search spaces

12 sept 2022

CSCI 373

train

rules

- change one letter at a time to get from train to prawn
- each intermediate word must be valid english

prawn

train
trail
trawl
brawl
brawn
prawn

train

your answer here

prawn

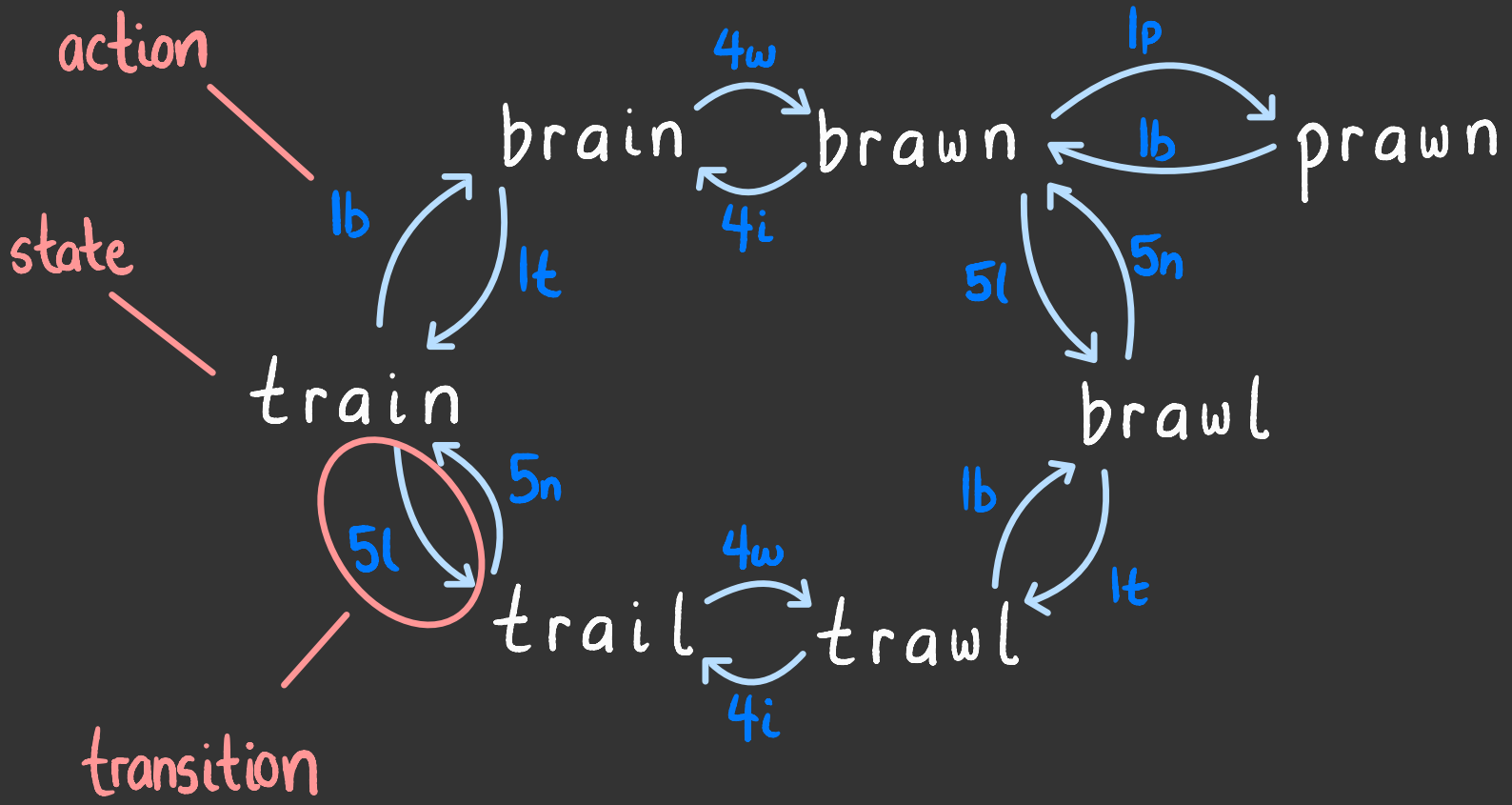
train

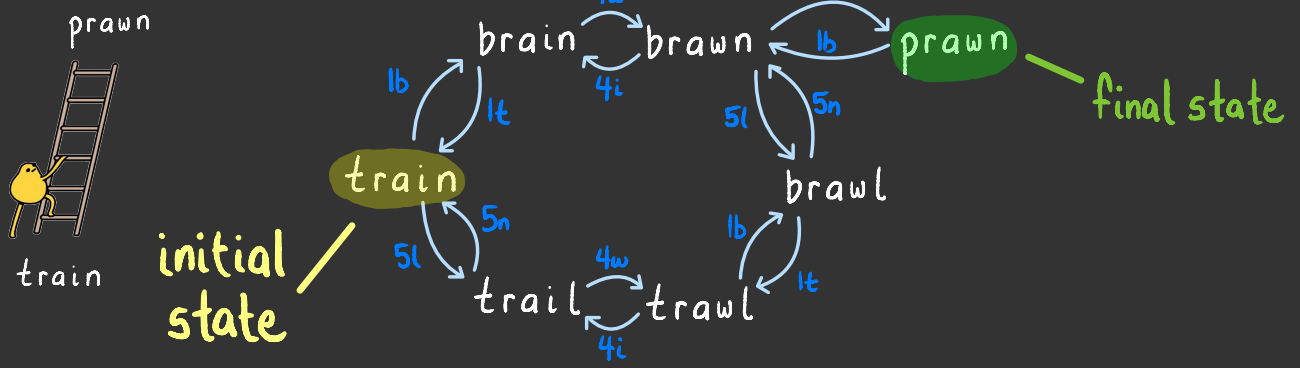
brain

brawn

prawn

how do we describe word ladder?


action
state
transition
brain
brawn
prawn
train
brawl
trail
trawl
4w
4i
1p
1b
1b
1t
5l
5n
5n
5l
1b
1t
4w
4i



define a state machine as a tuple $(Q, \Sigma, \Delta, q_0, F)$ where:

- Q is a set of states
- Σ is a set of actions
- $\Delta \subseteq Q \times \Sigma \times Q$ is a set of permitted transitions
- $q_0 \in Q$ is the initial state
- $F \subseteq Q$ is a set of final states

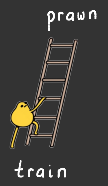
$Q = \{\text{train}, \text{brain}, \text{trail}, \ldots\}$

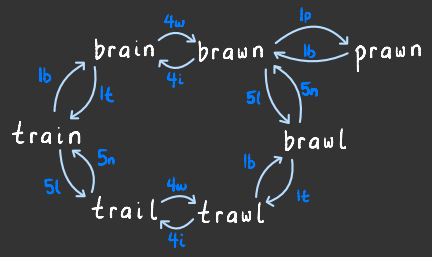
