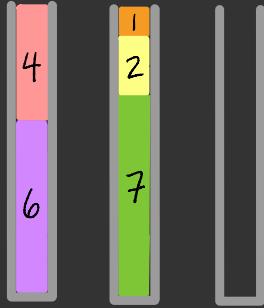


multiplayer
games

CSCI
373

train
brain
brawn
prawn



all of the
tasks we've
discussed so far
have involved a
**single
agent**

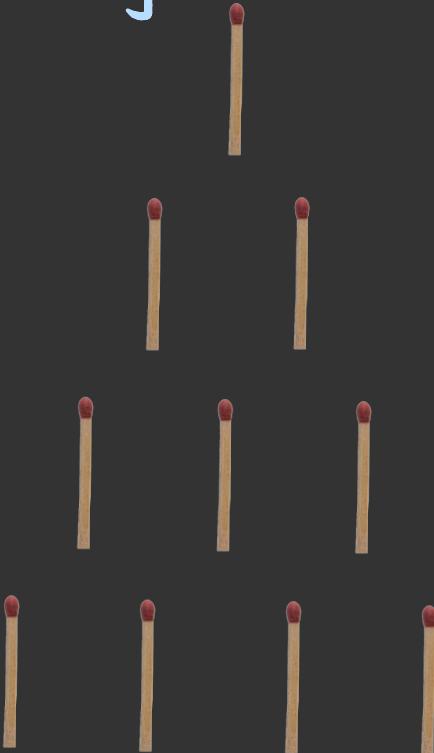
but what if others
want to stop us from
succeeding?



