resolution

CSCI 373

is $c_1 \wedge c_2 \wedge \cdots \wedge c_n$ unsatisfiable?

clauses

c_1

c_2

$\vdots$

c_n

strategy:

keep finding new clauses that are entailed by some subset of our current clauses until we entail $\perp$

$$(\neg A \lor \neg B) \land (A \lor C) \models \neg B \lor C$$

either $m(A) = 0$:	or $m(A) = 1$:
this literal is unsatisfied $(\neg A \lor \neg B) \land (A \lor C)$ so $m(C) = 1$	this literal is unsatisfied $(\neg A \lor \neg B) \land (A \lor C)$ so $m(B) = 0$

in general:

$$(\ell_1 \vee \ell_2) \wedge (\overline{\ell_1} \vee \ell_3) \models \ell_2 \vee \ell_3$$

where:

$\ell_1, \ell_2, \ell_3 \in$ literals(Σ)

$$\overline{\sigma} = \neg\sigma$$

$$\overline{\neg\sigma} = \sigma$$

e.g.

$$(\neg A \vee \neg B) \wedge (A \vee C) \models \neg B \vee C$$

$$(B \vee \neg C) \wedge (\neg B \vee A) \models \ ?$$

$$(B \vee \neg C) \wedge (\neg B \vee C) \models \ ?$$

$$(B \vee C) \wedge (\neg B \vee C) \models \ ?$$

in general:

$$(\ell_1 \vee \ell_2) \wedge (\overline{\ell_1} \vee \ell_3) \models \ell_2 \vee \ell_3$$

where:

$\ell_1, \ell_2, \ell_3 \in \text{literals}(\Sigma)$

$$\overline{\sigma} = \neg\sigma$$

$$\overline{\neg\sigma} = \sigma$$

e.g.

$$(\neg A \vee \neg B) \wedge (A \vee C) \models \neg B \vee C$$

$$(B \vee \neg C) \wedge (\neg B \vee A) \models \neg C \vee A$$

$$(B \vee \neg C) \wedge (\neg B \vee C) \models \neg C \vee C$$

$$(B \vee C) \wedge (\neg B \vee C) \models C \vee C$$

in general:

$$(\ell_1 \vee \ell_2) \wedge (\overline{\ell_1} \vee \ell_3) \models \ell_2 \vee \ell_3$$

where:

$$\ell_1, \ell_2, \ell_3 \in \text{literals}(\Sigma)$$

$$\overline{\sigma} = \neg\sigma$$

$$\overline{\neg\sigma} = \sigma$$

e.g.

$$(\neg A \vee \neg B) \wedge (A \vee C) \models \neg B \vee C$$

$$(B \vee \neg C) \wedge (\neg B \vee A) \models \neg C \vee A$$

$$(B \vee \neg C) \wedge (\neg B \vee C) \models \top$$

$$(B \vee C) \wedge (\neg B \vee C) \models C$$

the "resolution rule":

$$(\ell_1 \vee \ell_2) \wedge (\overline{\ell_1} \vee \ell_3) \models \ell_2 \vee \ell_3$$

where:

$\ell_1, \ell_2, \ell_3 \in \text{literals}(\Sigma)$

$$\overline{\sigma} = \neg\sigma$$

$$\overline{\neg\sigma} = \sigma$$

e.g.

$$(\neg A \vee \neg B) \wedge (A \vee C) \models \neg B \vee C$$

$$(B \vee \neg C) \wedge (\neg B \vee A) \models \neg C \vee A$$

$$(B \vee \neg C) \wedge (\neg B \vee C) \models \top$$

$$(B \vee C) \wedge (\neg B \vee C) \models C$$

theorem: $(\ell_1 \vee \ell_2) \wedge (\overline{\ell}_1 \vee \ell_3) \models \ell_2 \vee \ell_3$

