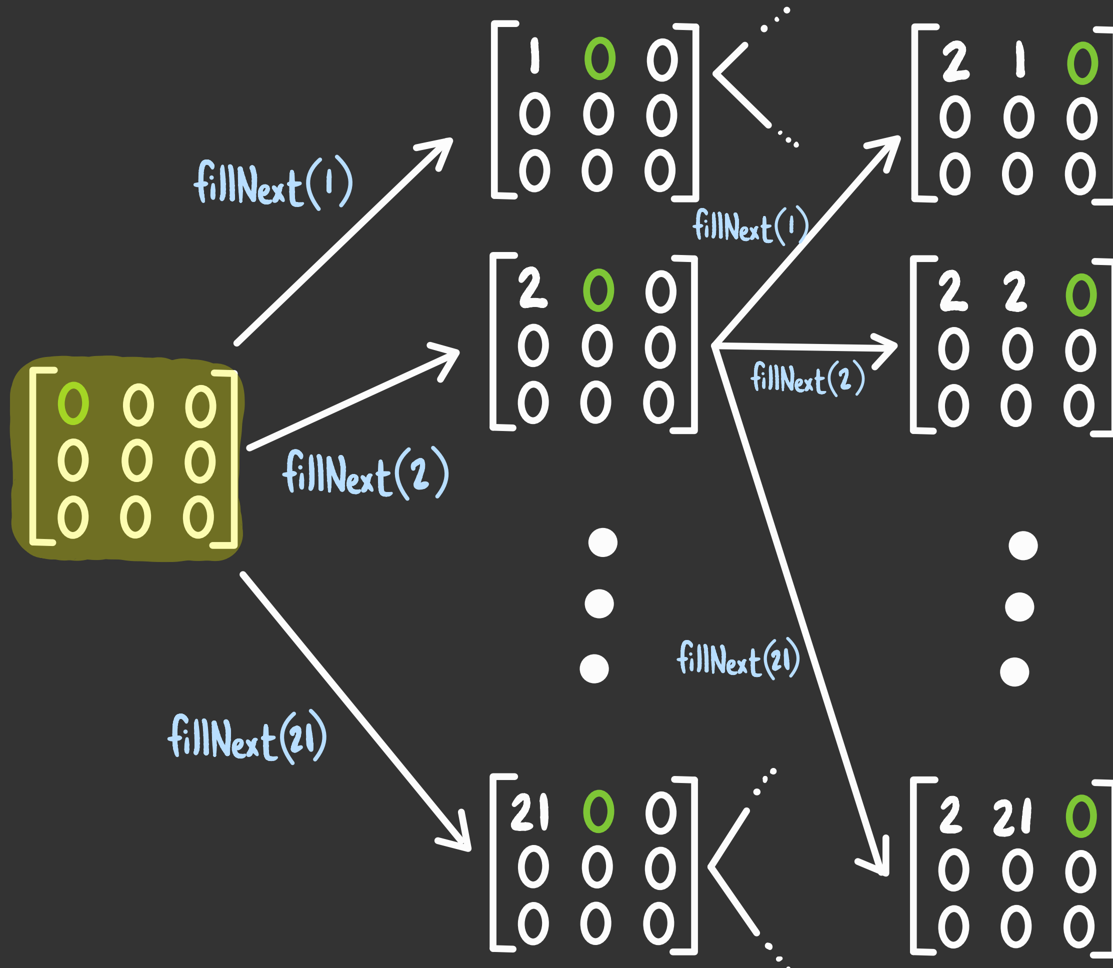
practice problem 1

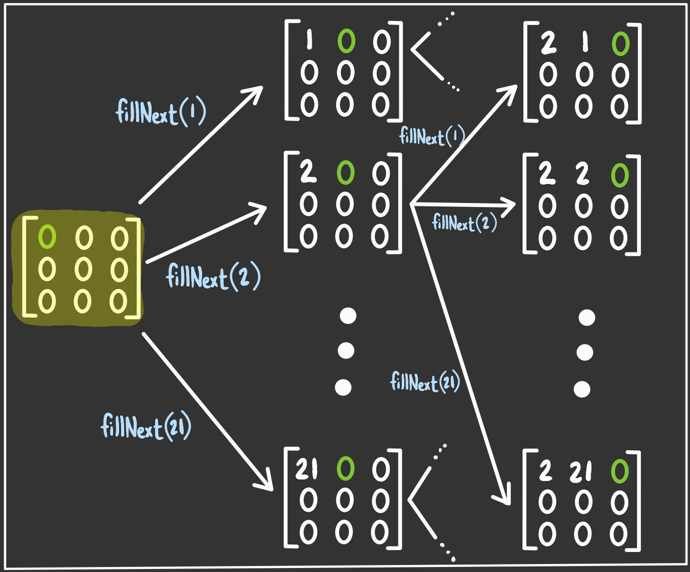
CSCI 373

formulation 1

(partial credit)

