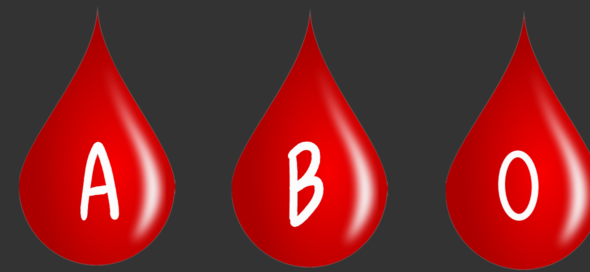
the value of information

CSCI 373

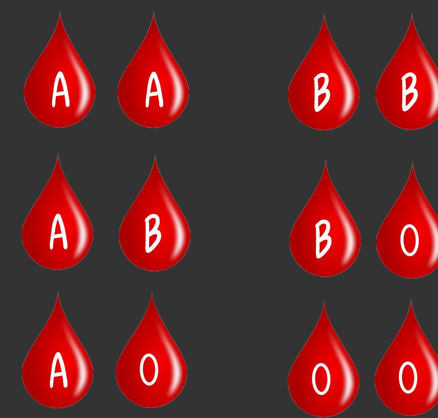
you are a vampire with a taste for type AB blood

you have an unscrupulous contact at the local hospital who provides you patient information... for a price

there are 3
blood type
genes

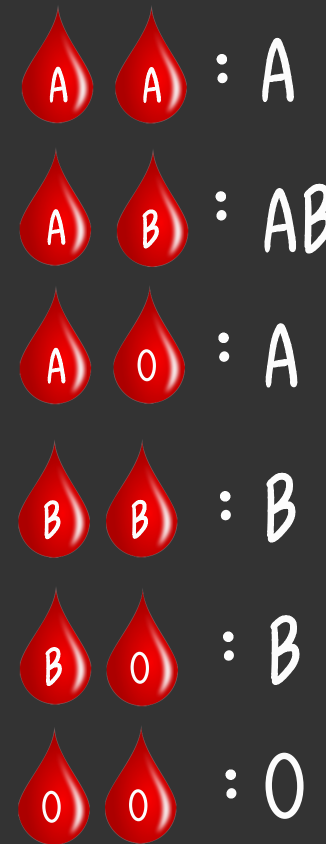


a person inherits one gene from each parent, resulting in 6 possible
genotypes



each genotype produces one of four possible

blood types



Yves

you are starting to get the sense that your neighbor Yves has "AB energy"

unfortunately Yves has never had his blood drawn at the local hospital, so your hospital contact cannot provide you his blood type

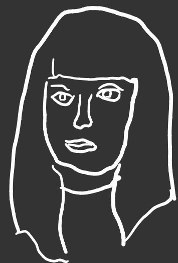
Xena

Yves

but your contact can offer you the blood type of Yves' wife **Xena**

should you purchase it?

your **answer** here



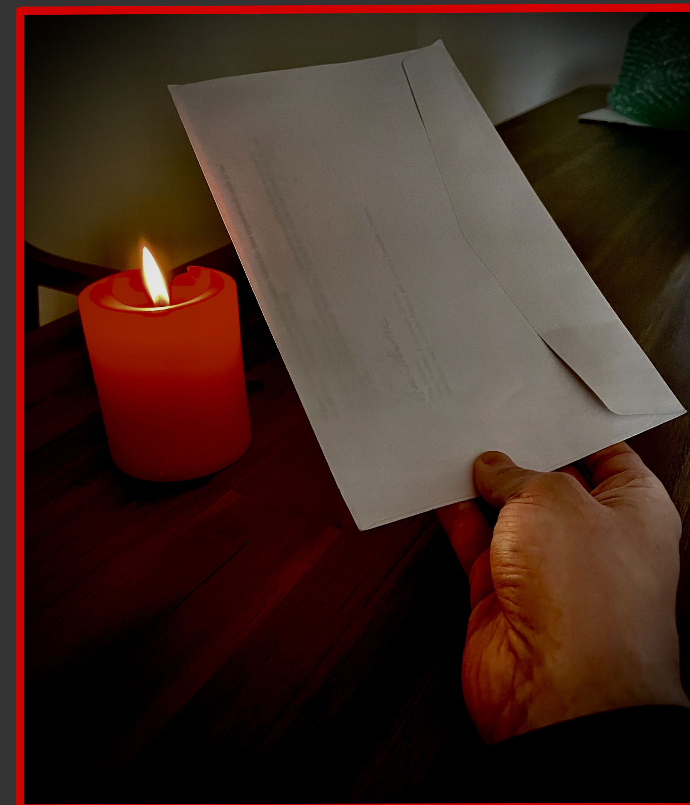
Xena

Yves

but your contact can offer you the blood type of Yves' wife Xena

should you purchase it?