proof:
$$
\begin{aligned}
& I\big((\ell_1 \vee \ell_2) \wedge (\overline{\ell}_1 \vee \ell_3)\big) \\
&= I(\ell_1 \vee \ell_2) \cap I(\overline{\ell}_1 \vee \ell_3) \\
&= \big(I(\ell_1) \cup I(\ell_2)\big) \cap \big(I(\overline{\ell}_1) \cup I(\ell_3)\big) \\
&= \big(I(\ell_1) \cap I(\overline{\ell}_1)\big) \cup \big(I(\ell_1) \cap I(\ell_3)\big) \cup \big(I(\ell_2) \cap I(\overline{\ell}_1)\big) \cup \big(I(\ell_2) \cap I(\ell_3)\big) \\
&= \big(I(\ell_1) \cap I(\ell_3)\big) \cup \big(I(\ell_2) \cap I(\overline{\ell}_1)\big) \cup \big(I(\ell_2) \cap I(\ell_3)\big) \\
&\subseteq I(\ell_3) \cup I(\ell_2) \cup I(\ell_2) \\
&= I(\ell_2) \cup I(\ell_3) \\
&= I(\ell_2 \vee \ell_3)
\end{aligned}
$$

theorem: $(\ell_1 \vee \ell_2) \wedge (\overline{\ell}_1 \vee \ell_3) \models \ell_2 \vee \ell_3$

proof: $I((\ell_1 \vee \ell_2) \wedge (\overline{\ell}_1 \vee \ell_3))$

$= I(\ell_1 \vee \ell_2) \cap I(\overline{\ell}_1 \vee \ell_3)$

$= (I(\ell_1) \cup I(\ell_2)) \cap (I(\overline{\ell}_1) \cup I(\ell_3))$

$= (I(\ell_1) \cap I(\overline{\ell}_1)) \cup (I(\ell_1) \cap I(\ell_3)) \cup (I(\ell_2) \cap I(\overline{\ell}_1)) \cup (I(\ell_2) \cap I(\ell_3))$

$= (I(\ell_1) \cap I(\ell_3)) \cup (I(\ell_2) \cap I(\overline{\ell}_1)) \cup (I(\ell_2) \cap I(\ell_3))$

$\subseteq I(\ell_3) \cup I(\ell_2) \cup I(\ell_2)$

$= I(\ell_2) \cup I(\ell_3)$

$= I(\ell_2 \vee \ell_3)$

theorem: $(\ell_1 \lor \ell_2) \land (\bar{\ell}_1 \lor \ell_3) \models \ell_2 \lor \ell_3$

proof:

$$
\begin{aligned}
& I\big((\ell_1 \lor \ell_2) \land (\bar{\ell}_1 \lor \ell_3)\big) \\
&= I(\ell_1 \lor \ell_2) \cap I(\bar{\ell}_1 \lor \ell_3) \\
&= \big(I(\ell_1) \cup I(\ell_2)\big) \cap \big(I(\bar{\ell}_1) \cup I(\ell_3)\big) \\
&= \big(I(\ell_1) \cap I(\bar{\ell}_1)\big) \cup \big(I(\ell_1) \cap I(\ell_3)\big) \cup \big(I(\ell_2) \cap I(\bar{\ell}_1)\big) \cup \big(I(\ell_2) \cap I(\ell_3)\big) \\
&= \big(I(\ell_1) \cap I(\ell_3)\big) \cup \big(I(\ell_2) \cap I(\bar{\ell}_1)\big) \cup \big(I(\ell_2) \cap I(\ell_3)\big) \\
&\subseteq I(\ell_3) \cup I(\ell_2) \cup I(\ell_2) \\
&= I(\ell_2) \cup I(\ell_3) \\
&= I(\ell_2 \lor \ell_3)
\end{aligned}
$$

theorem: $(\ell_1 \vee \ell_2) \wedge (\overline{\ell_1} \vee \ell_3) \models \ell_2 \vee \ell_3$

proof:
$$
\begin{aligned}
& I((\ell_1 \vee \ell_2) \wedge (\overline{\ell_1} \vee \ell_3)) \\
&= I(\ell_1 \vee \ell_2) \cap I(\overline{\ell_1} \vee \ell_3) \\
&= (I(\ell_1) \cup I(\ell_2)) \cap (I(\overline{\ell_1}) \cup I(\ell_3)) \\
&= (I(\ell_1) \cap I(\overline{\ell_1})) \cup (I(\ell_1) \cap I(\ell_3)) \cup (I(\ell_2) \cap I(\overline{\ell_1})) \cup (I(\ell_2) \cap I(\ell_3)) \\
&= (I(\ell_1) \cap I(\ell_3)) \cup (I(\ell_2) \cap I(\overline{\ell_1})) \cup (I(\ell_2) \cap I(\ell_3)) \\
&\subseteq I(\ell_3) \cup I(\ell_2) \cup I(\ell_2) \\
&= I(\ell_2) \cup I(\ell_3) \\
&= I(\ell_2 \vee \ell_3)
\end{aligned}
$$

