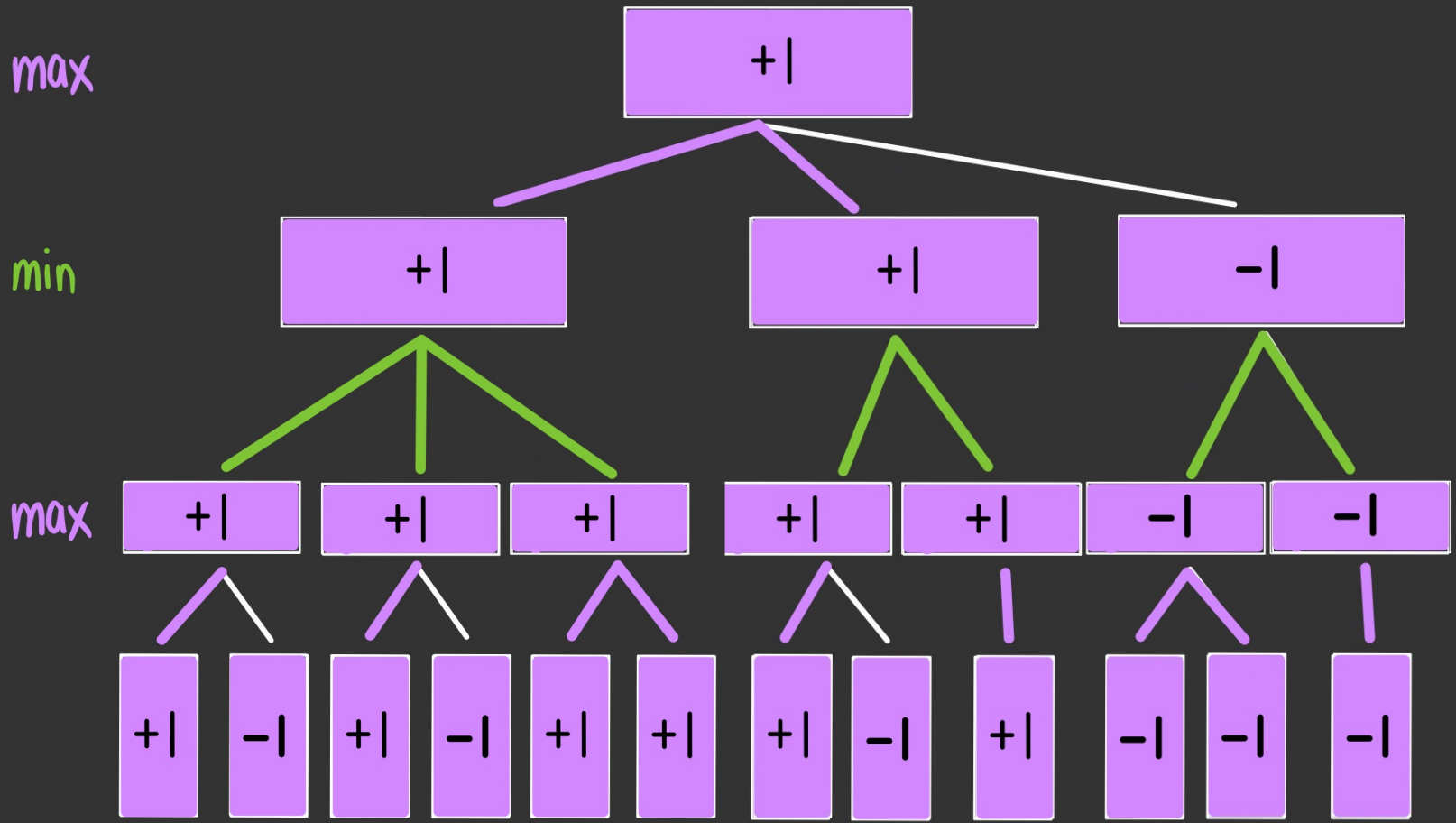
alpha-beta pruning

CSCI 373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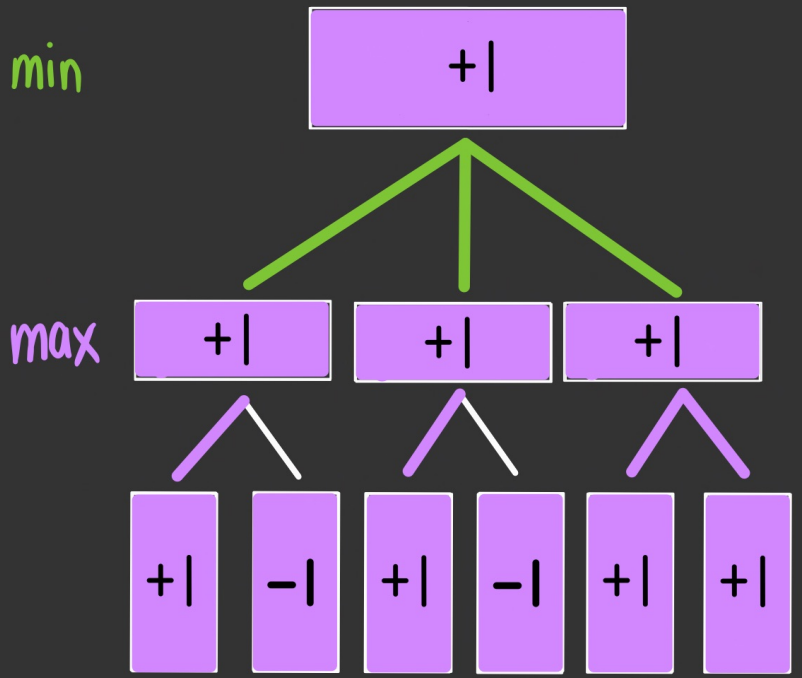
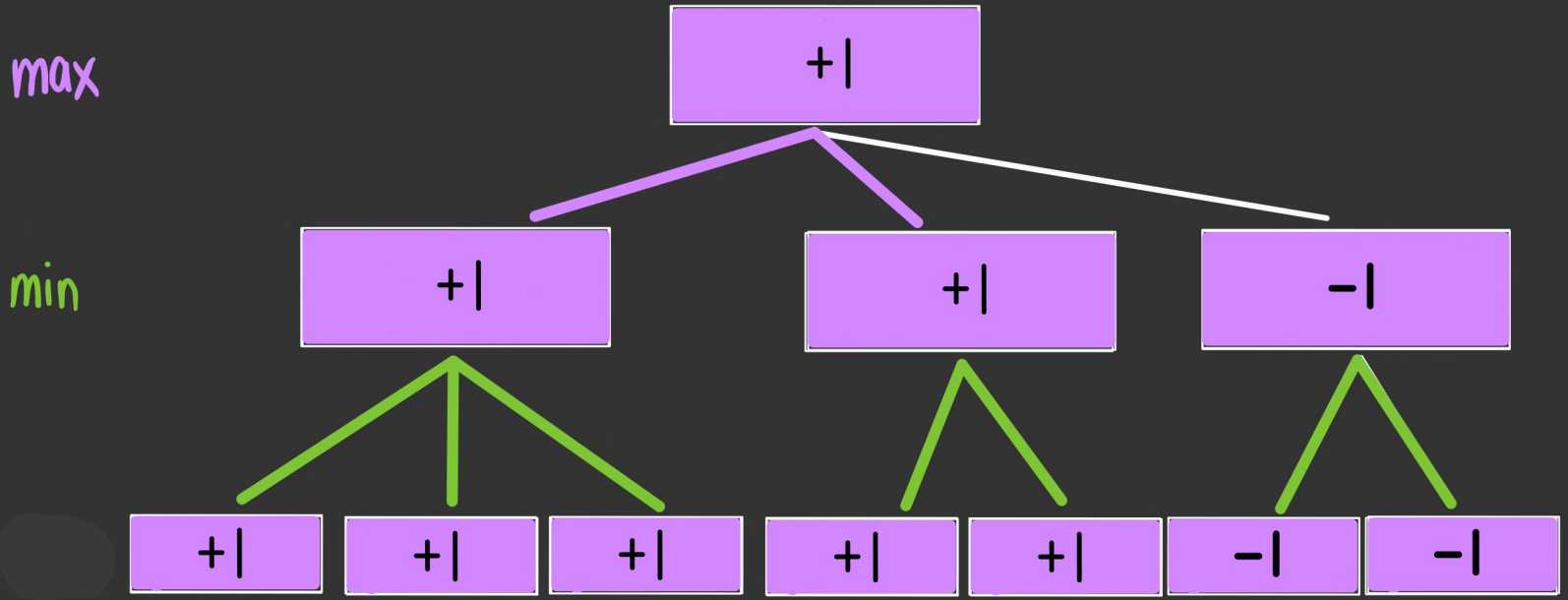


consider a min layer followed by a max layer



or a max layer followed by min layer

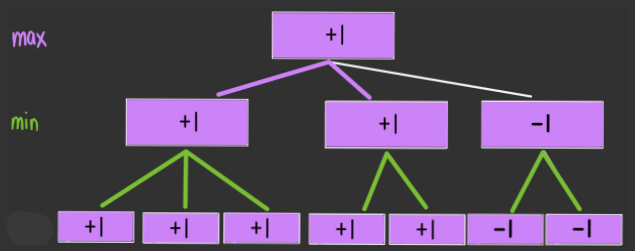
SCIENTIFIC AMERICAN MIND

BEHAVIOR

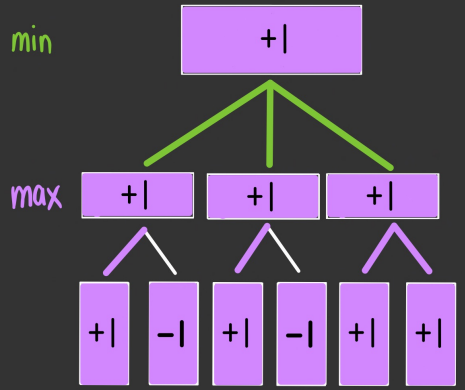
Are People Naturally Inclined to Cooperate or Be Selfish?

By Matthew Robison on September 1, 2014

It seems that human nature supports both prosocial and selfish traits. Genetic studies have made some progress toward identifying their biological roots. By comparing identical twins, who share nearly 100 percent of their genes, and fraternal twins, who share about half, researchers have found overwhelming evidence for genetic effects on behaviors such as sharing and empathy. In these twin studies, identical and fraternal twins are placed in hypothetical scenarios and asked, for example, to split a sum of money with a peer. Such studies often also rely on careful psychological assessments and DNA analysis.



max followed by min:
people are fundamentally
Selfish



min followed by max:
people are fundamentally
Selfless

max followed by min:
people are fundamentally
selfish

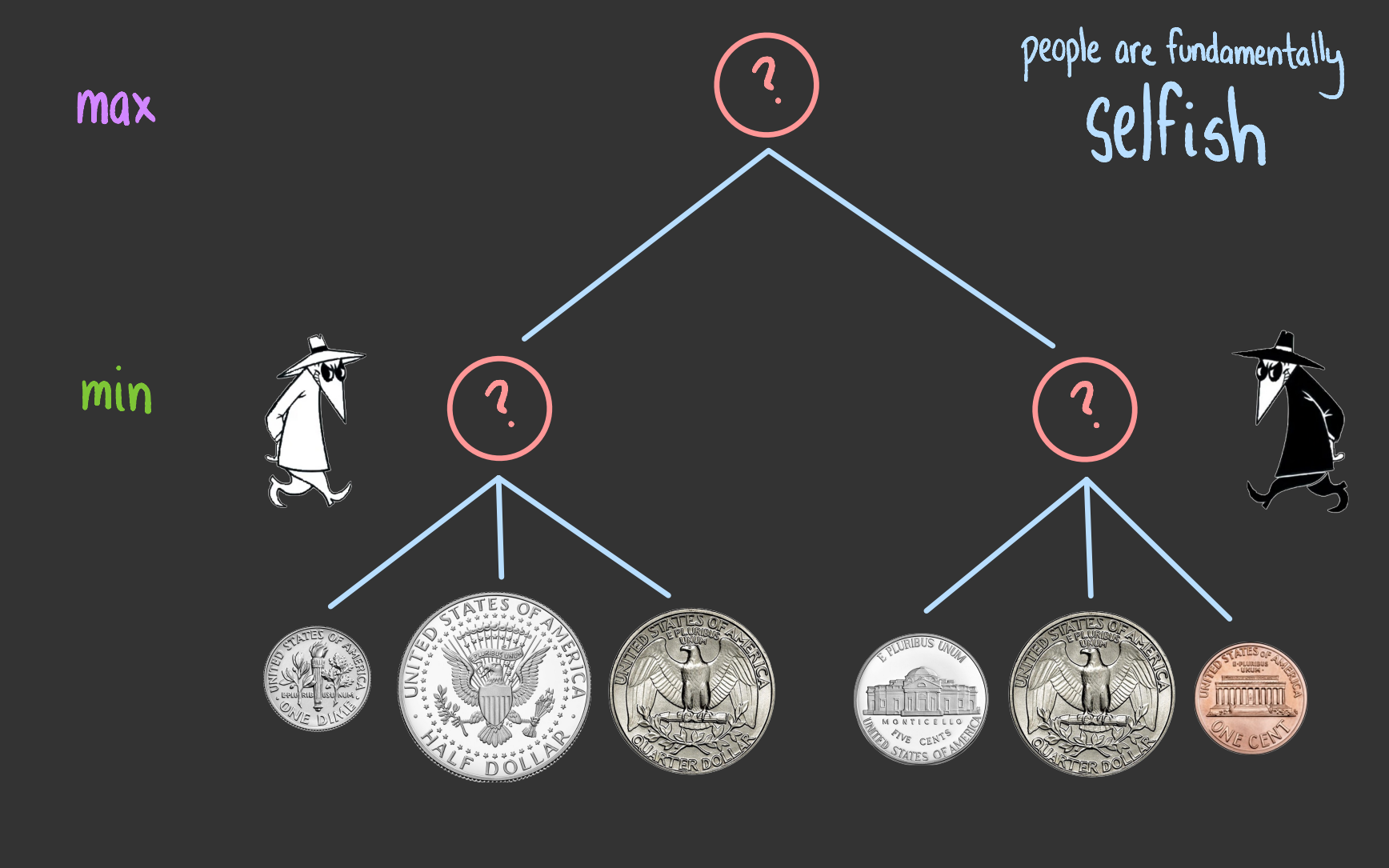
an edict comes down:
someone in the room
must give me a coin

