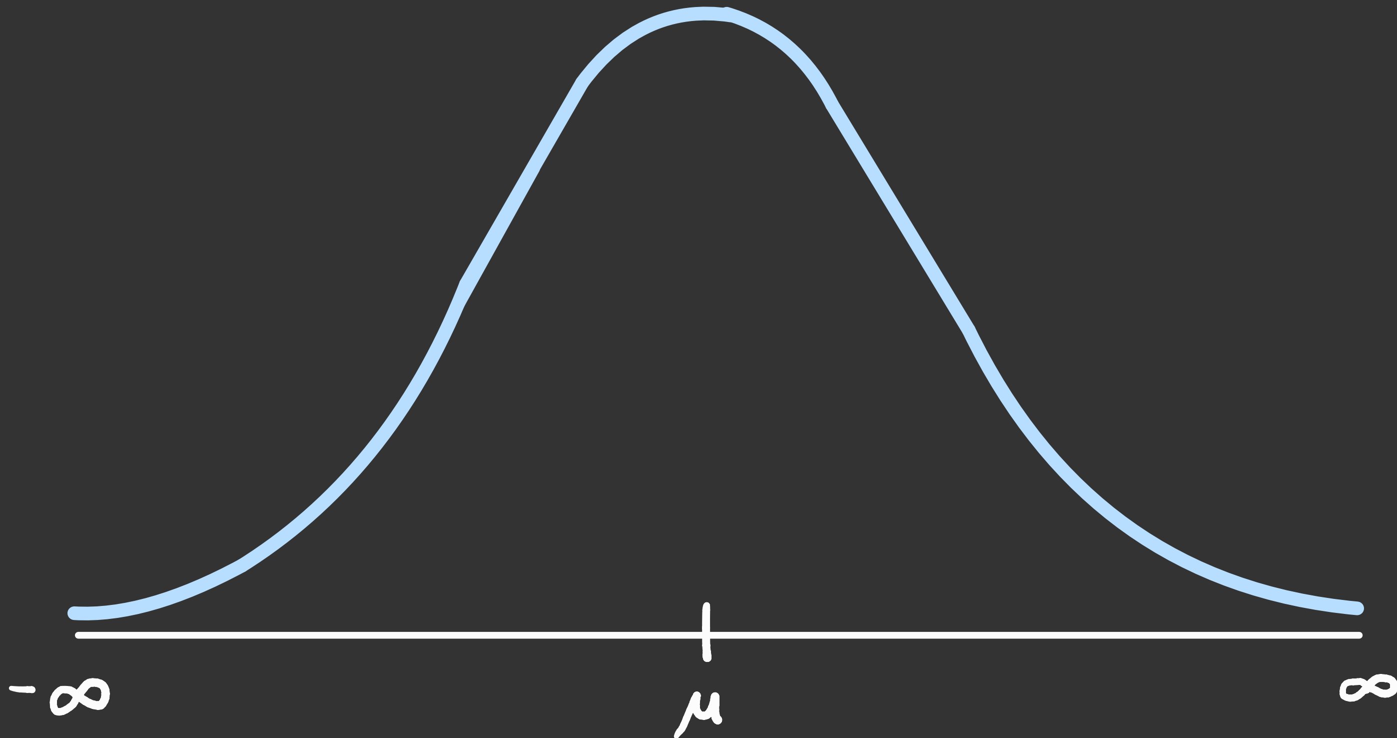
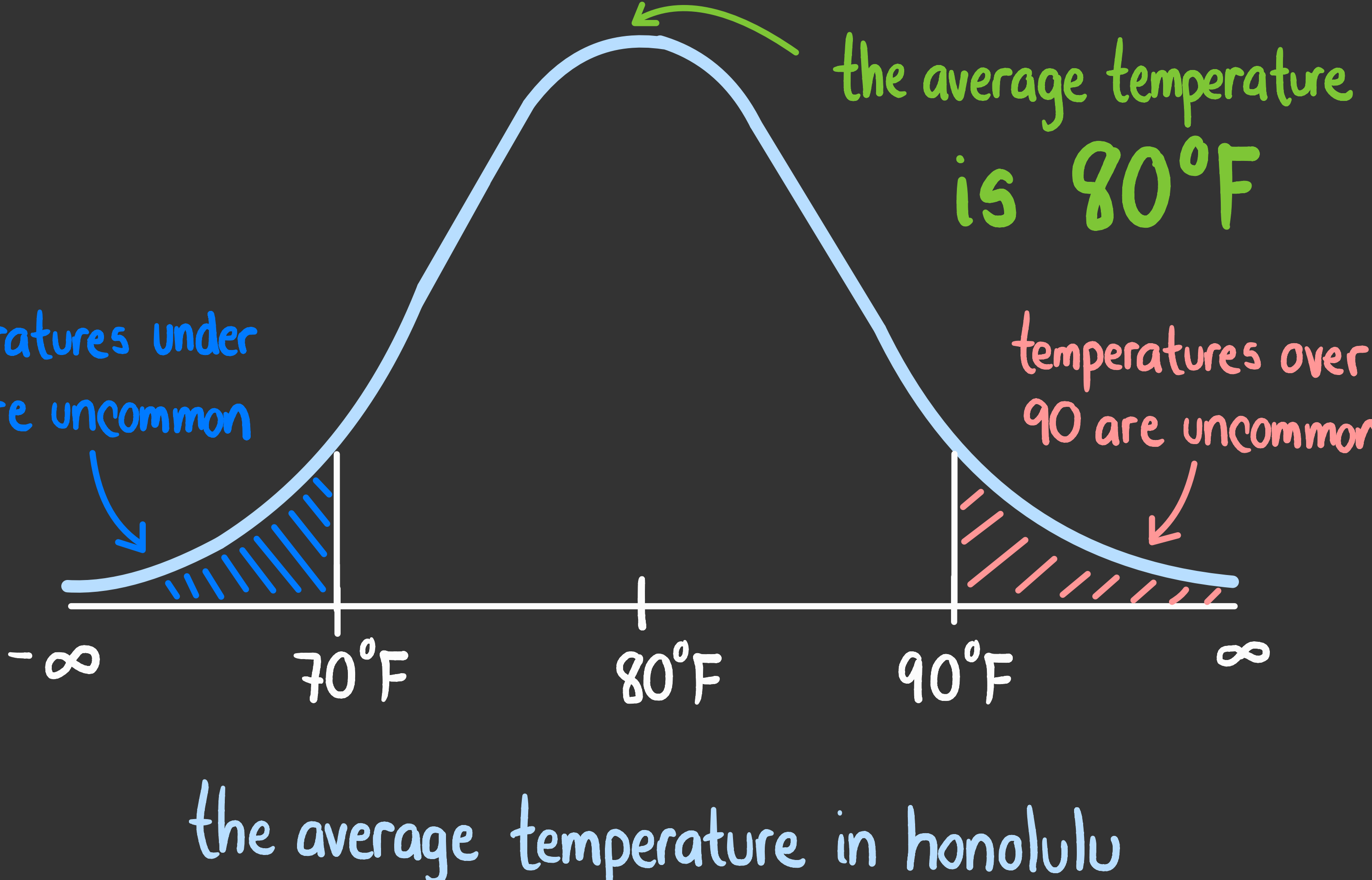
normal

