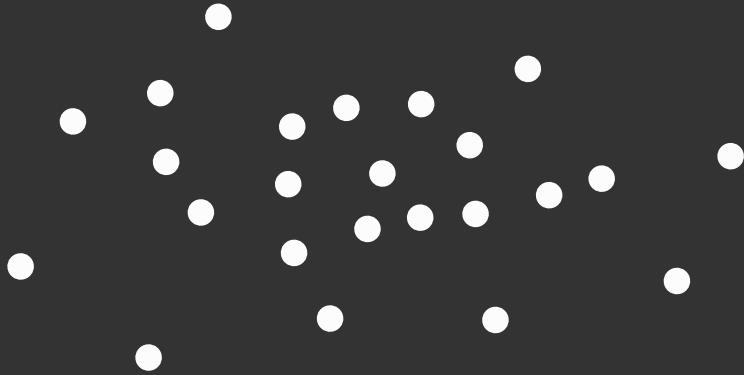


truth table  
inference

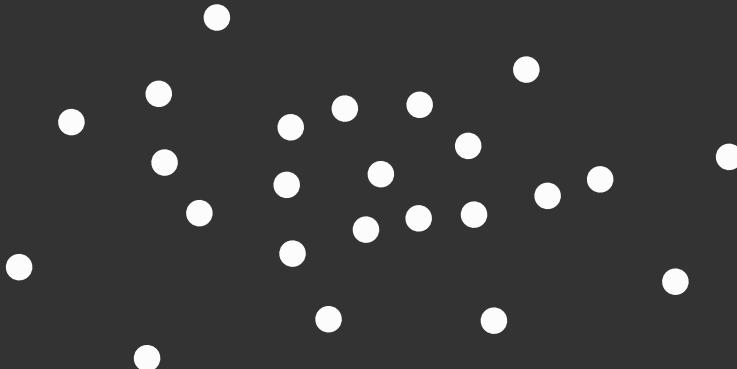
CSCI  
373

Consider a  
galaxy  
of organisms



each point  
represents  
some organism  
on earth

classify each point  
according to three  
boolean properties:

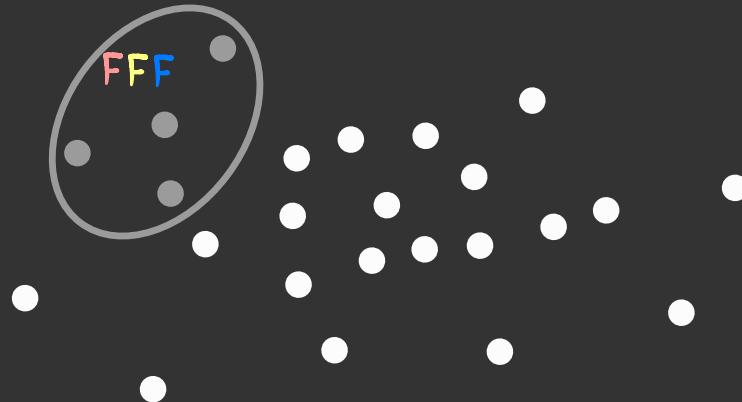


is it a bird?

is it a penguin?

can it fly?

classify each point  
according to three  
boolean properties:



is it a bird?

F

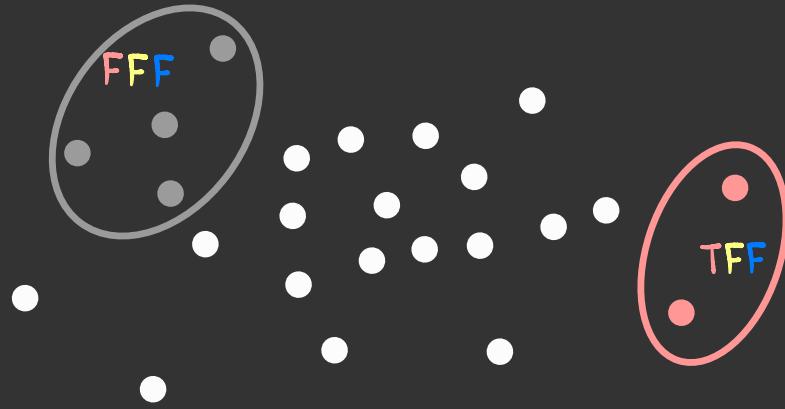
is it a penguin?

