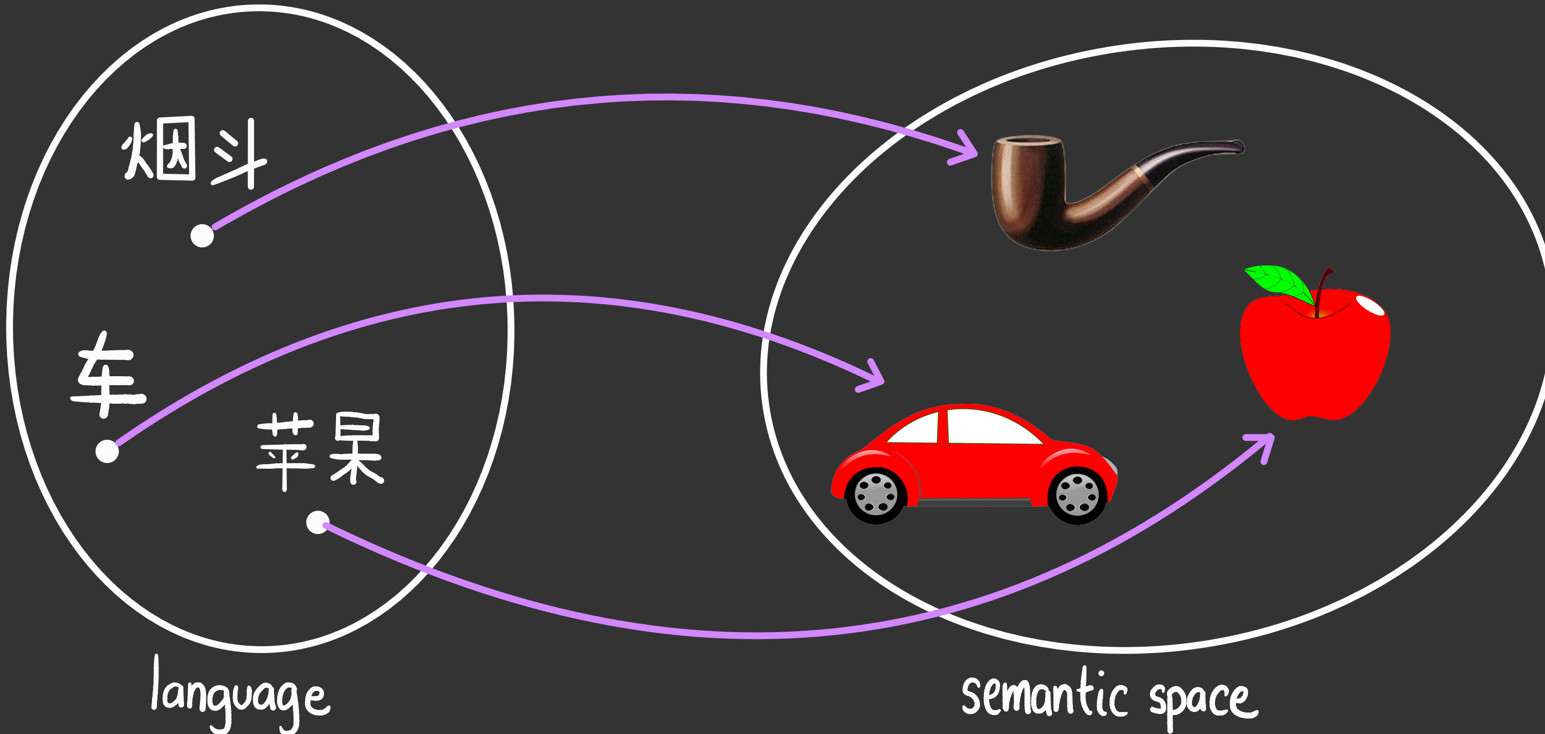