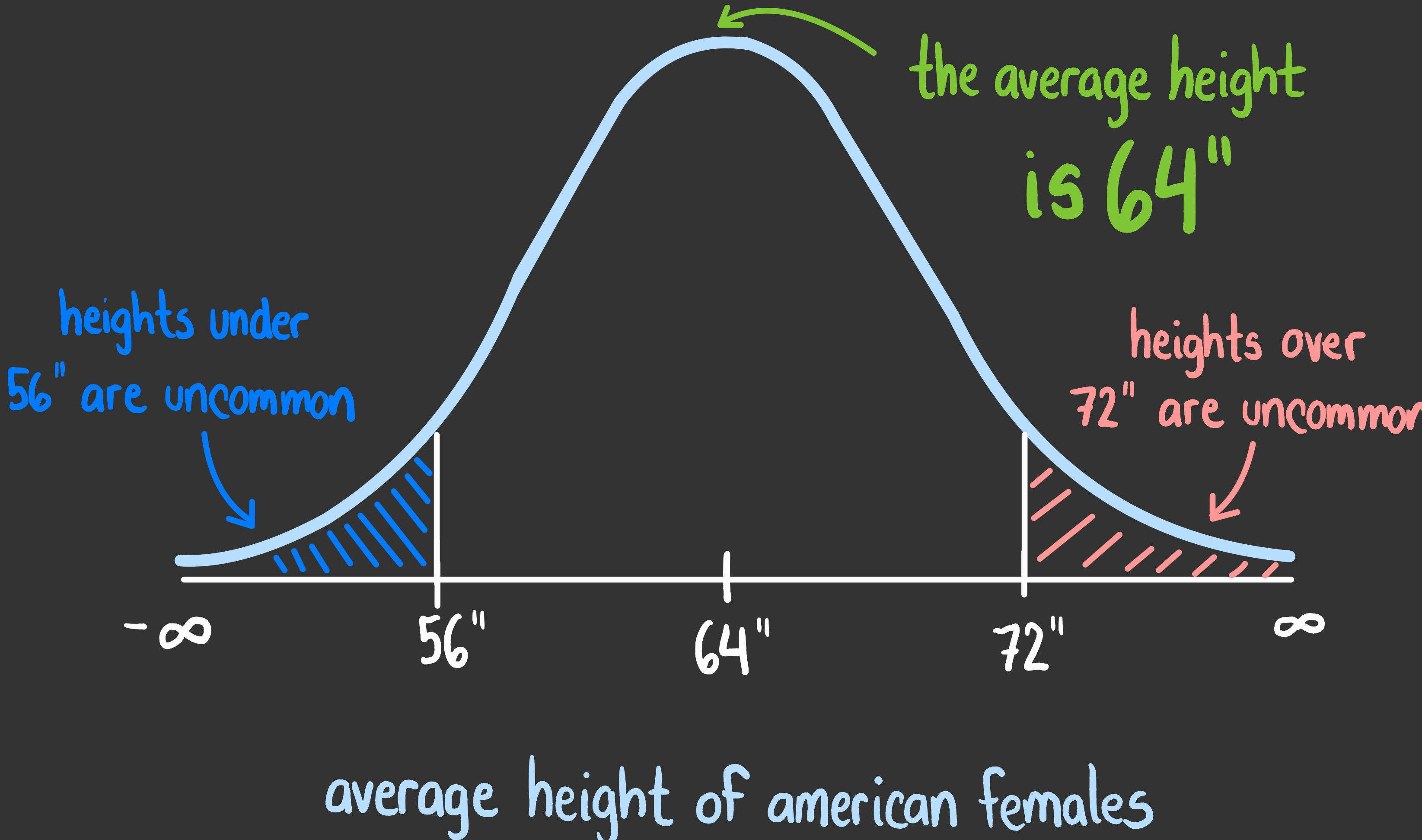
distributions

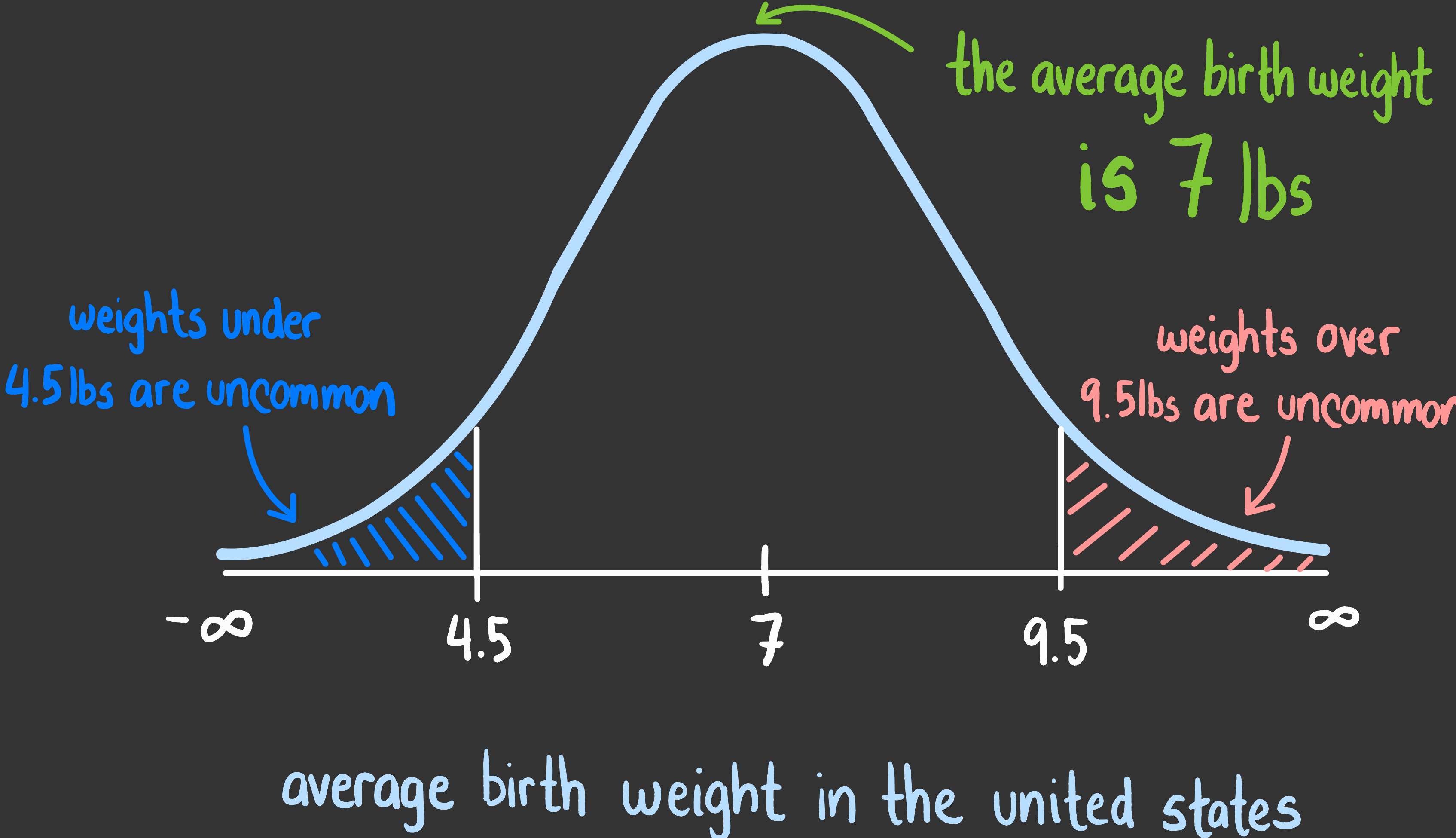
CSCI 373

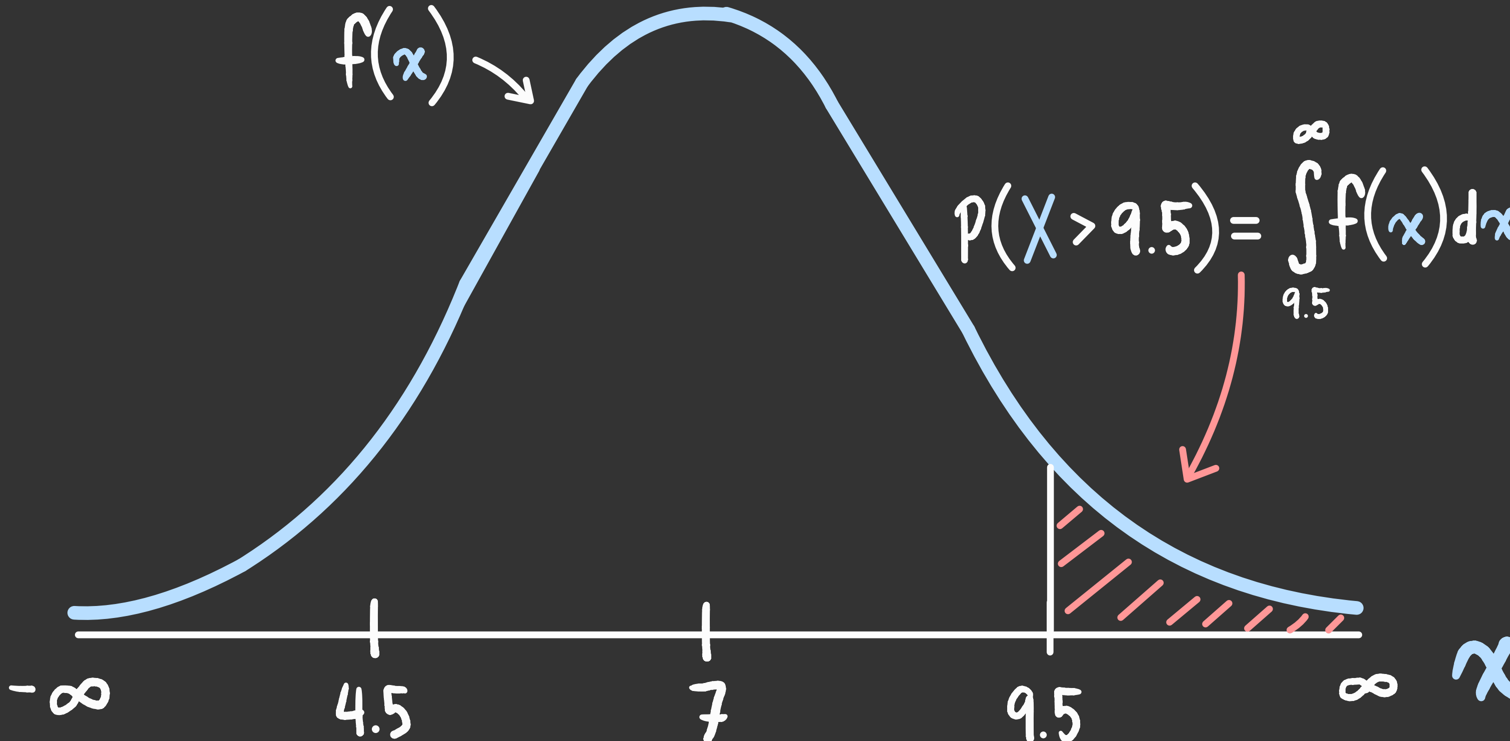


the normal distribution

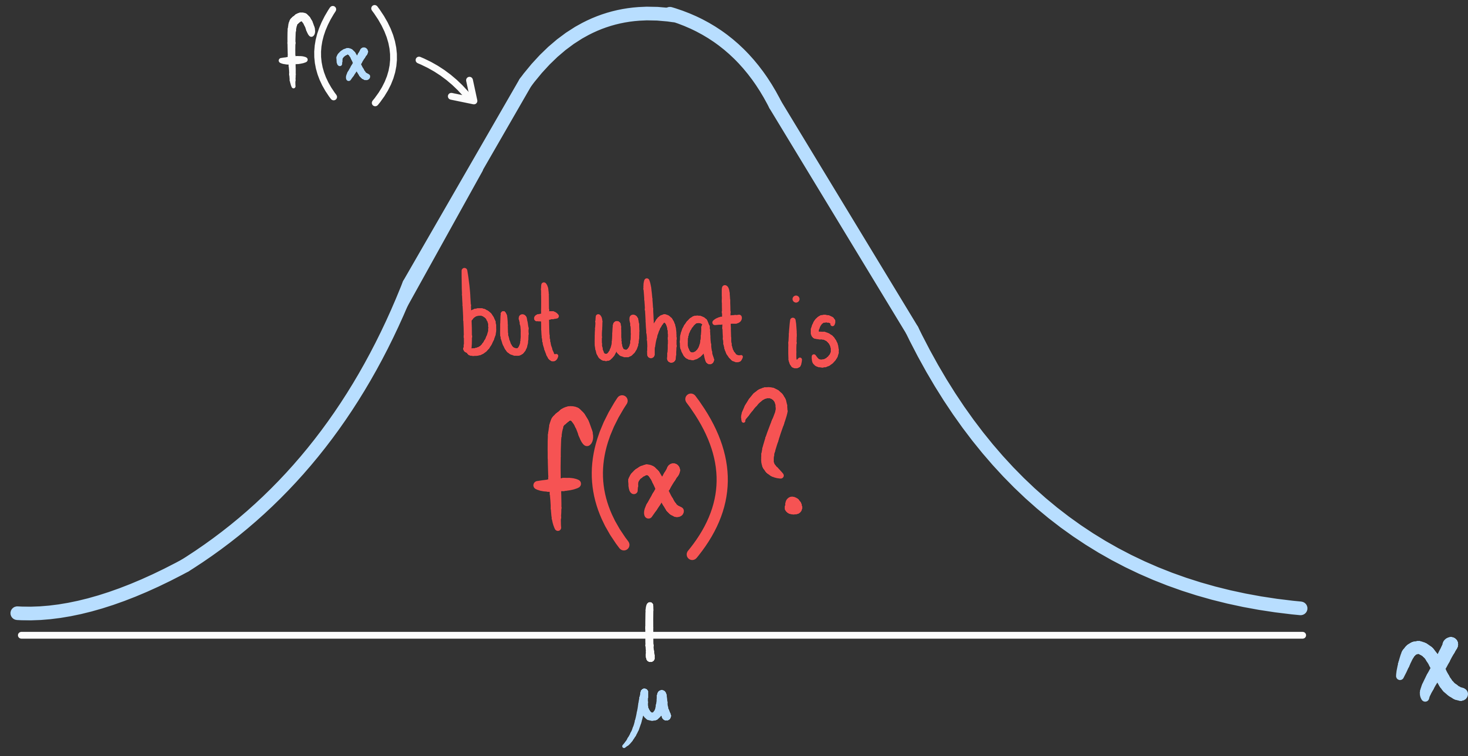

the average temperature is 80°F
temperatures under 70 are uncommon
temperatures over 90 are uncommon
-∞
70°F
80°F
90°F
∞
the average temperature in honolulu

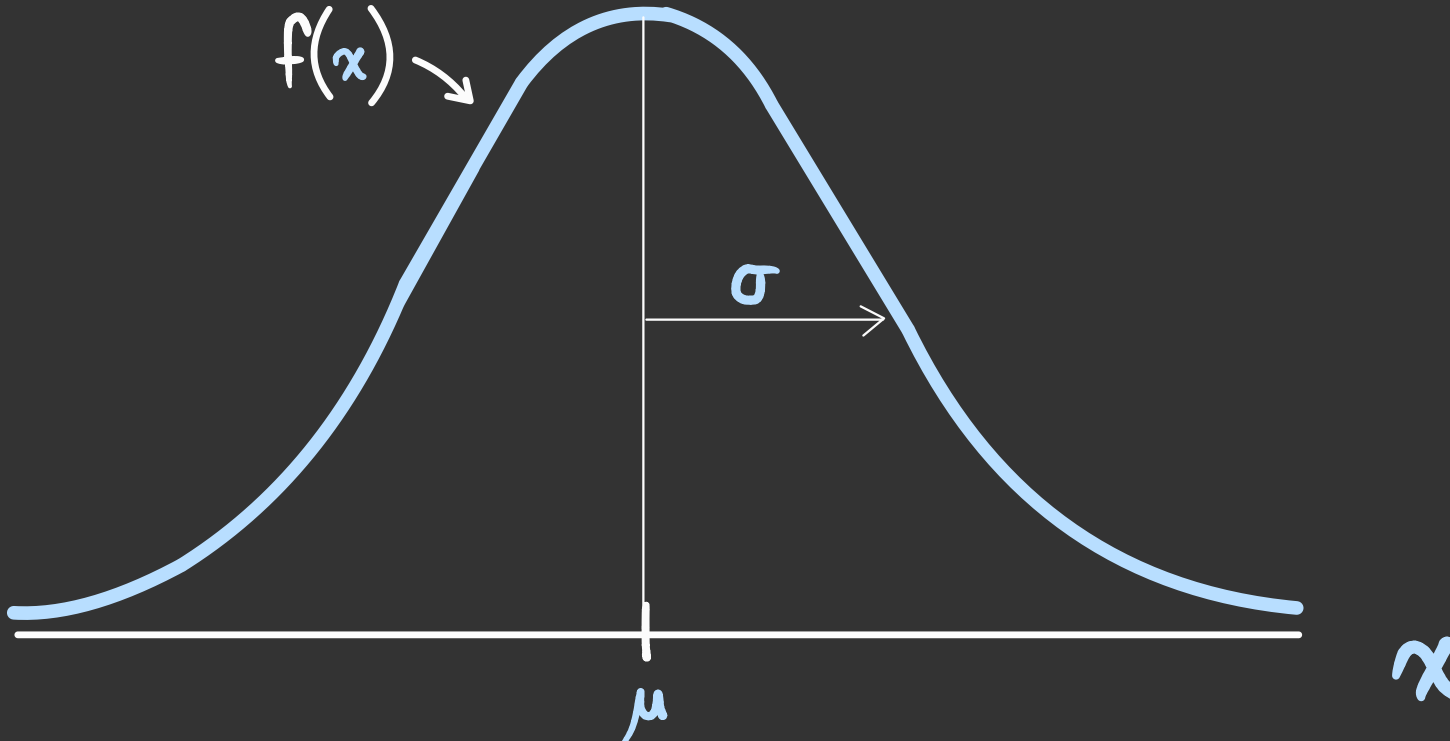

the average height is 64"
heights under 56" are uncommon
heights over 72" are uncommon
-∞
56"
64"
72"
∞
average height of american females


the average birth weight is 7 lbs
weights under 4.5 lbs are uncommon
weights over 9.5lbs are uncommon
-∞
4.5
7
9.5
∞
average birth weight in the united states

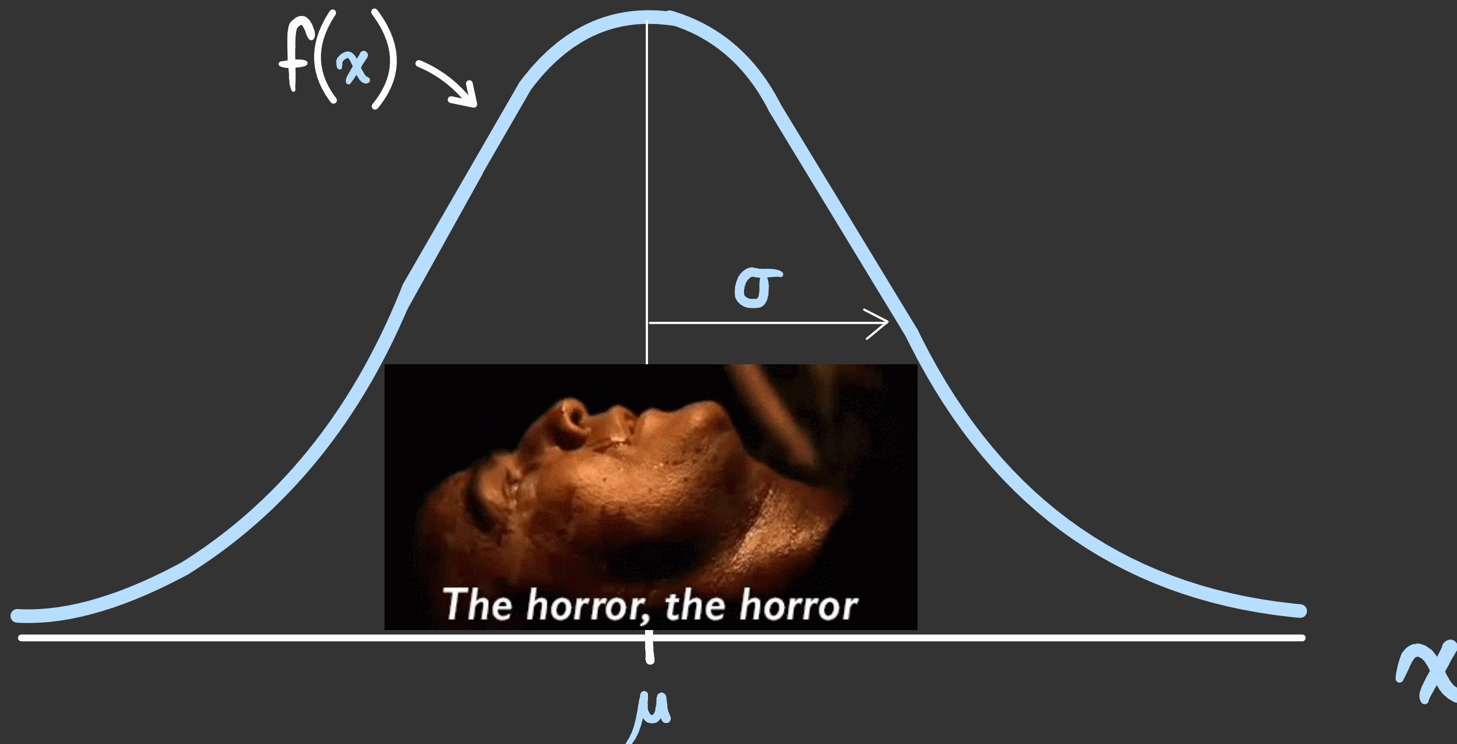


average birth weight in the united states


f(x)
but what is
f(x)?
μ
x



$$\boxed{f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}}$$



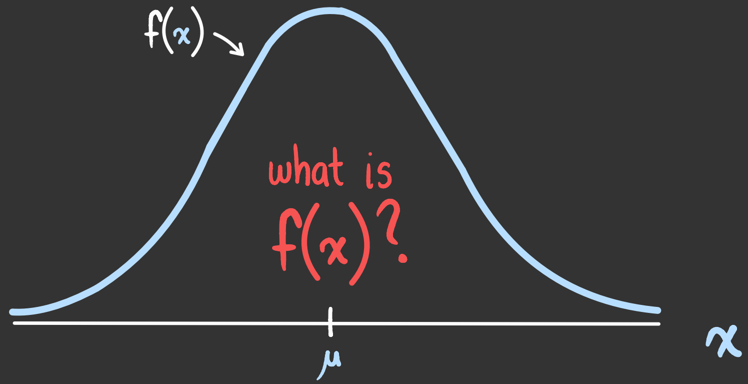
$$\boxed{f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}}$$

if god
did not exist,
it would be necessary
to invent him

– voltaire

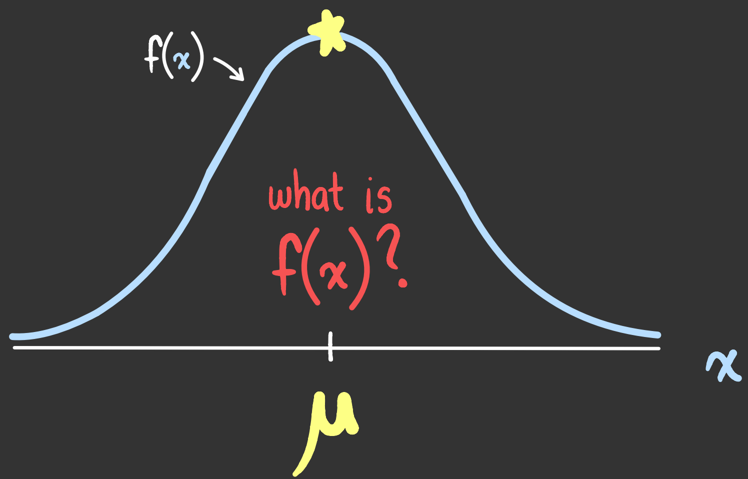
if the normal distribution
did not exist,
it would be necessary
to invent it