F

# Can it fly?

F

classify each point  
according to three  
boolean properties:



example?

is it a bird?

F T

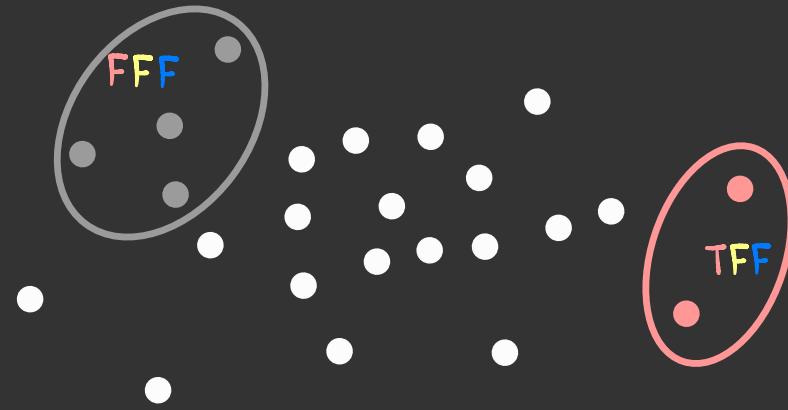
is it a penguin?

F F

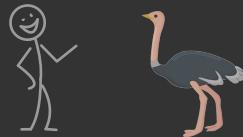
can it fly?

F F

classify each point  
according to three  
boolean properties:



is it a bird?



F



T

is it a penguin?

F

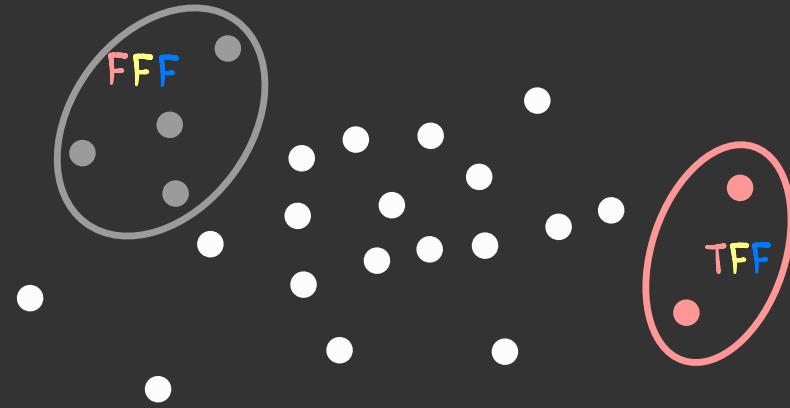
F

can it fly?

F

F

classify each point  
according to three  
boolean properties:



example?

is it a bird?

F T F

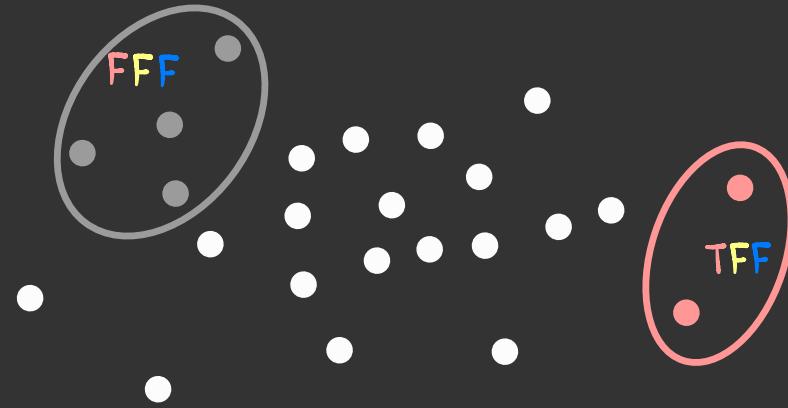
is it a penguin?

F F T

can it fly?