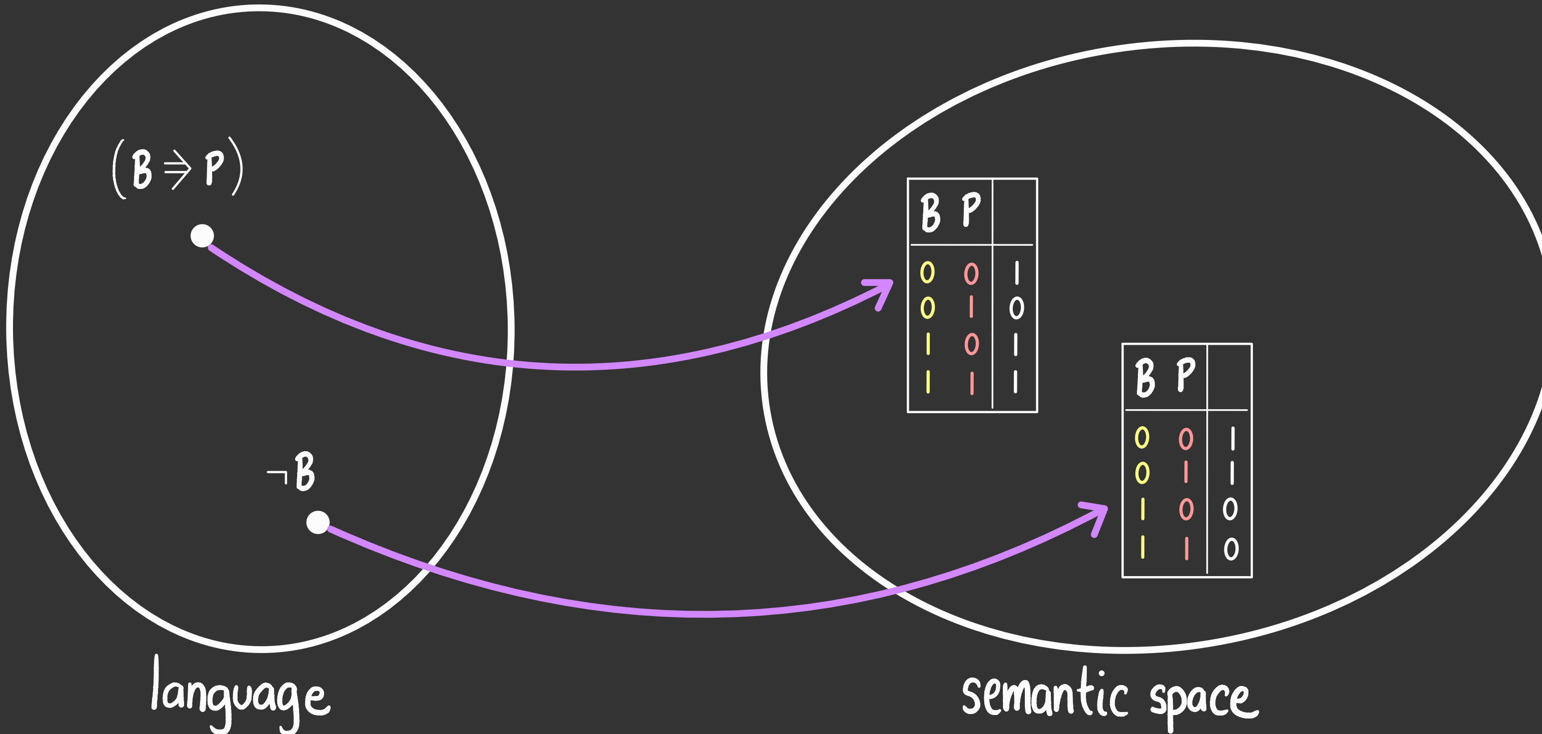
propositional

