

logic puzzle
workshop

CSCI
373

A Bargain Logic Puzzle

Benny, Carla and Daniella each picked up a bargain in the January sales with all three items having a reasonable saving on the original selling price - the biggest being \$11.50. From this information and the following clues, for each buyer, can you determine the item they bought, the color and the amount saved on the original selling price?

	Item			Color	Savings
	Trousers	Jacket	Umbrella		
Name	Benny				
	Carla				
	Daniella				
Savings	\$5.00				
	\$9.00				
	\$11.50				
Color	Red				
	Green				
	yellow				

1. The yellow jacket was not bought by Benny.
2. Daniella was pleased to have \$9.00 knocked off the item she bought.
3. The red item had more knocked off than the umbrella.
4. The trousers did not have \$5.00 knocked off and were not bought by Benny or Carla.

Source: ahapuzzles.com

let's solve this!

express the english as propositional logic

	Item			Color			Savings			
	Trousers	Jacket	Umbrella	Red	Green	yellow	\$5.00	\$9.00	\$11.50	
Name	Benny	T _B	J _B	U _B	R _B	G _B	Y _B	F _B	N _B	E _B
	Carla	T _C	J _C	U _C	R _C	G _C	Y _C	F _C	N _C	E _C
	Daniella	T _D	J _D	U _D	R _D	G _D	Y _D	F _D	N _D	E _D
Savings	\$5.00	T _S	J _S	U _S	R _S	G _S	Y _S			
	\$9.00	T _Q	J _Q	U _Q	R _Q	G _Q	Y _Q			
	\$11.50	T _{II}	J _{II}	U _{II}	R _{II}	G _{II}	Y _{II}			
Color	Red	T _R	J _R	U _R						
	Green	T _G	J _G	U _G						
	yellow	T _Y	J _Y	U _Y						

1. The yellow jacket was not bought by Benny.

your answer here

2. Daniella was pleased to have \$9.00 knocked off the item she bought.

your answer here

3. The red item had more knocked off than the umbrella.

your answer here

4. The trousers did not have \$5.00 knocked off and were not bought by Benny or Carla.

your answer here

express the english as propositional logic

	Item			Color			Savings			
	Trousers	Jacket	Umbrella	Red	Green	yellow	\$5.00	\$9.00	\$11.50	
Name	Benny	T_B	J_B	U_B	R_B	G_B	Y_B	F_B	N_B	E_B
	Carla	T_C	J_C	U_C	R_C	G_C	Y_C	F_C	N_C	E_C
	Daniella	T_D	J_D	U_D	R_D	G_D	Y_D	F_D	N_D	E_D
Savings	\$5.00	T_5	J_5	U_5	R_5	G_5	Y_5			
	\$9.00	T_9	J_9	U_9	R_9	G_9	Y_9			
	\$11.50	T_{11}	J_{11}	U_{11}	R_{11}	G_{11}	Y_{11}			
Color	Red	T_R	J_R	U_R						
	Green	T_G	J_G	U_G						
	yellow	T_Y	J_Y	U_Y						

1. The yellow jacket was not bought by Benny.

$$J_Y \wedge \neg(J_B \wedge Y_B)$$

2. Daniella was pleased to have \$9.00 knocked off the item she bought.

$$N_D$$

3. The red item had more knocked off than the umbrella.

$$(U_5 \Rightarrow (R_9 \vee R_{11})) \\ \wedge (U_9 \Rightarrow R_{11}) \\ \wedge \neg U_{11}$$

4. The trousers did not have \$5.00 knocked off and were not bought by Benny or Carla.

$$\neg T_5 \wedge \neg(T_B \vee T_C)$$

do these fully describe the puzzle?