$\Sigma = \{\text{la}, \ldots, \text{5z}\}$

$\Delta = \{(\text{train}, \text{lb}, \text{brain}), \ldots\}$

$q_0 = \text{train}$

$F = \{\text{prawn}\}$





define a state machine as a tuple $(Q, \Sigma, \Delta, q_0, F)$ where:

- Q is a set of states
- Σ is a set of actions
- $\Delta \subseteq Q \times \Sigma \times Q$ is a set of permitted transitions
- $q_0 \in Q$ is the initial state
- $F \subseteq Q$ is a set of final states

$Q = \{\text{train}, \text{brain}, \text{trail}, \dots\}$

$\Sigma = \{\text{la}, \dots, \text{5z}\}$

$\Delta = \{(\text{train}, \text{lb}, \text{brain}), \dots\}$

$q_0 = \text{train}$

$F = \{\text{prawn}\}$

define a weighted state machine as a tuple $(Q, \Sigma, \Delta, q_0, F, w)$ where:

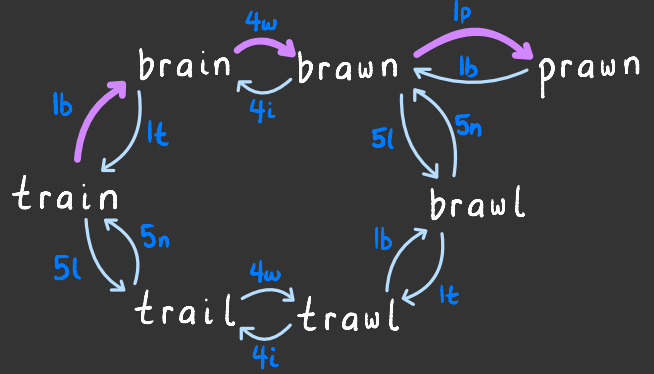
- $(Q, \Sigma, \Delta, q_0, F)$ is a state machine
- $w : \Delta \to \mathbb{R}$ assigns a real-valued weight to each transition

what is the weight function w for word ladder?

