Chapter 8: Feedback

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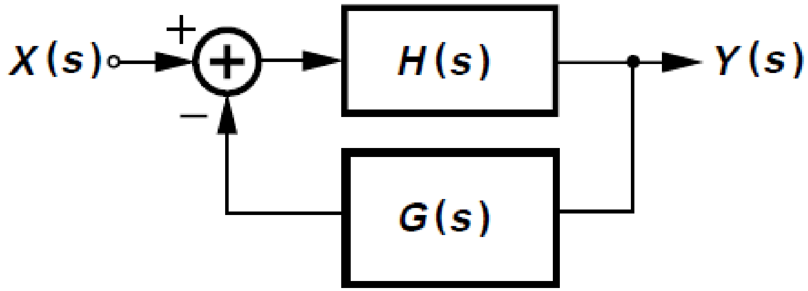
General Considerations



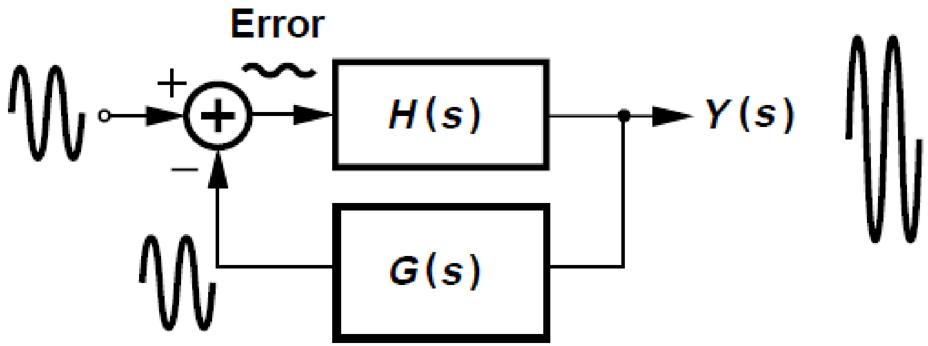
- Above figure shows a negative feedback system
- *H(s)* and *G(s)* are called the feedforward and forward networks respectively
- Feedback error is given by *X(s) – G(s)Y(s)*
- Thus

$$Y(s) = H(s)[X(s) - G(s)Y(s)]$$

$$\frac{Y(s)}{X(s)} = \frac{H(s)}{1 + G(s)H(s)}$$

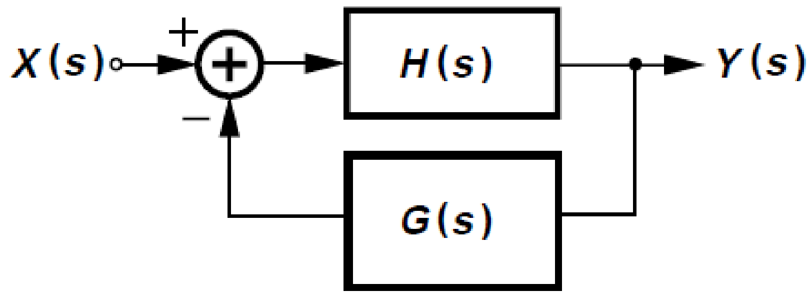
- *H(s)* is called the "open-loop" transfer function and *Y(s)/X(s)* is called the "closed-loop" transfer function

General Considerations



- In most cases, *H(s)* represents an amplifier and *G(s)* is a frequency-independent quantity
- In a well-designed negative feedback system, the error term is minimized, making the output of *G(s)* an "accurate" copy of the input and hence the output of the system a faithful (scaled) replica of the input
- *H(s)* is a "virtual ground" since the signal amplitude is small at this point
- In subsequent developments, *G(s)* is replaced by a frequency-independent quantity *β* called the *feedback factor*

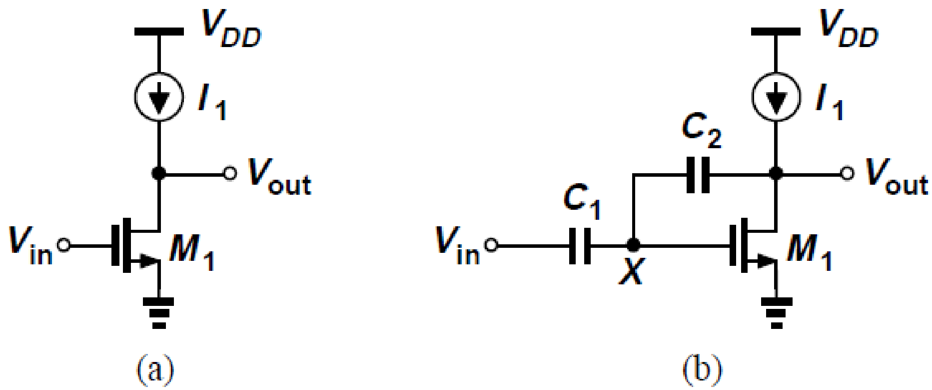
General Considerations



- Four elements of a feedback system
  - The feedforward amplifier
  - A means of sensing the output
  - The feedback network
  - A means of generating the feedback error, i.e., a subtractor (or an adder)
- These exist in every feedback system, though they may not be obvious in some cases

Properties of Feedback Circuits

- **Gain Desensitization:**

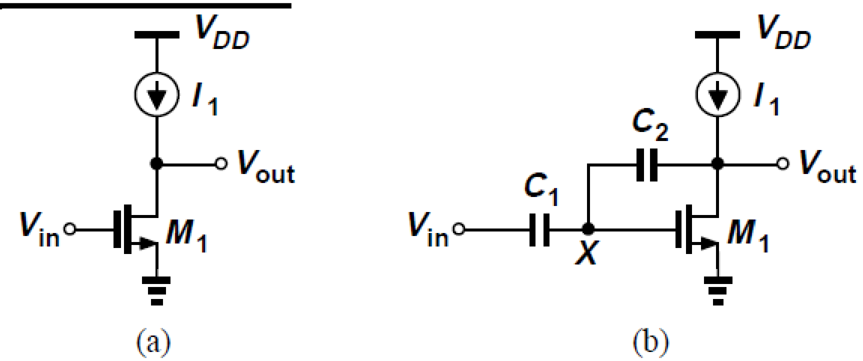


- **In Fig. (a) above, the CS stage has a gain of $g_{m1}r_{O1}$**
- **Gain is not well-defined since both g_{m1} and r_{O1} vary with process and temperature**
- **In the circuit of Fig. (b), the bias of M_1 is set by a means not shown, the overall voltage gain at low frequencies is given by**

$$\frac{V_{out}}{V_{in}} = -\frac{1}{\left(1+\frac{1}{g_{m1}r_{O1}}\right)\frac{C_2}{C_1}+\frac{1}{g_{m1}r_{O1}}}$$

Properties of Feedback Circuits

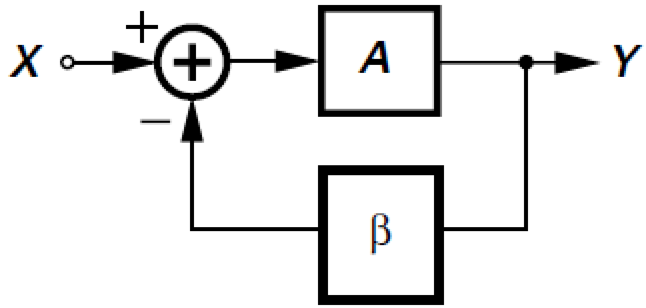
- **Gain Desensitization:**



- If $g_{m1}r_{O1}$ is sufficiently large, then

- Compared to $g_{m1}r_{O1}$, this gain can be controlled with higher accuracy since it is a *ratio* of two capacitors, relatively unaffected by process and temperature variations if C_1 and C_2 are made of the same material
- Closed-loop gain is less sensitive to device parameters than the open-loop gain, hence called "gain desensitization"

Properties of Feedback Circuits

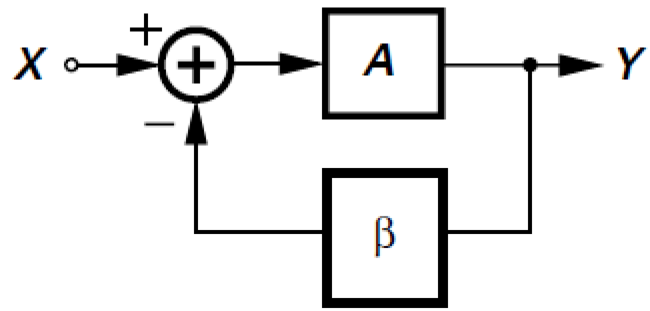


- **Frequency stability typically worsens as a result feedback**
- **For a more general case, gain desensitization is quantified by writing**

$$\frac{Y}{X} = \frac{A}{1+\beta A}$$
$$\approx \frac{1}{\beta}\left(1 - \frac{1}{\beta A}\right)$$

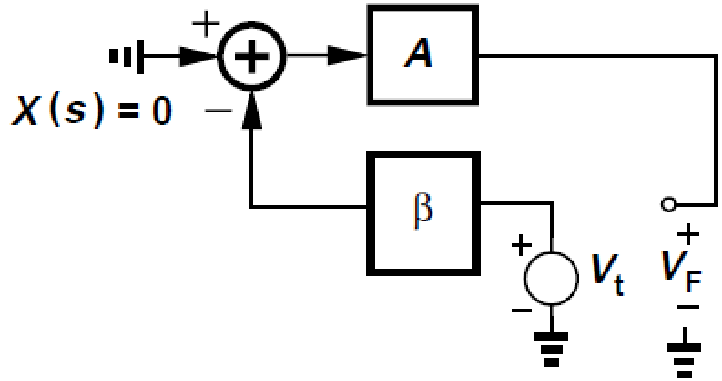
- **It is assumed *βA* >> 1; even if open-loop gain *A* varies by a factor of 2, *Y*/*X* varies by a small percentage since 1/(*βA*) << 1**

Properties of Feedback Circuits



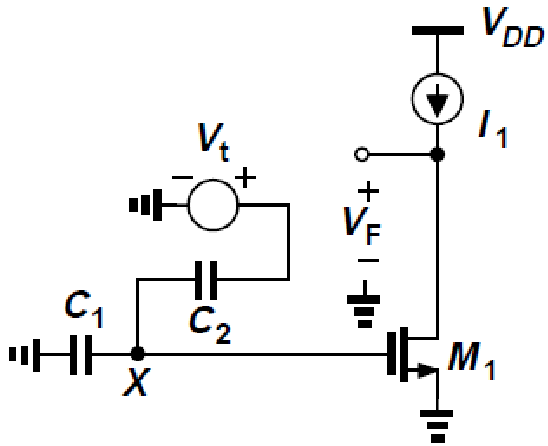
- Called the "loop gain", the quantity *βA* is important in feedback systems
- The higher *βA* is, the less sensitive *Y/X* is to variations in *A*, but closed-loop gain is reduced, i.e., tradeoff between precision and closed-loop gain
- The output of the feedback network is equal to $\beta Y = X \cdot \beta A/(1+\beta A)$ approaching *X* as *βA* becomes much greater than unity

Calculation of Loop Gain



- To calculate the loop gain:
  - Set the main input to (ac) zero
  - Inject a test signal in the "right" direction
  - Follow the signal around the loop and obtain the value that returns to the break point
  - Negative of the transfer function thus obtained is the loop gain
- Loop gain is a dimensionless quantity
- In above figure, $V_t \beta(-1) A = V_F$ and hence $V_F / V_t = -\beta A$

Calculation of Loop Gain: Example



- Applying the given procedure to find the loop gain in the circuit above, we can write

$$V_X = V_t C_2/(C_1 + C_2)$$

$$V_t \frac{C_2}{C_1 + C_2}(-g_{m1} r_{O1}) = V_F$$

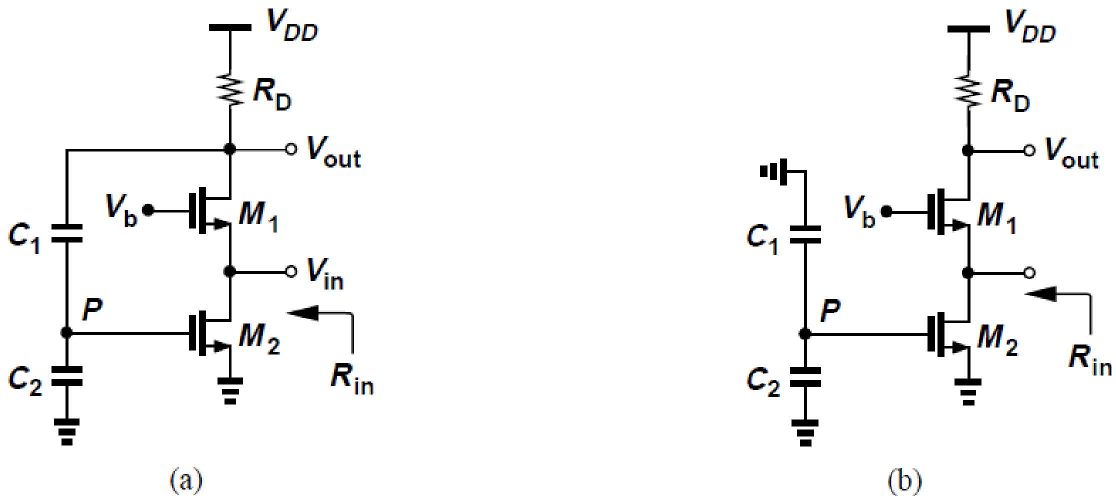
- That is,

$$\frac{V_F}{V_t} = -\frac{C_2}{C_1 + C_2} g_{m1} r_{O1}$$

- The current drawn by C_2 from the output is neglected

Properties of Feedback Circuits

- **Terminal Impedance Modification: Input Impedance**

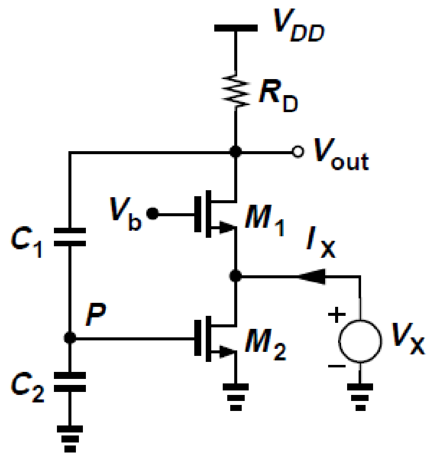


- **In the circuit of Fig. (a), a capacitive voltage divider senses the output voltage of a CG stage and applies the result to the gate of current source M_2 and hence returning a signal to the input**
- **Neglecting channel-length modulation and the current drawn by C_1 and breaking the circuit as in Fig. (b), we can write**

$$R_{in,open} = \frac{1}{g_{m1} + g_{mb1}}$$

Properties of Feedback Circuits

- **Terminal Impedance Modification: Input Impedance**



(c)

- For the closed-loop circuit of Fig. (c),

$$V_{out} = (g_{m1} + g_{mb1})V_X R_D$$

$$V_P = V_{out}\frac{C_1}{C_1 + C_2}$$

- Adding the small-signal drain currents of M_1 and M_2,

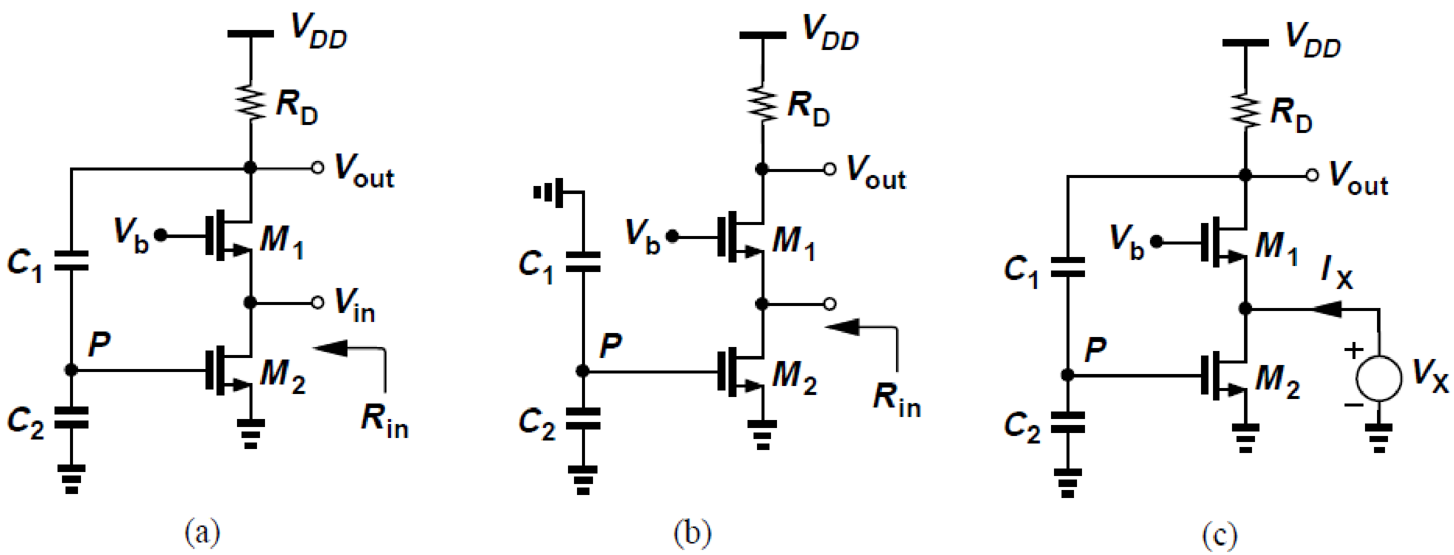
$$\begin{aligned} I_X &= (g_{m1} + g_{mb1})V_X + g_{m2}(g_{m1} + g_{mb1})\frac{C_1}{C_1 + C_2}R_D V_X \\ &= (g_{m1} + g_{mb1})\left(1 + g_{m2}R_D\frac{C_1}{C_1 + C_2}\right)V_X. \end{aligned}$$

- It follows that

$$\begin{aligned} R_{in,closed} &= V_X/I_X \\ &= \frac{1}{g_{m1} + g_{mb1}}\frac{1}{1 + g_{m2}R_D\frac{C_1}{C_1 + C_2}} \end{aligned}$$

Properties of Feedback Circuits

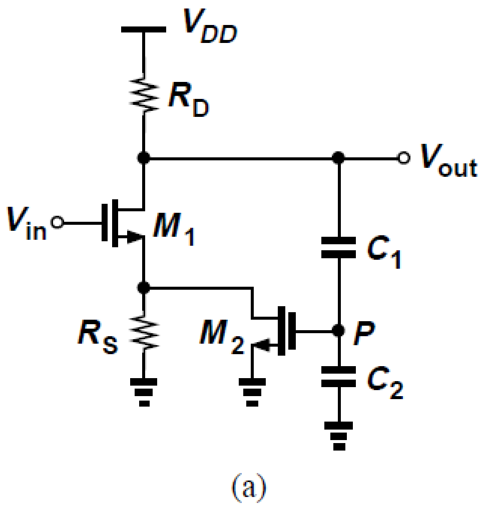
- **Terminal Impedance Modification: Input Impedance**



- **Feedback reduces the input impedance by a factor of** $1 + g_{m2}R_D C_1/(C_1 + C_2)$
- **It can be proved that** $g_{m2}R_D C_1/(C_1 + C_2)$ **is the loop gain**

Properties of Feedback Circuits

- **Terminal Impedance Modification: Output Impedance**

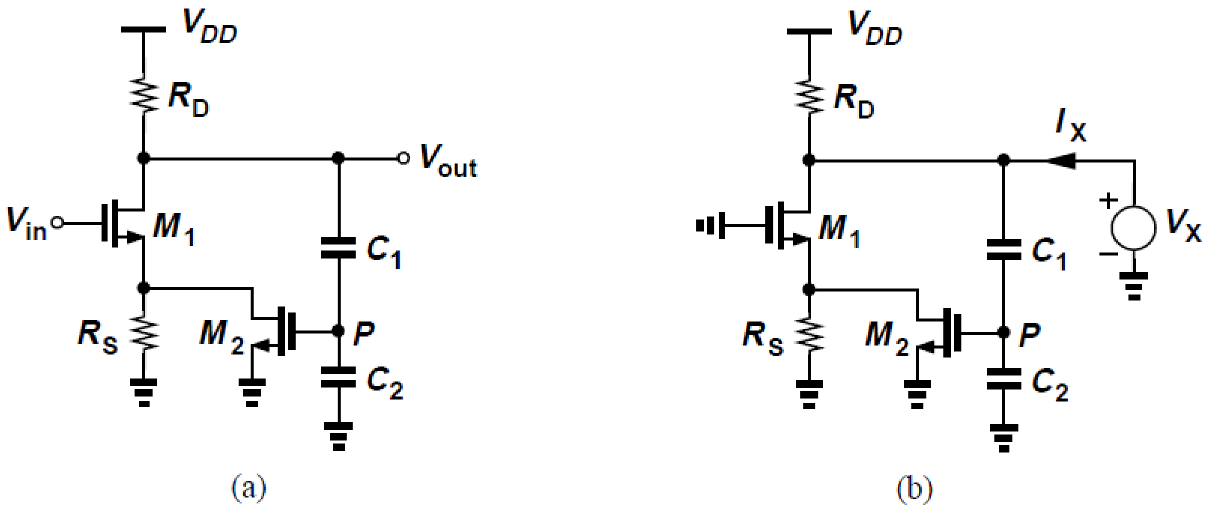


- **In the circuit of Fig. (a), *M*$_1$, *R*$_S$ and *R*$_D$ form a CS stage and *C*$_1$, *C*$_2$ and *M*$_2$ sense the output voltage, returning a current $[C_1/(C_1+C_2)]V_{out}g_{m2}$ to the source of *M*$_1$**
- **To find the output resistance at relatively low frequencies, the input is set to zero [Fig. (b)], so that**

$$I_{D1} = V_X \frac{C_1}{C_1+C_2} g_{m2} \frac{R_S}{R_S + \dfrac{1}{g_{m1}+g_{mb1}}}$$

Properties of Feedback Circuits

- **Terminal Impedance Modification: Output Impedance**



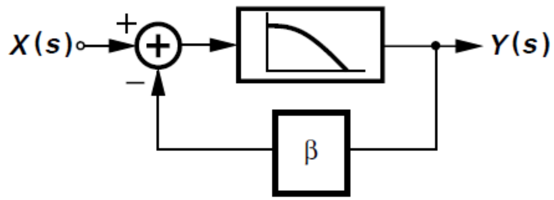
- **Since** $I_X = V_X/R_D + I_{D1}$, **we have**

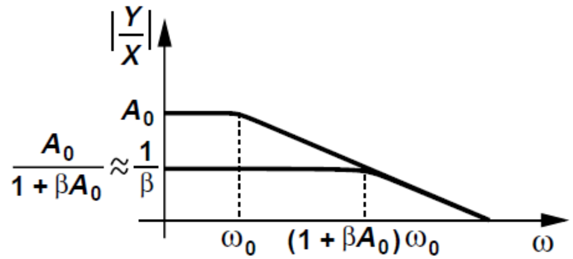
$$\frac{V_X}{I_X} = \frac{R_D}{1 + \frac{g_{m2}R_S(g_{m1}+g_{mb1})R_D}{(g_{m1}+g_{mb1})R_S+1}\frac{C_1}{C_1+C_2}}$$

- **This implies that negative feedback decreases the output impedance**
- **It can be verified that denominator is one plus the loop gain**

Properties of Feedback Circuits

- **Bandwidth Modification:**





- Suppose the feedforward amplifier above has a one-pole transfer function

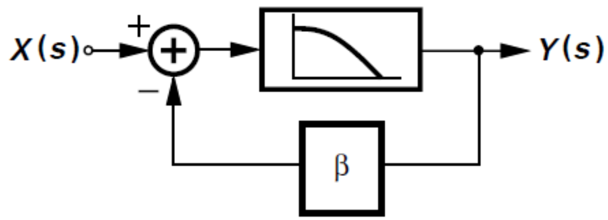
$$A(s) = \frac{A_0}{1 + \dfrac{s}{\omega_0}}$$

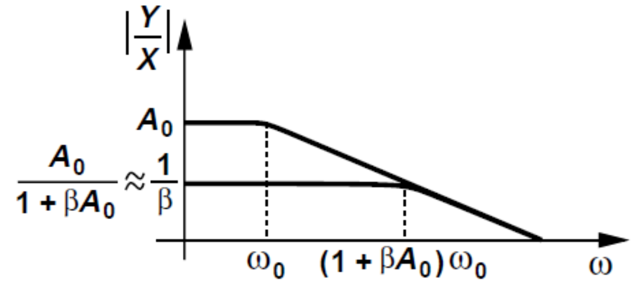
- A_0 is the low-frequency gain and ω_0 is the 3-dB bandwidth
- Transfer function of the closed-loop system is

$$\frac{Y}{X}(s) = \frac{\dfrac{A_0}{1+\dfrac{s}{\omega_0}}}{1+\beta\dfrac{A_0}{1+\dfrac{s}{\omega_0}}} = \frac{A_0}{1+\beta A_0+\dfrac{s}{\omega_0}} = \frac{\dfrac{A_0}{1+\beta A_0}}{1+\dfrac{s}{(1+\beta A_0)\omega_0}}$$