– not voltaire



we want a function $f(x)$ with the following properties:

- f has a unique maximum μ
- f decreases as we get further from μ
- $f(x) > 0$ for all $x \in \mathbb{R}$
- $\int_{-\infty}^{\infty} f(x)\, dx = 1$



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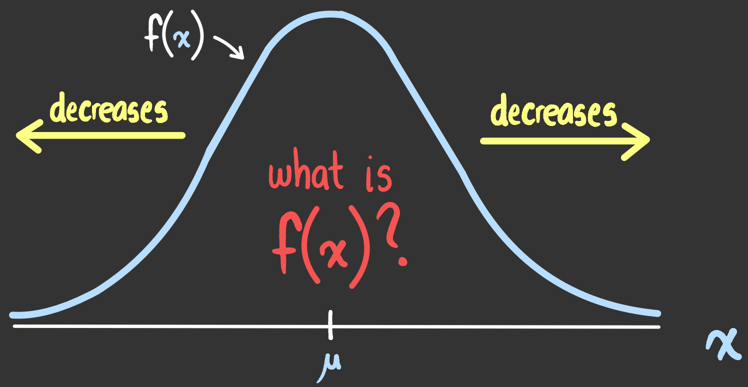

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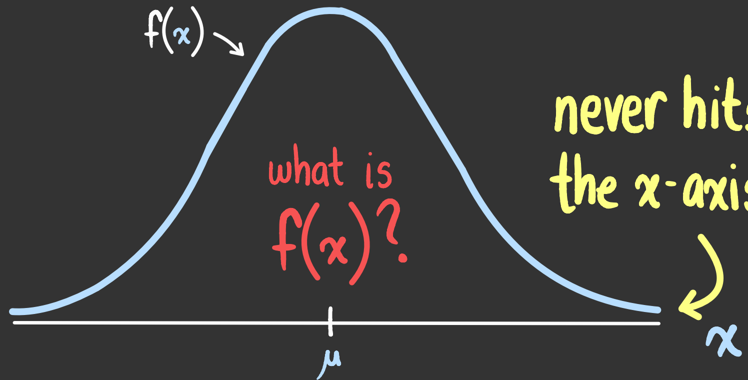

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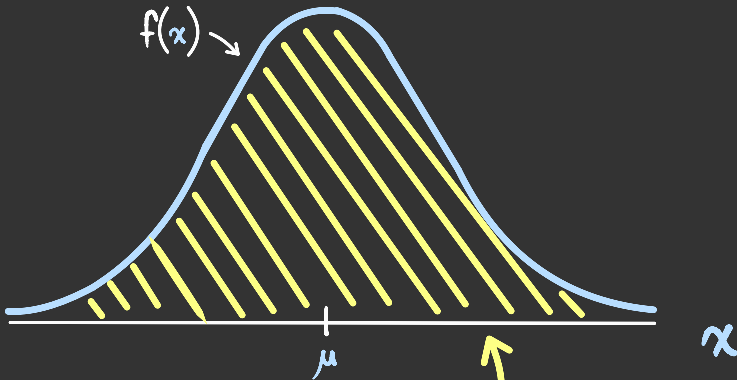

why is this important?

we want a function $f(x)$ with the following properties:

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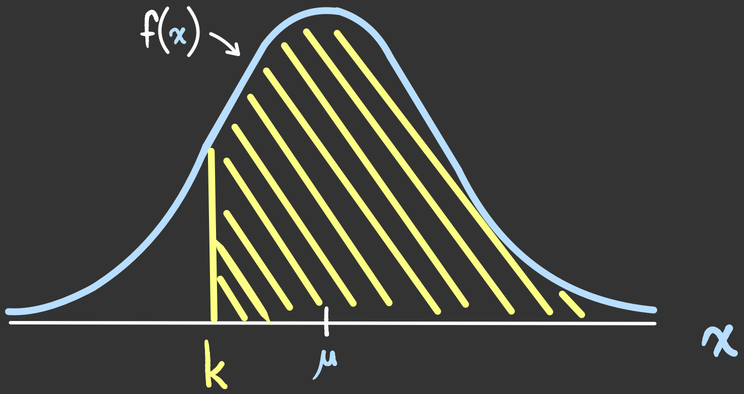
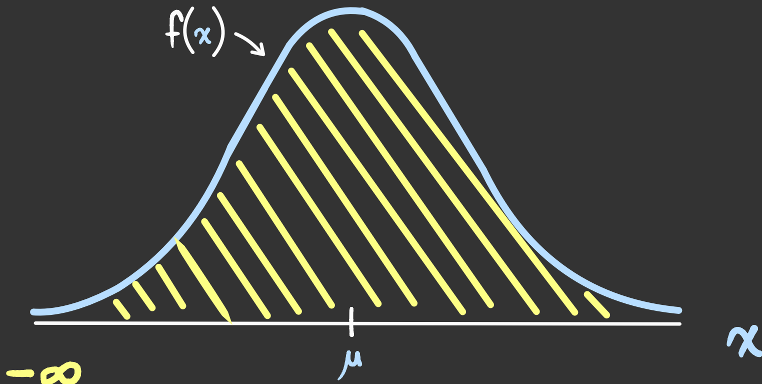


$$P(X > k) = \int_k^\infty f(x)\,dx$$

a probability must be at most one

we want a function $f(x)$ with the following properties:

- f has a unique maximum μ
- f decreases as we get further from μ
- $f(x) > 0$ for all $x \in \mathbb{R}$
- $\int_{-\infty}^{\infty} f(x)\,dx = 1$



$$P(X > -\infty) = \int_{-\infty}^{\infty} f(x)\,dx = 1$$

the probability over all values of x must be exactly one

we want a function $f(x)$ with the following properties:

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- $f(x) > 0$ for all $x \in \mathbb{R}$
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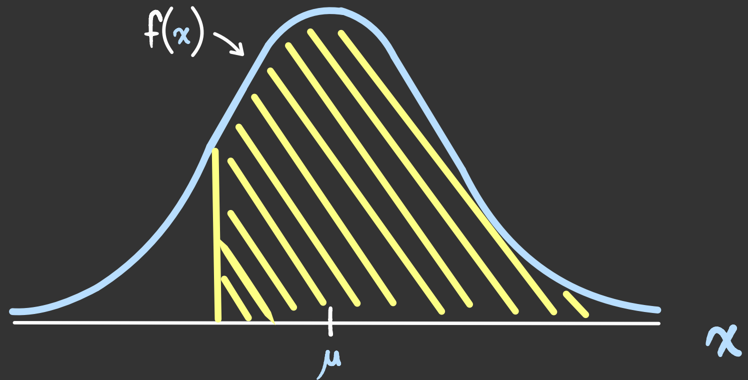


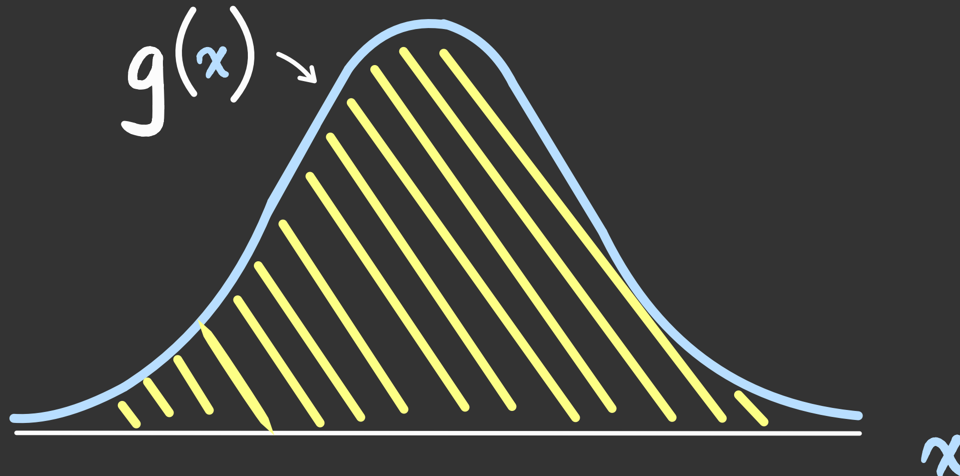
we want a function $f(x)$ with the following properties:

- f has a unique maximum μ
- f decreases as we get further from μ
- $f(x) > 0$ for all $x \in \mathbb{R}$
- $\int_{-\infty}^{\infty} f(x)\,dx = 1$

let's consider this property first

how do we create a function f such that:

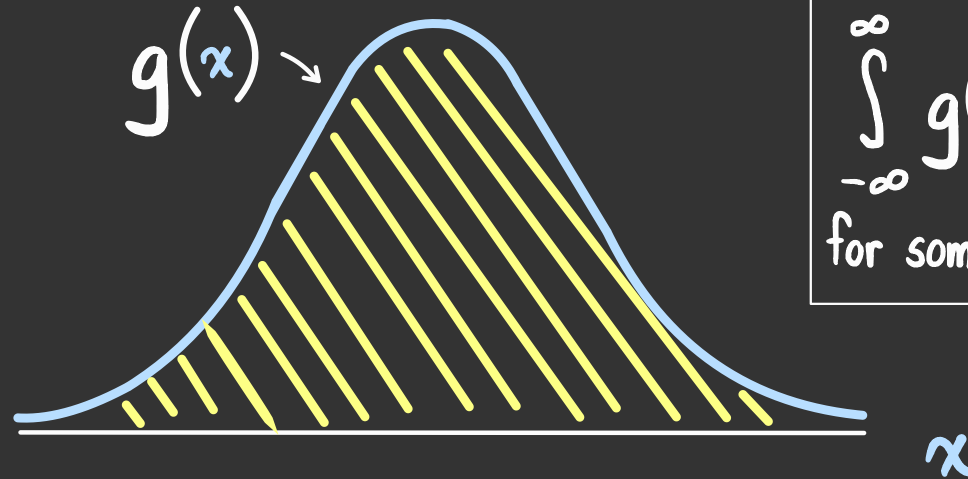
$$\int_{-\infty}^{\infty} f(x)\,dx = 1 \quad ?$$



suppose we have a positive function $g(x)$ such that

$$\int_{-\infty}^{\infty} g(x)\,dx = Z$$

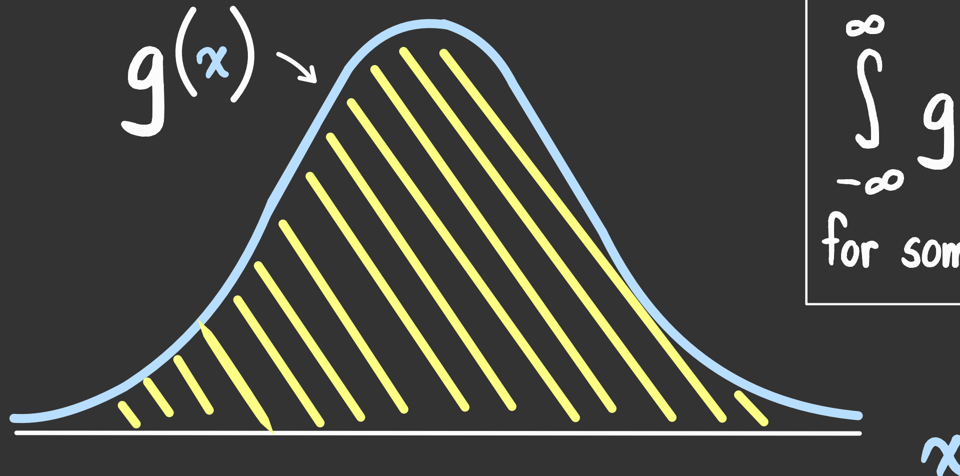
for some constant Z



$$\int_{-\infty}^{\infty} g(x)\,dx = Z$$

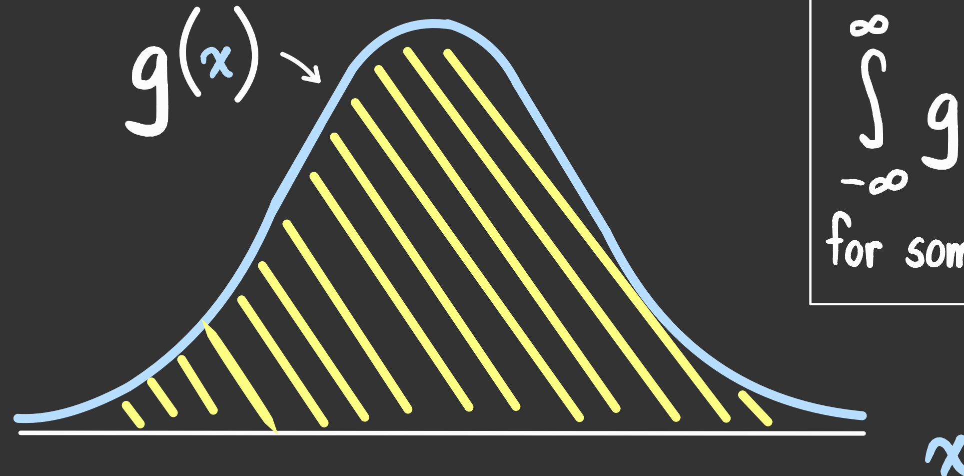
for some constant Z

$$\text{if } \int_{-\infty}^{\infty} g(x)\,dx = Z, \quad \text{then } \frac{1}{Z}\int_{-\infty}^{\infty} g(x)\,dx = 1$$