F F F

classify each point  
according to three  
boolean properties:



is it a bird?

F	T	F
---	---	---

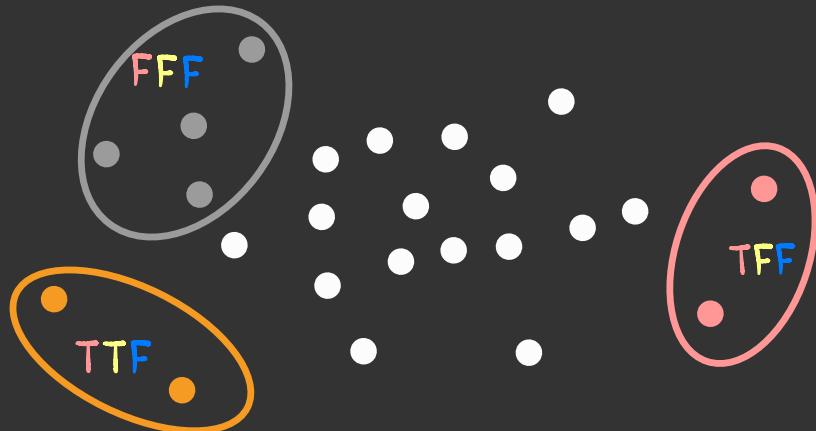
is it a penguin?

F	F	T
---	---	---

can it fly?

F	F	F
---	---	---

classify each point  
according to three  
boolean properties:



is it a bird?

F	T	F	T
---	---	---	---

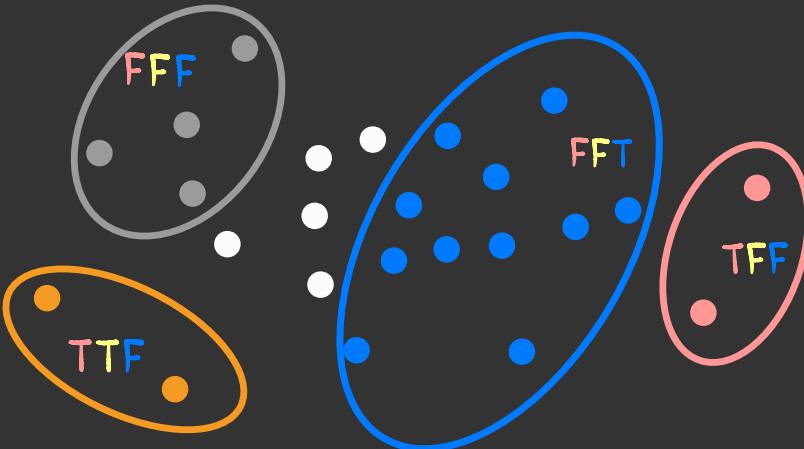
is it a penguin?

F	F	T	T
---	---	---	---

can it fly?

F	F	F	F
---	---	---	---

classify each point  
according to three  
boolean properties:



example?

is it a bird?

F	T	F	T	F
---	---	---	---	---

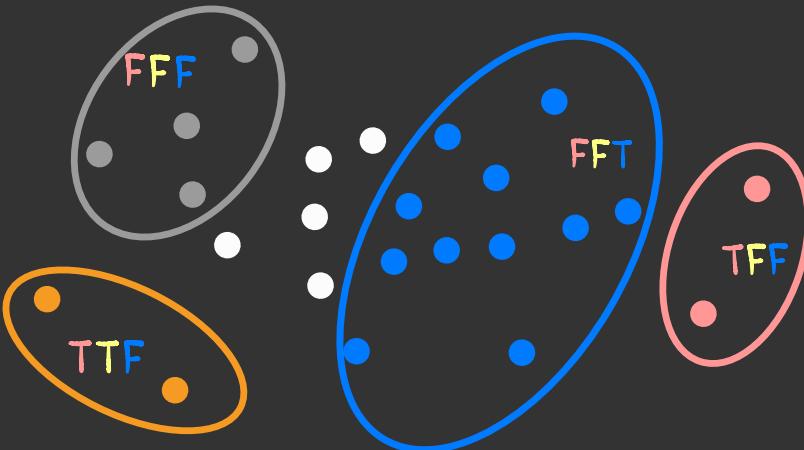
is it a penguin?