no, they're not blood relatives



Xena

Yves

Zelda

undaunted, your contact comes back with the blood type of Zelda, the daughter of Xena and Yves

should you purchase it?

your answer here

undaunted, your contact comes back with the blood type of Zelda, the daughter of Xena and Yves

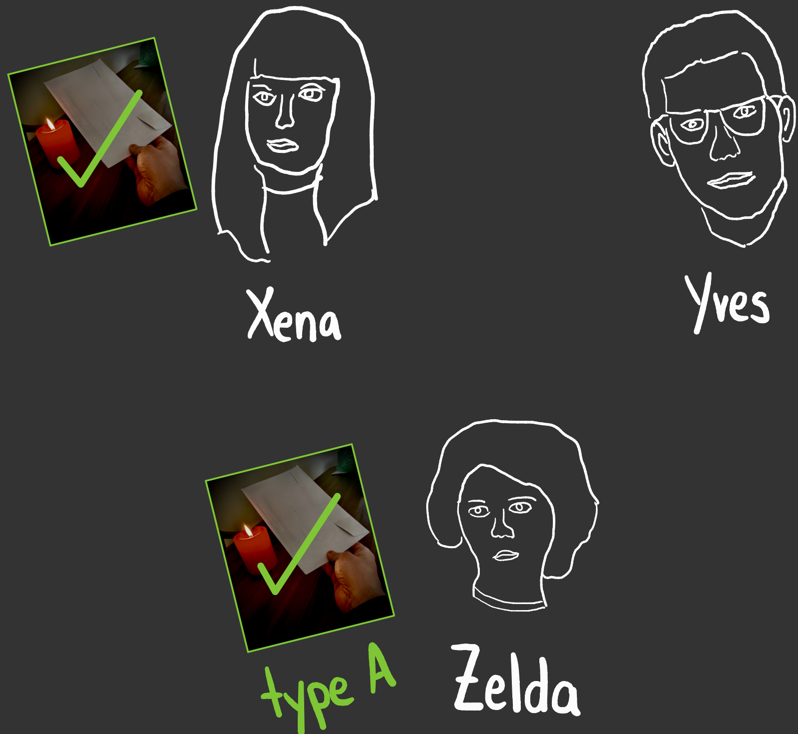
should you purchase it?

yes, because it might us information about Yves' bloodtype

at this point, your contact reminds you that Xena's blood type is still available for sale

should you purchase it now?

your answer here

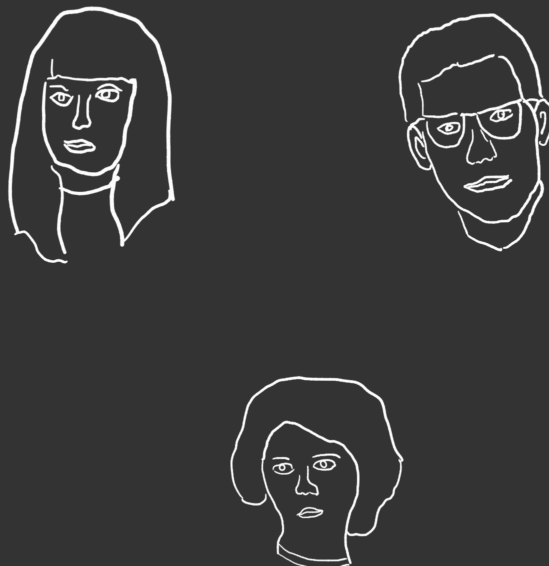


at this point, your contact reminds you that Xena's blood type is still available for sale

should you purchase it now?

yes, **now** it might us information about Yves' bloodtype

if Xena is type B, then Yves cannot be O,
given that Zelda is type A



the value of information is

Conditional

without Zelda's bloodtype, Xena's bloodtype gives us no information about Yves'

knowing Zelda's bloodtype, Xena's bloodtype can give us information about Yves'

 let X be Xena's blood type.

 let Y be Yves's blood type.

 let Z be Zelda's blood type.