$$w = \{\delta \mapsto 1 \mid \delta \in \Delta\}$$

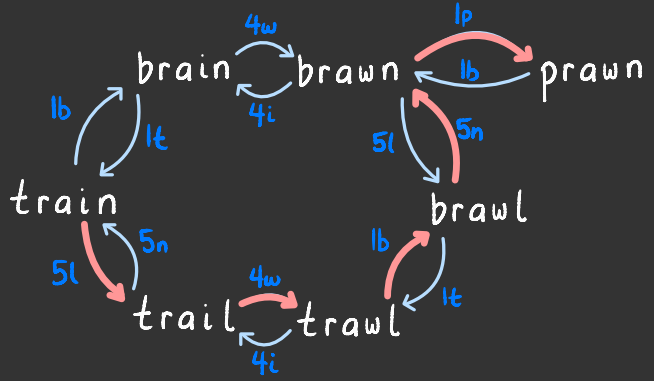
search paths

train
brain
brawn
prawn



formally: ⟨(train, lb, brain), (brain, 4w, brawn), (brawn, lp, prawn)⟩

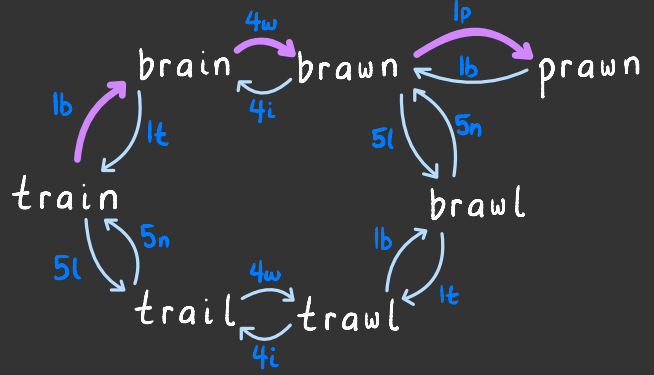
train
trail
trawl
brawl
brawn
prawn



formally: ⟨(train, 5l, trail), (trail, 4w, trawl), (trawl, lb, brawl), (brawl, 5n, brawn), (brawn, lp, prawn)⟩

search paths

train
brain
brawn
prawn



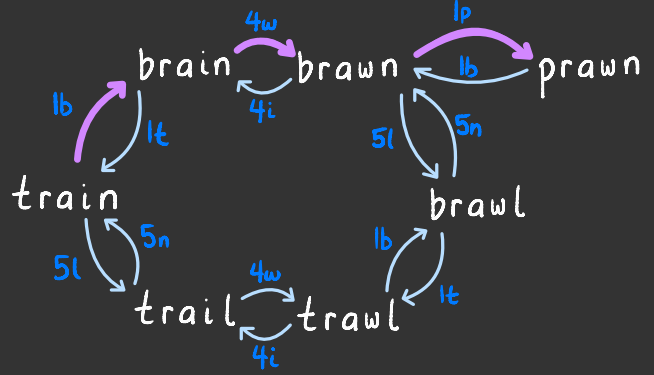
formally: $\langle (\text{train}, \text{lb}, \text{brain}), (\text{brain}, \text{4w}, \text{brawn}), (\text{brawn}, \text{lp}, \text{prawn}) \rangle$

states	$Q = \{\text{train}, \text{brain}, \text{trail}, \dots\}$
actions	$\Sigma = \{\text{la}, \dots, \text{5z}\}$
transitions	$\Delta = \{(\text{train}, \text{lb}, \text{brain}), \dots\}$
initial state	$q_0 = \text{train}$
final states	$F = \{\text{prawn}\}$
weight function	$w = \{\delta \mapsto 1 \mid \delta \in \Delta\}$

a **search path** is a sequence $\langle \delta_0, \dots, \delta_k \rangle$ of **transitions** from Δ such that there exist **states** $q_1, \dots, q_{k+1} \in Q$ and **actions** $\sigma_0, \dots, \sigma_k \in \Sigma$ such that $\delta_i = (q_i, \sigma_i, q_{i+1}) \quad \forall i \in \{0, \dots, k\}$