max

min

people are fundamentally

selfish



max

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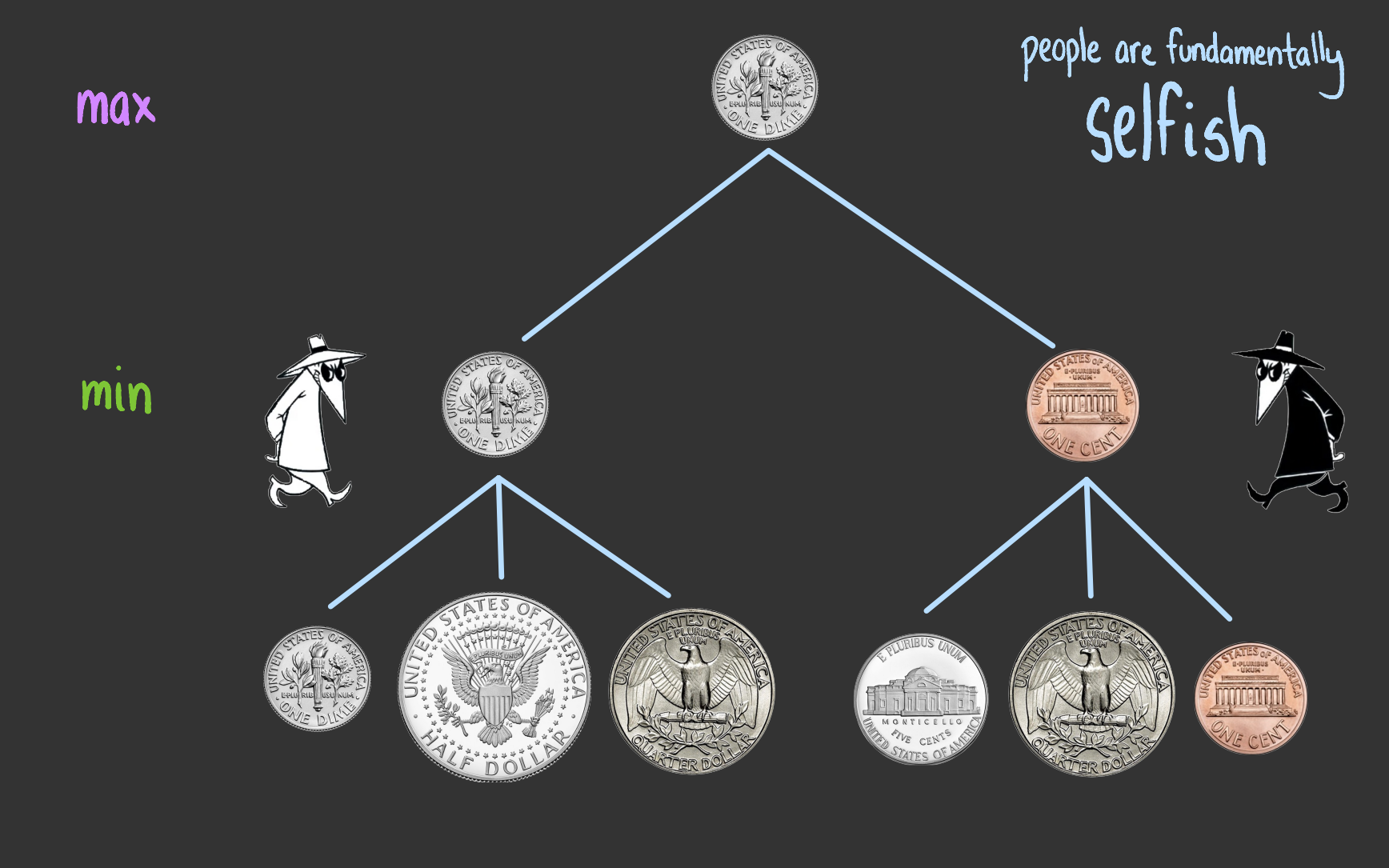
people are fundamentally

selfish

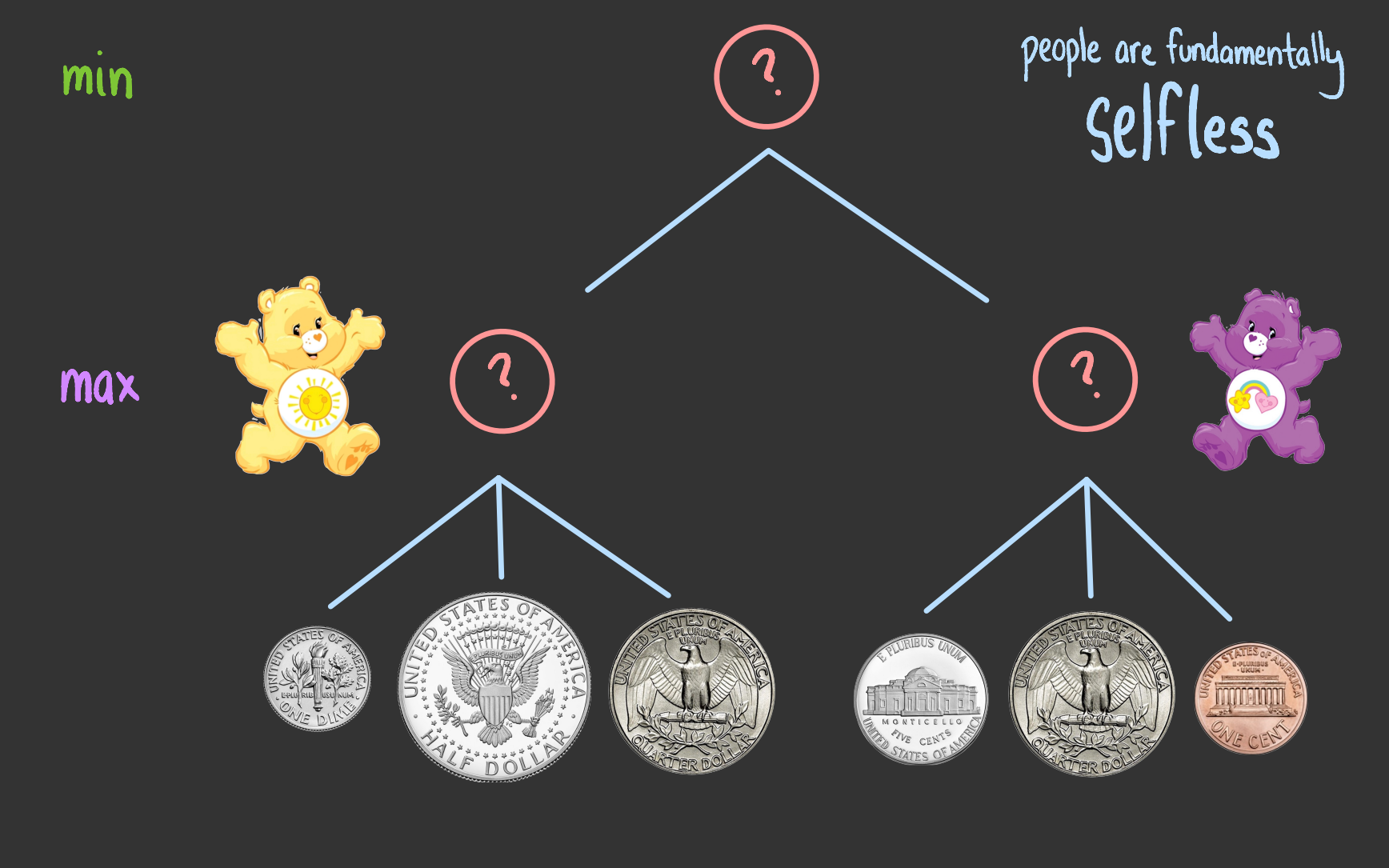


min

max

people are fundamentally

selfless



min

max

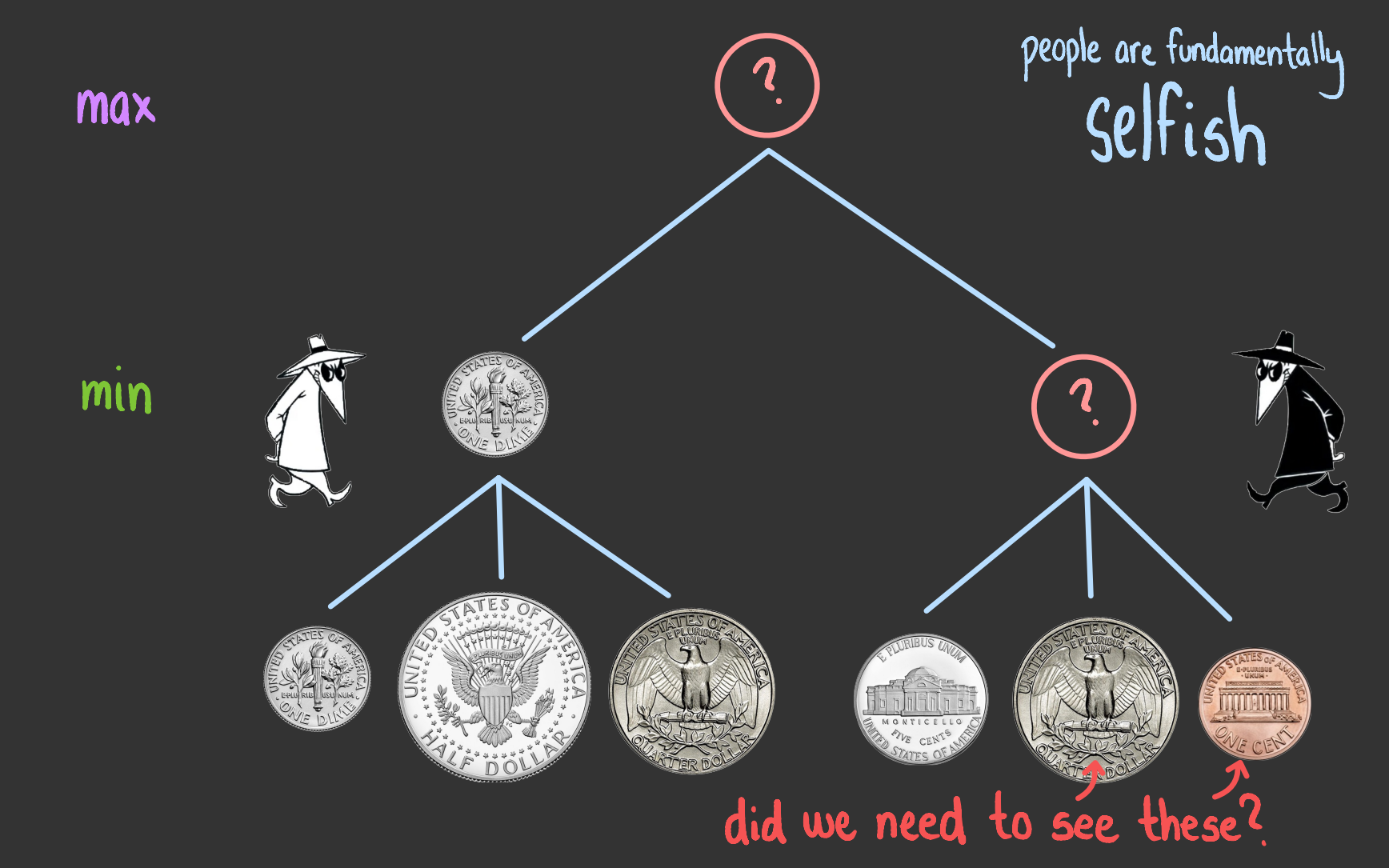
people are fundamentally

Selfless

max

min

people are fundamentally selfish



max

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people are fundamentally

selfish

?

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no

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max

people are fundamentally

selfless

did we need to see these?

