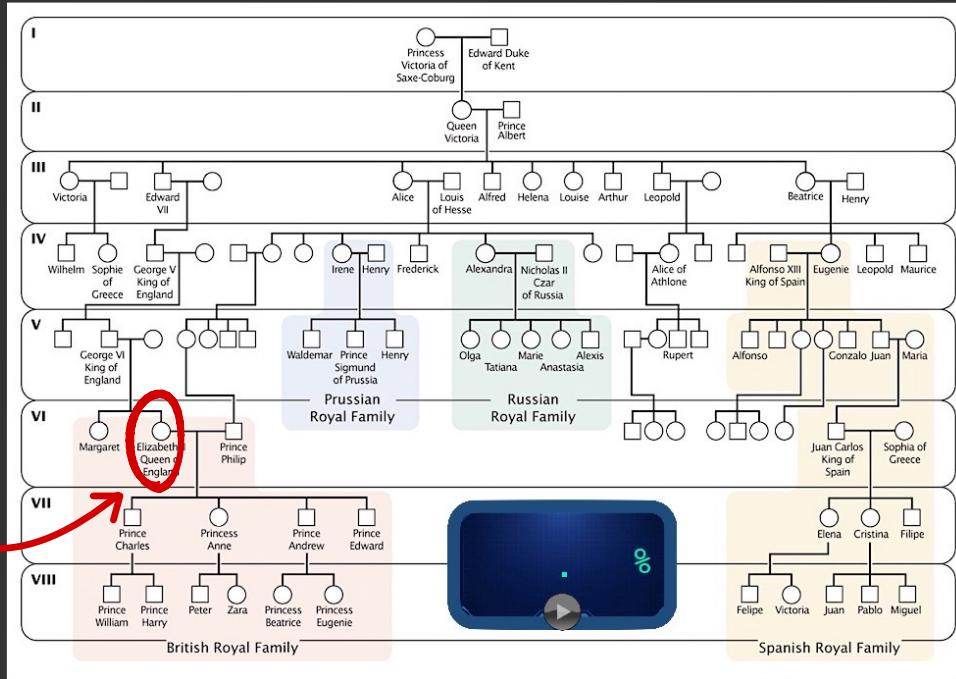


the junction tree algorithm

CSCI
373

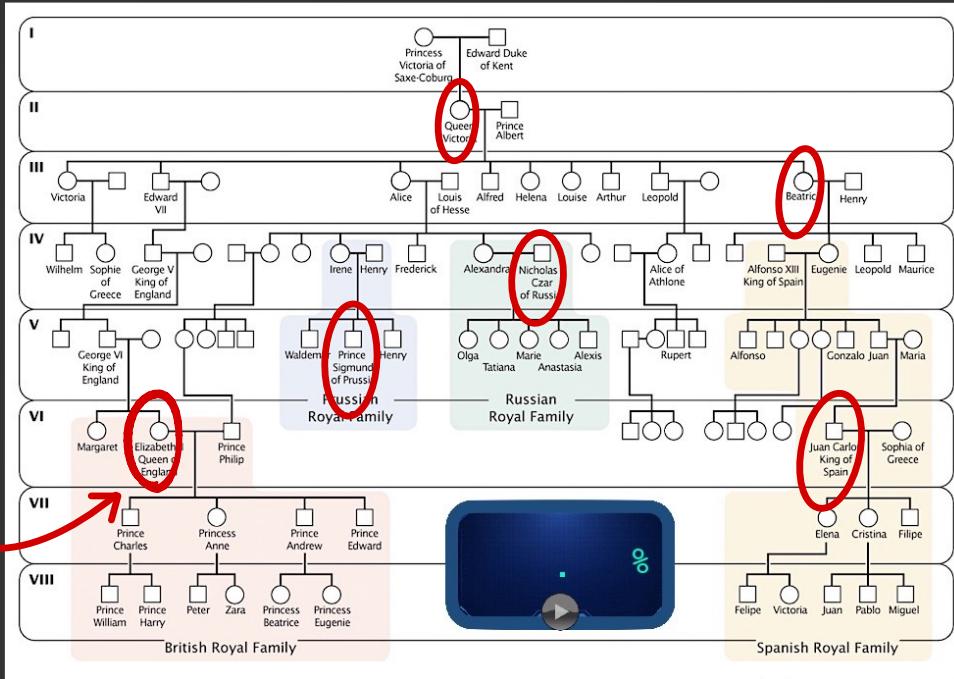
variable elimination computes a single marginal or conditional distribution

probability that elizabeth is a carrier for hemophilia



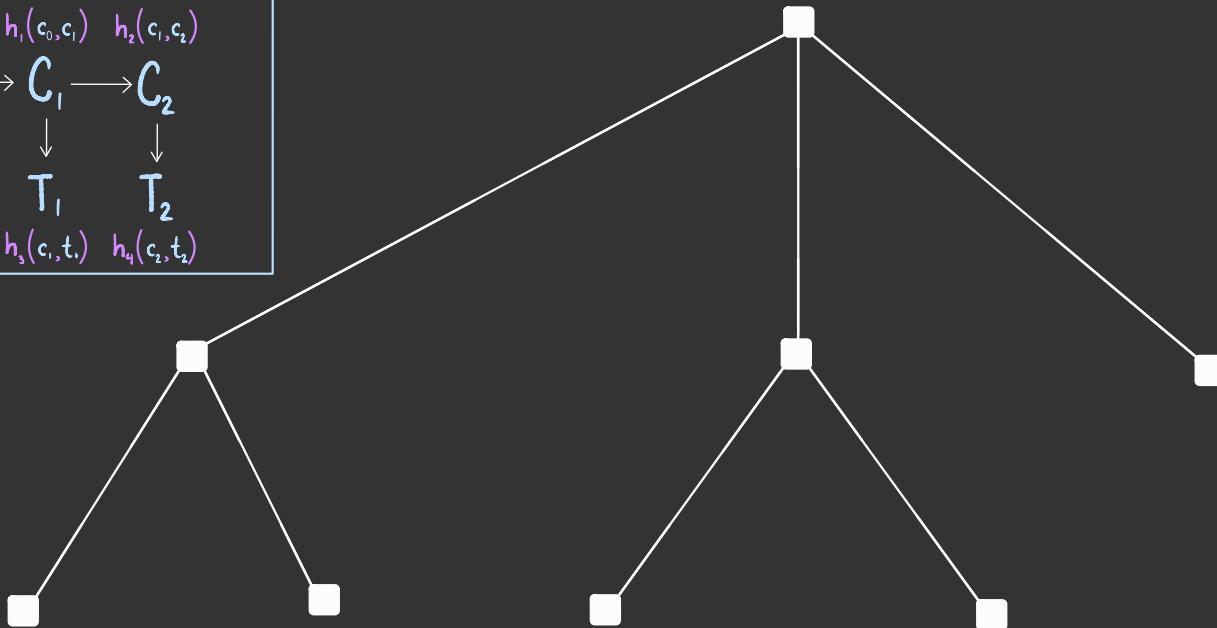
but what if wanted to compute these distributions for **every** variable of a network?

probability that
PERSON is a
carrier for
hemophilia

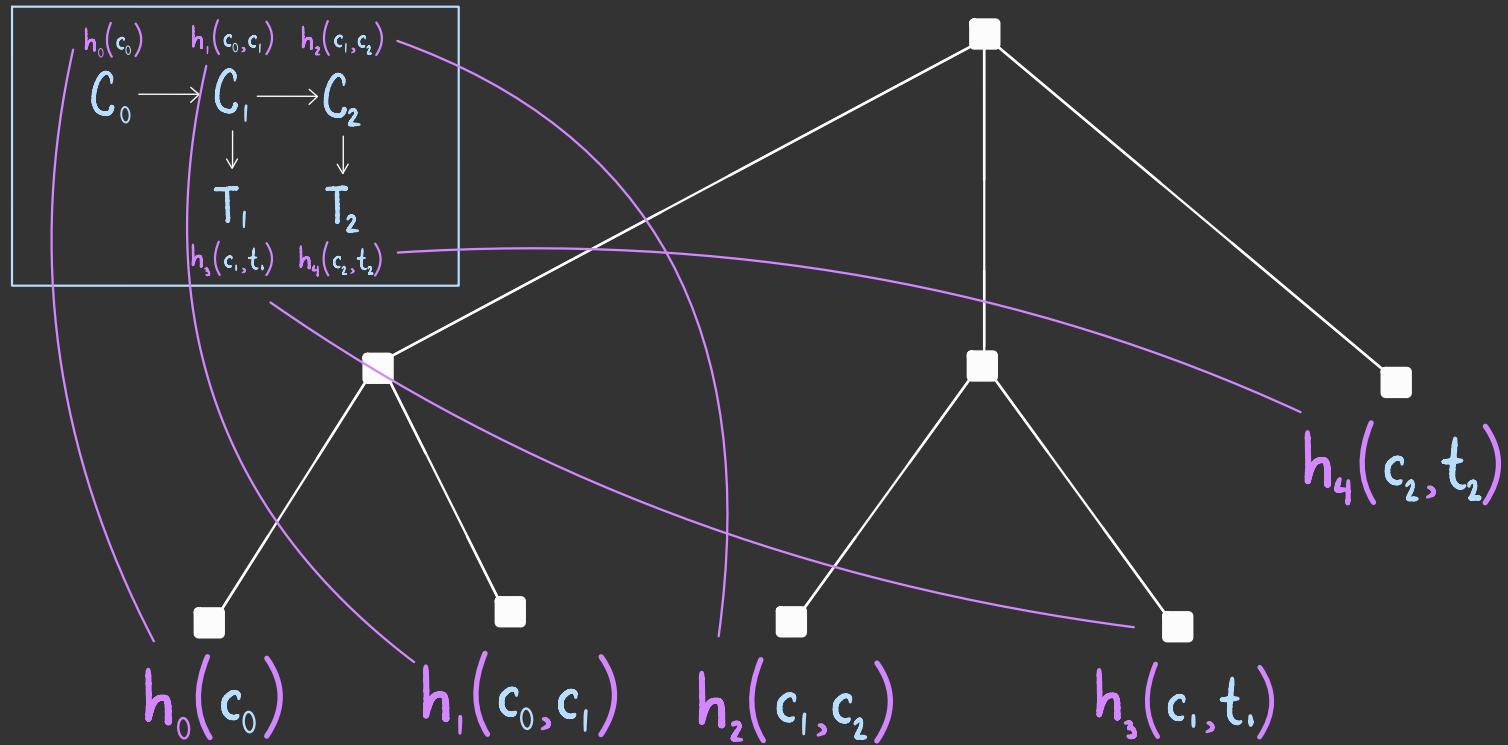


step one: start with a tree whose leaves are labeled with the factors of the bayesian network

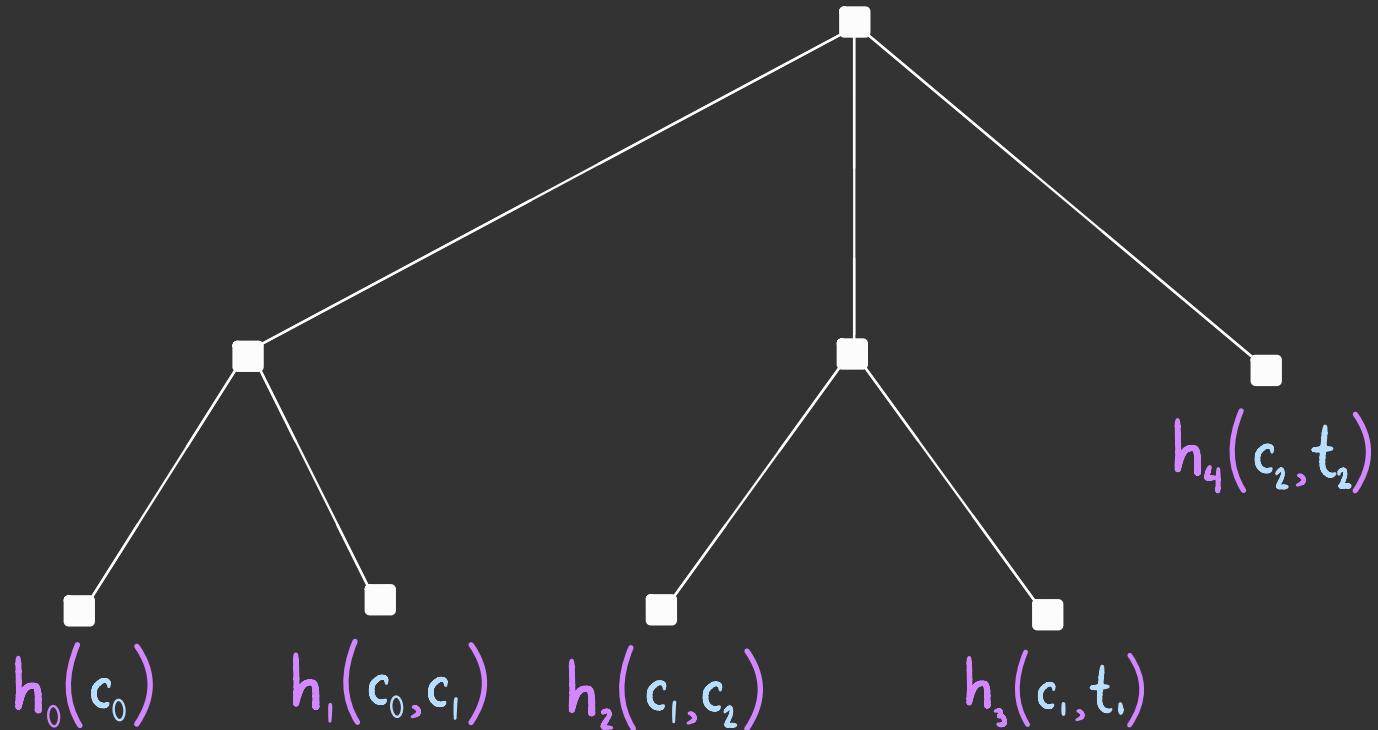
$$\begin{array}{c} h_0(c_0) \quad h_1(c_0, c_1) \quad h_2(c_1, c_2) \\ C_0 \longrightarrow C_1 \longrightarrow C_2 \\ \downarrow \qquad \downarrow \\ T_1 \qquad T_2 \\ h_3(c_1, t_1) \quad h_4(c_2, t_2) \end{array}$$



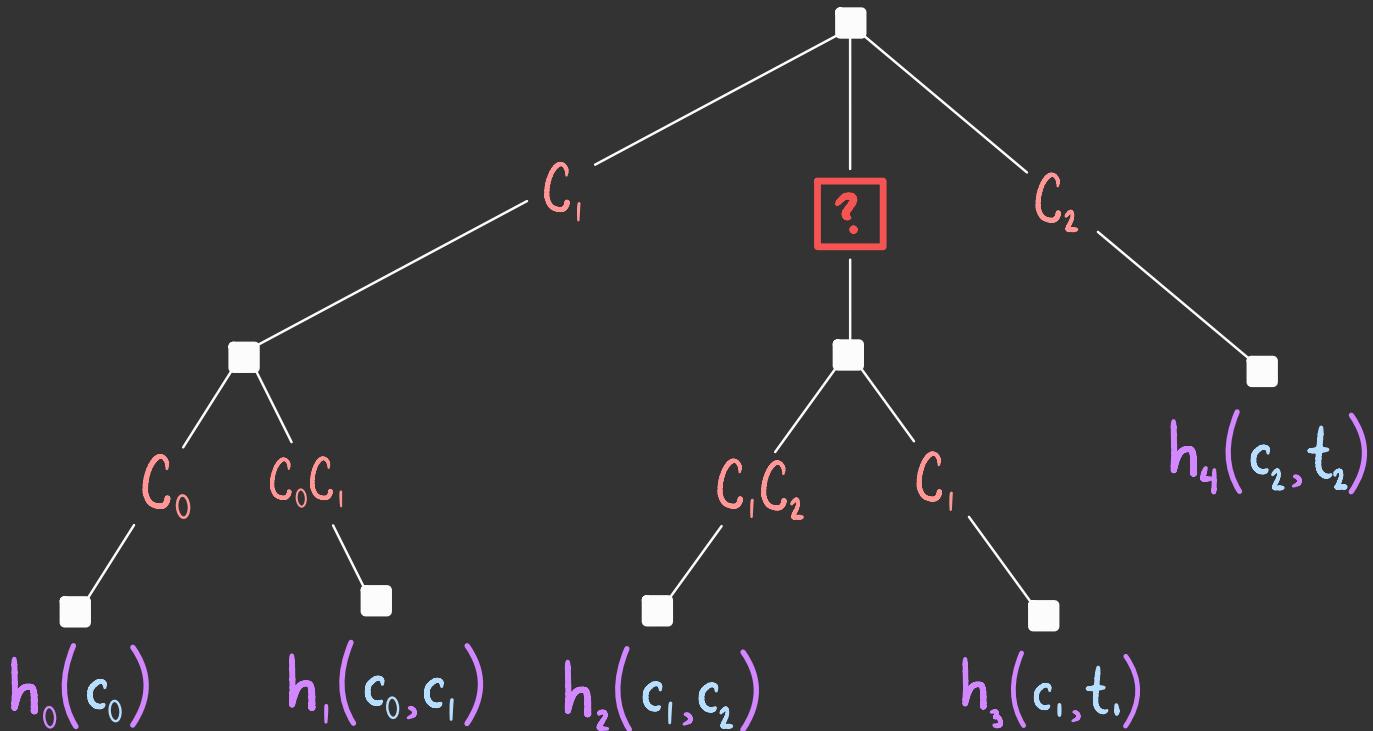
step one: start with a tree whose leaves are labeled with the factors of the bayesian network



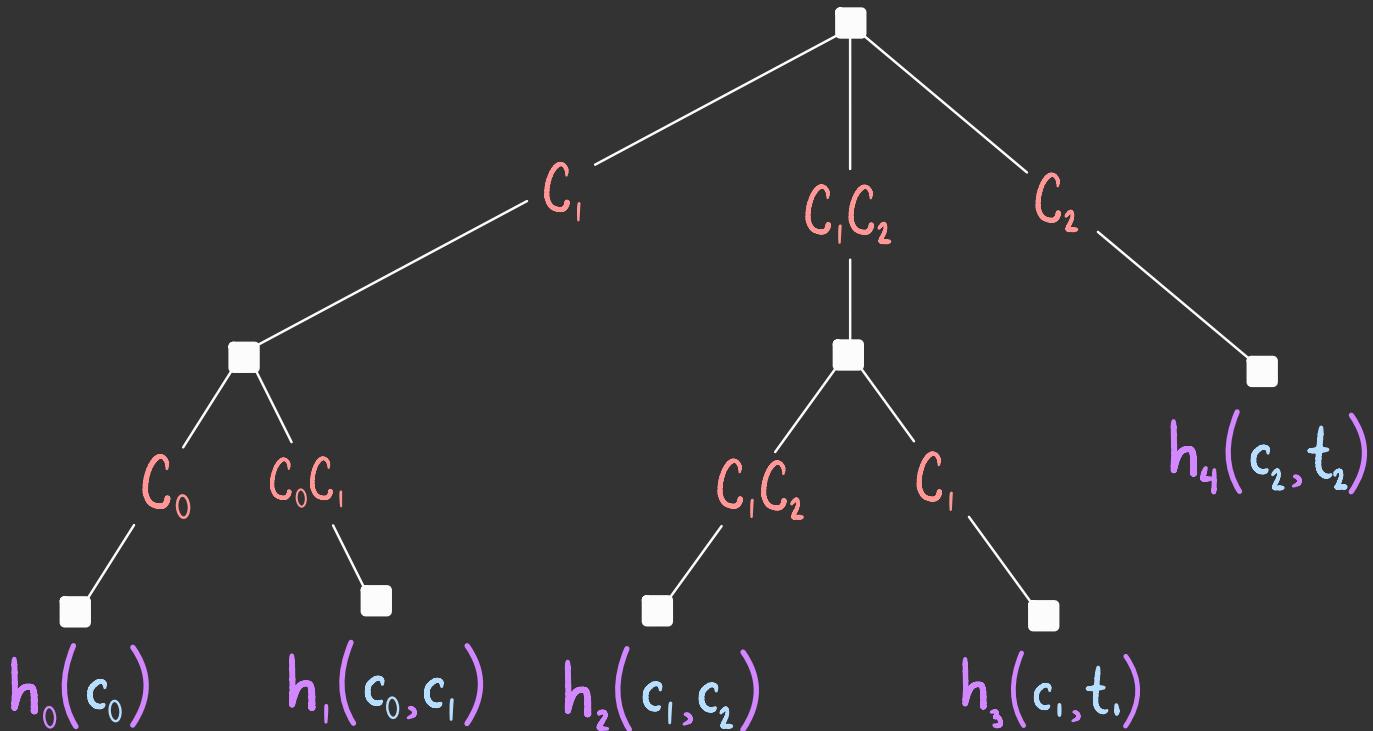
step one: start with a tree whose leaves are labeled with the factors of the bayesian network



step two: label the edges with their separators, the set of variables that appear in factors on both sides of the edge