"spy vs. spy"
mad magazine

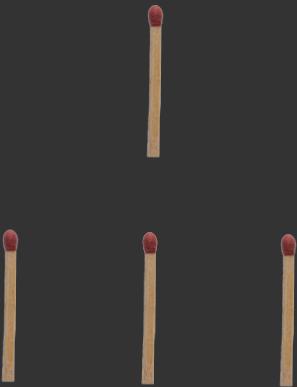
the game of nim

on your turn, you
must remove one or
more matchsticks
from a single row

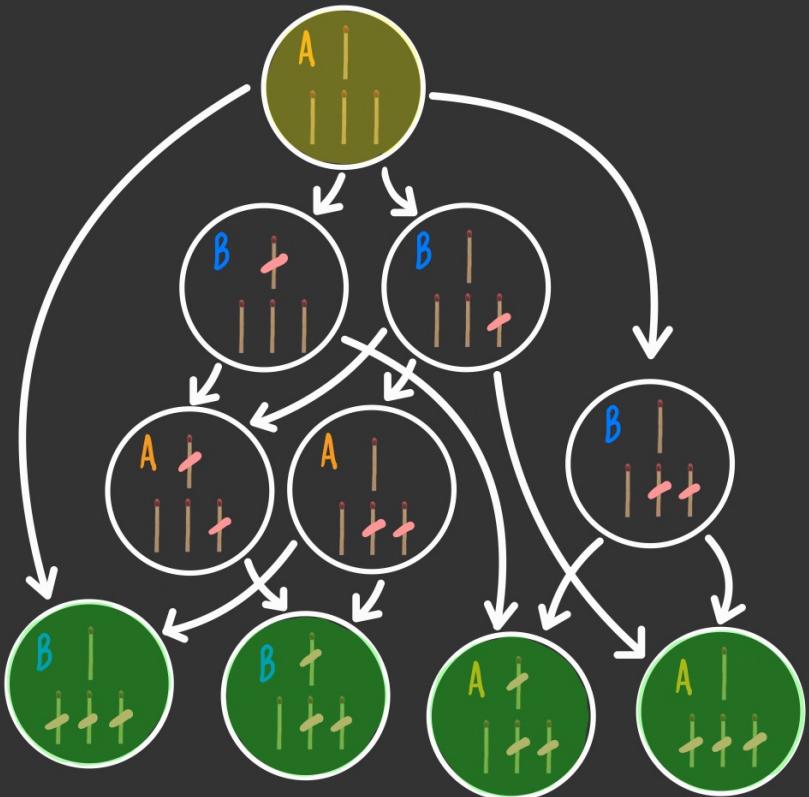


you lose if you
remove the last
matchstick

the game of nim

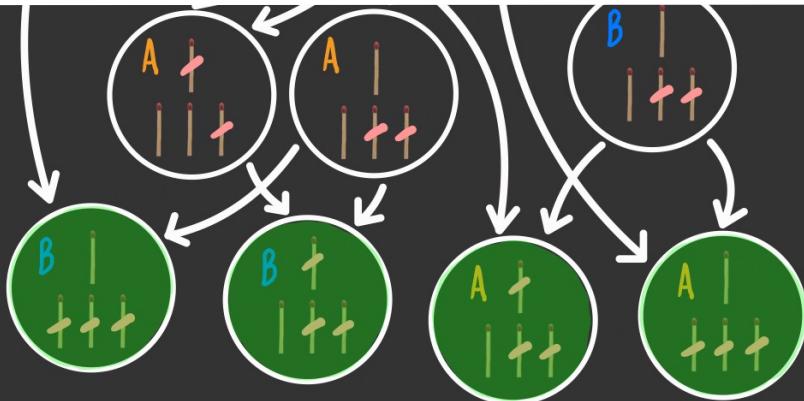


assume this initial nim board



- a game with players P is a tuple (m, U) where:
 - $M = (Q, \Sigma, \Delta, q_0, F)$ is a state machine where each state $q \in Q$ has the form $(p, q') \in P \times Q'$ for an auxiliary set Q' of states
 - utility function $U: F \times P \rightarrow \mathbb{R}$ which gives the value of each final state for each player

- S_0 : The **initial state**, which specifies how the game is set up at the start.
- $\text{TO-MOVE}(s)$: The player whose turn it is to move in state s .
- $\text{ACTIONS}(s)$: The set of legal moves in state s .
- $\text{RESULT}(s, a)$: The **transition model**, which defines the state resulting from taking action a in state s .
- $\text{IS-TERMINAL}(s)$: A **terminal test**, which is true when the game is over and false otherwise. States where the game has ended are called **terminal states**.
- $\text{UTILITY}(s, p)$: A **utility function** (also called an objective function or payoff function), which defines the final numeric value to player p when the game ends in terminal state s . In chess, the outcome is a win, loss, or draw, with values 1, 0, or $1/2$.² Some games have a wider range of possible outcomes—for example, the payoffs in backgammon range from 0 to 192.

