Propositional Logic

① Try to answer the following:

"The longest day of the year (in terms of number of daylight hours) occurs during which month?"

It's likely you might have a reflex answer to this question:

But then we might follow up with the question:

"In Australia, the longest day of the year occurs during which month?"

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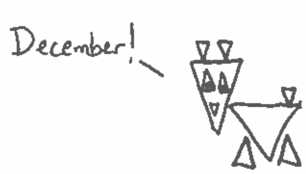
② At this point, some reasoning is required.

Australia is in the Southern hemisphere.

In the Southern hemisphere, the summer solstice occurs in December.

The summer solstice is the longest day of the year.

Thus, in Australia, the longest day of the year is in December.



③ What just happened? Well, one (unlikely) theory of cognition is that we just have a big table that stores answers to all questions we might be asked:

Place	Longest Day Occurs
Canada	June
Egypt	June
China	June
Australia	December
Fiji	December

Of course, then for any city, town, geographic feature, etc. we'd need a separate table entry.

④ A more acceptable theory is that we do store some set of facts, but we can derive more facts through the application of <u>rules of inference.</u>

known facts:

- Australia is in the Southern hemisphere
- In the Sothern hemisphere, the summer solstice occurs in December
- The summer solstice is the longest day of the year.

derived facts:

- In the Southern hemisphere, the longest day of the year occurs in December
- In Australia, the longest day of the year is in December

PROPOSITIONAL LOGIC

⑤ This reasoning seems very natural, but what are we actually doing? Can we create a mathematical model that replicates this process? This is what the study of logic attempts to do.

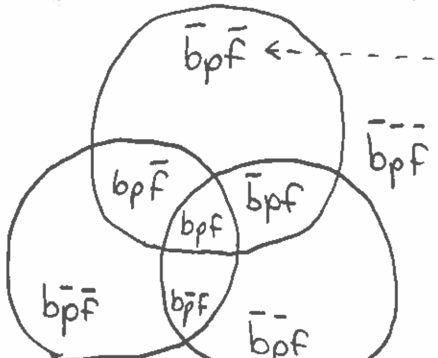
⑥ We'll begin by focusing on if-then statements. Consider the following simple example:

> If you are a penguin, then you cannot fly.
> Suppose you can fly.
> Are you a penguin?

⑦ Fundamentally, we can boil this down into eight "possible worlds" that consider three criteria:

- whether you are a BIRD
- whether you are a PENGUIN
- whether you can FLY

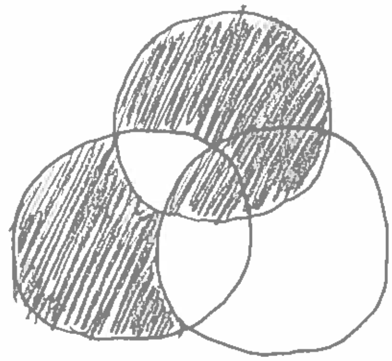
Using a Venn diagram, we can visualize the scenarios:



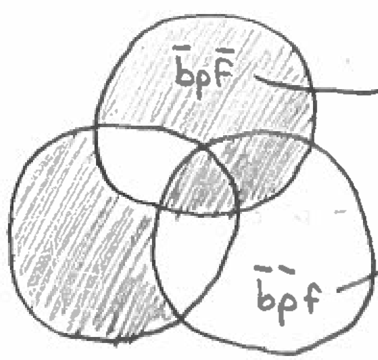
shorthand for
- NOT a bird ($\bar{b}$)
- IS a penguin (p)
- Can NOT fly ($\bar{f}$)

Propositional Logic

⑧ We might consider some scenarios to be impossible. For instance, $\bar{b}p\bar{f}$ is impossible because a penguin is a bird by definition (so it can't be a penguin but not a bird). We could color in impossible scenarios:



⑨ But it's more standard to use a truth table:



Bird	Penguin	Fly	
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

how many possible truth tables?

(10) A truth table can answer many questions:

- if you can fly, does that mean you are a bird?

B	P	F	
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

(i) consider the possible worlds

B	P	F
0	0	0
0	0	1
1	0	1
1	1	0

(ii) look at the worlds where you can fly

B	P	F
0	0	1
1	0	1

(iii) are you always a bird?

NO.

- if a bird cannot fly, does that mean it's a penguin?

B	P	F	
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

(i) consider the possible worlds

B	P	F
0	0	0
0	0	1
1	0	1
1	1	0

(ii) look at the worlds where you are a bird and you cannot fly

B	P	F
1	1	0

(iii) are you always a penguin?

YES.

PROPOSITIONAL LOGIC*

(11) Unfortunately, it's impractical to represent knowledge with truth tables:

A_1	A_2	...	A_n	
0	0		0	
0	0		1	
0	0		0	
0	0		1	
⋮	⋮		⋮	
1	1		0	
1	1		1	

2^n rows = too many to fit in memory for moderately large n

(12) Propositional logic allows us to represent knowledge more compactly and answer questions more efficiently. Essentially, it is a language for which every <u>sentence</u> encodes a truth table. We can view it as an encoding of the space of truth tables.

e.g. $B \wedge P$ encodes

B	P	F	
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

$(B \wedge \neg P) \Rightarrow F$ encodes

B	P	F	
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

Language is a choice of what we prioritize as important (and atomic). There is a reason why the word "in" is short and the word "disestablishmentarianism" is long.