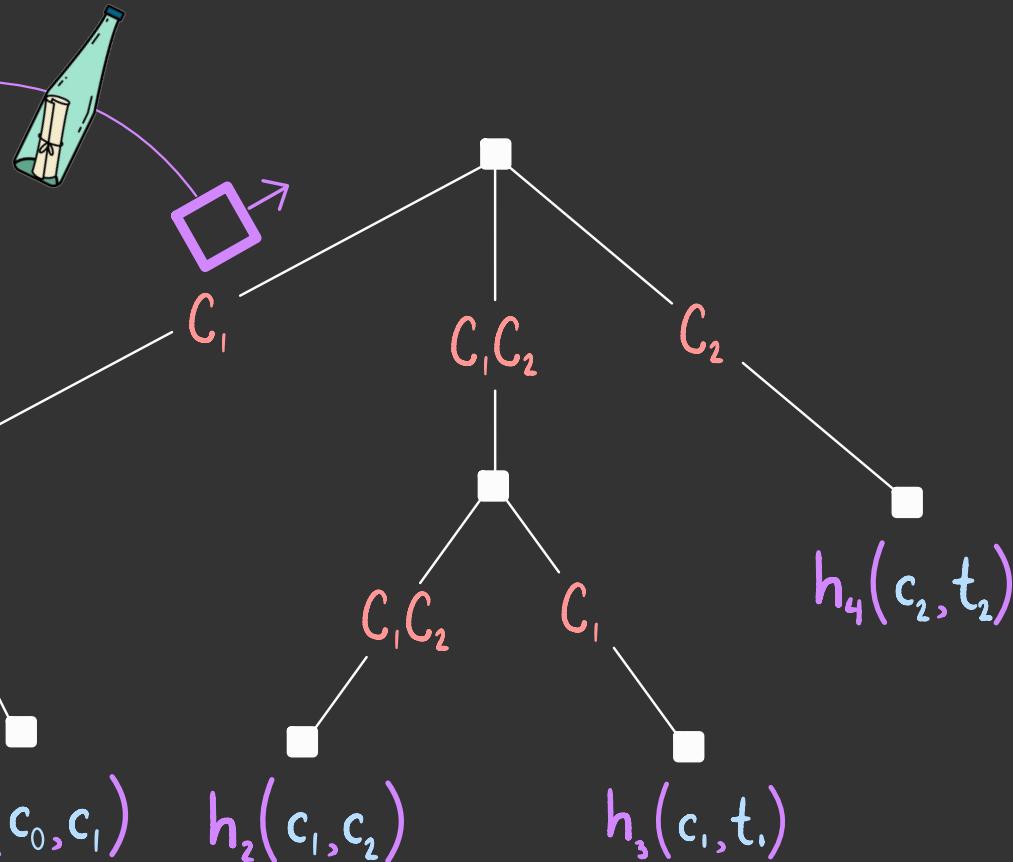


step two: label the edges with their separators, the set of variables that appear in factors on both sides of the edge



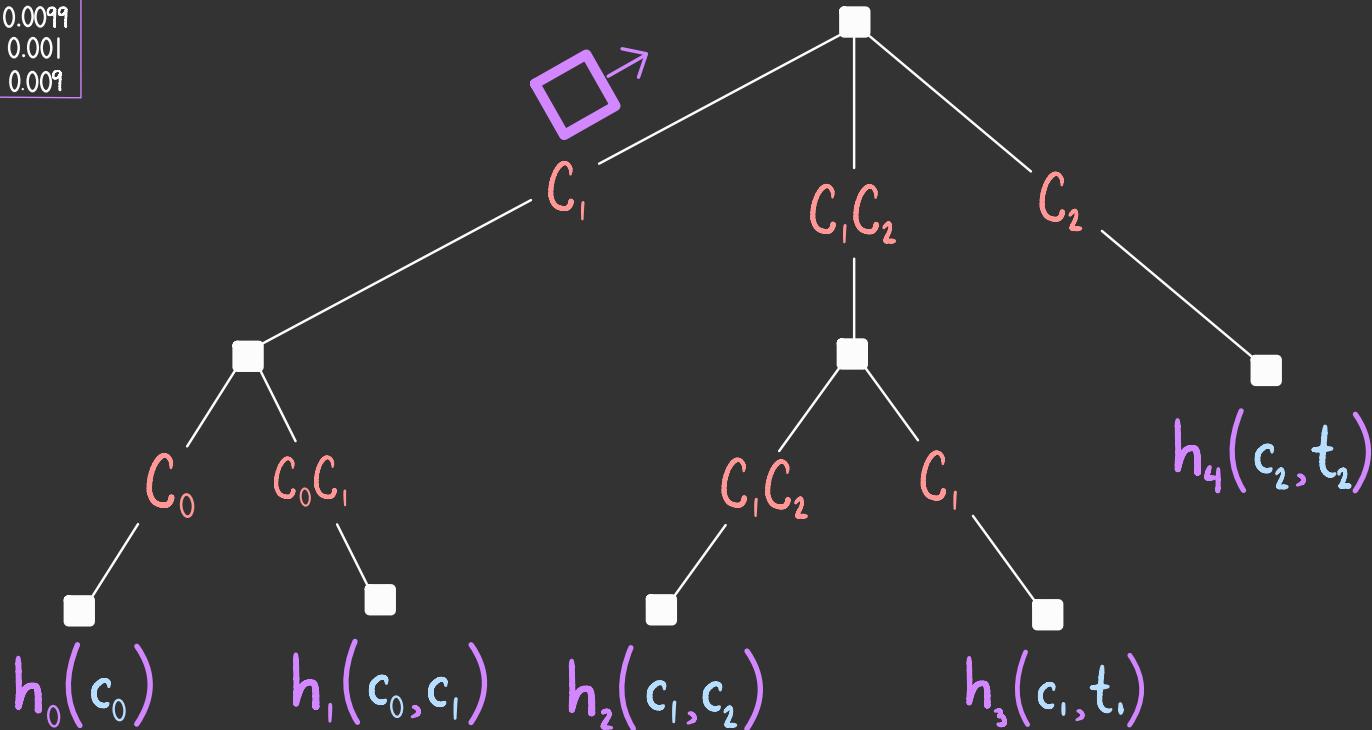
step three: the nodes start sending factors to their neighbors

C_0	C_1	$h(c_0, c_1)$
0	0	0.9801
0	1	0.0099
1	0	0.001
1	1	0.009



step three: but only the separator variables survive the journey
 the rest are marginalized out using 

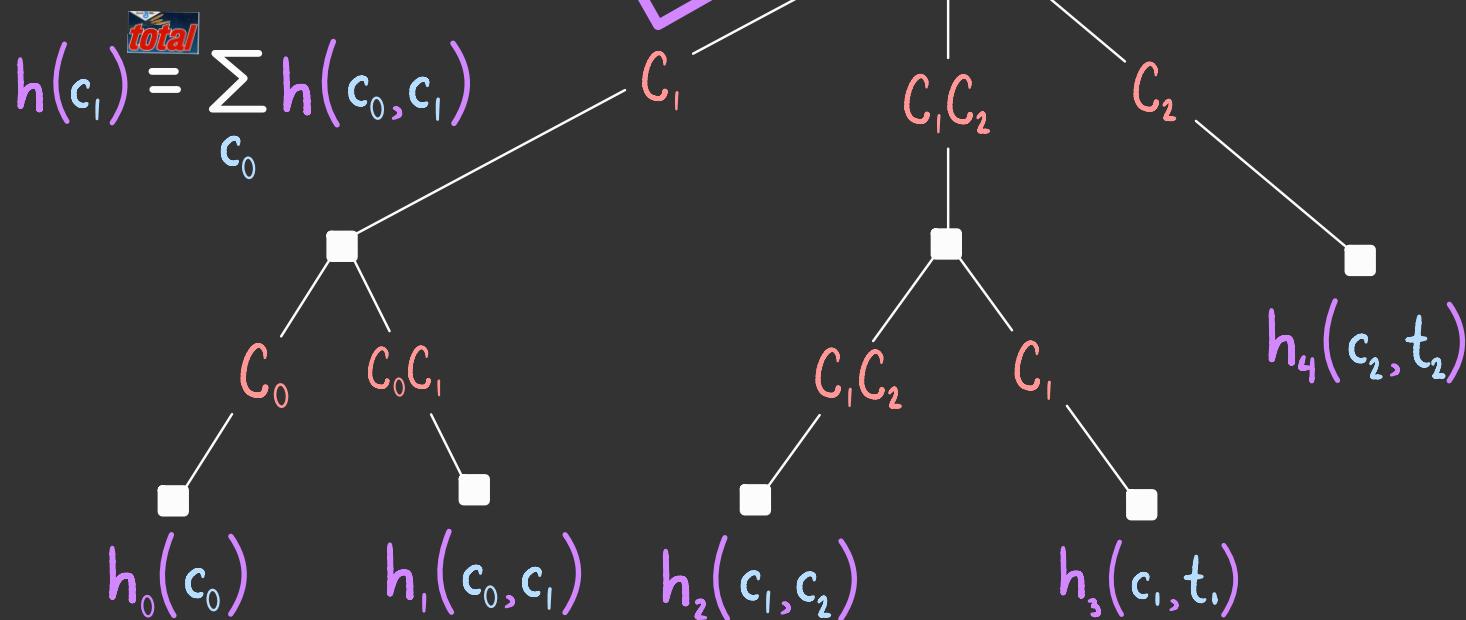
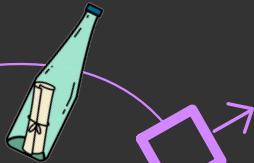
C_0	C_1	$h(c_0, c_1)$
0	0	0.9801
0	1	0.0099
1	0	0.001
1	1	0.009



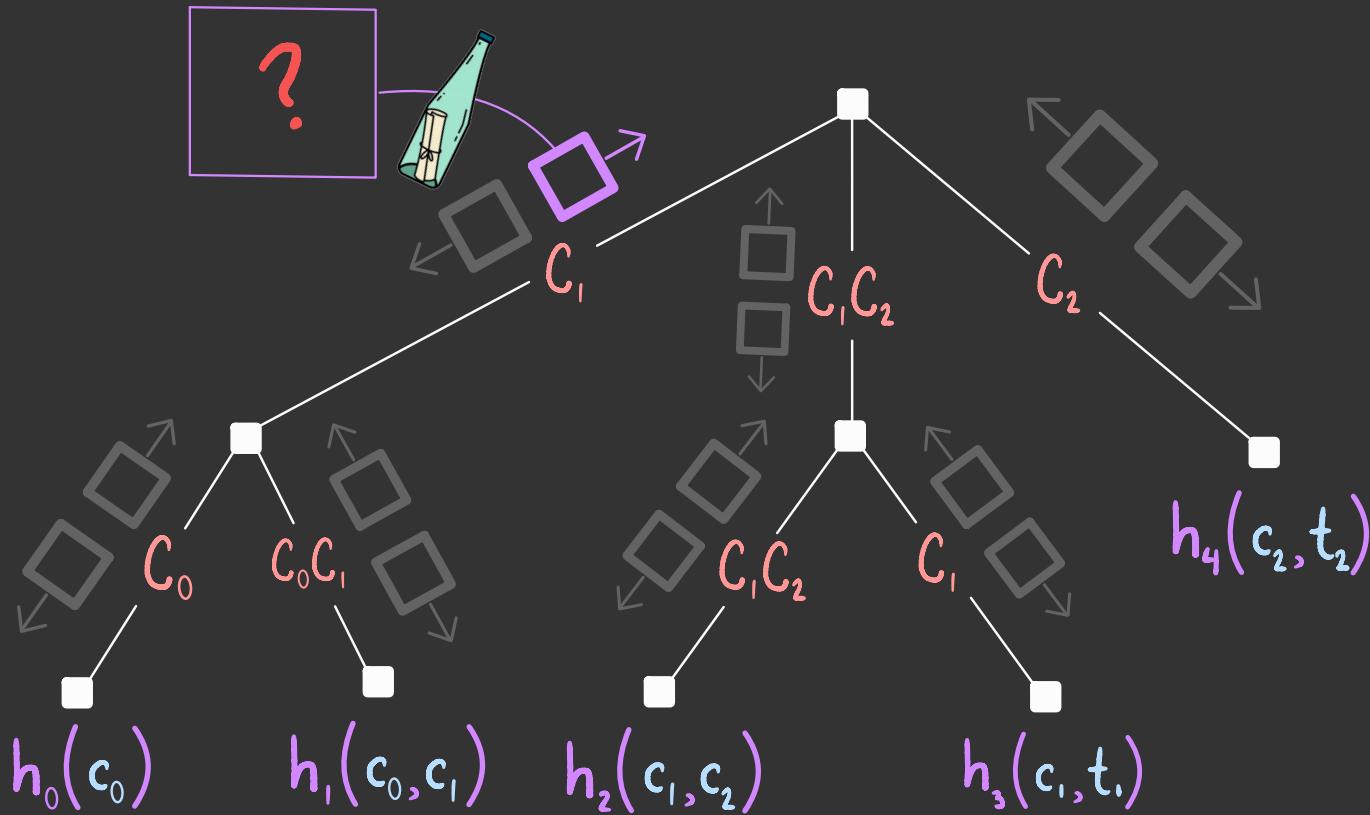
step three: but only the separator variables survive the journey
 the rest are marginalized out using 

C_0	C_1	$h(c_0, c_1)$
0	0	0.9801
0	1	0.0099
1	0	0.001
1	1	0.009

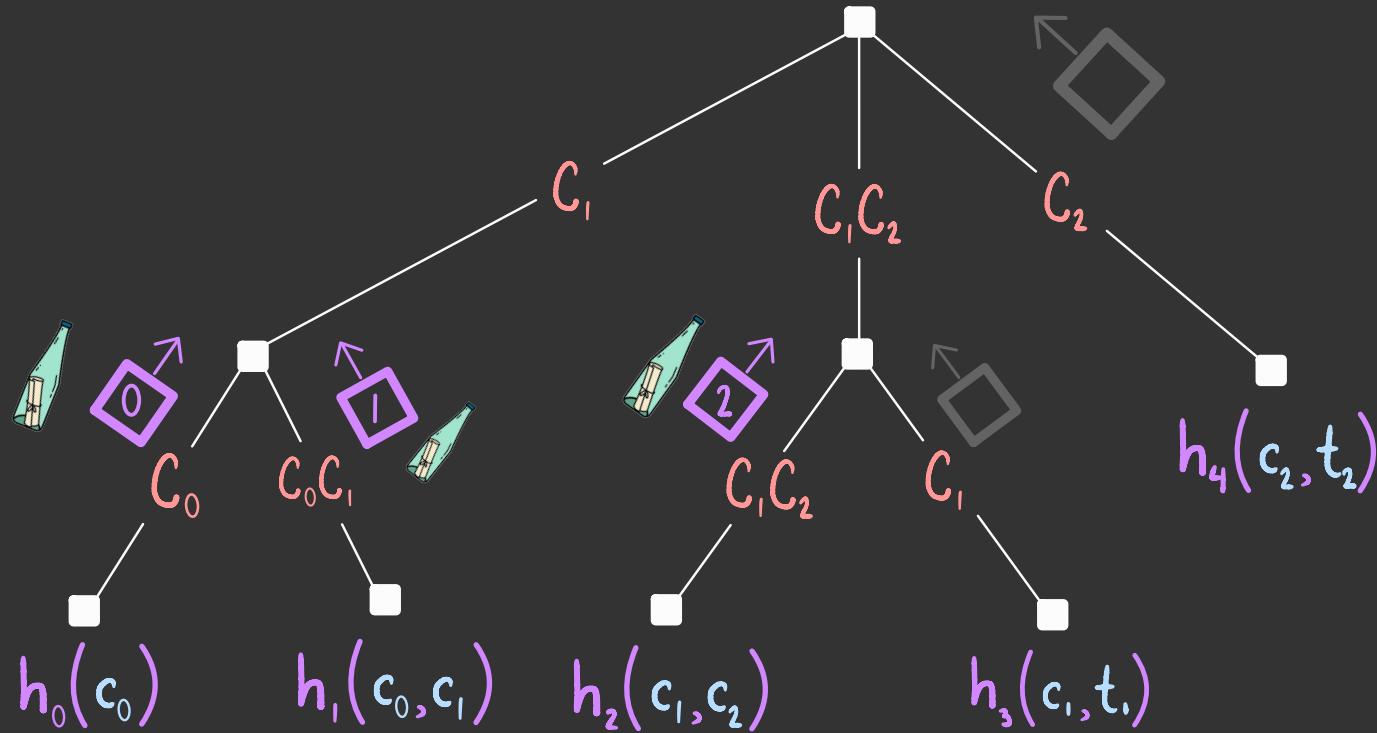
C_1	$h(c_1)$
0	0.9811
1	0.0189



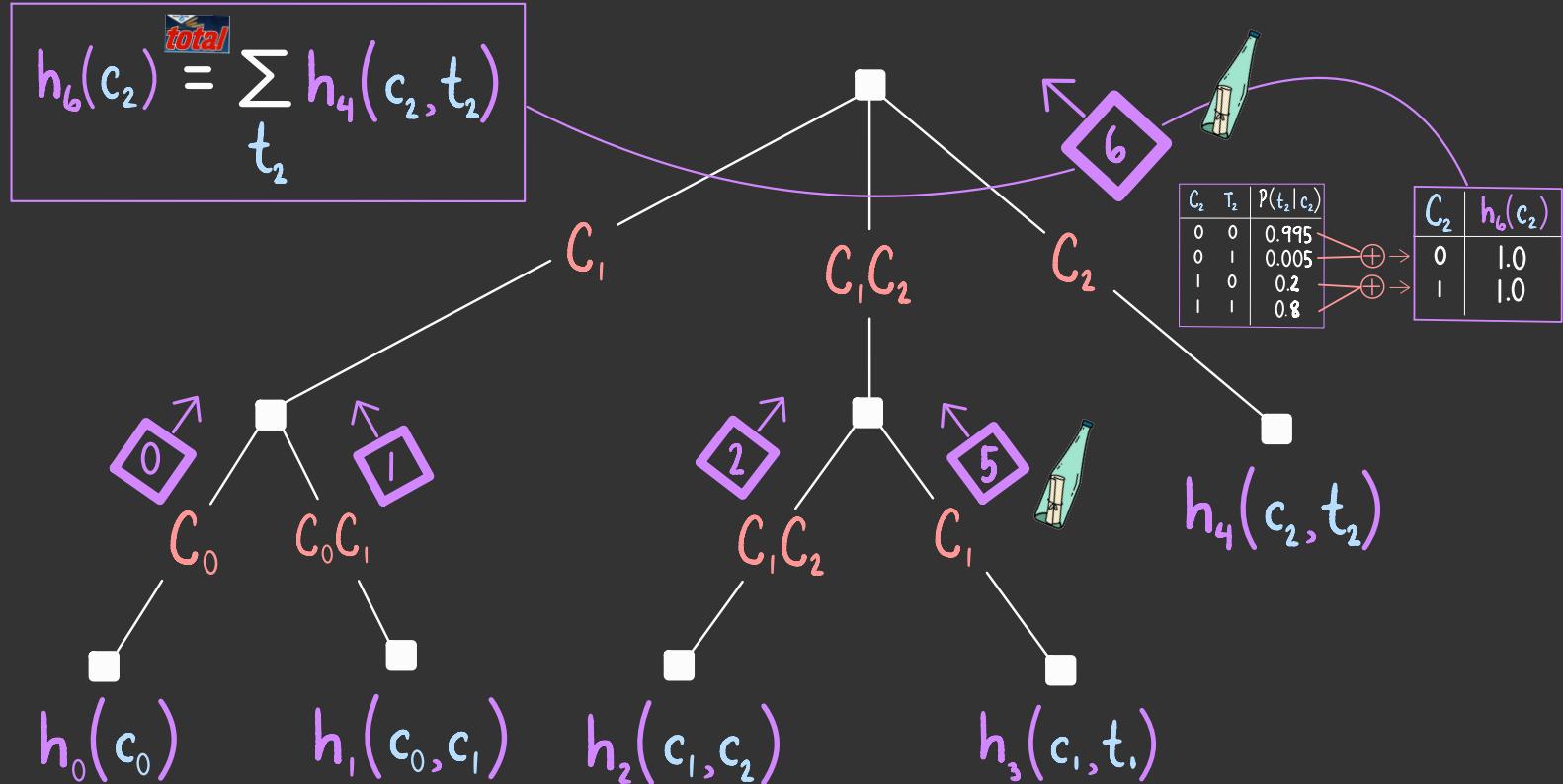
step three: but what factor does each node send?