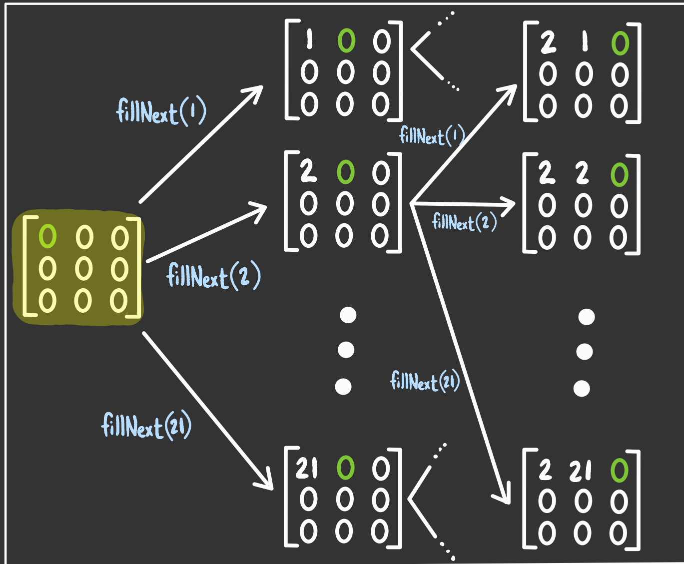


fillNext(1)
fillNext(2)
fillNext(21)
fillNext(1)
fillNext(2)
fillNext(21)



Q = all pairs (n, X) where
n is an int between 1 and 10,
and X is a 3x3 matrix
of non-negative integers

$$Q = \left\{ (n, X) \;\middle|\; \begin{array}{l} n \in \{1, \dots, 10\} \\ X \text{ is a } 3\times 3 \text{ int matrix} \end{array} \right\}$$



$$\Sigma = \{ \text{fillNext}(v) \mid v \in \{1, \dots, k\} \}$$

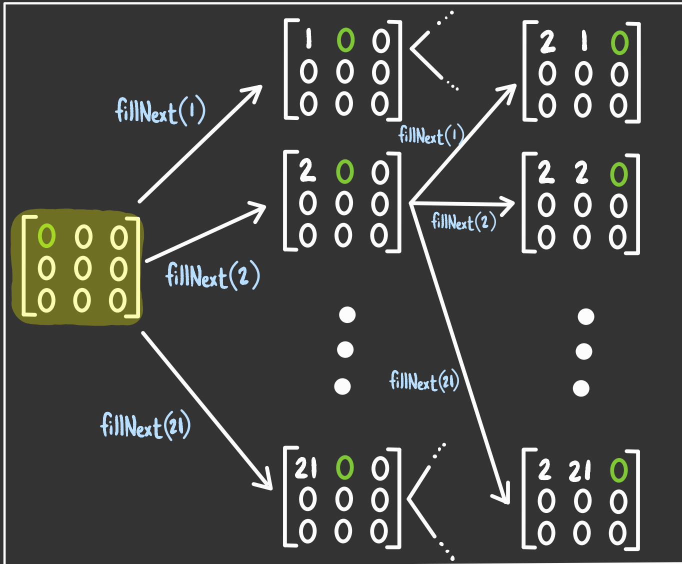
Δ = the set of triples of the form

$$\left\langle \left(1, \begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}\right), \text{fillNext}(v), \left(2, \begin{bmatrix}v&0&0\\0&0&0\\0&0&0\end{bmatrix}\right)\right\rangle$$