min

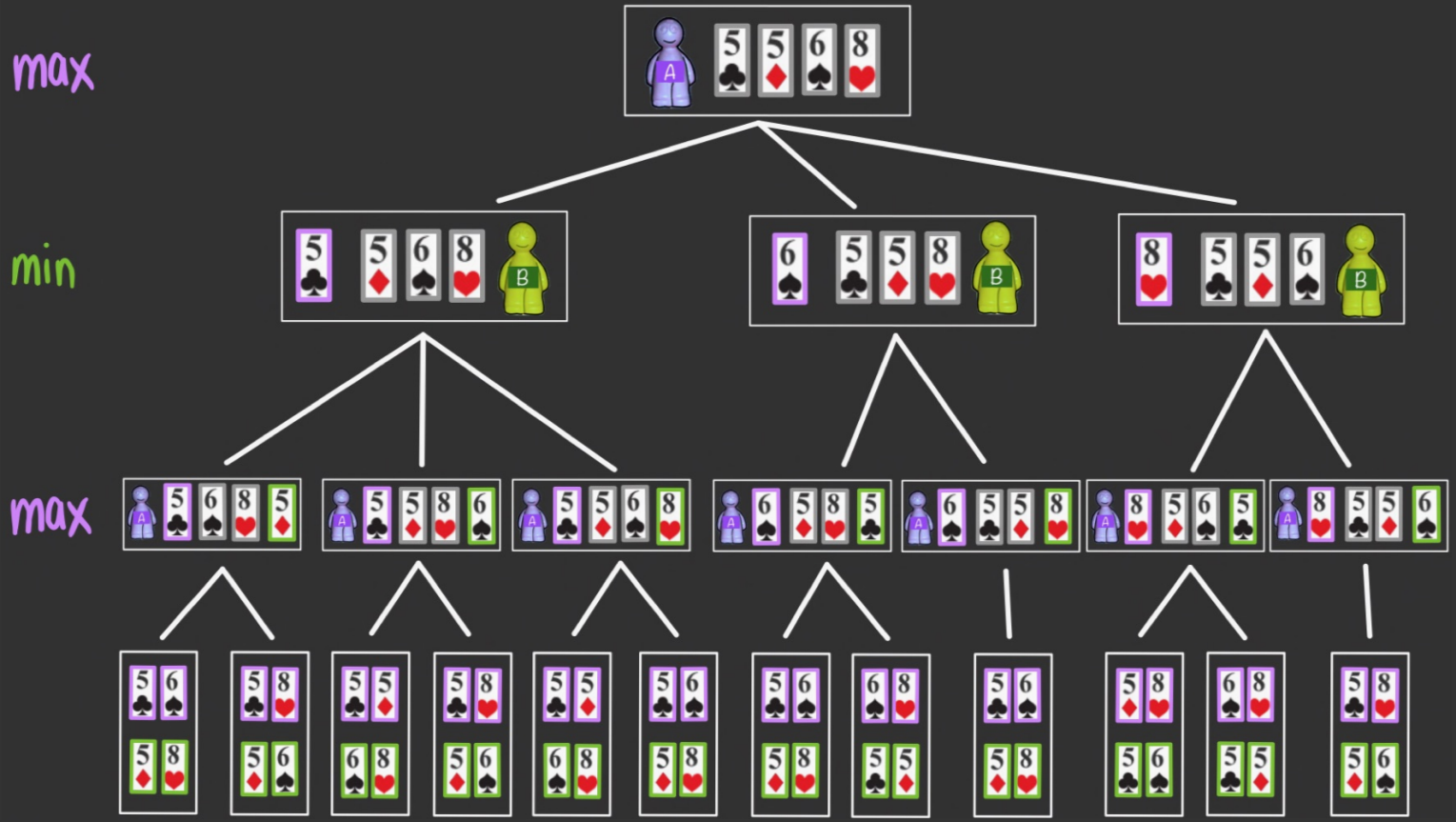
max

people are fundamentally

selfless

≥

no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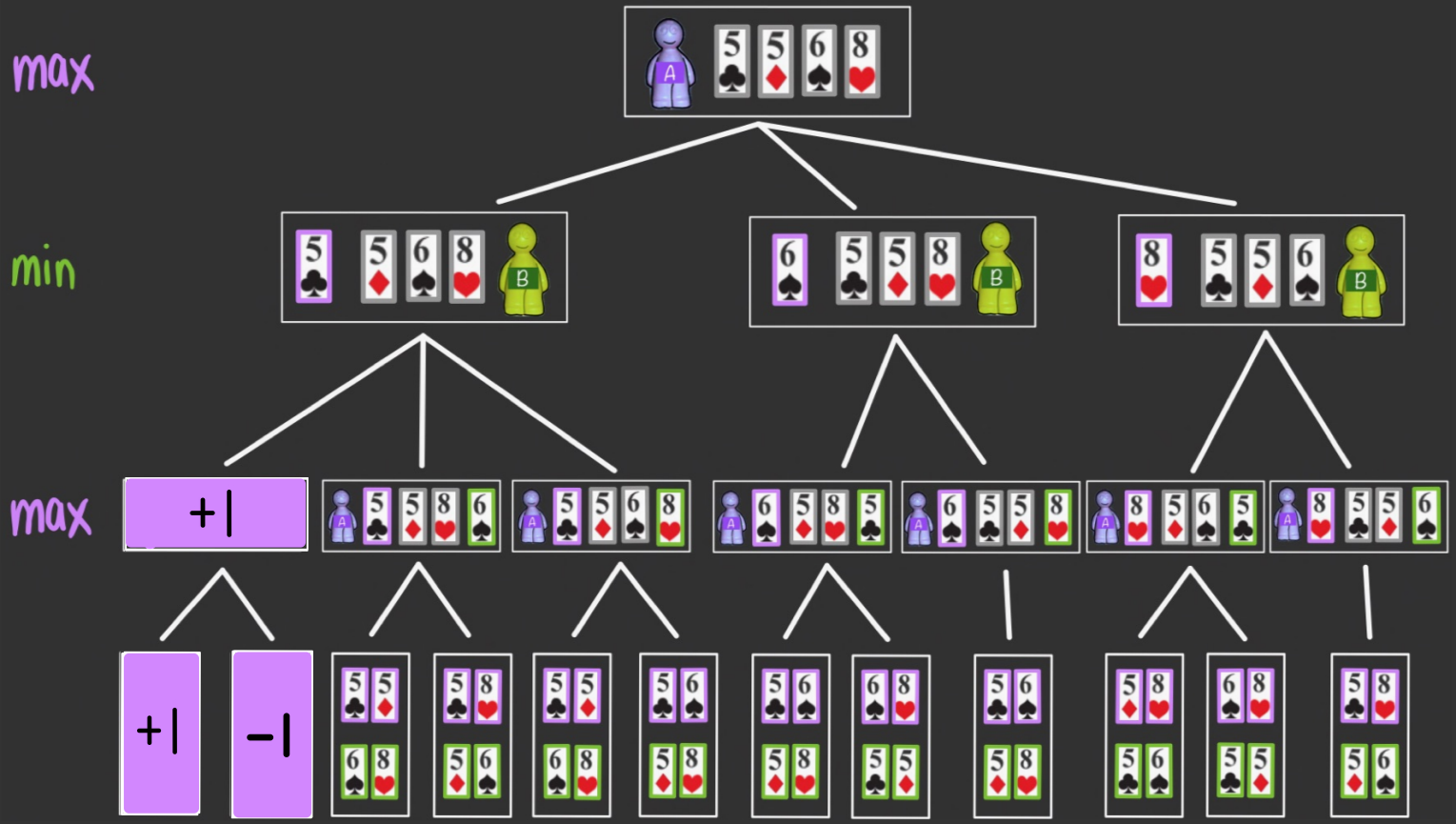
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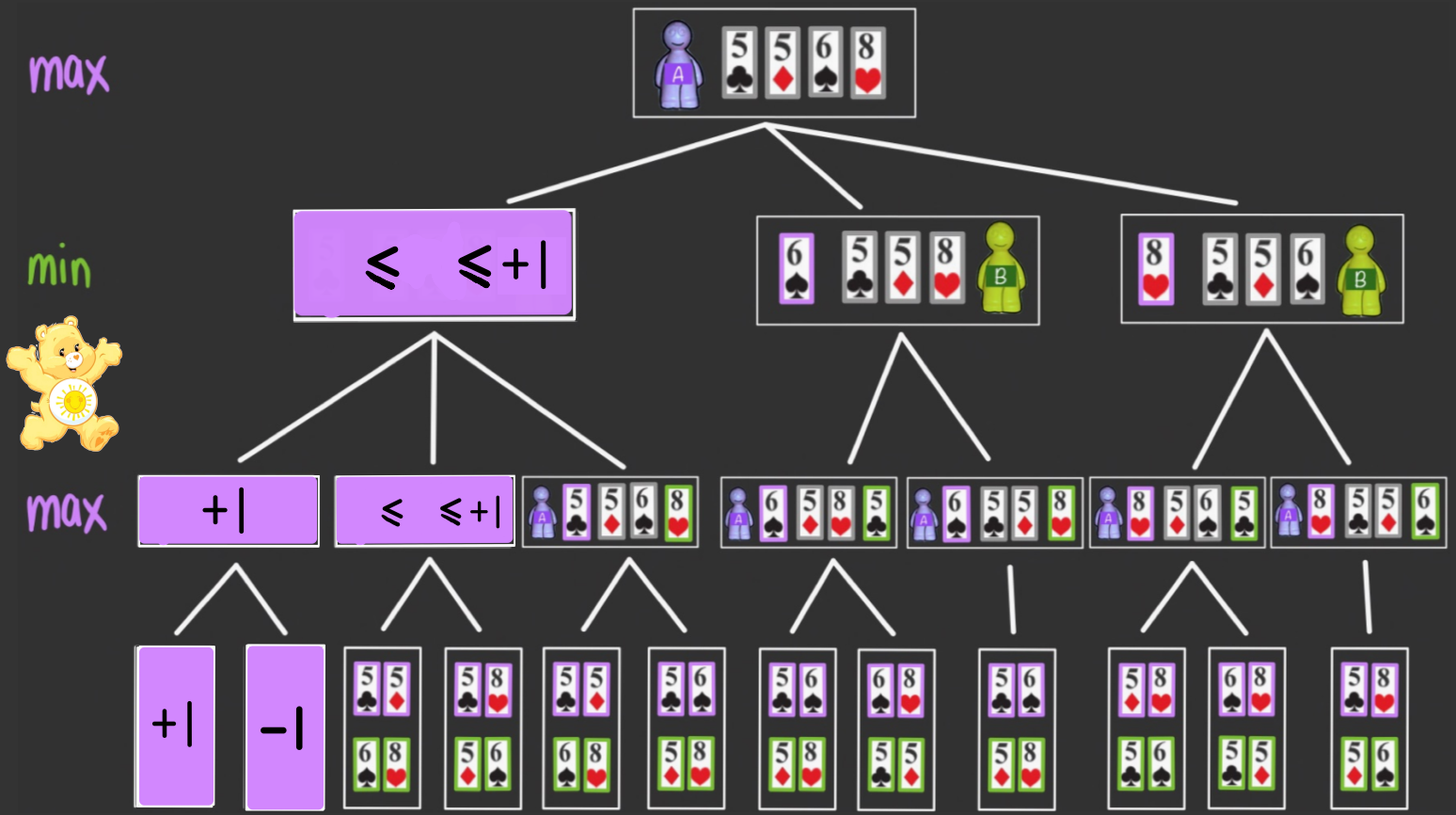
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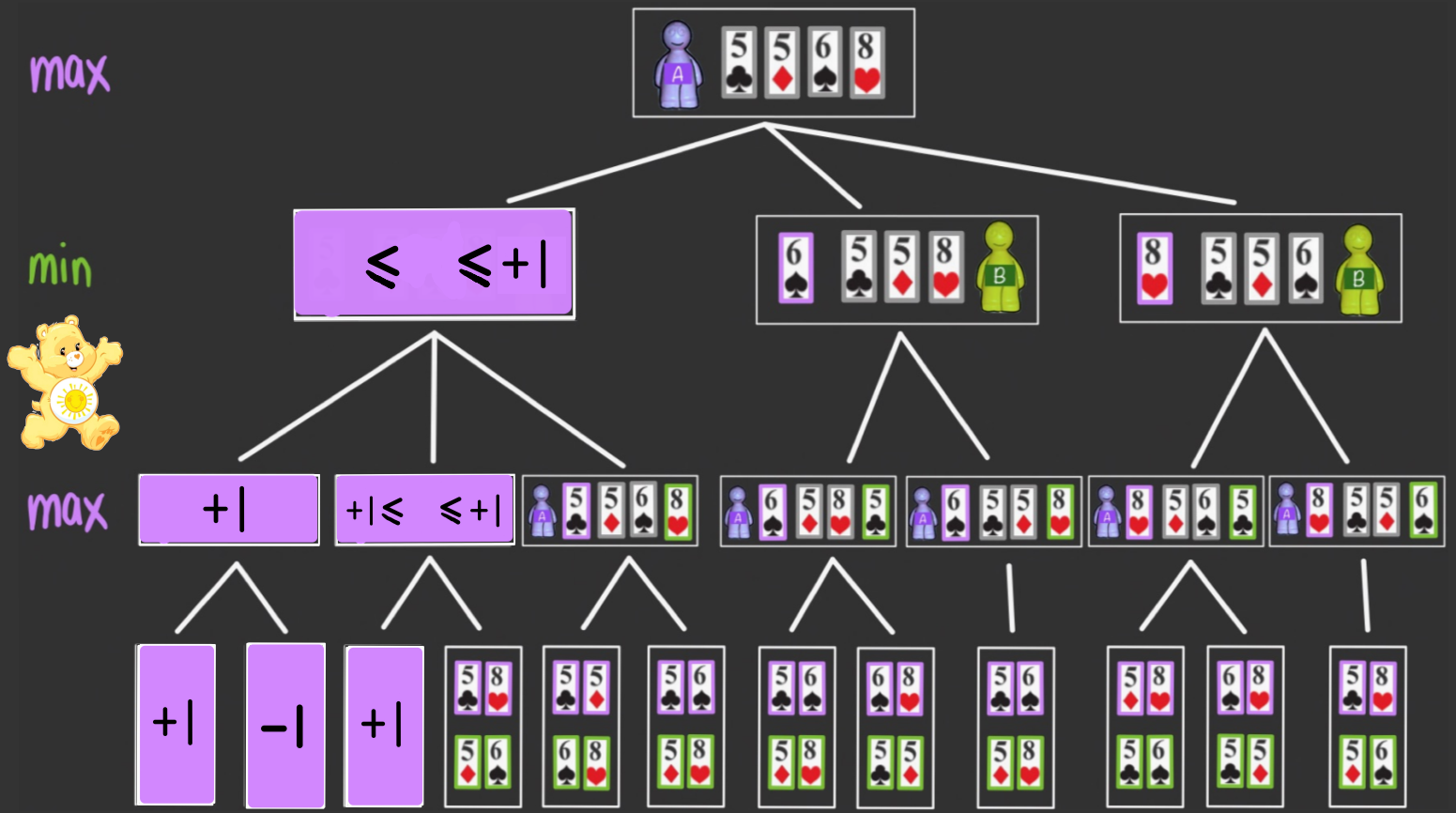
