HW: Heuristic Search

Prove that if H is a consistent heuristic function, then H is admissible. Hint: try induction on the length of the optimal completion path from state q.

Proof: Suppose H is consistent. Show $H(q) \leq H^*(q)$ $\forall q \in Q$.

base case: Let $q \in F$. Then $H(q) = H^*(q) = 0$, so $H(q) \leq H^*(q)$.

inductive step: Assume $H(q') \leq H^*(q')$ for any state $q' \in Q$ with an optimal completion path of length at most k.
Consider state q with optimal completion path $\langle \delta_1, \dots, \delta_{k+1} \rangle$, where transition $\delta_1 = (q, \sigma_1, q')$.
Observe that $\langle \delta_2, \dots, \delta_{k+1} \rangle$ is a completion path for state q', thus q' has an optimal completion path of length at most k.
By the inductive hypothesis:

$$H(q') \leq H^*(q')$$

Moreover:

$$\begin{aligned} H(q) &\leq w(q, \sigma_1, q') + H(q') && \text{[b/c } H \text{ is consistent]} \\ &\leq w(q, \sigma_1, q') + H^*(q') && \text{[inductive hypo]} \\ &= H^*(q) && \text{[b/c } \langle \delta_1, \dots, \delta_{k+1} \rangle \text{ is the optimal completion path for state } q' \text{]} \end{aligned}$$

Thus $H(q) \leq H^*(q)$ for any state q with an optimal completion path of length $k+1$.

By induction, $H(q) \leq H^*(q)$ for all states $q \in Q$. ■