Properties of Feedback Circuits

- **Bandwidth Modification:**

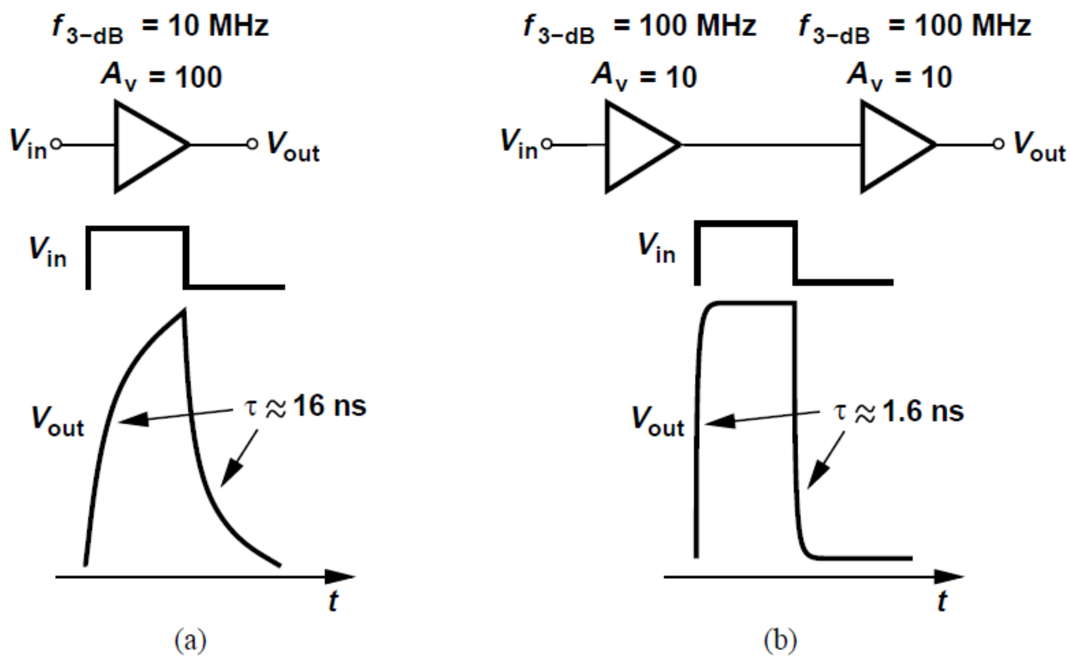




- **The closed-loop gain at low frequencies is reduced by a factor of $1 + \beta A_0$, and the 3-dB bandwidth is increased by the same factor, revealing a pole at $(1 + \beta A_0)\omega_0$**
- **If *A* is large enough, closed-loop gain remains approximately equal to 1/*β* even if *A* experiences substantial variations**
- **At high frequencies, A drops so that *βA* is comparable to unity and closed-loop gain falls below 1/*β***

Properties of Feedback Circuits

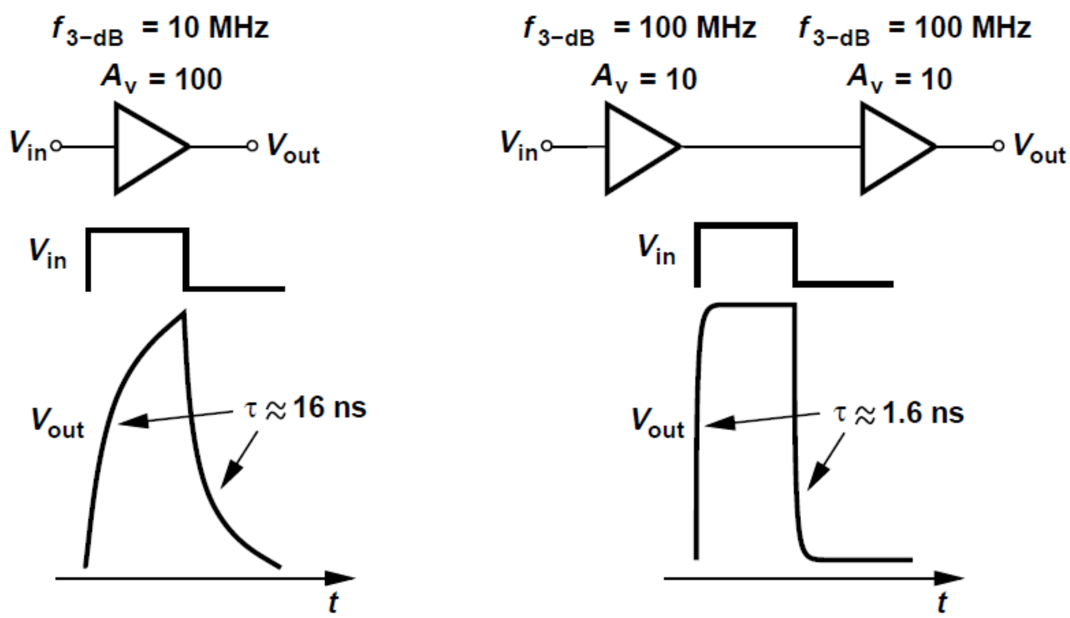
- **<u>Bandwidth Modification:</u>**
- **Gain-bandwidth product of a one-pole system is $A_0\omega_0$ and does not change with feedback**



- **For a single-pole amplifier with open loop gain of 100 and 3-dB bandwidth of 10 MHz, the response to a 20 MHz square wave exhibits long rise and fall times [Fig. (a)] with a time constant** $1/(2\pi f_{3-\text{dB}}) \approx 16\ \text{ns}$.

Properties of Feedback Circuits

- **Bandwidth Modification:**

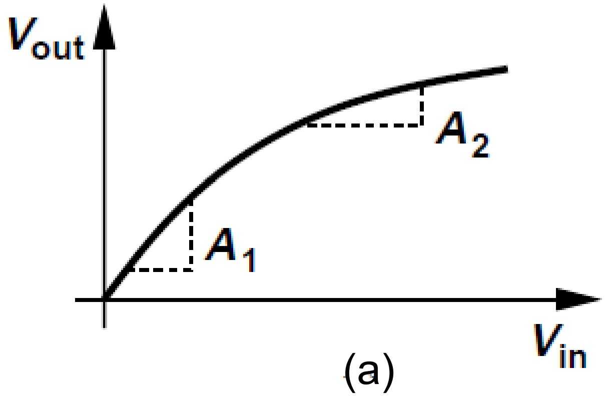


(a) (b)

- **If feedback is applied to the amplifier such that the gain and bandwidth are modified to 10 and 100 MHz respectively, two such amplifiers cascaded in series yield a much faster response [Fig. (b)], at the cost of double the power consumption**

Properties of Feedback Circuits

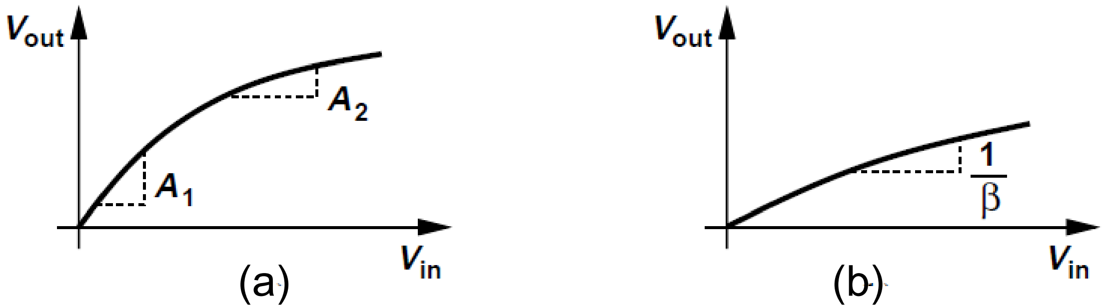
- **Nonlinearity Reduction:**
- **Negative feedback reduces nonlinearity in analog circuits**



- **A nonlinear characteristic departs from a straight line, i.e., its *slope* (or small-signal gain) varies [Fig. (a)]**
- **A closed-loop feedback system incorporating such an amplifier exhibits less gain variation and higher linearity [Fig. (b)]**

Properties of Feedback Circuits

- **Nonlinearity Reduction:**



- **In Fig. (a), open-loop gain ratios between regions 1 and 2 is**

$$r_{open} = \frac{A_2}{A_1}$$

- **Assuming $A_2 = A_1 - \Delta A$, we can write**

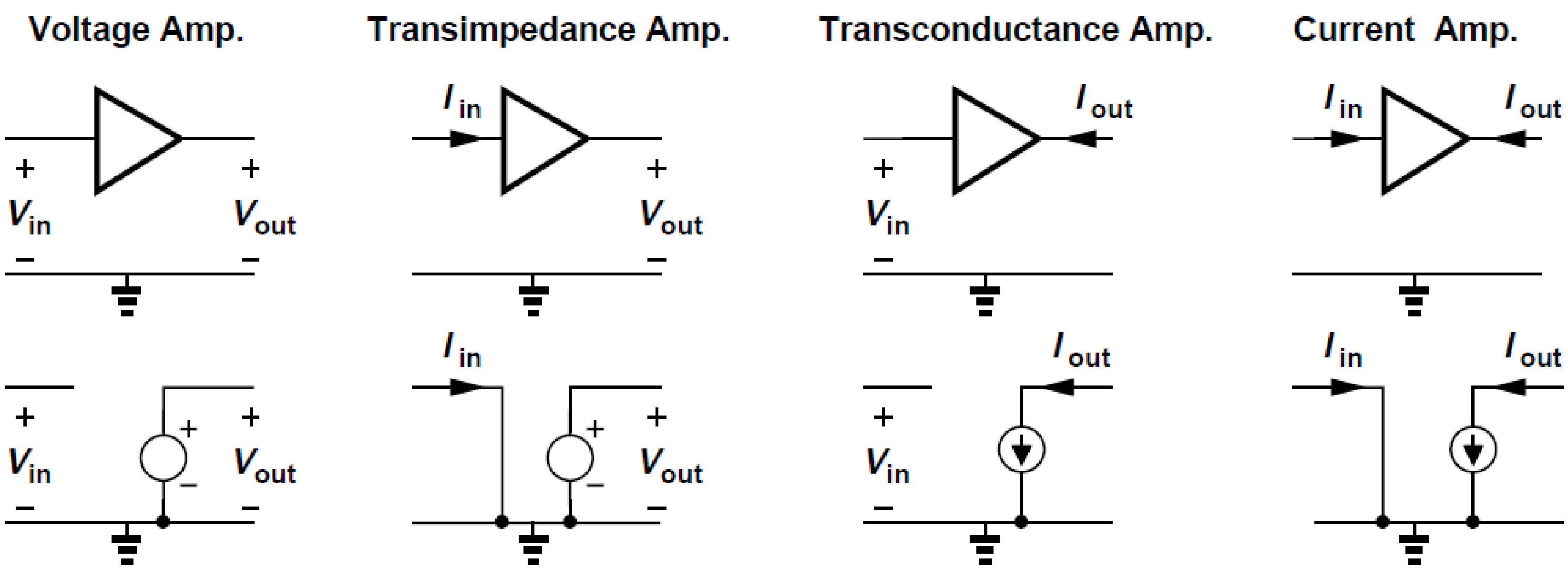
$$r_{open} = 1 - \frac{\Delta A}{A_1}$$

- **For the amplifier in negative feedback [Fig. (b)], the closed-loop gain ratio is much closer to 1 if the loop gain $1 + \beta A_2$, is large**

$$r_{closed} = \frac{\frac{A_2}{1+\beta A_2}}{\frac{A_1}{1+\beta A_1}} \approx 1 - \frac{\Delta A}{1+\beta A_2}\frac{1}{A_1}$$

Types of Amplifiers

- Four possible amplifier configurations depending on whether the input and output signals are voltage or current quantities



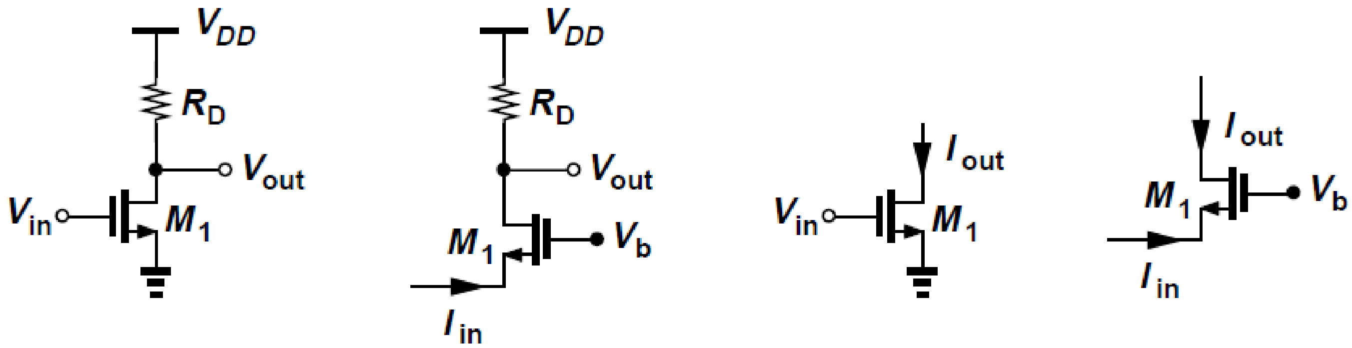
(a) (b) (c) (d)

- Figs. (a) – (d) show the four amplifier types with the corresponding idealized models

Types of Amplifiers

- The four configurations have quite different properties
- Circuits sensing a voltage must exhibit a high input impedance whereas those sensing a current must provide a low input impedance
- Circuits generating a voltage must exhibit a low output impedance while those generating a current must provide a high output impedance
- Gains of transimpedance and transconductance amplifiers have dimensions of resistance and conductance, respectively
- Sign conventions must be followed, taking into account the directions of I_{in} and I_{out} in transimpedance and transconductance amplifiers

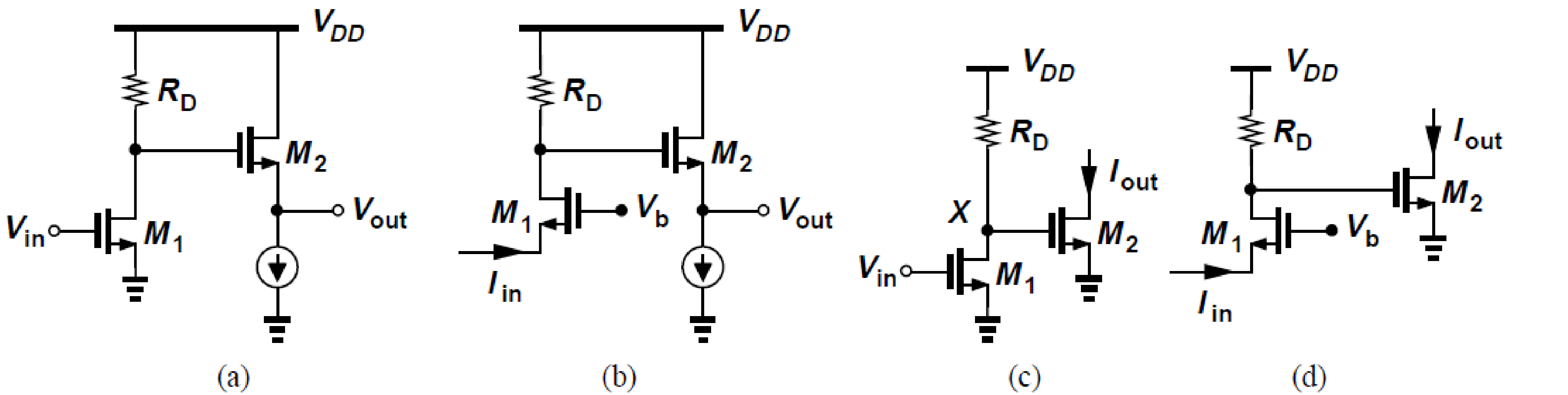
Types of Amplifiers



(a) (b) (c) (d)

- In Fig. (a), a common-source stage senses and produces voltages
- In Fig. (b), a common-gate stage serves as a transimpedance amplifier, converting the source current to a voltage at the drain
- In Fig. (c), a common-source transistor operates as a transconductance amplifier (or *V/I* converter), generating an output current in response to an input voltage
- In Fig. (d), a common-gate device senses and produces currents

Types of Amplifiers

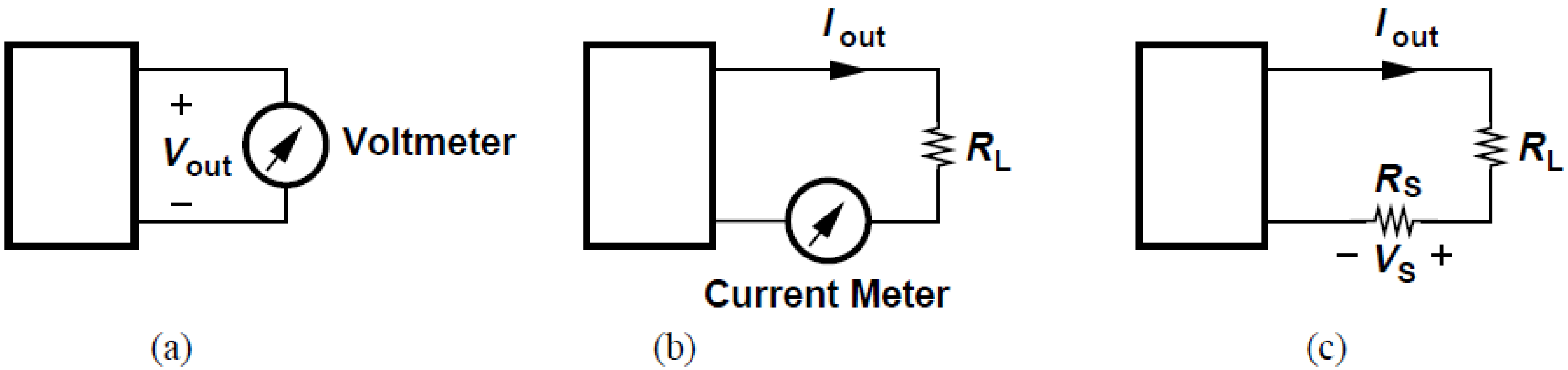


- Figs. (a) – (d) depict modifications to previous amplifier configurations to alter the output impedance or increase the gain

Sense and Return Mechanisms

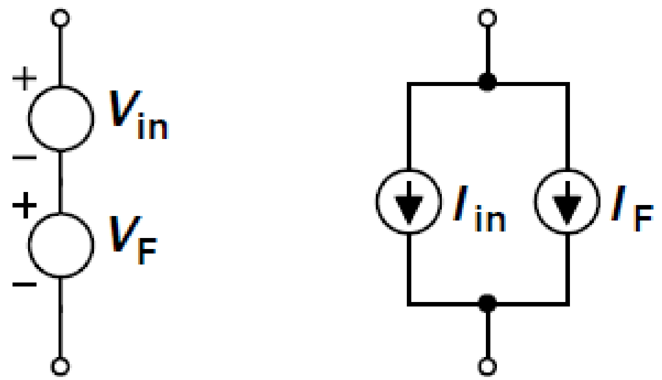
- **Placing a circuit in a feedback loop requires sensing an output signal and returning a fraction of it to the summing node at the input**
- **Four types of feedback**
  - **Voltage-Voltage**
  - **Voltage-Current**
  - **Current-Current**
  - **Current-Voltage**
- **First term is the quantity sensed at the output, and the second term is the type of signal returned to the input**

Sense and Return Mechanisms



- To sense a voltage, we place a voltmeter in parallel with the corresponding port [Fig. (a)], ideally introducing no loading, also called "shunt feedback"
- To sense a current, a current meter is inserted in series with the signal [Fig. (b)], ideally exhibiting zero resistance, also called "series feedback"
- In practice, the current meter is replaced by a small resistor [Fig. (c)], with the voltage drop as a measure of the output current

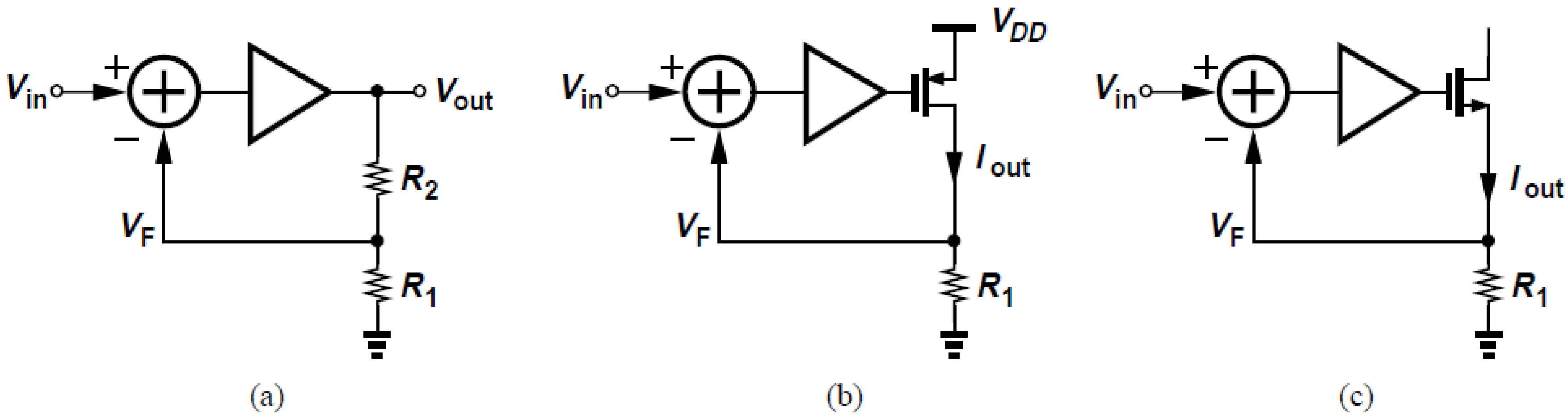
Sense and Return Mechanisms



(a) (b)

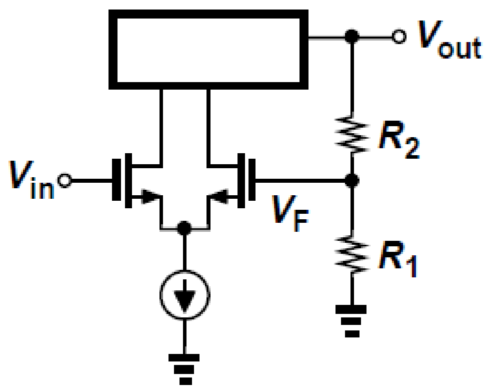
- **Addition of the feedback signal and the input signal can be performed in the voltage or current domains**
- **Voltages are added in series [Fig. (a)]**
- **Currents are added in parallel [Fig. (b)]**

Sense and Return Mechanisms

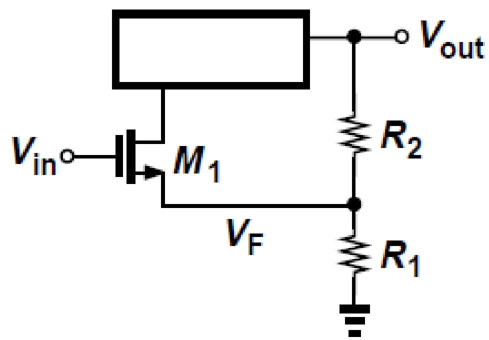


- **A voltage can be sensed by a resistive (or capacitive) divider in parallel with the port [Fig. (a)]**
- **A current can be sensed by placing a small resistor in series with the wire and sensing the voltage across it [Figs. (b) and (c)]**

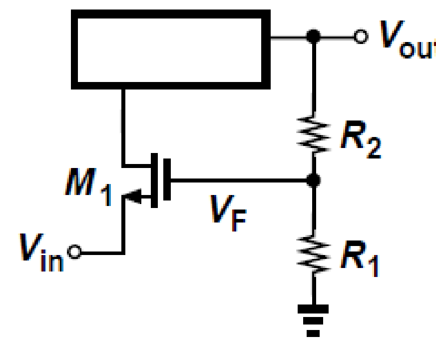
Sense and Return Mechanisms



(d)



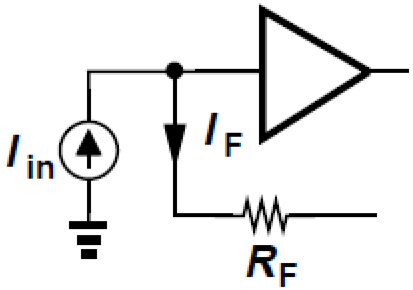
(e)