F	F	T	T	F
---	---	---	---	---

can it fly?

F	F	F	F	T
---	---	---	---	---

classify each point  
according to three  
boolean properties:



is it a bird?

F	T	F	T	F
---	---	---	---	---

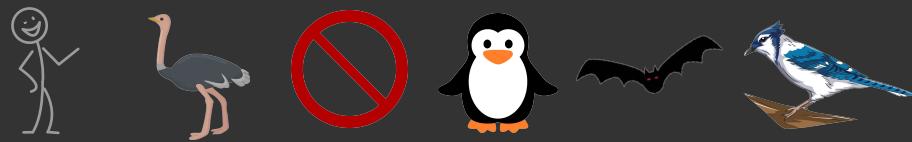
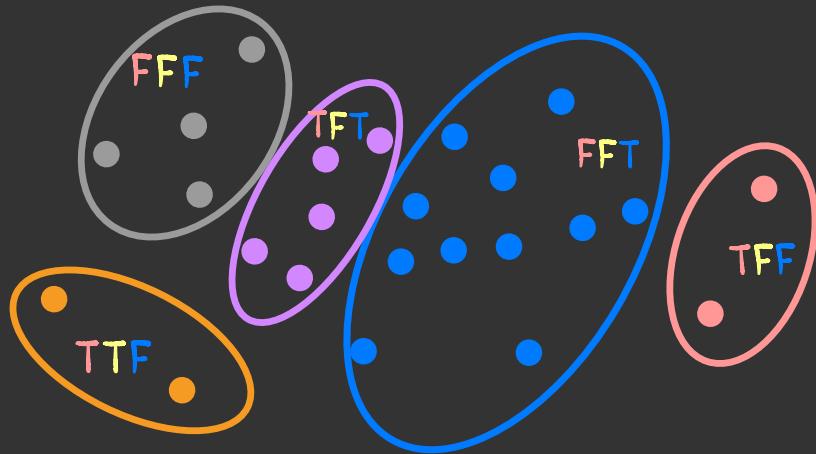
is it a penguin?

F	F	T	T	F
---	---	---	---	---

can it fly?

F	F	F	F	T
---	---	---	---	---

classify each point  
according to three  
boolean properties:



is it a bird?

F	T	F	T	F	T
---	---	---	---	---	---

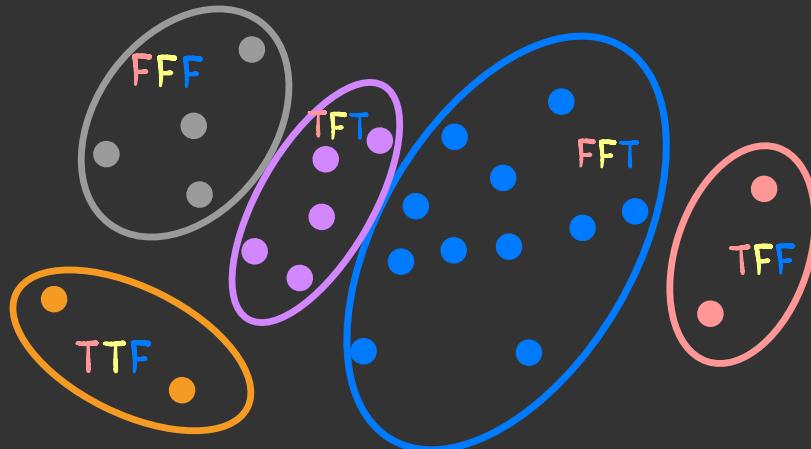
is it a penguin?

F	F	T	T	F	F
---	---	---	---	---	---

can it fly?

F	F	F	F	T	T
---	---	---	---	---	---

classify each point  
according to three  
boolean properties:



is it a bird?

F	T	F	T	F	T	F	T
---	---	---	---	---	---	---	---

is it a penguin?

F	F	T	T	F	F	T	T
---	---	---	---	---	---	---	---

can it fly?