semantics

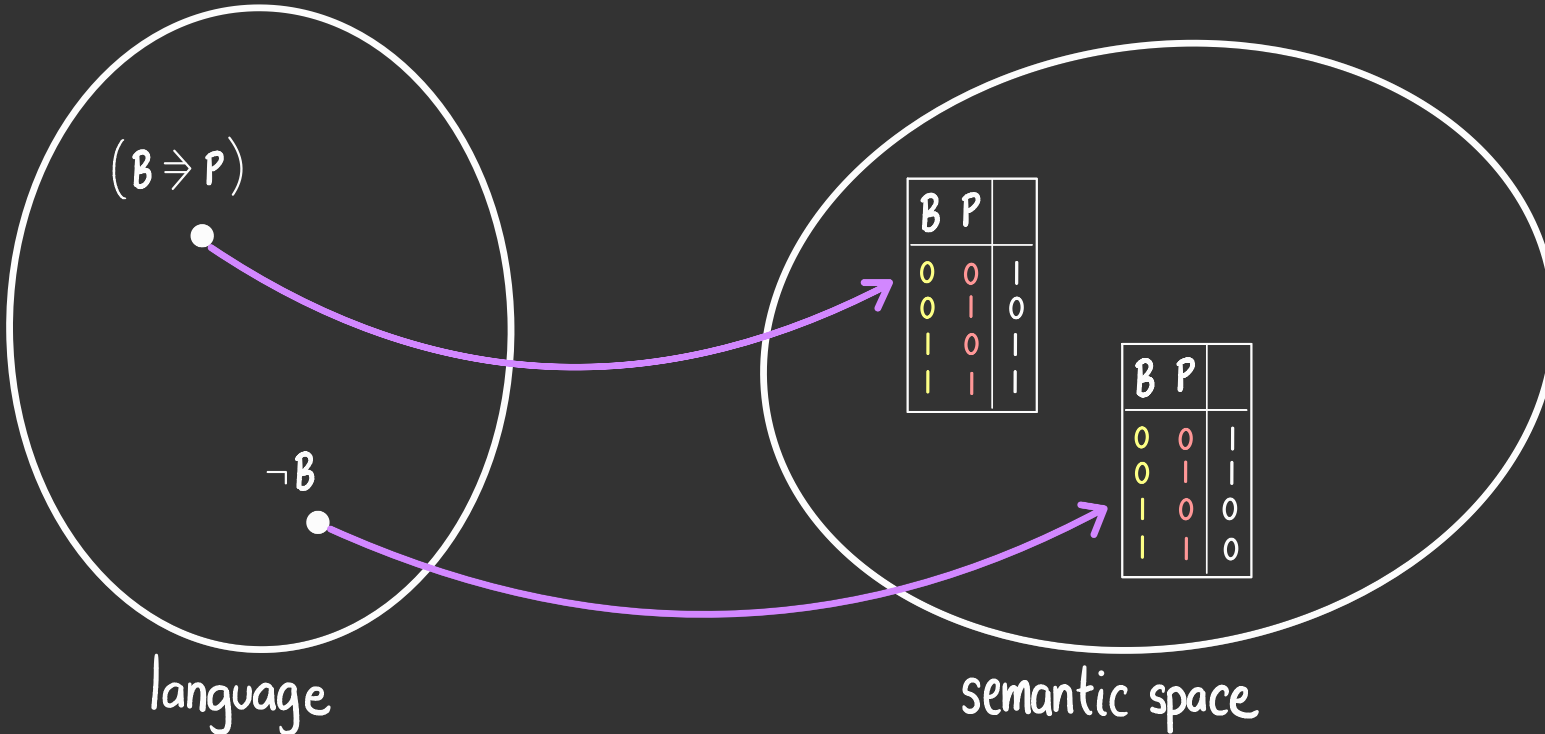
CSCI 373



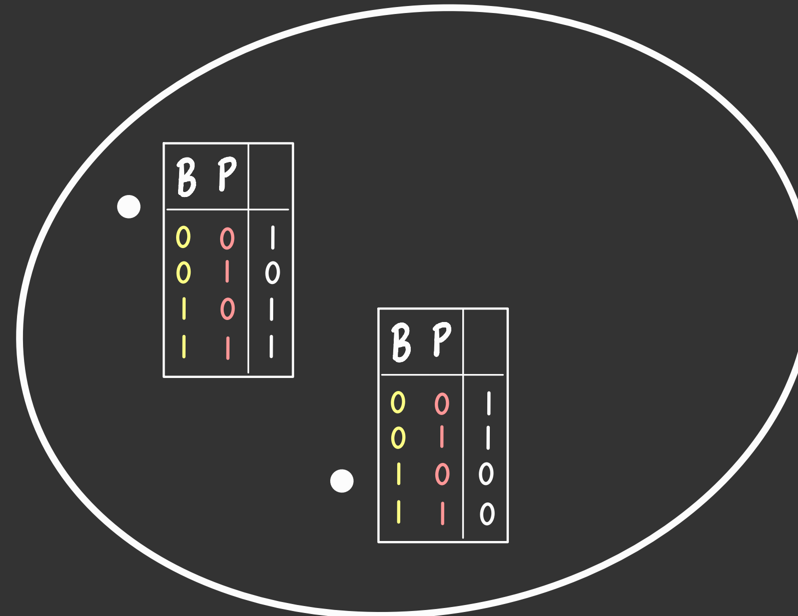
let's build a language for logical reasoning