theorem: $(\ell_1 \vee \ell_2) \wedge (\overline{\ell}_1 \vee \ell_3) \models \ell_2 \vee \ell_3$

proof: $I((\ell_1 \vee \ell_2) \wedge (\overline{\ell}_1 \vee \ell_3))$

$= I(\ell_1 \vee \ell_2) \cap I(\overline{\ell}_1 \vee \ell_3)$

$= (I(\ell_1) \cup I(\ell_2)) \cap (I(\overline{\ell}_1) \cup I(\ell_3))$

$= (I(\ell_1) \cap I(\overline{\ell}_1)) \cup (I(\ell_1) \cap I(\ell_3)) \cup (I(\ell_2) \cap I(\overline{\ell}_1)) \cup (I(\ell_2) \cap I(\ell_3))$

$= (I(\ell_1) \cap I(\ell_3)) \cup (I(\ell_2) \cap I(\overline{\ell}_1)) \cup (I(\ell_2) \cap I(\ell_3))$

$\subseteq I(\ell_3) \cup I(\ell_2) \cup I(\ell_2)$

$= I(\ell_2) \cup I(\ell_3)$

$= I(\ell_2 \vee \ell_3)$

theorem: $(\ell_1 \vee \ell_2) \wedge (\overline{\ell_1} \vee \ell_3) \models \ell_2 \vee \ell_3$

proof: $I((\ell_1 \vee \ell_2) \wedge (\overline{\ell_1} \vee \ell_3))$

$$= I(\ell_1 \vee \ell_2) \cap I(\overline{\ell_1} \vee \ell_3)$$

$$= (I(\ell_1) \cup I(\ell_2)) \cap (I(\overline{\ell_1}) \cup I(\ell_3))$$

$$= (I(\ell_1) \cap I(\overline{\ell_1})) \cup (I(\ell_1) \cap I(\ell_3)) \cup (I(\ell_2) \cap I(\overline{\ell_1})) \cup (I(\ell_2) \cap I(\ell_3))$$

$$= (I(\ell_1) \cap I(\ell_3)) \cup (I(\ell_2) \cap I(\overline{\ell_1})) \cup (I(\ell_2) \cap I(\ell_3))$$

$$\subseteq I(\ell_3) \cup I(\ell_2) \cup I(\ell_2)$$

$$= I(\ell_2) \cup I(\ell_3)$$

$$= I(\ell_2 \vee \ell_3)$$

theorem: $(\ell_1 \vee \ell_2) \wedge (\overline{\ell_1} \vee \ell_3) \models \ell_2 \vee \ell_3$

proof:

$$
\begin{aligned}
& I((\ell_1 \vee \ell_2) \wedge (\overline{\ell_1} \vee \ell_3)) \\
&= I(\ell_1 \vee \ell_2) \cap I(\overline{\ell_1} \vee \ell_3) \\
&= (I(\ell_1) \cup I(\ell_2)) \cap (I(\overline{\ell_1}) \cup I(\ell_3)) \\
&= (I(\ell_1) \cap I(\overline{\ell_1})) \cup (I(\ell_1) \cap I(\ell_3)) \cup (I(\ell_2) \cap I(\overline{\ell_1})) \cup (I(\ell_2) \cap I(\ell_3)) \\
&= (I(\ell_1) \cap I(\ell_3)) \cup (I(\ell_2) \cap I(\overline{\ell_1})) \cup (I(\ell_2) \cap I(\ell_3)) \\
&\subseteq I(\ell_3) \cup I(\ell_2) \cup I(\ell_2) \\
&= I(\ell_2) \cup I(\ell_3) \\
&= I(\ell_2 \vee \ell_3)
\end{aligned}
$$

theorem: $(\ell_1 \vee \ell_2) \wedge (\bar{\ell}_1 \vee \ell_3) \models \ell_2 \vee \ell_3$