search paths

train
brain
brawn
prawn



formally: ⟨(train, lb, brain), (brain, 4w, brawn), (brawn, lp, prawn)⟩

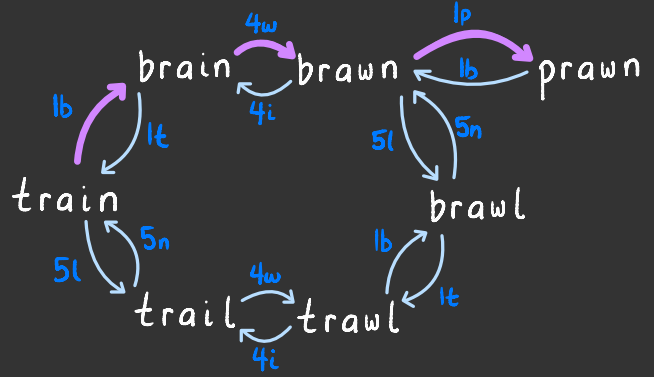
this must be the initial state

these must match

these must match

in a search path

train
brain
brawn
prawn



a search path is a sequence $\langle \delta_0, \dots, \delta_k \rangle$ of transitions from Δ such that there exist states $q_1, \dots, q_{k+1} \in Q$ and actions $\sigma_0, \dots, \sigma_k \in \Sigma$ such that $\delta_i = (q_i, \sigma_i, q_{i+1}) \quad \forall i \in \{0, \dots, k\}$

formally: $\langle (\text{train}, \text{lb}, \text{brain}), (\text{brain}, \text{4w}, \text{brawn}), (\text{brawn}, \text{lp}, \text{prawn}) \rangle$

