

Solution: Sudoku 1

- (a) Similar to the tic-tac-toe formulation, define variable $C_{i,j,d}$ as a binary variable representing whether cell (i,j) contains digit d . There are (for 2×2 sudoku) 4 rows, 4 columns, and 4 digits, thus there are $4^3 = 64$ such variables.
- (b) If there are 64 variables, then there are 2^{64} models. Observe that a model maps each variable to $\{0, 1\}$. We thus have two choices per variable, i.e. $\underbrace{2 \cdot \dots \cdot 2}_{64} = 2^{64}$ models.
- (c) Suppose we had $n=2$ variables: $\{A, B\}$. A truth table over 2 variables has four rows (models):

$$2^2 = 4 \text{ rows (models)} \left\{ \begin{array}{cc|c} \text{A} & \text{B} & \\ \hline 0 & 0 & ? \\ 0 & 1 & ? \\ 1 & 0 & ? \\ 1 & 1 & ? \end{array} \right.$$

and there are $2^{2^2} = 16$ truth tables of this form, namely:

$\begin{array}{cc c} \text{A} & \text{B} & \\ \hline 0 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{array}$	$\begin{array}{cc c} \text{A} & \text{B} & \\ \hline 0 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{array}$	$\begin{array}{cc c} \text{A} & \text{B} & \\ \hline 0 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{array}$	$\begin{array}{cc c} \text{A} & \text{B} & \\ \hline 0 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{array}$
$\begin{array}{cc c} \text{A} & \text{B} & \\ \hline 0 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{array}$	$\begin{array}{cc c} \text{A} & \text{B} & \\ \hline 0 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{array}$	$\begin{array}{cc c} \text{A} & \text{B} & \\ \hline 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{array}$	$\begin{array}{cc c} \text{A} & \text{B} & \\ \hline 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{array}$
$\begin{array}{cc c} \text{A} & \text{B} & \\ \hline 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{array}$	$\begin{array}{cc c} \text{A} & \text{B} & \\ \hline 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{array}$	$\begin{array}{cc c} \text{A} & \text{B} & \\ \hline 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 1 \end{array}$	$\begin{array}{cc c} \text{A} & \text{B} & \\ \hline 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{array}$
$\begin{array}{cc c} \text{A} & \text{B} & \\ \hline 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{array}$	$\begin{array}{cc c} \text{A} & \text{B} & \\ \hline 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{array}$	$\begin{array}{cc c} \text{A} & \text{B} & \\ \hline 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{array}$	$\begin{array}{cc c} \text{A} & \text{B} & \\ \hline 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{array}$

More generally, we can create 2^m truth tables over m models.

model 1	?	← 2 choices: 0 or 1
⋮		
model m	?	← 2 choices: 0 or 1
		$\overline{2^m}$ choices

Since there are $m = 2^n$ models over n variables, therefore there are 2^{2^n} truth tables over n variables.

For our variable set for (a), $n = 64$. Thus there are $2^{2^{64}}$ truth tables, which is approximately equal to "a lot", or "a whole bunch".