F	F	F	F	T	T	T	T
---	---	---	---	---	---	---	---

we have created a  
truth table

can fly?	penguin?	bird?	
F	F	F	
F	F	T	
F	T	F	
F	T	T	
T	F	F	
T	F	T	
T	T	F	
T	T	T	

we have created a  
truth table

can fly?	penguin?	bird?	
F	F	F	 I
F	F	T	 I
F	T	F	<del>O</del> O
F	T	T	 I
T	F	F	<del>I</del> I
T	F	T	 I
T	T	F	<del>O</del> O
T	T	T	<del>O</del> O

we have created a  
truth table

can fly?	penguin?	bird?	
F	F	<del>F</del> O	I
F	F	<del>T</del> I	I
F	T	<del>F</del> O	O
F	T	<del>T</del> I	I
T	F	<del>F</del> O	I
T	F	<del>T</del> I	I
T	T	<del>F</del> O	O
T	T	<del>T</del> I	O

if you can fly,  
are you a bird ?

can fly?	penguin?	bird?	
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	0

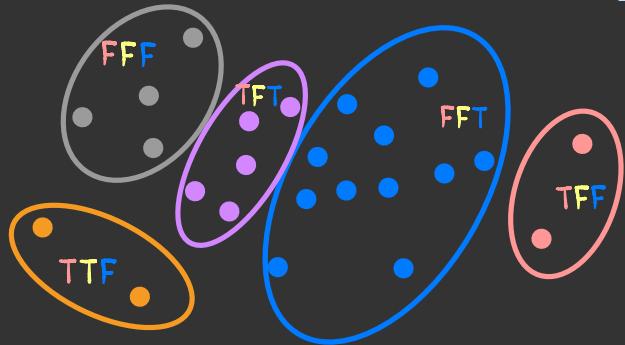
if you can fly,  
are you a bird?

1. eliminate the impossible

can fly?	penguin?	bird?	
0	0	0	
0	0		
0		0	0
0			
	0	0	
	0		
		0	0
			0

if you can fly,  
are you a bird?

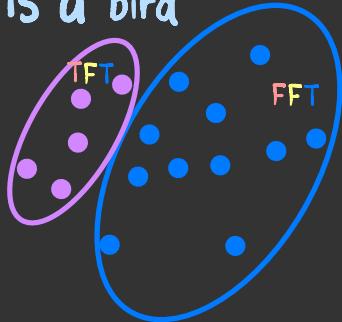
1. eliminate the impossible
2. consider the points that can fly



can fly?	penguin?	bird?	
0	0	0	
0	0		
0		0	0
0			
	0	0	
	0		
		0	0
			0

if you can fly,  
are you a bird?

1. eliminate the impossible
2. consider the points that can fly
3. see whether each remaining point is a bird



can fly?	penguin?	bird?	
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	0

if you can fly,  
are you a bird?

no



can fly?	penguin?	bird?	
0	0	0	
0	0		
0		0	0
0			
	0	0	
	0		
		0	0
			0

if you can fly,  
are you a bird?

if premise, then consequence?  
(you can fly) (you are a bird)

I. eliminate the impossible

can fly?	penguin?	bird?	
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	0

- eliminate rows that violate the premise

can	fly?	penguin?	bird?	
o	o	o	+	
o	o	+	+	
o	+	o	o	
o	+	+	+	
l	o	o	l	
l	o	l	l	
+	+	o	o	
+	+	+	o	

3. see if each remaining row satisfies the consequence

no, not  
a bird

if you are a bird  
that cannot fly,  
are you a penguin?

can fly?	penguin?	bird?	
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	0

if you are a bird  
that cannot fly,  
are you a penguin?

1. eliminate the  
impossible

can fly?	penguin?	bird?	
0	0	0	
0	0		
0		0	0
0			
	0	0	
	0		
		0	0
			0

if you are a bird  
that cannot fly,  
are you a penguin?

2. eliminate rows that  
violate the premise

can fly?	penguin?	bird?	
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	0

if you are a bird  
that cannot fly,  
are you a penguin?