	Item			Color			Savings			
	Trousers	Jacket	Umbrella	Red	Green	yellow	\$5.00	\$9.00	\$11.50	
Name	Benny	T_B	J_B	U_B	R_B	G_B	Y_B	F_B	N_B	E_B
	Carla	T_C	J_C	U_C	R_C	G_C	Y_C	F_C	N_C	E_C
	Daniella	T_D	J_D	U_D	R_D	G_D	Y_D	F_D	N_D	E_D
Savings	\$5.00	T_5	J_5	U_5	R_5	G_5	Y_5			
	\$9.00	T_9	J_9	U_9	R_9	G_9	Y_9			
	\$11.50	T_{11}	J_{11}	U_{11}	R_{11}	G_{11}	Y_{11}			
Color	Red	T_R	J_R	U_R						
	Green	T_G	J_G	U_G						
	yellow	T_Y	J_Y	U_Y						

1. The yellow jacket was not bought by Benny.

$$J_Y \wedge \neg(J_B \wedge Y_B)$$

2. Daniella was pleased to have \$9.00 knocked off the item she bought.

$$N_D$$

3. The red item had more knocked off than the umbrella.

$$\begin{aligned} & (U_5 \Rightarrow (R_9 \vee R_{11})) \\ & \wedge (U_9 \Rightarrow R_{11}) \\ & \wedge \neg U_{11} \end{aligned}$$

4. The trousers did not have \$5.00 knocked off and were not bought by Benny or Carla.

$$\neg T_5 \wedge \neg(T_B \vee T_C)$$

no!

exactly
one is
true

	Item			Color			Savings		
	Trousers	Jacket	Umbrella	Red	Green	yellow	\$5.00	\$9.00	\$11.50
Name	T _B	J _B	U _B	R _B	G _B	Y _B	F _B	N _B	E _B
Carla	T _C	J _C	U _C	R _C	G _C	Y _C	F _C	N _C	E _C
Daniella	T _D	J _D	U _D	R _D	G _D	Y _D	F _D	N _D	E _D
Savings	\$5.00	T _S	J _S	U _S	R _S	G _S	Y _S		
	\$9.00	T _Q	J _Q	U _Q	R _Q	G _Q	Y _Q		
	\$11.50	T _{II}	J _{II}	U _{II}	R _{II}	G _{II}	Y _{II}		
Color	Red	T _R	J _R	U _R					
	Green	T _G	J _G	U _G					
	yellow	T _Y	J _Y	U _Y					

exactly
one is
true

1. The yellow jacket was not bought by Benny.

$$J_Y \wedge \neg(J_B \wedge Y_B)$$

2. Daniella was pleased to have \$9.00 knocked off the item she bought.

$$N_D$$

3. The red item had more knocked off than the umbrella.

$$(U_5 \Rightarrow (R_q \vee R_{II})) \\ \wedge (U_q \Rightarrow R_{II}) \\ \wedge \neg U_{II}$$

4. The trousers did not have \$5.00 knocked off and were not bought by Benny or Carla.

$$\neg T_5 \wedge \neg(T_B \vee T_C)$$

but how do we say "exactly one" in propositional logic?

	Item			Color			Savings		
	Trousers	Jacket	Umbrella	Red	Green	yellow	\$5.00	\$9.00	\$11.50
Name	T _B	J _B	U _B	R _B	G _B	Y _B	F _B	N _B	E _B
Carla	T _C	J _C	U _C	R _C	G _C	Y _C	F _C	N _C	E _C
Daniella	T _D	J _D	U _D	R _D	G _D	Y _D	F _D	N _D	E _D
Savings	\$5.00	T _S	J _S	U _S	R _S	G _S	Y _S		
	\$9.00	T ₉	J ₉	U ₉	R ₉	G ₉	Y ₉		
	\$11.50	T ₁₁	J ₁₁	U ₁₁	R ₁₁	G ₁₁	Y ₁₁		
Color	Red	T _R	J _R	U _R					
	Green	T _G	J _G	U _G					
	yellow	T _Y	J _Y	U _Y					

exactly
one is
true

exactly
one is
true

thoughts?

but how do we say "exactly one" in propositional logic?