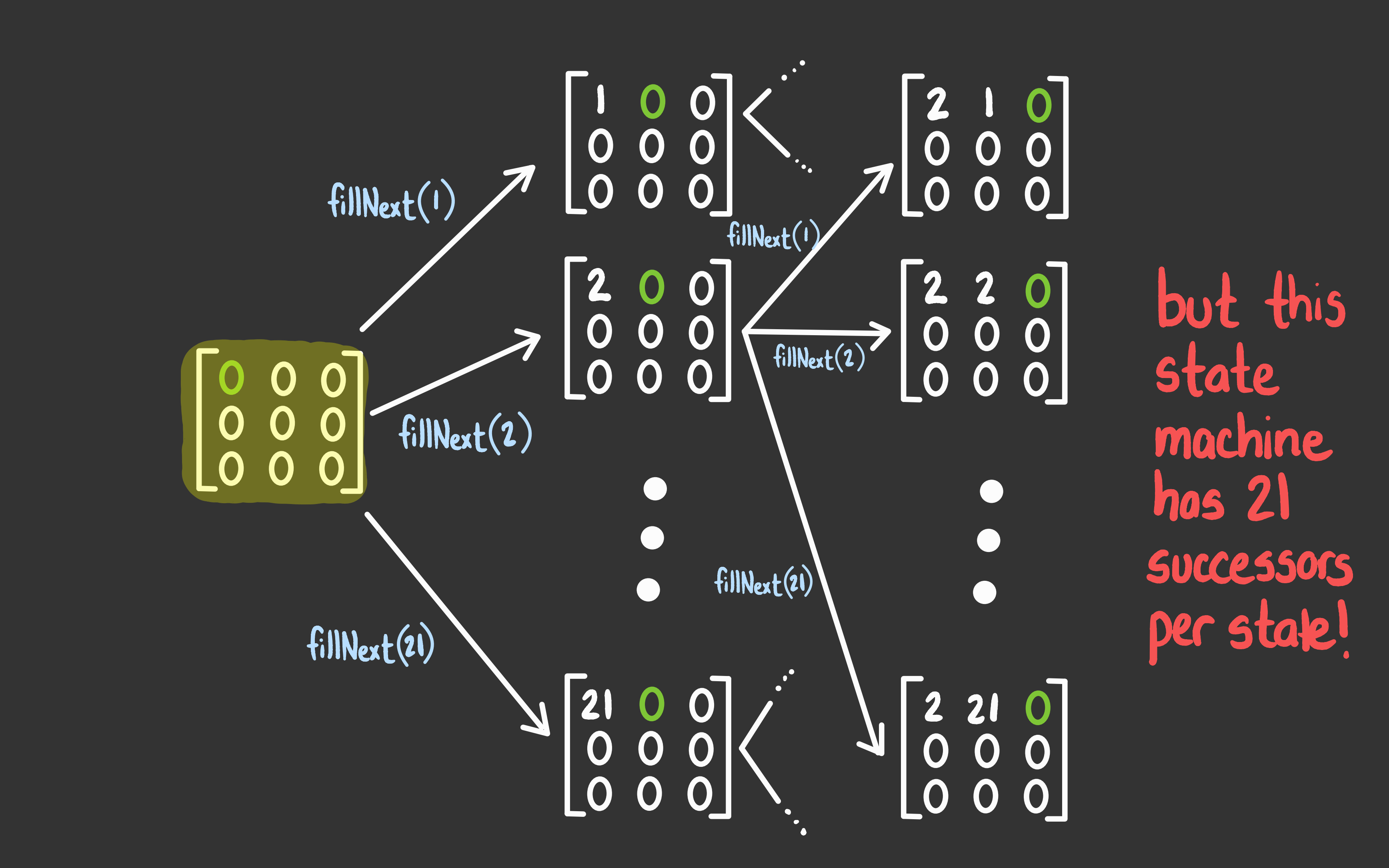
or $$\left\langle \left(2, \begin{bmatrix}a&0&0\\0&0&0\\0&0&0\end{bmatrix}\right), \text{fillNext}(v), \left(3, \begin{bmatrix}a&v&0\\0&0&0\\0&0&0\end{bmatrix}\right)\right\rangle$$

or $$\left\langle \left(3, \begin{bmatrix}a&b&0\\0&0&0\\0&0&0\end{bmatrix}\right), \text{fillNext}(v), \left(4, \begin{bmatrix}a&b&v\\0&0&0\\0&0&0\end{bmatrix}\right)\right\rangle$$