states $Q = \{\text{train}, \text{brain}, \text{trail}, \dots\}$

actions $\Sigma = \{\text{la}, \dots, \text{5z}\}$

transitions $\Delta = \{(\text{train}, \text{lb}, \text{brain}), \dots\}$

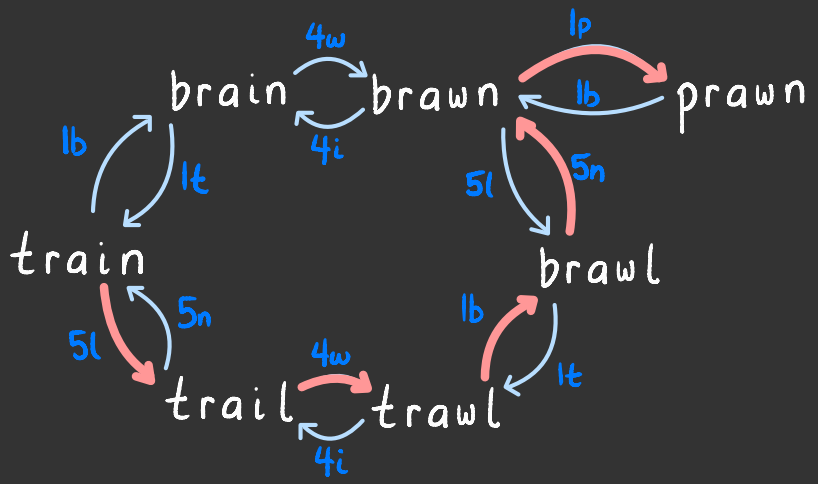
initial state $q_0 = \text{train}$

final states $F = \{\text{prawn}\}$

weight function $w = \{\delta \mapsto 1 \mid \delta \in \Delta\}$

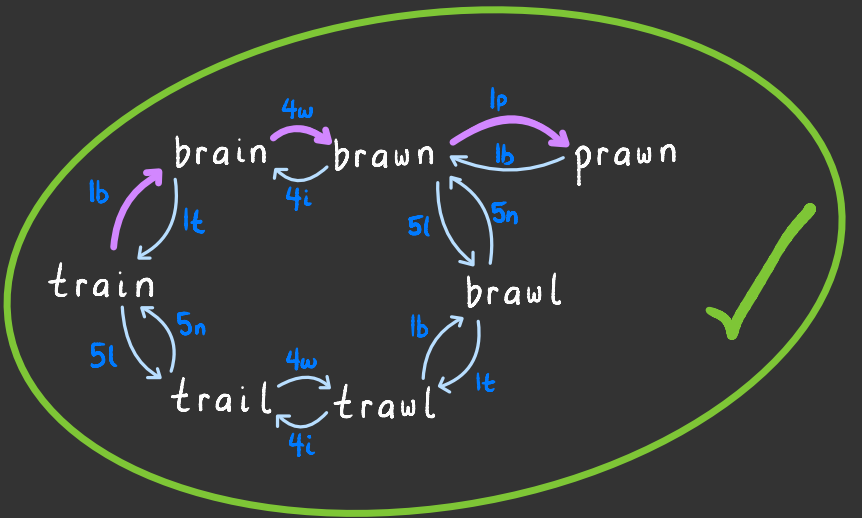
the cost of a search path is the sum of the weights of the transitions

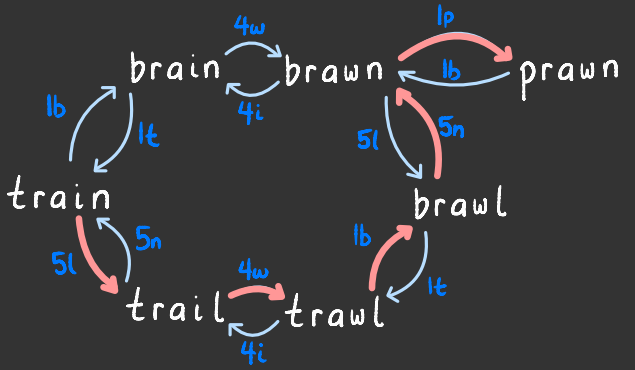
what is the cost of this search path?



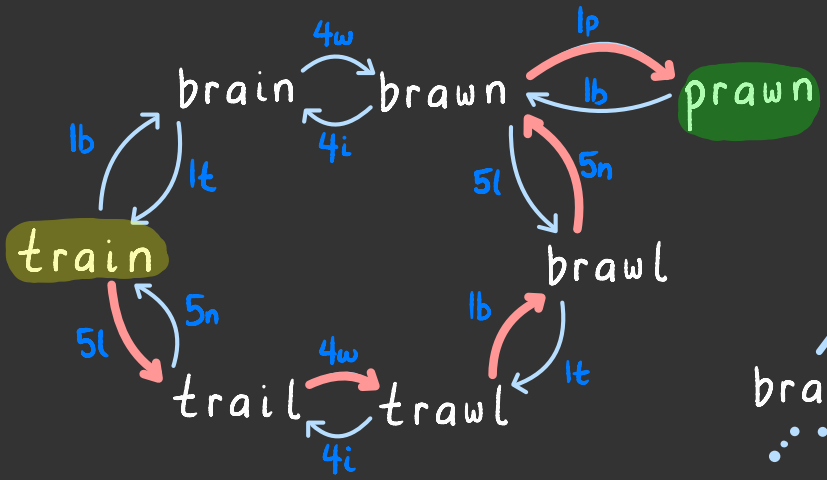
$$
\begin{aligned}
& w((\text{train}, 5\text{l}, \text{trail})) && 1 \\
+\ & w((\text{trail}, 4\text{w}, \text{trawl})) && +\ 1 \\
+\ & w((\text{trawl}, 1\text{b}, \text{brawl})) && +\ 1 \\
+\ & w((\text{brawl}, 5\text{n}, \text{brawn})) && +\ 1 \\
+\ & w((\text{brawn}, 1\text{p}, \text{prawn})) && +\ 1 \\
\hline
& && 5
\end{aligned}
$$