proof:
$$
\begin{aligned}
&I((\ell_1 \vee \ell_2) \wedge (\bar{\ell}_1 \vee \ell_3)) \\
&= I(\ell_1 \vee \ell_2) \cap I(\bar{\ell}_1 \vee \ell_3) \\
&= (I(\ell_1) \cup I(\ell_2)) \cap (I(\bar{\ell}_1) \cup I(\ell_3)) \\
&= (I(\ell_1) \cap I(\bar{\ell}_1)) \cup (I(\ell_1) \cap I(\ell_3)) \cup (I(\ell_2) \cap I(\bar{\ell}_1)) \cup (I(\ell_2) \cap I(\ell_3)) \\
&= (I(\ell_1) \cap I(\ell_3)) \cup (I(\ell_2) \cap I(\bar{\ell}_1)) \cup (I(\ell_2) \cap I(\ell_3)) \\
&\subseteq I(\ell_3) \cup I(\ell_2) \cup I(\ell_2) \\
&= I(\ell_2) \cup I(\ell_3) \\
&= I(\ell_2 \vee \ell_3)
\end{aligned}
$$

resolution :

$$(l_1 \vee l_2) \quad \wedge \quad (\overline{l_1} \vee l_3) \quad \models \quad l_2 \vee l_3$$

generalized resolution :

$$(l_1 \vee \cdots \vee l_j \vee \cdots \vee l_m) \wedge (l_{m+1} \vee \cdots \vee l_k \vee \cdots \vee l_n) \models_R \bigvee_{\substack{i=1 \\ i \neq j,\, i \neq k}}^{n} l_i$$

$$\text{if } \; l_j = \overline{l_k}$$

generalized resolution:

$$\left(\bigvee_{i=1}^{m} l_i\right) \wedge \left(\bigvee_{i=m+1}^{n} l_i\right) \vdash_R \bigvee_{\substack{i=1 \\ i\neq j,\, i\neq k}}^{n} l_i$$

where: $j \in \{1, \ldots, m\}$

$k \in \{m+1, \ldots, n\}$

$l_j = \overline{l_k}$

e.g. $(\neg A \vee \neg B \vee \neg D) \wedge (A \vee C) \vdash_R \neg B \vee \neg D \vee C$

$(\neg A \vee \neg B \vee \neg D) \wedge (\neg A \vee B \vee E) \vdash_R \neg A \vee \neg D \vee E$

$(A \vee \neg B \vee \neg D) \wedge (\neg A \vee B \vee E) \vdash_R T$

generalized resolution:

$$\left(\bigvee_{i=1}^{m} l_i\right) \wedge \left(\bigvee_{i=m+1}^{n} l_i\right) \vdash_R \bigvee_{\substack{i=1 \\ i \neq j,\, i \neq k}}^{n} l_i$$

where: $j \in \{1, \dots, m\}$

$k \in \{m+1, \dots, n\}$

$l_j = \overline{l_k}$

e.g. $(\neg A \vee \neg B \vee \neg D) \wedge (A \vee C) \vdash_R \neg B \vee \neg D \vee C$

$(\neg A \vee \neg B \vee \neg D) \wedge (\neg A \vee B \vee E) \vdash_R \neg A \vee \neg D \vee E$

$(A \vee \neg B \vee \neg D) \wedge (\neg A \vee B \vee E) \vdash_R A \vee \neg A \vee \neg D \vee E$

$\vdash_R \neg B \vee B \vee \neg D \vee E$

generalized resolution:

$$\left(\bigvee_{i=1}^{m} l_i\right) \wedge \left(\bigvee_{i=m+1}^{n} l_i\right) \vdash_R \bigvee_{\substack{i=1 \\ i \neq j,\, i \neq k}}^{n} l_i$$

where:
$j \in \{1, \ldots, m\}$
$k \in \{m+1, \ldots, n\}$
$l_j = \overline{l_k}$

e.g.
$$(\neg A \vee \neg B \vee \neg D) \wedge (A \vee C) \vdash_R \neg B \vee \neg D \vee C$$

$$(\neg A \vee \neg B \vee \neg D) \wedge (\neg A \vee B \vee E) \vdash_R \neg A \vee \neg D \vee E$$

$$(A \vee \neg B \vee \neg D) \wedge (\neg A \vee B \vee E) \vdash_R \top$$

generalized resolution :

$$\left(\bigvee_{i=1}^{m} l_i\right) \wedge \left(\bigvee_{i=m+1}^{n} l_i\right) \vdash_R \bigvee_{\substack{i=1 \\ i\neq j,\, i\neq k}}^{n} l_i$$

where : $j \in \{1, \dots, m\}$

$k \in \{m+1, \dots, n\}$

$l_j = \overline{l_k}$

e.g.