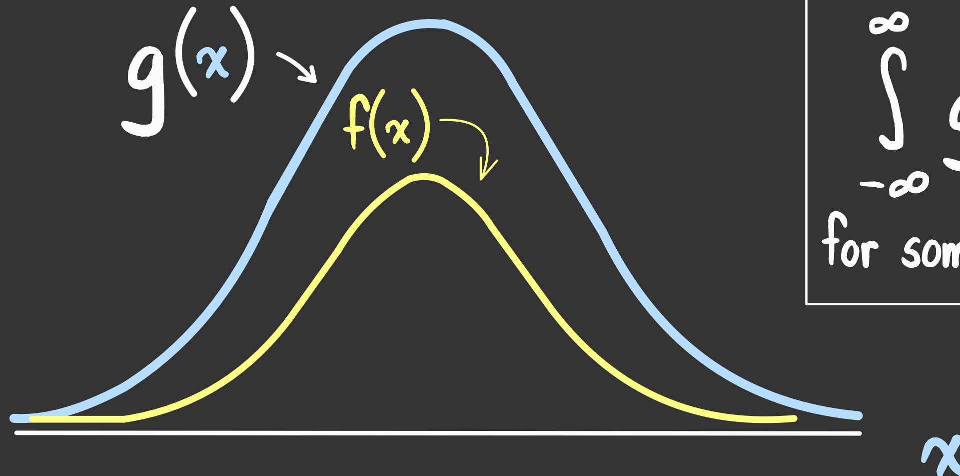
$$\boxed{\begin{array}{l}\displaystyle\int_{-\infty}^{\infty} g(x)\,dx = Z \\ \text{for some constant } Z\end{array}}$$

$$\text{if } \int_{-\infty}^{\infty} g(x)\,dx = Z, \quad \text{then } \int_{-\infty}^{\infty} \frac{g(x)}{Z}\,dx = 1$$



$$\boxed{\begin{array}{l}\displaystyle\int_{-\infty}^{\infty} g(x)\,dx = Z \\ \text{for some constant } Z\end{array}}$$

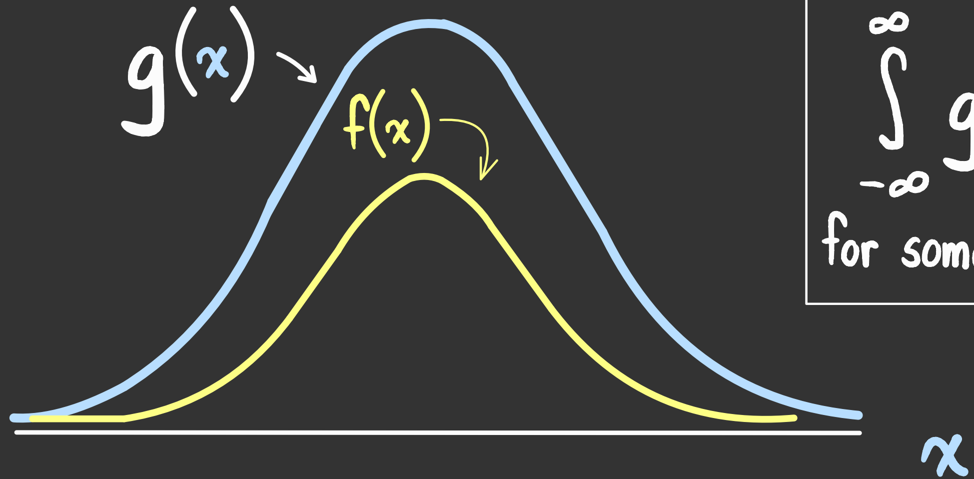
if $\displaystyle\int_{-\infty}^{\infty} g(x)\,dx = Z$, then $\displaystyle\int_{-\infty}^{\infty} \underbrace{\frac{g(x)}{Z}}_{f(x)}\,dx = 1$



$$\int_{-\infty}^{\infty} g(x) = Z$$

for some constant Z

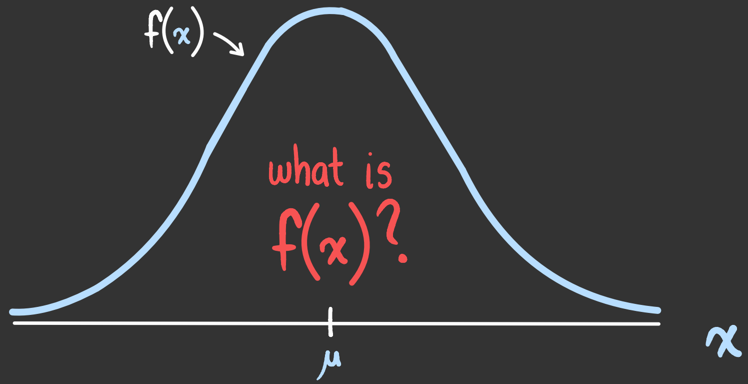
let $f(x) = \dfrac{g(x)}{\int_{-\infty}^{\infty} g(x)}$. then $\int_{-\infty}^{\infty} f(x)\,dx = 1$



$$\boxed{\int_{-\infty}^{\infty} g(x) = Z \quad \text{for some constant } Z}$$

only works if this holds

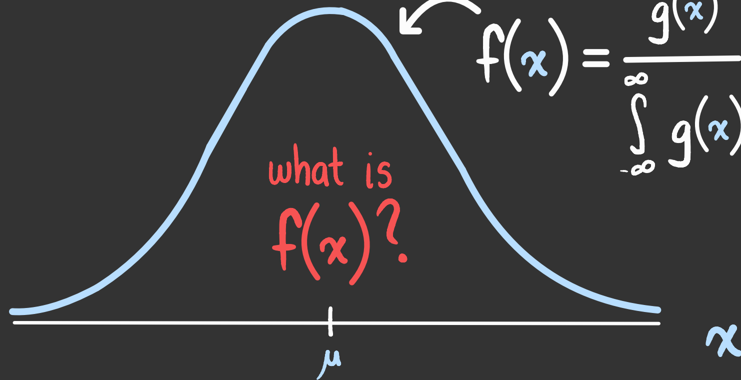
let $f(x) = \dfrac{g(x)}{\int_{-\infty}^{\infty} g(x)}$. then $\int_{-\infty}^{\infty} f(x)\,dx = 1$



so let's revisit
our desired
properties

we want a function $f(x)$
with the following properties:

- f has a unique maximum μ
- f decreases as we get further from μ
- $f(x) > 0$ for all $x \in \mathbb{R}$
- $\int_{-\infty}^{\infty} f(x)\,dx = 1$

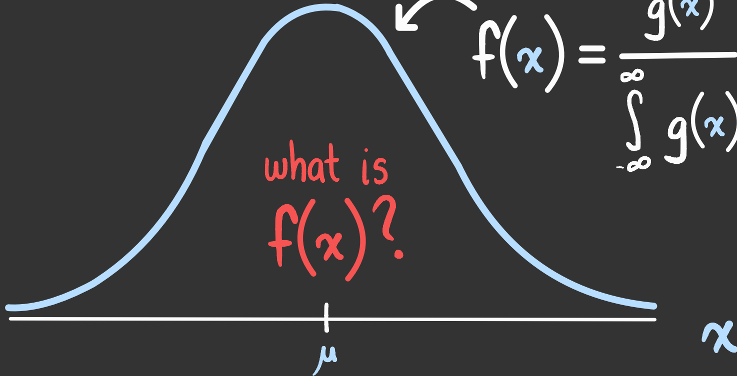


$$f(x) = \frac{g(x)}{\int_{-\infty}^{\infty} g(x)}$$

so let's revisit our desired properties

we want a function $\cancel{f(x)}$ $g(x)$ with the following properties:

- $\cancel{f}$ g has a unique maximum μ
- $\cancel{f}$ g decreases as we get further from μ
- $\cancel{f}g(x) > 0$ for all $x \in \mathbb{R}$
- $\int_{-\infty}^{\infty} \cancel{f}g(x)\,dx = \cancel{Z}\,Z$ for some constant Z



$$f(x) = \frac{g(x)}{\int_{-\infty}^{\infty} g(x)}$$

so let's revisit our desired properties

we want a function $g(x)$ with the following properties:

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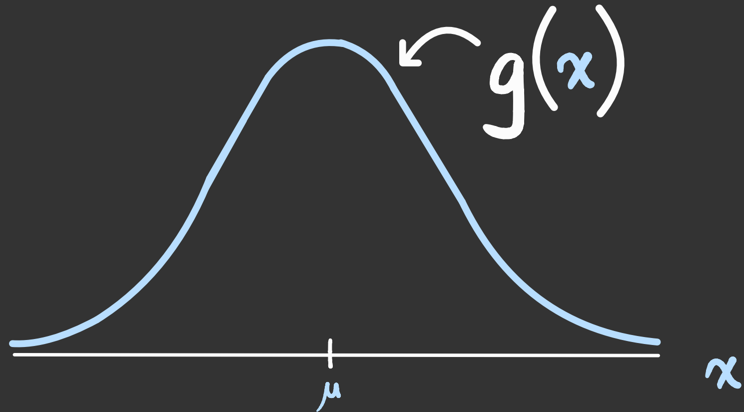
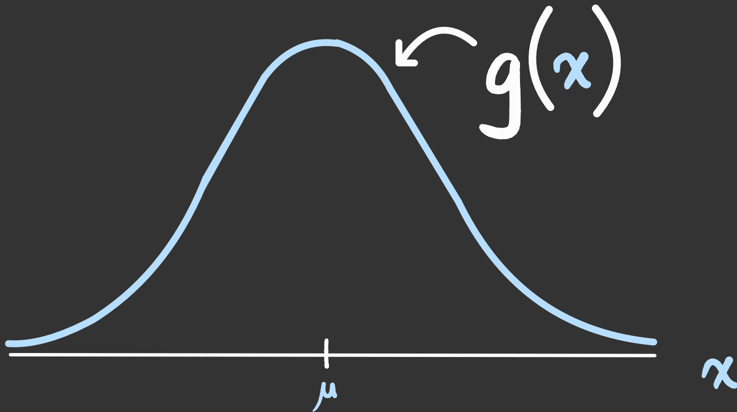
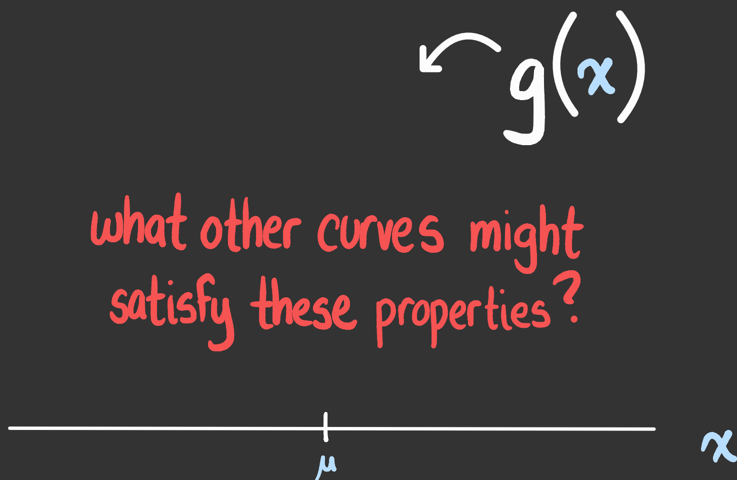


now let's consider the first three properties

we want a function $g(x)$ with the following properties:

- g has a unique maximum μ
- g decreases as we get further from μ
- $g(x) > 0$ for all $x \in \mathbb{R}$
- $\int_{-\infty}^{\infty} g(x)\,dx = Z$ for some constant Z





what other curves might satisfy these properties?

we want a function $g(x)$ with the following properties:

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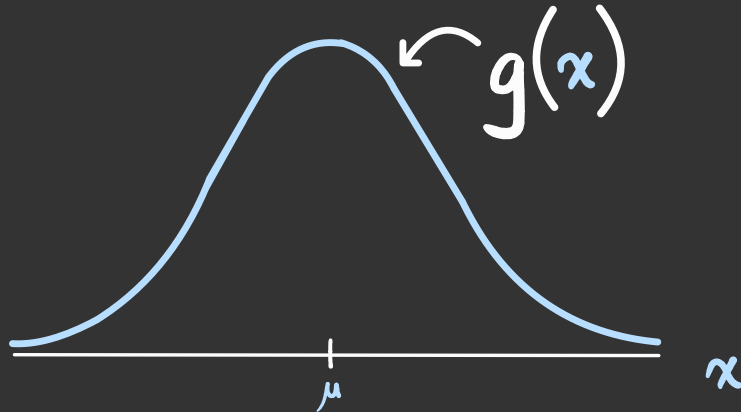
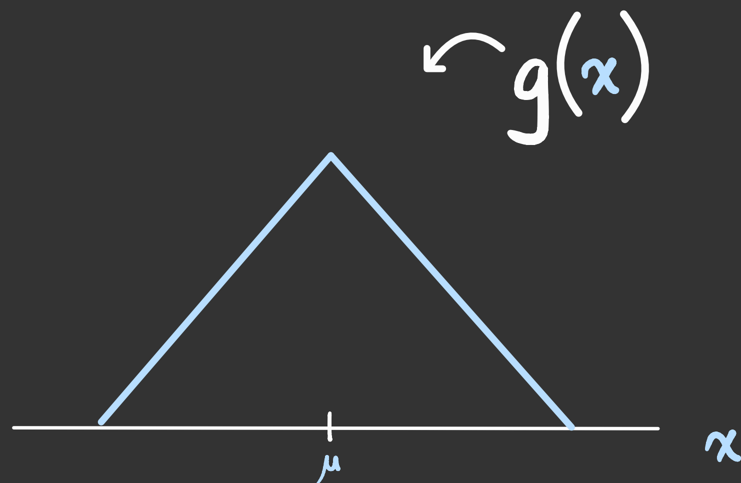
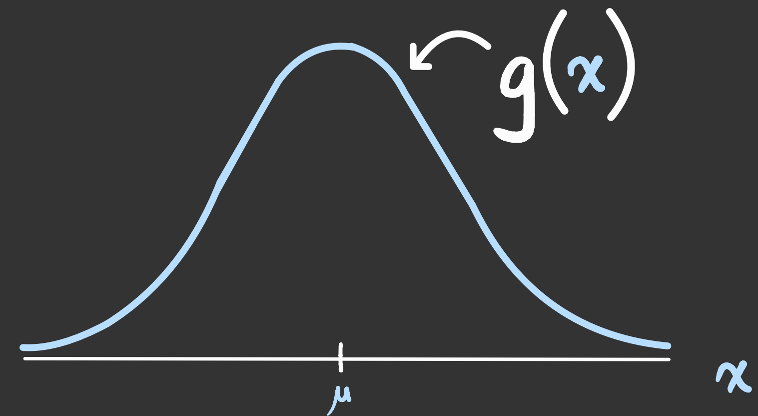
- $\int_{-\infty}^{\infty} g(x)\,dx = Z$ for some constant Z

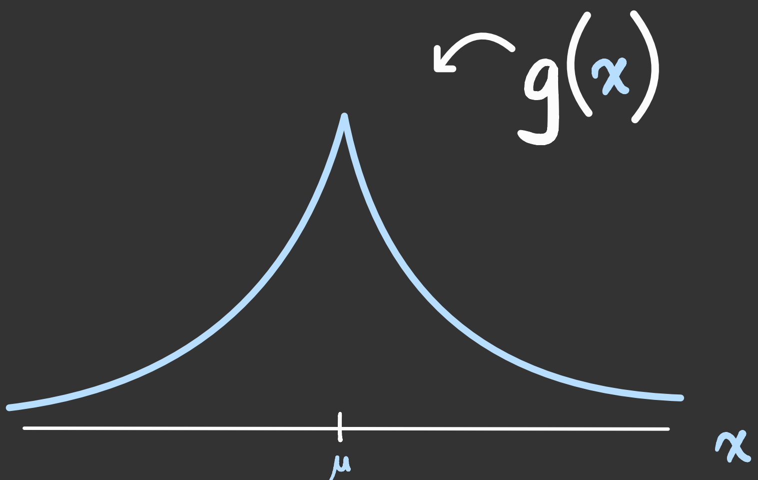




we want a function $g(x)$ with the following properties:

- g has a unique maximum μ
- g decreases as we get further from μ
- $g(x) \overset{\geq}{\cancel{>}} 0$ for all $x \in \mathbb{R}$
- $\int_{-\infty}^{\infty} g(x)\,dx = Z$ for some constant Z





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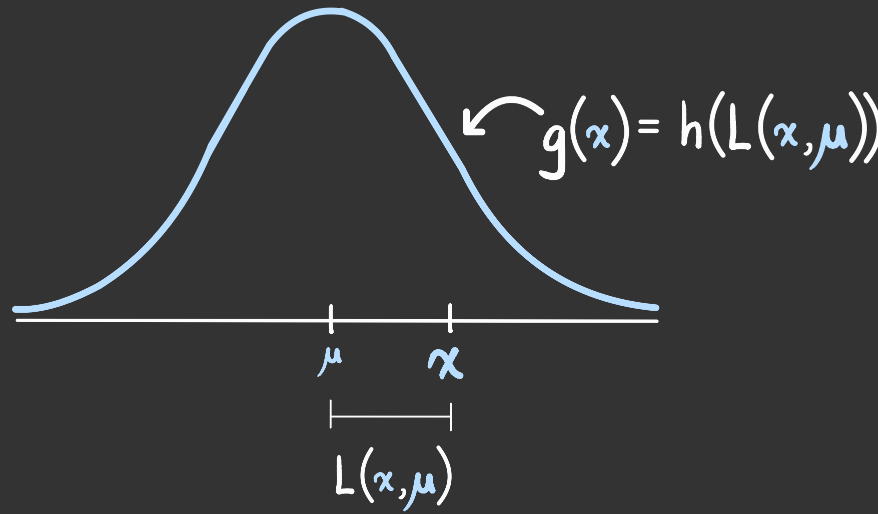


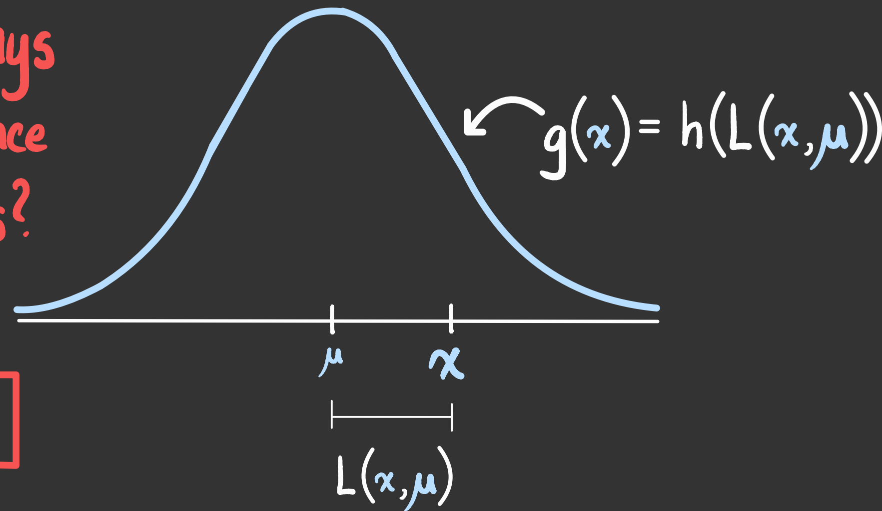
we want a function $g(x)$ with the following properties:

- g has a unique maximum μ
- g decreases as we get further from μ

one way to satisfy these conditions:

define $g(x)$ as a function h of the distance $L(x,\mu)$ between x and μ

what are some ways to measure the distance between two numbers?



$L(x,\mu) = \boxed{?}$

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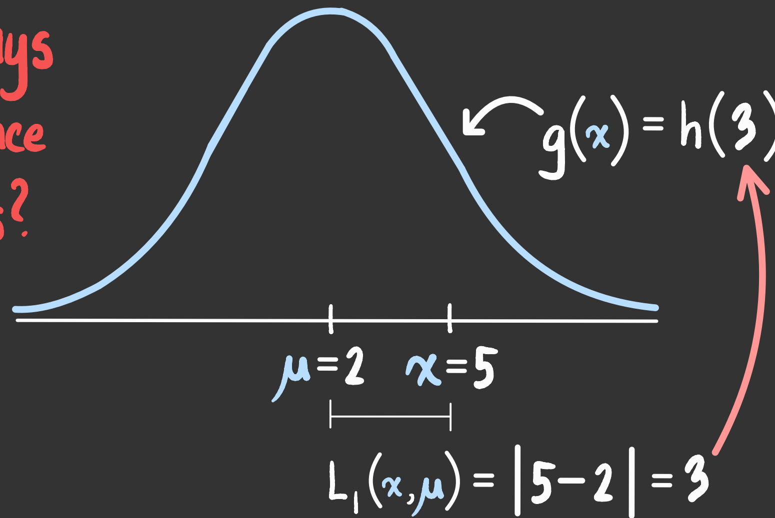

$$L_1(x,\mu) = |x-\mu|$$

we want a function $g(x)$ with the following properties:

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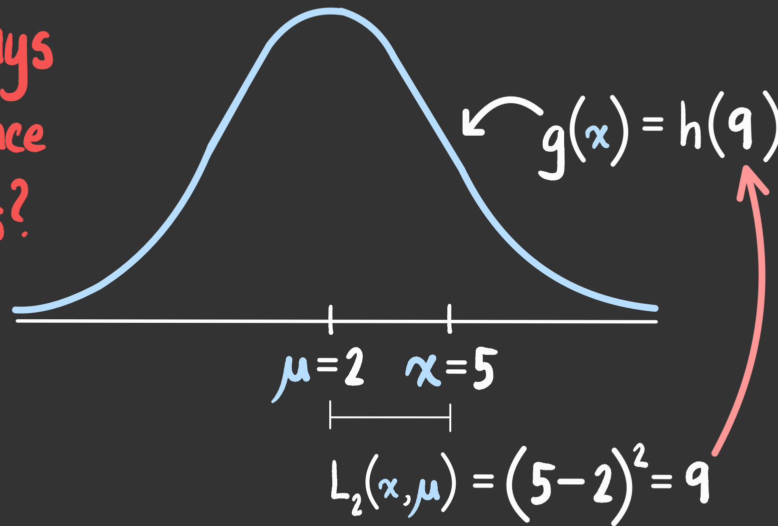

$$L_2(x,\mu) = (x-\mu)^2$$

we want a function $g(x)$ with the following properties:

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one way to satisfy these conditions:

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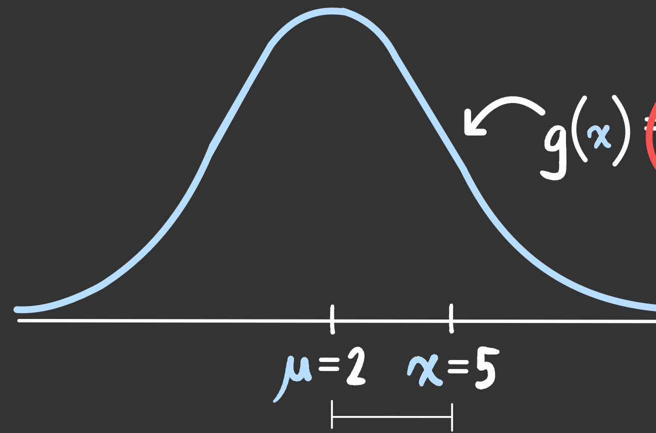
$$L_1(x,\mu) = |x-\mu|$$



$$L_1(x,\mu) = |5-2| = 3$$

$$g(x) = h(3)$$

what about this?

we want a function $g(x)$ with the following properties:

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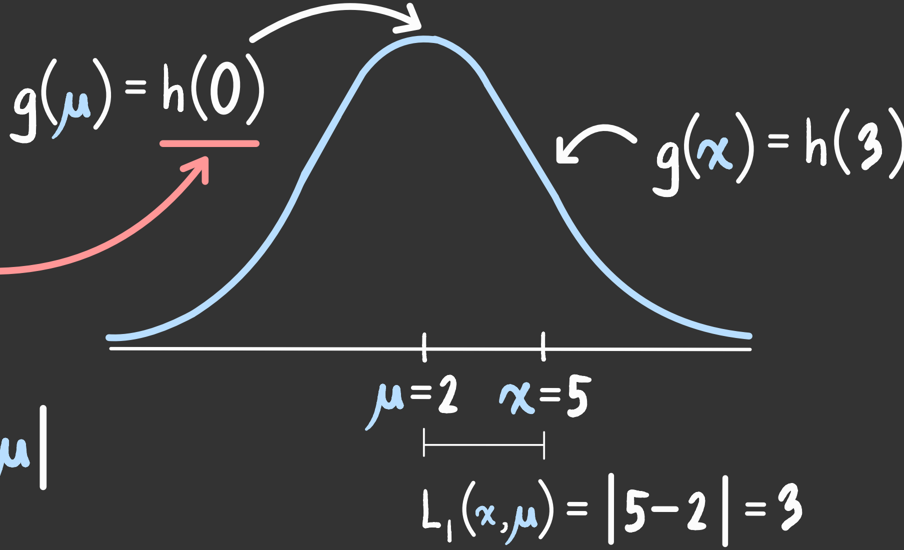
define $g(x)$ as a function h of the distance $L(x,\mu)$ between x and μ

want $h(0)$ to be finite



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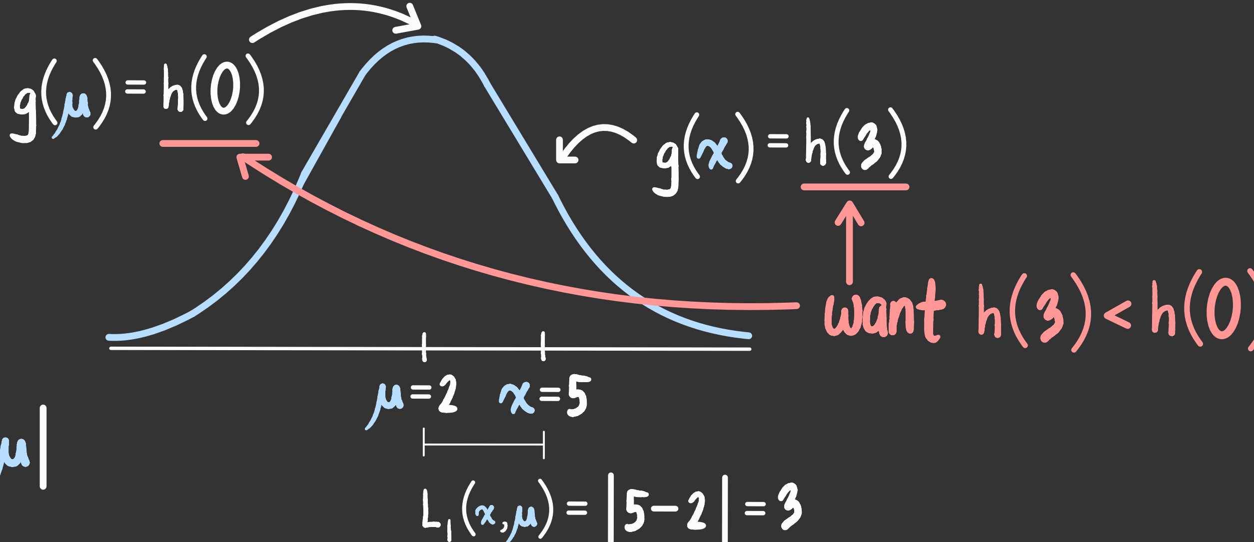

$$L_1(x,\mu) = |x - \mu|$$

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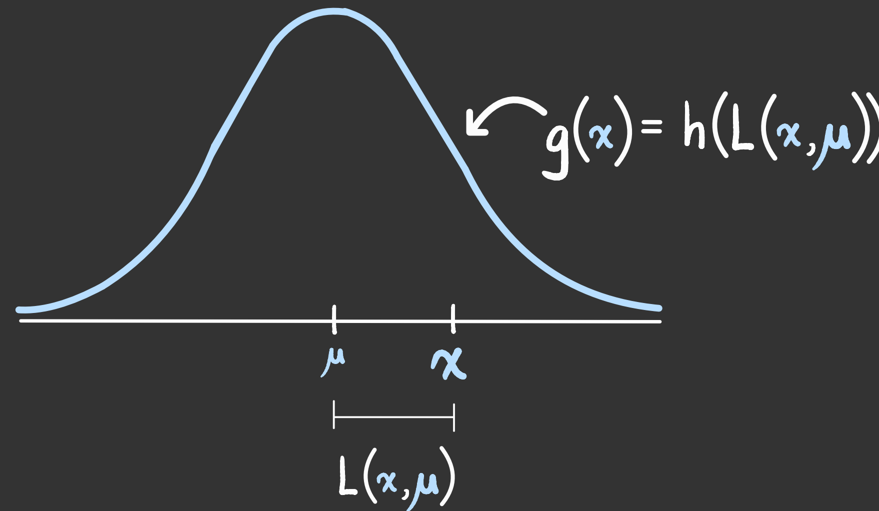
> - g decreases as we get further from μ

one way to satisfy these conditions:

define $g(x)$ as a function h of the distance $L(x,\mu)$ between x and μ

so we want to define h as a positive function such that:

- its domain is $(0, \infty)$
- $h(0)$ is finite
- as ℓ increases, $h(\ell)$ decreases



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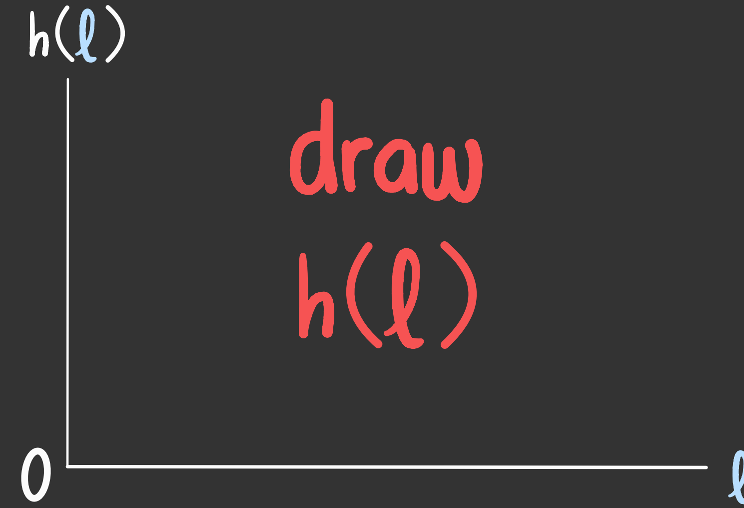