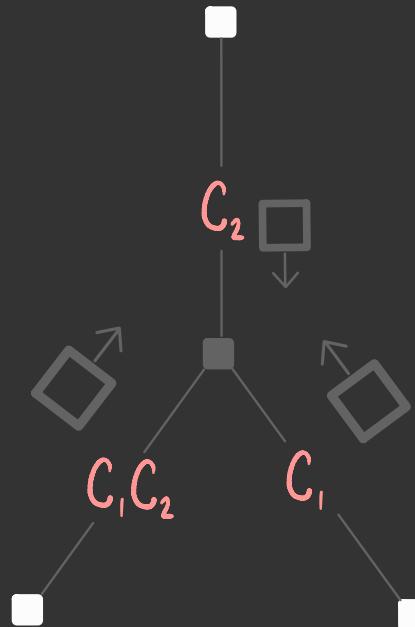
step three: leaf nodes send their factor to their only neighbor



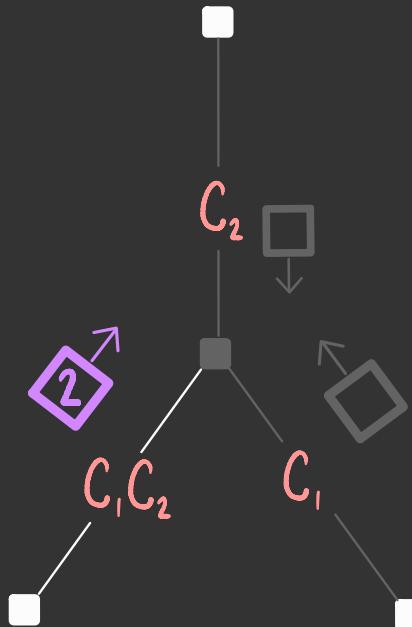
step three: leaf nodes send their factor to their only neighbor, after marginalizing out non-separator variables



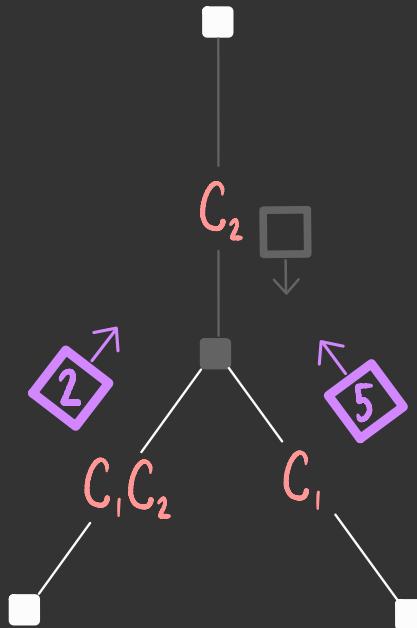
step three: each internal node waits to receive factors from its neighbors



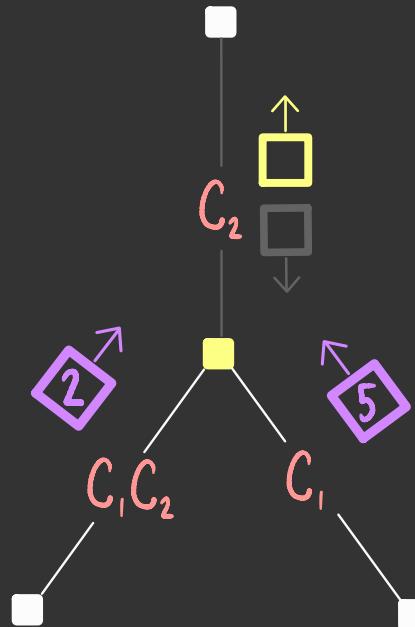
step three: each internal node waits to receive factors from its neighbors



step three: each internal node waits to receive factors from its neighbors

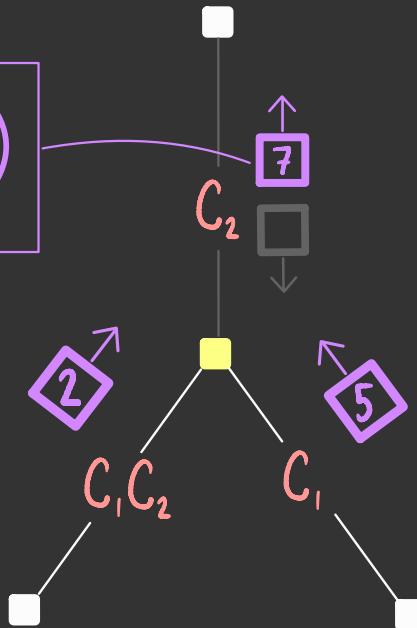


step three: once the node has received factors from all neighbors but one, it can send a factor to the remaining neighbor



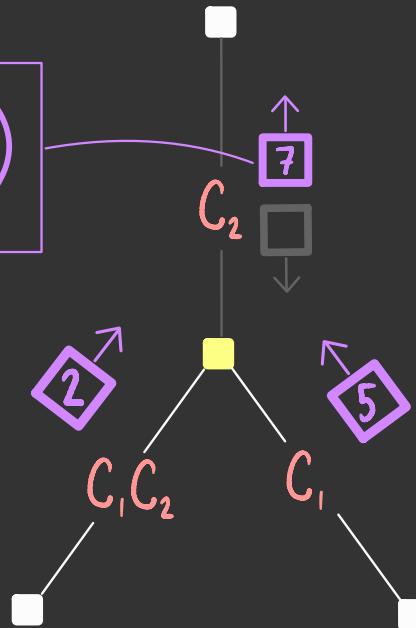
step three: this new factor will be the product of the received factors...

$$h_7(c_1, c_2) = h_2(c_1, c_2)h_5(c_1)$$

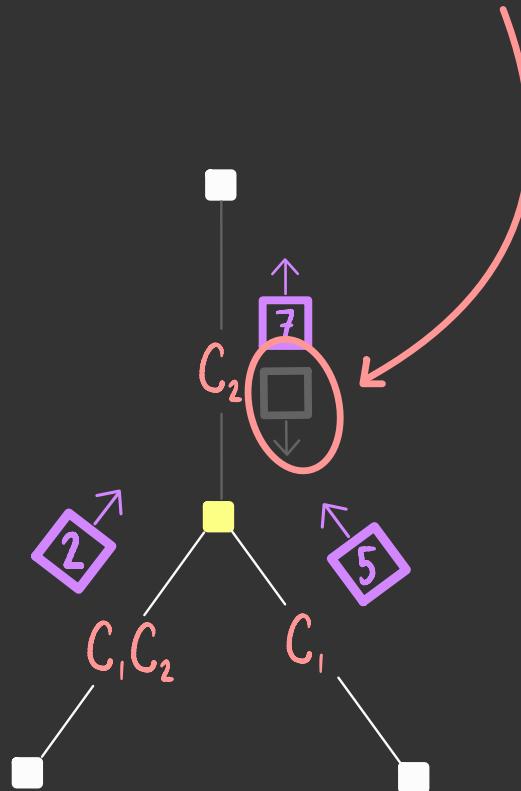


step three: this new factor will be the product of the received factors...
after marginalizing out variables not in the separator with that neighbor

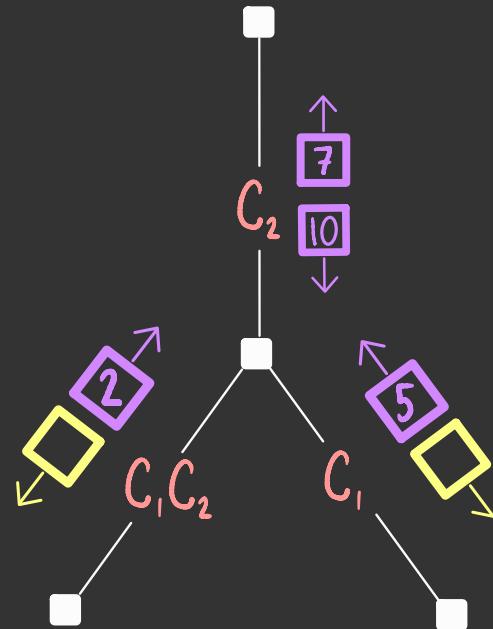
$$h_7(c_2) = \sum_{c_1} h_2(c_1, c_2) h_5(c_1)$$



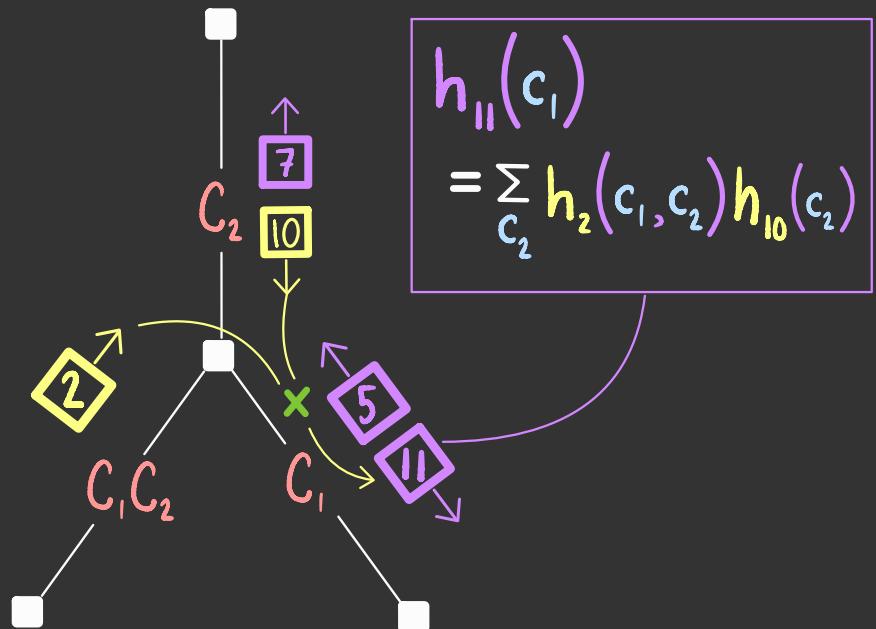
step three: once we receive the factor from the final neighbor...



step three: once we receive the factor from the final neighbor...
we can send analogous factors to our other neighbors

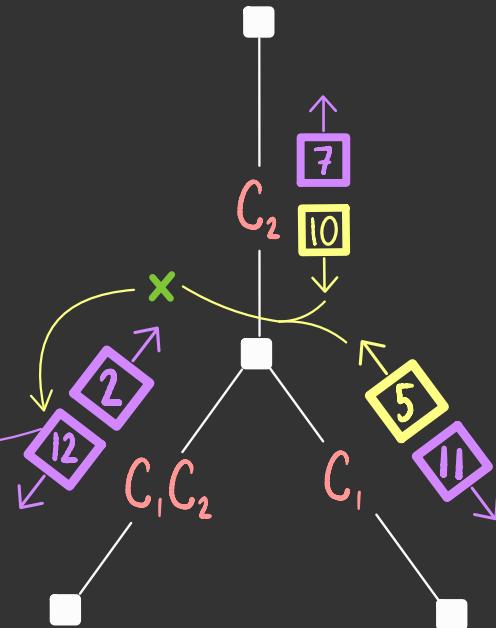


step three: once we receive the factor from the final neighbor...
we can send analogous factors to our other neighbors



step three: once we receive the factor from the final neighbor...
we can send analogous factors to our other neighbors

$$h_{12}(c_1, c_2) \\ = h_5(c_1)h_{10}(c_2)$$



- 5** $\sum_{t_i} h_3(c_i, t_i)$

6 $\sum_{c_0} h_0(c_0) h_1(c_0, c_1)$

7 $h_2(c_1, c_2) h_5(c_1)$

8 $\sum_{c_1} h_6(c_1) h_7(c_1, c_2)$

9 $\sum_{t_2} h_4(c_2, t_2)$

10 $h_9(c_2) h_6(c_1)$

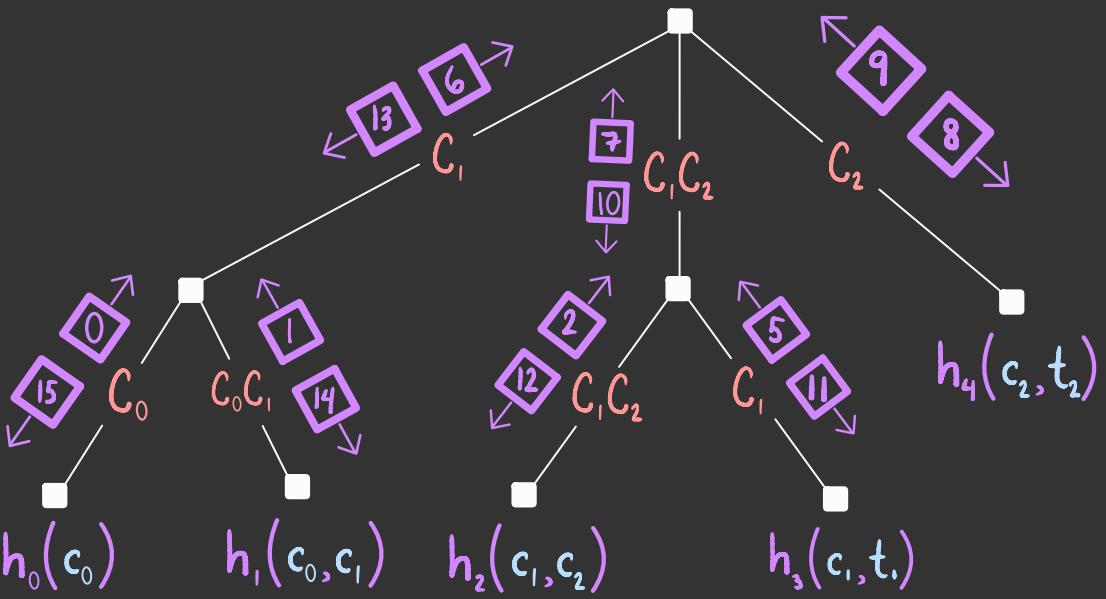
11 $\sum_{c_2} h_2(c_1, c_2) h_{10}(c_1, c_2)$

12 $h_5(c_1) h_{10}(c_1, c_2)$

13 $\sum_{c_2} h_7(c_1, c_2) h_9(c_2)$

14 $h_0(c_0) h_{13}(c_1)$

15 $\sum_{c_1} h_1(c_0, c_1) h_{13}(c_1)$



running this algorithm (the junction tree algorithm) gives us every factor we need to compute all the single-variable marginals

5

$$\sum_{t_i} h_3(c_i, t_i)$$

6

$$\sum_{c_0} h_0(c_0) h_1(c_0, c_1)$$

7

$$h_2(c_1, c_2) h_5(c_1)$$

8

$$\sum_{c_1} h_6(c_1) h_7(c_1, c_2)$$

9

$$\sum_{t_2} h_4(c_2, t_2)$$

10

$$h_9(c_2) h_6(c_1)$$

11

$$\sum_{c_2} h_2(c_1, c_2) h_{10}(c_1, c_2)$$

12

$$h_5(c_1) h_{10}(c_1, c_2)$$

13

$$\sum_{c_2} h_7(c_1, c_2) h_9(c_2)$$

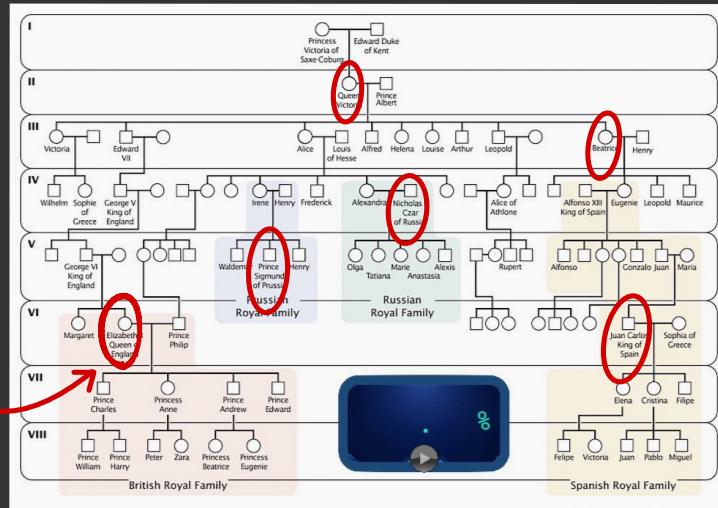
14

$$h_0(c_0) h_{13}(c_1)$$

15

$$\sum_{c_1} h_1(c_0, c_1) h_{13}(c_1)$$

probability that
PERSON is a
carrier for
hemophilia



running this algorithm (the junction tree algorithm) gives us every factor we need to compute all the single-variable marginals