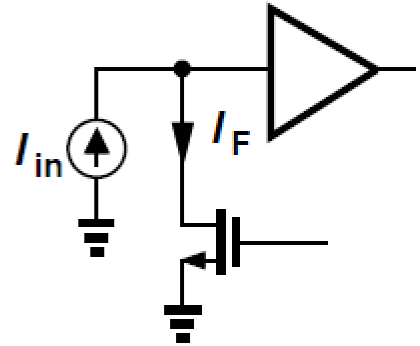
(f)

- **To subtract two voltages, a differential pair can be used [Fig. (d)]**
- **A single transistor can also perform voltage subtraction [Figs. (e) and (f)] since I_{D1} is a function of $V_{in} - V_F$**

Sense and Return Mechanisms



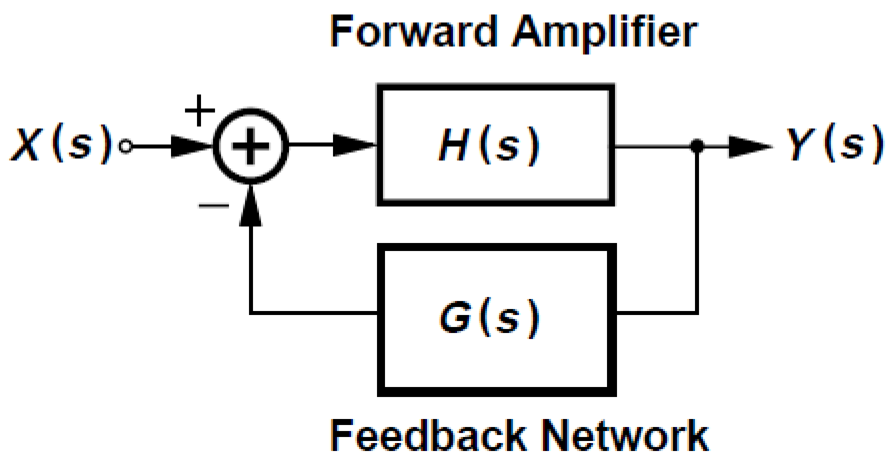
(g)



(h)

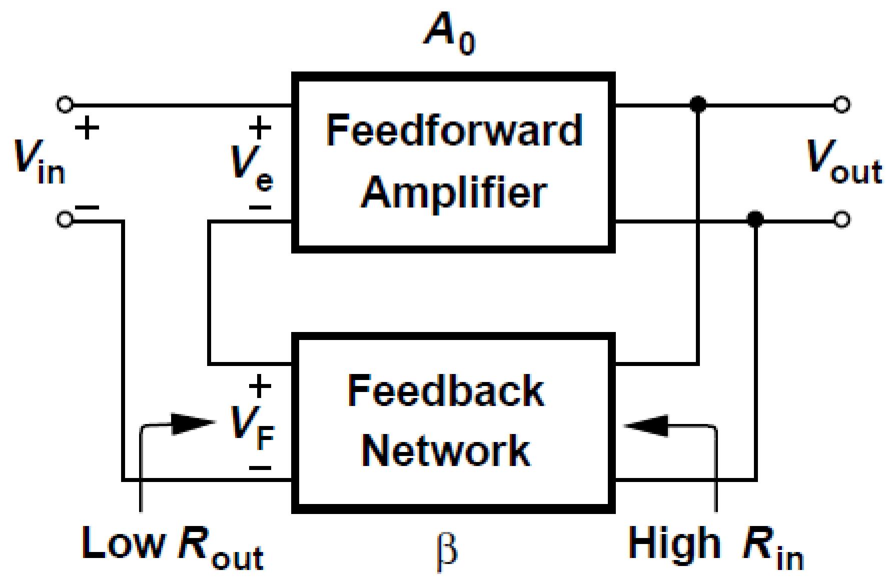
- **Current subtraction can be performed as shown in Figs. (g) and (h)**
- **For voltage subtraction, the input and feedback signals are applied to two distinct nodes**
- **For current subtraction, the input and feedback signals are applied to a single node**

Feedback Topologies



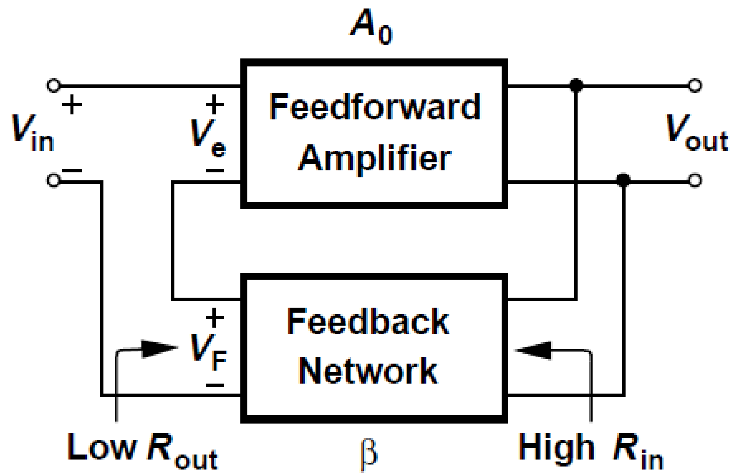
- In the above figure, *X* and *Y* can be a current or a voltage quantity
- Main amplifier is called "feedforward" or simply "forward" amplifier around which feedback is applied
- Four "canonical" topologies result from placing each of the four amplifier types in negative feedback

Voltage-Voltage Feedback



- This topology senses the output voltage and returns the feedback signal as a voltage
- Feedback network is connected in *parallel* with the output and in *series* with the input
- An ideal feedback network in this case has infinite input impedance (ideal voltmeter) and zero output impedance (ideal voltage source)

Voltage-Voltage Feedback

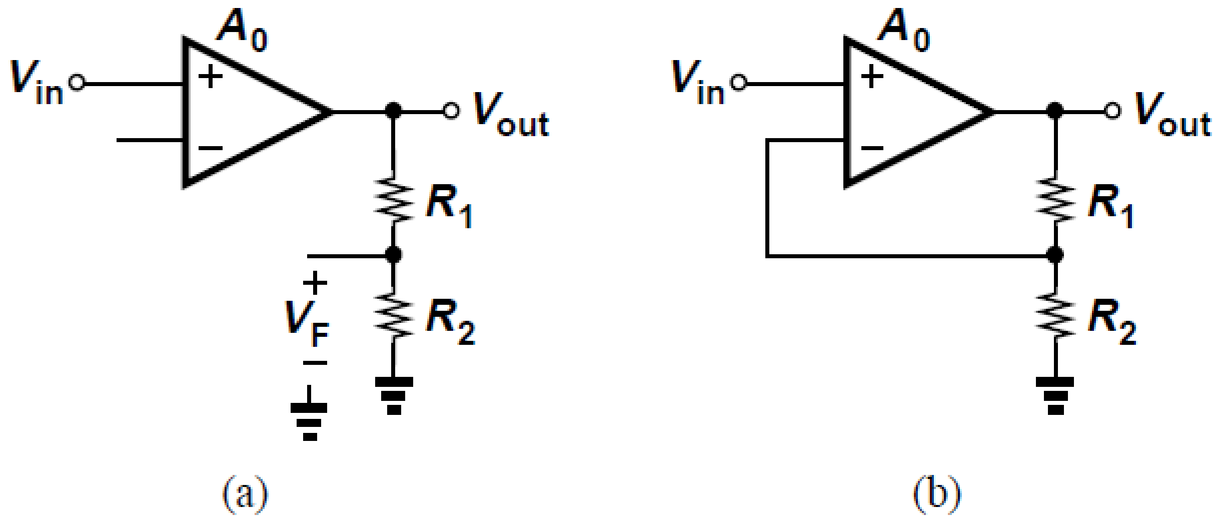


- **Also called "series-shunt" feedback; first term refers to the *input* connection and second to the *output* connection**
- **We can write $V_F = \beta V_{out}$, $V_e = V_{in} - V_F$, $V_{out} = A_0(V_{in} - \beta V_{out})$, and hence**

$$\frac{V_{out}}{V_{in}} = \frac{A_0}{1 + \beta A_0}$$

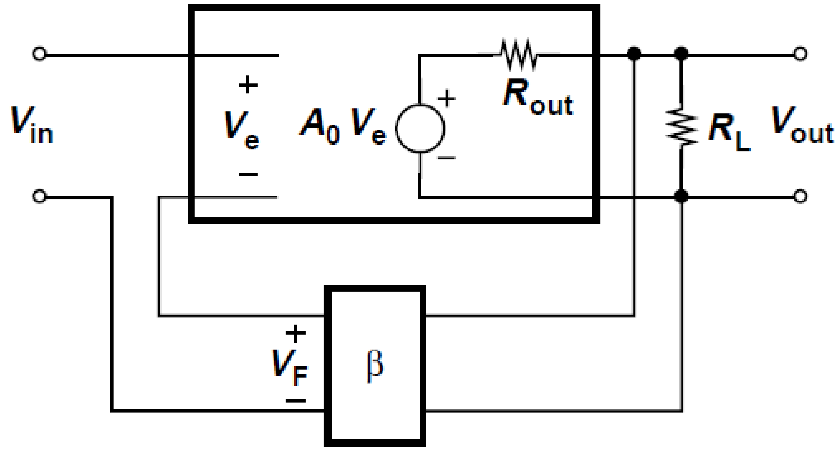
- **βA_0 is the loop gain and the overall gain has dropped by $1+ \beta A_0$**

Voltage-Voltage Feedback



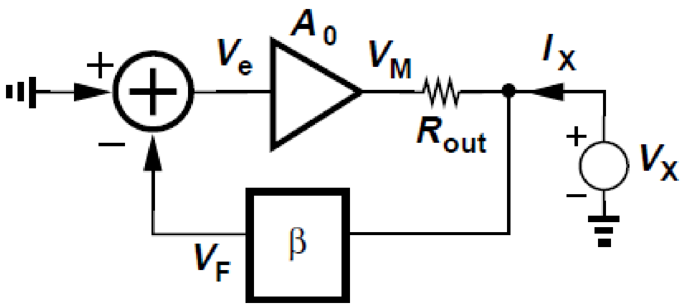
- **As an example of voltage-voltage feedback, a differential voltage amplifier with single-ended output can be used as the forward amplifier and a resistive divider as the feedback network [Fig. (a)]**
- **The sensed voltage V_F is placed in series with the input to perform subtraction of voltages**

Voltage-Voltage Feedback: Output Resistance



- If output is loaded by resistor R_L, in open-loop configuration, output decreases in proportion to $R_L/(R_L+R_{out})$
- In closed-loop V_{out} is maintained as a constant replica of V_{in} regardless of R_L as long as loop gain is much greater than unity
- Circuit "stabilizes" output voltage despite load variations, behaves as a voltage source and exhibits low output impedance

Voltage-Voltage Feedback: Output Resistance

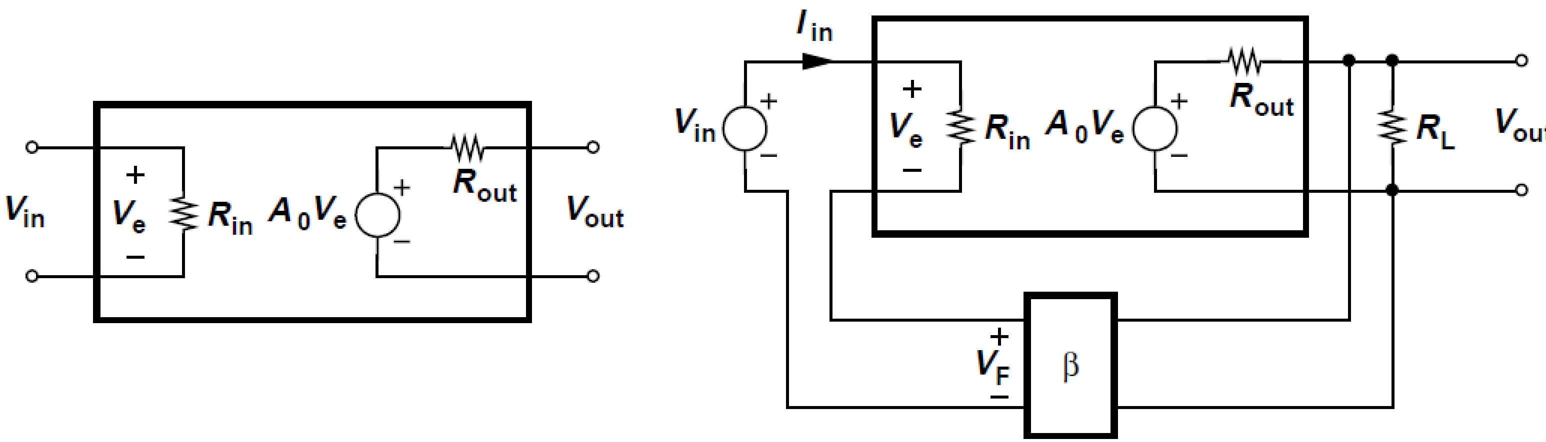


- In the above model, R_{out} represents the output impedance of the feedforward amplifier
- Setting input to zero and applying a voltage at the output, we write $V_F = \beta V_X$, $V_e = \beta V_X$, $V_M = \beta A_0 V_X$ and hence $I_X = [V_X - (-\beta A_0 V_X)]/R_{out}$ (if current drawn by feedback network is neglected)
- It follows that

$$\frac{V_X}{I_X} = \frac{R_{out}}{1 + \beta A_0}$$

- Output impedance and gain are lowered by same factor

Voltage-Voltage Feedback: Input Resistance

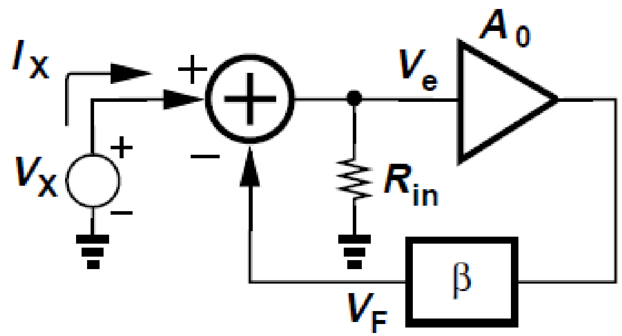


(a)

(b)

- **Voltage-voltage feedback also modifies input impedance**
- **In Fig. (a) [open-loop], R_{in} of the forward amplifier sustains the entire V_{in}, whereas only a fraction in Fig. (b) [closed-loop]**
- **I_{in} is less in the feedback topology compared to open-loop system, suggesting increase in the input impedance**

Voltage-Voltage Feedback: Input Resistance

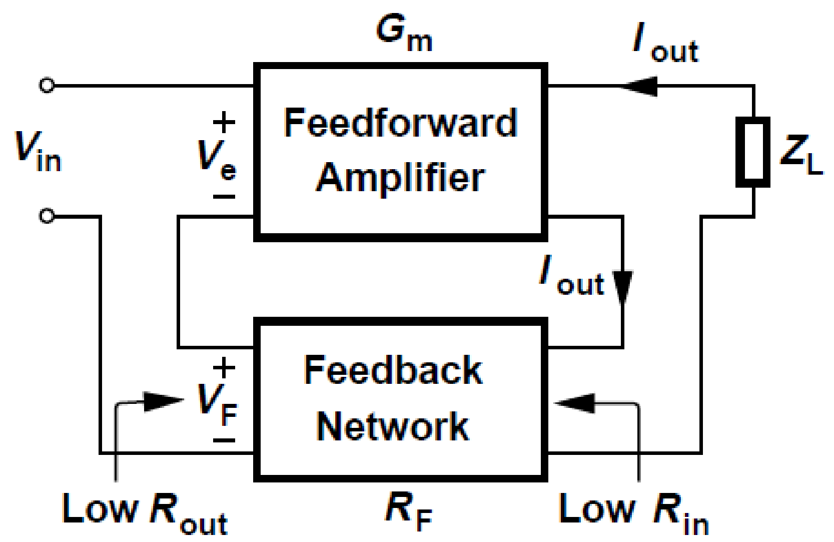


- In the above model, $V_e = I_X R_{in}$ and $V_F = \beta A_0 I_X R_{in}$
- Thus, we have $V_e = V_X - V_F = V_X - \beta A_0 I_X R_{in}$
- Hence, $I_X R_{in} = V_X - \beta A_0 I_X R_{in}$ and

$$\frac{V_X}{I_X} = R_{in}(1 + \beta A_0)$$

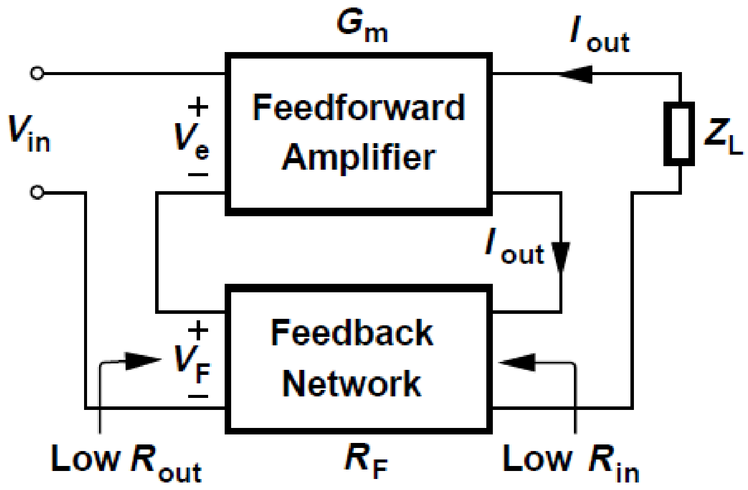
- Input impedance increases by the factor $1+\beta A_0$, bringing the circuit closer to an ideal voltage amplifier
- Voltage-voltage feedback decreases output impedance and increases input impedance, useful as a buffer stage

Current-Voltage Feedback



- This topology senses the output current and returns a voltage as the feedback signal
- The current is sensed by measuring the voltage drop across a (small) resistor placed in series with the output
- Feedback factor β has the dimension of resistance and is hence denoted by R_F

Current-Voltage Feedback



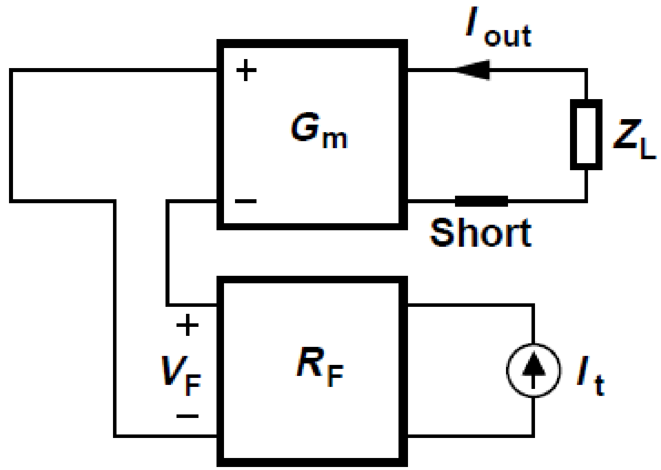
- A G_m stage must be terminated by a finite impedance to ensure it can deliver its output current
- If $Z_L = \infty$, an ideal G_m stage would sustain an infinite output voltage

- We write $V_F = R_F I_{out}$, $V_e = V_{in} - R_F I_{out}$ and hence $I_{out} = G_m(V_{in} - R_F I_{out})$
- It follows that

$$\frac{I_{out}}{V_{in}} = \frac{G_m}{1 + G_m R_F}$$

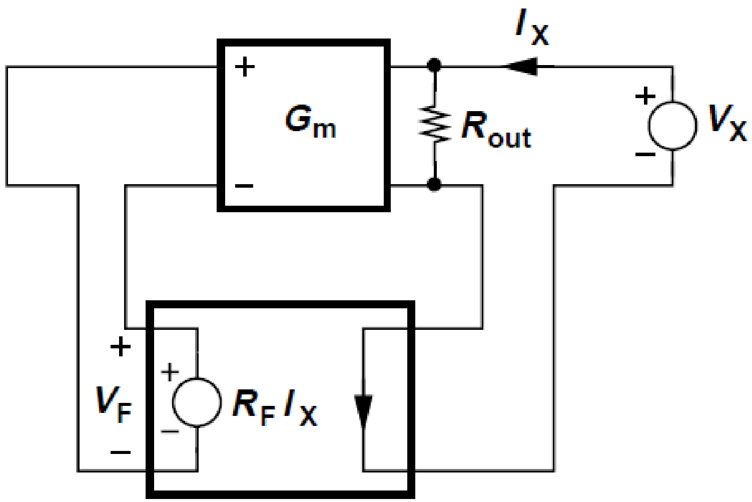
- Ideal feedback network in this case exhibits zero input and output impedances

Current-Voltage Feedback: Loop Gain



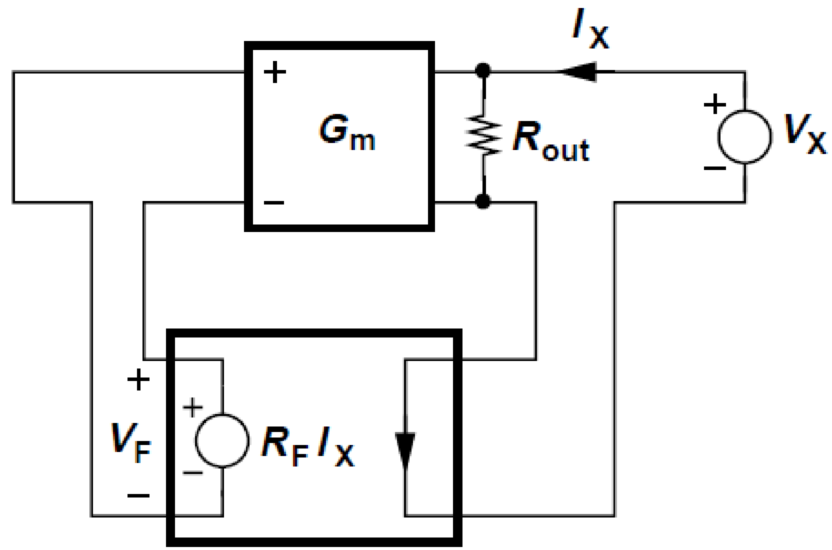
- To calculate the loop gain, the input is set to zero and the loop is broken by disconnecting the feedback network from the output and replacing it with a short at the output (if the feedback network is ideal)
- Test signal It is injected, producing $V_F = R_F I_t$ and hence $I_{out} = -G_m R_F I_t$
- Thus, loop gain is $G_m R_F$ and transconductance of the amplifier is reduced by $1+G_m R_F$ when feedback is applied

Current-Voltage Feedback: Output Resistance



- Sensing the current at the output increases the output impedance
- System delivers the same current waveform as the load varies, approaching an ideal current source which exhibits a high output impedance
- In the above figure, R_{out} represents the finite output impedance of the feedforward amplifier
- Feedback network produces V_F proportional to I_X, i.e., $V_F = R_F I_X$

Current-Voltage Feedback: Output Resistance

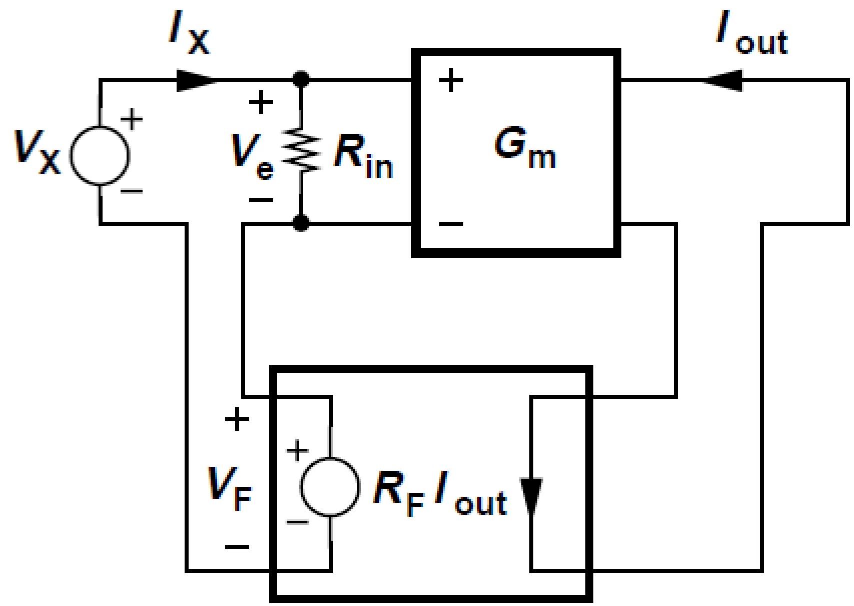


- The current generated by G_m equals $-R_FI_XG_m$
- As a result, $-R_FI_XG_m = I_X - V_X/R_{out}$, yielding

$$\frac{V_X}{I_X} = R_{out}(1 + G_m R_F)$$

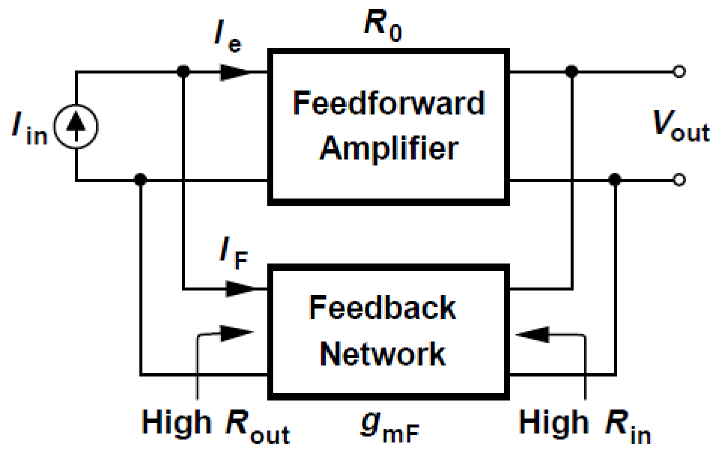
- The output impedance therefore increases by a factor of $1+G_mR_F$