Name	Item			Color			Savings		
	Trousers	Jacket	Umbrella	Red	Green	yellow	\$5.00	\$9.00	\$11.50
Benny	T_B	J_B	U_B	R_B	G_B	Y_B	F_B	N_B	E_B
Carla	T_C	J_C	U_C	R_C	G_C	Y_C	F_C	N_C	E_C
Daniella	T_D	J_D	U_D	R_D	G_D	Y_D	F_D	N_D	E_D
	\$5.00	T_S	J_S	U_S	R_S	G_S	Y_S		
	\$9.00	T_Q	J_Q	U_Q	R_Q	G_Q	Y_Q		
	\$11.50	T_{II}	J_{II}	U_{II}	R_{II}	G_{II}	Y_{II}		
Color	Red	T_R	J_R	U_R					
	Green	T_G	J_G	U_G					
	yellow	T_Y	J_Y	U_Y					

"at least one of
 T_B, J_B, U_B is true"

and

"at most one of
 T_B, J_B, U_B is true"

but how do we say "exactly one" in propositional logic?

Name	Item			Color			Savings		
	Trousers	Jacket	Umbrella	Red	Green	yellow	\$5.00	\$9.00	\$11.50
Name	T_B	J_B	U_B	R_B	G_B	Y_B	F_B	N_B	E_B
Benny	T_B	J_B	U_B	R_B	G_B	Y_B	F_B	N_B	E_B
Carla	T_C	J_C	U_C	R_C	G_C	Y_C	F_C	N_C	E_C
Daniella	T_D	J_D	U_D	R_D	G_D	Y_D	F_D	N_D	E_D
Savings	\$5.00	T_S	J_S	U_S	R_S	G_S	Y_S		
\$9.00		T_Q	J_Q	U_Q	R_Q	G_Q	Y_Q		
\$11.50		T_{II}	J_{II}	U_{II}	R_{II}	G_{II}	Y_{II}		
Color	Red	T_R	J_R	U_R					
Green		T_G	J_G	U_G					
yellow		T_Y	J_Y	U_Y					

"at least one of
 T_B, J_B, U_B is true"

express in
propositional logic

"at most one of
 T_B, J_B, U_B is true"

express in
propositional logic

but how do we say "exactly one" in propositional logic?

exactly one is true

Name	Item			Color			Savings		
	Trousers	Jacket	Umbrella	Red	Green	yellow	\$5.00	\$9.00	\$11.50
Benny	T_B	J_B	U_B	R_B	G_B	Y_B	F_B	N_B	E_B
Carla	T_C	J_C	U_C	R_C	G_C	Y_C	F_C	N_C	E_C
Daniella	T_D	J_D	U_D	R_D	G_D	Y_D	F_D	N_D	E_D
	\$5.00	T_5	J_5	U_5	R_5	G_5	Y_5		
	\$9.00	T_9	J_9	U_9	R_9	G_9	Y_9		
	\$11.50	T_{11}	J_{11}	U_{11}	R_{11}	G_{11}	Y_{11}		
Color	Red	T_R	J_R	U_R					
	Green	T_G	J_G	U_G					
	yellow	T_Y	J_Y	U_Y					

exactly one is true

"at least one of T_B, J_B, U_B is true"

$$T_B \vee J_B \vee U_B$$

"at most one of T_B, J_B, U_B is true"

$$(\neg T_B \vee \neg J_B) \wedge (\neg T_B \vee \neg U_B) \wedge (\neg J_B \vee \neg U_B)$$

"exactly one of T_B, J_B, U_B is true"

$$(T_B \vee J_B \vee U_B) \wedge$$

$$((\neg T_B \vee \neg J_B) \wedge (\neg T_B \vee \neg U_B) \wedge (\neg J_B \vee \neg U_B))$$

a useful shorthand

"at most one of A,B,C,D is true"

$$(\neg A \vee \neg B) \wedge (\neg A \vee \neg C) \wedge (\neg A \vee \neg D) \wedge (\neg B \vee \neg C) \wedge (\neg B \vee \neg D) \wedge (\neg C \vee \neg D)$$

$$\bigwedge (\neg x \vee \neg y)$$

$$x, y \in \{A, B, C, D\}:$$

$$x \neq y$$