$$(\neg A \vee \neg B) \wedge A \vdash_R ?$$

generalized resolution :

$$\left(\bigvee_{i=1}^{m} l_i\right) \wedge \left(\bigvee_{i=m+1}^{n} l_i\right) \vdash_R \bigvee_{\substack{i=1\\ i\neq j,\, i\neq k}}^{n} l_i$$

where : $j \in \{1, \dots, m\}$

$k \in \{m+1, \dots, n\}$

$l_j = \overline{l_k}$

e.g.

$$\boxed{\begin{aligned} I(\alpha \vee \bot) &= I(\alpha) \cup I(\bot) \\ &= I(\alpha) \cup \{\} \\ &= I(\alpha) \end{aligned}}$$

$$\begin{aligned} & (\neg A \vee \neg B) \wedge A \\ \equiv\ & (\neg A \vee \neg B) \wedge (A \vee \bot) \\ \vdash_R\ & \neg B \vee \bot \\ \equiv\ & \neg B \end{aligned}$$

generalized resolution :

$$\left(\bigvee_{i=1}^{m} l_i\right) \bigwedge \left(\bigvee_{i=m+1}^{n} l_i\right) \vdash_R \bigvee_{\substack{i=1 \\ i\neq j,\, i\neq k}}^{n} l_i$$

where : $j \in \{1, \dots, m\}$

$k \in \{m+1, \dots, n\}$

$l_j = \overline{l_k}$

e.g. $\neg A \wedge A \vdash_R$?

generalized resolution :

$$\left(\bigvee_{i=1}^{m} l_i\right) \wedge \left(\bigvee_{i=m+1}^{n} l_i\right) \vdash_R \bigvee_{\substack{i=1 \\ i\neq j,\, i\neq k}}^{n} l_i$$

where : $j \in \{1, \dots, m\}$

$k \in \{m+1, \dots, n\}$

$l_j = \overline{l_k}$

e.g.

$$\boxed{\begin{aligned} I(\alpha \vee \bot) &= I(\alpha) \cup I(\bot) \\ &= I(\alpha) \cup \{\} \\ &= I(\alpha) \end{aligned}}$$

$$\begin{aligned} &\neg A \wedge A \\ &\equiv (\neg A \vee \bot) \wedge (A \vee \bot) \\ &\vdash_R \bot \vee \bot \\ &\equiv \bot \end{aligned}$$

generalized resolution:

$$\left(\bigvee_{i=1}^{m} \ell_i\right) \bigwedge \left(\bigvee_{i=m+1}^{n} \ell_i\right) \vdash_R \bigvee_{\substack{i=1 \\ i\neq j,\, i\neq k}}^{n} \ell_i$$

where: $j \in \{1, \ldots, m\}$

$k \in \{m+1, \ldots, n\}$

$\ell_j = \overline{\ell_k}$

special cases:

$$\ell_1 \wedge \overline{\ell_1} \vdash_R \perp$$

$$\ell_1 \wedge (\overline{\ell_1} \vee \ell_2) \vdash_R \ell_2$$

generalized resolution :

$$\left(\bigvee_{i=1}^{m} l_i\right) \wedge \left(\bigvee_{i=m+1}^{n} l_i\right) \vdash_R \bigvee_{\substack{i=1 \\ i\neq j,\, i\neq k}}^{n} l_i$$

where : $j \in \{1, \ldots, m\}$

$k \in \{m+1, \ldots, n\}$

$l_j = \overline{l_k}$

generalized resolution is sound :

if $\alpha \vdash_R \beta$, then $\alpha \models \beta$

is $c_1 \wedge c_2 \wedge \cdots \wedge c_n$ unsatisfiable?

clauses

c_1

c_2

$\vdots$

c_n

strategy:

keep finding new clauses that are entailed by some subset of our current clauses until we entail $\perp$

is $C_1 \wedge C_2 \wedge \cdots \wedge C_n$ unsatisfiable?

clauses

C_1

C_2

$\vdots$

C_n

strategy:

keep finding new clauses that are entailed by

resolution

until we entail $\perp$

is $(\neg A \vee \neg B) \wedge (A \vee C) \wedge B \wedge \neg C$ unsatisfiable?

clauses

$\neg A \vee \neg B$

$A \vee C$

B

$\neg C$

$\neg B \vee C$

C

$\perp$

which clauses are entailed by resolution?