3. see if each remaining  
row satisfies the  
consequence

can fly?	penguin?	bird?	
0	0	0	
0	no! 0		
0		0	0
0			
	0	0	
	0		
		0	0
			0

automating logical  
deduction is

easy

with a truth table

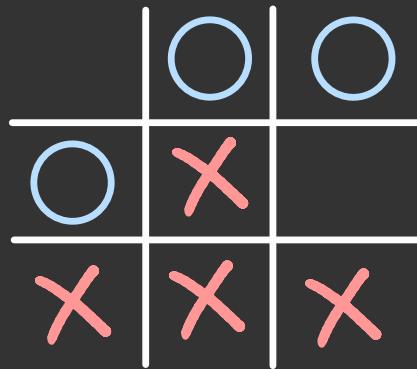
automating logical  
deduction is

easy

with a truth table

but there is a downside

Suppose we want to  
reason logically about  
tic-tac-toe

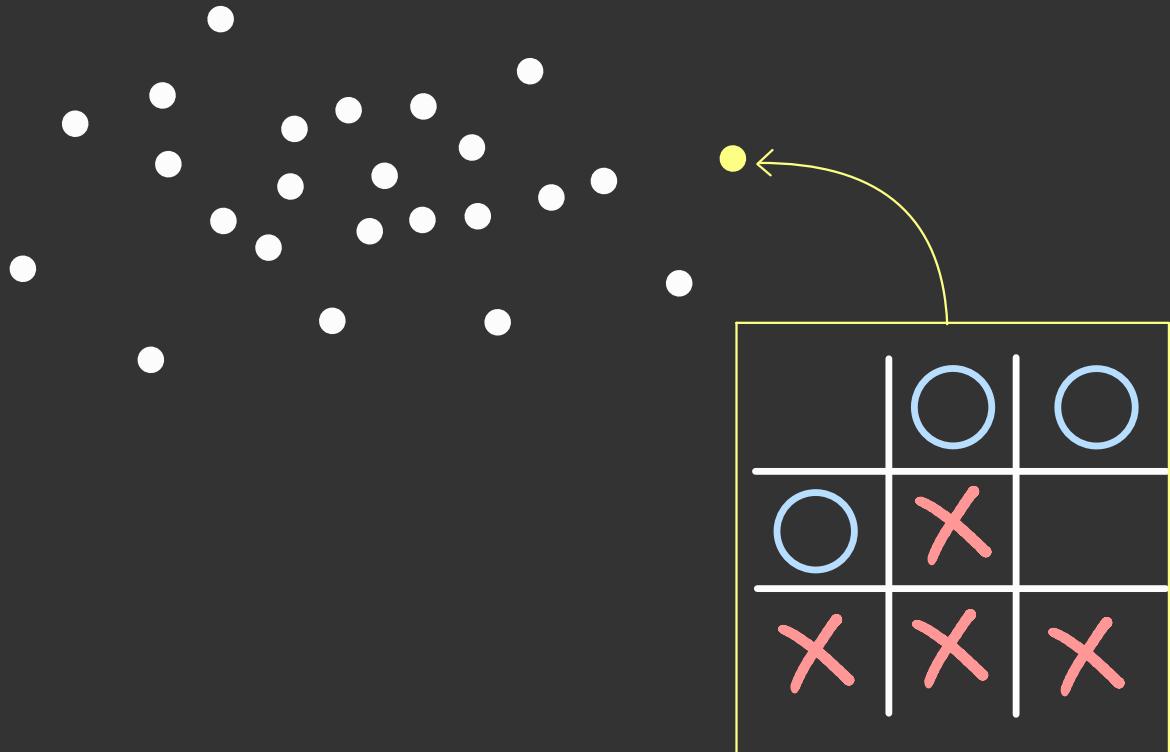


Suppose we want to  