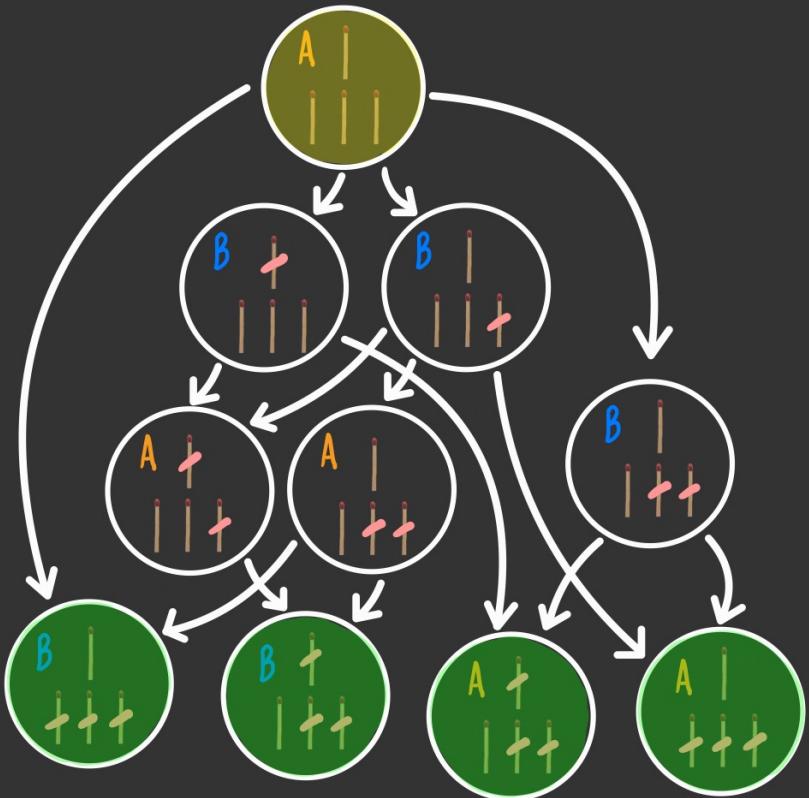


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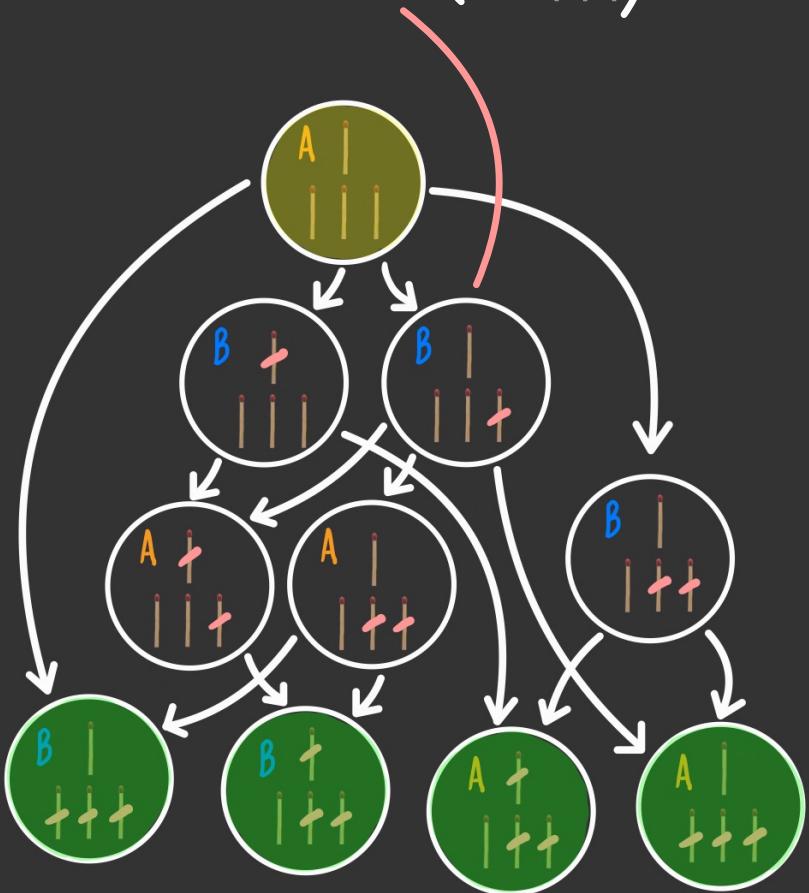
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$$P = \{A, B\}$$



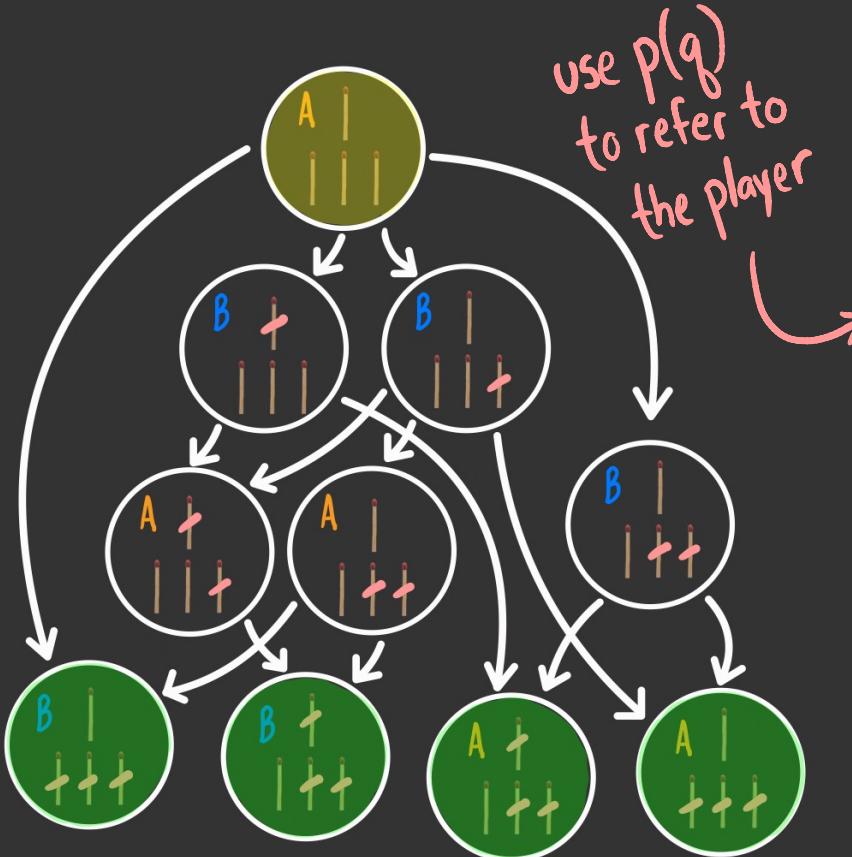
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shorthand for $(B, \text{ ||+ })$

