{g_{m2}}{g_{m1}^2} \right) \end{aligned}$$

- Source followers provide a voltage gain less than unity

# Cascode Stage



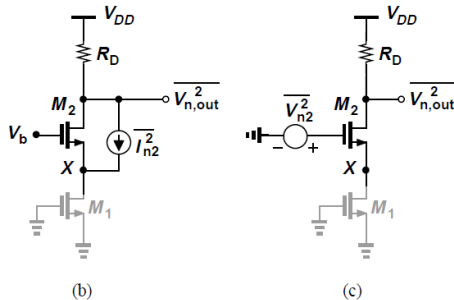
(a)

- In the cascode stage of Fig. (a), the noise currents of *M1* and *RD* flow mostly through *RD* at low frequencies
- Noise contributed by *M1* and *RD* is quantified in a common-source stage

$$\overline{V_{n,in}^2}|_{M1,RD} = 4kT \left( \frac{\gamma}{g_{m1}} + \frac{1}{g_{m1}^2 R_D} \right)$$

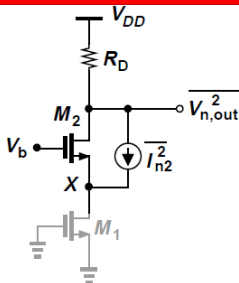


# Cascode Stage

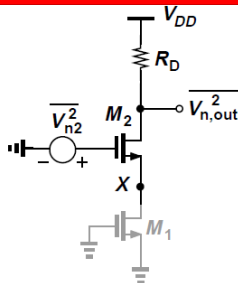


- As shown in Fig. (b),  $M2$  contributes negligibly to noise at the output, especially at low frequencies
- If channel-length modulation of  $M1$  is neglected, then  $I_{n2} + I_{D2} = 0$  and hence  $M2$  does not affect  $V_{n,out}$
- From another perspective, in the equivalent circuit of Fig. (c), voltage gain from  $V_{n2}$  to the output is small if impedance at node  $X$  is large

# Cascode Stage



(b)



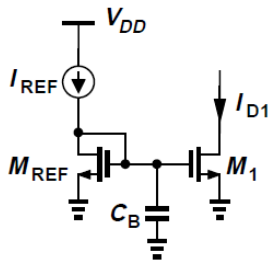
(c)

- At high frequencies, the total capacitance at node X,  $C_X$  gives rise to a gain, increasing the output noise

$$\frac{V_{n,out}}{V_{n2}} \approx \frac{-R_D}{1/g_{m2} + 1/(C_X s)}$$

- This capacitance also reduces the gain from the main input to the output by shunting the signal current produced by  $M1$  to ground

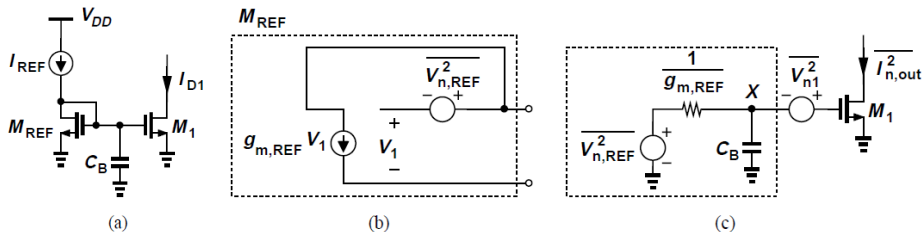
# Noise in Current Mirrors



(a)

- In the above current-mirror topology,  $(W/L)_1 = N(W/L)_{REF}$
- The factor  $N$  is in the range of 5 to 10 to minimize power consumed by reference branch
- To determine the flicker noise in  $I_{D1}$ , we assume  $\lambda = 0$  and  $I_{REF}$  is noiseless