is $(\neg A \lor \neg B) \land (A \lor C) \land B \land \neg C$ unsatisfiable?

clauses

$\boxed{\neg A \lor \neg B}$

$\boxed{A \lor C}$

B

$\neg C$

$$(\neg A \lor \neg B) \land (A \lor C) \vdash_R (\neg B \lor C)$$

so add $(\neg B \lor C)$ to our clauses

is $(\neg A \lor \neg B) \land (A \lor C) \land B \land \neg C$ unsatisfiable?

clauses

$\neg A \lor \neg B$

$A \lor C$

$\boxed{B}$

$\neg C$

$\boxed{\neg B \lor C}$

$B \land (\neg B \lor C) \vdash_R C$

so add C to our clauses

is $(\neg A \vee \neg B) \wedge (A \vee C) \wedge B \wedge \neg C$ unsatisfiable?

clauses

$\neg A \vee \neg B$

$A \vee C$

B

$\boxed{\neg C}$

$\neg B \vee C$

$\boxed{C}$

$C \wedge \neg C \vdash_R \perp$

so add $\perp$ to our clauses

is $(\neg A \vee \neg B) \wedge (A \vee C) \wedge B \wedge \neg C$ unsatisfiable?

clauses

$\neg A \vee \neg B$

$A \vee C$

B

$\neg C$

$\neg B \vee C$

C

$\boxed{\perp}$

the original sentence entails $\perp$, so it is

unsatisfiable

is $(\neg A \vee \neg B) \wedge (A \vee C) \wedge (B \vee \neg C)$ unsatisfiable?

clauses

$\neg A \vee \neg B$

$A \vee C$

$B \vee \neg C$

$\neg B \vee C$

$A \vee B$

which clauses are entailed by resolution?

$$\boxed{\text{is } (\neg A \lor \neg B) \land (A \lor C) \land (B \lor \neg C) \text{ unsatisfiable?}}$$

clauses

$\boxed{\neg A \lor \neg B}$

$\boxed{A \lor C}$

$B \lor \neg C$

$$(\neg A \lor \neg B) \land (A \lor C) \vdash_R (\neg B \lor C)$$

so add $(\neg B \lor C)$
to our clauses

$$\boxed{\text{is } (\neg A \lor \neg B) \land (A \lor C) \land (B \lor C) \text{ unsatisfiable?}}$$

clauses

$\boxed{\neg A \lor \neg B}$

$A \lor C$

$\boxed{B \lor C}$

$\neg B \lor C$

$$(\neg A \lor \neg B) \land (B \lor C) \vdash_R (\neg A \lor C)$$

so add $(\neg A \lor C)$ to our clauses

is $(\neg A \vee \neg B) \wedge (A \vee C) \wedge (B \vee C)$ unsatisfiable?

clauses

$\neg A \vee \neg B$

$\boxed{A \vee C}$

$B \vee C$

$\neg B \vee C$

$\boxed{\neg A \vee C}$

$$(A \vee C) \wedge (\neg A \vee C) \vdash_R C$$

so add C to our clauses

is $(\neg A \vee \neg B) \wedge (A \vee C) \wedge (B \vee C)$ unsatisfiable?

clauses

$\neg A \vee \neg B$

$A \vee C$

$B \vee C$

$\neg B \vee C$

$\neg A \vee C$

C

nothing more to resolve

is $(\neg A \vee \neg B) \wedge (A \vee C) \wedge (B \vee C)$ unsatisfiable?

clauses

$\neg A \vee \neg B$

$A \vee C$

$B \vee C$

$\neg B \vee C$

$\neg A \vee C$

C

does this mean the original sentence is satisfiable?

is $C_1 \land C_2 \land \cdots \land C_n$ unsatisfiable?

soundness	completeness
if we keep applying resolution to the clauses until we obtain $\perp$, then the sentence is unsatisfiable	if we keep applying resolution to the clauses and never obtain $\perp$, then the sentence is satisfiable
✓	?

is $c_1 \wedge c_2 \wedge \cdots \wedge c_n$ unsatisfiable?

soundness

if we keep applying resolution to the clauses until we obtain $\perp$, then the sentence is unsatisfiable ✓

completeness

if we keep applying resolution to the clauses and never obtain $\perp$, then the sentence is satisfiable ✓