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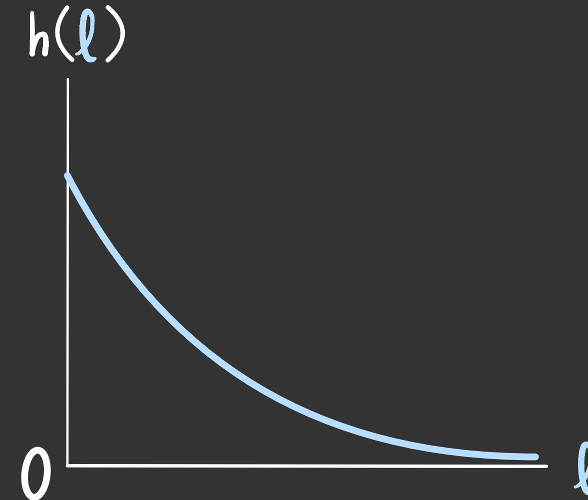

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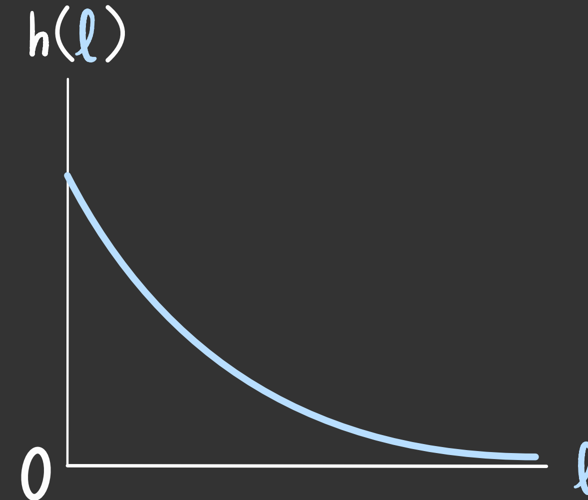
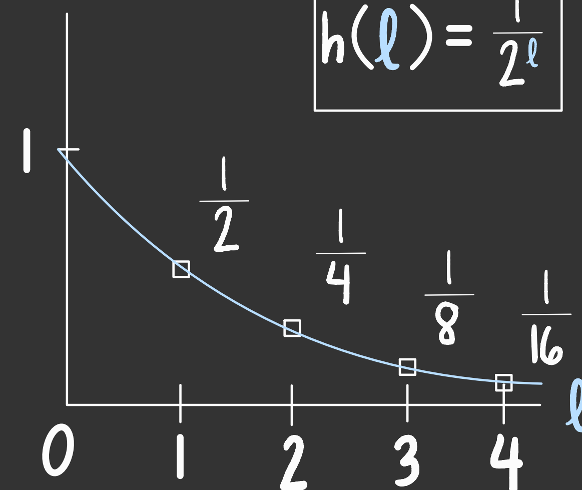


$$h(\ell) = \frac{1}{2^{\ell}}$$



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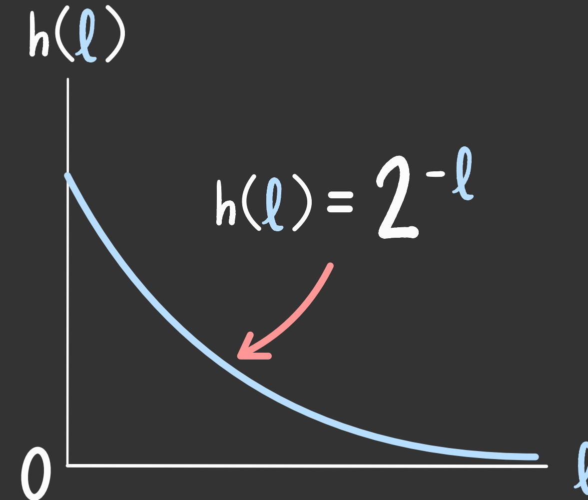

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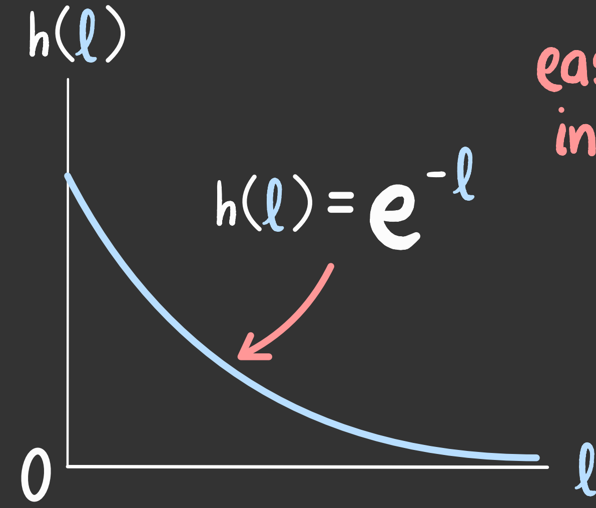

easier to integrate!

$$\int e^{-\ell} \, d\ell = \boxed{?}$$

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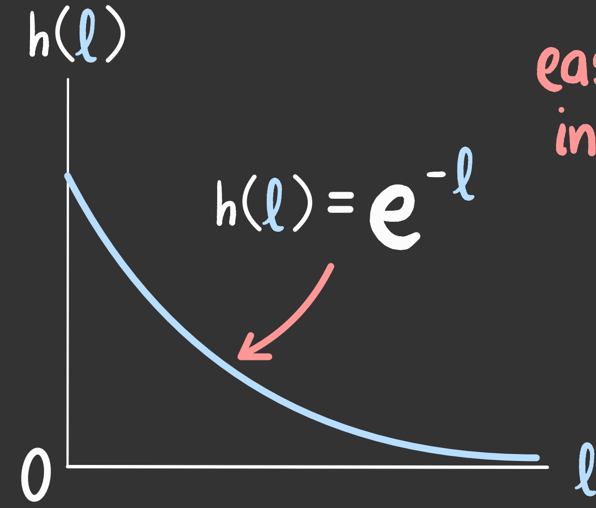
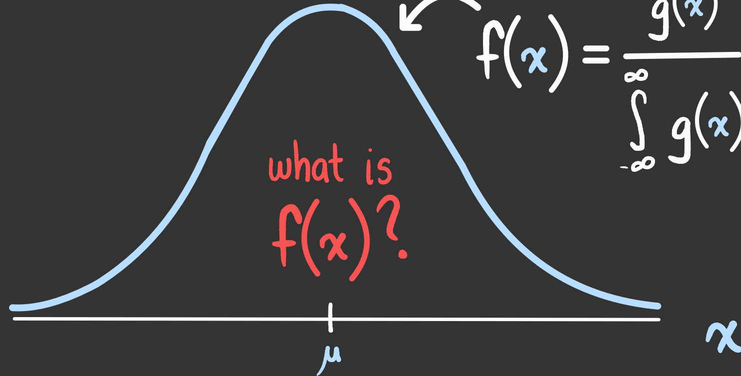
- $h(0)$ is finite
- as ℓ increases, $h(\ell)$ decreases



easier to integrate!

$$\int e^{-\ell}\, d\ell = -e^{-\ell}$$

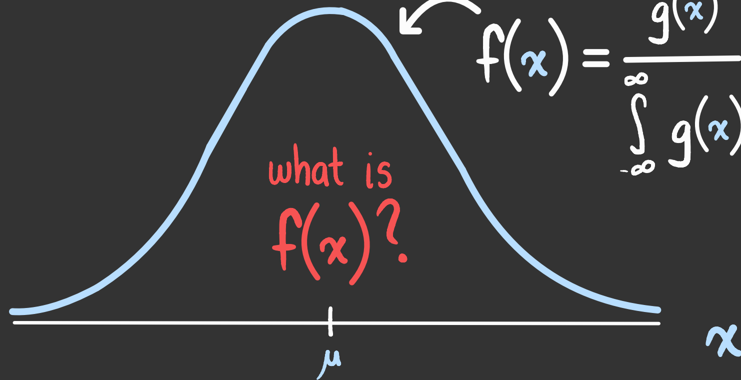
$$f(x) = \frac{g(x)}{\int_{-\infty}^{\infty} g(x)\,dx}$$



we want a function $g(x)$ with the following properties:

- g has a unique maximum μ
- g decreases as we get further from μ
- $g(x) > 0$ for all $x \in \mathbb{R}$
- $\int_{-\infty}^{\infty} g(x)\,dx = Z$ for some constant Z

what have we learned?



$$f(x) = \frac{g(x)}{\int_{-\infty}^{\infty} g(x)\,dx}$$

we want a function $g(x)$ with the following properties:

- ~~g has a unique maximum μ~~
- ~~g decreases as we get further from μ~~
- ~~$g(x) > 0$ for all $x \in \mathbb{R}$~~
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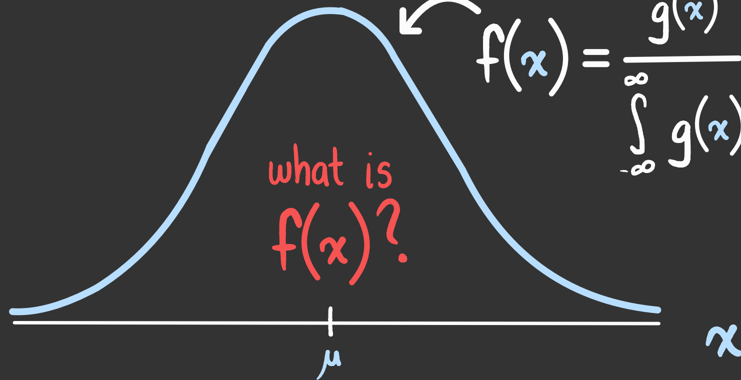

$$f(x) = \frac{g(x)}{\int_{-\infty}^{\infty} g(x)\,dx}$$

what have we learned?

we want a function $g(x)$ with the following properties:

- $g(x) = h(L(x, \mu))$ where
  - $L(x, \mu)$ is the distance between x and μ
  - $h(\ell) = e^{-\ell}$
- $\int_{-\infty}^{\infty} g(x)\,dx = Z$ for some constant Z

the only decision we haven't fully settled on is the distance function $L(x,\mu)$

we want a function $g(x)$ with the following properties:

- $g(x) = h(L(x,\mu))$ where
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- $\int_{-\infty}^{\infty} g(x)\,dx = Z$ for some constant Z

distance functions

option one

$$L_1(x,\mu) = |x-\mu|$$

$\mu=2$ $\quad$ $x=5$

$$L_1(x,\mu) = |5-2| = 3$$

option two

$$L_2(x,\mu) = (x-\mu)^2$$

$\mu=2$ $\quad$ $x=5$

$$L_2(x,\mu) = (5-2)^2 = 9$$

so far we've seen two options

distance functions

option one

$$L_1(x,\mu) = |x-\mu|$$

$\mu=2$ $\quad x=5$

$$L_1(x,\mu) = |5-2| = 3$$

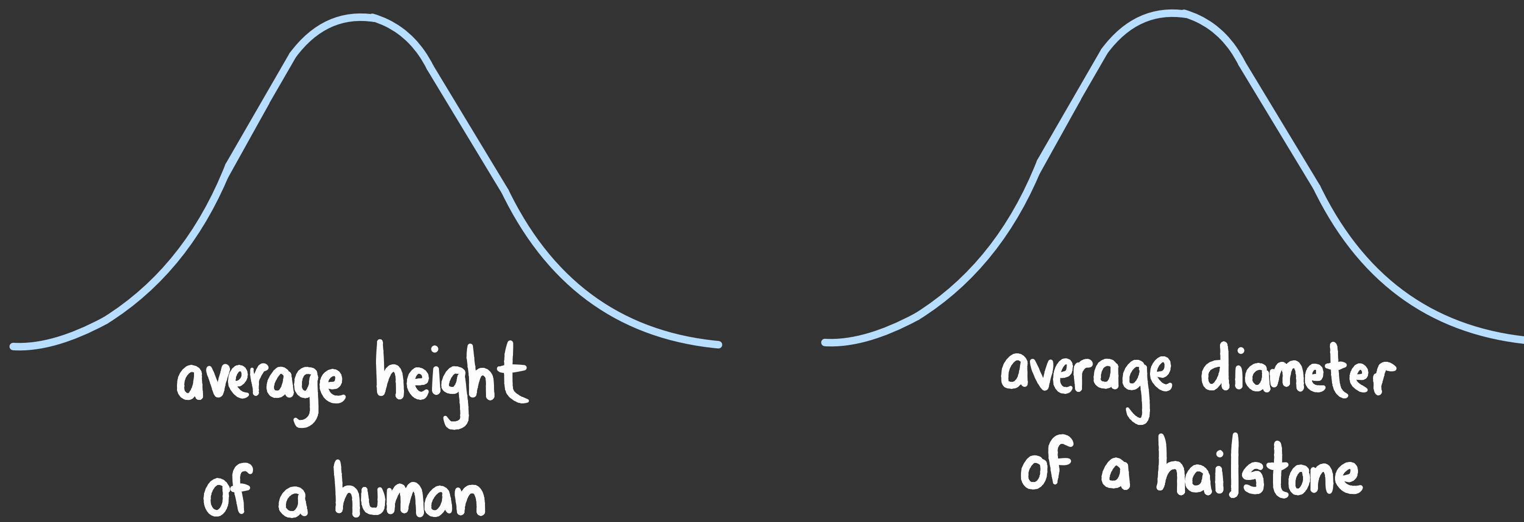
option two

$$L_2(x,\mu) = (x-\mu)^2$$

$\mu=2$ $\quad x=5$

$$L_2(x,\mu) = (5-2)^2 = 9$$

neither option addresses the matter of **scale**

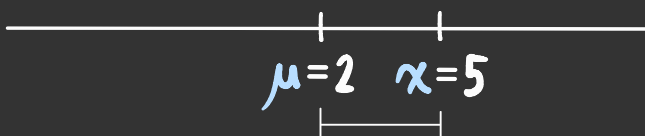


same pattern, different scales

distance functions

option one

$$L_1(x,\mu) = |x-\mu|$$



$$L_1(x,\mu) = |5-2| = 3$$

3 km?
3 cm?
3 km?

neither option addresses the matter of **scale**

distance functions

option one

$$L_1(x,\mu) = \frac{|x-\mu|}{b}$$

$\mu = 2$ $x = 5$

$$L_1(x,\mu) = \frac{|5-2|}{.001} = 3000$$

3 km?
3 cm?
3 km?

to specify **scale**, we will introduce a **scaling factor**

distance functions

option one

$$L_1(x,\mu) = \frac{|x-\mu|}{b}$$

$\mu = 2$ $\quad x = 5$

$$L_1(x,\mu) = \frac{|5-2|}{100} = .03$$

3 km?

3 cm?

3 mm?

to specify **scale**, we will introduce a **scaling factor**

distance functions

option one

$$L_1(x,\mu) = \frac{|x-\mu|}{b}$$

$\mu = 2$ $\quad$ $x = 5$

$$L_1(x,\mu) = \frac{|5-2|}{1000} = .003$$

3 km?

3 cm?

