

HW: Sudoku 2

Now we have a language for logical reasoning, let's use it to express the rules of a valid Sudoku solution. We'll adopt the following signature (slightly modified from the solution to Sudoku 1):

$$\Sigma = \{ Y_{ad} \mid a \in \text{ADDRESSES}, d \in \{1, 2, 3, 4\} \}$$

("addresses")

where ADDRESSES contains the 16 cell locations \forall in a 2×2 board:

$$\text{ADDRESSES} = \{ (1, 1), (1, 2), (1, 3), (1, 4), \\ (2, 1), (2, 2), (2, 3), (2, 4), \\ (3, 1), (3, 2), (3, 3), (3, 4), \\ (4, 1), (4, 2), (4, 3), (4, 4) \}$$

Your job is to come up with a sentence $\gamma \in \mathcal{L}(\Sigma)$ from the propositional language such that its interpretation $I(\gamma)$ is exactly the models of $M(\Sigma)$ that correspond to valid Sudoku boards.

Hint: It may help to build the sentence incrementally by defining its components separately, then putting them together as a conjunction. For instance, you could define a sentence $\gamma_{\text{only1}}(d, A)$ such that $I(\gamma_{\text{only1}}(d, A))$ contains the models corresponding to boards where digit $d \in \{1, 2, 3, 4\}$ appears only once in the specified set A of cell addresses.