# Noise in Current Mirrors

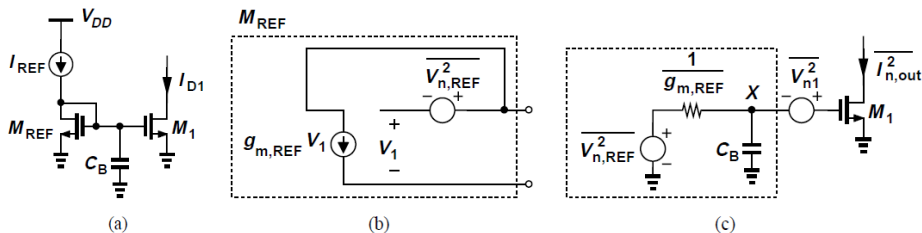


- In the Thevenin equivalent for  $M_{REF}$  and its flicker noise, the open-circuit voltage is  $V_{n,REF}$  [Fig. (b)] and the Thevenin resistance is  $1/g_{m,REF}$  [Fig. (c)]
- The noise voltage at node X and  $V_{n1}$  add and drive the gate of  $M_1$ , producing

$$\overline{I_{n,out}^2} = \left( \frac{g_{m,REF}^2}{C_B^2 \omega^2 + g_{m,REF}^2} \overline{V_{n,REF}^2} + \overline{V_{n1}^2} \right) g_{m1}^2$$

- Since  $(W/L)_1 = N(W/L)_{REF}$  and typically  $L_1 = L_{REF}$ , we observe that  $\overline{V_{n,REF}^2} = N \overline{V_{n1}^2}$

# Noise in Current Mirrors



- It follows that

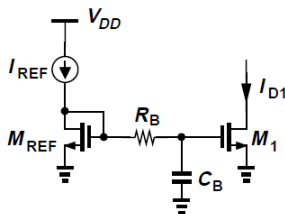
$$\overline{I_{n,out}^2} = \left( \frac{N g_{m,REF}^2}{C_B^2 \omega^2 + g_{m,REF}^2} + 1 \right) g_{m1}^2 \overline{V_{n1}^2}$$

- For the noise of the diode-connected device to be small, we need  $(N - 1)g_{m,REF}^2 \ll C_B^2 \omega^2$  and hence

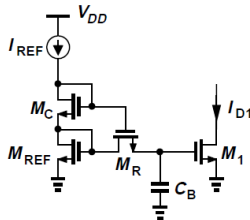
$$C_B^2 \gg \frac{(N - 1)g_{m,REF}^2}{\omega^2}$$

- This can lead to  $C_B$  being very high

# Noise in Current Mirrors



(a)



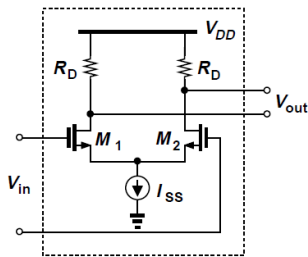
(b)

- In order to reduce noise contributed by  $M_{REF}$  and avoid a large capacitor, we can insert a resistance between its gate and  $C_B$ , so that

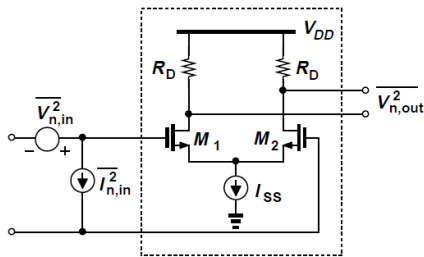
$$\overline{I_{n,out}^2} = \left[ \frac{g_{m,REF}^2}{(1 + g_{m,REF} R_B)^2 C_B^2 \omega^2 + g_{m,REF}^2} (\overline{V_{n,REF}^2} + \overline{V_{n,RB}^2}) + \overline{V_{n1}^2} \right] g_{m1}^2$$

- $R_B$  lowers the filter cutoff frequency but also contributes its own noise
- The MOS device  $M_R$  with a small overdrive provides a high resistance and occupies a moderate area [Fig. (b)]

# Noise in Differential Pairs



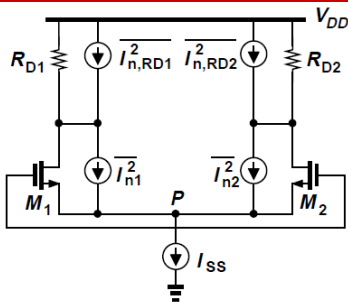
(a)



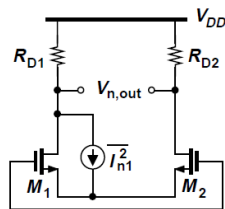
(b)

- As shown in Fig. (a), a differential pair can be viewed as a two-port circuit
- It is possible to model the overall noise as depicted in Fig. (b)
- For low-frequency operation,  $\overline{I_{n,in}^2}$  is negligible