3 mm?

to specify **scale**, we will introduce a **scaling factor**

distance functions

option one

$$L_1(x,\mu) = \frac{|x-\mu|}{b}$$

option two

$$L_2(x,\mu) = \frac{(x-\mu)^2}{b}$$

scaling factor

to specify **scale**, we will introduce a **scaling factor**

distance functions

option one

$$L_1(x,\mu) = \frac{|x-\mu|}{b}$$

option two

$$L_2(x,\mu) = \frac{(x-\mu)^2}{b}$$

let's try option one

if we use distance $L_1(x,\mu) = \dfrac{|x-\mu|}{b}$,

what do we get for $f(x)$?

first, compute $g(x)$:

$$g(x) = h(L(x,\mu))$$
$$= h\left(\frac{|x-\mu|}{b}\right)$$
$$= e^{-\frac{|x-\mu|}{b}}$$

- $g(x) = h(L(x,\mu))$ where
  - $L(x,\mu)$ is the distance between x and μ
  - $h(\ell) = e^{-\ell}$
- $\int_{-\infty}^{\infty} g(x)\,dx = Z$ for some constant Z
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\end{aligned}
$$

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\end{aligned}
$$

to compute $f(x)$, we need $\int_{-\infty}^{\infty} g(x)\,dx$

- $g(x) = h(L(x,\mu))$ where
  - $L(x,\mu)$ is the distance between x and μ
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$$= h\left(\frac{|x-\mu|}{b}\right)$$
$$= e^{-\frac{|x-\mu|}{b}}$$

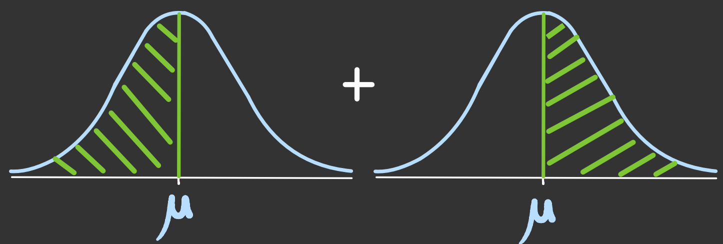
next, compute $\int_{-\infty}^{\infty} g(x)\,dx$:

$$\int_{-\infty}^{\infty} g(x)\,dx = \int_{-\infty}^{\infty} e^{-\frac{|x-\mu|}{b}}\,dx$$
$$= ?$$

$$\int_{-\infty}^{\infty} g(x)\,dx = \int_{-\infty}^{\infty} e^{-\frac{|x-\mu|}{b}}\,dx$$

$$\int_{-\infty}^{\infty} g(x)\,dx = \int_{-\infty}^{\infty} e^{-\frac{|x-\mu|}{b}}\,dx$$

$$= \int_{-\infty}^{\mu} e^{-\frac{|x-\mu|}{b}}\,dx + \int_{\mu}^{\infty} e^{-\frac{|x-\mu|}{b}}\,dx$$



$x < \mu$

so $|x-\mu| = ?$

$\mu - x$

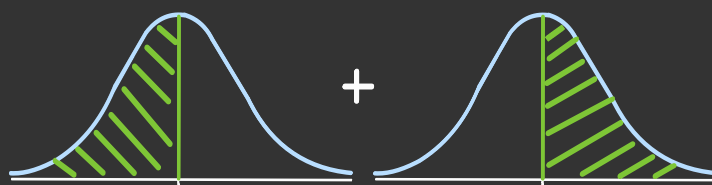
$x > \mu$

so $|x-\mu| = ?$

$x - \mu$

$$\int_{-\infty}^{\infty} g(x)\,dx = \int_{-\infty}^{\infty} e^{-\frac{|x-\mu|}{b}}\,dx$$

$$= \int_{-\infty}^{\mu} e^{-\frac{\mu-x}{b}}\,dx + \int_{\mu}^{\infty} e^{-\frac{x-\mu}{b}}\,dx$$



$x < \mu$

so $|x-\mu| = \mu - x$

$x > \mu$

so $|x-\mu| = x - \mu$

$$\int_{-\infty}^{\infty} g(x)\,dx = \int_{-\infty}^{\infty} e^{-\frac{|x-\mu|}{b}}\,dx$$

$$= \int_{-\infty}^{\mu} e^{\frac{x-\mu}{b}}\,dx + \int_{\mu}^{\infty} e^{\frac{\mu-x}{b}}\,dx$$

$$\int_{-\infty}^{\infty} g(x)\,dx = \int_{-\infty}^{\infty} e^{-\frac{|x-\mu|}{b}}\,dx$$

$$= \int_{-\infty}^{\mu} e^{\frac{x-\mu}{b}}\,dx + \int_{\mu}^{\infty} e^{\frac{\mu-x}{b}}\,dx$$

$$= \lim_{k\to-\infty} \int_{k}^{\mu} e^{\frac{x-\mu}{b}}\,dx + \lim_{k\to\infty} \int_{\mu}^{k} e^{\frac{\mu-x}{b}}\,dx$$

$$\int_{-\infty}^{\infty} g(x)\,dx = \int_{-\infty}^{\infty} e^{-\frac{|x-\mu|}{b}}\,dx$$

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$$= \lim_{k\to-\infty} \int_{k}^{\mu} e^{\frac{x-\mu}{b}}\,dx + \lim_{k\to\infty} \int_{\mu}^{k} e^{\frac{\mu-x}{b}}\,dx$$

$$= \lim_{k\to-\infty} \left[be^{\frac{x-\mu}{b}}\right]_{k}^{\mu} + \lim_{k\to\infty} \left[-be^{\frac{\mu-x}{b}}\right]_{\mu}^{k}$$

$$
\begin{aligned}
\int_{-\infty}^{\infty} g(x)\,dx &= \int_{-\infty}^{\infty} e^{-\frac{|x-\mu|}{b}}\,dx \\
&= \int_{-\infty}^{\mu} e^{\frac{x-\mu}{b}}\,dx + \int_{\mu}^{\infty} e^{\frac{\mu-x}{b}}\,dx \\
&= \lim_{k\to-\infty} \int_{k}^{\mu} e^{\frac{x-\mu}{b}}\,dx + \lim_{k\to\infty} \int_{\mu}^{k} e^{\frac{\mu-x}{b}}\,dx \\
&= \lim_{k\to-\infty} \left[be^{\frac{x-\mu}{b}} \right]_{k}^{\mu} + \lim_{k\to\infty} \left[-be^{\frac{\mu-x}{b}} \right]_{\mu}^{k} \\
&= \lim_{k\to-\infty} \left(be^{\frac{\mu-\mu}{b}} - be^{\frac{k-\mu}{b}} \right) + \lim_{k\to\infty} \left(-be^{\frac{\mu-k}{b}} + be^{\frac{\mu-\mu}{b}} \right)
\end{aligned}
$$

$$\int_{-\infty}^{\infty} g(x)\,dx = \int_{-\infty}^{\infty} e^{-\frac{|x-\mu|}{b}}\,dx$$

$$= \int_{-\infty}^{\mu} e^{\frac{x-\mu}{b}}\,dx + \int_{\mu}^{\infty} e^{\frac{\mu-x}{b}}\,dx$$

$$= \lim_{k\to-\infty} \int_{k}^{\mu} e^{\frac{x-\mu}{b}}\,dx + \lim_{k\to\infty} \int_{\mu}^{k} e^{\frac{\mu-x}{b}}\,dx$$

$$= \lim_{k\to-\infty} \left[b e^{\frac{x-\mu}{b}} \right]_{k}^{\mu} + \lim_{k\to\infty} \left[-b e^{\frac{\mu-x}{b}} \right]_{\mu}^{k}$$

$$= \lim_{k\to-\infty} \left(b e^{0} - b e^{\frac{k-\mu}{b}} \right) + \lim_{k\to\infty} \left(-b e^{\frac{\mu-k}{b}} + b e^{0} \right)$$

$$
\begin{aligned}
\int_{-\infty}^{\infty} g(x)\,dx &= \int_{-\infty}^{\infty} e^{-\frac{|x-\mu|}{b}}\,dx \\
&= \int_{-\infty}^{\mu} e^{\frac{x-\mu}{b}}\,dx + \int_{\mu}^{\infty} e^{\frac{\mu-x}{b}}\,dx \\
&= \lim_{k\to-\infty} \int_{k}^{\mu} e^{\frac{x-\mu}{b}}\,dx + \lim_{k\to\infty} \int_{\mu}^{k} e^{\frac{\mu-x}{b}}\,dx \\
&= \lim_{k\to-\infty} \left[b e^{\frac{x-\mu}{b}} \right]_{k}^{\mu} + \lim_{k\to\infty} \left[-b e^{\frac{\mu-x}{b}} \right]_{\mu}^{k} \\
&= \lim_{k\to-\infty} \left(b - b e^{\frac{k-\mu}{b}} \right) + \lim_{k\to\infty} \left(-b e^{\frac{\mu-k}{b}} + b \right)
\end{aligned}
$$

$$
\begin{aligned}
\int_{-\infty}^{\infty} g(x)\,dx &= \int_{-\infty}^{\infty} e^{-\frac{|x-\mu|}{b}}\,dx \\
&= \int_{-\infty}^{\mu} e^{\frac{x-\mu}{b}}\,dx + \int_{\mu}^{\infty} e^{\frac{\mu-x}{b}}\,dx \\
&= \lim_{k\to-\infty} \int_{k}^{\mu} e^{\frac{x-\mu}{b}}\,dx + \lim_{k\to\infty} \int_{\mu}^{k} e^{\frac{\mu-x}{b}}\,dx \\
&= \lim_{k\to-\infty} \left[be^{\frac{x-\mu}{b}} \right]_{k}^{\mu} + \lim_{k\to\infty} \left[-be^{\frac{\mu-x}{b}} \right]_{\mu}^{k} \\
&= \lim_{k\to-\infty} \left(b - be^{\frac{k-\mu}{b}} \right) + \lim_{k\to\infty} \left(-be^{\frac{\mu-k}{b}} + b \right) \\
&= \lim_{k\to-\infty} b - \lim_{k\to-\infty} be^{\frac{k-\mu}{b}} + \lim_{k\to\infty} b - \lim_{k\to\infty} be^{\frac{\mu-k}{b}}
\end{aligned}
$$

$$
\begin{aligned}
\int_{-\infty}^{\infty} g(x)\,dx &= \int_{-\infty}^{\infty} e^{-\frac{|x-\mu|}{b}}\,dx \\
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&= \lim_{k\to-\infty} \left[be^{\frac{x-\mu}{b}} \right]_{k}^{\mu} + \lim_{k\to\infty} \left[-be^{\frac{\mu-x}{b}} \right]_{\mu}^{k} \\
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&= \lim_{k\to-\infty} b - \lim_{k\to-\infty} be^{\frac{k-\mu}{b}} + \lim_{k\to\infty} b - \lim_{k\to\infty} be^{\frac{\mu-k}{b}} \\
&= b - 0 + b - 0
\end{aligned}
$$

$$
\begin{aligned}
\int_{-\infty}^{\infty} g(x)\,dx &= \int_{-\infty}^{\infty} e^{-\frac{|x-\mu|}{b}}\,dx \\
&= \int_{-\infty}^{\mu} e^{\frac{x-\mu}{b}}\,dx + \int_{\mu}^{\infty} e^{\frac{\mu-x}{b}}\,dx \\
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&= \lim_{k\to-\infty} \left(b - be^{\frac{k-\mu}{b}} \right) + \lim_{k\to\infty} \left(-be^{\frac{\mu-k}{b}} + b \right) \\
&= \lim_{k\to-\infty} b - \lim_{k\to-\infty} be^{\frac{k-\mu}{b}} + \lim_{k\to\infty} b - \lim_{k\to\infty} be^{\frac{\mu-k}{b}} \\
&= 2b
\end{aligned}
$$

if we use distance $L_1(x,\mu) = \dfrac{|x-\mu|}{b}$,

what do we get for $f(x)$?

first, compute $g(x)$:

$$g(x) = h(L(x,\mu))$$
$$= h\left(\frac{|x-\mu|}{b}\right)$$
$$= e^{-\frac{|x-\mu|}{b}}$$

next, compute $\int_{-\infty}^{\infty} g(x)\,dx$:

$$\int_{-\infty}^{\infty} g(x)\,dx = ?$$

if we use distance $L_1(x,\mu) = \dfrac{|x-\mu|}{b}$,

what do we get for $f(x)$?

first, compute $g(x)$:

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\begin{aligned}
g(x) &= h(L(x,\mu)) \\
&= h\left(\frac{|x-\mu|}{b}\right) \\
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\end{aligned}
$$

next, compute $\int_{-\infty}^{\infty} g(x)\,dx$:

$$\int_{-\infty}^{\infty} g(x)\,dx = 2b$$

if we use distance $L_1(x,\mu) = \frac{|x-\mu|}{b}$,

what do we get for $f(x)$?

first, compute $g(x)$:

$$g(x) = h(L(x,\mu)) = h\left(\frac{|x-\mu|}{b}\right) = e^{-\frac{|x-\mu|}{b}}$$

next, compute $\int_{-\infty}^{\infty} g(x)\,dx$:

$$\int_{-\infty}^{\infty} g(x)\,dx = 2b$$

finally, compute $f(x)$:

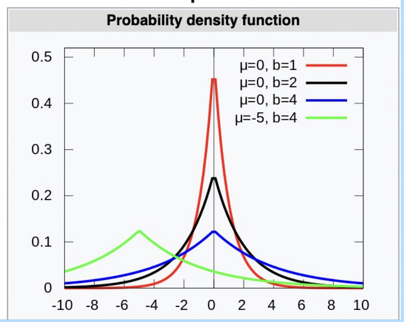
$$f(x) = \frac{g(x)}{\int_{-\infty}^{\infty} g(x)} = \frac{e^{-\frac{|x-\mu|}{b}}}{2b}$$

Laplace distribution

From Wikipedia, the free encyclopedia

In probability theory and statistics, the **Laplace distribution** is a continuous probability distribution named after Pierre-Simon Laplace. It is also sometimes called the **double exponential distribution**, because it can be thought of as two exponential distributions (with an additional location parameter) spliced together along the abscissa, although the term is also sometimes used to refer to the Gumbel distribution. The difference between

Laplace



this is called the laplace distribution

next, compute $\int_{-\infty}^{\infty} g(x)\,dx$:

$$\int_{-\infty}^{\infty} g(x)\,dx = 2b$$

finally, compute $f(x)$:

$$\boxed{f(x) = \frac{e^{-\frac{|x-\mu|}{b}}}{2b}}$$

distance functions

~~option one~~

$$\cancel{L_1(x,\mu) = \frac{|x-\mu|}{b}}$$

option two 👀

$$L_2(x,\mu) = \frac{(x-\mu)^2}{b}$$

if we use distance $L_2(x,\mu) = \frac{(x-\mu)^2}{b}$,

what do we get for $f(x)$?

first, compute $g(x)$:

$$g(x) = h(L(x,\mu))$$

$$= h\left(\frac{(x-\mu)^2}{b}\right)$$

$$= e^{-\frac{(x-\mu)^2}{b}}$$

- $g(x) = h(L(x,\mu))$ where
  - $L(x,\mu)$ is the distance between x and μ
  - $h(\ell) = e^{-\ell}$
- $\int_{-\infty}^{\infty} g(x) = Z$ for some constant Z
- $f(x) = \frac{g(x)}{\int_{-\infty}^{\infty} g(x)}$

if we use distance $L_2(x,\mu) = \frac{(x-\mu)^2}{b}$,

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to compute $f(x)$, we need $\int_{-\infty}^{\infty} g(x)$