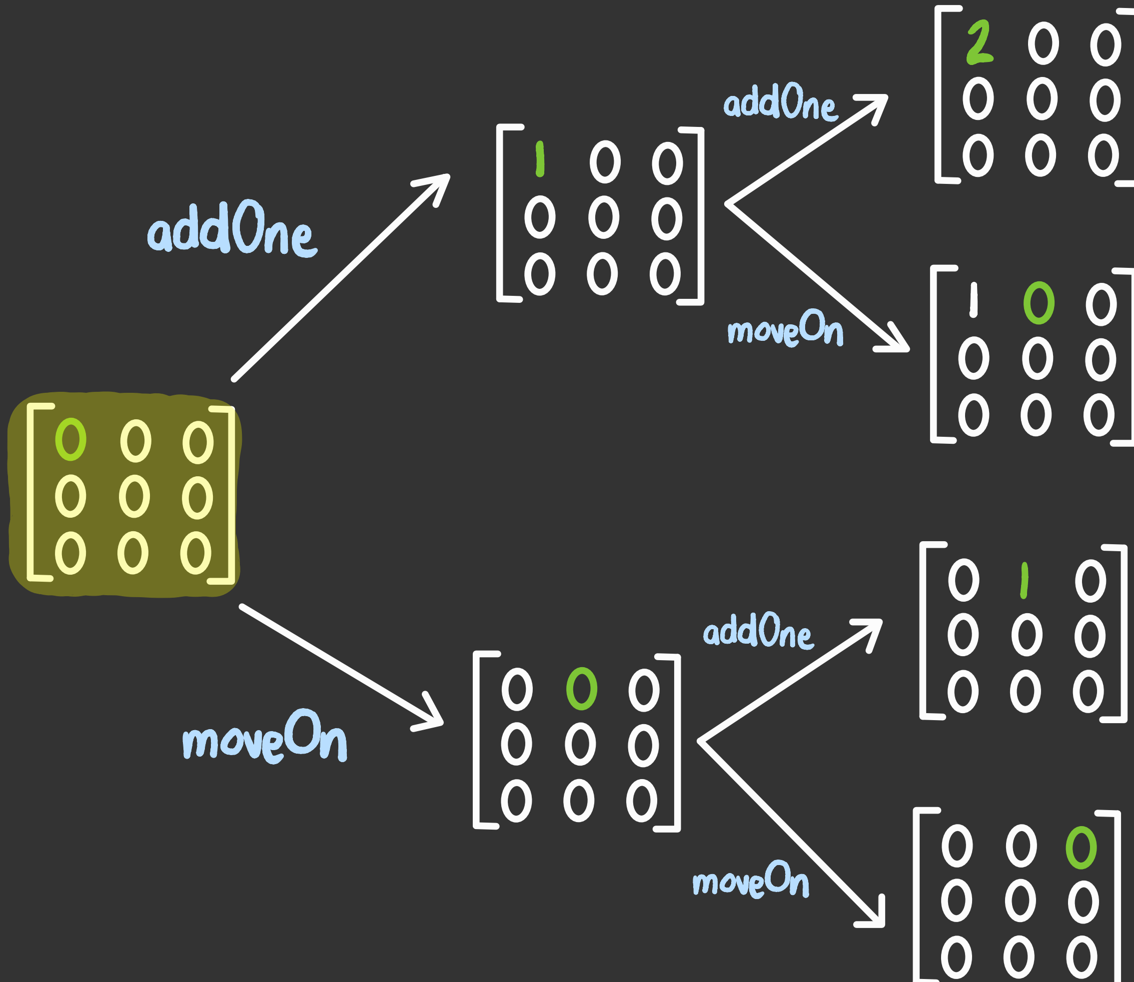
etc.



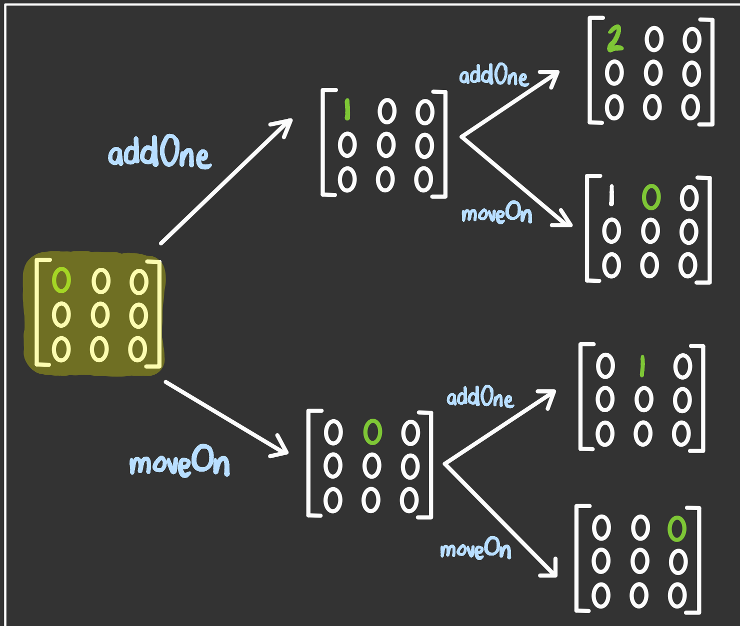
$$F = \left\{ \left(10, \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \right) \middle| \begin{array}{l} a, b, \ldots, i \text{ distinct} \\ a+b+c=k \\ d+e+f=k \\ \text{etc.} \end{array} \right\}$$


fillNext(1)
fillNext(2)
fillNext(21)
fillNext(1)
fillNext(2)
fillNext(21)
but this state machine has 21 successors per state!