5

$$\sum_{t_i} h_3(c_i, t_i)$$

6

$$\sum_{c_0} h_0(c_0) h_1(c_0, c_1)$$

7

$$h_2(c_1, c_2) h_5(c_1)$$

8

$$\sum_{c_1} h_6(c_1) h_7(c_1, c_2)$$

9

$$\sum_{t_2} h_4(c_2, t_2)$$

10

$$h_9(c_2) h_6(c_1)$$

11

$$\sum_{c_2} h_2(c_1, c_2) h_{10}(c_1, c_2)$$

12

$$h_5(c_1) h_{10}(c_1, c_2)$$

13

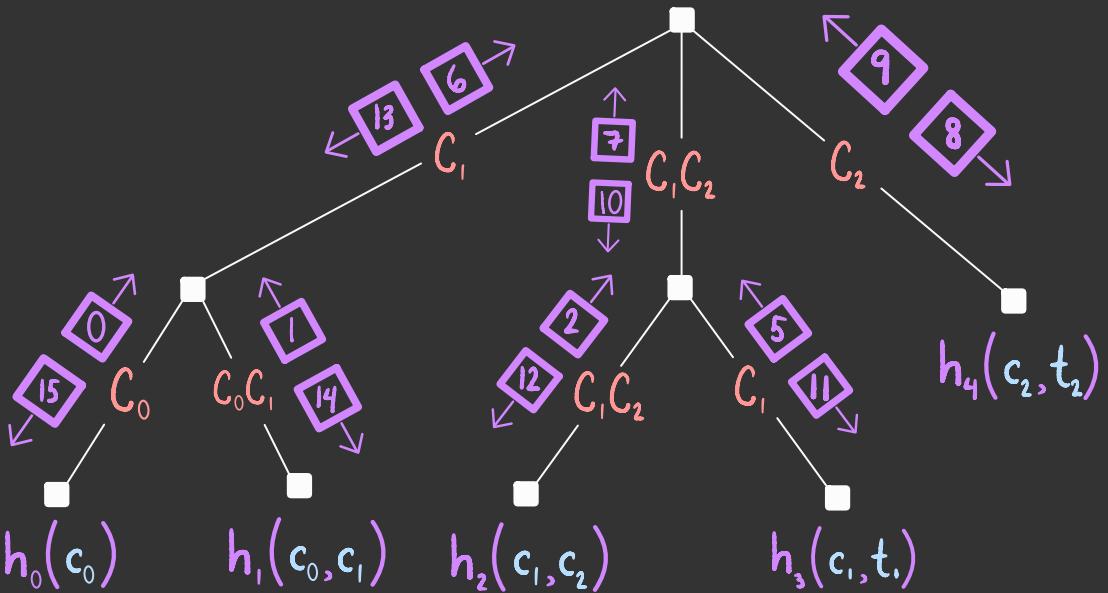
$$\sum_{c_2} h_7(c_1, c_2) h_9(c_2)$$

14

$$h_0(c_0) h_{13}(c_1)$$

15

$$\sum_{c_1} h_1(c_0, c_1) h_{13}(c_1)$$



to compute a single-variable marginal:

5

$$\sum_{t_i} h_3(c_i, t_i)$$

6

$$\sum_{c_0} h_0(c_0) h_1(c_0, c_1)$$

7

$$h_2(c_1, c_2) h_5(c_1)$$

8

$$\sum_{c_1} h_6(c_1) h_7(c_1, c_2)$$

9

$$\sum_{t_2} h_4(c_2, t_2)$$

10

$$h_9(c_2) h_6(c_1)$$

11

$$\sum_{c_2} h_2(c_1, c_2) h_{10}(c_1, c_2)$$

12

$$h_5(c_1) h_{10}(c_1, c_2)$$

13

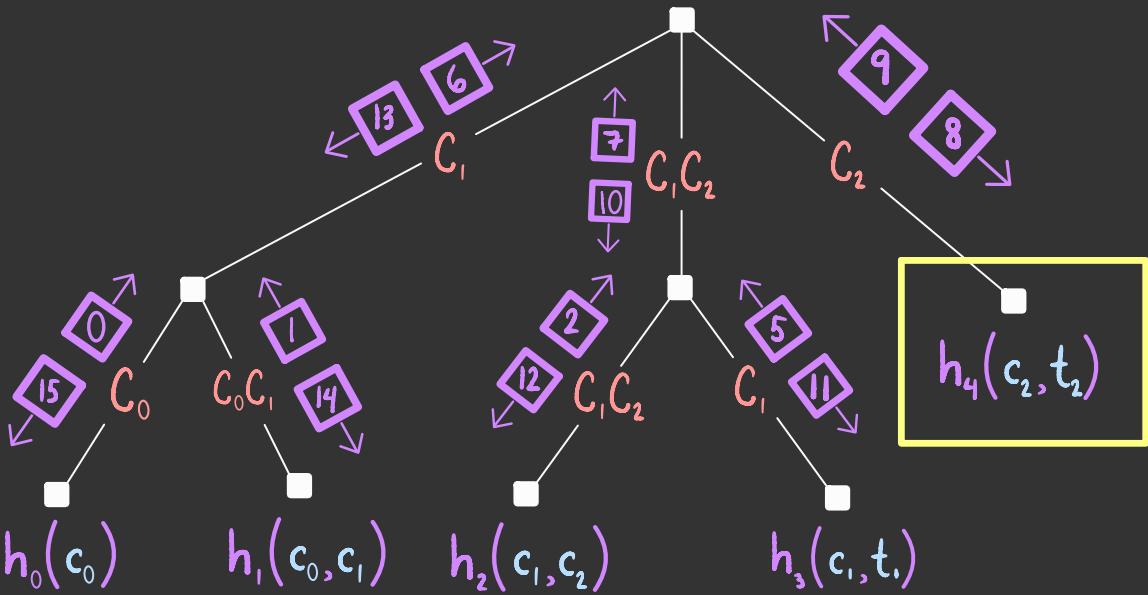
$$\sum_{c_2} h_7(c_1, c_2) h_9(c_2)$$

14

$$h_0(c_0) h_{13}(c_1)$$

15

$$\sum_{c_1} h_1(c_0, c_1) h_{13}(c_1)$$



to compute a single-variable marginal:

- choose a leaf containing the variable

5

$$\sum_{t_i} h_3(c_i, t_i)$$

6

$$\sum_{c_0} h_0(c_0) h_1(c_0, c_1)$$

7

$$h_2(c_1, c_2) h_5(c_1)$$

8

$$\sum_{c_1} h_6(c_1) h_7(c_1, c_2)$$

9

$$\sum_{t_2} h_4(c_2, t_2)$$

10

$$h_9(c_2) h_6(c_1)$$

11

$$\sum_{c_2} h_2(c_1, c_2) h_{10}(c_1, c_2)$$

12

$$h_5(c_1) h_{10}(c_1, c_2)$$

13

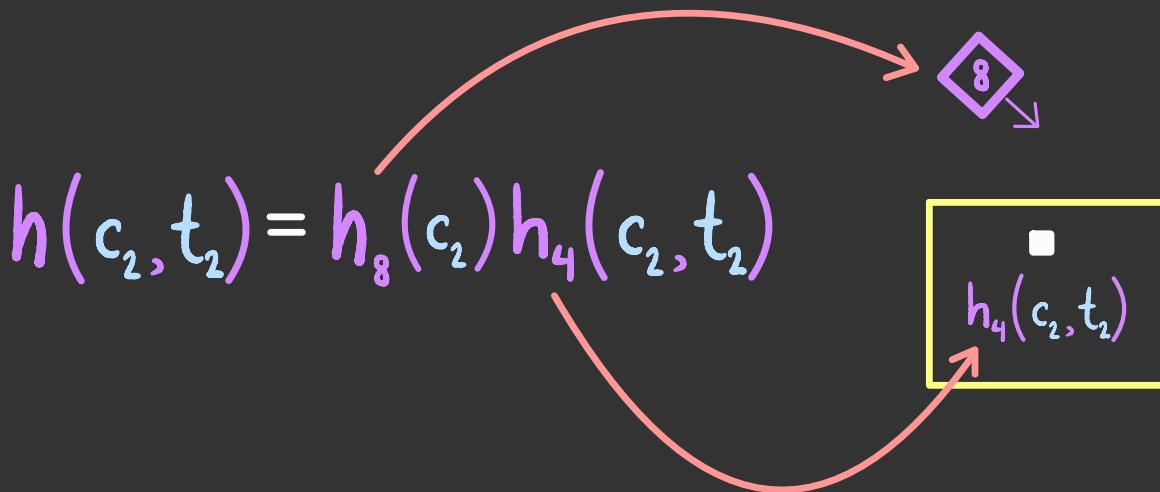
$$\sum_{c_2} h_7(c_1, c_2) h_9(c_2)$$

14

$$h_0(c_0) h_{13}(c_0)$$

15

$$\sum_{c_1} h_1(c_0, c_1) h_{13}(c_1)$$



to compute a single-variable marginal:

- choose a leaf containing the variable
- multiply its factor with its received "message"

5

$$\sum_{t_i} h_3(c_i, t_i)$$

6

$$\sum_{c_0} h_0(c_0) h_1(c_0, c_1)$$

7

$$h_2(c_1, c_2) h_5(c_1)$$

8

$$\sum_{c_1} h_6(c_1) h_7(c_1, c_2)$$

9

$$\sum_{t_2} h_4(c_2, t_2)$$

10

$$h_9(c_2) h_6(c_1)$$

11

$$\sum_{c_2} h_2(c_1, c_2) h_{10}(c_1, c_2)$$

12

$$h_5(c_1) h_{10}(c_1, c_2)$$

13

$$\sum_{c_2} h_7(c_1, c_2) h_9(c_2)$$

14

$$h_0(c_0) h_{13}(c_0)$$

15

$$\sum_{c_1} h_1(c_0, c_1) h_{13}(c_1)$$

we obtain the marginal



$$P(t_2) = \sum_{c_2} h_8(c_2) h_4(c_2, t_2)$$

8

■
h₄(c₂, t₂)

to compute a single-variable marginal:

- choose a leaf containing the variable
- multiply its factor with its received "message"
- marginalize out other variables

5

$$\sum_{t_1} h_3(c_1, t_1)$$

6

$$\sum_{c_0} h_0(c_0) h_1(c_0, c_1)$$

7

$$h_2(c_1, c_2) h_5(c_1)$$

8

$$\sum_{c_1} h_6(c_1) h_7(c_1, c_2)$$

9

$$\sum_{t_2} h_4(c_2, t_2)$$

10

$$h_9(c_2) h_6(c_1)$$

11

$$\sum_{c_2} h_2(c_1, c_2) h_{10}(c_1, c_2)$$

12

$$h_5(c_1) h_{10}(c_1, c_2)$$

13

$$\sum_{c_2} h_7(c_1, c_2) h_9(c_2)$$

14

$$h_0(c_0) h_{13}(c_1)$$

15

$$\sum_{c_1} h_1(c_0, c_1) h_{13}(c_1)$$

why does this
work?

to compute a single-variable marginal:

- choose a leaf containing the variable
- multiply its factor with its received "message"
- marginalize out other variables

5

$$\sum_{t_i} h_3(c_i, t_i)$$

6

$$\sum_{c_0} h_0(c_0) h_1(c_0, c_1)$$

7

$$h_2(c_1, c_2) h_5(c_1)$$

8

$$\sum_{c_1} h_6(c_1) h_7(c_1, c_2)$$

9

$$\sum_{t_2} h_4(c_2, t_2)$$

10

$$h_9(c_2) h_6(c_1)$$

11

$$\sum_{c_2} h_2(c_1, c_2) h_{10}(c_1, c_2)$$

12

$$h_5(c_1) h_{10}(c_1, c_2)$$

13

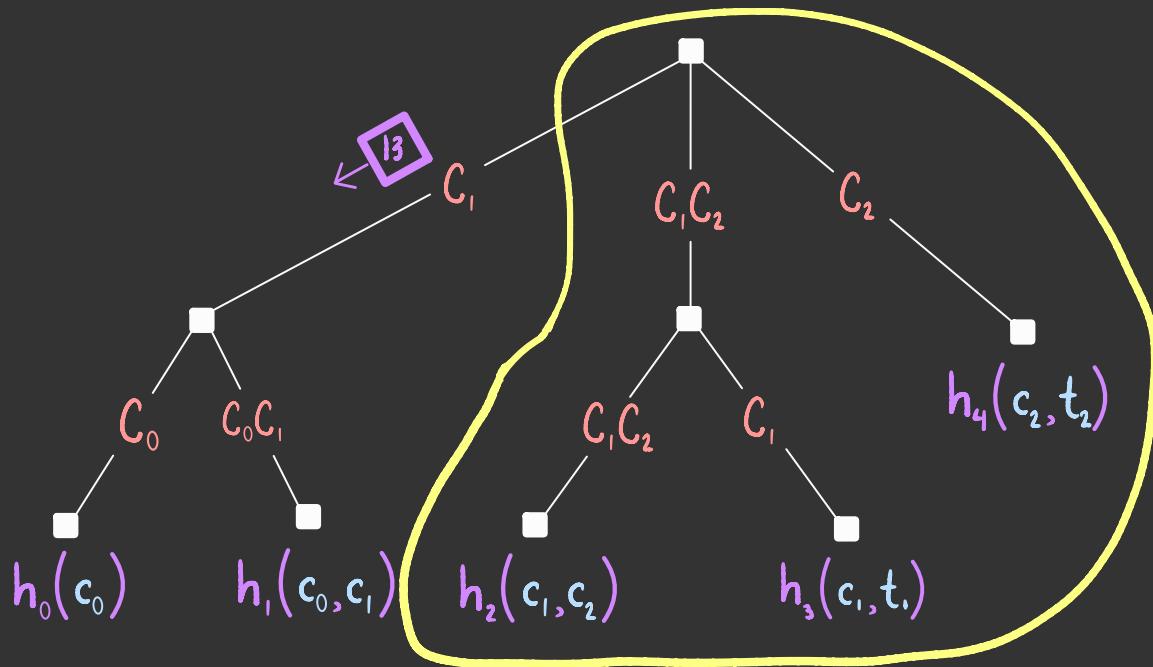
$$\sum_{c_2} h_7(c_1, c_2) h_9(c_2)$$

14

$$h_0(c_0) h_{13}(c_1)$$

15

$$\sum_{c_1} h_1(c_0, c_1) h_{13}(c_1)$$



the message that a node sends its neighbor is the product of the factors on its side of the edge...

5

$$\sum_{t_i} h_3(c_i, t_i)$$

6

$$\sum_{c_0} h_0(c_0) h_1(c_0, c_1)$$

7

$$h_2(c_1, c_2) h_5(c_1)$$

8

$$\sum_{c_1} h_6(c_1) h_7(c_1, c_2)$$

9

$$\sum_{t_2} h_4(c_2, t_2)$$

10

$$h_9(c_2) h_6(c_1)$$

11

$$\sum_{c_2} h_2(c_1, c_2) h_{10}(c_1, c_2)$$

12

$$h_5(c_1) h_{10}(c_1, c_2)$$

13

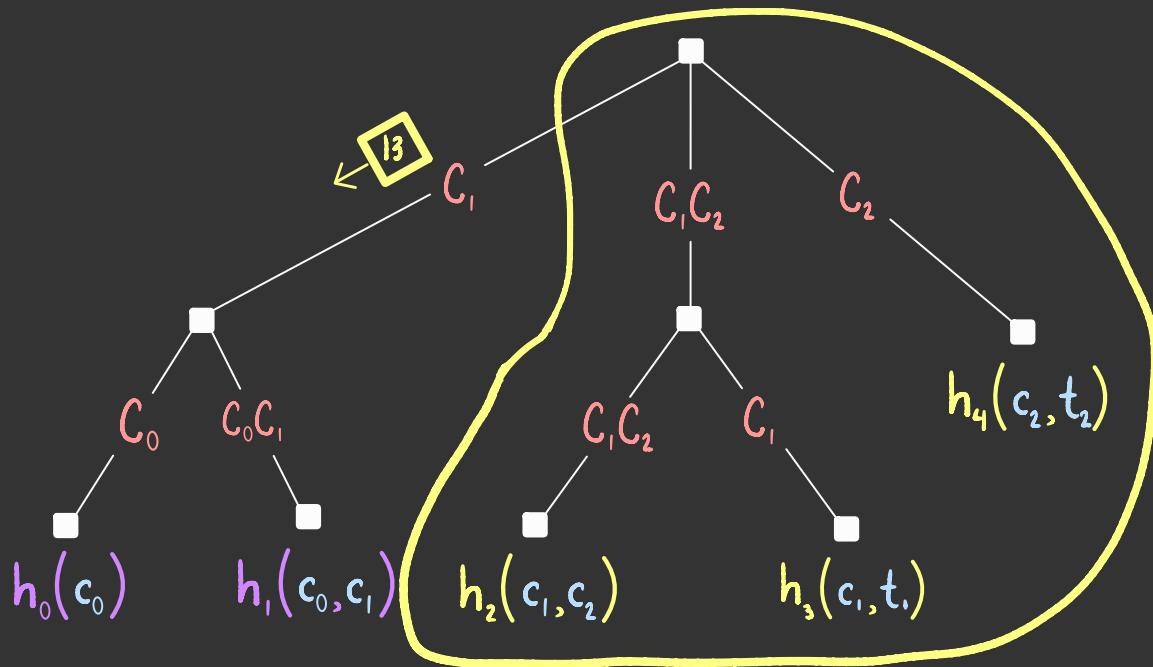
$$\sum_{c_2} h_7(c_1, c_2) h_9(c_2)$$

14

$$h_0(c_0) h_{13}(c_1)$$

15

$$\sum_{c_1} h_1(c_0, c_1) h_{13}(c_1)$$



the message that a node sends its neighbor is the product of the factors on its side of the edge...

13

$$h_2(c_1, c_2) h_3(c_1, t_1) h_4(c_2, t_2)$$

5

$$\sum_{t_i} h_3(c_i, t_i)$$

6

$$\sum_{c_0} h_0(c_0) h_1(c_0, c_1)$$

7

$$h_2(c_1, c_2) h_5(c_1)$$

8

$$\sum_{c_1} h_6(c_1) h_7(c_1, c_2)$$

9

$$\sum_{t_2} h_4(c_2, t_2)$$

10

$$h_9(c_2) h_6(c_1)$$

11

$$\sum_{c_2} h_2(c_1, c_2) h_{10}(c_1, c_2)$$

12

$$h_5(c_1) h_{10}(c_1, c_2)$$

13

$$\sum_{c_2} h_7(c_1, c_2) h_9(c_2)$$

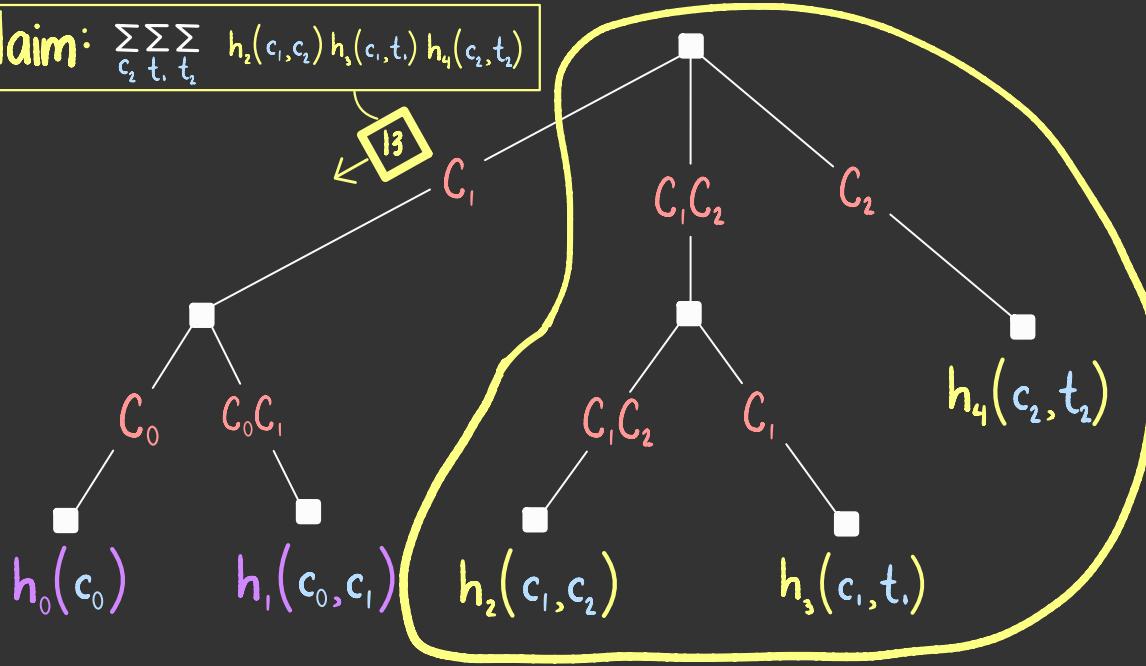
14

$$h_0(c_0) h_{13}(c_1)$$

15

$$\sum_{c_1} h_1(c_0, c_1) h_{13}(c_1)$$

claim: $\sum_{c_2} \sum_{t_1} \sum_{t_2} h_2(c_1, c_2) h_3(c_1, t_1) h_4(c_2, t_2)$



the message that a node sends its neighbor is the product of the factors on its side of the edge... with the non-separator variables marginalized out