- $g(x) = h(L(x,\mu))$ where
  - $L(x,\mu)$ is the distance between x and μ
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$$\int_{-\infty}^{\infty} g(x)\,dx = ?$$

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first, compute $g(x)$:

$$g(x) = h(L(x,\mu))$$
$$= h\left(\frac{(x-\mu)^2}{b}\right)$$
$$= e^{-\frac{(x-\mu)^2}{b}}$$

next, compute $\int_{-\infty}^{\infty} g(x)\,dx$:

$$\int_{-\infty}^{\infty} g(x)\,dx = \sqrt{b\pi}$$

we'll just look this one up online

if we use distance $L_2(x,\mu) = \frac{(x-\mu)^2}{b}$,

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first, compute $g(x)$:

$$g(x) = h(L(x,\mu))$$

$$= h\left(\frac{(x-\mu)^2}{b}\right)$$

$$= e^{-\frac{(x-\mu)^2}{b}}$$

next, compute $\int_{-\infty}^{\infty} g(x)\,dx$:

$$\int_{-\infty}^{\infty} g(x)\,dx = \sqrt{b\pi}$$

finally, compute $f(x)$:

$$f(x) = \frac{g(x)}{\int_{-\infty}^{\infty} g(x)} = \frac{e^{-\frac{(x-\mu)^2}{b}}}{\sqrt{b\pi}}$$

if we use distance $L_2(x,\mu) = \frac{(x-\mu)^2}{b}$, what do we get for $f(x)$?

what distribution is this?

first, compute $g(x)$:

$$g(x) = h(L(x,\mu))$$
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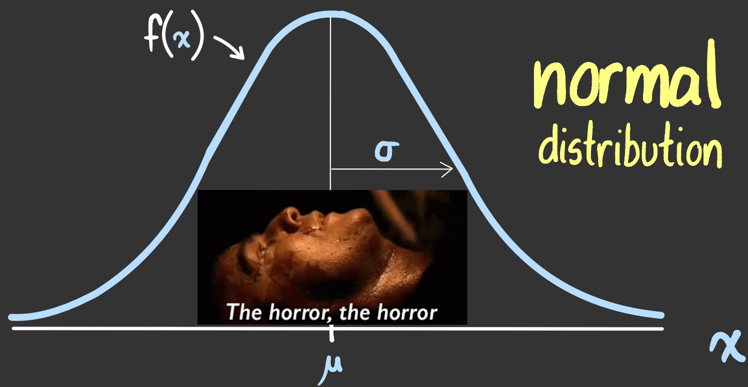
$$\boxed{f(x) = \frac{e^{-\frac{(x-\mu)^2}{b}}}{\sqrt{b\pi}}}$$

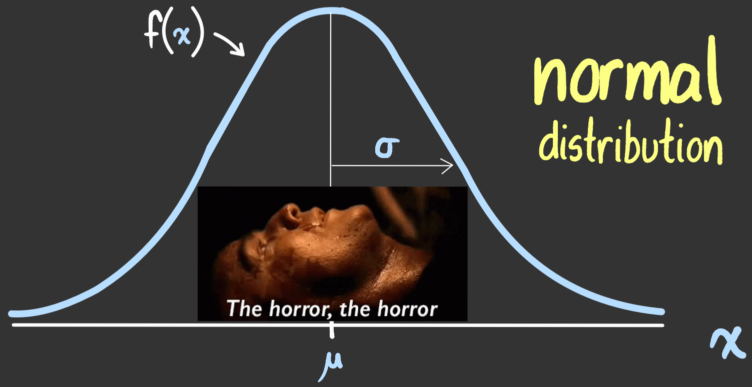


$$\boxed{f(x) = \frac{1}{\sqrt{2\pi\sigma^2}}\, e^{\frac{-(x-\mu)^2}{2\sigma^2}}}$$

what distribution is this?

$$\boxed{f(x) = \frac{e^{-\frac{(x-\mu)^2}{b}}}{\sqrt{b\pi}}}$$

normal distribution



$$\boxed{f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(x-\mu)^2}{2\sigma^2}}}$$

it's the normal distribution

rewrite b as $2\sigma^2$

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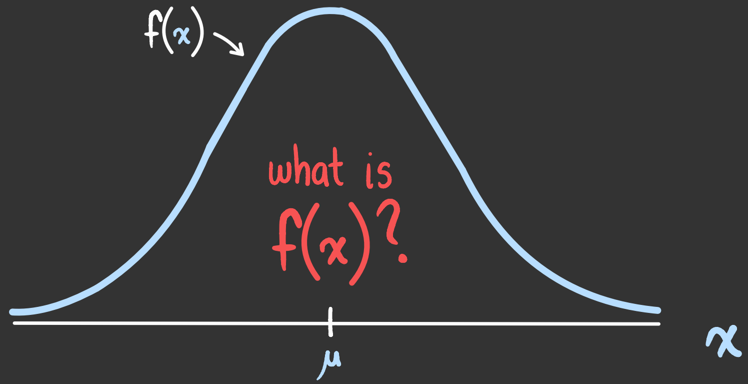
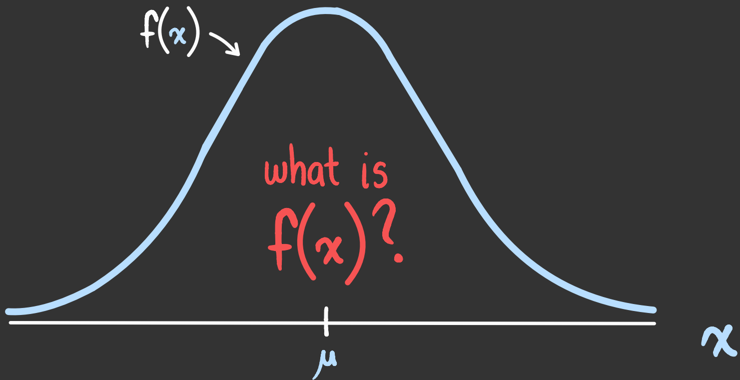


$$f(x) =$$

we want a function $f(x)$ with the following properties:

- f has a unique maximum μ
- f decreases as we get further from μ
- $f(x) > 0$ for all $x \in \mathbb{R}$
- $\int_{-\infty}^{\infty} f(x)\,dx = 1$



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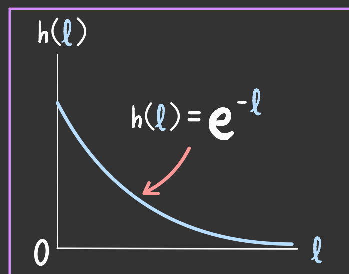
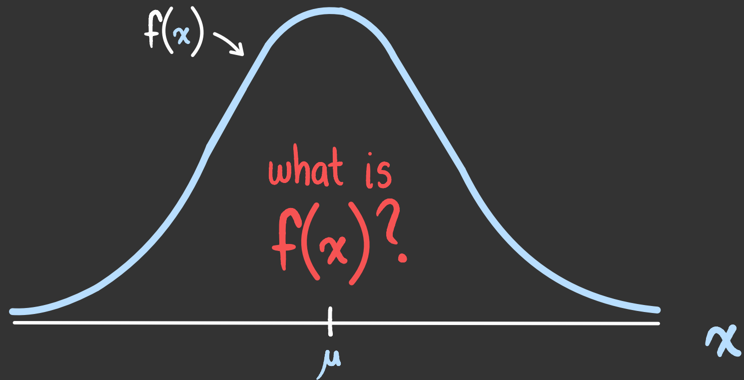
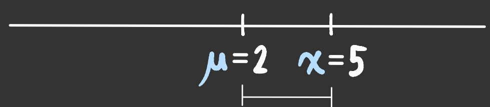


$$\boxed{f(x) = e^{-L(x,\mu)}}$$





$$L_2(x,\mu) = (5-2)^2 = 9$$

$$f(x) = e^{-\frac{(x-\mu)^2}{b}}$$

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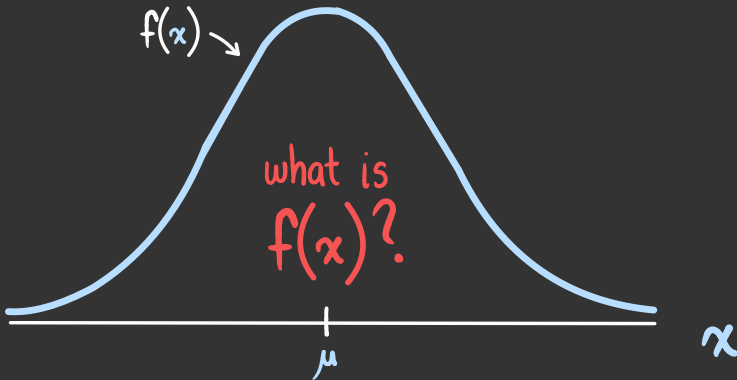

$$f(x) = \frac{g(x)}{\int_{-\infty}^{\infty} g(x)}$$

$$\boxed{f(x) = \frac{1}{\sqrt{b\pi}} e^{-\frac{(x-\mu)^2}{b}}}$$

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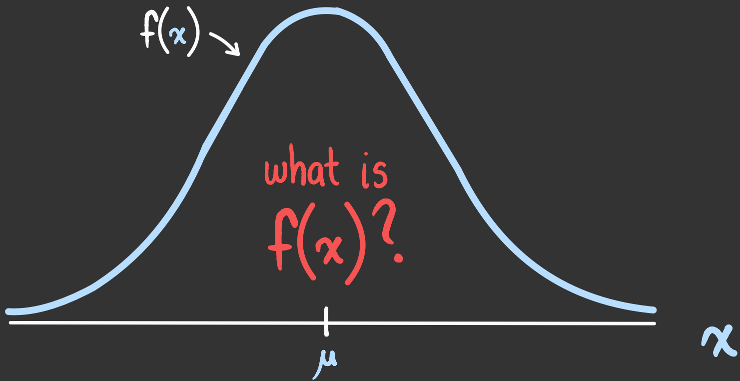

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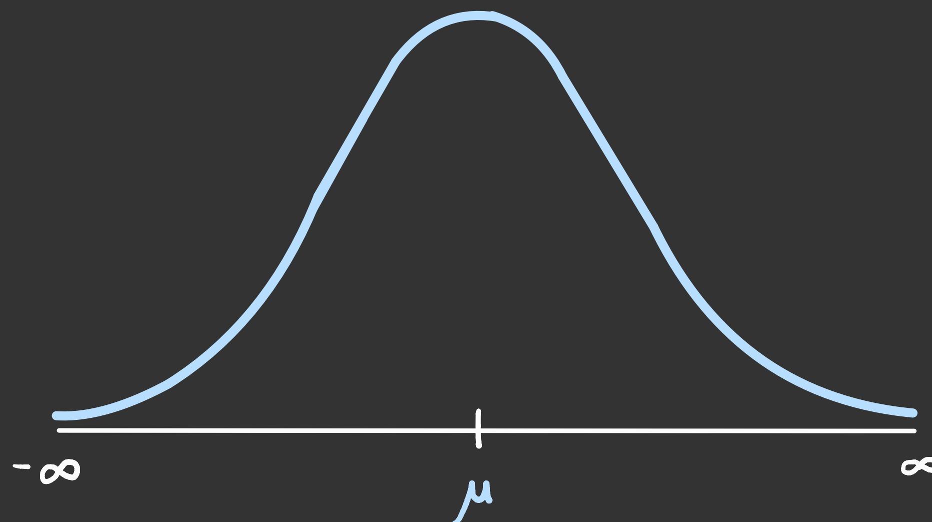
rewrite b as $2\sigma^2$

$$\boxed{f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}}$$

the normal distribution has two **parameters**:



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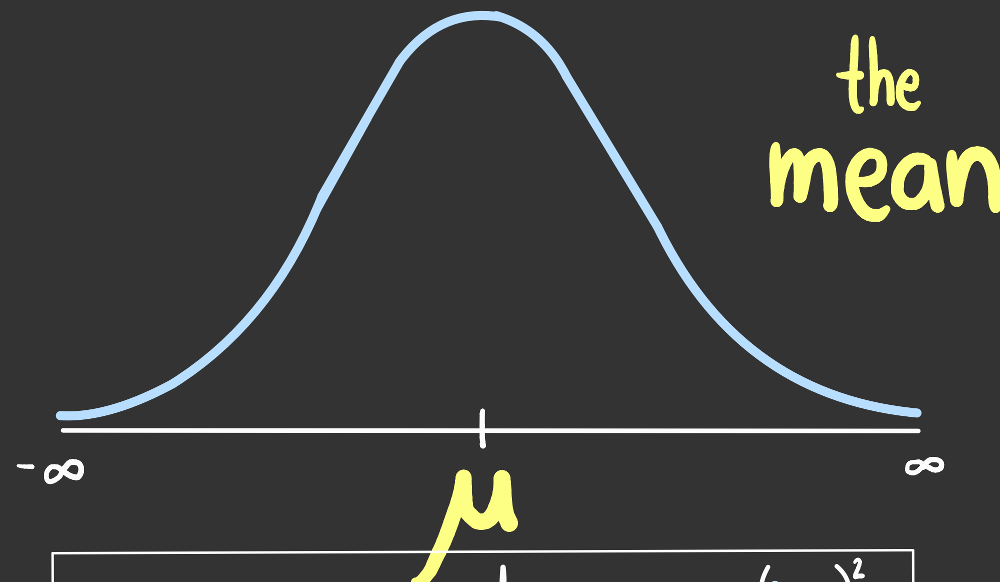

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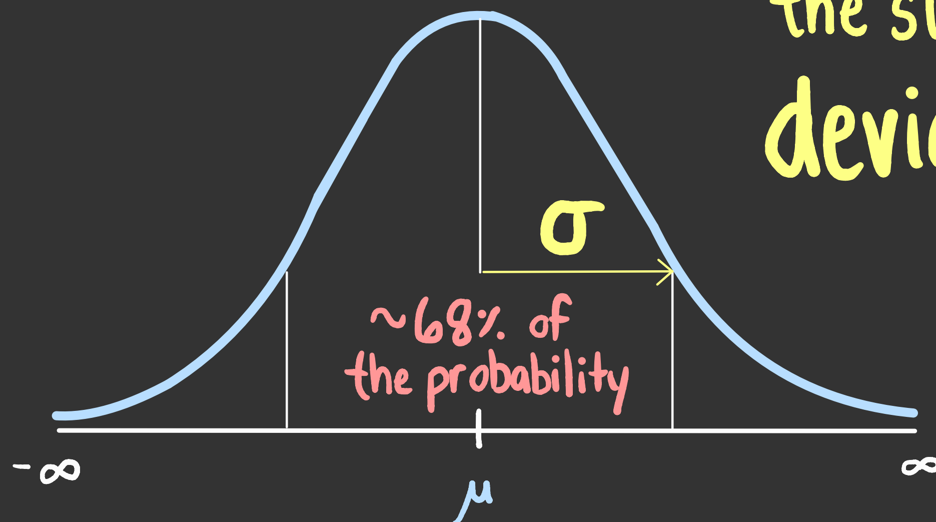
the normal distribution has two parameters:

the standard deviation



$$\boxed{f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}}$$