the big picture: we'll develop a language where each sentence corresponds to a truth table



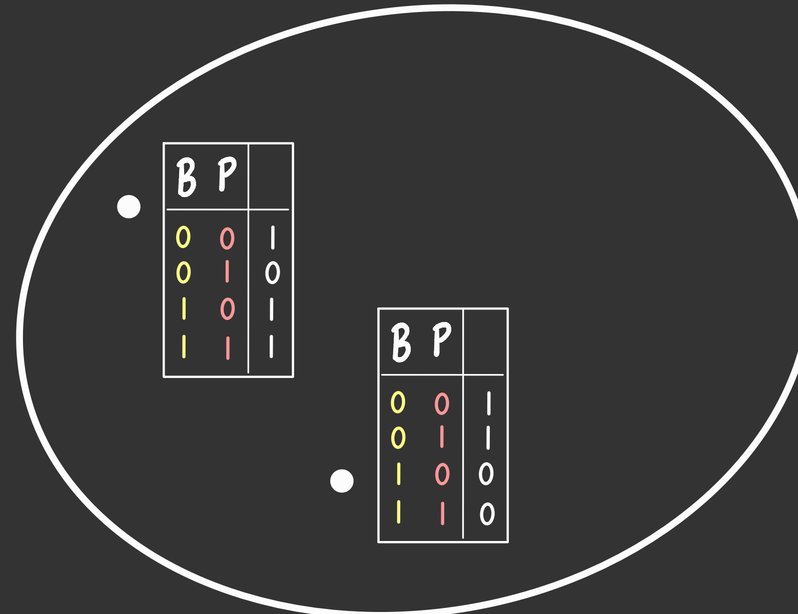
this will allow us to **talk** and **reason** about truth tables without explicitly representing them each point in the semantic space is a **truth table**