# Noise in Differential Pairs



(a)

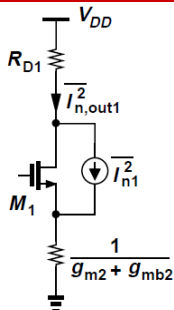


(b)

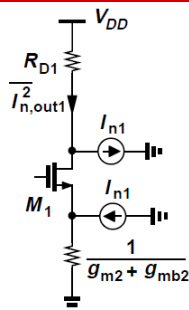
- To calculate the thermal component  $\overline{V_{n, in}^2}$ , we first obtain the total output noise with the inputs shorted together [Fig. (a)]
- Since  $I_{n1}$  and  $I_{n2}$  are uncorrelated, node  $P$  cannot be considered a virtual ground, so cannot use half-circuit concept
- Derive contribution of each source individually [Fig. (b)]



# Noise in Differential Pairs



(c)

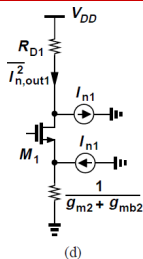
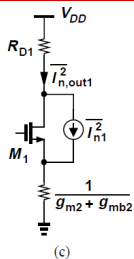


(d)

- In Fig. (c), to calculate the contribution of  $I_{n1}$ , neglecting channel-length modulation, it can be proved that half of  $I_{n1}$  flows through  $R_{D1}$  and the other half through  $M_2$  and  $R_{D2}$  [Fig. (d)]
- The differential output noise due to  $M_1$  is equal to

$$V_{n,out}|_{M1} = \frac{I_{n1}}{2} R_{D1} + \frac{I_{n1}}{2} R_{D2}$$

# Noise in Differential Pairs



- If  $RD1 = RD2 = RD$ ,

$$\overline{V_{n,out}^2}|_{M1} = \overline{I_{n1}^2} R_D^2 \quad \overline{V_{n,out}^2}|_{M2} = \overline{I_{n2}^2} R_D^2$$

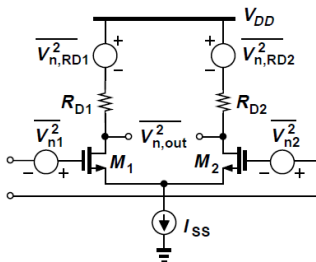
- Thus
- Taking into account the noise of  $RD1$  and  $RD2$ ,

$$\begin{aligned} \overline{V_{n,out}^2} &= (\overline{I_{n1}^2} + \overline{I_{n2}^2}) R_D^2 + 2(4kTR_D) \\ &= 8kT \left( \gamma g_m R_D^2 + R_D \right). \end{aligned}$$

- Dividing by the square of the differential gain  $g_m^2 R_D^2$ ,

$$\overline{V_{n,in}^2} = 8kT \left( \frac{\gamma}{g_m} + \frac{1}{g_m^2 R_D} \right)$$

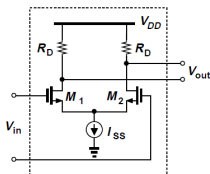
# Noise in Differential Pairs



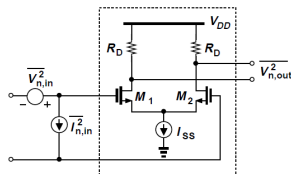
- Input-referred noise voltage can also be calculated using the previous lemma
- The noise of  $M_1$  and  $M_2$  can be modeled as a voltage source in series with their gates
- The noise of  $RD1$  and  $RD2$  is divided by  $g_m^2 R_D^2$  resulting in previously obtained equation
- Including  $1/f$  noise,

$$\overline{V_{n,in,tot}^2} = 8kT \left( \frac{\gamma}{g_m} + \frac{1}{g_m^2 R_D} \right) + \frac{2K}{C_{ox} W L f}$$

# Noise in Differential Pairs



(a)



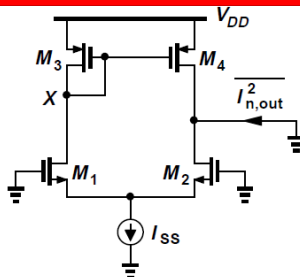
(b)

