Solution: Propositional Logic
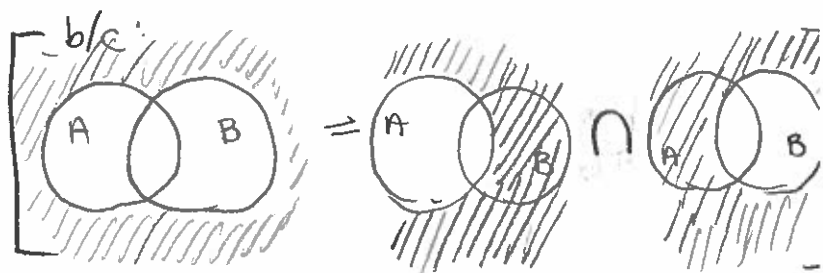
①   ✓ A

     — $B \lor C$

     ✓ $(B \Rightarrow C)$

     — $(B \Rightarrow \text{True}) \land (\text{False} \Rightarrow \text{True})$

     — $(B \Rightarrow$

     — $(B \lor C) \models C$

     — $(A \land B \land \neg C)$

     ✓ $((A \lor B) \land (B \lor \neg C))$

     ✓ $\neg (A \lor B)$

     — $(A \Rightarrow D)$

     ✓ $\neg\neg\neg B$

② $I\big(((A \Rightarrow B) \lor (B \Rightarrow \neg C))\big) = I\big((A \Rightarrow B)\big) \cup I\big((B \Rightarrow \neg C)\big)$

$$= \big(I(\neg A) \cup I(B)\big) \cup \big(I(\neg B) \cup I(\neg C)\big)$$

$$= \big(\overline{I(A)} \cup I(B)\big) \cup \big(\overline{I(B)} \cup \overline{I(C)}\big)$$

$$= \overline{I(A)} \cup \big(I(B) \cup \overline{I(B)}\big) \cup \overline{I(C)}$$

[because set union is associative]

$$= \overline{I(A)} \cup M(\Sigma) \cup \overline{I(C)}$$

$$= M(\Sigma)$$

③ $I(\neg(A \vee B)) = \overline{I((A \vee B))}$

$= \overline{I(A) \cup I(B)}$

$= \overline{I(A)} \cap \overline{I(B)}$

$= I(\neg A) \cap I(\neg B)$

$= I((\neg A \wedge \neg B))$