Current-Voltage Feedback: Input Resistance



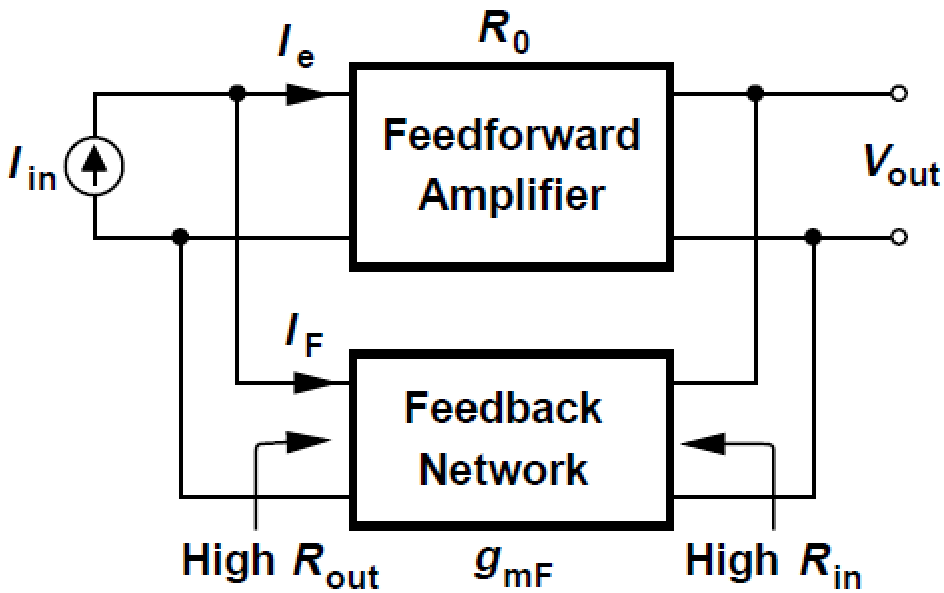
- Current-voltage feedback increases the input impedance by a factor of one plus the loop gain
- As shown in the above figure, we have $I_X R_{in} G_m = I_{out}$
- Thus, $V_e = V_X - G_m R_F I_X I_{in}$ and
- Current-voltage feedback increases both the input and output impedances while decreasing the feedforward transconductance

Voltage-Current Feedback



- **In this type of feedback, the output voltage is sensed and a proportional current is returned to the input summing point**
- **Feedforward path incorporates a transimpedance amplifier with gain R_0 and the feedback factor g_{mF} has a dimension of conductance**
- **Feedback network ideally exhibits infinite input and output impedances**
- **Also called "shunt-shunt" feedback**

Voltage-Current Feedback

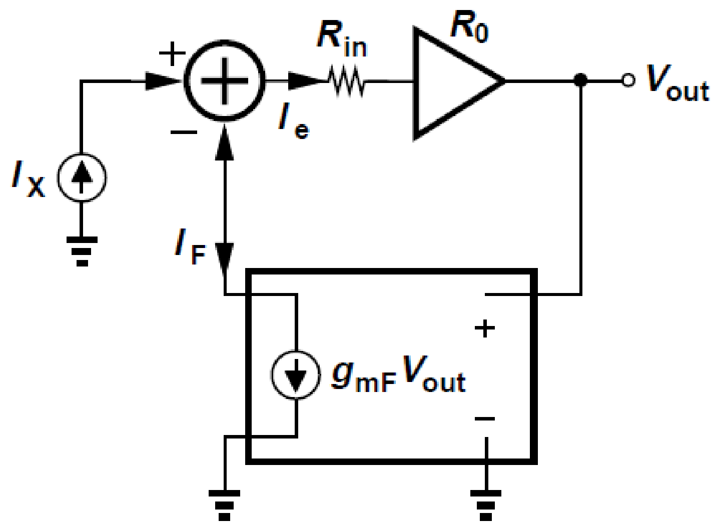


- Since $I_F = g_{mF}V_{out}$ and $I_e = I_{in} - I_F$, we have $V_{out} = R_0 I_e = R_0(I_{in} - g_{mF}V_{out})$
- It follows that

$$\frac{V_{out}}{I_{in}} = \frac{R_0}{1 + g_{mF}R_0}$$

- This feedback lowers the transimpedance by a factor of one plus the loop gain

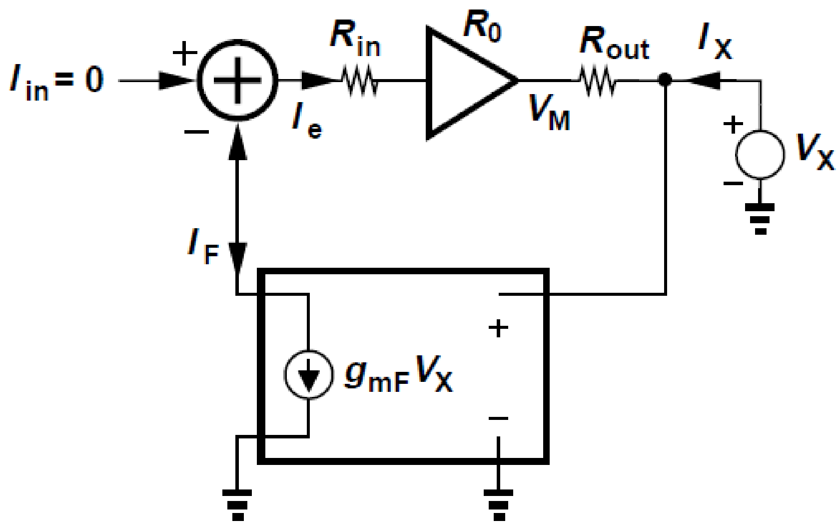
Voltage-Current Feedback: Output Impedance



- Voltage-current feedback decreases the output impedance
- Input resistance R_{in} of R_0 appears in series with the input port
- We write $I_F = I_X - V_X/R_{in}$ and $(V_X/R_{in})R_0 g_{mF} = I_F$
- Thus,

$$\frac{V_X}{I_X} = \frac{R_{in}}{1 + g_{mF}R_0}$$

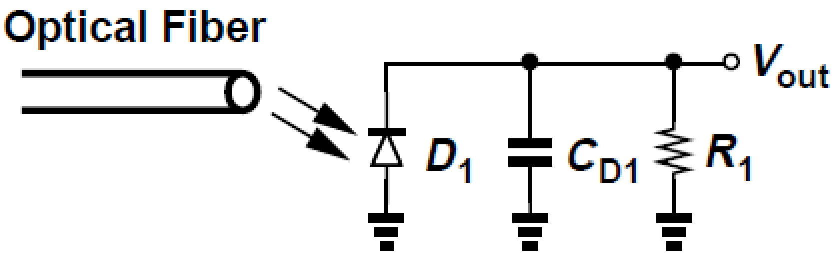
Voltage-Current Feedback: Input Impedance



- Voltage-current feedback decreases the input impedance too
- From the figure, we have $I_F = V_X g_{mF}$, $I_e = -I_F$, and $V_M = -R_0 g_{mF} V_X$
- Neglecting the input current of the feedback network, $I_X = (V_X - V_M)/R_{out} = (V_X + g_{mF} R_0 V_X)/R_{out}$
- Thus,

$$\frac{V_X}{I_X} = \frac{R_{out}}{1 + g_{mF} R_0}$$

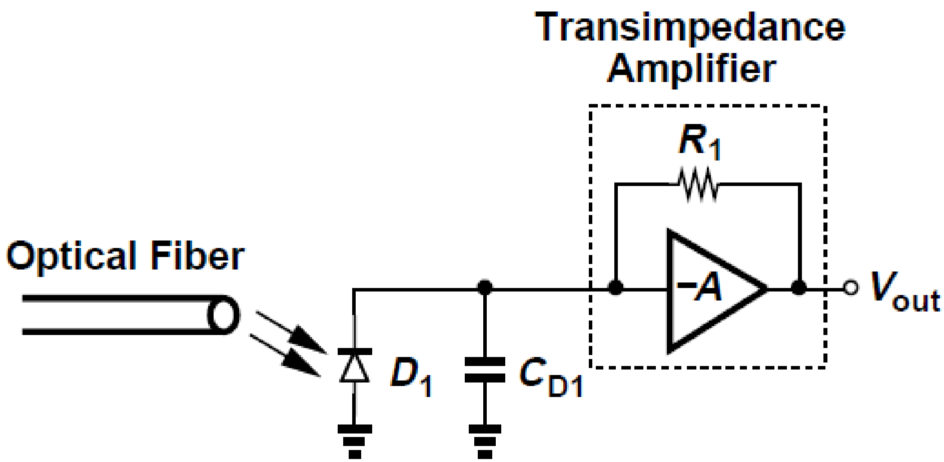
Voltage-Current Feedback: Applications



(a)

- **Amplifiers with low input impedance are used in fiber optic receivers, where light received through a fiber is converted to a current by a reverse-biased photodiode**
- **This current is converted to a voltage for processing by subsequent stages**
- **Fig. (a) show this conversion using a resistor at the cost of bandwidth due to large junction capacitance C_{D1} of the diode**

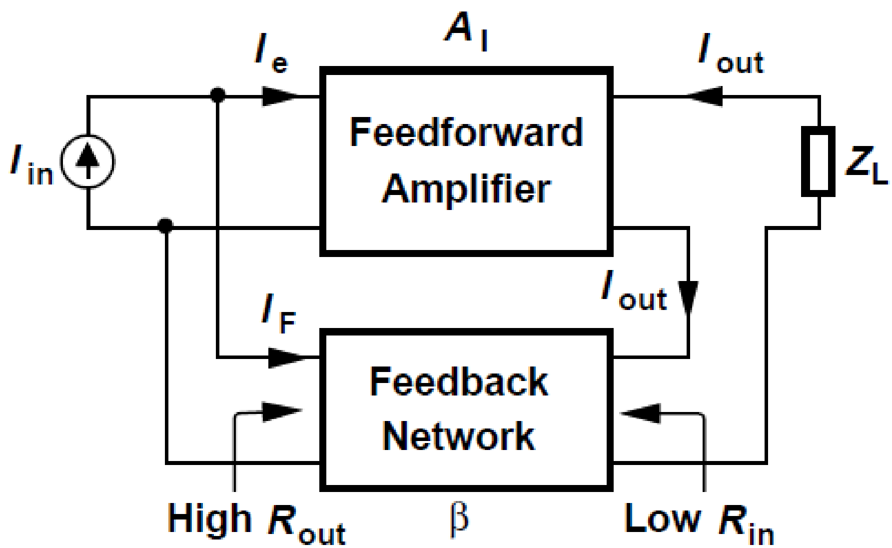
Voltage-Current Feedback: Applications



(b)

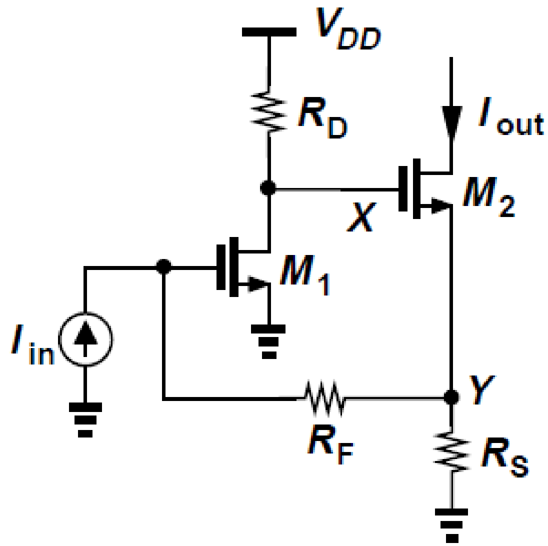
- To improve performance, the feedback topology of Fig. (b) is employed, where R_1 is placed around the voltage amplifier A to form a "transimpedance amplifier" (TIA)
- The input impedance is $R_1/(1+A)$ and output voltage is approximately R_1I_{D1}
- Bandwidth thus increases from $1/(2\pi R_1C_{D1})$ to $(1+A)/(2\pi R_1C_{D1})$ if A itself is a wideband amplifier

Current-Current Feedback



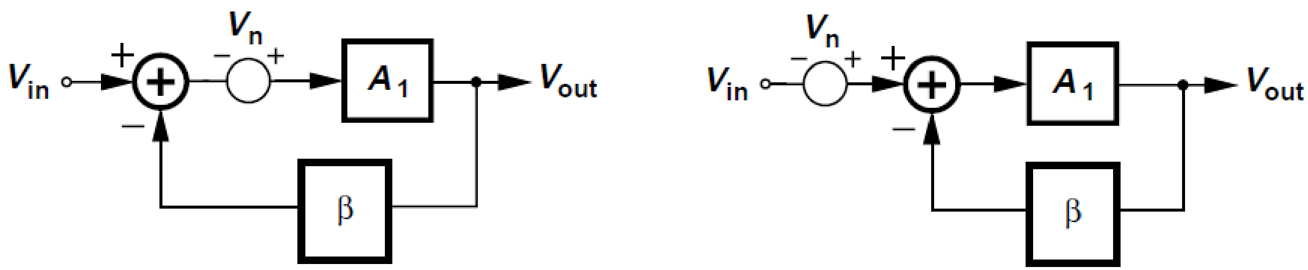
- **Output voltage is sensed and a proportional current is returned**
- **Feedforward amplifier is characterized by a current gain A_I and feedback network by a current ratio β**
- **It can be proved that the closed-loop current gain is equal to $A_I/(1+\beta A_I)$, the input impedance is divided by $1+\beta A_I$, and the output impedance is multiplied by $1+\beta A_I$**

Current-Current Feedback: Example



- Above figure shows an example of current-current feedback
- Since the source and drain currents of M_1 are equal (at low frequencies), resistor R_S is inserted in the source network to monitor the output current
- Resistor R_F senses the output voltage and returns a current to the input

Effect of Feedback on Noise



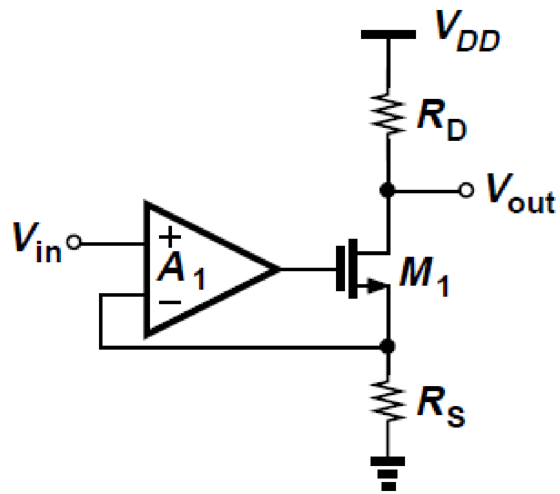
(a) (b)

- Feedback does not improve noise performance of circuits
- In Fig. (a), the open-loop amplifier A_1 is characterized by only an input-referred noise voltage and the feedback network is assumed to be noiseless
- We have $(V_{in} - \beta V_{out} + V_n)A_1 = V_{out}$, and hence

$$V_{out} = (V_{in} + V_n)\frac{A_1}{1+\beta A_1}$$

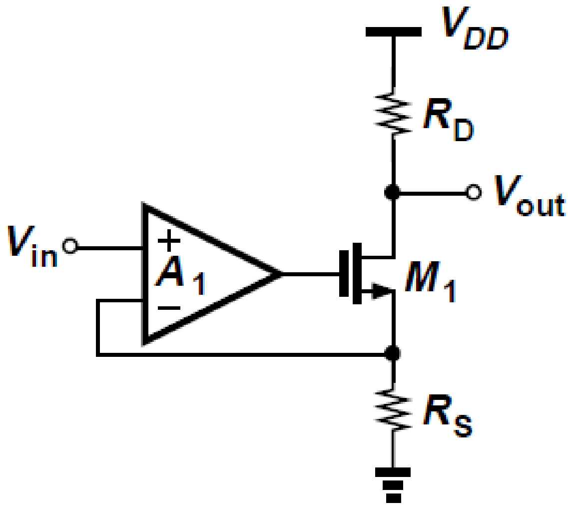
- Circuit can be modified as in Fig. (b), input-referred noise is still V_n

Effect of Feedback on Noise



- Output of interest may not always be the quantity sensed by the feedback network
- In above circuit, output is at the drain of M_1 whereas the feedback network senses source voltage of M_1
- Here, input-referred noise of the closed-loop circuit is not equal to that of the open-loop circuit even if the feedback network is noiseless

Effect of Feedback on Noise



- Consider only the noise of R_D, $V_{n,RD}$ in this circuit
- Closed-loop voltage gain of the circuit is

$$-A_1 g_m R_D/[1+(1+A_1)g_m R_S]$$

if $\lambda = \gamma = 0$

- Input-referred noise voltage due to R_D is

$$|V_{n,in,closed}| = \frac{|V_{n,RD}|}{A_1 R_D}\left[\frac{1}{g_m} + (1+A_1)R_S\right]$$

- Input-referred noise of the open-loop circuit is

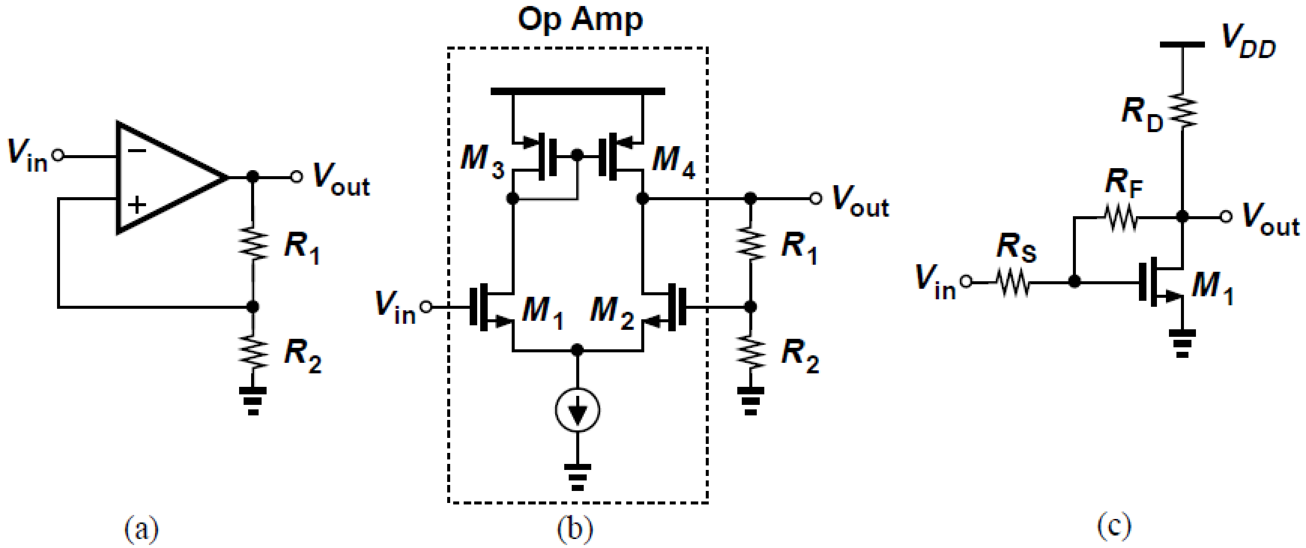
$$|V_{n,in,open}| = \frac{|V_{n,RD}|}{A_1 R_D}\left[\frac{1}{g_m} + R_S\right]$$

- As $A_1 \to \infty$, $|V_{n,in,closed}| \to |V_{n,RD}|R_S/R_D$,whereas $|V_{n,in,open}| \to 0$

Feedback Analysis Difficulties

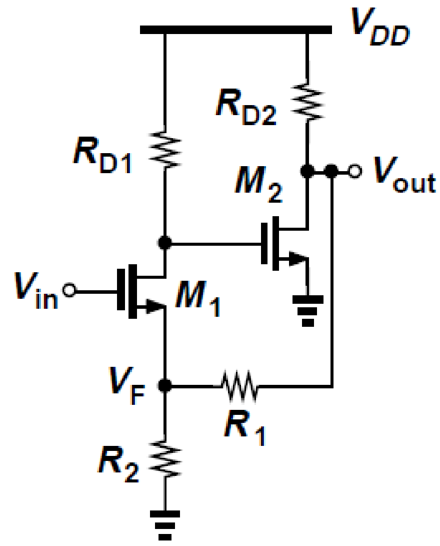
- Analysis approach used proceeds as follows:
  - Break the loop and obtain the open-loop gain and input and output impedances
  - Determine the loop gain, βA_0 and hence the closed-loop parameters from their open-loop counterparts
  - Use the loop gain to study properties such as stability, etc.
- The simplifying assumptions made may not hold in all circuits
- Five difficulties arising in the analysis of feedback circuits are discussed subsequently

Feedback Analysis Difficulties: (1)



- In the non-inverting amplifier of Fig. (a) and its simple implementation in Fig. (b), the feedback branch consisting of R_1 and R_2 may draw significant *signal* current from the output, reducing its *open-loop* gain
- In the circuit of Fig. (c), the open-loop gain of the forward CS stage falls if R_F is not very large
- In all cases, the "output" loading results from non-ideal input impedance of the feedback network

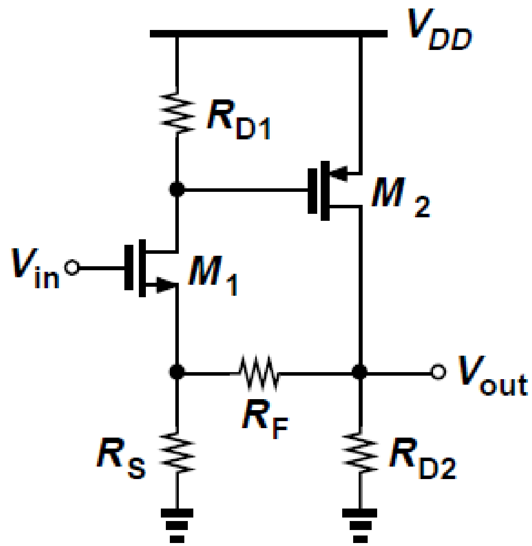
Feedback Analysis Difficulties: (1)



(d)

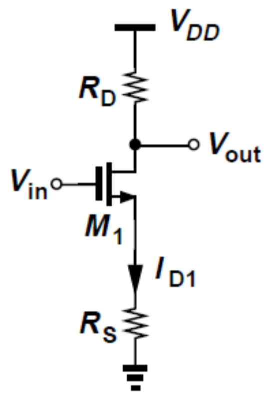
- In the circuit of Fig. (d), R_1 and R_2 sense V_{out} and return a voltage to the source of M_1
- Since the output impedance of the feedback network may not be sufficiently small, we surmise that M_1 is degenerated considerably even as far as the open-loop forward amplifier is concerned
- This is a case of "input loading" due to non-ideal output impedance of the feedback network

Feedback Analysis Difficulties: (2)

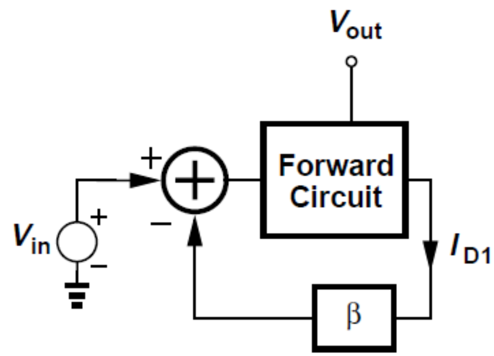


- Some circuits cannot be clearly decomposed into a forward amplifier and a feedback network
- In the above two-stage network, it is unclear whether R_{D2} belongs to the feedforward amplifier or the feedback network
- The former may be chosen, reasoning that M_2 needs a load to operate as a voltage amplifier, although this choice is arbitrary