13

$$\sum_{c_2} \sum_{t_1} \sum_{t_2} h_2(c_1, c_2) h_3(c_1, t_1) h_4(c_2, t_2)$$

5

$$\sum_{t_i} h_3(c_i, t_i)$$

6

$$\sum_{c_0} h_0(c_0) h_1(c_0, c_1)$$

7

$$h_2(c_1, c_2) h_5(c_1)$$

8

$$\sum_{c_1} h_6(c_1) h_7(c_1, c_2)$$

9

$$\sum_{t_2} h_4(c_2, t_2)$$

10

$$h_q(c_2) h_6(c_1)$$

11

$$\sum_{c_2} h_2(c_1, c_2) h_{10}(c_1, c_2)$$

12

$$h_5(c_1) h_{10}(c_1, c_2)$$

13

$$\sum_{c_2} h_7(c_1, c_2) h_q(c_2)$$

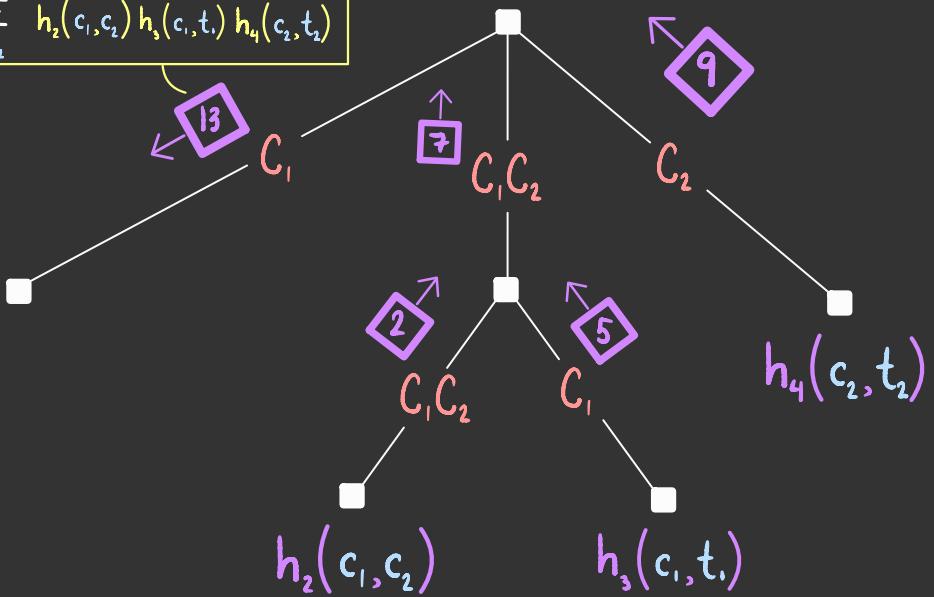
14

$$h_0(c_0) h_{13}(c_1)$$

15

$$\sum_{c_1} h_1(c_0, c_1) h_{13}(c_1)$$

claim: $\sum_{c_2} \sum_{t_i} \sum_{t_2} h_2(c_1, c_2) h_3(c_i, t_i) h_4(c_2, t_2)$



13 $\sum_{c_2} h_7(c_1, c_2) h_q(c_2) =$

5

$$\sum_{t_i} h_3(c_i, t_i)$$

6

$$\sum_{c_0} h_0(c_0) h_1(c_0, c_1)$$

7

$$h_2(c_1, c_2) h_5(c_1)$$

8

$$\sum_{c_1} h_6(c_1) h_7(c_1, c_2)$$

9

$$\sum_{t_2} h_4(c_2, t_2)$$

10

$$h_9(c_2) h_6(c_1)$$

11

$$\sum_{c_2} h_2(c_1, c_2) h_{10}(c_1, c_2)$$

12

$$h_5(c_1) h_{10}(c_1, c_2)$$

13

$$\sum_{c_2} h_7(c_1, c_2) h_9(c_2)$$

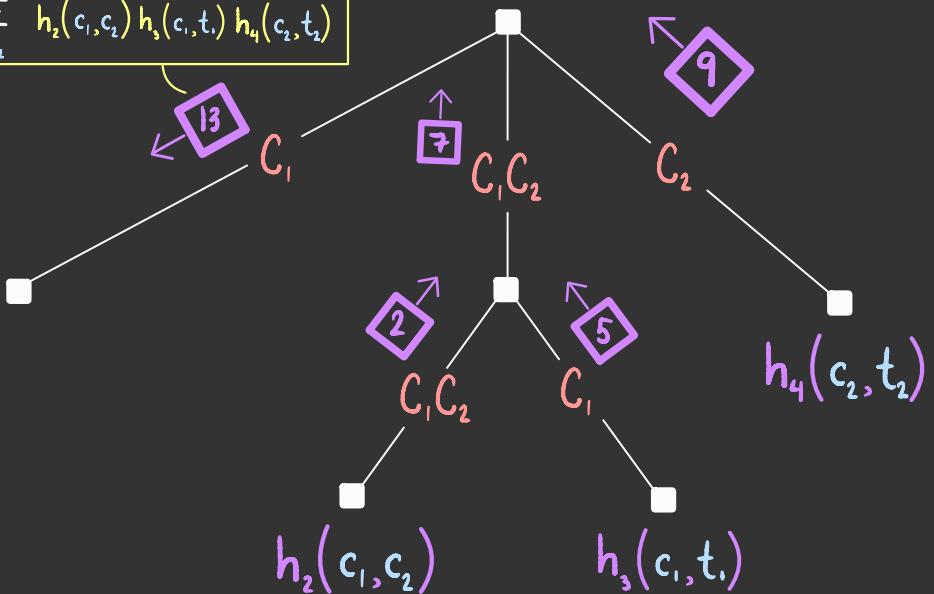
14

$$h_0(c_0) h_{13}(c_1)$$

15

$$\sum_{c_1} h_1(c_0, c_1) h_{13}(c_1)$$

claim: $\sum_{c_2} \sum_{t_i} h_2(c_1, c_2) h_3(c_i, t_i) h_4(c_2, t_2)$



13 $\sum_{c_2} h_7(c_1, c_2) h_9(c_2) = \sum_{c_2} h_2(c_1, c_2) h_5(c_1) h_9(c_2)$

5

$$\sum_{t_i} h_3(c_i, t_i)$$

6

$$\sum_{c_0} h_0(c_0) h_1(c_0, c_1)$$

7

$$h_2(c_1, c_2) h_5(c_1)$$

8

$$\sum_{c_1} h_6(c_1) h_7(c_1, c_2)$$

9

$$\sum_{t_2} h_4(c_2, t_2)$$

10

$$h_q(c_2) h_6(c_1)$$

11

$$\sum_{c_2} h_2(c_1, c_2) h_{10}(c_1, c_2)$$

12

$$h_5(c_1) h_{10}(c_1, c_2)$$

13

$$\sum_{c_2} h_7(c_1, c_2) h_q(c_2)$$

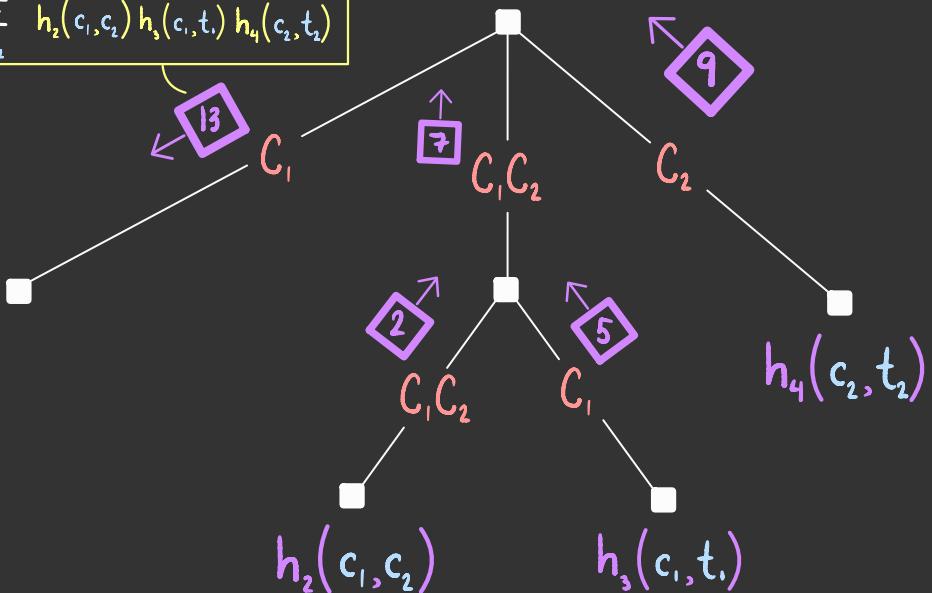
14

$$h_0(c_0) h_{13}(c_1)$$

15

$$\sum_{c_1} h_1(c_0, c_1) h_{13}(c_1)$$

claim: $\sum_{c_2} \sum_{t_i} h_2(c_1, c_2) h_3(c_i, t_i) h_4(c_2, t_2)$



13 $\sum_{c_2} h_7(c_1, c_2) h_q(c_2) = \sum_{c_2} h_2(c_1, c_2) h_5(c_1) h_q(c_2)$

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$$\sum_{c_2} h_2(c_1, c_2) h_{10}(c_1, c_2)$$

12

$$h_5(c_1) h_{10}(c_1, c_2)$$

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$$\sum_{c_2} h_7(c_1, c_2) h_q(c_2)$$

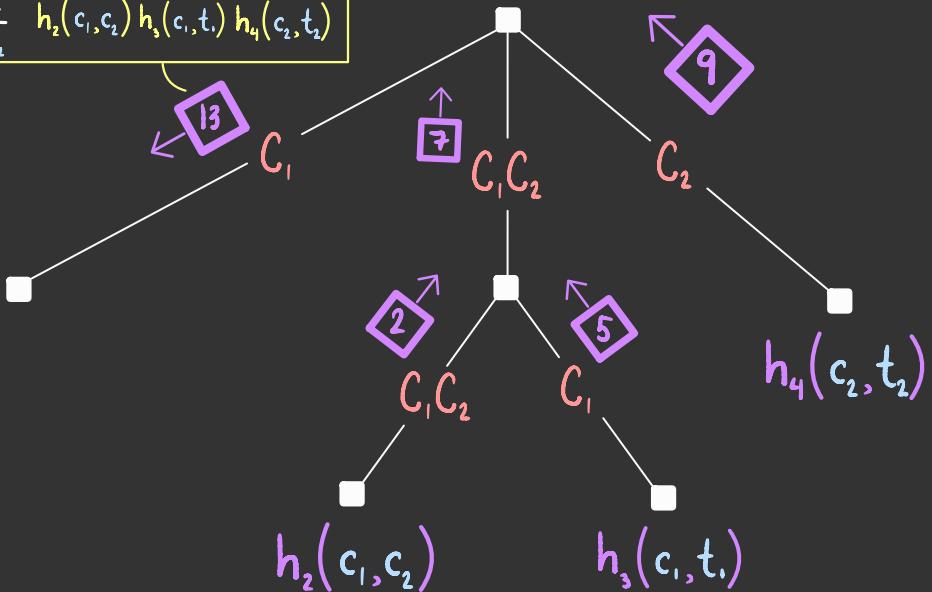
14

$$h_0(c_0) h_{13}(c_1)$$

15

$$\sum_{c_1} h_1(c_0, c_1) h_{13}(c_1)$$

claim: $\sum_{c_2} \sum_{t_i} \sum_{t_2} h_2(c_1, c_2) h_3(c_i, t_i) h_4(c_2, t_2)$



$$\begin{aligned}
 13 \quad \sum_{c_2} h_7(c_1, c_2) h_q(c_2) &= \sum_{c_2} h_2(c_1, c_2) h_5(c_1) h_q(c_2) \\
 &= \sum_{c_2} h_2(c_1, c_2) \sum_{t_i} h_3(c_i, t_i) h_q(c_2)
 \end{aligned}$$

5

$$\sum_{t_i} h_3(c_i, t_i)$$

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$$\sum_{c_0} h_0(c_0) h_1(c_0, c_1)$$

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$$h_2(c_1, c_2) h_5(c_1)$$

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$$\sum_{c_1} h_6(c_1) h_7(c_1, c_2)$$

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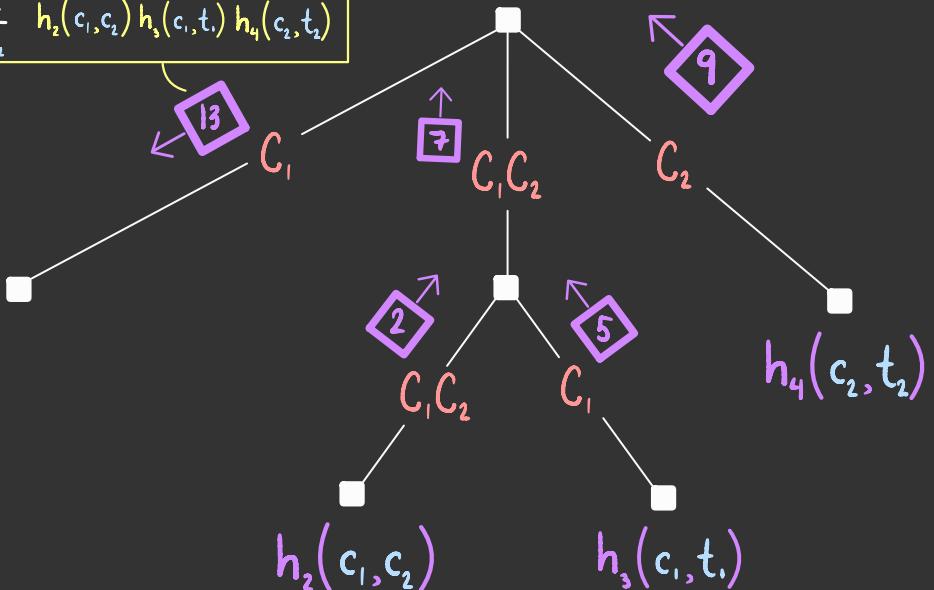
14

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claim: $\sum_{c_2} \sum_{t_i} \sum_{t_2} h_2(c_1, c_2) h_3(c_1, t_i) h_4(c_2, t_2)$



13

$$\begin{aligned}
 \sum_{c_2} h_7(c_1, c_2) h_q(c_2) &= \sum_{c_2} h_2(c_1, c_2) h_5(c_1) h_q(c_2) \\
 &= \sum_{c_2} h_2(c_1, c_2) \sum_{t_i} h_3(c_1, t_i) \sum_{t_2} h_4(c_2, t_2)
 \end{aligned}$$

5

$$\sum_{t_i} h_3(c_i, t_i)$$

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$$\sum_{c_0} h_0(c_0) h_1(c_0, c_1)$$

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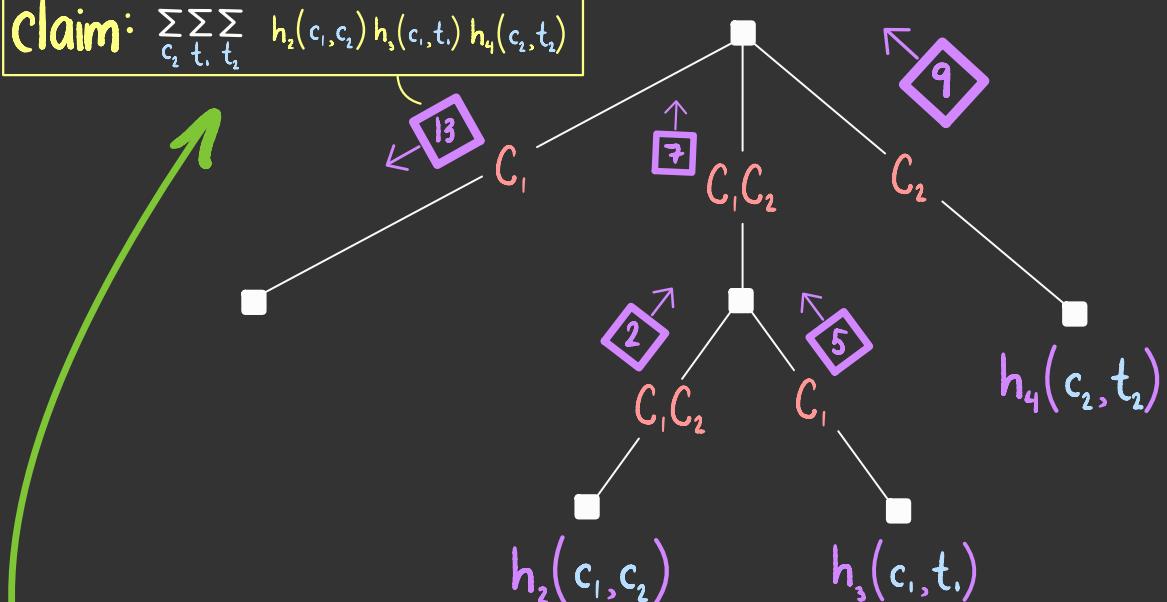
14

$$h_0(c_0) h_{13}(c_1)$$

15

$$\sum_{c_1} h_1(c_0, c_1) h_{13}(c_1)$$

claim: $\sum_{c_2} \sum_{t_i} \sum_{t_2} h_2(c_1, c_2) h_3(c_i, t_i) h_4(c_2, t_2)$



13

$$\begin{aligned}
 \sum_{c_2} h_7(c_1, c_2) h_q(c_2) &= \sum_{c_2} h_2(c_1, c_2) h_5(c_1) h_q(c_2) \\
 &= \sum_{c_2} h_2(c_1, c_2) \sum_{t_i} h_3(c_i, t_i) \sum_{t_2} h_4(c_2, t_2) \\
 &= \sum_{c_2} \sum_{t_i} \sum_{t_2} h_2(c_1, c_2) h_3(c_i, t_i) h_4(c_2, t_2)
 \end{aligned}$$

5

$$\sum_{t_i} h_3(c_i, t_i)$$

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$$\sum_{c_0} h_0(c_0) h_1(c_0, c_1)$$

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$$h_2(c_1, c_2) h_5(c_1)$$

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$$\sum_{c_2} h_7(c_1, c_2) h_9(c_2)$$

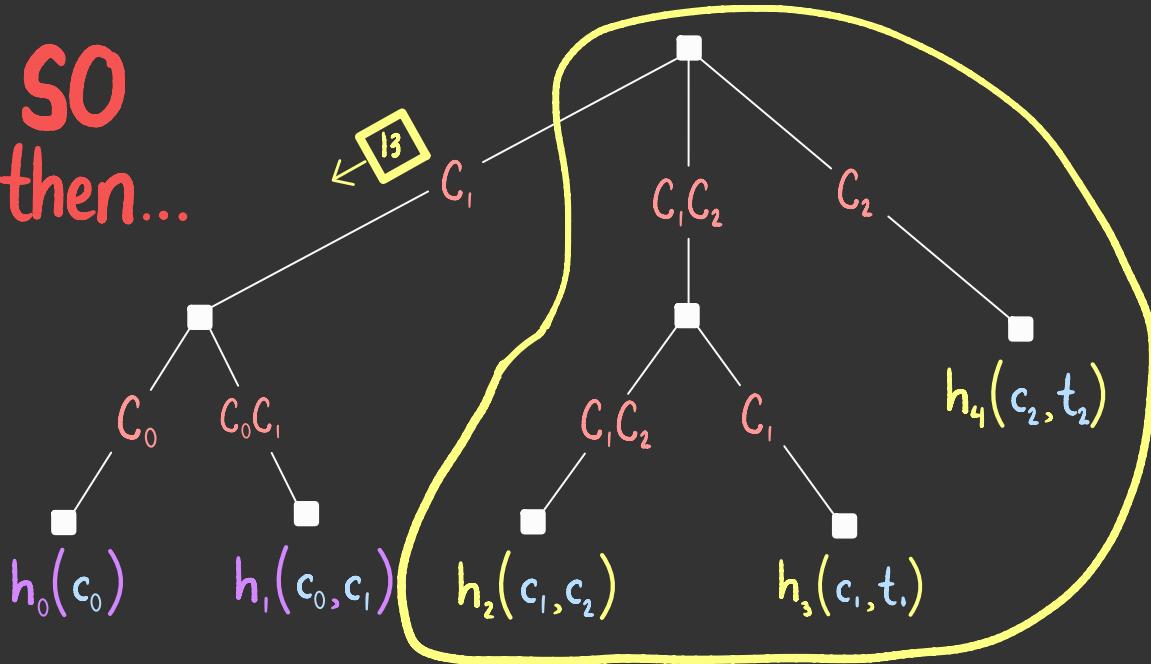
14

$$h_0(c_0) h_{13}(c_1)$$

15

$$\sum_{c_1} h_1(c_0, c_1) h_{13}(c_1)$$

so
then...