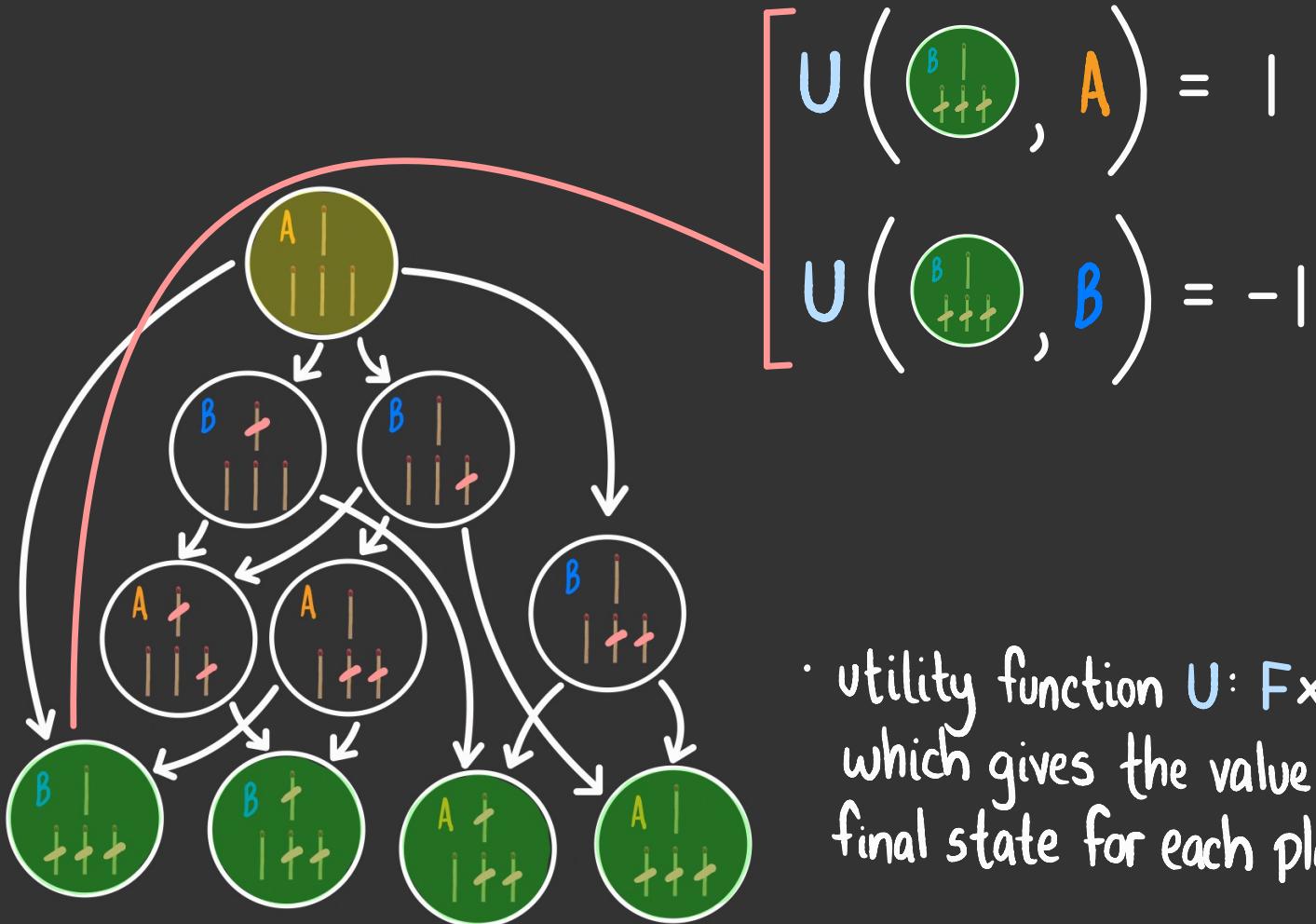


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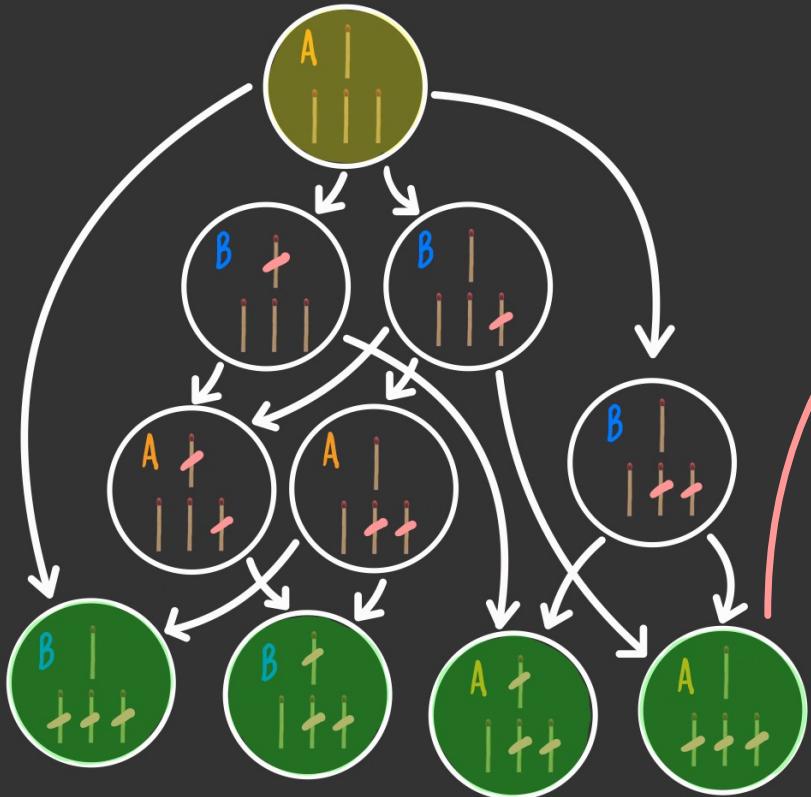
e.g. $P\left(\bigcirc \begin{array}{|c|c|c|} \hline B & | & | \\ \hline | & | & + \\ \hline \end{array}\right) = B$