Feedback Analysis Difficulties: (3)



(a)

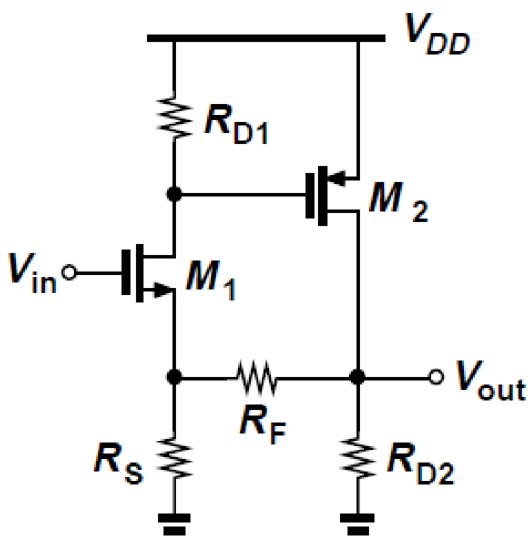


(b)

- **Some circuits do not readily map to the four canonical topologies**
- **A simple degenerated CS stage does not contain feedback because the source resistance measures the drain current, converts it to a voltage, and subtracts the result from the input [Fig. (a)]**
- **It is not immediately clear which feedback topology represents this arrangement because the sensed quantity, I_{D1} is different from the output of interest, V_{out} [Fig. (b)]**

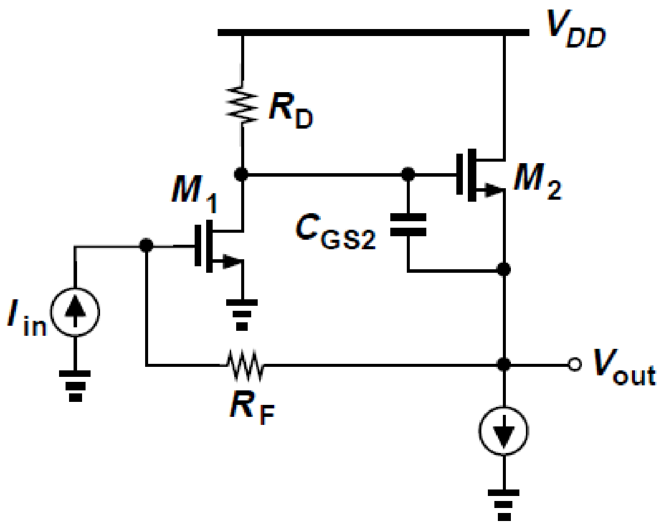
Feedback Analysis Difficulties: (4)

- General feedback system thus far assumes unilateral stages, i.e., signal propagation in only one direction around the loop
- In practice, the loop may contain bilateral circuits, allowing signals to flow from the input, through the feedback network, to the output
- In the circuit below, the input travels through R_F and alters V_{out}



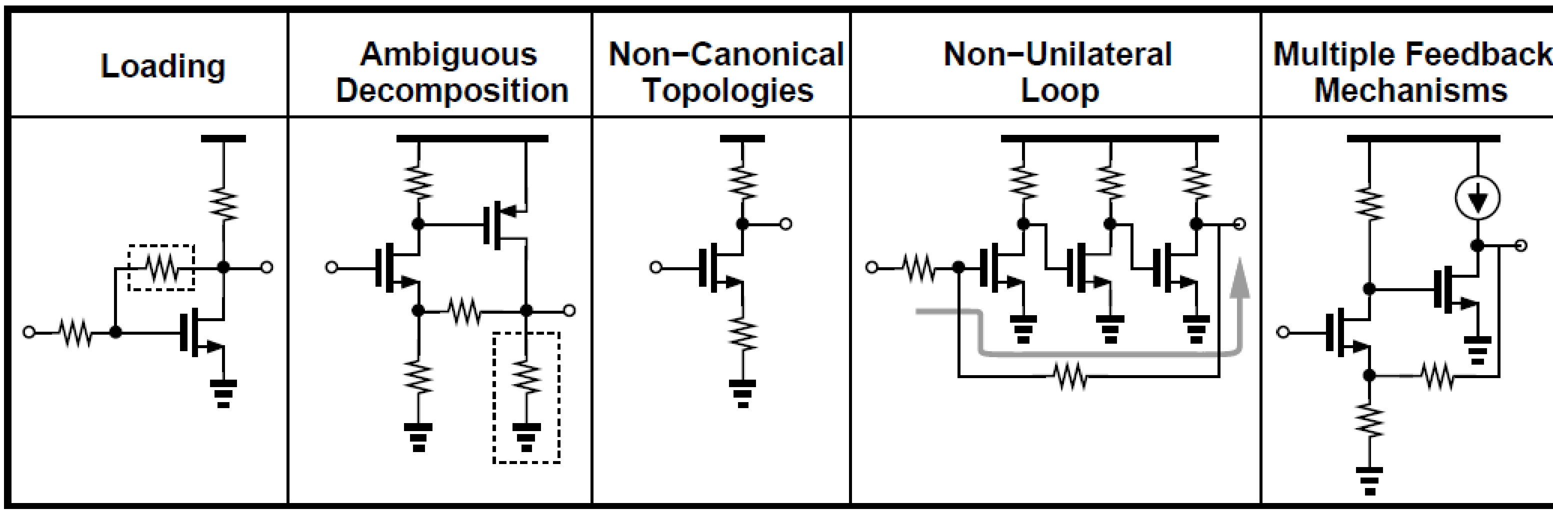
Feedback Analysis Difficulties: (5)

- Some circuits contain multiple feedback mechanisms (loosely called “multiloop circuits”)
- In the topology below, for example, R_F provides feedback around the circuit, and C_{GS2} around M_2
- It might be said that the source follower itself contains degeneration and hence feedback
- It is not exactly clear which loop should be broken and the meaning of “loop gain”



Feedback Analysis Difficulties: Summary

- The five difficulties in the analysis of feedback circuits are summarized below



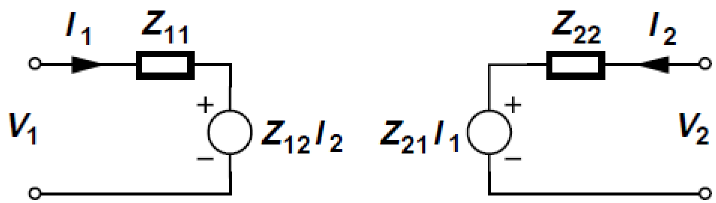
Feedback Analysis Methods

- We introduce two methods of feedback circuit analysis
  - Two-port method
  - Bode's method
- The details of the two methods are outlined below

Two−Port Method	Bode's Method
• Computes open−loop and closed−loop quantities and the loop gain. • Includes loading effects. • Neglects feedforward through feedback network. • Can be applied recursively to multiple feedback mechanisms. • Does not apply to non−canonical topologies.	• Computes closed−loop quantities without breaking the loop. • Applies to any topology. • Provides loop gain only if one feedback mechanism is present.

Review of Two-Port Network Models

- A two-port linear (and time-invariant) network can be represented by any one of four two-port network models
- The "Z model" in Fig. (a) consists of input and output impedances in series with current-dependent voltage sources



(a)

- The Z model is described by two equations

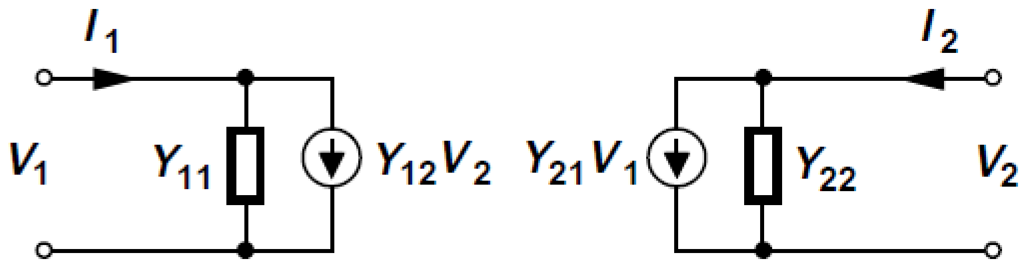
$$V_1 = Z_{11}I_1 + Z_{12}I_2$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2$$

- Each Z parameter has a dimension of impedance and is obtained by leaving one port open, e.g., $Z_{11} = V_1/I_1$ when $I_2 = 0$

Review of Two-Port Network Models

- The "Y model" in Fig. (b) comprises input and output admittances in parallel with voltage-dependent current sources



(b)

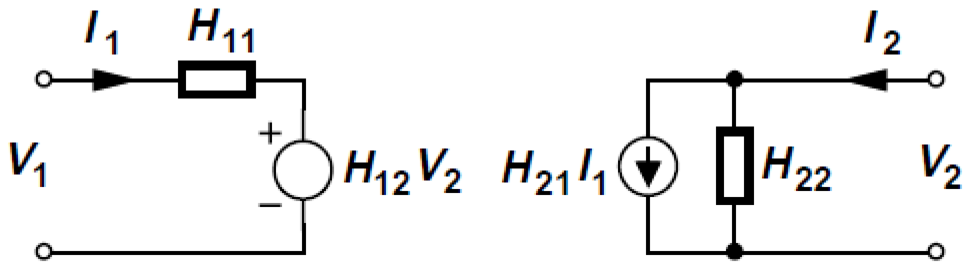
- The Y model is described by

$$
\begin{aligned}
I_1 &= Y_{11}V_1 + Y_{12}V_2 \\
I_2 &= Y_{21}V_1 + Y_{22}V_2
\end{aligned}
$$

- Each Y parameter is calculated by shorting one port, e.g., $Y_{11} = I_1/V_1$ when $V_2 = 0$

Review of Two-Port Network Models

- The "H model" in Fig. (c) incorporates a combination of impedances and admittances and voltage and current sources



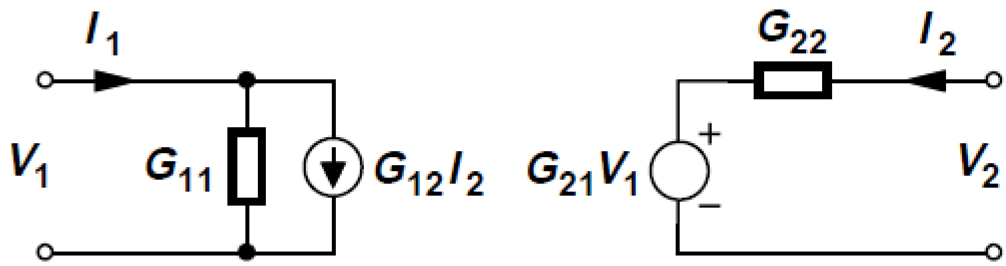
(c)

- The H model is described by

$$
\begin{aligned}
V_1 &= H_{11}I_1 + H_{12}V_2 \\
I_2 &= H_{21}I_1 + H_{22}V_2
\end{aligned}
$$

Review of Two-Port Network Models

- The “G model” in Fig. (d) is also a “hybrid model” and is characterized by a combination of impedances and admittances and voltage and current sources



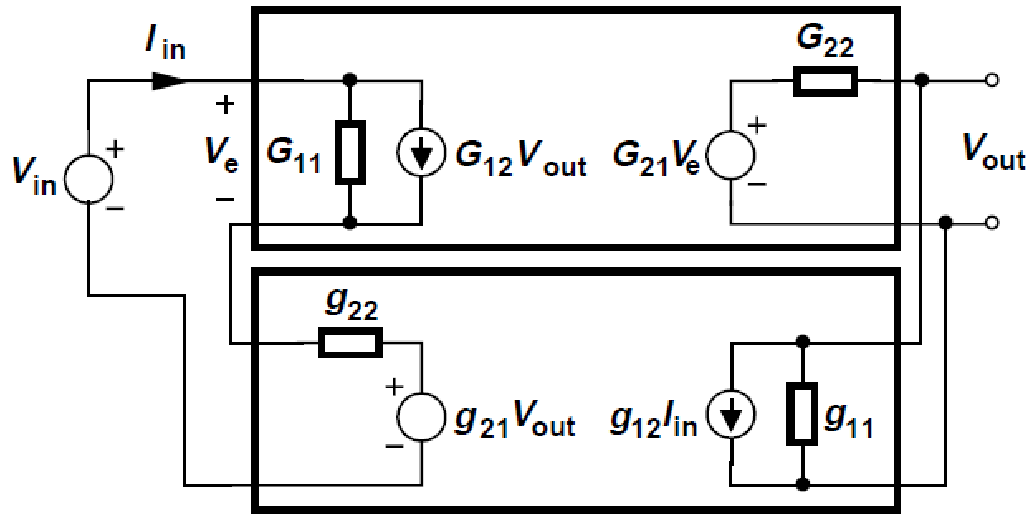
(d)

- The G model is described by

$$
\begin{aligned}
I_1 &= G_{11}V_1 + G_{12}I_2 \\
V_2 &= G_{21}V_1 + G_{22}I_2
\end{aligned}
$$

Loading in Voltage-Voltage Feedback

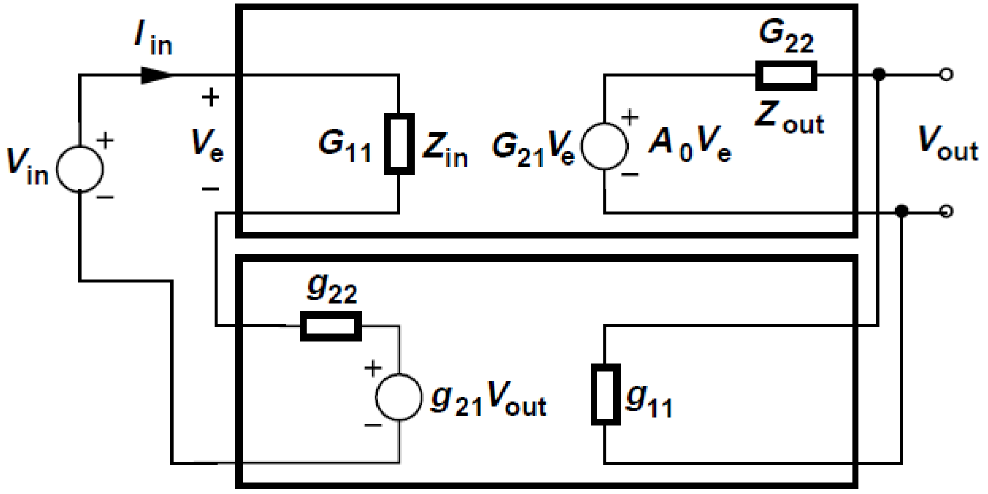
- The Z and H models fail to represent voltage amplifiers if the input current is very small – as in a simple CS stage, therefore the G model is chosen
- Fig. (a) shows the complete equivalent circuit, with the forward and feedback network parameters denoted by upper-case and lower-case letters, respectively



(a)

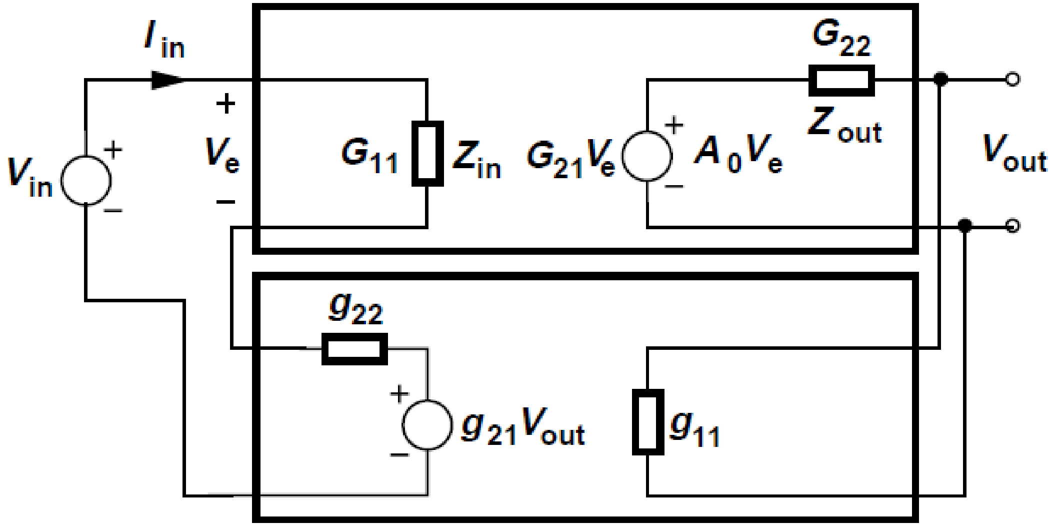
Loading in Voltage-Voltage Feedback

- The analysis is simplified by neglecting two quantities:
  - The amplifier's internal feedback, $G_{12}V_{out}$
  - The "forward" propagation of the input signal through the feedback network, $g_{12}I_{in}$
- The loop is "unilateralized"
- Fig. (b) shows the resulting circuit with intuitive amplifier notations



(b)

Loading in Voltage-Voltage Feedback



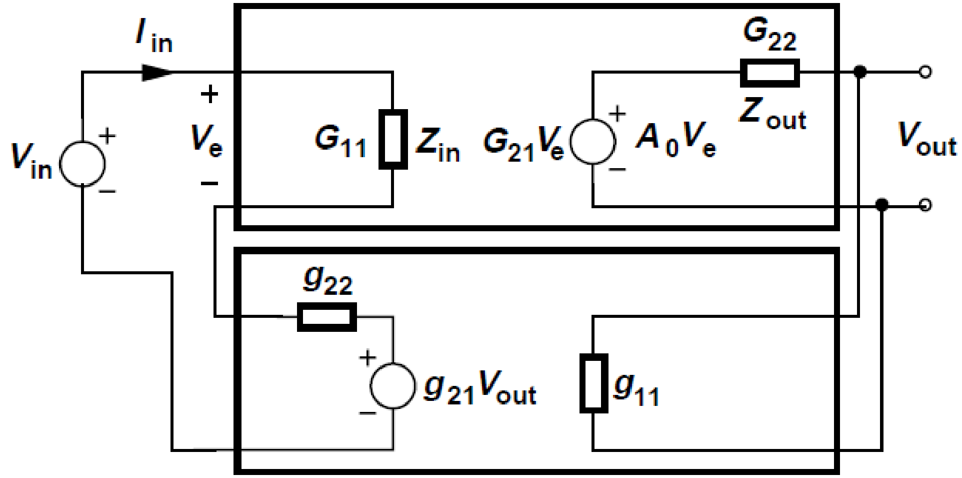
(b)

- **The closed-loop voltage gain is directly computed recognizing that g_{11} is an admittance and g_{22} is an impedance, and by writing a KVL around the input network and a KCL at the output node**

$$V_{in} = V_e + g_{22}\frac{V_e}{Z_{in}} + g_{21}V_{out}$$

$$g_{11}V_{out} + \frac{V_{out} - A_0V_e}{Z_{out}} = 0.$$

Loading in Voltage-Voltage Feedback



(b)

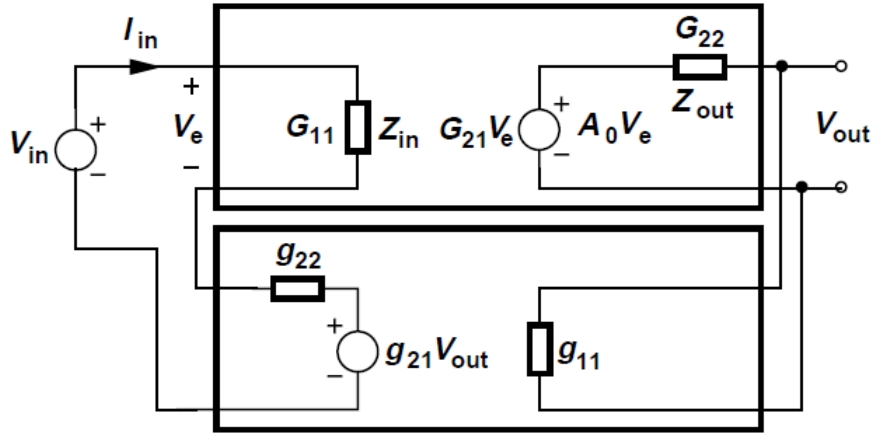
- **Eliminating V_e,**

$$\frac{V_{out}}{V_{in}} = \frac{A_0}{(1+\frac{g_{22}}{Z_{in}})(1+g_{11}Z_{out})+g_{21}A_0}$$

- **Expressing this in the form of $A_{v,open}/(1+\beta A_{v,open})$,**

$$\frac{V_{out}}{V_{in}} = \frac{\dfrac{A_0}{(1+\frac{g_{22}}{Z_{in}})(1+g_{11}Z_{out})}}{1+g_{21}\dfrac{A_0}{(1+\frac{g_{22}}{Z_{in}})(1+g_{11}Z_{out})}}$$

Loading in Voltage-Voltage Feedback



(b)

- **We can thus write,**

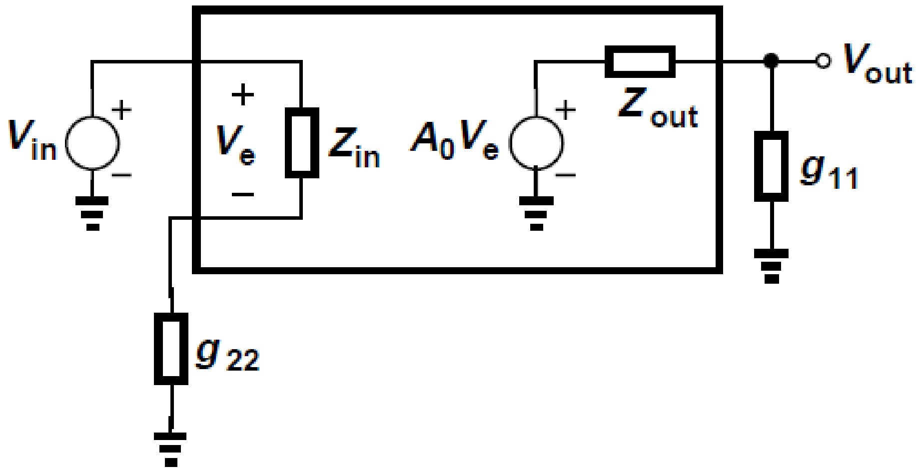
$$A_{v,open} = \frac{A_0}{(1+\frac{g_{22}}{Z_{in}})(1+g_{11}Z_{out})}$$

$$\beta = g_{21}.$$

- **The equivalent open-loop gain contains a factor A0, i.e., the original amplifier's voltage gain (before immersion in feedback)**
- **This gain is attenuated by two factors,** $1+g_{22}/Z_{in}$ **and** $1+g_{11}Z_{out}$

Loading in Voltage-Voltage Feedback

- The loaded forward amplifier is as shown below, excluding the two generators $G_{12}V_{out}$ and $g_{12}I_{in}$



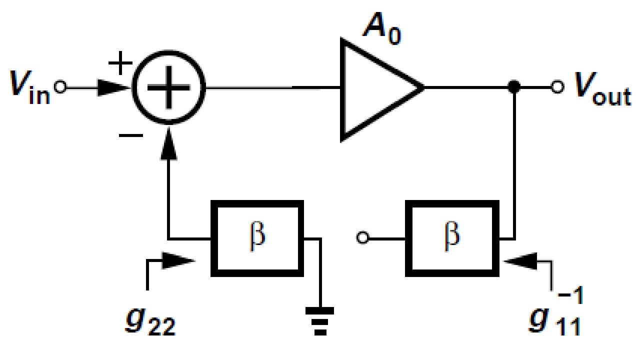
- Allows a quick and intuitive understanding not possible from direct analysis
- The finite input and output impedances of the feedback network reduce the output voltage and the voltage seen at the input of the main amplifier respectively

Loading in Voltage-Voltage Feedback

- g_{11} and g_{22} are computed as follows:

$$g_{11} = \left.\frac{I_1}{V_1}\right|_{I2=0}$$
$$g_{22} = \left.\frac{V_2}{I_2}\right|_{V1=0}$$

