inference

CSCI 373


let X be Xena's blood type.

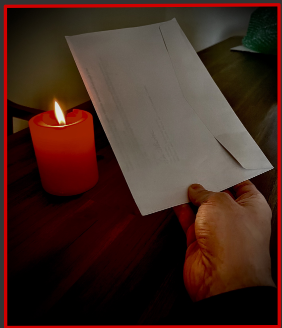
let $X_♀$, $X_♂$ be the genes Xena inherited from her parents


let Y be Yves's blood type.

let $Y_♀$, $Y_♂$ be the genes Yves inherited from his parents


let Z be Zelda's blood type.

let $Z_♀$, $Z_♂$ be the genes Zelda inherited from Xena and Yves



what is $P(Y=AB \mid Z=B)$, the probability that Yves is type AB, given we learn that Zelda is type B? assume we know how to compute joint probability $P(x_♀, x_♂, x, y_♀, y_♂, y, z_♀, z_♂, z)$

$$P(Y=AB \mid Z=B)$$

$$= \frac{P(Y=AB, Z=B)}{P(Z=B)}$$

$$= \frac{\sum_{x_{♀}} \sum_{x_{♂}} \sum_{x} \sum_{y_{♀}} \sum_{y_{♂}} \sum_{z_{♀}} \sum_{z_{♂}} P(x_{♀}, x_{♂}, x, y_{♀}, y_{♂}, Y=AB, z_{♀}, z_{♂}, Z=B)}{\sum_{x_{♀}} \sum_{x_{♂}} \sum_{x} \sum_{y_{♀}} \sum_{y_{♂}} \sum_{y} \sum_{z_{♀}} \sum_{z_{♂}} P(x_{♀}, x_{♂}, x, y_{♀}, y_{♂}, y, z_{♀}, z_{♂}, Z=B)}$$

$$P(Y=AB \mid Z=B)$$

$$= \frac{P(Y=AB, Z=B)}{P(Z=B)}$$

$$= \frac{\sum_{x_\female}\sum_{x_\male}\sum_{x}\sum_{y_\female}\sum_{y_\male}\sum_{z_\female}\sum_{z_\male} P(x_\female, x_\male, x, y_\female, y_\male, Y=AB, z_\female, z_\male, Z=B)}{\sum_{x_\female}\sum_{x_\male}\sum_{x}\sum_{y_\female}\sum_{y_\male}\sum_{y}\sum_{z_\female}\sum_{z_\male} P(x_\female, x_\male, x, y_\female, y_\male, y, z_\female, z_\male, Z=B)}$$

$$P(Y=AB \mid Z=B)$$

$$= \frac{P(Y=AB, Z=B)}{P(Z=B)}$$

$$= \frac{\sum_{x_{\female}} \sum_{x_{\male}} \sum_{x} \sum_{y_{\female}} \sum_{y_{\male}} \sum_{z_{\female}} \sum_{z_{\male}} P(x_{\female}, x_{\male}, x, y_{\female}, y_{\male}, Y=AB, z_{\female}, z_{\male}, Z=B)}{\sum_{x_{\female}} \sum_{x_{\male}} \sum_{x} \sum_{y_{\female}} \sum_{y_{\male}} \sum_{y} \sum_{z_{\female}} \sum_{z_{\male}} P(x_{\female}, x_{\male}, x, y_{\female}, y_{\male}, y, z_{\female}, z_{\male}, Z=B)}$$

$$P(Y = AB \mid Z = B)$$

$$= \frac{P(Y = AB, Z = B)}{P(Z = B)}$$

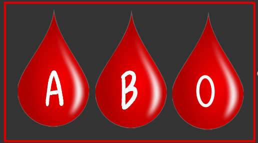
$$= \frac{\sum_{x_{\text{♀}}} \sum_{x_{\text{♂}}} \sum_{x} \sum_{y_{\text{♀}}} \sum_{y_{\text{♂}}} \sum_{z_{\text{♀}}} \sum_{z_{\text{♂}}} P(x_{\text{♀}}, x_{\text{♂}}, x, y_{\text{♀}}, y_{\text{♂}}, Y = AB, z_{\text{♀}}, z_{\text{♂}}, Z = B)}{\boxed{\sum_{x_{\text{♀}}} \sum_{x_{\text{♂}}} \sum_{x} \sum_{y_{\text{♀}}} \sum_{y_{\text{♂}}} \sum_{y} \sum_{z_{\text{♀}}} \sum_{z_{\text{♂}}} P(x_{\text{♀}}, x_{\text{♂}}, x, y_{\text{♀}}, y_{\text{♂}}, y, z_{\text{♀}}, z_{\text{♂}}, Z = B)}}$$

how many terms are in this summation?