- If the differential input is zero and the circuit is symmetric, then the noise in  $I_{SS}$  divides equally between  $M1$  and  $M2$ , and produces only a common-mode noise voltage at the output
- For a small differential input  $\Delta V_{in}$ , we have

$$\begin{aligned}\Delta I_{D1} - \Delta I_{D2} &= g_{m1} \Delta V_{in} \\ &= \sqrt{2\mu_n C_{ox} \frac{W}{L} \left( \frac{I_{SS} + I_n}{2} \right)} \Delta V_{in}\end{aligned}$$

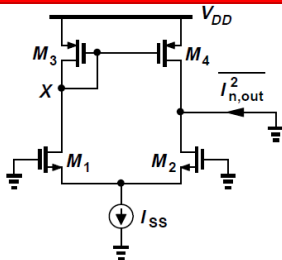
- $I_n$  denotes the noise in  $I_{SS}$  and  $I_n \ll I_{SS}$
- As circuit departs from equilibrium,  $I_n$  is more unevenly divided, generating differential output noise

# Noise in five-transistor OTA



- The Norton noise equivalent is sought by first computing the output short-circuit noise current
- This is then multiplied by the output resistance and divided by the gain to get input-referred noise voltage
- Transconductance is approximately  $gm_{1,2}$
- Output noise current due to  $M1$  and  $M2$  is this transconductance multiplied by gate-referred noises of  $M1$  and  $M2$ , i.e.,  $g_{m1,2}^2(4kT\gamma/g_{m1} + 4kT\gamma/g_{m2})$

# Noise in five-transistor OTA



- The noise current of  $M3$  primarily circulates through the diode-connected impedance  $1/g_{m3}$ , producing a voltage at the gate of  $M4$  with spectral density  $4kT\gamma/g_{m3}$

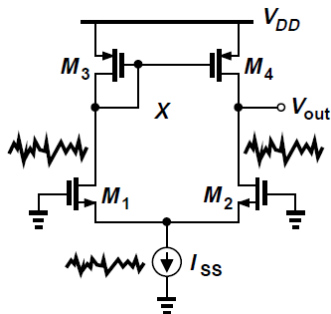
- This noise is multiplied by  $g_{m42}$  as it emerges from the drain of  $M4$ ; the noise current of  $M4$  also flows directly through the output short-circuit, thus

$$\overline{I_{n,out}^2} = 4kT\gamma(2g_{m1,2} + 2g_{m3,4})$$

- Multiplying this noise by  $R_{out}^2 \approx (r_{O1,2} || r_{O3,4})^2$  and dividing the result by  $A_v^2 = G_m^2 R_{out}^2$ , the total input-referred noise is

$$\overline{V_{n,in}^2} = 8kT\gamma \left( \frac{1}{g_{m1,2}} + \frac{g_{m3,4}}{g_{m1,2}^2} \right)$$

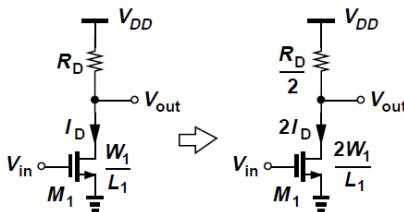
# Noise in five-transistor OTA



- The output voltage in the OTA  $V_{out}$  is equal to  $V_X$
- If  $I_{SS}$  fluctuates, so do  $V_X$  and  $V_{out}$
- Since the tail noise current  $I_n$  splits equally between  $M_1$  and  $M_2$ , the noise voltage at  $X$  is given by  $\overline{I_n^2}/(4g_{m3}^2)$  and so is the noise voltage at the output

# Noise-Power Tradeoff

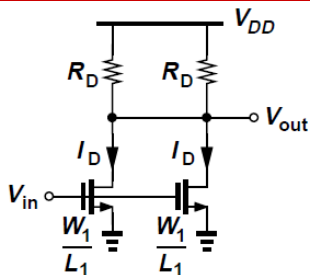
- Noise contributed by transistors “in the signal path” is inversely proportional to their transconductance
- Suggests a tradeoff between noise and power consumption



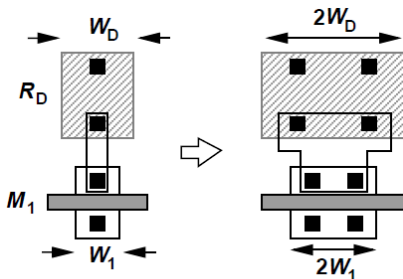
- In the simple CS stage of fig. (a), we double  $W/L$  and  $ID1$  and halve  $RD$ , maintaining voltage gain and output swing
- Input-referred thermal and flicker noise power is halved, at the cost of power consumption