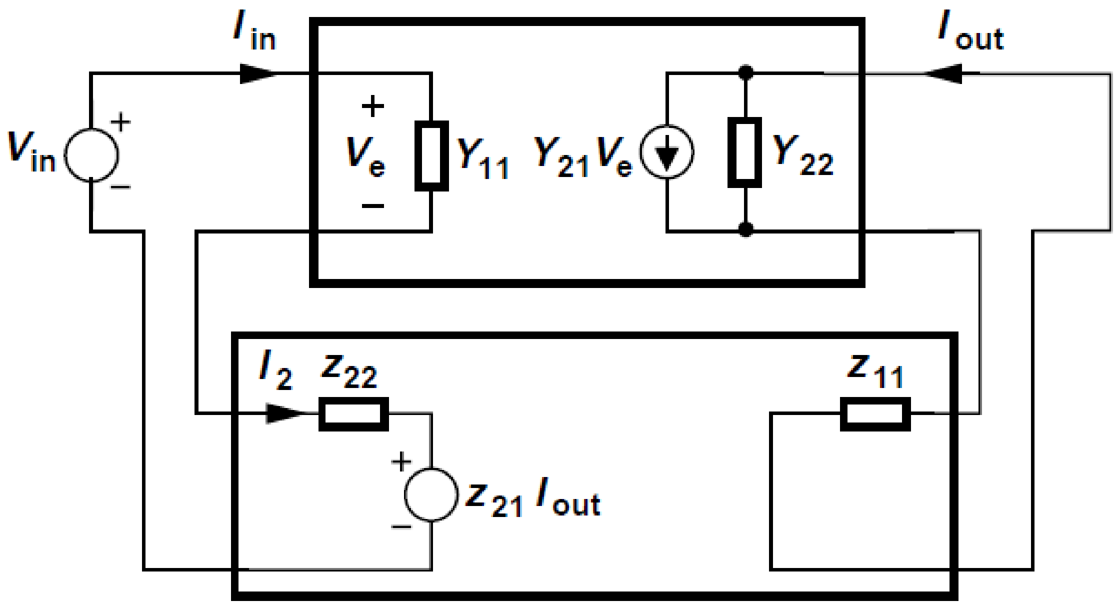
- As shown below, g_{11} is obtained by leaving the output of the feedback network open whereas g_{22} is calculated by shorting the input of the feedback network



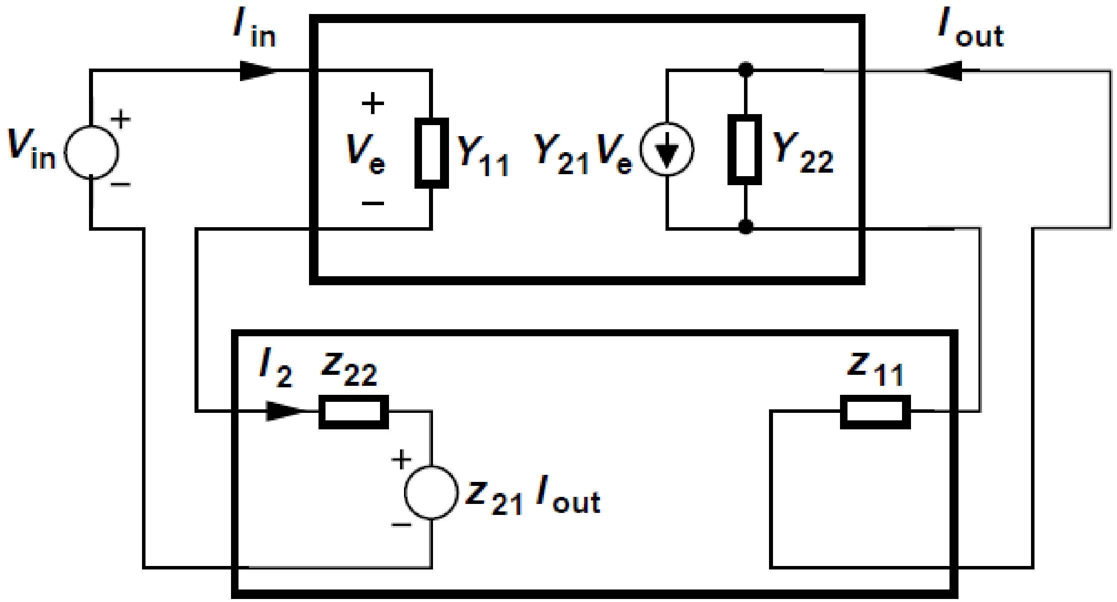
- Loop gain is simply the loaded open-loop gain multiplied by g_{21}
- Open-loop input and output impedances are scaled by $1 + g_{21}A_{v,open}$ to yield closed-loop values

Loading in Current-Voltage Feedback

- In this case, the feedback network appears in series with the output to sense the current
- Forward amplifier and feedback network are represented by Y and Z models respectively, neglecting the generators $Y_{12}V_{out}$ and $z_{12}I_{in}$, as shown below:



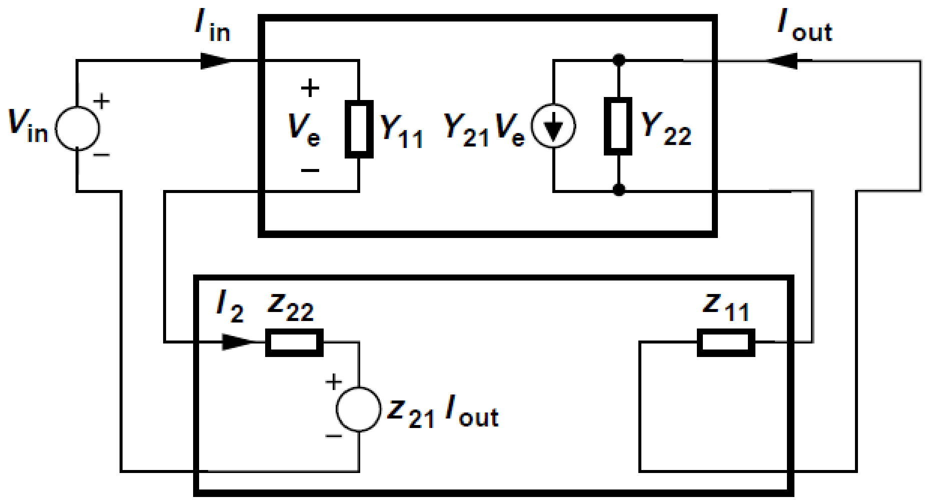
Loading in Current-Voltage Feedback



- **To compute the closed-loop gain I_{out}/V_{in}, and obtain open-loop parameters in the presence of loading, we note that $I_{in} = Y_{11}V_e$ and $I_2 = I_{in}$ and write two KVLs:**

$$\begin{aligned} V_{in} &= V_e + Y_{11}V_e z_{22} + z_{21}I_{out} \\ -I_{out}z_{11} &= \frac{I_{out} - Y_{21}V_e}{Y_{22}}. \end{aligned}$$

Loading in Current-Voltage Feedback



- **Eliminating V_e, we get**

$$\frac{I_{out}}{V_{in}} = \frac{\dfrac{Y_{21}}{(1+z_{22}Y_{11})(1+z_{11}Y_{22})}}{1 + z_{21}\dfrac{Y_{21}}{(1+z_{22}Y_{11})(1+z_{11}Y_{22})}}$$

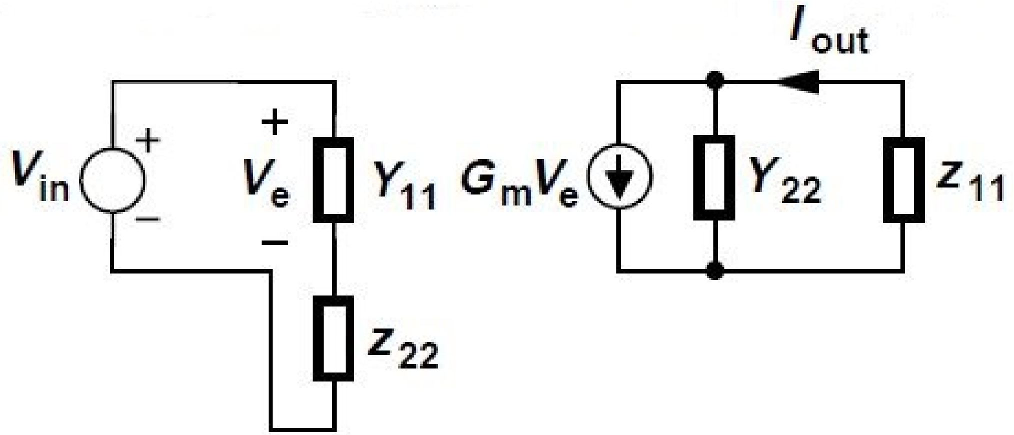
- **The loaded open-loop gain and feedback factor can be seen to be**

$$G_{m,open} = \frac{Y_{21}}{(1+z_{22}Y_{11})(1+z_{11}Y_{22})}$$

$$\beta = z_{21}.$$

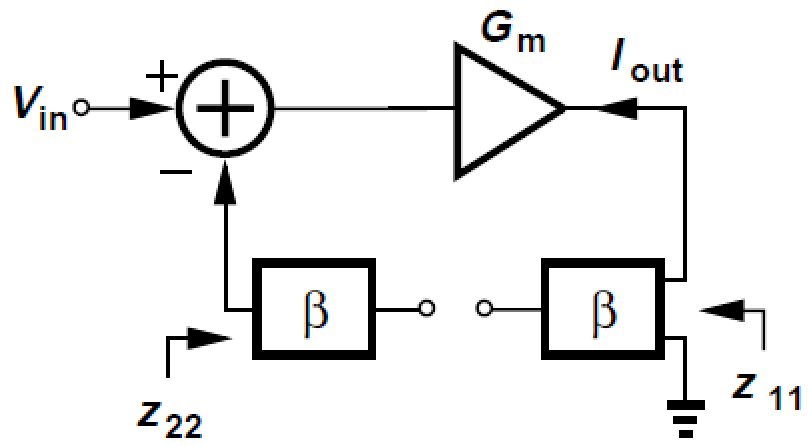
Loading in Current-Voltage Feedback

- Y_{21}, the transconductance gain of the original amplifier is attenuated by $(1 + z_{22}Y_{11})^{-1}$ and $(1 + z_{11}Y_{22})^{-1}$, which respectively correspond to voltage division at the input and current division at the output
- The loaded open-loop amplifier can be pictured as below



Loading in Current-Voltage Feedback

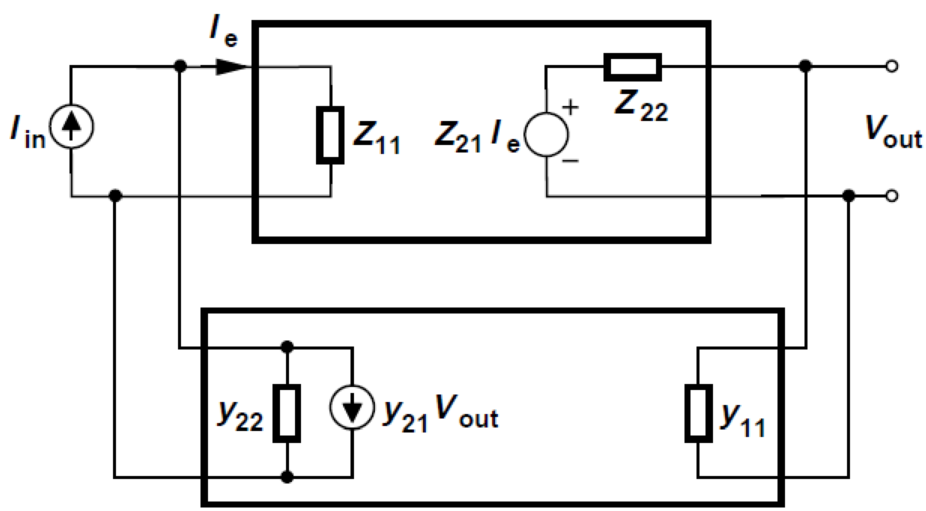
- Since $z_{22} = V_2/I_2$ with $I_1 = 0$ and $z_{11} = V_1/I_1$ with $I_2 = 0$, the conceptual picture below shows how to properly break the feedback



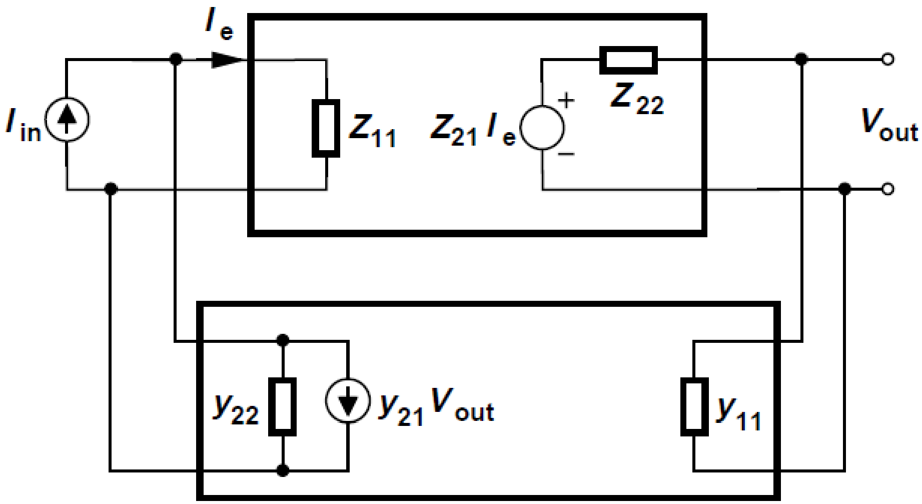
- The loop gain is $z_{21}G_{m,open}$

Loading in Voltage-Current Feedback

- In this configuration, the forward (transimpedance) amplifier generates an output voltage in response to the input current and can thus be represented by a Z model
- Feedback network lends itself to a Y model since it senses the output voltage and returns a proportional current
- The equivalent circuit below ignores the effect of Z_{12} and y_{12}



Loading in Voltage-Current Feedback



- **We compute the closed-loop gain, V_{out}/I_{in}, by writing two equations**

$$I_{in} = I_e + I_e Z_{11} y_{22} + y_{21} V_{out}$$

$$y_{11} V_{out} + \frac{V_{out} - Z_{21} I_e}{Z_{22}} = 0.$$

- **Eliminating I_e, we get**

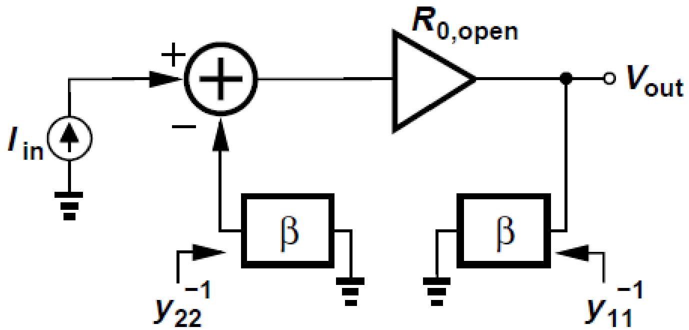
$$\frac{V_{out}}{I_{in}} = \frac{\dfrac{Z_{21}}{(1 + y_{22}Z_{11})(1 + y_{11}Z_{22})}}{1 + y_{21}\dfrac{Z_{21}}{(1 + y_{22}Z_{11})(1 + y_{11}Z_{22})}}$$

Loading in Voltage-Current Feedback

- **Thus, the equivalent open-loop gain and feedback factor are given by**

$$R_{0,open} = \frac{Z_{21}}{(1 + y_{22}Z_{11})(1 + y_{11}Z_{22})}$$
$$\beta = y_{21}.$$

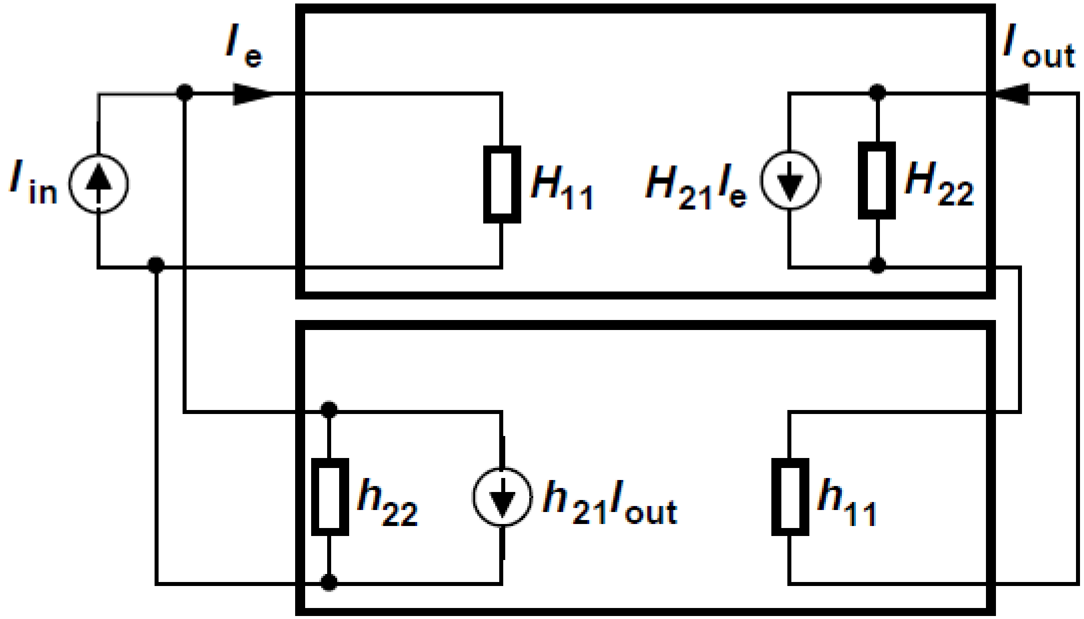
- **Interpreting the attenuation factors in $R_{0,open}$ as current division at the input and voltage division at the output, we arrive at the conceptual view shown below**



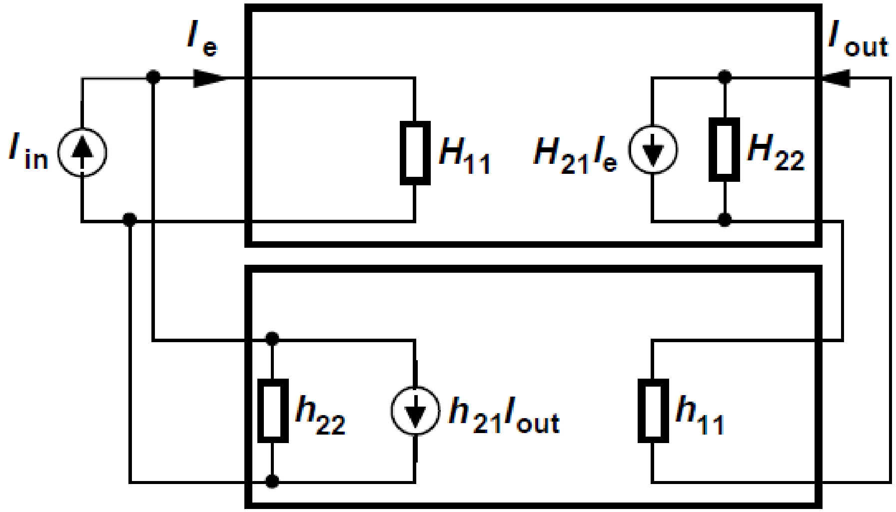
- **The loop gain is given by $y_{21}R_{0,open}$**

Loading in Current-Current Feedback

- The forward amplifier in this case generates an output current in response to the input current and can be represented by an H model and so can the feedback network
- The equivalent circuit with the H_{12} and h_{12} generators is shown below



Loading in Current-Current Feedback



- **We can write**

$$
\begin{aligned}
I_{in} &= I_e H_{11} h_{22} + h_{21} I_{out} + I_e \\
I_{out} &= -I_{out} h_{11} H_{22} + H_{21} I_e
\end{aligned}
$$

- **Eliminating I_e, we get the closed-loop gain I_{out}/I_{in}**

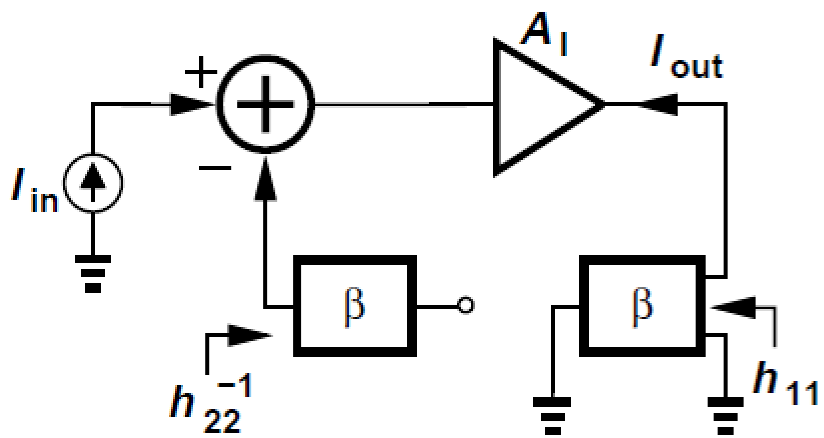
$$
\frac{I_{out}}{I_{in}} = \frac{\dfrac{H_{21}}{(1+h_{22}H_{11})(1+h_{11}H_{22})}}{1 + h_{21}\dfrac{H_{21}}{(1+h_{22}H_{11})(1+h_{11}H_{22})}}
$$

Loading in Current-Current Feedback

- **As with previous topologies, we define the equivalent open-loop current gain and the feedback factor as**

$$\begin{aligned} A_{I,open} &= \frac{H_{21}}{(1+h_{22}H_{11})(1+h_{11}H_{22})} \\ \beta &= h_{21}. \end{aligned}$$

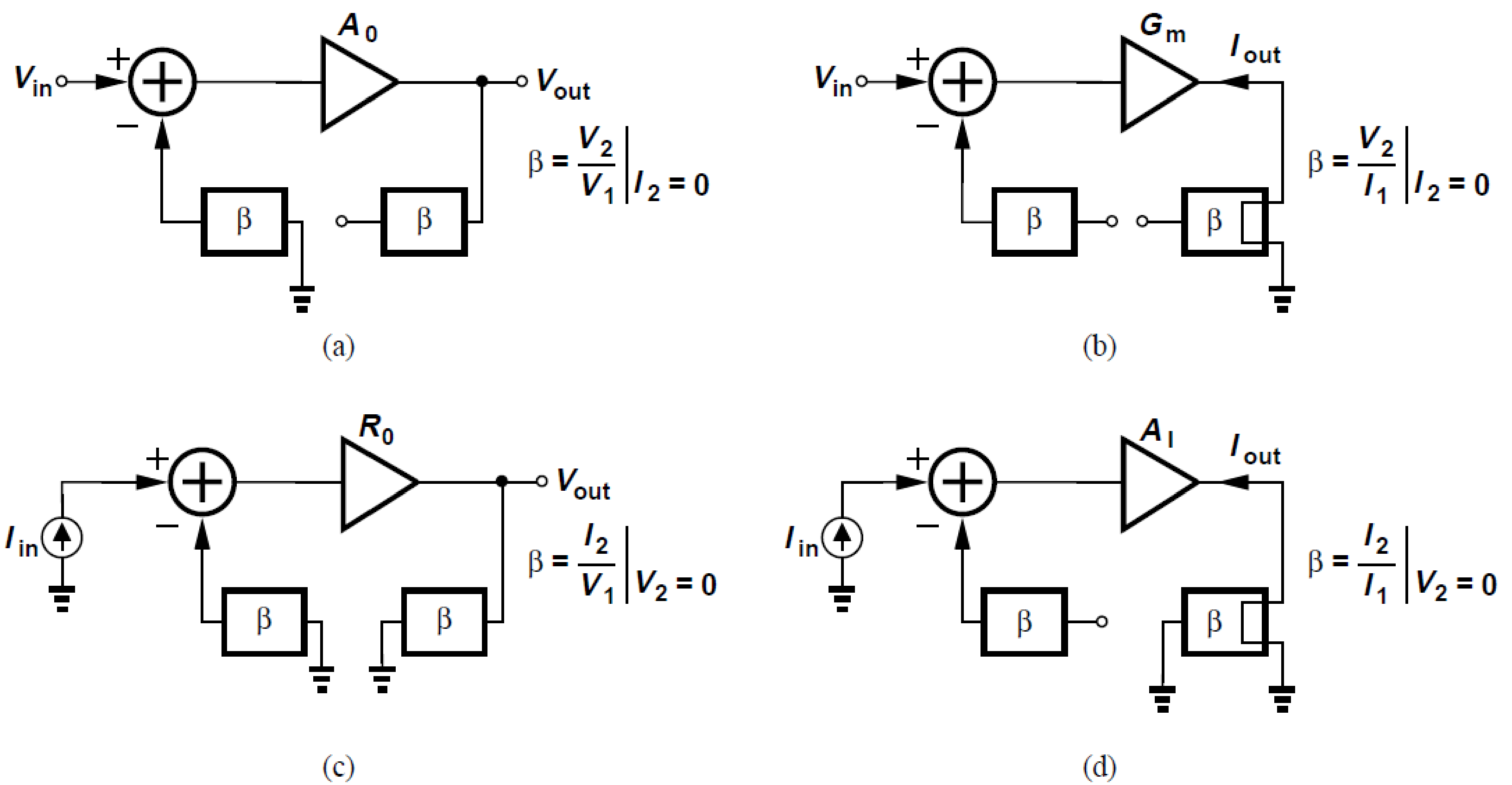
- **The conceptual view of the broken loop is shown below**