formulation 2



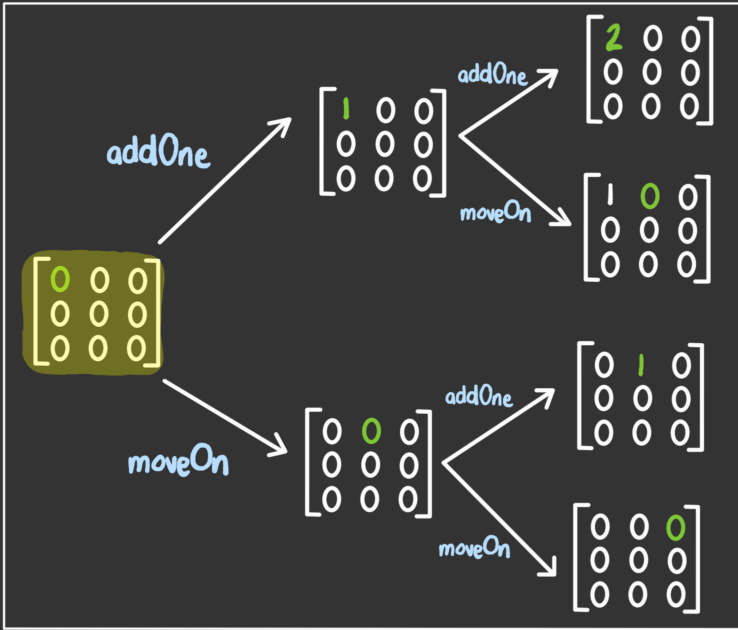
addOne
moveOn
addOne
moveOn
addOne
moveOn



$Q =$ all pairs (n, X) where n is an int between 1 and 9, and X is a 3×3 matrix of non-negative integers

$$Q = \left\{ (n, X) \;\middle|\; \begin{array}{l} n \in \{1, \dots, 9\} \\ X \text{ is a } 3\times 3 \text{ int matrix} \end{array} \right\}$$

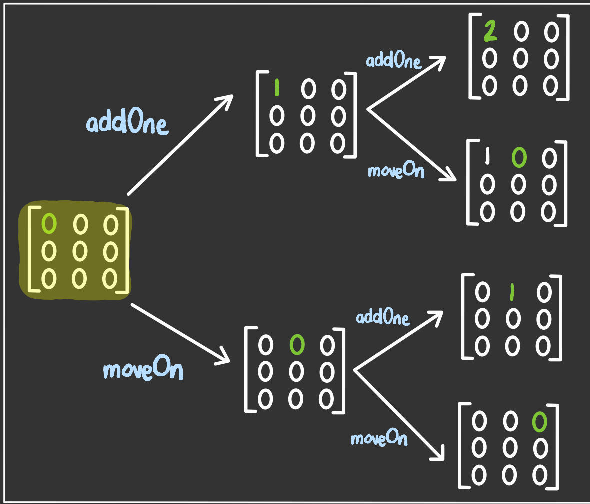
same as formulation one



$$F = \left\{ \left(10, \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \right) \middle| \begin{array}{l} a, b, \ldots, i \text{ distinct} \\ a+b+c=k \\ d+e+f=k \\ \text{etc.} \end{array} \right\}$$

$$\Sigma = \{ addOne, moveOn \}$$

same as formulation one



$$\Sigma = \{ \text{moveOn} , \text{addOne} \}$$

$$\Delta = \{ \langle (n,M), \text{moveOn}, (n+1,M) \rangle \mid (n,M),(n+1,M) \in Q \}$$

$$\cup \{ \langle (n,M), \text{addOne}, (n,M') \rangle \mid M' \text{ is identical to } m, \text{ except square } n \text{ has been incremented by } 1 \}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

$$\left\{ \left\langle (n,M),\ \texttt{addOne},\ (n,M') \right\rangle \;\middle|\; \text{M' is identical to m, except square } n \text{ has been incremented by 1} \right\}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

can also be expressed:

$$\left\{ \left\langle (n,M),\ \texttt{addOne},\ (n,M+E_{ij}) \right\rangle \;\middle|\; \begin{array}{l} (n,M) \in Q, \\ i = 1 + \left\lfloor \frac{n-1}{3} \right\rfloor \\ j = 1 + n-1 \bmod 3 \end{array} \right\}$$

where E_{ij} is a 3x3 matrix s.t. entry (i,j) is 1, and all other entries are zeroes