# Noise-Power Tradeoff



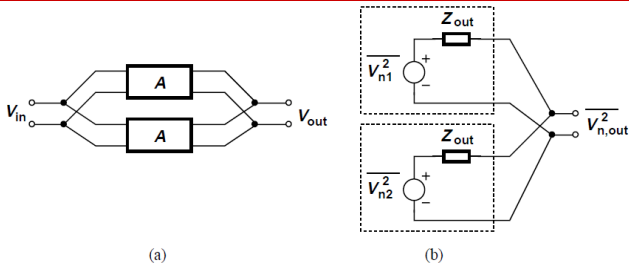
(b)



(c)

- Called “linear scaling”, the earlier transformation can be viewed as placing two instances of the original circuit in *parallel* [Fig. (b)]
- Alternatively, it can be said that the widths of the transistor and the resistor are doubled [Fig. (c)]

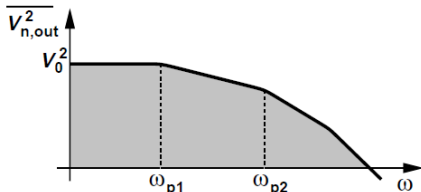
# Noise-Power Tradeoff



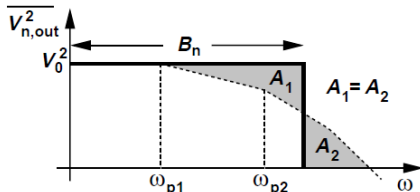
- In general, if two instances of a circuit are placed in parallel, the output noise power is halved [Fig. (a)]
- Proved by setting the input to zero and constructing a Thevenin equivalent for each [Fig. (b)]
- Since  $V_{n1,out}$  and  $V_{n2,out}$  are uncorrelated, we can use superposition of powers to write

$$\begin{aligned}\overline{V_{n,out}^2} &= \frac{\overline{V_{n1,out}^2}}{4} + \frac{\overline{V_{n2,out}^2}}{4} \\ &= \frac{\overline{V_{n1,out}^2}}{2}.\end{aligned}$$

# Noise Bandwidth



(a)



(b)

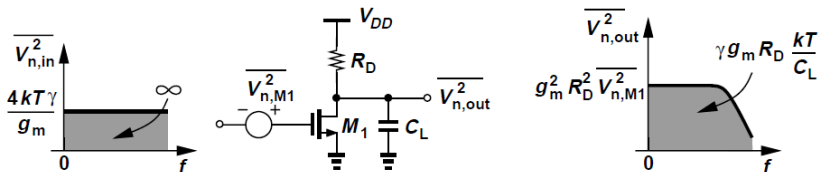
- Total noise corrupting a signal in a circuit results from all frequency components in the bandwidth of the circuit
- For a multipole system with noise spectrum as in Fig.(a), total output noise is

$$\overline{V_{n,out,tot}^2} = \int_0^{\infty} \overline{V_{n,out}^2} df.$$

- As shown in Fig.(b), the total noise can also be expressed as  $V_0^2 \cdot B_n$ , where the bandwidth  $B_n$ , called the “noise bandwidth” chosen is such that

$$V_0^2 \cdot B_n = \int_0^{\infty} \overline{V_{n,out}^2} df$$

# Problem of Input Noise Integration



- In the CS stage above, where it is assumed  $\lambda = 0$  and noise of  $R_D$  is neglected with only thermal noise of  $M_1$  considered
- Output noise spectrum is the amplified and low-pass filtered noise of  $M_1$ , easily leading to integration
- Input-referred noise voltage, however, is simply  $\overline{V_{n,M1}^2}$ , carrying an *infinite* power and prohibiting integration
- For fair comparison of different designs, we can divide the integrated output noise by the low-frequency gain, for example,

$$\overline{V_{n,in,tot}^2} = \gamma g_m R_D \frac{kT}{C_L} \cdot \frac{1}{g_m^2 R_D^2} = \frac{\gamma}{g_m R_D} \frac{kT}{C_L}$$