Xena's bloodtype gives us no information about Yves' bloodtype $\quad P(y|x) = P(y) \quad$ for **all** $y \in D(Y), x \in D(X)$

if Zelda is type O, Yves cannot be type AB $\quad P(y|z) \neq P(y) \quad$ for some $y \in D(Y), z \in D(Z)$

e.g. $P(Y=AB|Z=O) \neq P(Y=AB)$

if Xena is type B, then Yves cannot be O, given that Zelda is type A $\quad P(y|z,x) \neq P(y|z) \quad$ for some $y \in D(Y), z \in D(Z), x \in D(X)$

e.g. $P(Y=O|Z=A, X=B) \neq P(Y=O|Z=A)$

 let X be Xena's blood type.

 let Y be Yves's blood type.

 let Z be Zelda's blood type.

Y is marginally independent of X if

$$P(y|x) = P(y)$$

for all $y \in D(Y), x \in D(X)$

Xena's bloodtype gives us no information about Yves' bloodtype

$P(y|x) = P(y)$ for all $y \in D(Y), x \in D(X)$

if Zelda is type O, Yves cannot be type AB

$P(y|z) \neq P(y)$ for some $y \in D(Y), z \in D(Z)$

e.g. $P(Y=AB \mid Z=O) \neq P(Y=AB)$

if Xena is type B, then Yves cannot be O, given that Zelda is type A

$P(y|z,x) \neq P(y|z)$ for some $y \in D(Y), z \in D(Z), x \in D(X)$

e.g. $P(Y=O \mid Z=A, X=B) \neq P(Y=O \mid Z=A)$


let X be Xena's blood type.


let Y be Yves's blood type.


let Z be Zelda's blood type.

$Y \perp\!\!\!\perp X$ if

$$P(y|x) = P(y)$$

for all $y \in D(Y), x \in D(X)$

Xena's bloodtype gives us no information about Yves' bloodtype

$$P(y|x) = P(y) \quad \text{for all } y \in D(Y), x \in D(X)$$

if Zelda is type O, Yves cannot be type AB

$$P(y|z) \neq P(y) \quad \text{for some } y \in D(Y), z \in D(Z)$$

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e.g. $P(Y=O \mid Z=A, X=B) \neq P(Y=O \mid Z=A)$


let X be Xena's blood type.


let Y be Yves's blood type.


let Z be Zelda's blood type.

$Y \not\perp\!\!\!\perp Z$ if

$$P(y|z) \neq P(y)$$

for some $y \in D(Y), x \in D(Z)$

Xena's bloodtype gives us no information about Yves' bloodtype

$P(y|x) = P(y)$ for all $y \in D(Y), x \in D(X)$

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$P(y|z,x) \neq P(y|z)$ for some $y \in D(Y), z \in D(Z), x \in D(X)$

e.g. $P(Y=O \mid Z=A, X=B) \neq P(Y=O \mid Z=A)$


let X be Xena's blood type.


let Y be Yves's blood type.


let Z be Zelda's blood type.

Y is conditionally independent of X given Z if

$$P(y \mid z, x) = P(y \mid z)$$

for all $y \in D(Y)$, $z \in D(Z)$, $x \in D(X)$

Xena's bloodtype gives us no information about Yves' bloodtype

$$P(y \mid x) = P(y)$$

for all $y \in D(Y)$, $x \in D(X)$

if Zelda is type O, Yves cannot be type AB

$$P(y \mid z) \neq P(y)$$

for some $y \in D(Y)$, $z \in D(Z)$

e.g. $P(Y = AB \mid Z = O) \neq P(Y = AB)$

if Xena is type B, then Yves cannot be O, given that Zelda is type A

$$P(y \mid z, x) \neq P(y \mid z)$$

for some $y \in D(Y)$, $z \in D(Z)$, $x \in D(X)$

e.g. $P(Y = O \mid Z = A, X = B) \neq P(Y = O \mid Z = A)$

 let X be Xena's blood type.

 let Y be Yves's blood type.

 let Z be Zelda's blood type.

$Y \perp\!\!\!\perp X \mid Z$ if

$$P(y \mid z, x) = P(y \mid z)$$

for all $y \in D(Y), z \in D(Z), x \in D(X)$

Xena's bloodtype gives us no information about Yves' bloodtype

$P(y \mid x) = P(y)$ for all $y \in D(Y), x \in D(X)$

if Zelda is type O, Yves cannot be type AB

$P(y \mid z) \neq P(y)$ for some $y \in D(Y), z \in D(Z)$

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$P(y \mid z, x) \neq P(y \mid z)$ for some $y \in D(Y), z \in D(Z), x \in D(X)$

e.g. $P(Y=O \mid Z=A, X=B) \neq P(Y=O \mid Z=A)$


let X be Xena's blood type.


let Y be Yves's blood type.


let Z be Zelda's blood type.