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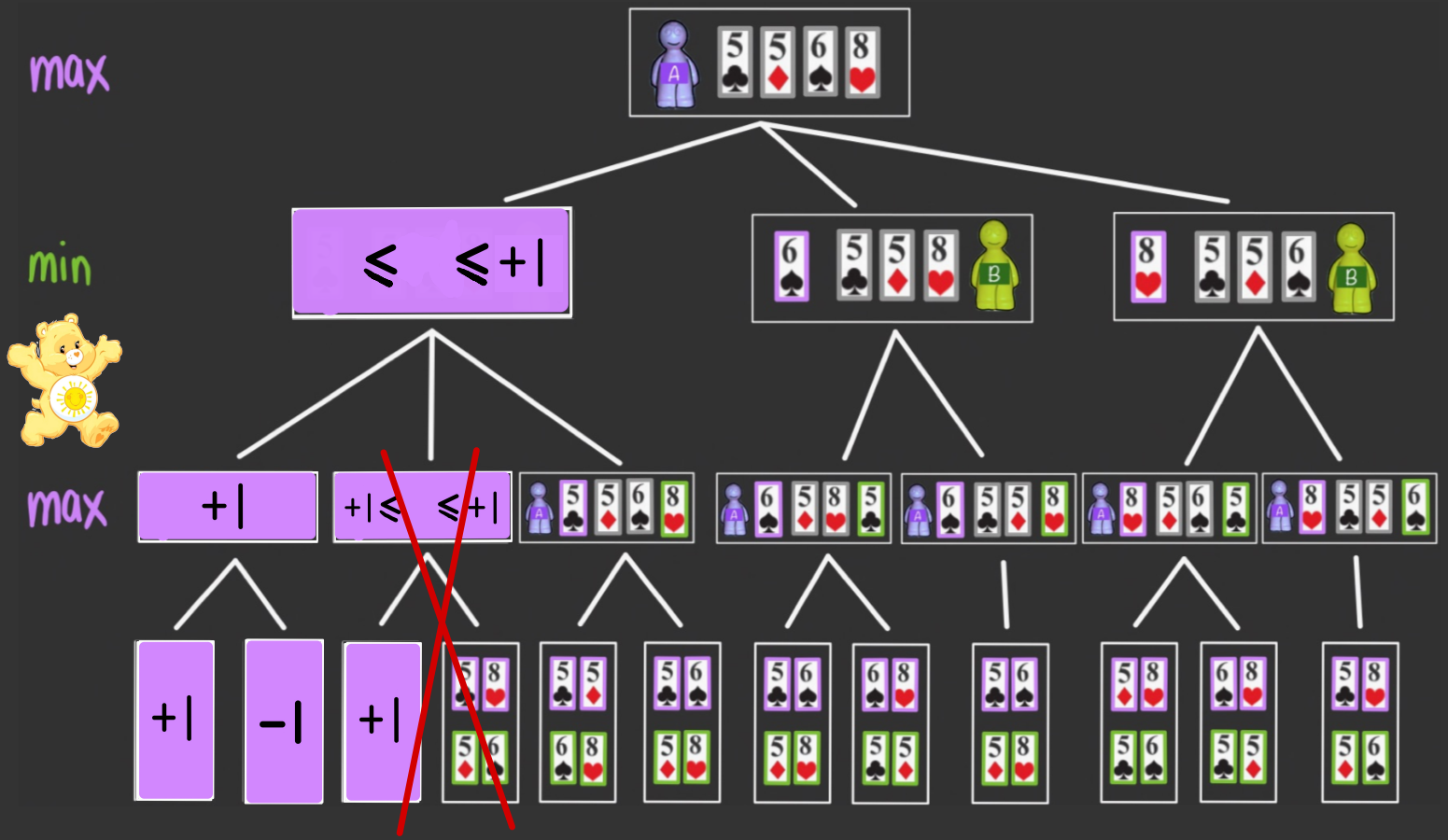
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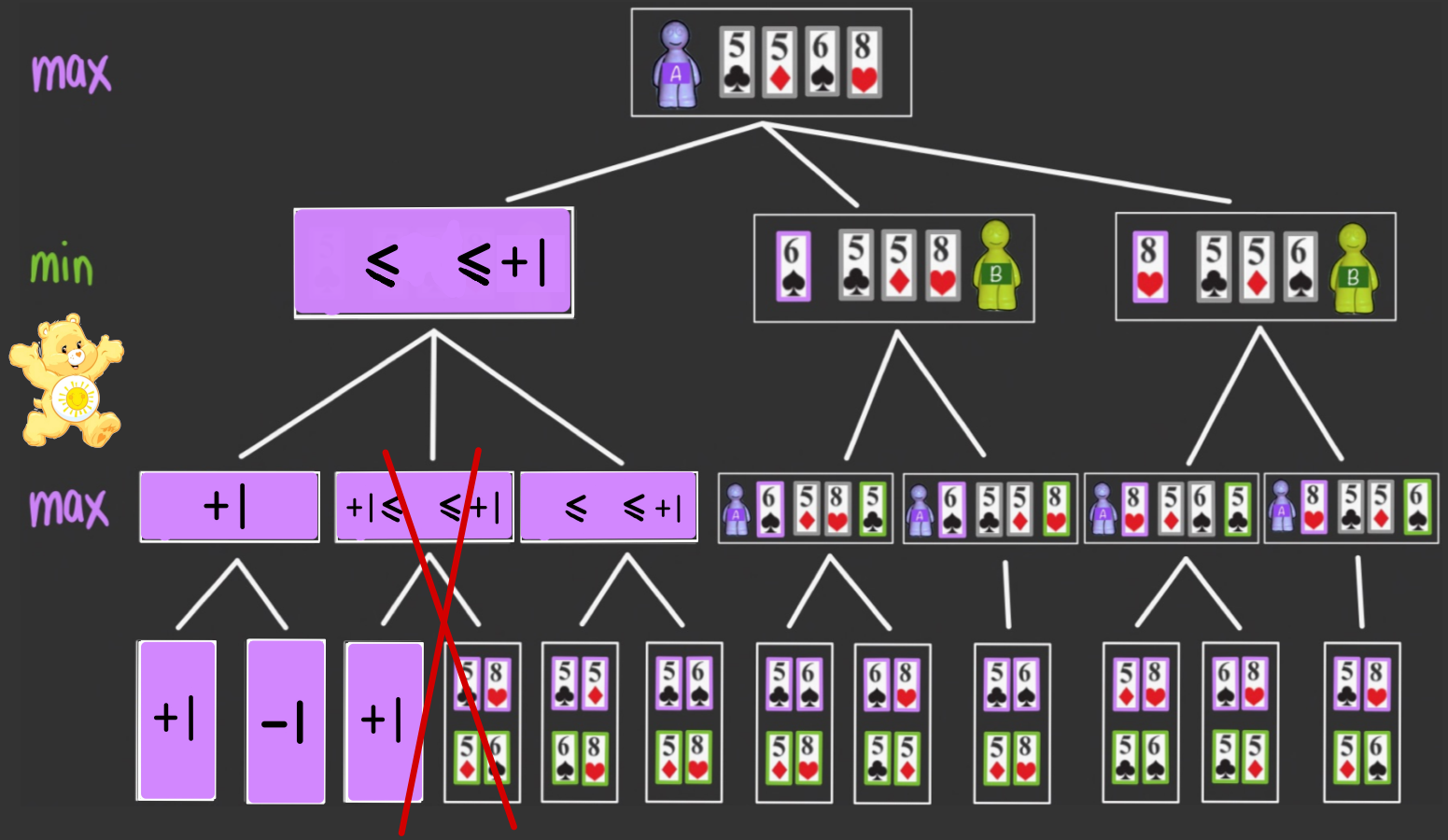
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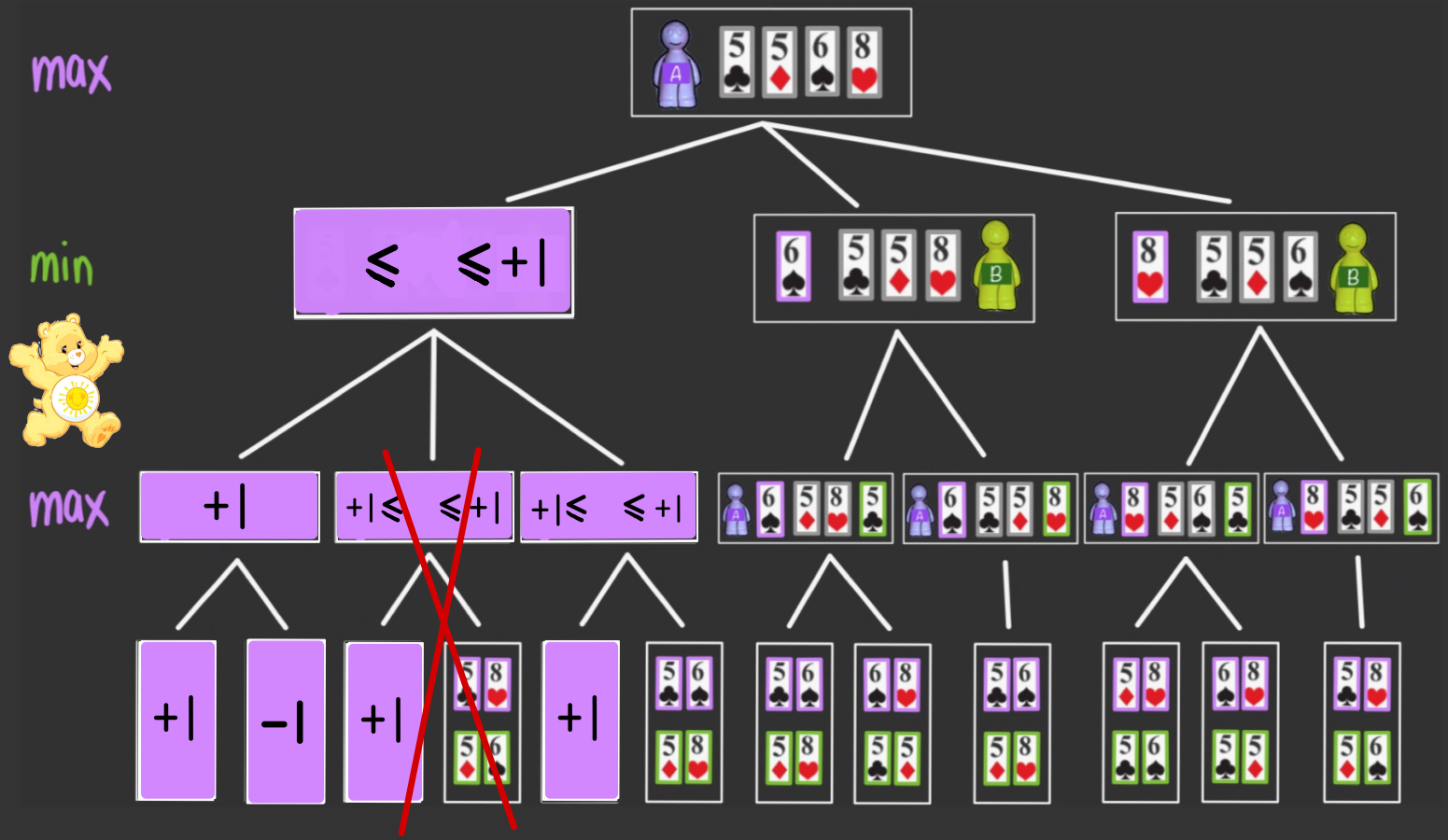
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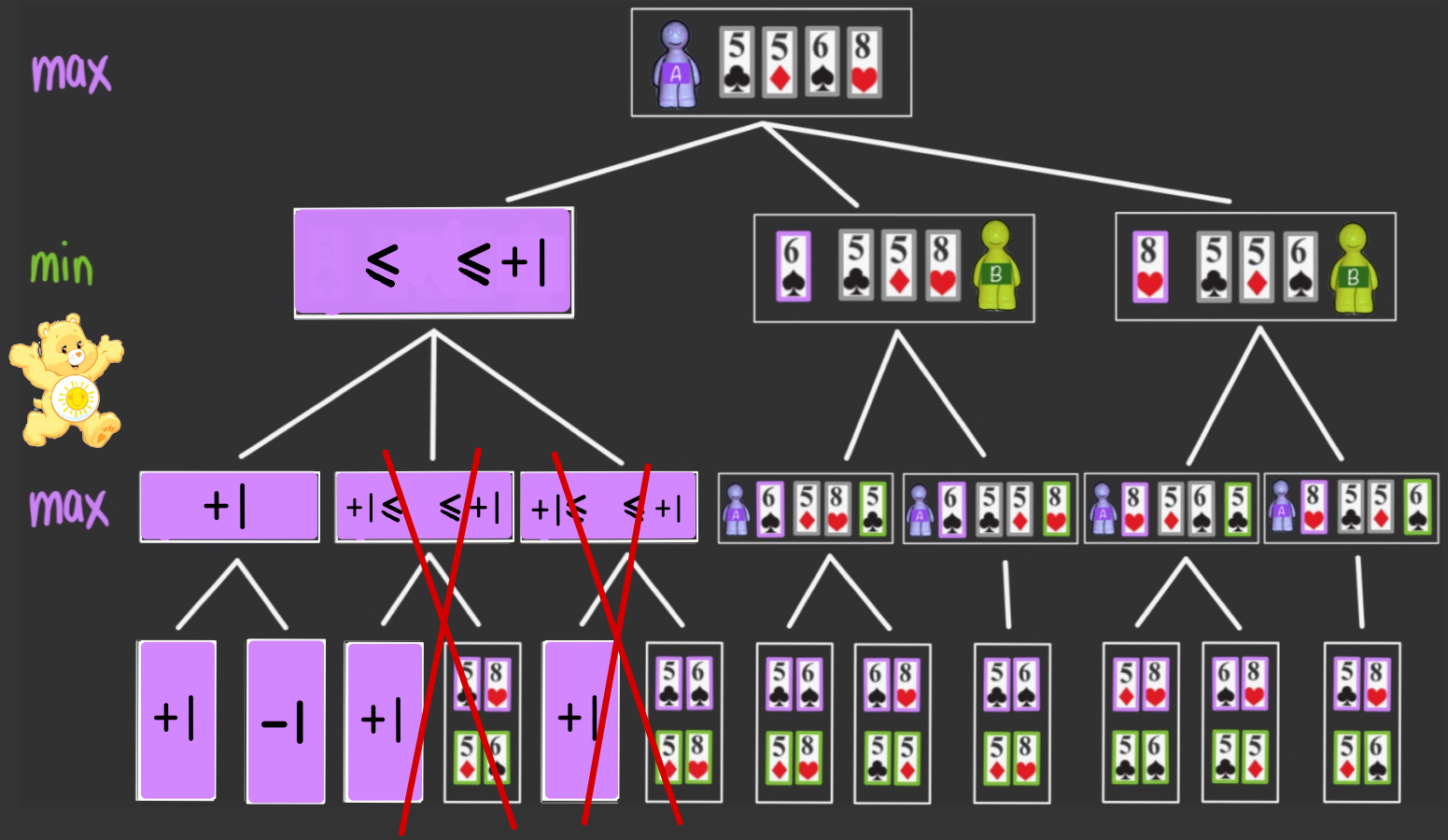
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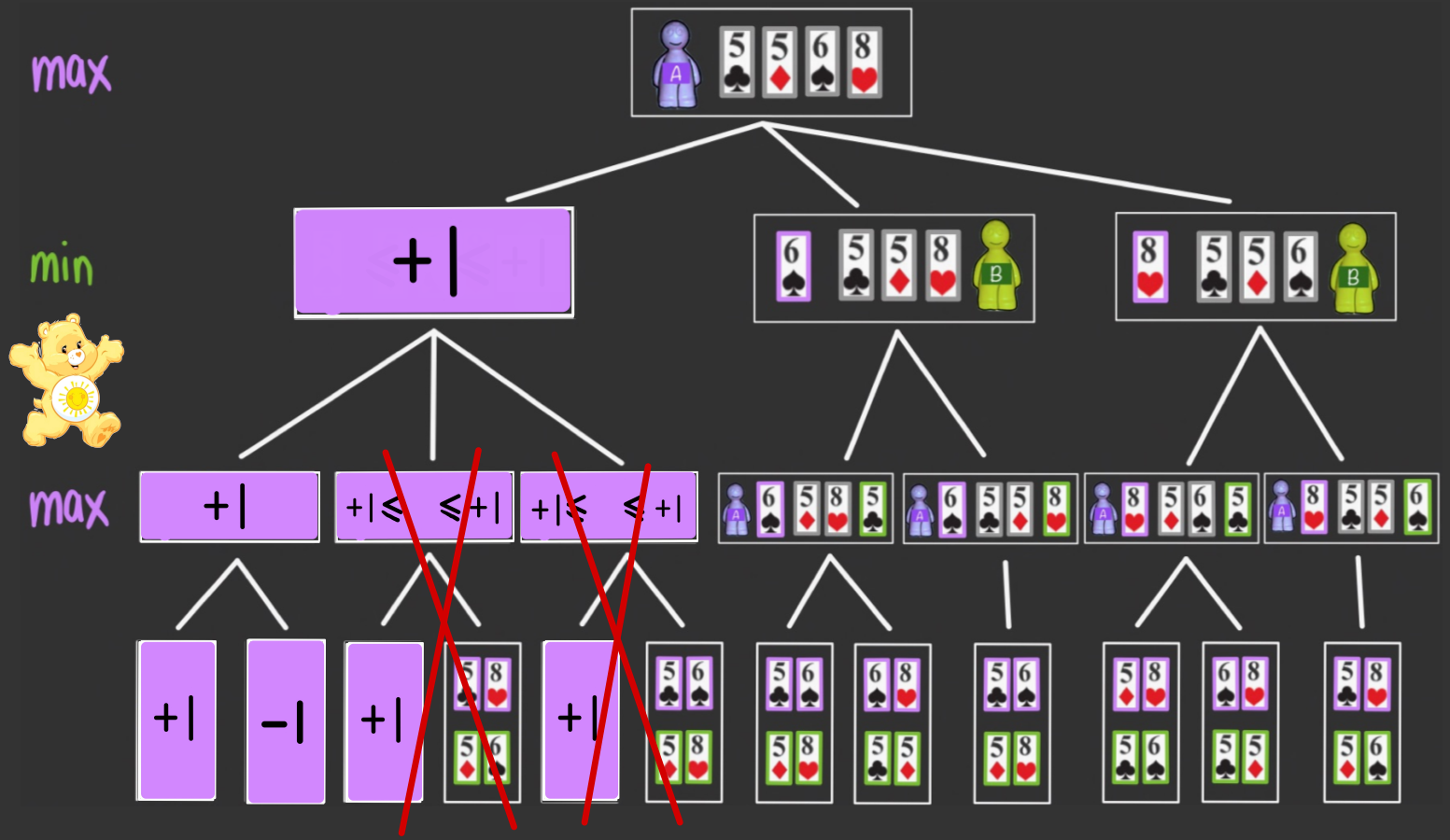