reason logically about  
tic-tac-toe

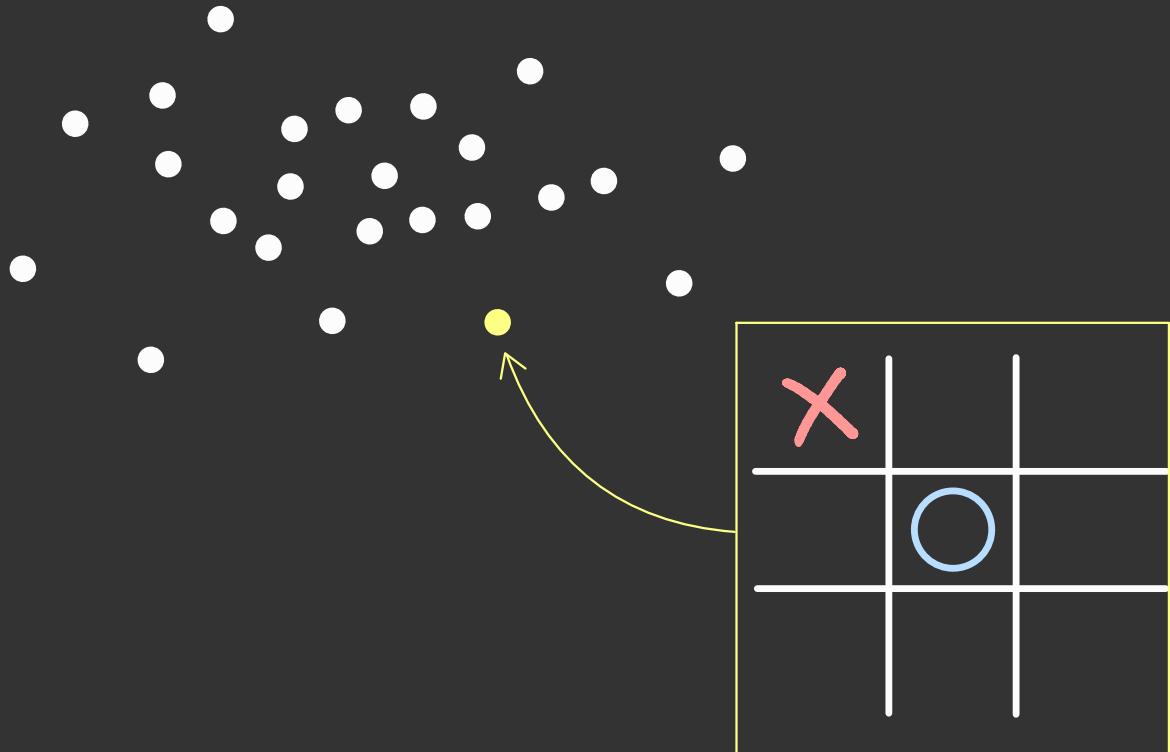
1	2	3
4	5	6
7	8	9

let  $x_i$  be whether square  $i$  has an  $\times$   
let  $o_i$  be whether square  $i$  has an  $\circ$

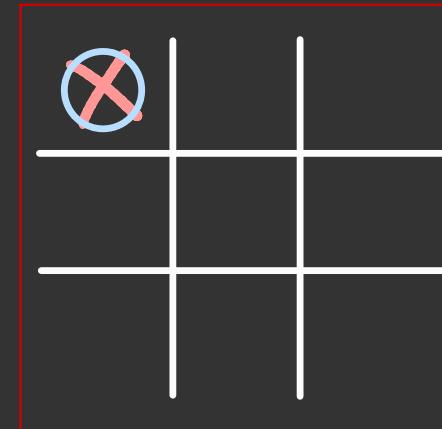
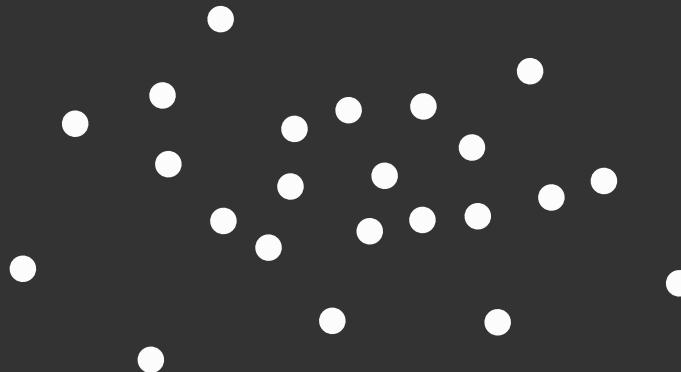
1	2	3	$X_1$	$X_2$	$X_3$	$X_4$	$X_5$	$X_6$	$X_7$	$X_8$	$X_9$	$O_1$	$O_2$	$O_3$	$O_4$	$O_5$	$O_6$	$O_7$	$O_8$	$O_9$
4	5	6	0	0	0	0	1	0	1	1	1	0	1	1	1	0	0	0	0	
7	8	9																	1	