- **The loop gain is equal to $h_{21}A_{I,open}$**

Summary of Loading Effects

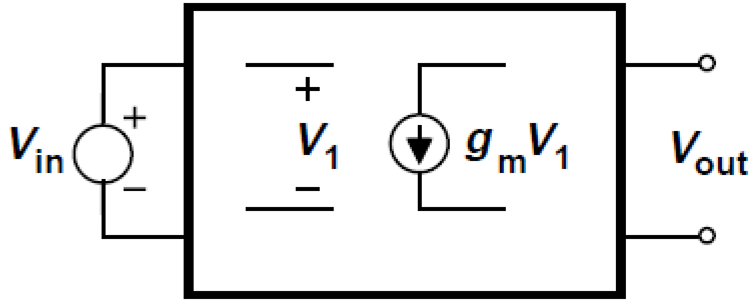
- Figs. (a) – (d) summarize the loading effects in all four topologies



Summary of Loading Effects

- **The analysis of loading is carried out in three steps:**
  1) **Open the loop with proper loading and calculate the open-loop gain, A_{OL}, and the open-loop input and output impedances**
  2) **Determine the feedback ratio β, and hence the loop gain, βA_{OL}**
  3) **Calculate the closed-loop gain and input and output impedances by scaling the open-loop values by a factor of $1+\beta A_{OL}$**
- **In the equations defining β, the subscripts 1 and 2 refer to the input and output ports of the feedback network, respectively**

Bode's Analysis of Feedback Circuits: Observations

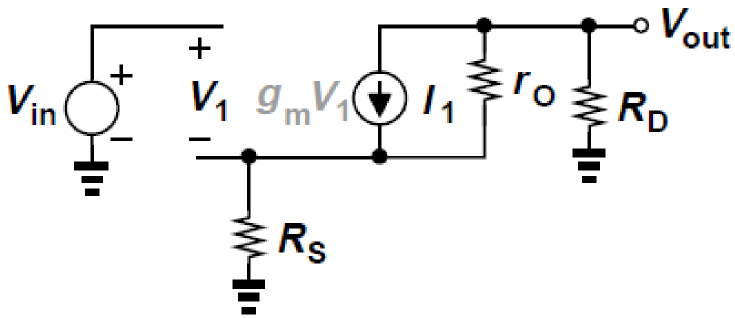


(a)

- Consider the general circuit in Fig. (a), where one transistor is explicitly shown in its ideal form
- From previous analysis, V_{out} can eventually be expressed as $A_v V_{in}$ or $H(s)V_{in}$
- If the dependent current source is denoted by I_1 and we do not make the substitution $I_1 = g_m V_1$, then V_{out} is obtained as a function of both V_{in} and I_1:

$$V_{out} = AV_{in} + BI_1$$

Bode's Analysis of Feedback Circuits: Observations



(b)

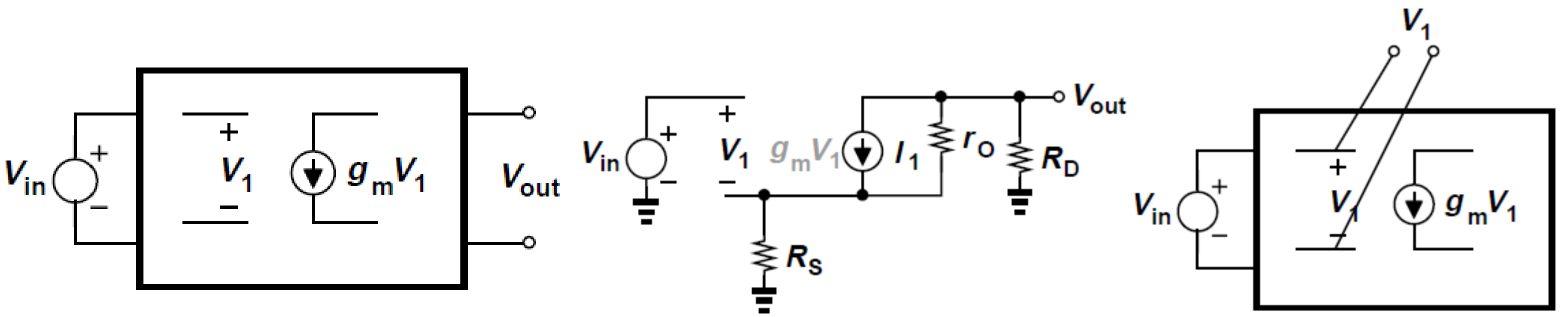
- As an example, in the degenerated CS stage of Fig. (b), we note that the current flowing upward through R_D (and downward through R_S) is $-V_{out}/R_D$ and hence the voltage drop across r_O is $(-V_{out}/R_D - I_1)r_O$
- KVL around the output network gives

$$V_{out} = \left(-\frac{V_{out}}{R_D} - I_1\right) r_O - \frac{V_{out}}{R_D} R_S$$

$$V_{out} = \frac{-r_O}{1 + \dfrac{r_O + R_S}{R_D}} I_1$$

- In this case, $A = 0$ and $B = -r_O R_D / (R_D + r_O + R_S)$

Bode's Analysis of Feedback Circuits: Observations



(a) (b) (c)

- **Next, consider V_1 as the signal of interest, i.e., we wish to compute V_1 as a function of V_{in} in the form of A_vV_{in} or $H(s)V_{in}$**
- **We can pretend that V_1 is the "output", as in Fig. (c)**
- **In a similar manner, V_1 can be written, if we temporarily forget that $I_1 = g_mV_1$, as**

$$V_1 = CV_{in} + DI_1$$

- **KVL around the output network gives**

$$V_1 = V_{in} - \frac{r_O R_S}{R_D + r_O + R_S} I_1$$

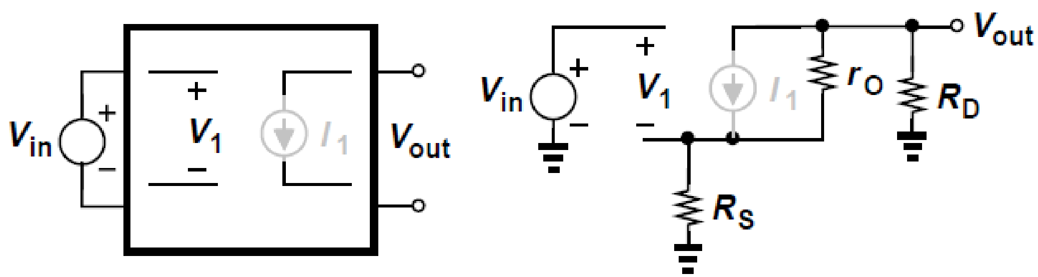
- **Hence,** $C = 1$ **and** $D = -r_O R_S/(R_D + r_O + R_S)$

Interpretation of Coefficients

- A is given by

$$A = \frac{V_{out}}{V_{in}} \text{ with } I_1 = 0$$

- A is obtained as the voltage gain of the circuit if the dependent current source is set to zero, by setting $g_m = 0$
- V_{out} in this case can be considered the "feedthrough" of the input signal (in the absence of the ideal transistor) [Fig. (a)]



(a)

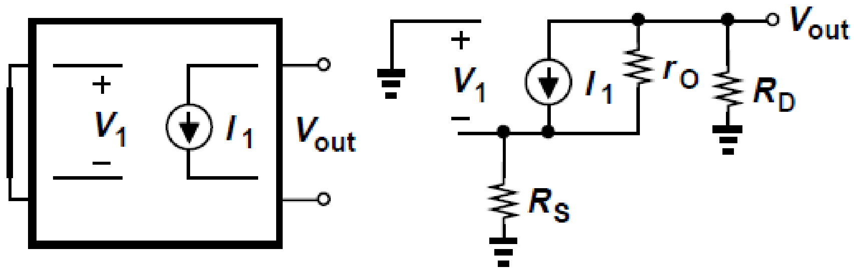
- In the CS example, $V_{out} = 0$ if $I_1 = 0$ because no current flows through R_S, r_O, and R_D, i.e., $A = 0$

Interpretation of Coefficients

- As for the *B* coefficient, we have

$$B = \frac{V_{out}}{I_1} \text{ with } V_{in} = 0$$

- We set the input to zero and compute V_{out} as a result of I_1 [Fig. (b)], pretending that I_1 is an independent source



(b)

- In the CS example,

$$\left(-\frac{V_{out}}{R_D} - I_1\right) r_O - \frac{V_{out}}{R_D} R_S = V_{out} \qquad V_{out} = \frac{-r_O R_D}{R_D + r_O + R_S} I_1$$

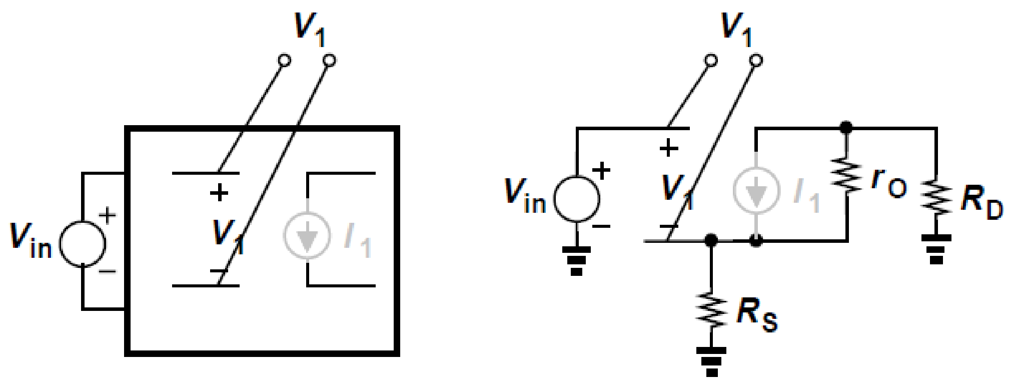
- Thus, $B = -r_O R_D / (R_D + r_O + R_S)$

Interpretation of Coefficients

- The C coefficient is interpreted as

$$C = \frac{V_1}{V_{in}} \text{ with } I_1 = 0$$

- This is the transfer function from the input to V_1 with the transistor's g_m set to zero [Fig. (c)]



(c)

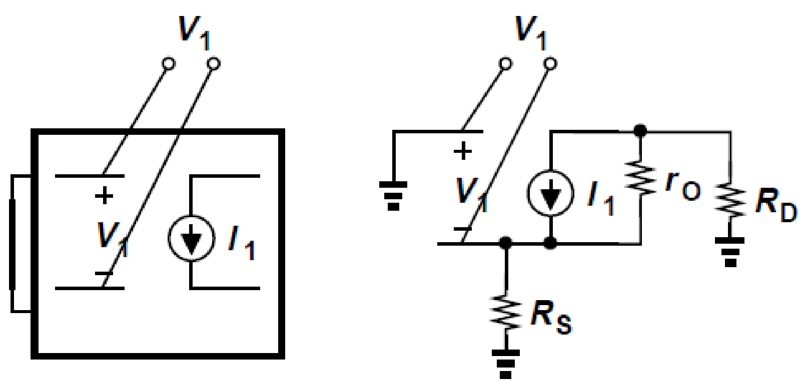
- In the CS circuit, no current flows through R_S under this condition, yielding $V_1 = V_{in}$ and $C = 1$

Interpretation of Coefficients

- Lastly, the D coefficient is obtained as

$$D = \frac{V_1}{I_1} \text{ with } V_{in} = 0$$

- As shown in Fig. (d), this represents the transfer function from I_1 to V_1 with the input at zero



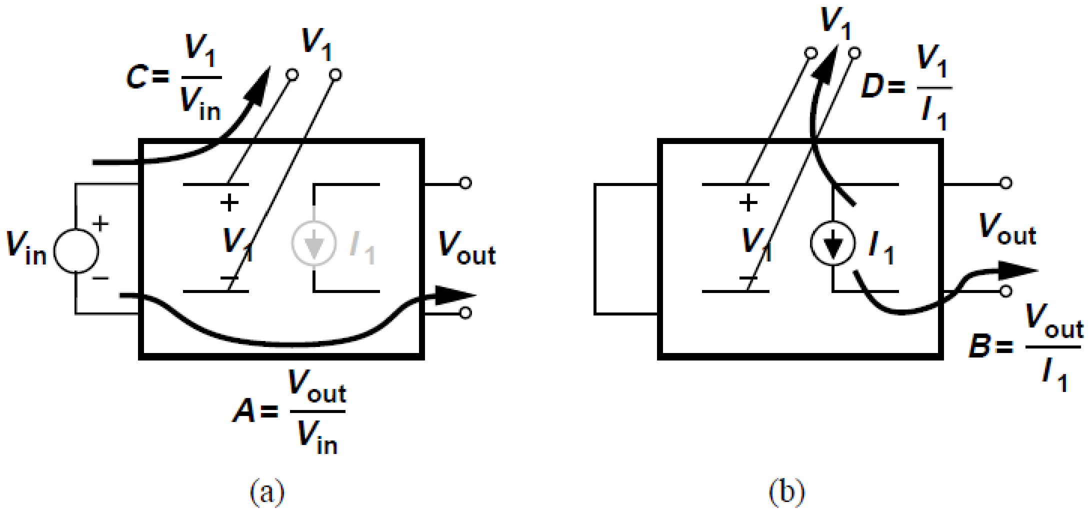
(d)

- In the CS example, under the above condition,

$$-V_1 - \left(\frac{V_1}{R_S} + I_1\right) r_O = \frac{V_1}{R_S} R_D \qquad V_1 = -\frac{r_O R_S}{R_D + r_O + R_S} I_1$$

- Hence, $D = -r_O R_S / (R_D + r_O + R_S)$

Interpretation of Coefficients: Summary



- In summary, the *A*-*D* coefficients are computed as shown in Figs. (a) and (b)
- We disable the transistor by setting its g_m to zero and obtain *A* and *C* as feedthroughs from V_{in} to V_{out} and to V_1 respectively
- We set the input to zero and calculate *B* and *D* as the gain from I_1 to V_{out} and to V_1 respectively
- The former step finds responses to V_{in} with $g_m = 0$ and the latter to I_1 with $V_{in} = 0$

Bode's Analysis

- V_{out}/V_{in} is expressed in terms of *A-D* coefficients
- Since

$$\begin{aligned} V_{out} &= AV_{in} + BI_1 \\ V_1 &= CV_{in} + DI_1 \end{aligned}$$

and in the actual circuit, $I_1 = g_m V_1$, we have

$$V_1 = \frac{C}{1 - g_m D} V_{in}$$

- The closed-loop gain is therefore equal to

$$\frac{V_{out}}{V_{in}} = A + \frac{g_m BC}{1 - g_m D}$$

- The first term represents the input-output feedthrough when $g_m = 0$
- We can also write

$$\frac{V_{out}}{V_{in}} = \frac{A + g_m(BC - AD)}{1 - g_m D}$$

Bode's Analysis: Observations

$$\frac{V_{out}}{V_{in}} = A + \frac{g_m BC}{1 - g_m D}$$

- **If $A = 0$, then closed-loop gain equation yields $V_{out}/V_{in} = g_m BC/(1-g_m D)$, which resembles the generic feedback equation $A_0/(1+\beta A_0)$**
- **$g_m BC$ is loosely called the "open-loop" gain**

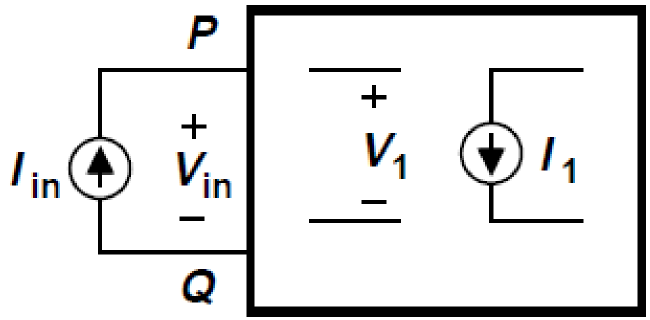
Bode's Analysis: Return Ratio and Loop Gain

$$\frac{V_{out}}{V_{in}} = \frac{A + g_m(BC - AD)}{1 - g_m D}$$

- The closed-loop gain expression above may suggest that *$1 - g_m D = 1 +$ loop gain* and hence *loop gain $= -g_m D$*
- In both cases, we set the main input to zero, break the loop by replacing the dependent source with an independent one, and compute the returned quantity
- In Bode's original treatment, the term "return ratio" *(RR)* is used to refer to $-g_m D$ and is ascribed to a given dependent source in the circuit
- *RR* appears to be the same as the true loop gain even if the loop cannot be completely broken
- *RR* is equal to the loop gain if the circuit contains only one feedback mechanism and the loop traverses the transistor of interest

Blackman's Impedance Theorem

- Blackman's theorem determines the impedance seen at any port of a general circuit
  - Can be proved using Bode's approach



(a)

- In the general circuit of Fig. (a), the impedance between nodes *P* and *Q* is of interest
- One of the transistors is explicitly shown by the voltage-dependent current source I_1

Blackman's Impedance Theorem

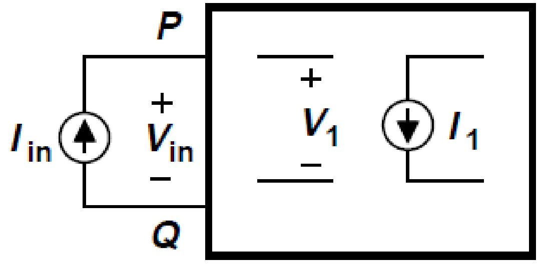
- Let us pretend that I_{in} is the input signal and V_{in} the output signal so that we can utilize Bode's results:

$$\begin{aligned} V_{in} &= AI_{in} + BI_1 \\ V_1 &= CI_{in} + DI_1 \end{aligned}$$

- It follows that

$$Z_{in} = \frac{V_{in}}{I_{in}} = A + \frac{g_m BC}{1 - g_m D}$$

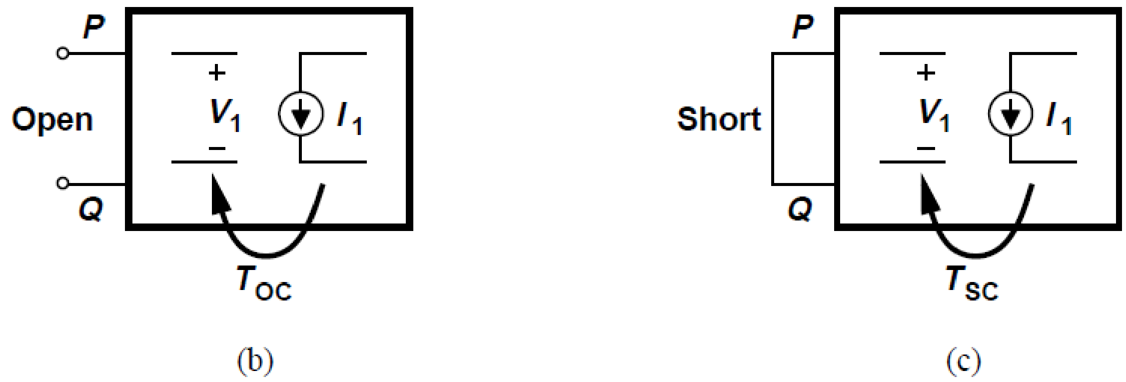
where g_m denotes the transconductance of the transistor modeled in Fig. (a)



(a)

Blackman's Impedance Theorem

- Recognizing that $V_1/I_1 = D$ if $I_{in} = 0$, we call $-g_m D$ the "open-circuit loop gain" (because the port of interest is left *open*) and denote it by T_{oc} [Fig. (b)]



- If $V_{in} = 0$, then $I_{in} = (-B/A)I_1$ and hence

$$\frac{V_1}{I_1} = \frac{AD - BC}{A}$$

- We call $-g_m$ times this quantity the "short-circuit" loop gain (because $V_{in} = 0$) and denote it by T_{sc} [Fig. (c)]

Blackman's Impedance Theorem

- Both T_{oc} and T_{sc} can be viewed as return ratios associated with I_1 for two circuit topologies

$$T_{oc} = -g_m \frac{V_1}{I_1}|_{Iin=0}$$

$$T_{sc} = -g_m \frac{V_1}{I_1}|_{Vin=0}$$

- In the third step, we use T_{oc} and T_{sc} to rewrite

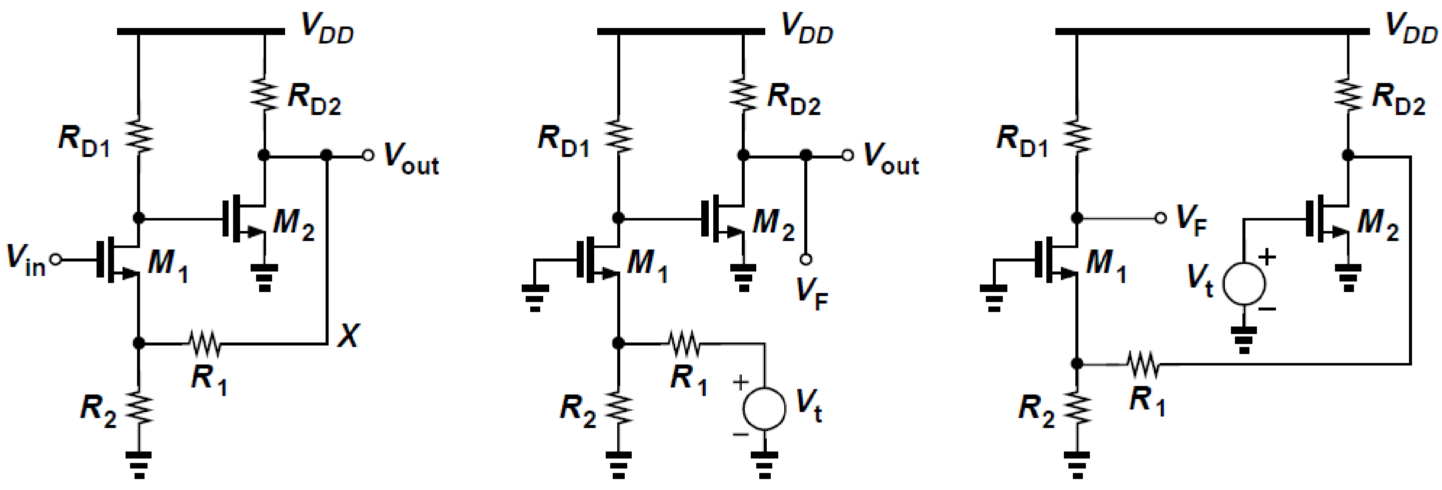
$$Z_{in} = \frac{V_{in}}{I_{in}} = \frac{A - g_m(BC - AD)}{1 - g_m D} = A\frac{1 + T_{sc}}{1 + T_{oc}}.$$

- *A* can be roughly viewed as the "open-loop" impedance without the transistor in the feedback loop
- In addition, if $|T_{sc}| \ll 1$ then $Z_{in} \approx A/(1 + T_{oc})$ and if $|T_{oc}| \ll 1$, then $Z_{in} \approx A(1 + T_{sc})$
- Closed-loop impedance cannot be expressed as Z_{in} multiplied or divided by *(1 + loop gain)*

Loop Gain Calculation Issues

- Loop gain plays a central role in feedback systems
- If poles and zeros in the loop are considered, then the loop gain [called "loop transmission" *T(s)* in this case] reveals circuit's stability properties
- Loop gain calculation proceeds as
  - Break the loop at some point, apply a test signal, follow it around the loop (in the proper direction), and obtain the returned signal
- This elicits two questions:
  1) Can the loop be broken at any arbitrary point?
  2) Should the test signal be a voltage or current?
- In such a test, the actual input and output disappear