the message that a node sends its neighbor is the product of the factors on its side of the edge...
with the non-separator variables marginalized out

13 $\sum_{c_2} \sum_{t_1} \sum_{t_2} h_2(c_1, c_2) h_3(c_1, t_1) h_4(c_2, t_2)$

5

$$\sum_{t_i} h_3(c_i, t_i)$$

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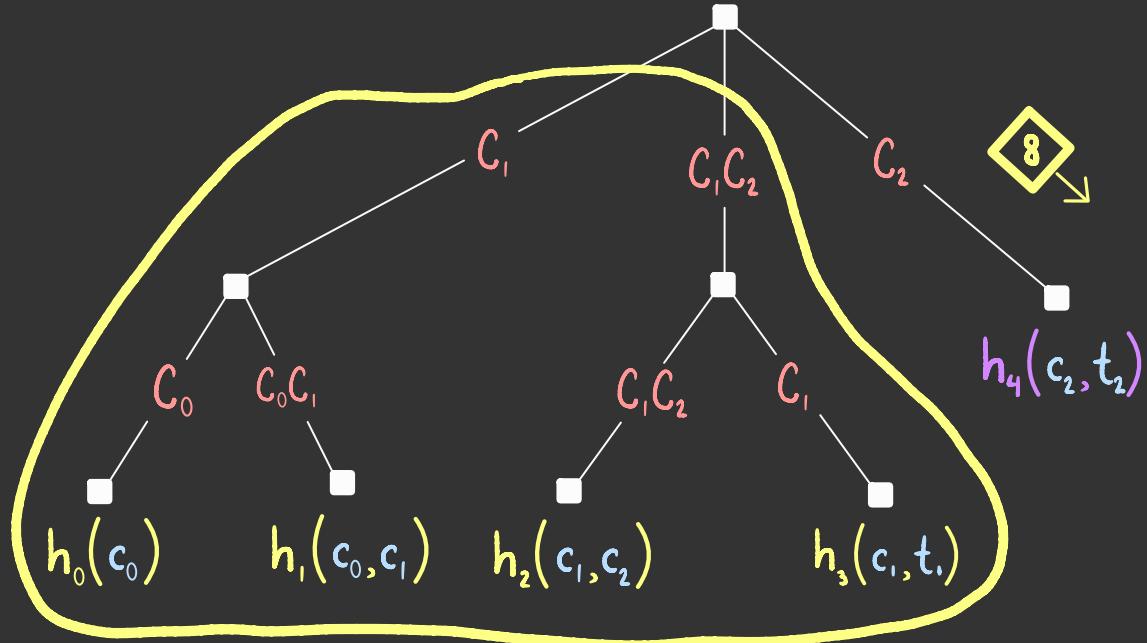
$$\sum_{c_2} h_7(c_1, c_2) h_9(c_2)$$

14

$$h_0(c_0) h_{13}(c_1)$$

15

$$\sum_{c_1} h_1(c_0, c_1) h_{13}(c_1)$$



the message that a node sends its neighbor is the product of the factors on its side of the edge... with the non-separator variables marginalized out

8

$$\sum_{t_i} \sum_{c_1} \sum_{c_0} h_0(c_0) h_1(c_0, c_1) h_2(c_1, c_2) h_3(c_1, t_i)$$

5

$$\sum_{t_i} h_3(c_i, t_i)$$

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$$\sum_{c_0} h_0(c_0) h_1(c_0, c_1)$$

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$$h_2(c_1, c_2) h_5(c_1)$$

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$$\sum_{c_1} h_6(c_1) h_7(c_1, c_2)$$

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$$\sum_{t_2} h_4(c_2, t_2)$$

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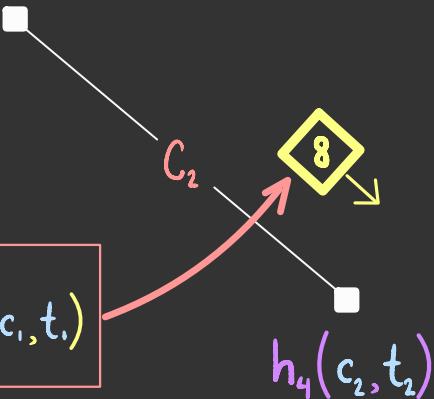
$$\sum_{c_2} h_7(c_1, c_2) h_9(c_2)$$

14

$$h_0(c_0) h_{13}(c_1)$$

15

$$\sum_{c_1} h_1(c_0, c_1) h_{13}(c_1)$$



$$\sum_{t_i} \sum_{c_1} \sum_{c_0} h_0(c_0) h_1(c_0, c_1) h_2(c_1, c_2) h_3(c_1, t_i)$$

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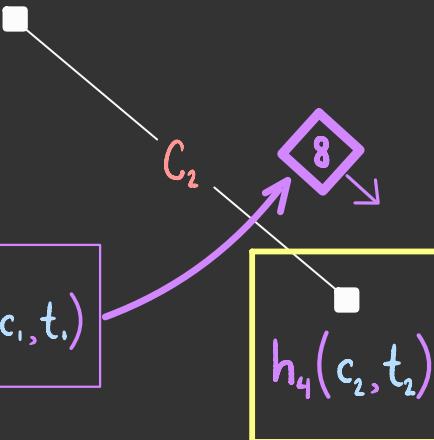
14

$$h_0(c_0) h_{13}(c_1)$$

15

$$\sum_{c_1} h_1(c_0, c_1) h_{13}(c_1)$$

$$\sum_{t_i} \sum_{c_1} \sum_{c_0} h_0(c_0) h_1(c_0, c_1) h_2(c_1, c_2) h_3(c_1, t_i)$$



to compute single-variable marginal $P(t_2)$:

- choose a leaf containing variable T_2
- multiply its factor with its received "message"
- marginalize out other variables

5

$$\sum_{t_i} h_3(c_i, t_i)$$

6

$$\sum_{c_0} h_0(c_0) h_1(c_0, c_1)$$

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$$h_2(c_1, c_2) h_5(c_1)$$

8

$$\sum_{c_1} h_6(c_1) h_7(c_1, c_2)$$

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$$\sum_{t_2} h_4(c_2, t_2)$$

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$$h_9(c_2) h_6(c_1)$$

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$$\sum_{t_i} \sum_{c_i} \sum_{c_0} h_0(c_0) h_1(c_0, c_1) h_2(c_1, c_2) h_3(c_1, t_i)$$

$$= \sum_{t_i} \sum_{c_i} \sum_{c_0} h_0(c_0) h_1(c_0, c_1) h_2(c_1, c_2) h_3(c_1, t_i) h_4(c_2, t_i)$$

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14

$$h_0(c_0) h_{13}(c_1)$$

15

$$\sum_{c_1} h_1(c_0, c_1) h_{13}(c_1)$$

$$\sum_{t_i} \sum_{c_1} \sum_{c_0} h_0(c_0) h_1(c_0, c_1) h_2(c_1, c_2) h_3(c_1, t_i) h_4(c_2, t_2)$$

$$= \sum_{c_2} \sum_{t_i} \sum_{c_1} \sum_{c_0} h_0(c_0) h_1(c_0, c_1) h_2(c_1, c_2) h_3(c_1, t_i) h_4(c_2, t_2)$$

to compute single-variable marginal $P(t_2)$:

- choose a leaf containing variable T_2
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- 5** $\sum_{t_i} h_3(c_i, t_i)$
- 6** $\sum_{c_0} h_0(c_0) h_1(c_0, c_1)$
- 7** $h_2(c_1, c_2) h_5(c_1)$
- 8** $\sum_{c_1} h_6(c_1) h_7(c_1, c_2)$
- 9** $\sum_{t_2} h_4(c_2, t_2)$
- 10** $h_9(c_2) h_6(c_1)$
- 11** $\sum_{c_2} h_2(c_1, c_2) h_{10}(c_1, c_2)$
- 12** $h_5(c_1) h_{10}(c_1, c_2)$
- 13** $\sum_{c_2} h_7(c_1, c_2) h_9(c_2)$
- 14** $h_0(c_0) h_{13}(c_1)$
- 15** $\sum_{c_1} h_1(c_0, c_1) h_{13}(c_1)$

$$P(t_2)$$



$$= \sum_{c_2} \sum_{t_2} \sum_{c_1} \sum_{c_0} h_0(c_0) h_1(c_0, c_1) h_2(c_1, c_2) h_3(c_1, t_2) h_4(c_2, t_2)$$

to compute single-variable marginal $P(t_2)$:

- choose a leaf containing variable T_2
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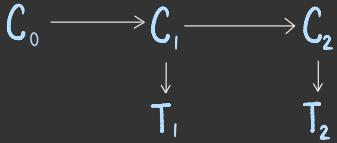
it's as if we took a bayesian network...

intuition

C_0	C_1	$P(c_0)$
0	0	0.99
0	1	0.99

C_0	C_1	$P(c_1 c_0)$
0	0	0.99
0	1	0.01
1	0	0.1
1	1	0.9

C_1	C_2	$P(c_2 c_1)$
0	0	0.99
0	1	0.01
1	0	0.1
1	1	0.9

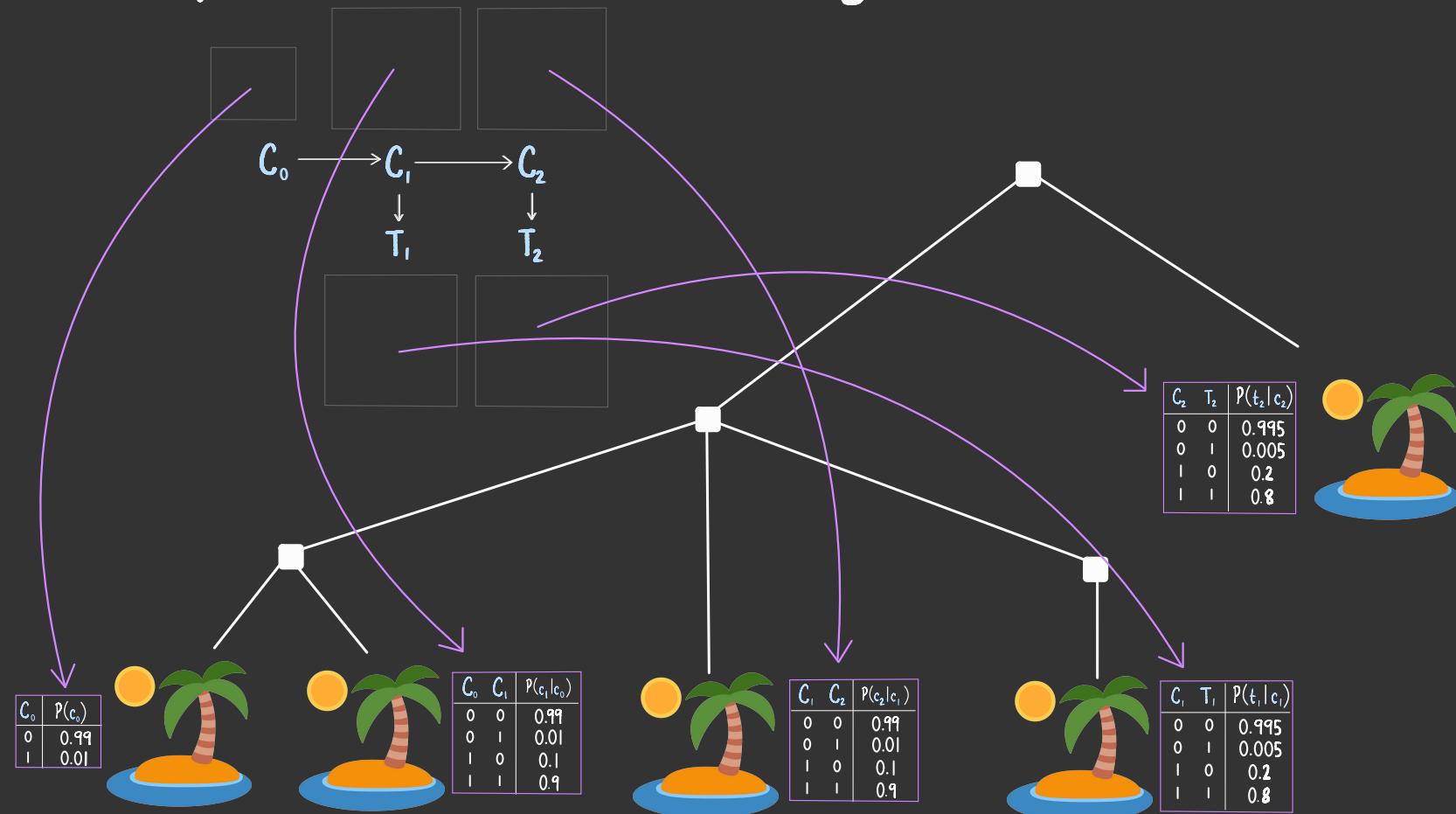


C_1	T_1	$P(t_1 c_1)$
0	0	0.995
0	1	0.005
1	0	0.2
1	1	0.8

C_2	T_2	$P(t_2 c_2)$
0	0	0.995
0	1	0.005
1	0	0.2
1	1	0.8



...and dispersed its conditional probability tables to remote islands



on our island, we know conditional probabilities $P(t_2 | c_2)$
but we have dreams... dreams of things beyond our ken...

c_2	t_2	$P(t_2 c_2)$
0	0	0.995
0	1	0.005
1	0	0.2
1	1	0.8

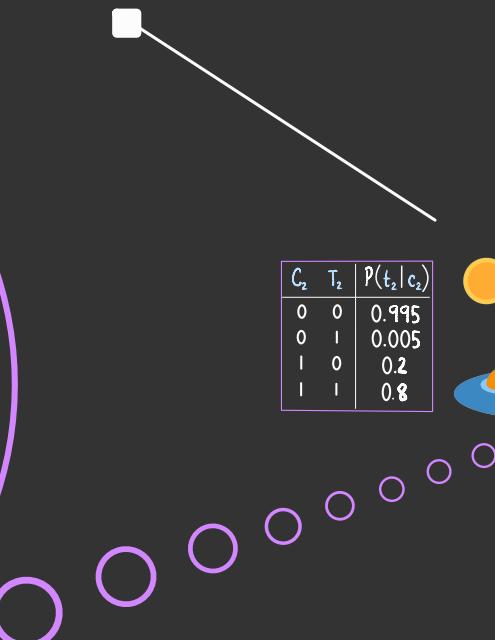
c_2	t_2	$P(t_2 c_2)$
0	0	0.995
0	1	0.005
1	0	0.2
1	1	0.8



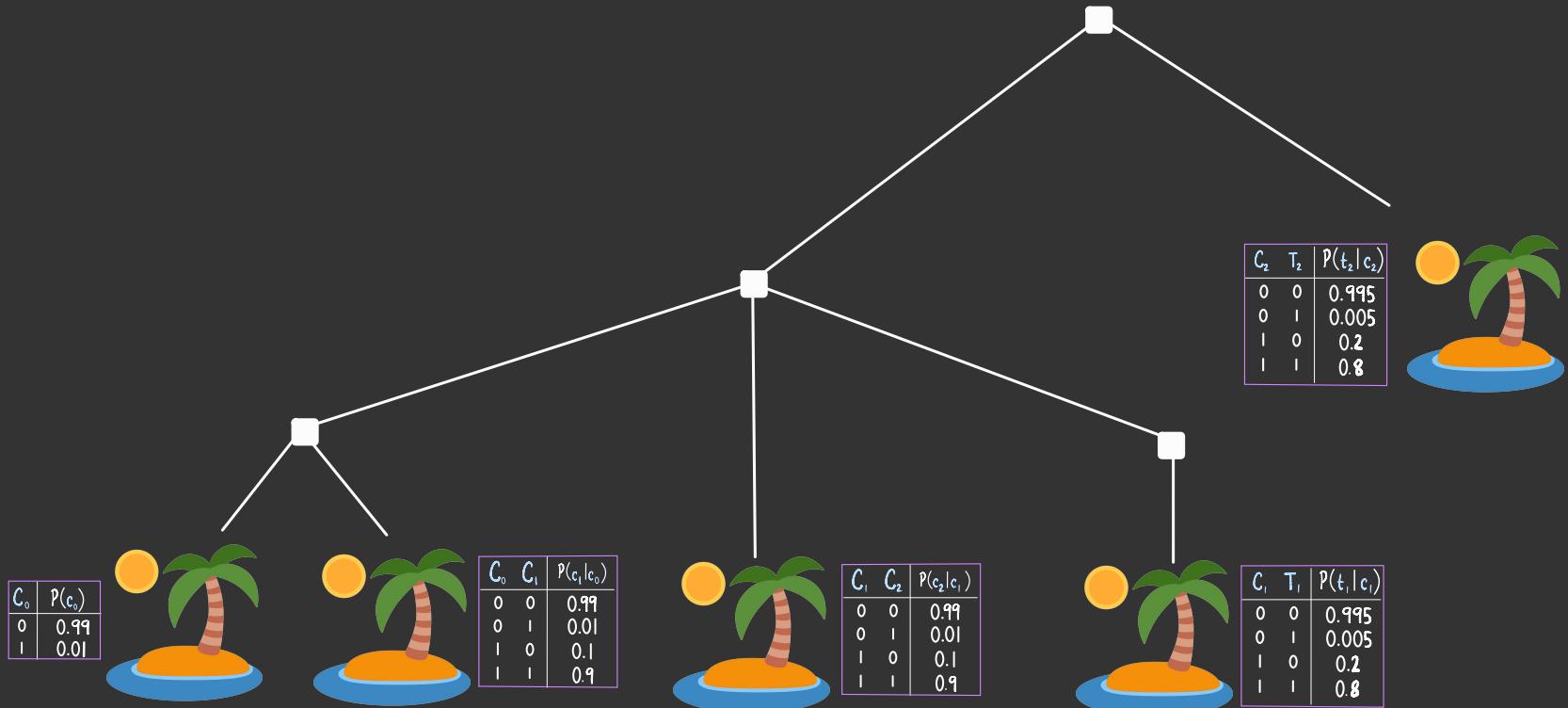
...dreams of knowing the marginal probabilities $P(c_2, t_2)$

c_2	t_2	$P(c_2, t_2)$
0	0	?
0	1	?
1	0	?
1	1	?