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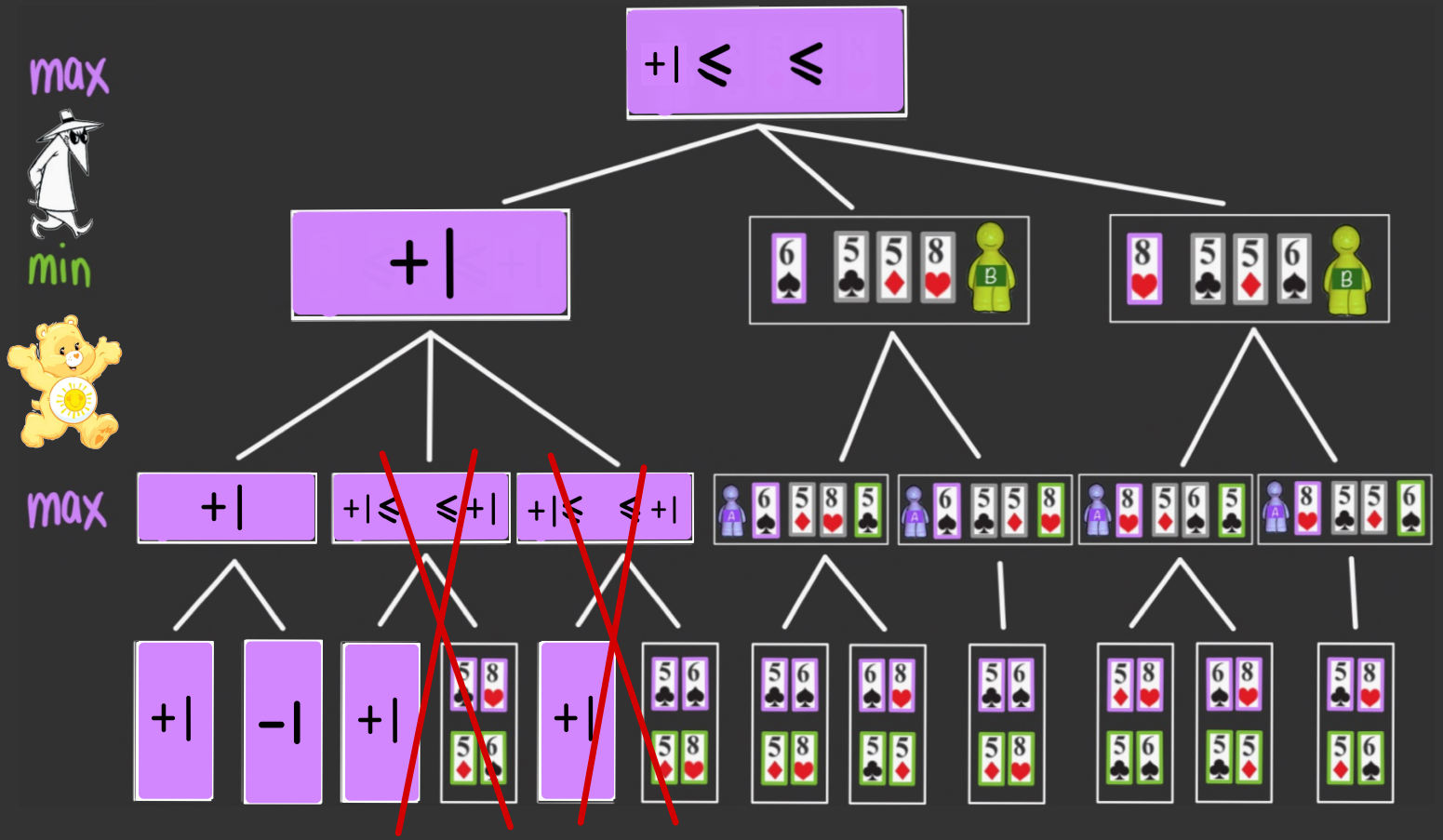
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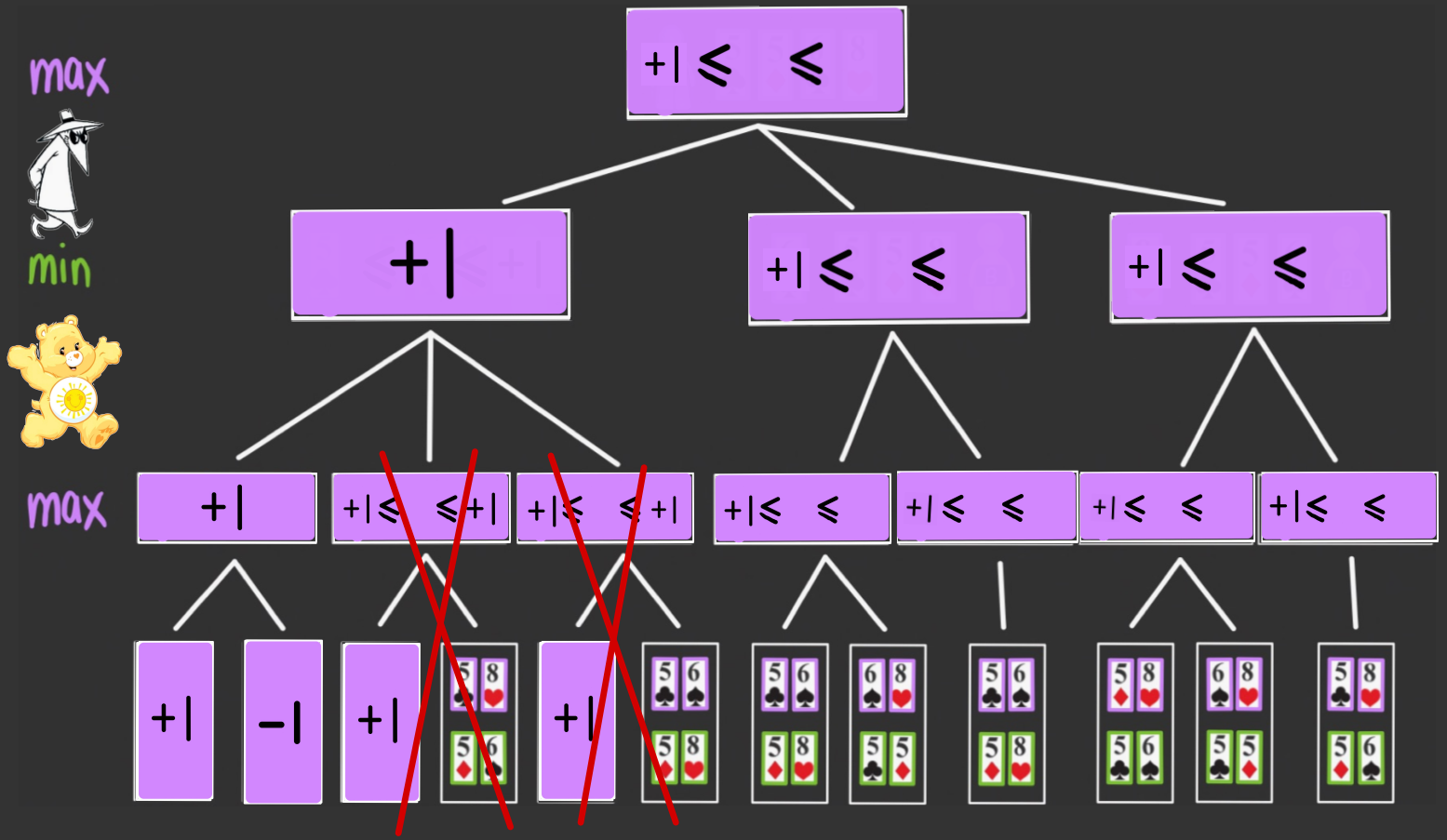
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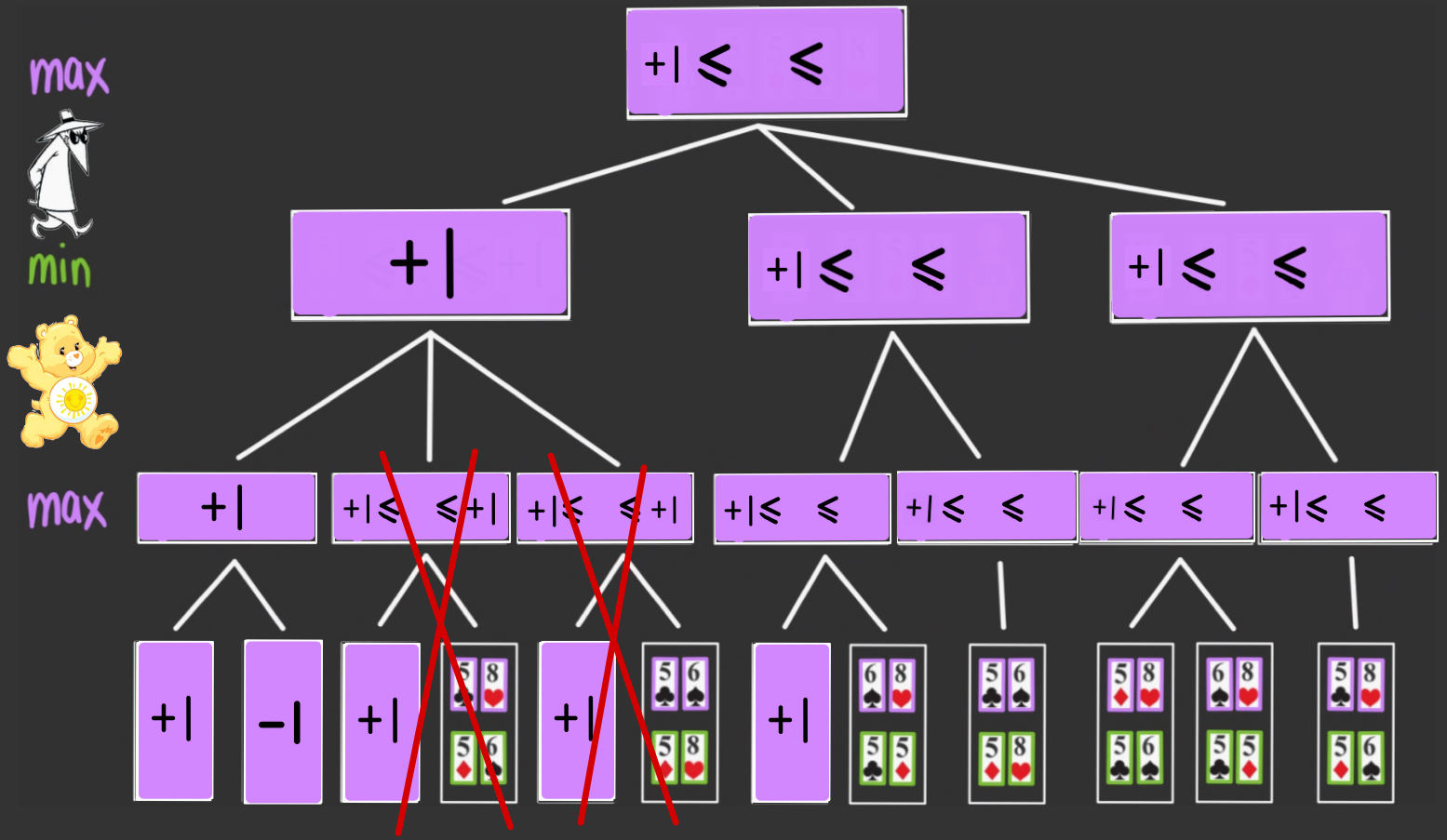
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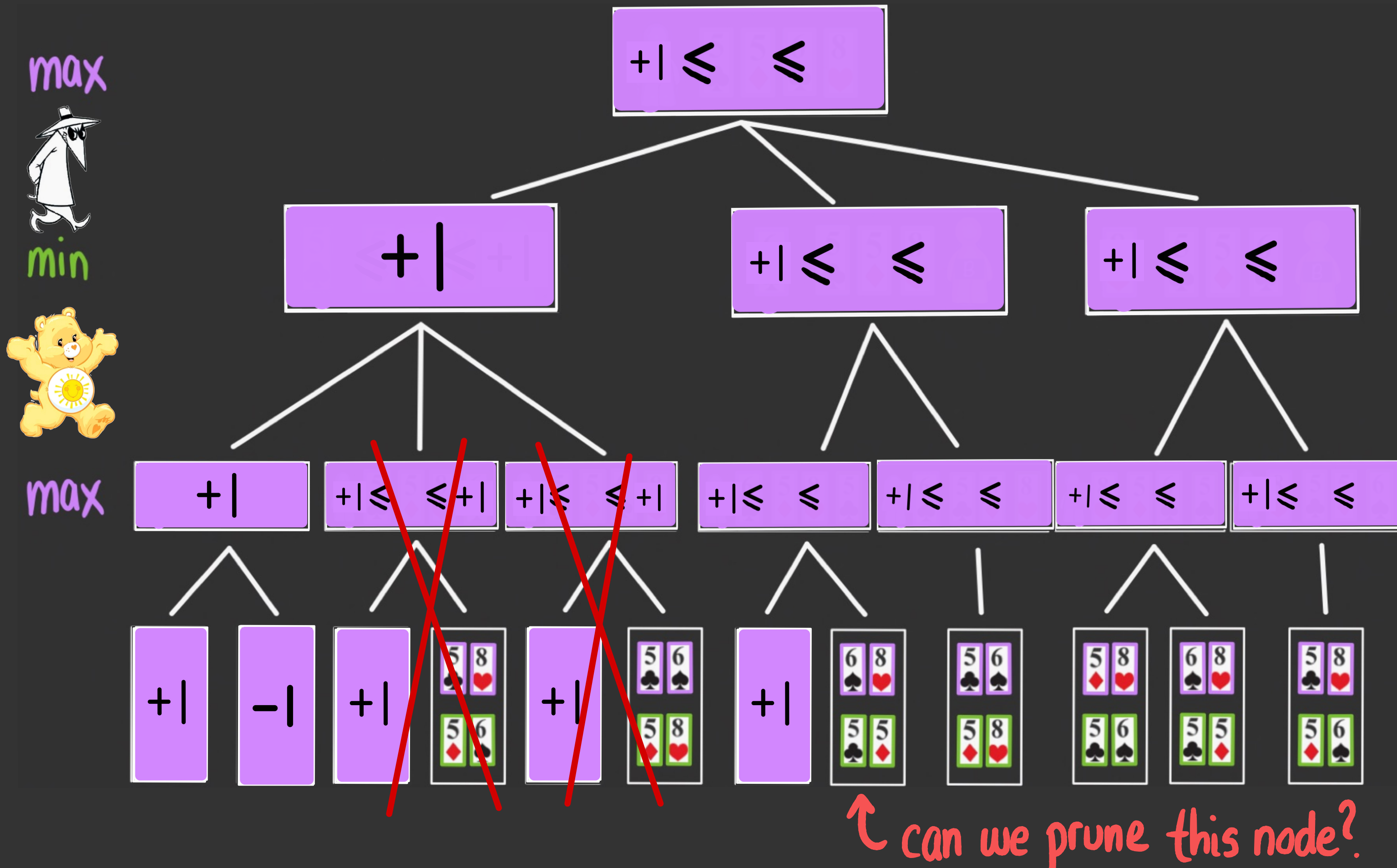