semantic space

define a **signature** Σ as a set of **boolean** variables

e.g. $\Sigma = \{B, P\}$



semantic space $S(\Sigma)$

the set of truth tables over signature Σ

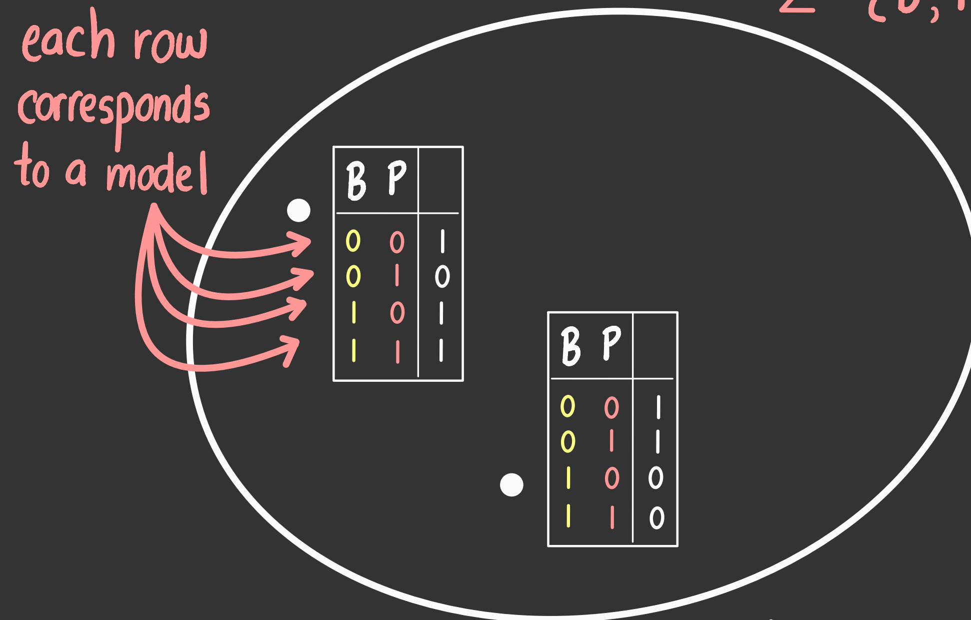
$\Sigma = \{B, P\}$

a model

of signature Σ is a function

$$\Sigma \to \{0, 1\}$$

e.g. $\{B \mapsto 0, P \mapsto 1\}$



semantic space $S(\Sigma)$

---

the set of truth tables over signature Σ

$\Sigma = \{B, P\}$

a **model**

of signature Σ is a function

$$\Sigma \to \{0, 1\}$$

each row corresponds to a model

B	P	
0	0	1
0	1	0
1	0	1
1	1	1

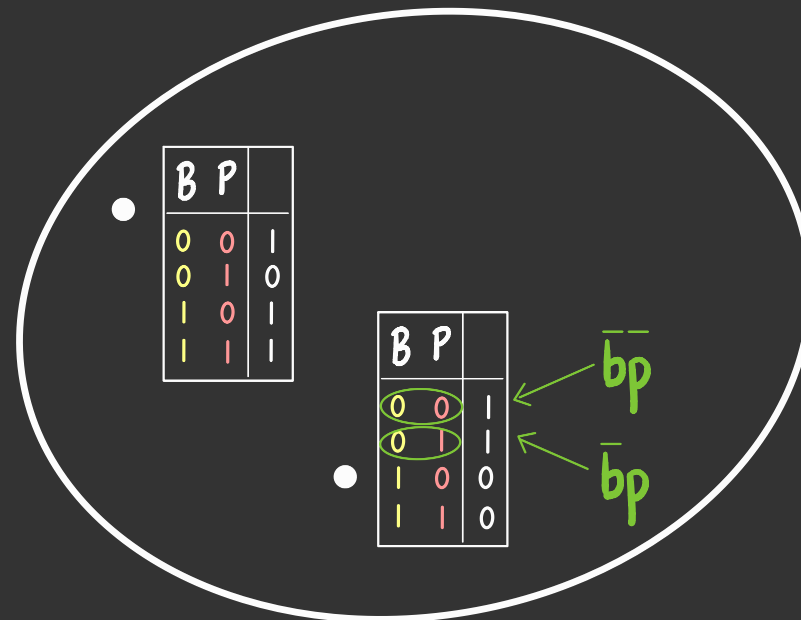
B	P	
0	0	1
0	1	1
1	0	0
1	1	0

semantic space $S(\Sigma)$

the set of truth tables over signature Σ

	shorthand
e.g. $\{B \mapsto 0, P \mapsto 0\}$	$\bar{b}\bar{p}$
$\{B \mapsto 0, P \mapsto 1\}$	$\bar{b}p$
$\{B \mapsto 1, P \mapsto 0\}$	$b\bar{p}$
$\{B \mapsto 1, P \mapsto 1\}$	bp

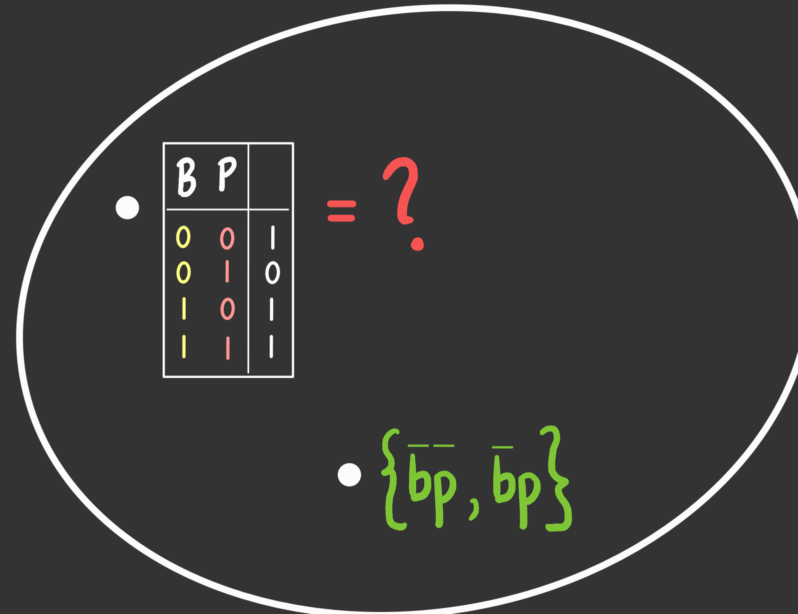
an alternative way to represent a truth table is as its set of **possible models**, i.e. the models it maps to 1