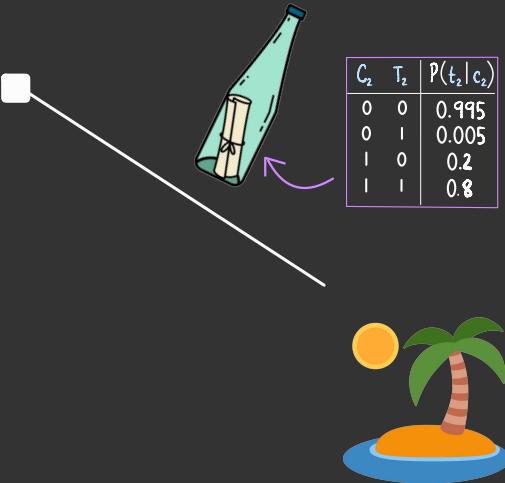
c_2	t_2	$P(t_2 c_2)$
0	0	0.995
0	1	0.005
1	0	0.2
1	1	0.8



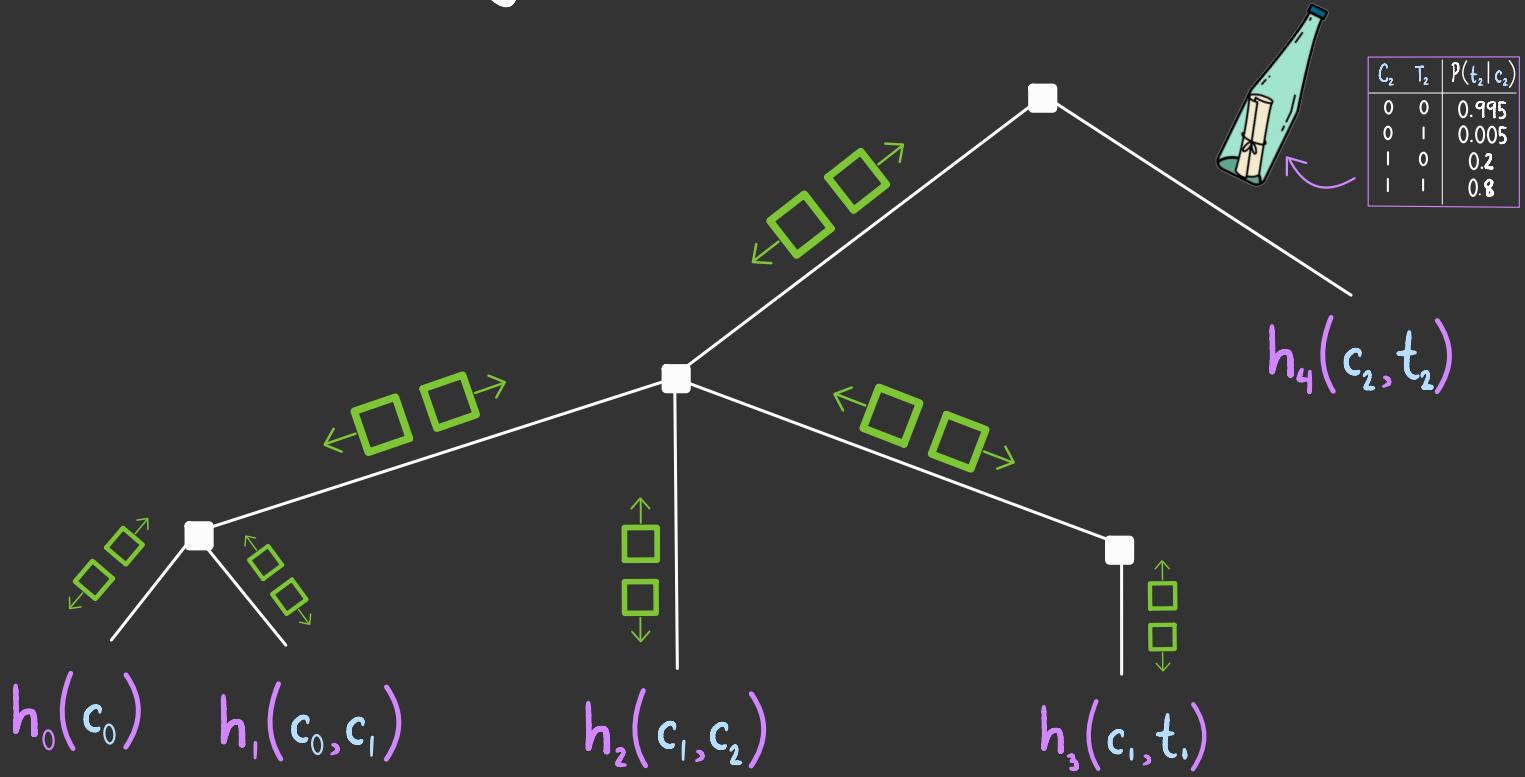
but computing the marginal probabilities $P(c_2, t_2)$
requires information possessed by other islands



So we send our conditional probability table adrift
on the waves as a message in a bottle

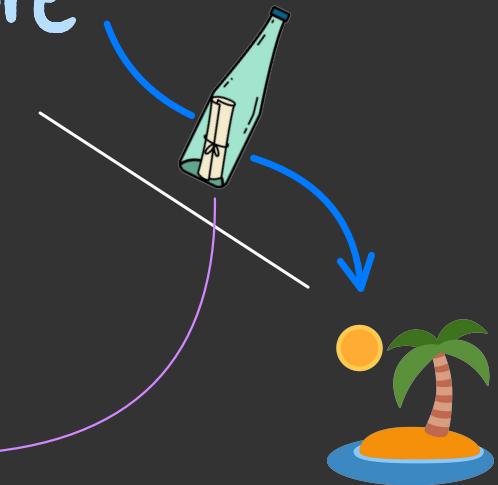


the other islands all cooperate, sending their own messages according to the belief propagation step



eventually a factor
washes up on our shore

c_2	T_2	$h(c_2, t_2)$
0	0	•
0	1	•
1	0	•
1	1	•

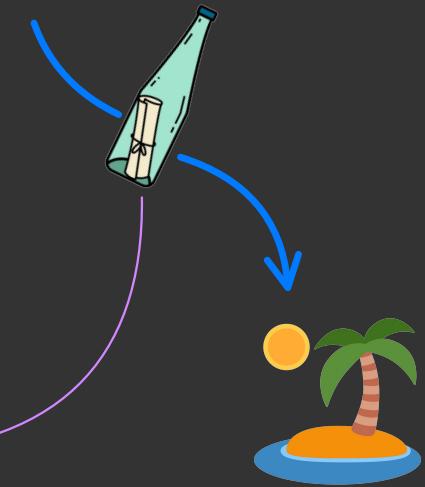


multiplying it with our original factor gives us a marginal

C_2	T_2	$P(t_2 c_2)$
0	0	0.995
0	1	0.005
1	0	0.2
1	1	0.8

×

C_2	T_2	$h(c_2, t_2)$
0	0	•
0	1	•
1	0	•
1	1	•



multiplying it with our original factor gives us a marginal

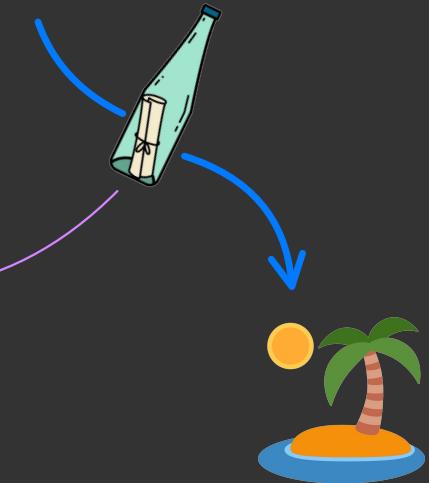
C_2	T_2	$P(t_2 c_2)$
0	0	0.995
0	1	0.005
1	0	0.2
1	1	0.8

×

C_2	T_2	$h(c_2, t_2)$
0	0	•
0	1	•
1	0	•
1	1	•

=

C_2	T_2	$P(c_2, t_2)$
0	0	•
0	1	•
1	0	•
1	1	•



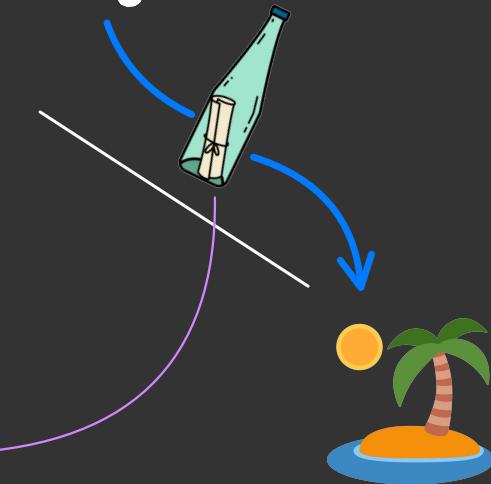
with this, we can compute single variable marginals for any variable that appeared in our original factor

$$P(c_2) = \sum_{t_2} P(c_2, t_2)$$

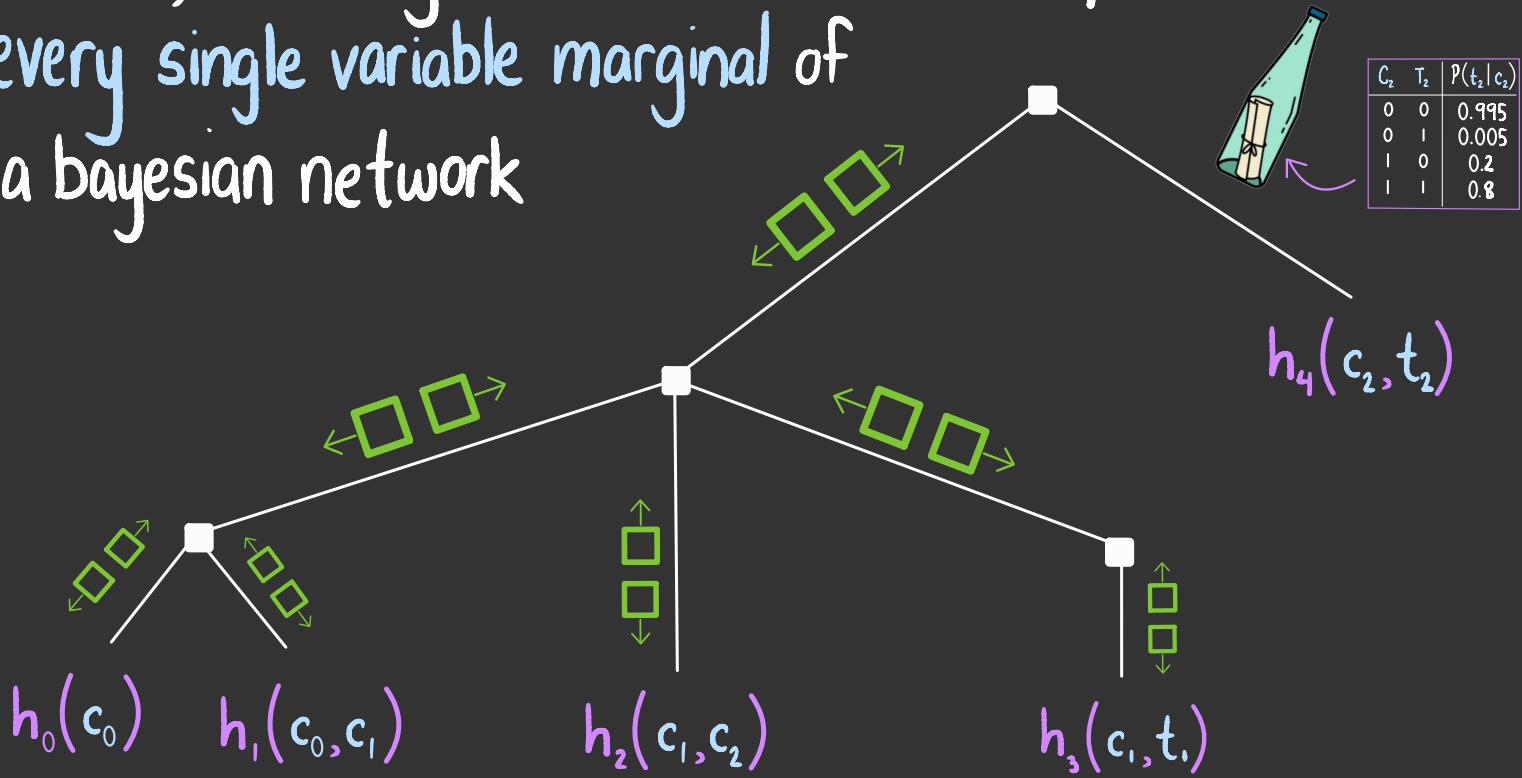
or

$$P(t_2) = \sum_{c_2} P(c_2, t_2)$$

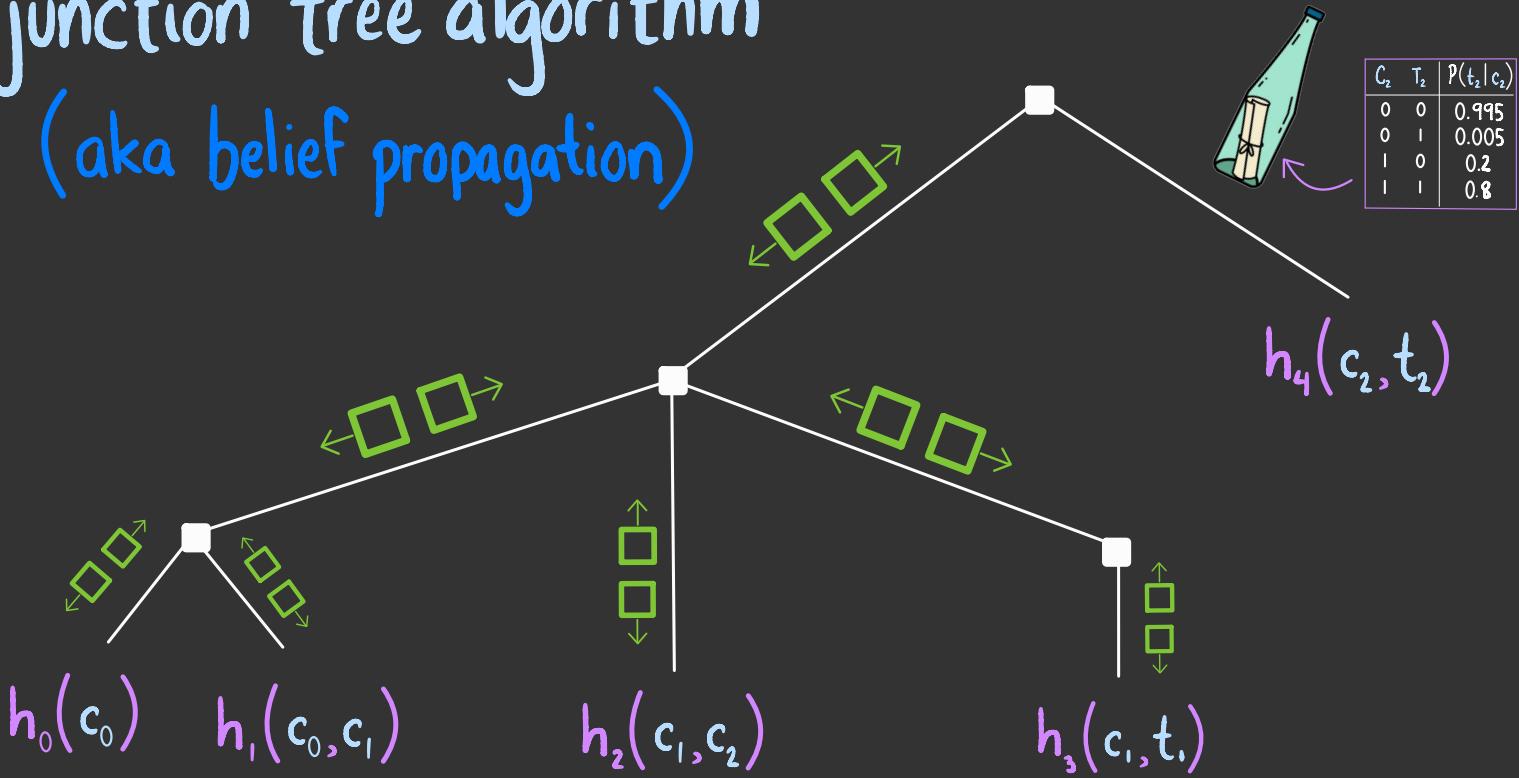
C_2	T_2	$P(c_2, t_2)$
0	0	•
0	1	•
1	0	•
1	1	•



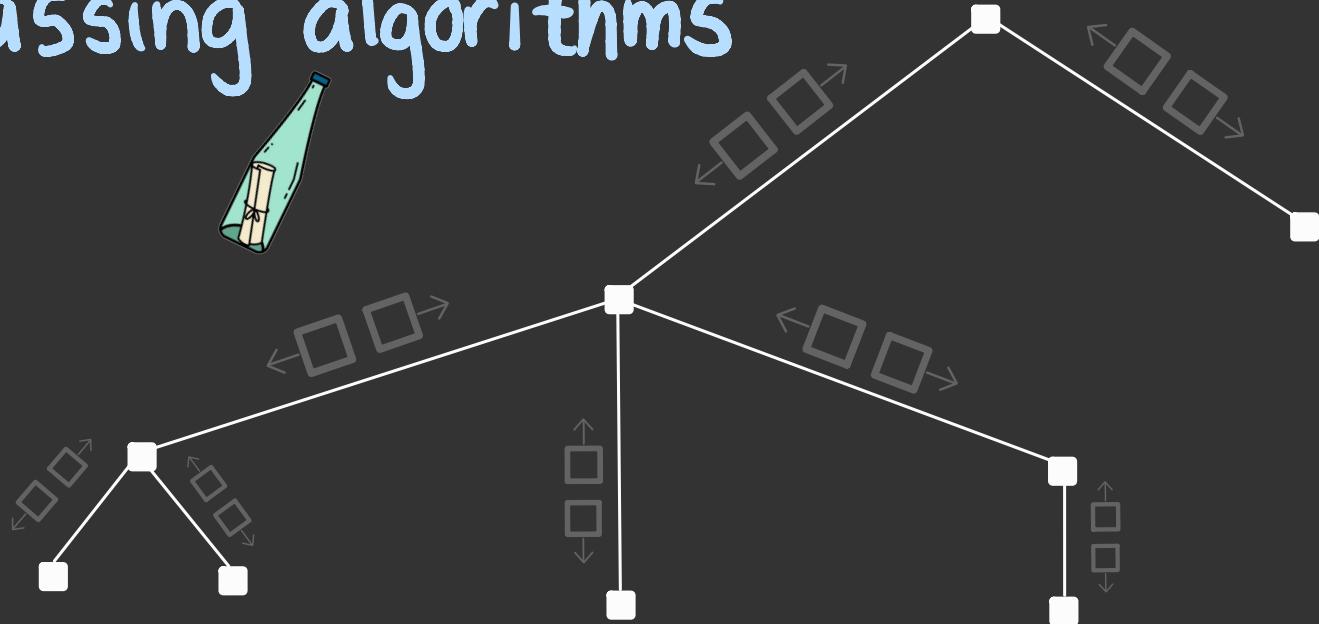
because every variable appears on at least one island, this algorithm allows us to compute every single variable marginal of a bayesian network