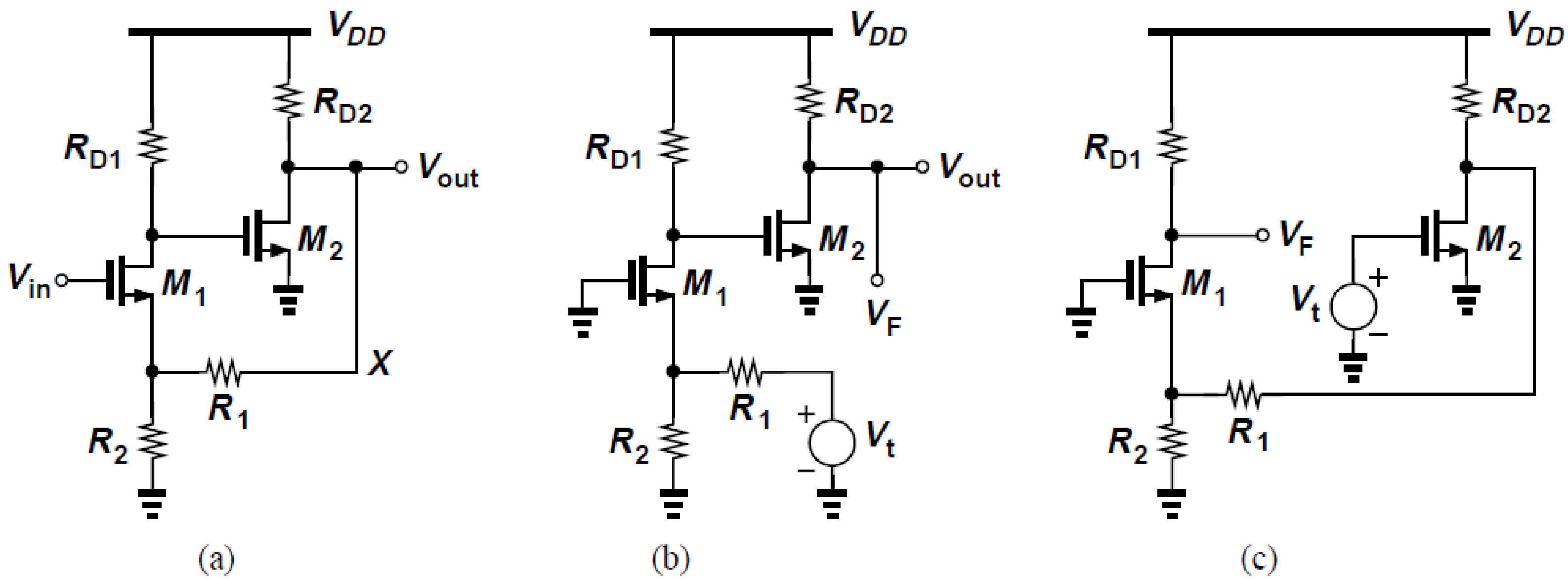
Loop Gain Calculation Issues



(a) (b) (c)

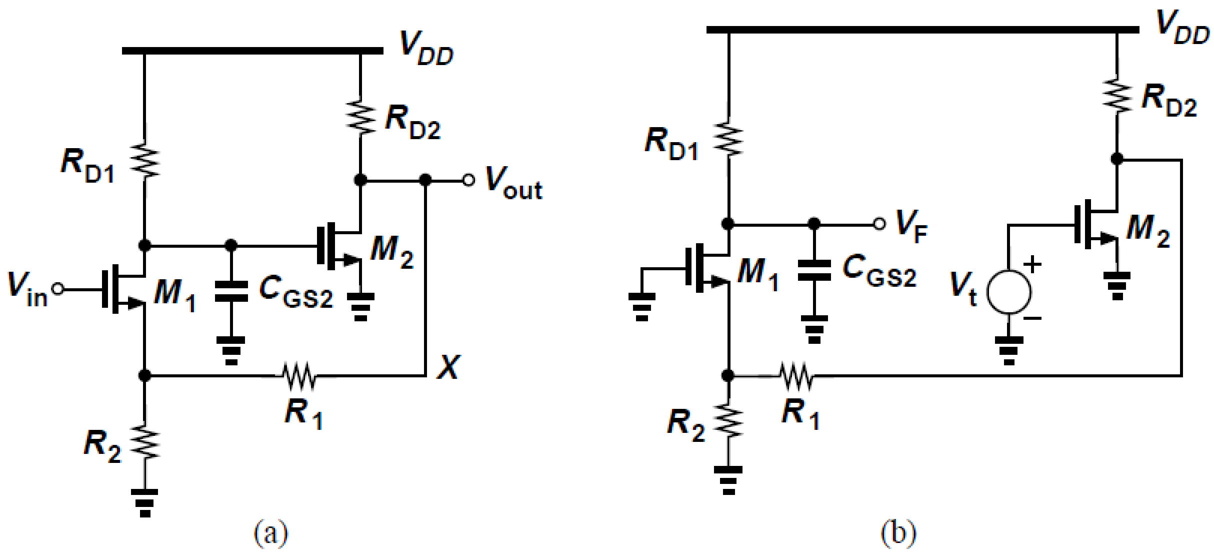
- In the two-stage amplifier of Fig. (a), resistive divider consisting of R_1 and R_2 senses output voltage and returns a fraction to source of M_1
- As shown in Fig. (b), we set V_{in} to zero, break the loop at node X, apply a test signal to the right terminal of R_1 and measure the resulting V_F
- In circuit of Fig. (a), R_1 draws an ac current from R_{D2} but in Fig. (b), it does not
- Gain of second CS stage has been altered

Loop Gain Calculation Issues



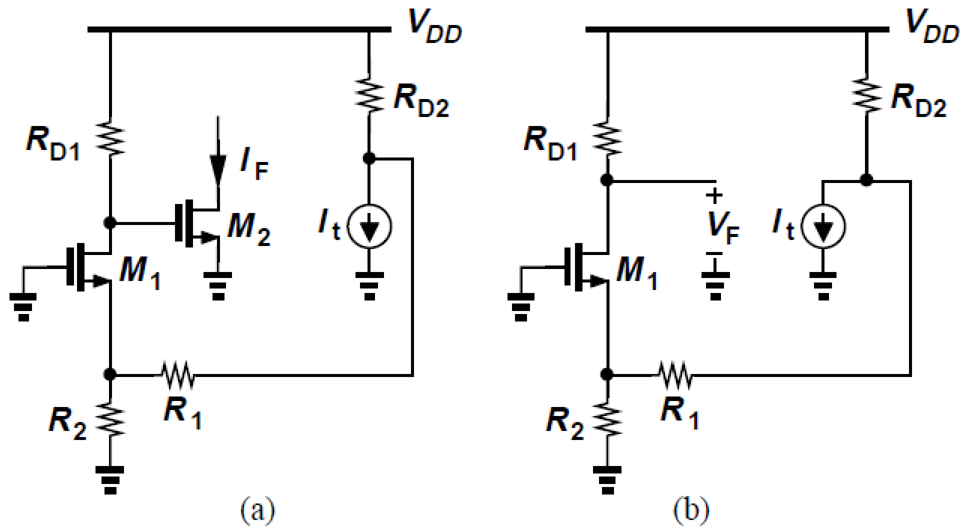
- **It is best to break the loop at the *gate* of a MOSFET**
- **We can break the loop at the gate of M_2 [Fig. (c)] and thus not alter the gain associated with first stage at low frequencies**

Loop Gain Calculation Issues



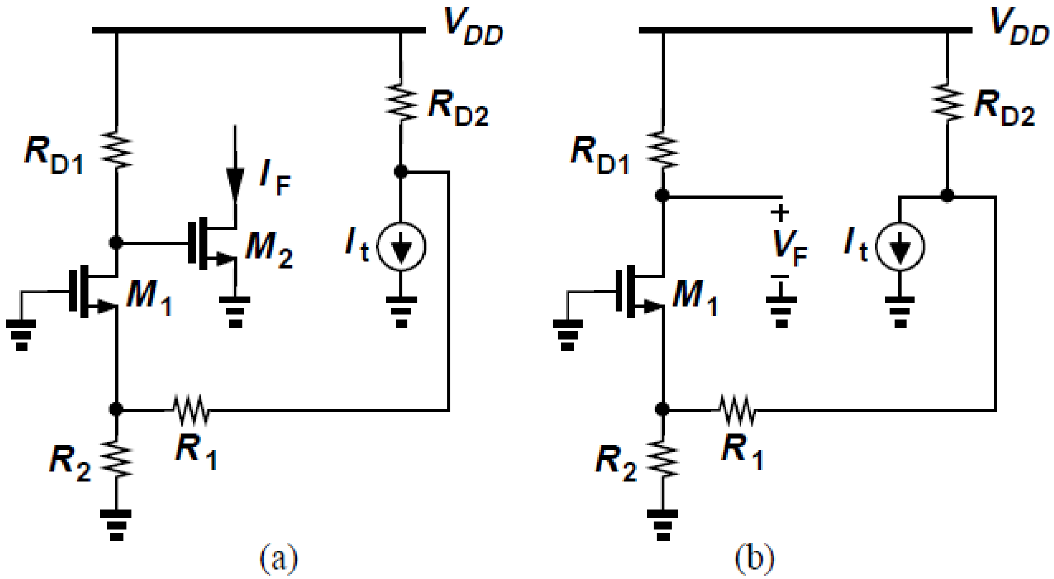
- To include C_{GS} of M_2 [Fig. (a)], we break the loop after C_{GS2} [Fig. (b)] to ensure that the load seen by M_1 remains unchanged
- It is always possible to break the loop at the gate of a MOSFET
- For the feedback to be negative, the signal must be sensed by at least one gate in the loop because only the common-source topology inverts signals

Loop Gain Calculation Issues



- **Can we apply a test current instead of a test voltage?**
- **We can break the loop at the drain of M_2, inject a current I_t, and measure the current returned by M_2 [Fig. (a)]**
- **If drain of M_2 is tied to ac ground, this node does not experience voltage excursions as in closed-loop circuit – when r_{O2} is taken into account**
- **In general, cannot inject I_t without altering some aspects of the circuit**

Loop Gain Calculation Issues



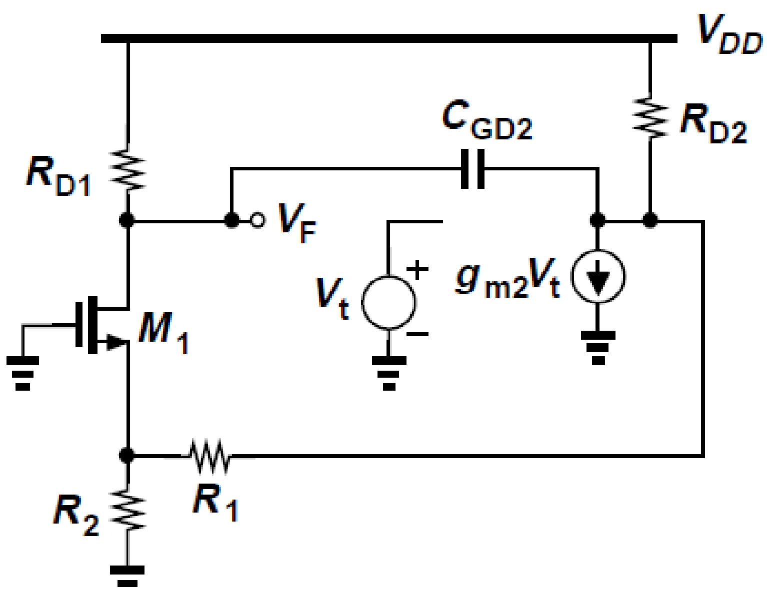
- **If controlled current source of M_2 is replaced with an independent current source It, and compute the returned V_{GS} as V_F [Fig. (b)]**
- **Since in the original circuit, the dependent source and V_{GS2} were related by a factor of g_{m2}, the loop gain can be written as $(-V_F/I_t) \times g_{m2}$**
- **This approach is feasible even if M_2 is degenerated**
- **This result is the same as return ratio of M_2**

Loop Gain Calculation Issues

- **In summary, the "best" place to break a feedback loop is**
  - **The gate-source of a MOSFET if voltage injection is desired**
  - **The dependent current source of a MOSFET if current injection is desired (provided that the returned quantity is VGS of the MOSFET)**
- **These two methods are related because they differ only by a factor of g_m**

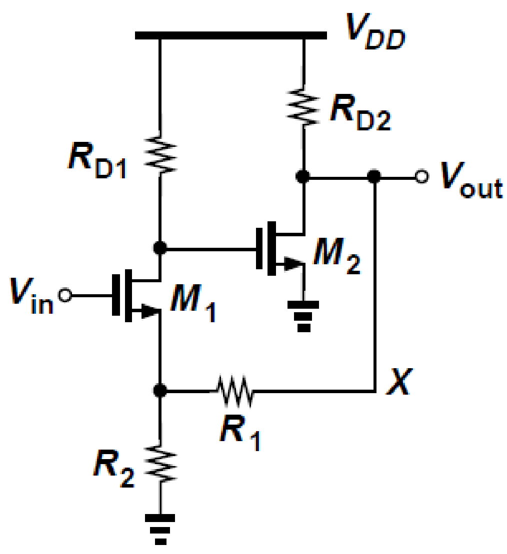
Loop Gain Calculation Issues

- **If we include C_{GD2} in previous circuit and inject a test voltage or current, C_{GD2} does not allow a "clean break"**
- **As shown below, even though gate-source voltage is provided by the independent source V_t, C_{GD2} creates "local" feedback from the drain of M_2 to its gate, raising the question whether loop gain should be obtained by nulling all feedback mechanisms**



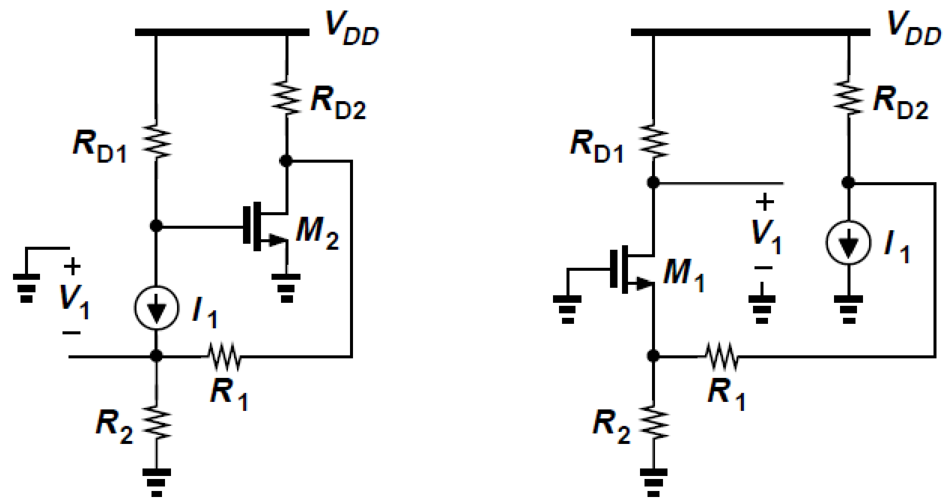
Difficulties with Return Ratio

- We may view the return ratio associated with a given dependent source as the loop gain
- Circuits containing more than one feedback mechanism exhibit different return ratios for different ratios
- In circuit of Fig. (a) below, R_1 and R_2 provide both "global" and "local" feedback (by degenerating M_1)



(a)

Difficulties with Return Ratio



(a) (b)

- Using equivalent circuits of Figs. (a) and (b), it can be shown that return ratios for M_1 and M_2 are given by

$$\text{Return Ratio}|_{M1} = \frac{g_{m1}R_2(R_1 + R_{D2} + g_{m2}R_{D2}R_{D1})}{R_1 + R_2 + R_{D2}}$$

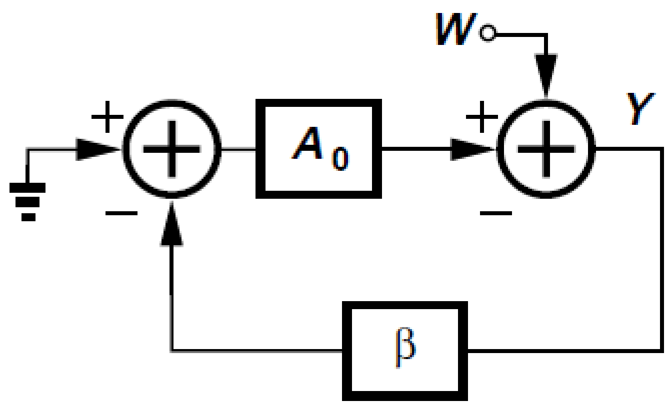
$$\text{Return Ratio}|_{M2} = \frac{g_{m1}g_{m2}R_2R_{D1}R_{D2}}{(1 + g_{m1}R_2)(R_1 + R_{D2}) + R_2}$$

- Different return ratios obtained because disabling M_1 removes both feedback mechanisms while disabling M_2 still retains degeneration experienced by M_1

Difficulties with Return Ratio

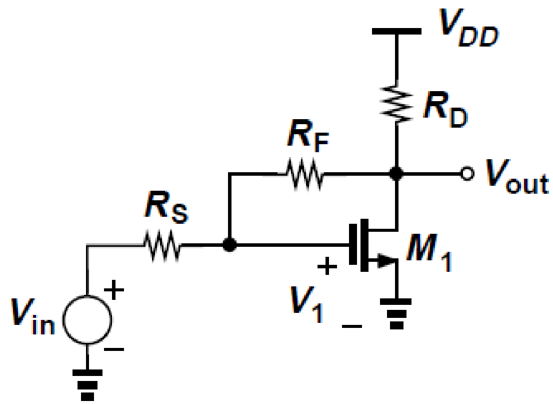
- **Another method of loop gain calculation is to inject a signal without breaking the loop as shown in figure below and write $Y/W = 1/(1 + \beta A_0)$ and hence**

$$\text{Loop Gain} = \left(\frac{Y}{W}\right)^{-1} - 1$$

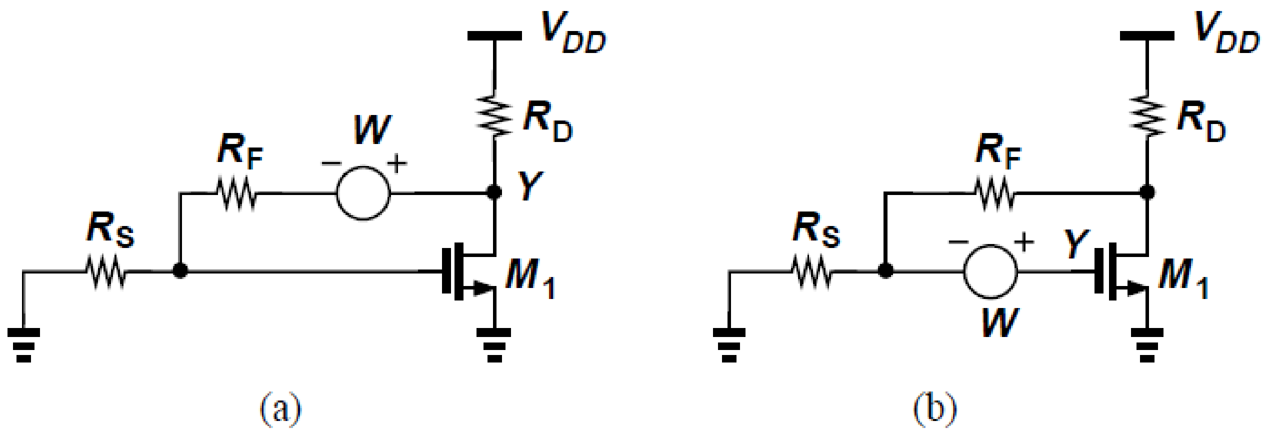


- **This method assumes a unilateral loop, yielding different loop gains for different injection points if the loop is not unilateral**

Difficulties with Return Ratio



- As an example, above circuit can be excited as in Figs. (a) or (b), producing different values for $(Y/W)^{-1} - 1$



Alternative Interpretations of Bode's Method

- **<u>Asymptotic Gain Form:</u>**
- **From Bode's results,** $V_{out}/V_{in} = A + g_m BC/(1 - g_m D)$ **and** $V_{out}/V_{in} = A$ if $g_m = 0$ **(the dependent source is disabled) and** $V_{out}/V_{in} = A - BC/D$ if $g_m \rightarrow \infty$ **(the dependent source is "very strong")**
- **We denote these values of V_{out}/V_{in} by H_0 and H_∞ respectively, and $-g_m D$ by T**
- **H_0 can be considered as the direct feedthrough and H_∞ as the "ideal gain". i.e., if the dependent source were infinitely strong (or if the loop gain were infinite)**
- **It follows that**

$$\begin{aligned}
\frac{V_{out}}{V_{in}} &= H_0 + \frac{g_m BC}{1+T} \\
&= H_0 \frac{1+T}{1+T} + \frac{g_m BC}{1+T} \\
&= \frac{H_0}{1+T} + \frac{T(H_0 + g_m BC/T)}{1+T}
\end{aligned}$$

Alternative Interpretations of Bode's Method

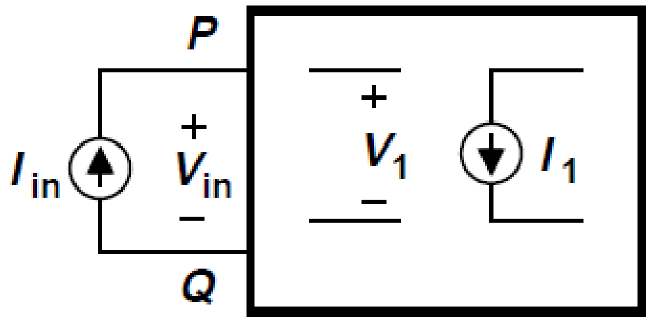
- **<u>Asymptotic Gain Form (contd.):</u>**
- **Since** $H_0 + g_m BC/T = A - g_m BC/(g_m D) = A - BC/D = H_\infty$ **we have,**

$$\frac{V_{out}}{V_{in}} = H_\infty \frac{T}{1+T} + H_0 \frac{1}{1+T}$$

- **Called the "asymptotic gain equation", this form reveals that the gain consists of an ideal value multiplied by *T/(1 + T)* and a direct feedthrough multiplied by *1/(1 + T)***
- **Calculations are simpler here if we recognize from** $V_1 = CV_{in} + DI_1$ and $I_1 = g_m V_1$ **that** $V_1 = CV_{in}/(1 - g_m D) \to 0$ if $g_m \to \infty$ (provided that $V_{in} < \infty$).
- **This is similar to how a virtual ground is created if the loop gain is large**

Alternative Interpretations of Bode's Method

- **Double Null Method:**
- **From Blackman's Impedance Theorem, we recognize that [refer Fig. (a)]**
  - T_{oc} **is the return ratio with** $I_{in} = 0$**, i.e.,** T_{oc} **denotes the RR with the input set to zero**
  - T_{sc} **is the RR with** $V_{in} = 0$**, i.e.,** T_{sc} **represents the RR with the output forced to zero**



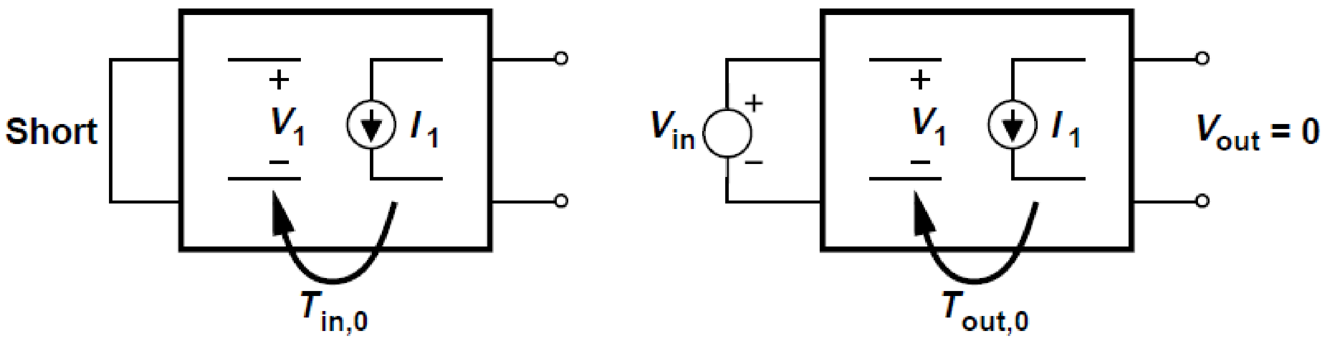
(a)

Alternative Interpretations of Bode's Method

- **Double Null Method (contd.):**
- **Making a slight change in our notation, we postulate that the transfer function of a given circuit can be written as**

$$\frac{V_{out}}{V_{in}} = A\frac{1+T_{out,0}}{1+T_{in,0}}$$

- **Where $A = V_{out}/V_{in}$ with the dependent source set to zero, and Tout,0 and Tin,0 respectively denote the return ratios for $V_{out} = 0$ and $V_{in} = 0$**



(a) (b)

Alternative Interpretations of Bode's Method

- <u>**Double Null Method (proof):**</u>
- **Beginning from**

$$\begin{aligned} V_{out} &= AV_{in} + BI_1 \\ V_1 &= CV_{in} + DI_1 \end{aligned}$$

- **We observe that if**

  $V_{in} = 0$, then $V_1/I_1 = D$ and hence $T_{in,0} = -g_m D$

- **On the other hand, if**

  $V_{out} = 0$, then $V_{in} = (-B/A)I_1$ and hence $V_1/I_1 = (AD - BC)/A$
  i.e., $T_{out,0} = -g_m(AD - BC)/A$

- **Combining these results yields**

$$\frac{V_{out}}{V_{in}} = A\frac{1 + T_{out,0}}{1 + T_{in,0}}$$

- **Division by *A* in these calculations assumes *A ≠ 0***