$Y \not\perp\!\!\!\perp X \mid Z$ if

$$P(y \mid z, x) \neq P(y \mid z)$$

for **some** $y \in D(Y), z \in D(Z), x \in D(X)$

Xena's bloodtype gives us no information about Yves' bloodtype

$P(y \mid x) = P(y)$ for all $y \in D(Y), x \in D(X)$

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if Zelda is type O, Yves cannot be type AB

$P(y \mid z) \neq P(y)$ for some $y \in D(Y), z \in D(Z)$

e.g. $P(Y=AB \mid Z=O) \neq P(Y=AB)$

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if Xena is type B, then Yves cannot be O, given that Zelda is type A

$P(y \mid z, x) \neq P(y \mid z)$ for some $y \in D(Y), z \in D(Z), x \in D(X)$

e.g. $P(Y=O \mid Z=A, X=B) \neq P(Y=O \mid Z=A)$

$$\boxed{\begin{array}{l} Y \perp\!\!\!\perp X \text{ if} \\ \quad P(y|x) = P(y) \\ \text{for all } y \in D(Y),\ x \in D(X) \end{array}}$$

marginal independence is commutative:

if $Y \perp\!\!\!\perp X$, then $X \perp\!\!\!\perp Y$

$$\boxed{\begin{array}{l} Y \perp\!\!\!\perp X \mid Z \text{ if} \\ \quad P(y|z,x) = P(y|z) \\ \text{for all } y \in D(Y),\ z \in D(Z),\ x \in D(X) \end{array}}$$

conditional independence is commutative:

if $Y \perp\!\!\!\perp X \mid Z$, then $X \perp\!\!\!\perp Y \mid Z$

C	S	$P(c,s)$
red	●	$\frac{3}{25}$
red	■	$\frac{2}{25}$
red	▲	$\frac{2}{25}$
white	●	$\frac{2}{25}$
white	■	$\frac{6}{25}$
white	▲	$\frac{10}{25}$

is $C \perp\!\!\!\perp S$?

$Y \perp\!\!\!\perp X$ if

$$P(y|x) = P(y)$$

for all $y \in D(Y), x \in D(X)$

C	S	$P(c,s)$
red	$\bullet$	$\frac{3}{25}$
red	$\blacksquare$	$\frac{2}{25}$
red	$\blacktriangle$	$\frac{2}{25}$
white	$\bullet$	$\frac{2}{25}$
white	$\blacksquare$	$\frac{6}{25}$
white	$\blacktriangle$	$\frac{10}{25}$

$$P(C=\text{red} \mid S=\bullet) \stackrel{?}{=} P(C=\text{red})$$

$$P(C=\text{red} \mid S=\blacksquare) \stackrel{?}{=} P(C=\text{red})$$

$$P(C=\text{red} \mid S=\blacktriangle) \stackrel{?}{=} P(C=\text{red})$$

$$P(C=\text{white} \mid S=\bullet) \stackrel{?}{=} P(C=\text{white})$$

$$P(C=\text{white} \mid S=\blacksquare) \stackrel{?}{=} P(C=\text{white})$$

$$P(C=\text{white} \mid S=\blacktriangle) \stackrel{?}{=} P(C=\text{white})$$

$Y \perp\!\!\!\perp X$ if

$$P(y \mid x) = P(y)$$

for all $y \in D(Y)$, $x \in D(X)$

$$P(C=\text{red} \mid S=\bullet) \overset{?}{=} P(C=\text{red})$$

C	S	P(c, s)
red	$\bullet$	$\frac{3}{25}$
red	$\blacksquare$	$\frac{2}{25}$
red	$\blacktriangle$	$\frac{2}{25}$
white	$\bullet$	$\frac{2}{25}$
white	$\blacksquare$	$\frac{6}{25}$
white	$\blacktriangle$	$\frac{10}{25}$

$$P(C=\text{red} \mid S=\bullet)$$

$$\overset{y|x}{=} \frac{P(C=\text{red}, S=\bullet)}{P(S=\bullet)}$$

$$\overset{\text{total}}{=} \frac{P(C=\text{red}, S=\bullet)}{\sum_{c} P(c, S=\bullet)}$$

$$= \frac{\frac{3}{25}}{\frac{3}{25} + \frac{2}{25}}$$

$$= \frac{3}{5}$$

$$P(C=\text{red})$$

$$\overset{\text{total}}{=} \sum_{s} P(C=\text{red}, s)$$

$$= \frac{3}{25} + \frac{2}{25} + \frac{2}{25}$$

$$= \frac{7}{25}$$

not equal, so

$$C \not\perp\!\!\!\perp S$$