max
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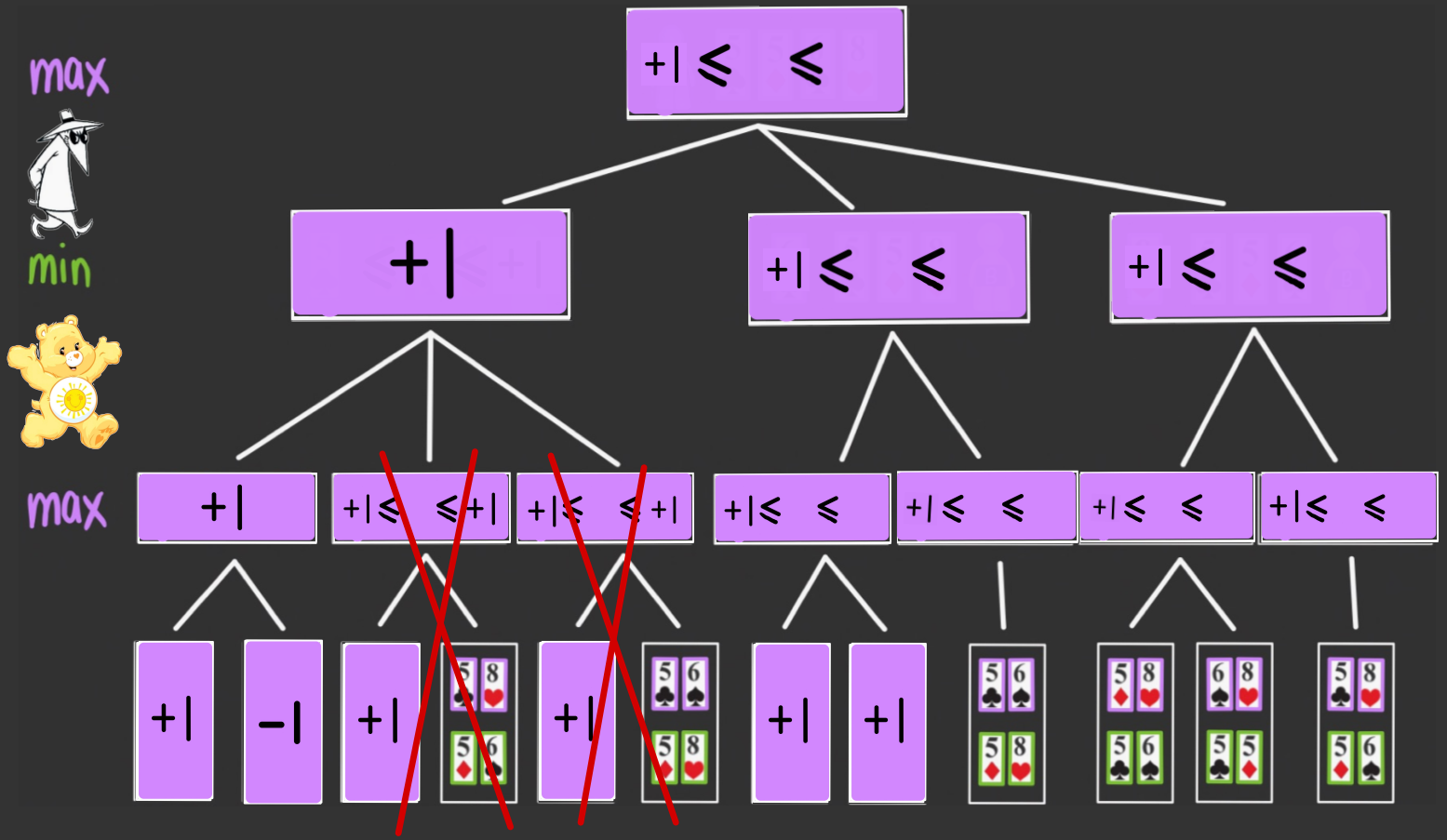
max
can we prune this node?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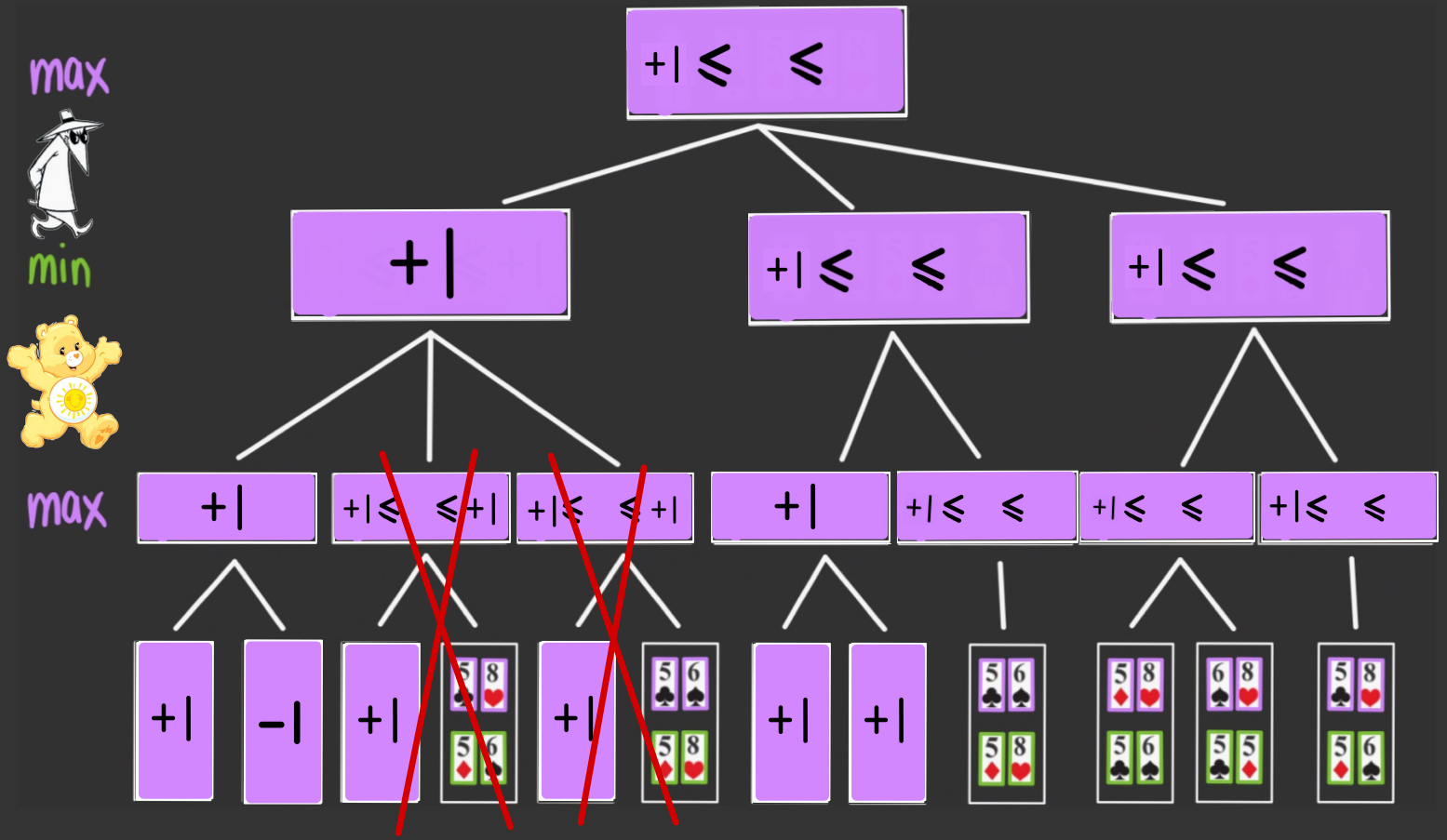
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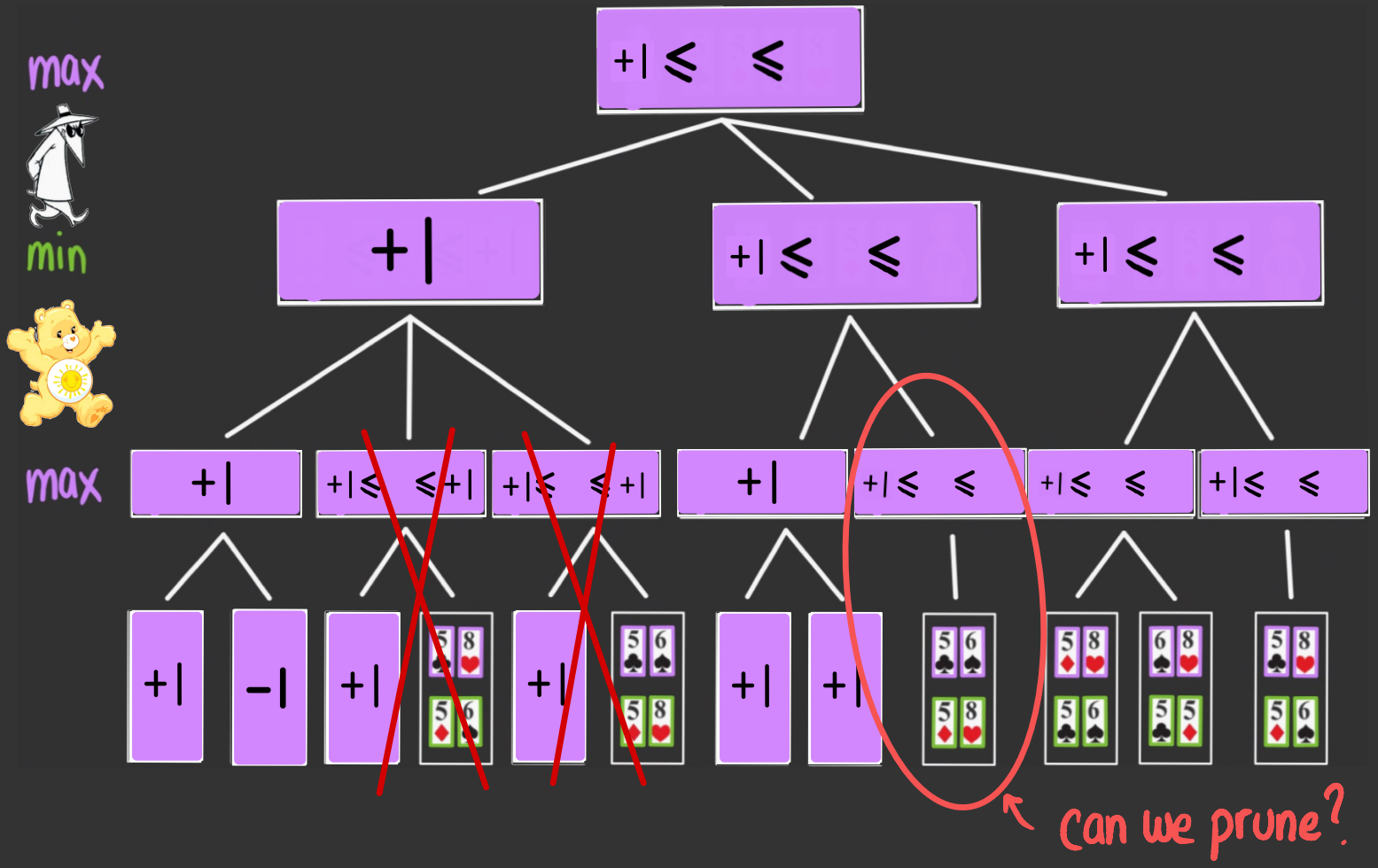
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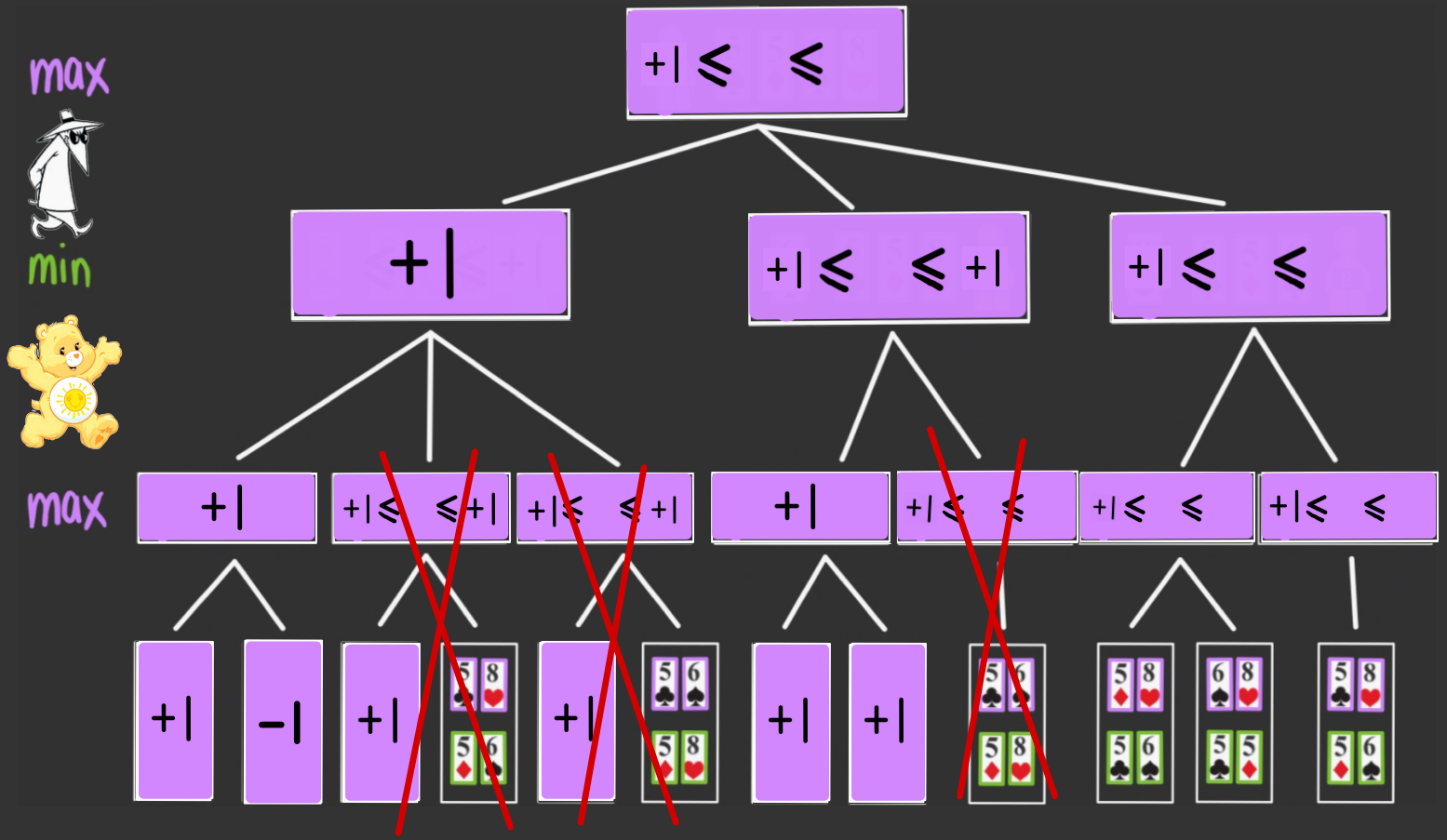
Can we prune?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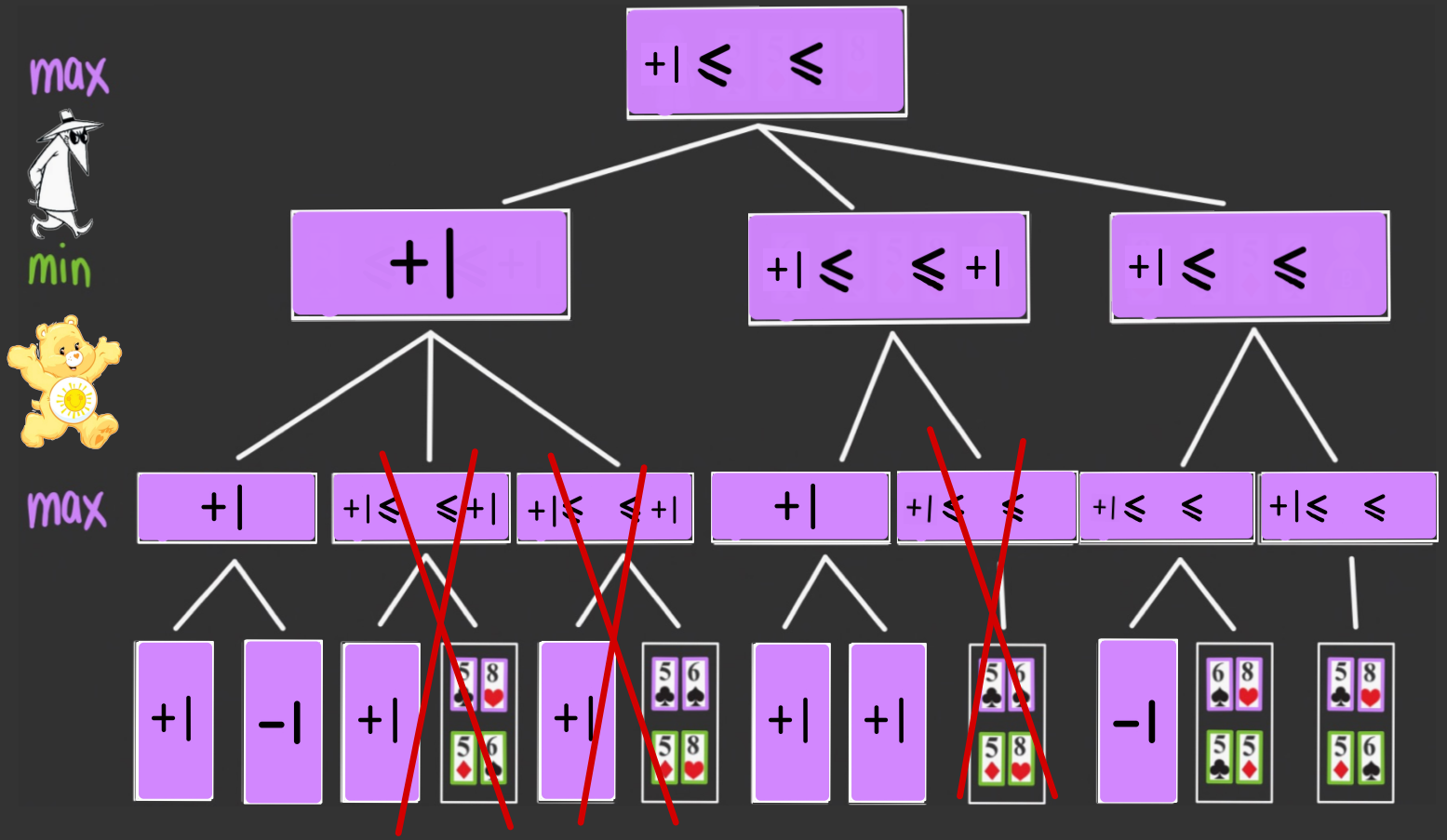
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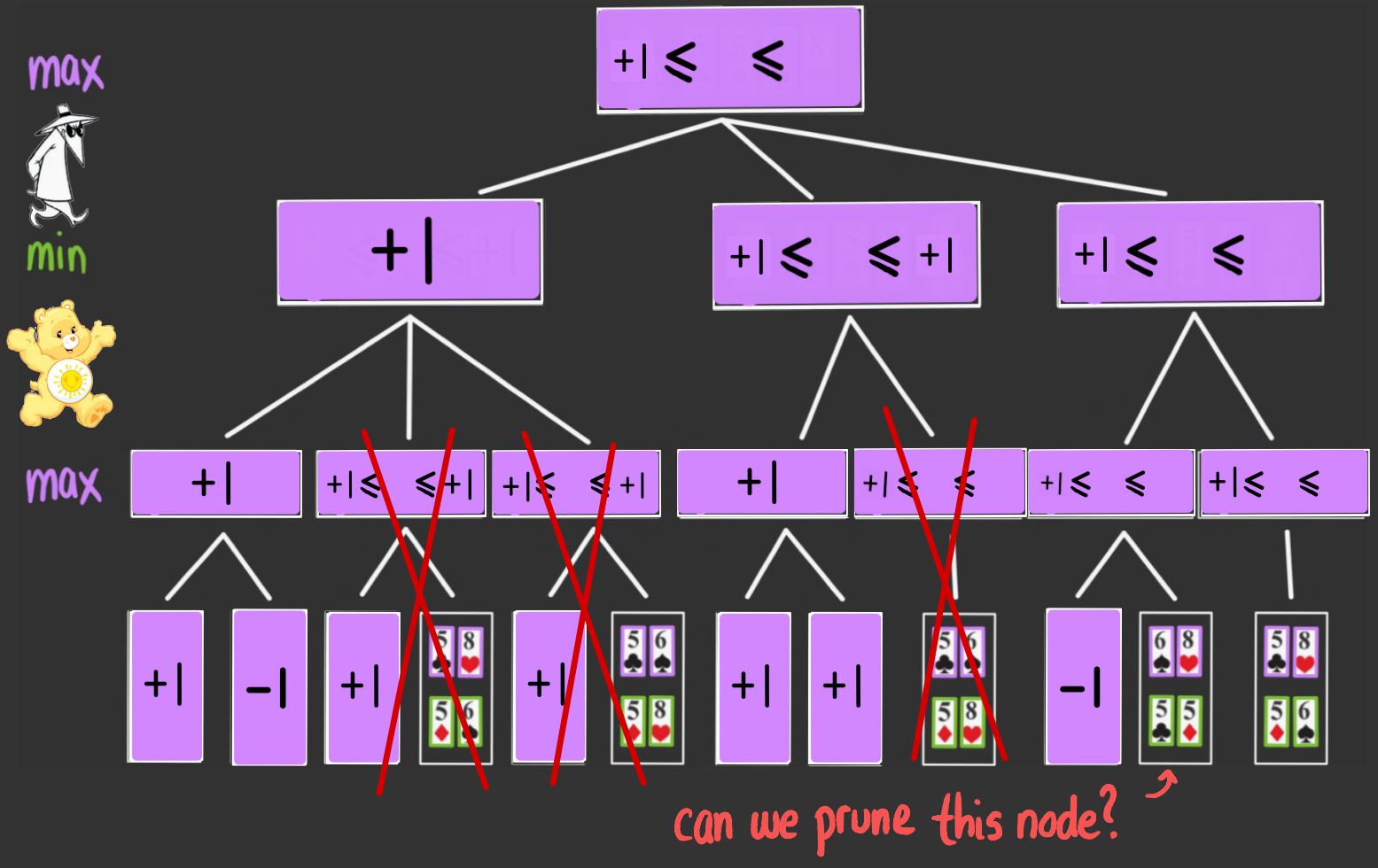