semantic space

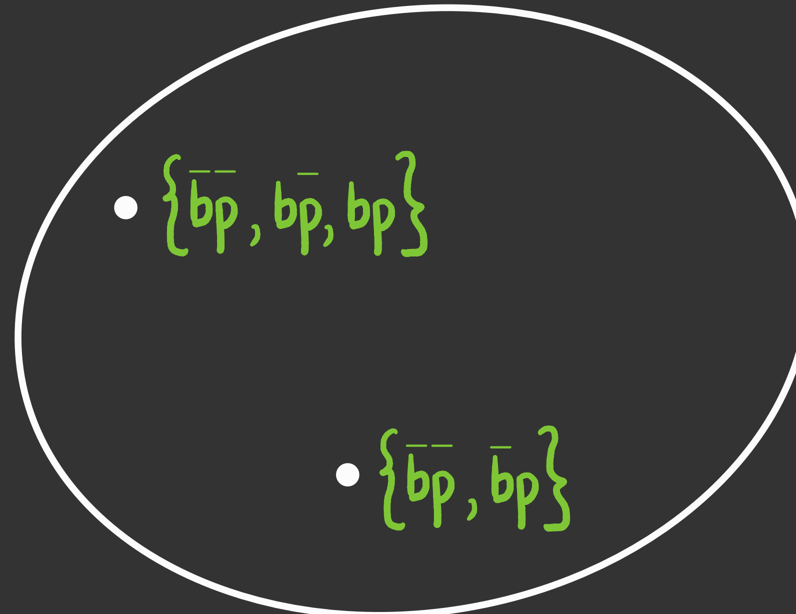
an alternative way to represent a truth table is as its set of **possible models**,

i.e. the models it maps to 1



semantic space

let $M(\Sigma)$ be the space of all models of signature Σ



$$\frac{\text{semantic space } S(\Sigma)}{\text{the set of subsets of model space } M(\Sigma)}$$

let signature $\Sigma = \{A, C\}$

model space $M(\Sigma) =$?

the space of all models of signature Σ

semantic space $S(\Sigma) =$?

the set of subsets of model space $M(\Sigma)$

let signature $\Sigma = \{A, C\}$

model space $M(\Sigma) = \{\bar{a}\bar{c}, a\bar{c}, \bar{a}c, ac\}$

semantic space $S(\Sigma) =$

$$\{\bar{a}\bar{c}, a\bar{c}, \bar{a}c, ac\}$$

$$\{a\bar{c}, \bar{a}c, ac\} \quad \{\bar{a}\bar{c}, \bar{a}c, ac\} \quad \{\bar{a}\bar{c}, a\bar{c}, ac\} \quad \{\bar{a}\bar{c}, a\bar{c}, \bar{a}c\}$$

$$\{\bar{a}c, ac\} \quad \{a\bar{c}, ac\} \quad \{a\bar{c}, \bar{a}c\} \quad \{\bar{a}\bar{c}, a\bar{c}\} \quad \{\bar{a}\bar{c}, \bar{a}c\} \quad \{\bar{a}\bar{c}, ac\}$$

$$\{\bar{a}\bar{c}\} \quad \{a\bar{c}\} \quad \{\bar{a}c\} \quad \{ac\}$$

$$\{\}$$

a one-to-one correspondence with the space of truth tables