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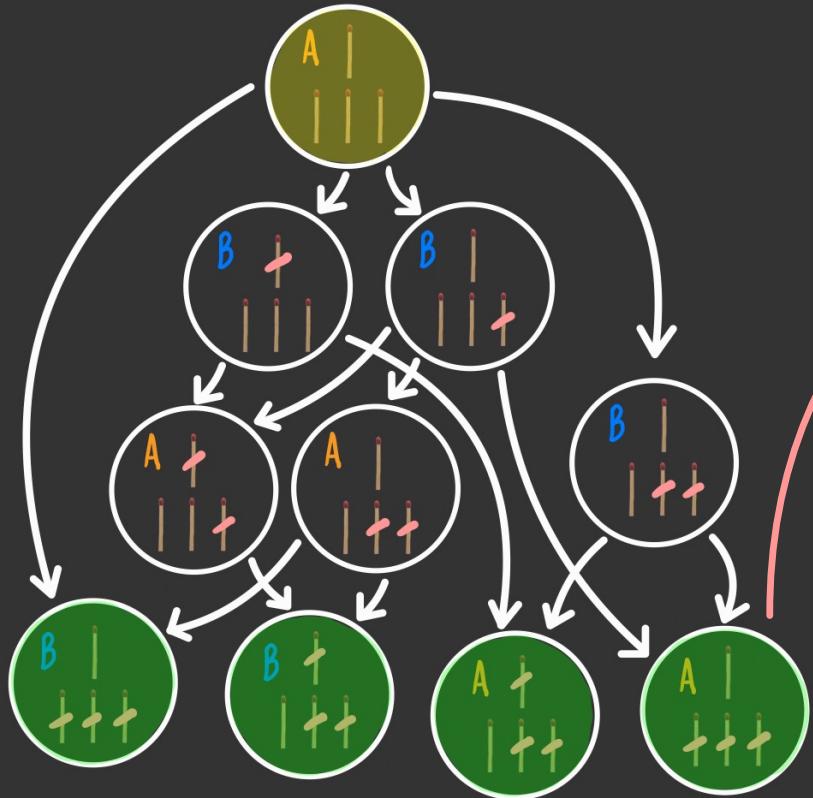
- utility function $U: F \times P \rightarrow \mathbb{R}$
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$$U(A | \text{Initial State}, A) = ?$$