$$P(Y=AB \mid Z=B)$$

$$= \frac{P(Y=AB,\ Z=B)}{P(Z=B)}$$

$$= \frac{\sum\limits_{x_\female}\sum\limits_{x_\male}\sum\limits_{x}\sum\limits_{y_\female}\sum\limits_{y_\male}\sum\limits_{z_\female}\sum\limits_{z_\male} P(x_\female, x_\male, x, y_\female, y_\male, Y=AB, z_\female, z_\male, Z=B)}{\sum\limits_{\substack{x_\female\\3}}\sum\limits_{\substack{x_\male\\3}}\sum\limits_{x}\sum\limits_{\substack{y_\female\\3}}\sum\limits_{\substack{y_\male\\3}}\sum\limits_{y}\sum\limits_{\substack{z_\female\\3}}\sum\limits_{\substack{z_\male\\3}} P(x_\female, x_\male, x, y_\female, y_\male, y, z_\female, z_\male, Z=B)}$$



$$P(Y=AB \mid Z=B)$$

$$= \frac{P(Y=AB,\ Z=B)}{P(Z=B)}$$

$$= \frac{\sum\limits_{x_{♀}}\sum\limits_{x_{♂}}\sum\limits_{x}\sum\limits_{y_{♀}}\sum\limits_{y_{♂}}\sum\limits_{z_{♀}}\sum\limits_{z_{♂}} P(x_{♀}, x_{♂}, x, y_{♀}, y_{♂}, Y=AB, z_{♀}, z_{♂}, Z=B)}{\sum\limits_{x_{♀}}\sum\limits_{x_{♂}}\sum\limits_{x}\sum\limits_{y_{♀}}\sum\limits_{y_{♂}}\sum\limits_{y}\sum\limits_{z_{♀}}\sum\limits_{z_{♂}} P(x_{♀}, x_{♂}, x, y_{♀}, y_{♂}, y, z_{♀}, z_{♂}, Z=B)}$$

3 3 4 3 3 4 3 3

A, B, AB, O

$$P(Y=AB \mid Z=B)$$

$$= \frac{P(Y=AB,\ Z=B)}{P(Z=B)}$$

$$= \frac{\displaystyle\sum_{x_{\text{♀}}}\sum_{x_{\text{♂}}}\sum_{x}\sum_{y_{\text{♀}}}\sum_{y_{\text{♂}}}\sum_{z_{\text{♀}}}\sum_{z_{\text{♂}}} P(x_{\text{♀}}, x_{\text{♂}}, x, y_{\text{♀}}, y_{\text{♂}}, Y=AB, z_{\text{♀}}, z_{\text{♂}}, Z=B)}{\displaystyle\sum_{x_{\text{♀}}}\sum_{x_{\text{♂}}}\sum_{x}\sum_{y_{\text{♀}}}\sum_{y_{\text{♂}}}\sum_{y}\sum_{z_{\text{♀}}}\sum_{z_{\text{♂}}} P(x_{\text{♀}}, x_{\text{♂}}, x, y_{\text{♀}}, y_{\text{♂}}, y, z_{\text{♀}}, z_{\text{♂}}, Z=B)}$$

$$3 \times 3 \times 4 \times 3 \times 3 \times 4 \times 3 \times 3$$

$$= 11664 \text{ terms}$$

one approach to automating probability computations

query $P(Y=AB \mid Z=B)$

joint distribution

$X_♀$	$X_♂$	X	$Y_♀$	$Y_♂$	Y	$Z_♀$	$Z_♂$	Z	
A	A	A	A	A	A	A	A	A	p_1
A	A	A	A	A	A	A	A	B	p_2
				⋮					⋮
O	O	O	O	O	O	O	O	O	p_{46656}

→ **inference algorithm** → .18

$$P(Y=AB \mid Z=B)$$

$$= \frac{P(Y=AB, Z=B)}{P(Z=B)}$$

$$= \frac{\sum_{x_♀}\sum_{x_♂}\sum_{x}\sum_{y_♀}\sum_{y_♂}\sum_{z_♀}\sum_{z_♂} P(x_♀, x_♂, x, y_♀, y_♂, Y=AB, z_♀, z_♂, Z=B)}{\sum_{x_♀}\sum_{x_♂}\sum_{x}\sum_{y_♀}\sum_{y_♂}\sum_{y}\sum_{z_♀}\sum_{z_♂} P(x_♀, x_♂, x, y_♀, y_♂, y, z_♀, z_♂, Z=B)}$$

what is a drawback of this approach?