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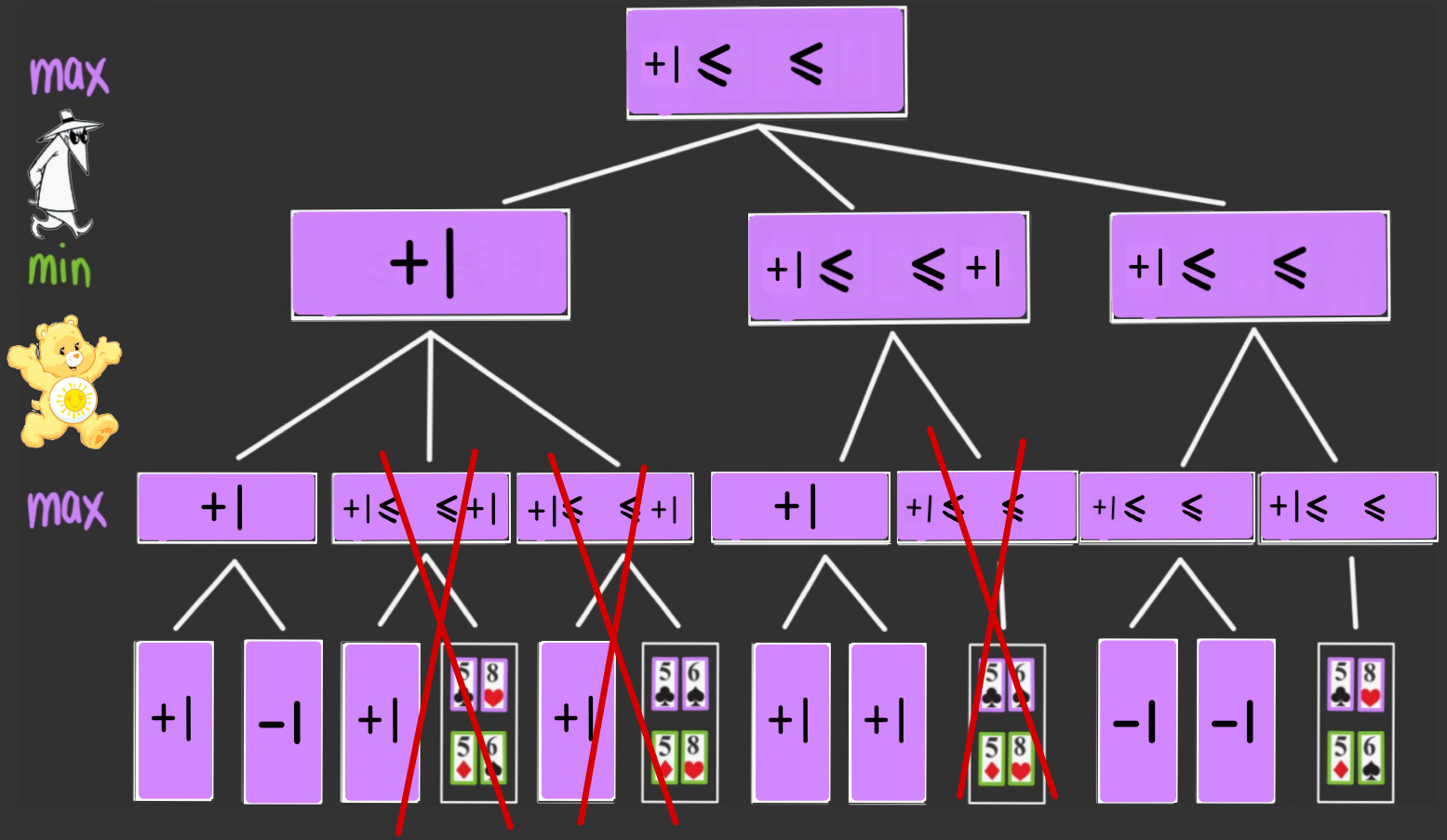
can we prune this node?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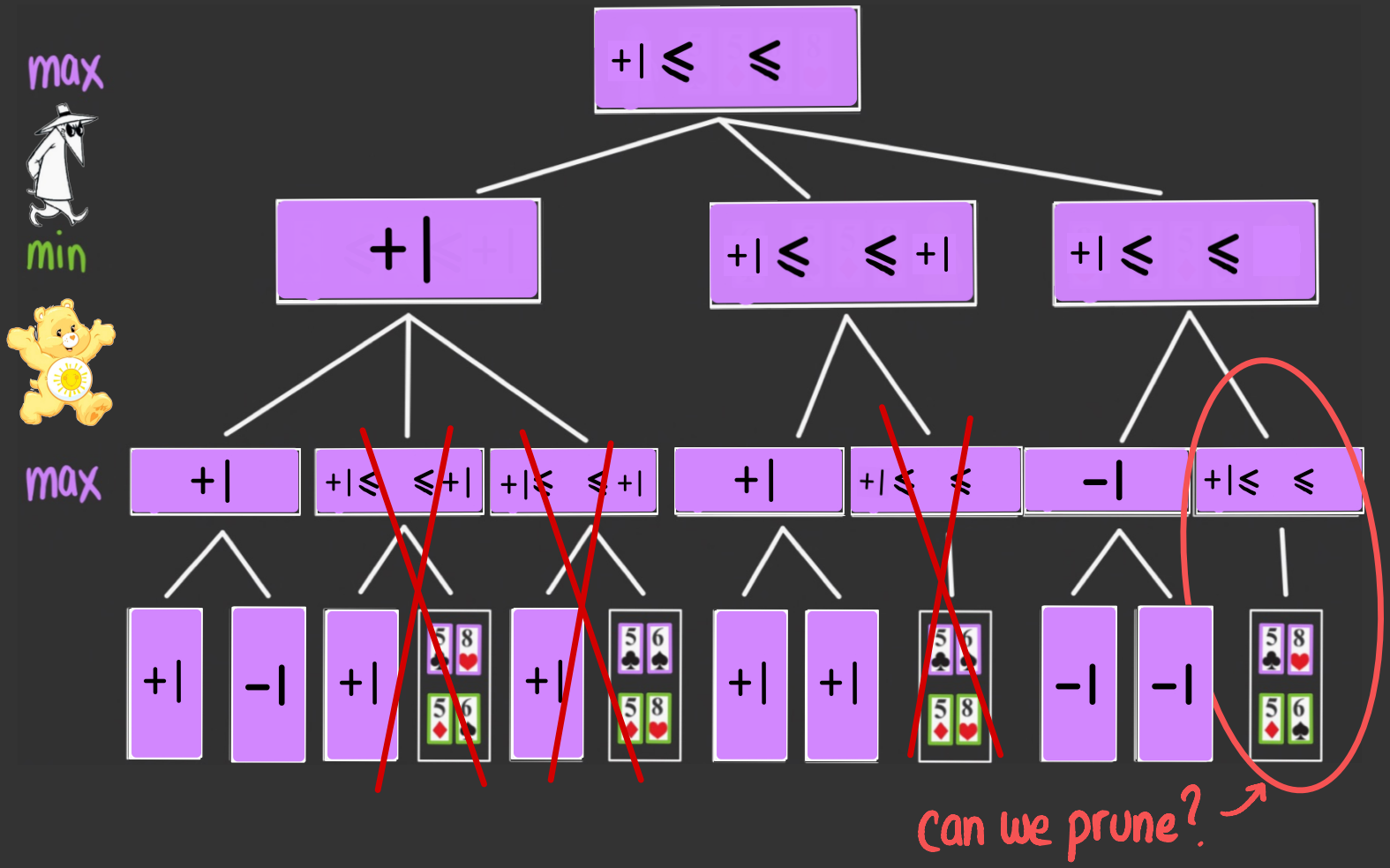
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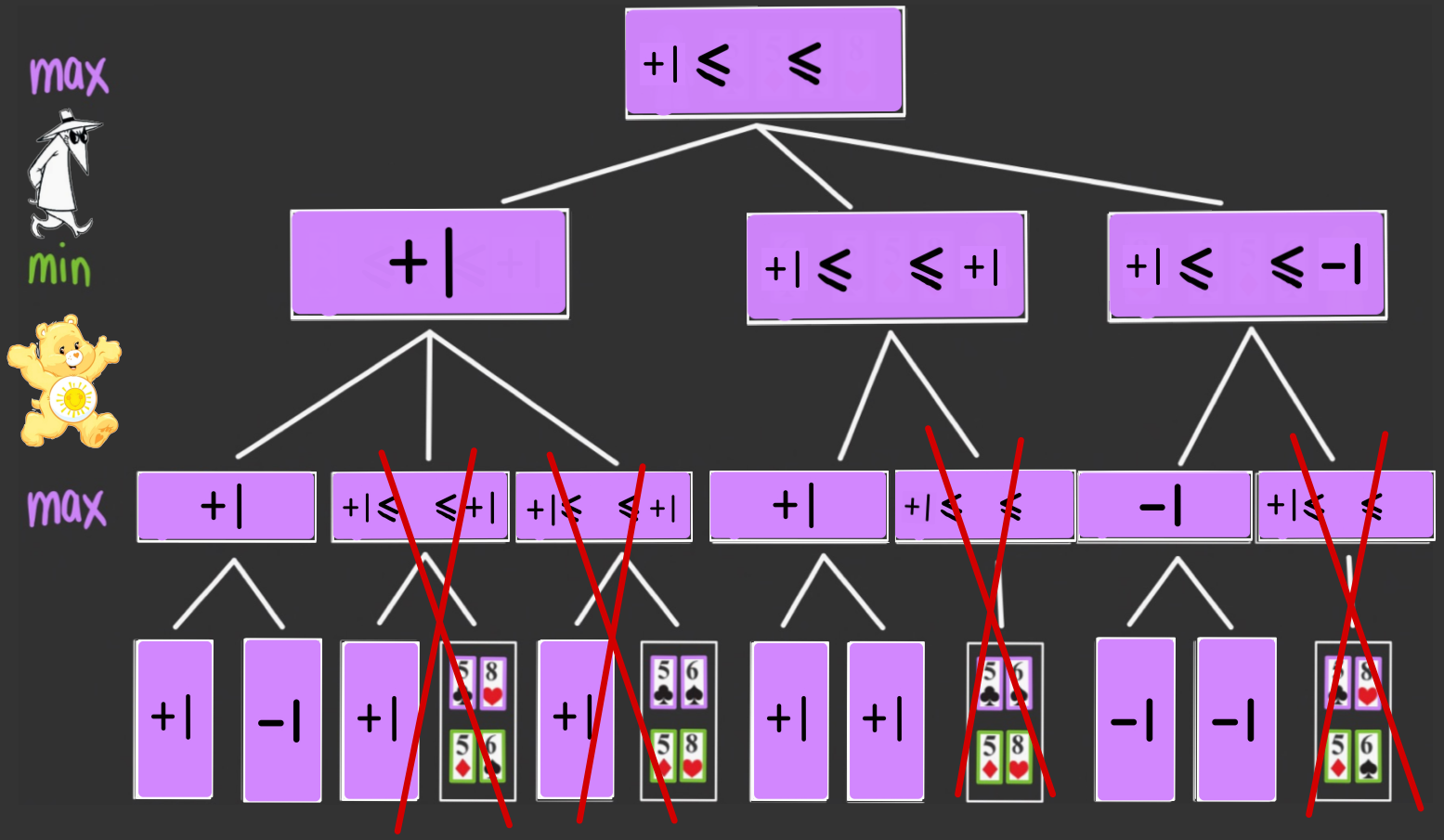
Can we prune?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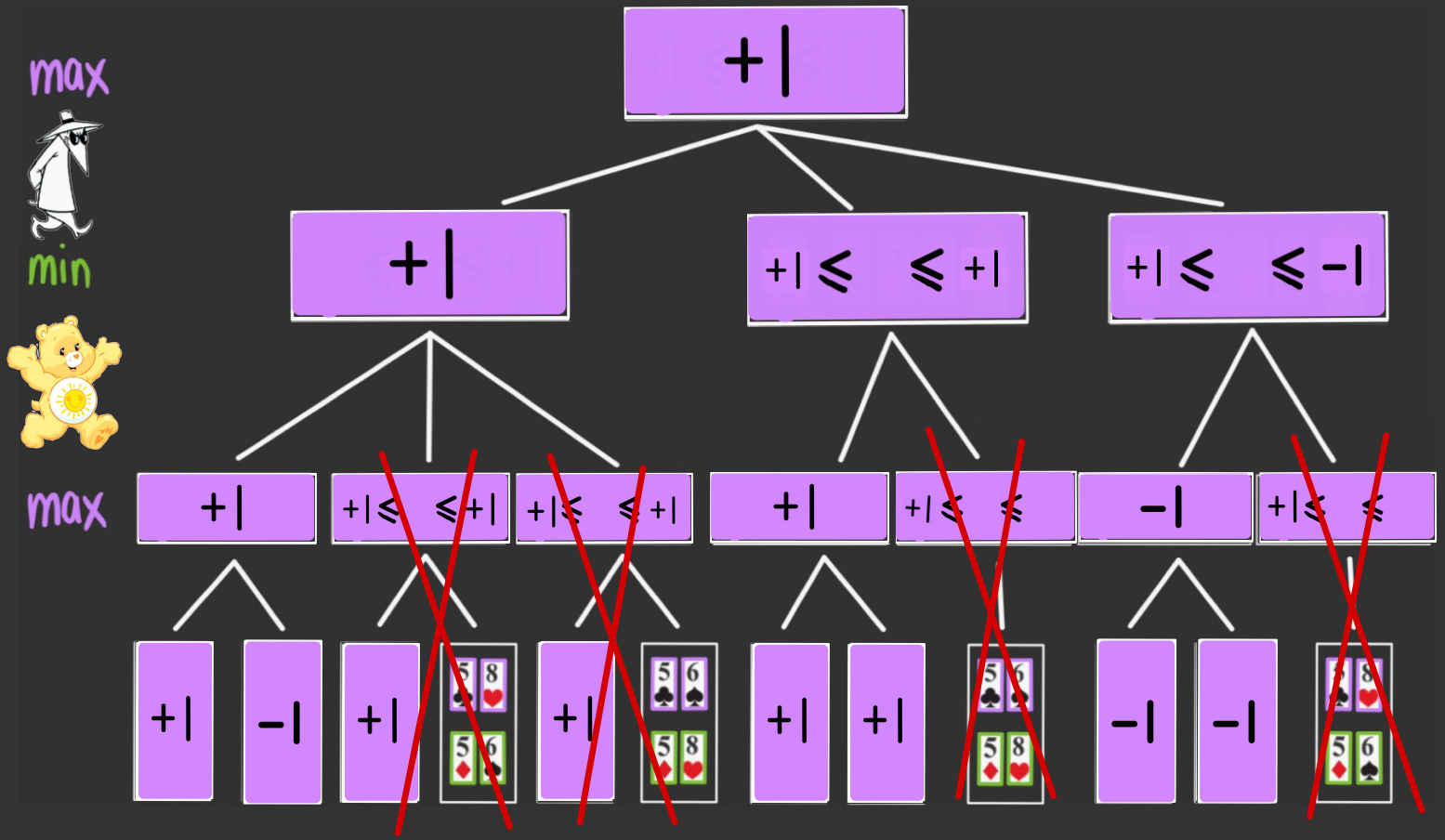
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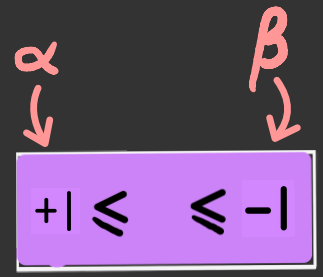
these bounds we maintain are called α and β, and therefore this optimization to minimax is called