$$U(A | \text{Initial State}, B) = ?$$

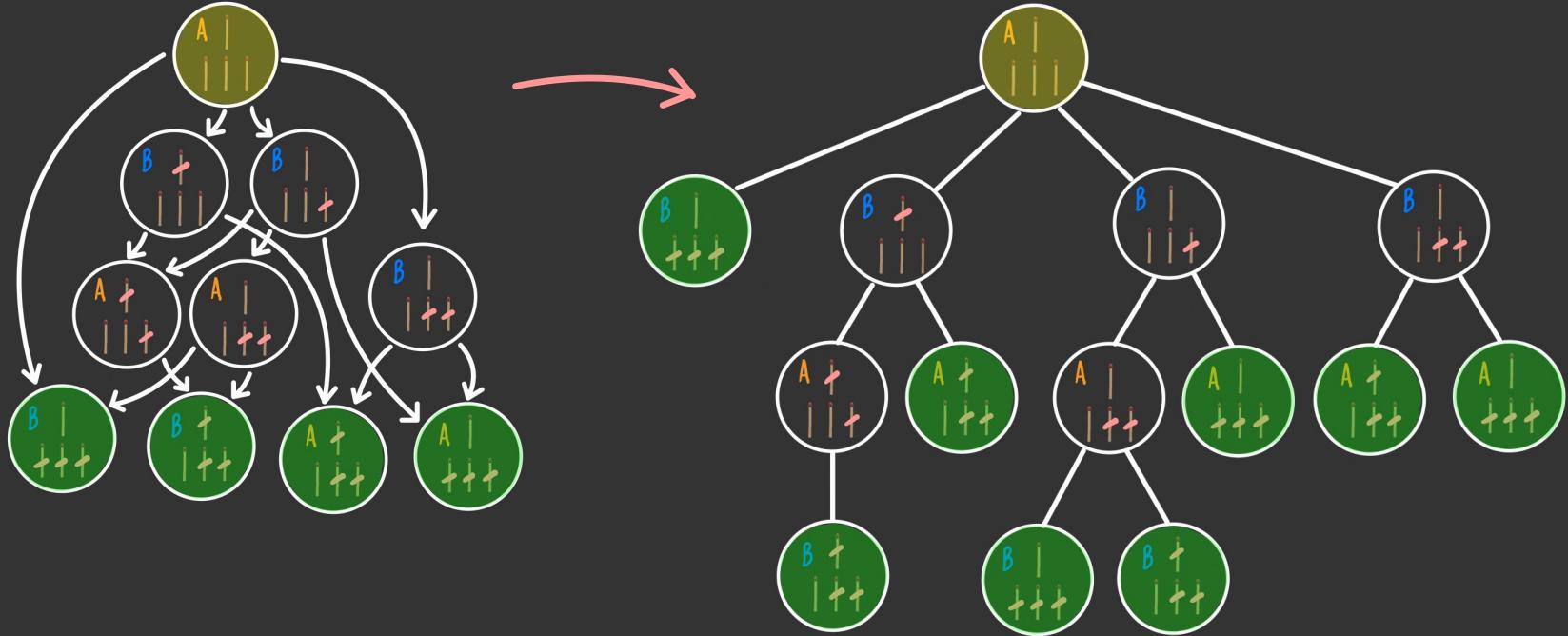
- utility function $U: F \times P \rightarrow \mathbb{R}$
which gives the value of each final state for each player



$$U(A, A) = -1$$

$$U(A, B) = 1$$

- utility function $U: F \times P \rightarrow \mathbb{R}$
which gives the value of each final state for each player



analogous to single-agent search, a game can be unraveled into a search tree