one approach to automating probability computations

query $P(Y=AB \mid Z=B)$

joint distribution

$X_{♀}$	$X_{♂}$	X	$Y_{♀}$	$Y_{♂}$	Y	$Z_{♀}$	$Z_{♂}$	Z	
A	A	A	A	A	A	A	A	A	p_1
A	A	A	A	A	A	A	A	B	p_2
				⋮					⋮
O	O	O	O	O	O	O	O	O	p

46656

→ **inference algorithm** → .18

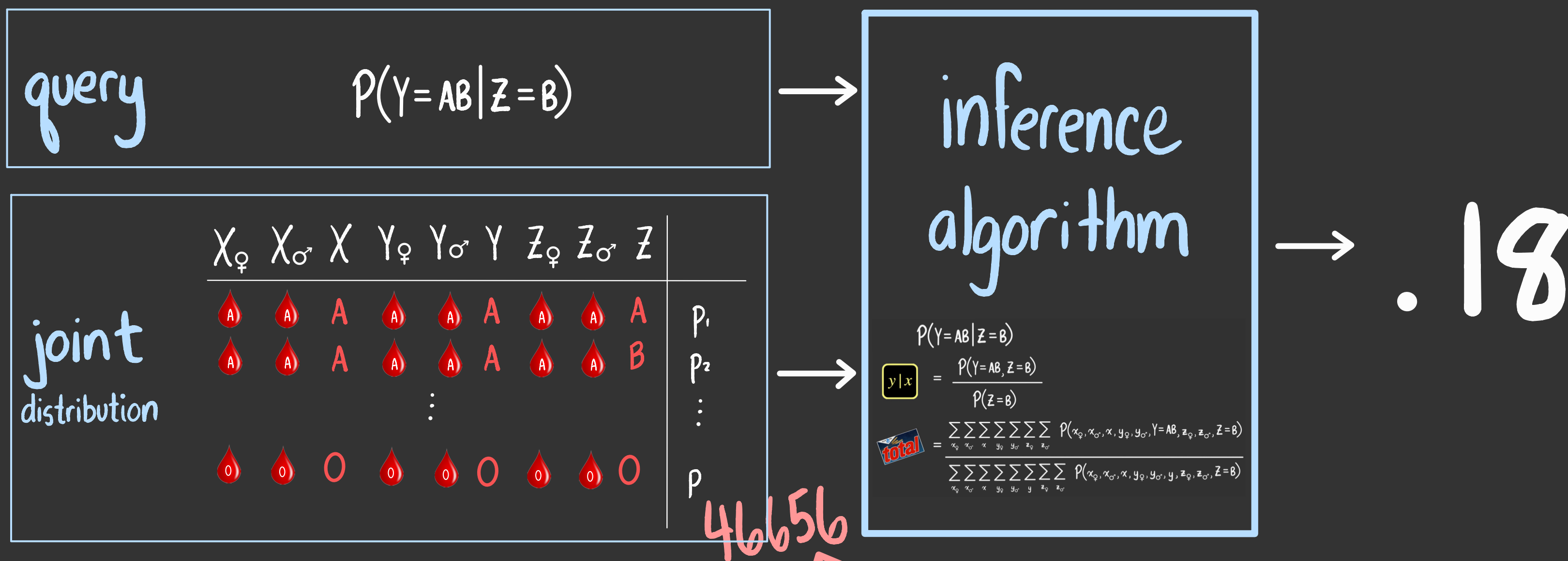
$$P(Y=AB \mid Z=B)$$

$$y|x \quad = \frac{P(Y=AB, Z=B)}{P(Z=B)}$$

$$= \frac{\sum_{x_{♀}}\sum_{x_{♂}}\sum_{x}\sum_{y_{♀}}\sum_{y_{♂}}\sum_{z_{♀}}\sum_{z_{♂}} P(x_{♀}, x_{♂}, x, y_{♀}, y_{♂}, Y=AB, z_{♀}, z_{♂}, Z=B)}{\sum_{x_{♀}}\sum_{x_{♂}}\sum_{x}\sum_{y_{♀}}\sum_{y_{♂}}\sum_{y}\sum_{z_{♀}}\sum_{z_{♂}} P(x_{♀}, x_{♂}, x, y_{♀}, y_{♂}, y, z_{♀}, z_{♂}, Z=B)}$$

joint distributions are huge

one approach to automating probability computations



joint distributions are huge

what can we do about that?