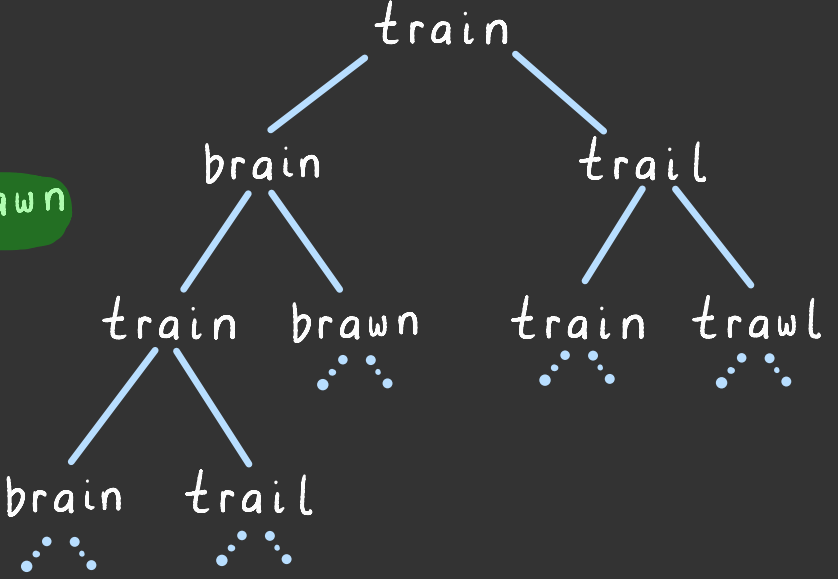
the goal of search:
to find a search path
of minimal cost whose
last transition leads to
a final state





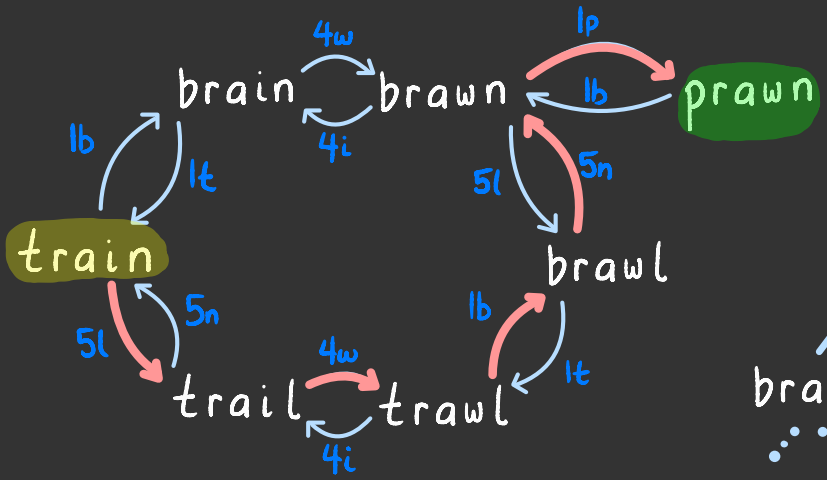
a state machine can be unraveled into a search tree

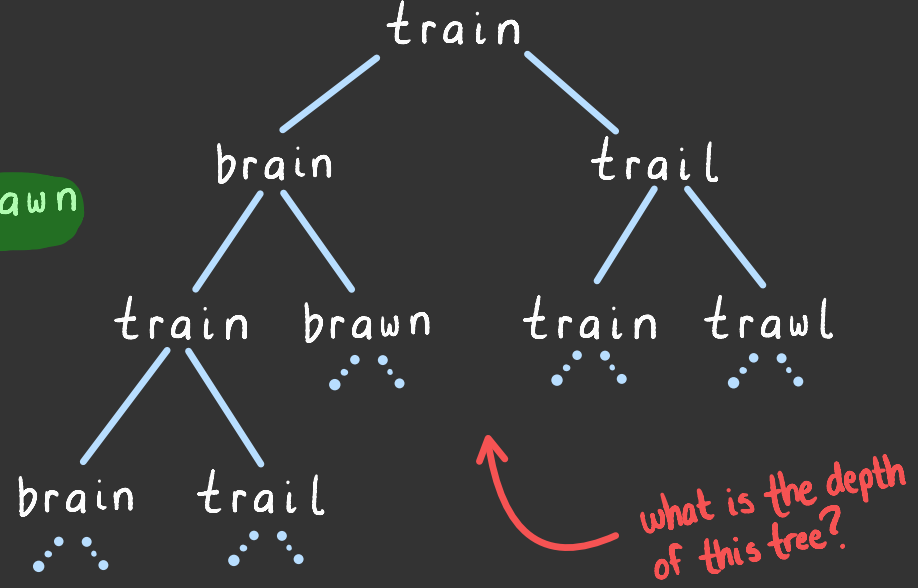




the search tree is a compact representation of all the search paths

a state machine can be unraveled into a search tree





infinite. in general, if a state machine has reachable cycles, then its search tree has infinite depth