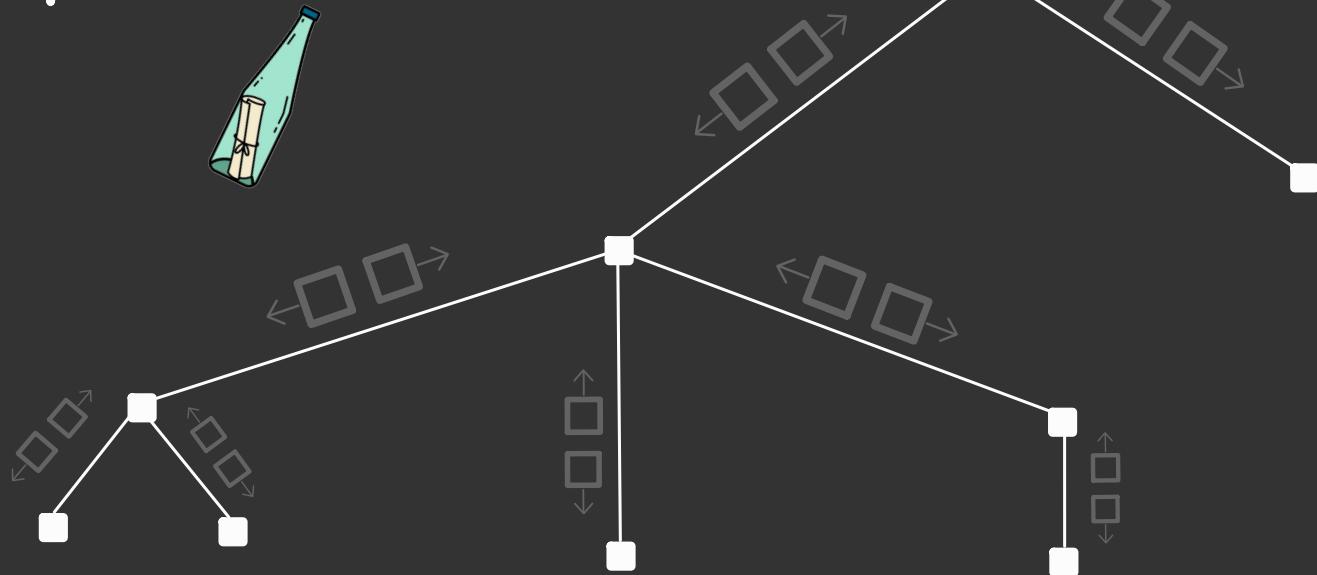
this algorithm is called the
junction tree algorithm
(aka belief propagation)

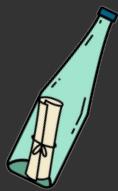


it is an example of a family
of algorithms called message
passing algorithms

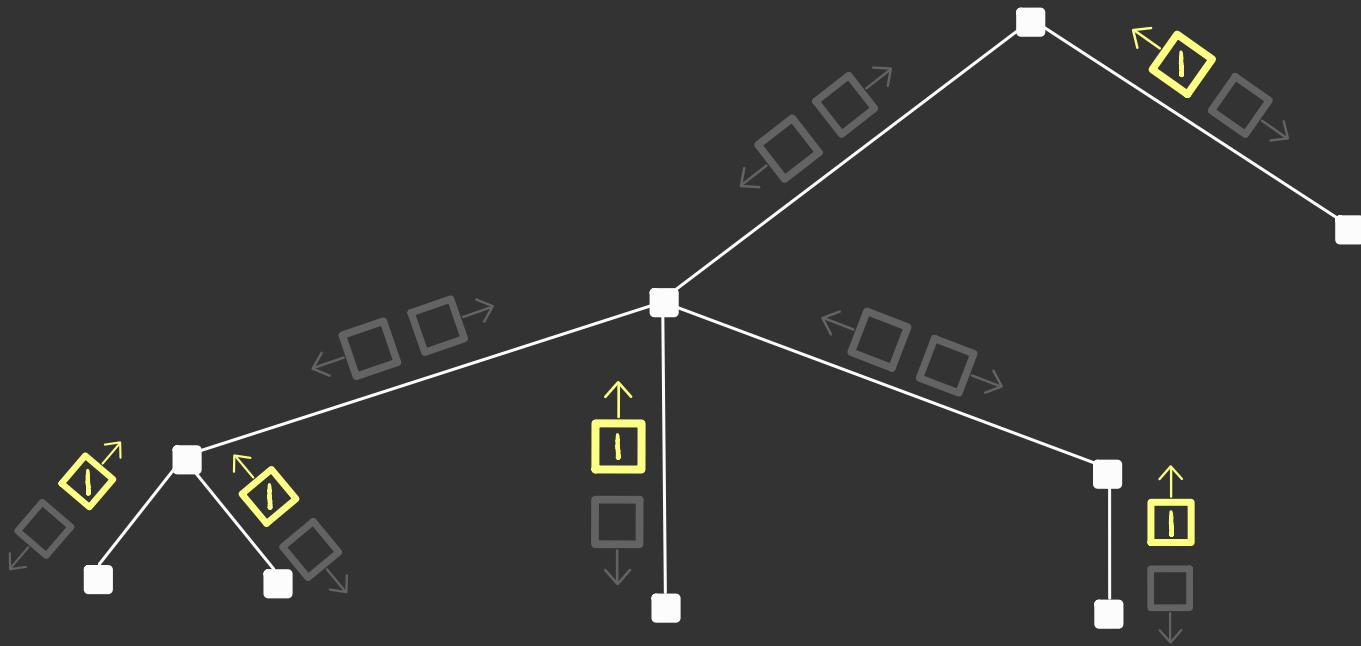


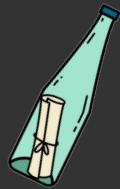
by changing the messages,
we can compute other
properties of the tree



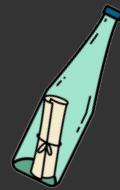


leaf : send the number 1

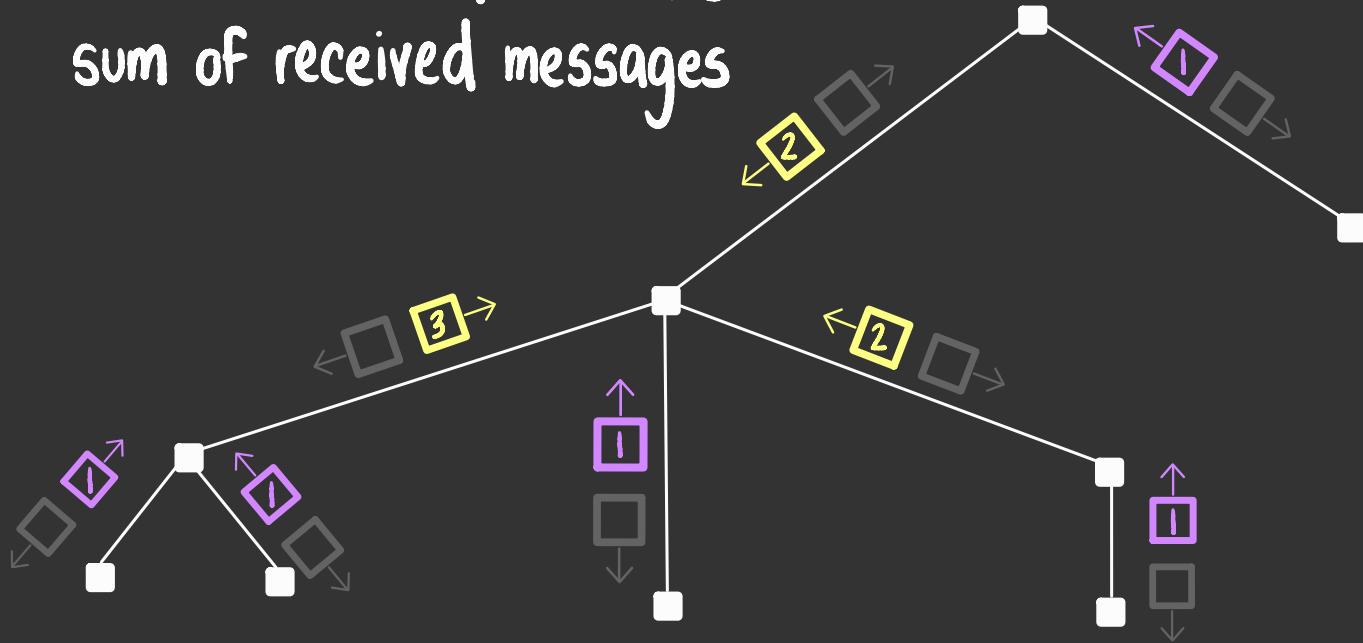


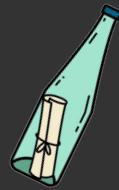


leaf : send the number 1

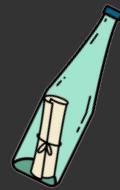


internal : send $1 + \text{the sum of received messages}$

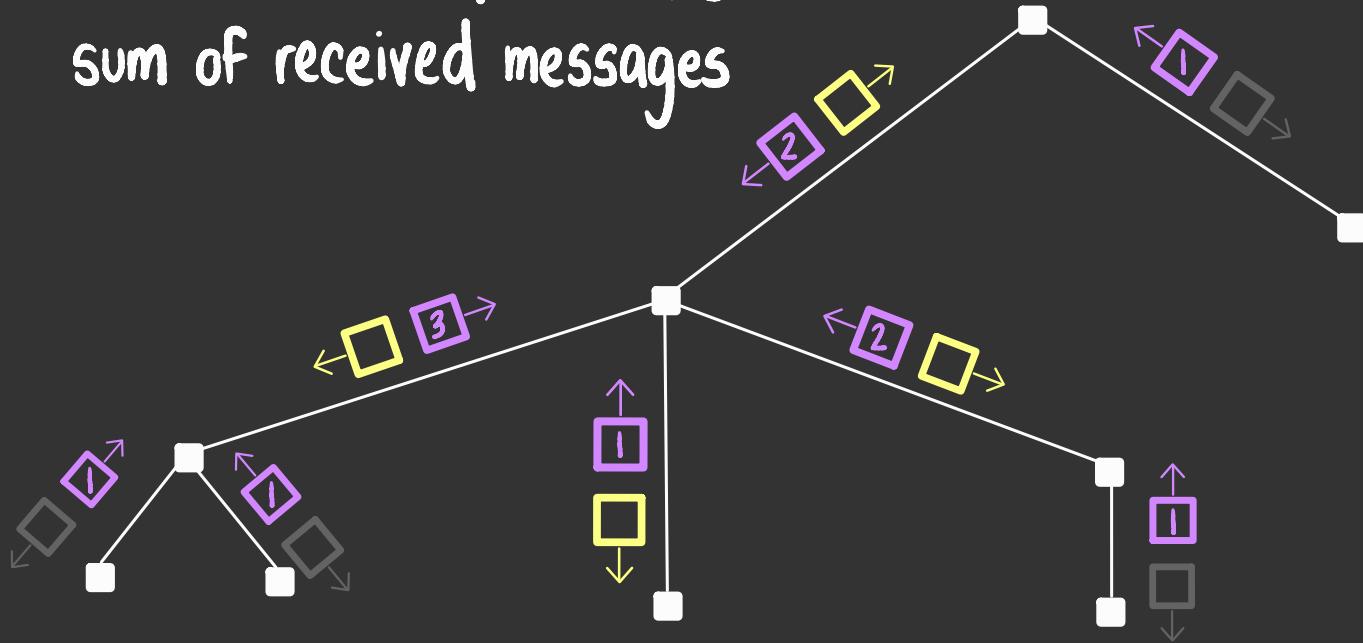


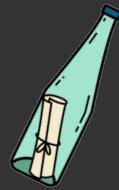


leaf : send the number 1

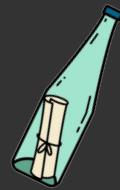


internal : send $1 + \text{the sum of received messages}$

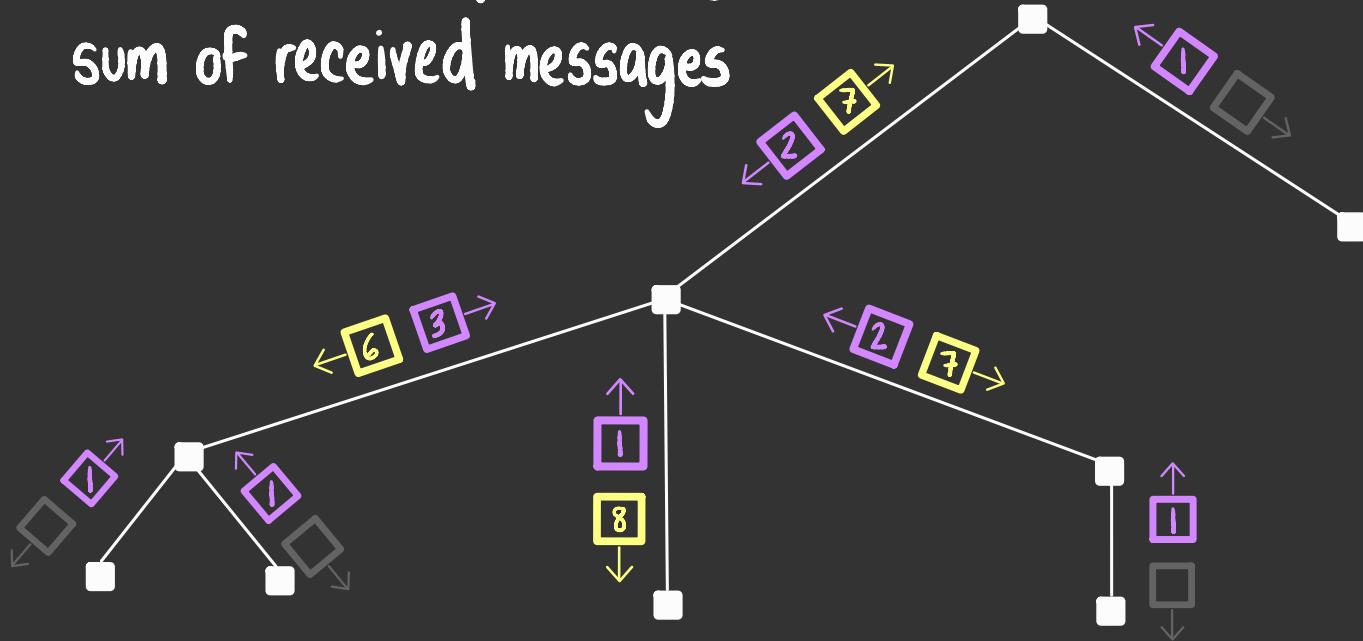


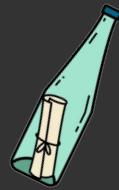


leaf : send the number 1

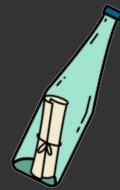


internal : send $1 + \text{the sum of received messages}$

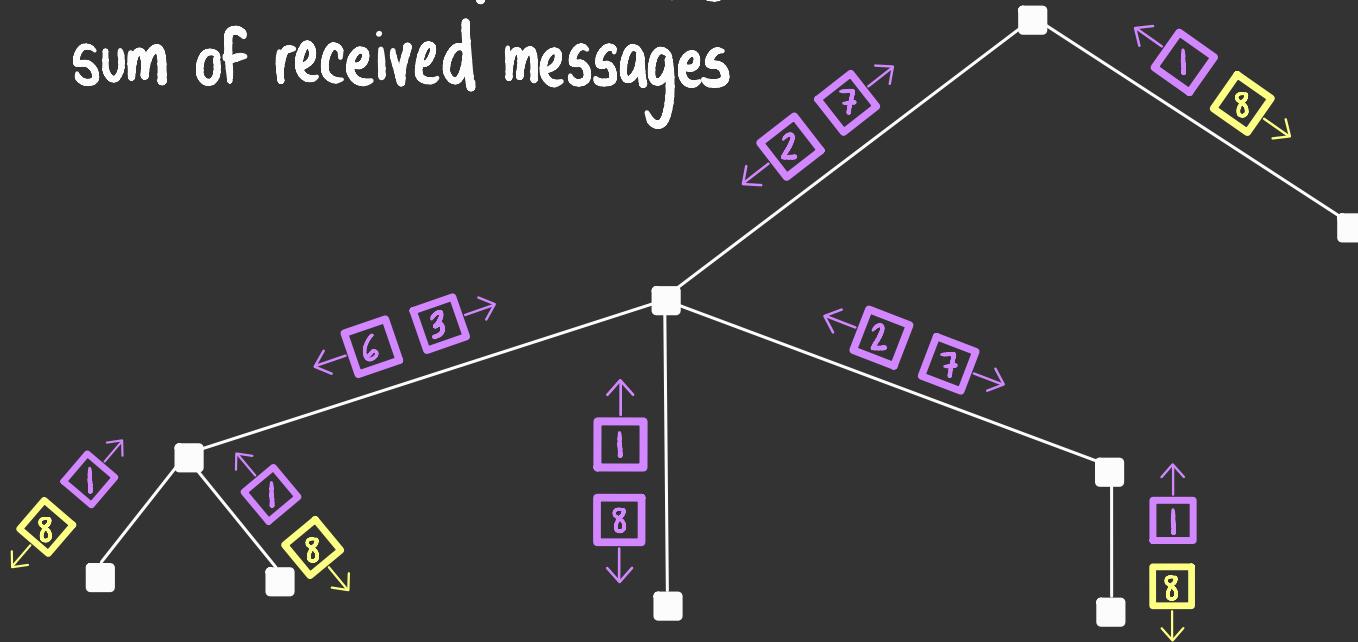


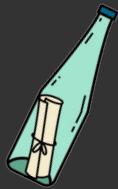


leaf : send the number 1

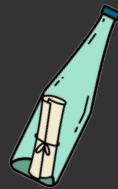


internal : send $1 + \text{the sum of received messages}$



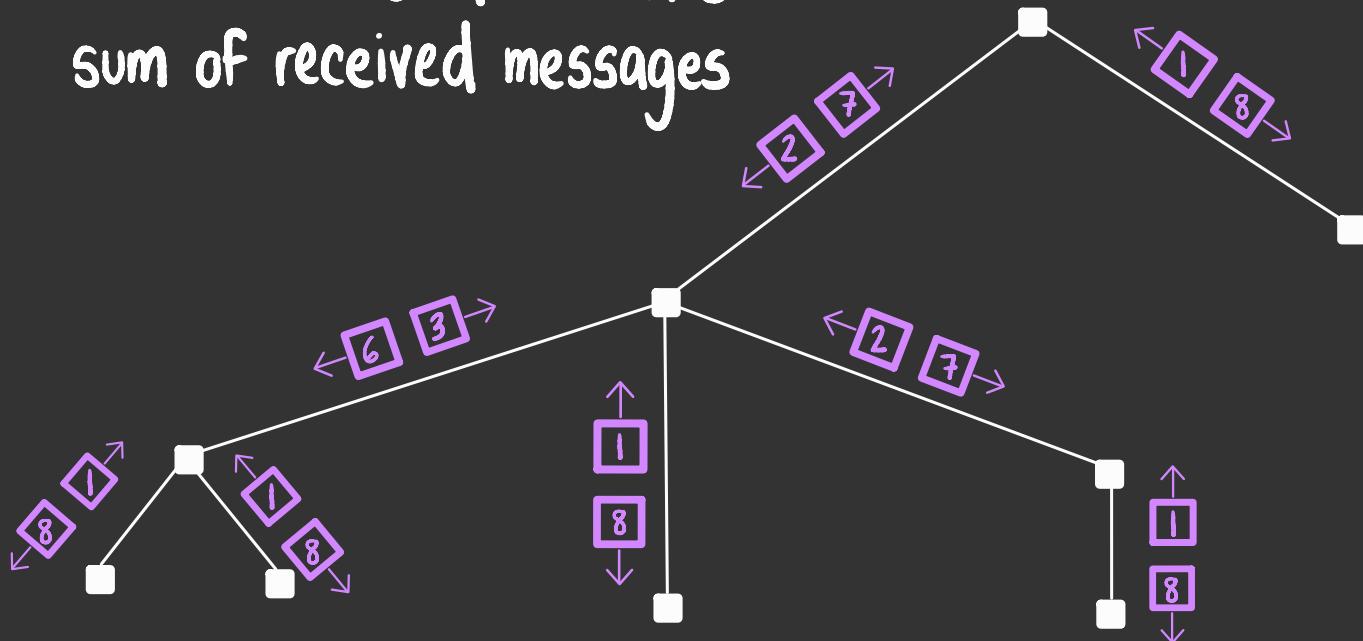


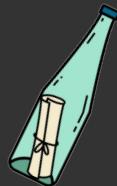
leaf : send the number 1



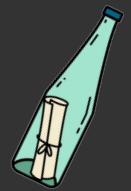
internal : send $1 + \text{the sum of received messages}$

what is this?
computing





leaf : send the number 1



internal : send $1 + \text{the sum of received messages}$

the number of nodes in the tree