1	2	3	$X_1$	$X_2$	$X_3$	$X_4$	$X_5$	$X_6$	$X_7$	$X_8$	$X_9$	$O_1$	$O_2$	$O_3$	$O_4$	$O_5$	$O_6$	$O_7$	$O_8$	$O_9$
4	5	6	0	0	0	0	1	0	1	1	1	0	1	1	1	0	0	0	0	
7	8	9	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	
																		1	1	



1	2	3	$X_1$	$X_2$	$X_3$	$X_4$	$X_5$	$X_6$	$X_7$	$X_8$	$X_9$	$O_1$	$O_2$	$O_3$	$O_4$	$O_5$	$O_6$	$O_7$	$O_8$	$O_9$
4	5	6	0	0	0	0	1	0	1	1	1	0	1	1	1	0	0	0	0	
7	8	9	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	
			1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	



1	2	3	$X_1$	$X_2$	$X_3$	$X_4$	$X_5$	$X_6$	$X_7$	$X_8$	$X_9$	$O_1$	$O_2$	$O_3$	$O_4$	$O_5$	$O_6$	$O_7$	$O_8$	$O_9$
4	5	6	0	0	0	0	1	0	1	1	1	0	1	1	1	0	0	0	0	1
7	8	9	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1
			1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0

how many rows are  
in this truth table?

1	2	3	$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	$x_6$	$x_7$	$x_8$	$x_9$	$O_1$	$O_2$	$O_3$	$O_4$	$O_5$	$O_6$	$O_7$	$O_8$	$O_9$
4	5	6	0	0	0	0	1	0	1	1	1	0	1	1	1	0	0	0	0	1
7	8	9	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	1
			1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0

how many rows are  
in this truth table?

$$2^{18} = 262,144$$

1	2	3	$X_1$	$X_2$	$X_3$	$X_4$	$X_5$	$X_6$	$X_7$	$X_8$	$X_9$	$O_1$	$O_2$	$O_3$	$O_4$	$O_5$	$O_6$	$O_7$	$O_8$	$O_9$
4	5	6	0	0	0	0	1	0	1	1	1	0	1	1	1	0	0	0	0	1
7	8	9	1	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
			1	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0

and if we were playing  
4x4 tic-tac-toe?

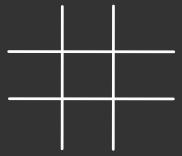
1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	16

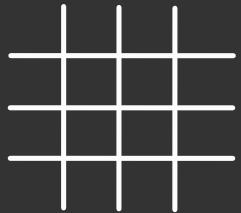
$x_1$	$\dots$	$\dots$	$x_{16}$	$o_1$	$\dots$	$\dots$	$o_{16}$	$\mid$
0	$\dots$	$\dots$	1	0	$\dots$	$\dots$	0	1
1	$\dots$	$\dots$	0	0	$\dots$	$\dots$	0	1
1	$\dots$	$\dots$	0	1	$\dots$	$\dots$	0	0

and if we were playing  
 4x4 tic-tac-toe?

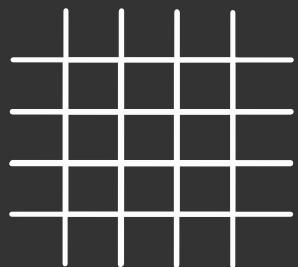
$$2^{32} = 4,294,947,296 \text{ rows}$$



$$2^{18} = 262,144 \text{ rows}$$



$$2^{32} = 4,294,947,296 \text{ rows}$$



$$2^{50} >$$



how can we perform  
logical reasoning  
more efficiently?