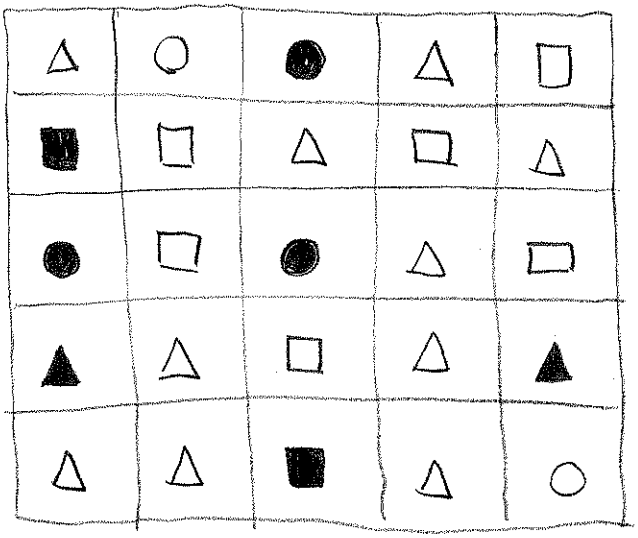
A Quick Review of Basic Probability Formulas – M. Hopkins

(1) Suppose we have a 5×5 cabinet of drawers, each of which contains a single object:



The drawers are closed and we don't know what's inside them.

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(2) Suppose we now open a drawer. Let C represent the color of the object we find. Let S be the shape of the object we find. It is easy to express the joint probability P(C, S) as a table:

C	S	P(C, S)
b	△	$\frac{2}{25} = .08$
b	○	$\frac{3}{25} = .12$
b	□	$\frac{2}{25} = .08$
w	△	$\frac{10}{25} = .40$
w	○	$\frac{2}{25} = .08$
w	□	$\frac{6}{25} = .24$

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(3) The law of total probability allows us to express $P(C)$ in terms of the joint probability $P(C,S)$:

$$P(C) = P(C, S=\Delta) + P(C, S=\bigcirc) + P(C, S=\square)$$

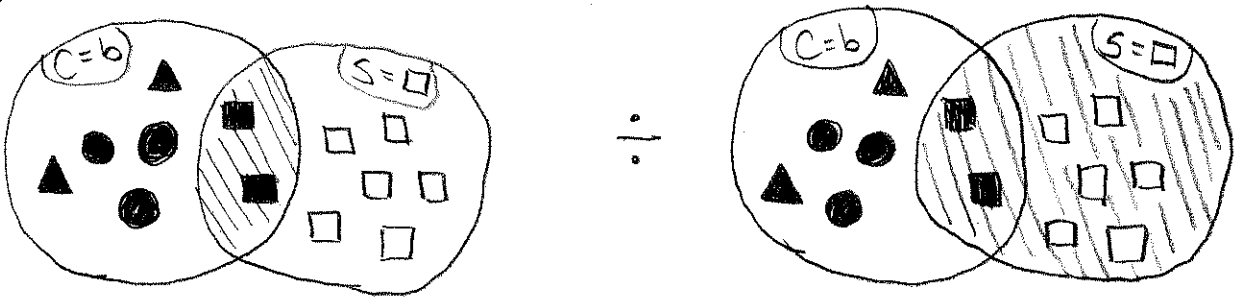
In other words, we can compute the probability that the object's color is white as a <u>case analysis</u>.

C	S	P(C,S)	P(C)
b	△	.08	
b	○	.12	= .28
b	□	.08	
w	△	.40	
w	○	.08	= .72
w	□	.24	

(4) We define the conditional probability $P(C|S)$ as:

$$P(C|S) \triangleq \frac{P(C,S)}{P(S)}$$

e.g. $P(C=b|S=\square) =$



$P(C=b, S=\square)$ $\qquad$ $P(S=\square)$

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(5) From the definition of conditional probability, we see:

$$P(C|S) = \frac{P(C,S)}{P(S)} \Rightarrow P(C,S) = P(C|S)P(S)$$

$$P(S|C) = \frac{P(S,C)}{P(C)} \Rightarrow P(S,C) = P(S|C)P(C)$$

Because $P(C,S) = P(S,C)$, this means that

$$P(C|S)P(S) = P(S|C)P(C)$$

A simple rearrangement gives us Bayes rule:

$$P(C|S) = \frac{P(S|C)P(C)}{P(S)}$$