# Solution: Sudoku 2

First, define the zones of the board:

$$\text{Rows} = \{\{(i,j) \mid j \in \{1,2,3,4\}\} \mid i \in \{1,2,3,4\}\}$$

$$\text{Cols} = \{\{(i,j) \mid i \in \{1,2,3,4\}\} \mid j \in \{1,2,3,4\}\}$$

$$\begin{aligned}\text{Boxes} = \{&\{(1,1),(1,2),(2,1),(2,2)\},\\ &\{(1,3),(1,4),(2,3),(2,4)\},\\ &\{(3,1),(3,2),(4,1),(4,2)\},\\ &\{(3,3),(3,4),(4,3),(4,4)\}\}\end{aligned}$$

$$\text{Zones} = \text{Rows} \cup \text{Cols} \cup \text{Boxes}$$

Next, create sentences corresponding to various constraints.

"digit $d$ appears at least once in each zone"

$$\Delta_{\geq 1}(d, A) = \{Y_{a_1,d} \vee \cdots \vee Y_{a_n,d}\} \quad \text{where } d \in \{1,2,3,4\},\ A = \{a_1, \ldots, a_n\} \subseteq \text{Addresses}$$

"digit $d$ appears at most once in each zone"

$$\Delta_{\leq 1}(d, A) = \{\neg Y_{ad} \vee \neg Y_{a'd} \mid a, a' \in A, a \neq a'\}$$

where $d \in \{1,2,3,4\}$, $A \subseteq \text{Addresses}$

"no cell is empty"

$$\Delta_{\text{nonempty}} = \{Y_{a1} \vee Y_{a2} \vee Y_{a3} \vee Y_{a4} \mid a \in \text{Addresses}\}$$

Now, put all the sentences together into a single set.

$$\Delta = \Delta_{\text{nonempty}} \cup \left( \bigcup_{\substack{A \in \text{Zones} \\ d \in \{1,2,3,4\}}} \Delta_{\geq 1}(d, A) \right) \cup \left( \bigcup_{\substack{A \in \text{Zones} \\ d \in \{1,2,3,4\}}} \Delta_{\leq 1}(d, A) \right)$$

Finally, conjoin them into a single sentence!

$$\bigwedge_{\sigma \in \Delta} \sigma$$