alpha-beta pruning



MINIMAX$(q, p, (m, U))$:

- if $q \in F$: return $U(q, p)$
- children = $\{ \text{minimax}(q', p, (m, U)) \mid \langle q, \sigma, q' \rangle \in \Delta(m) \}$
- return max(children) if $p(q) = p$ else min(children)

let's re-express MINIMAX so that the search tree is processed left-to-right and bottom-up

```
MINIMAX(q, p, (M, U)):   # M = (Q, Σ, Δ, q₀, F)
  ▸ if q ∈ F: return U(q, p)
  ▸ bestvalue = [?] if p(q) = p else [?]
  ▸ for ⟨q, σ, q'⟩ ∈ Δ:
      ▸ childvalue = MINIMAX(q', p, (M, U))
      ▸ if p(q) = p:
          ▸ bestvalue = [?](bestvalue, childvalue)
      ▸ else
          ▸ bestvalue = [?](bestvalue, childvalue)
      ▸ return bestvalue
```

$\textsc{Minimax}(q, p, (M, U))$: # $M = (Q, \Sigma, \Delta, q_0, F)$

- if $q \in F$: return $U(q, p)$
- bestvalue = $-\infty$ if $p(q) = p$ else ∞
- for $\langle q, \sigma, q' \rangle \in \Delta$:
  - childvalue = $\textsc{Minimax}(q', p, (M, U))$
  - if $p(q) = p$:
    - bestvalue = max (bestvalue, childvalue)
  - else
    - bestvalue = min (bestvalue, childvalue)
  - return bestvalue

$\text{MINIMAX}(q, p, (M, U), \alpha, \beta)$: # $M = (Q, \Sigma, \Delta, q_0, F)$

- if $q \in F$: return $U(q, p)$
- bestvalue = $-\infty$ if $p(q) = p$ else ∞
- for $\langle q, \sigma, q' \rangle \in \Delta$:
  - ?
  - childvalue = $\text{MINIMAX}(q', p, (M, U), \alpha, \beta)$
  - if $p(q) = p$:
    - bestvalue = max (bestvalue, childvalue)
    - ?
  - else:
    - bestvalue = min (bestvalue, childvalue)
    - ?
  - return bestvalue

$\text{MINIMAX}(q, p, (M, U), \alpha, \beta)$: # $M = (Q, \Sigma, \Delta, q_0, F)$

- if $q \in F$: return $U(q, p)$
- bestvalue = $-\infty$ if $p(q) = p$ else ∞
- for $\langle q, \sigma, q' \rangle \in \Delta$:
  - ?
  - childvalue = $\text{MINIMAX}(q', p, (M, U), \alpha, \beta)$
  - if $p(q) = p$:
    - bestvalue = max (bestvalue, childvalue)
    - α = max (α, childvalue)
  - else:
    - bestvalue = min (bestvalue, childvalue)
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  - return bestvalue

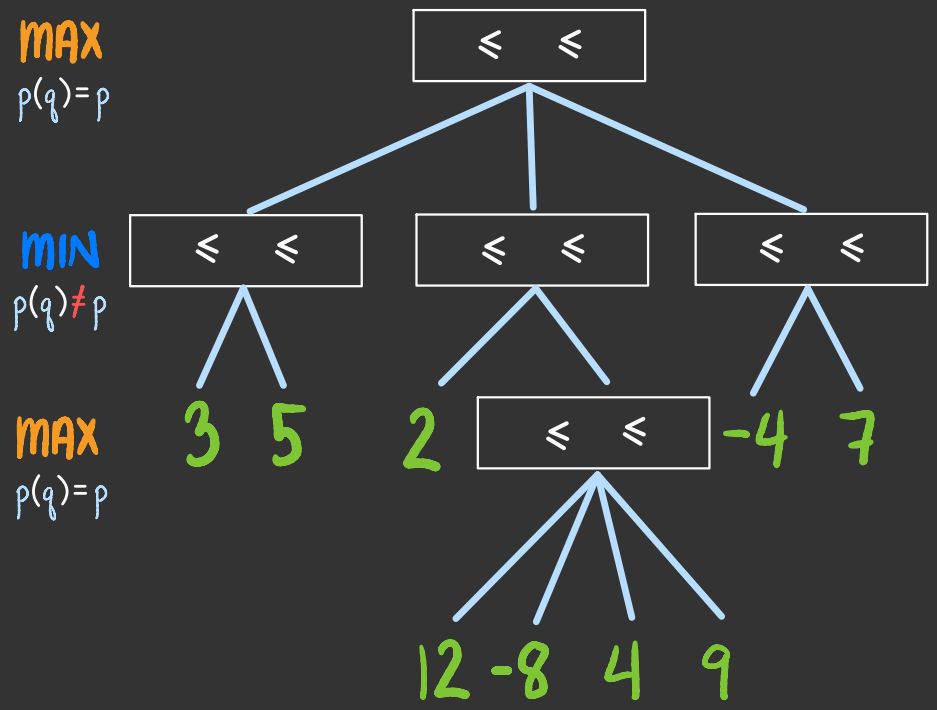
$\text{MINIMAX}(q, p, (M, U), \alpha, \beta)$: # $M = (Q, \Sigma, \Delta, q_0, F)$

- if $q \in F$: return $U(q, p)$
- bestvalue = $-\infty$ if $p(q) = p$ else ∞
- for $\langle q, \sigma, q' \rangle \in \Delta$:
  - if $\alpha \geq \beta$: return bestvalue
  - childvalue = $\text{MINIMAX}(q', p, (M, U), \alpha, \beta)$
  - if $p(q) = p$:
    - bestvalue = max (bestvalue, childvalue)
    - α = max (α, childvalue)
  - else:
    - bestvalue = min (bestvalue, childvalue)
    - β = min (β, childvalue)
  - return bestvalue

try alpha-beta yourself

$\text{MINIMAX}(q, p, (M, U), \alpha, \beta)$: # $M = (Q, \Sigma, \Delta, q_0, F)$

- if $q \in F$: return $U(q, p)$
- bestvalue = $-\infty$ if $p(q) = p$ else ∞
- for $\langle q, \sigma, q' \rangle \in \Delta$:
  - if $\alpha \geq \beta$: return bestvalue
  - childvalue = $\text{MINIMAX}(q', p, (M, U), \alpha, \beta)$
  - if $p(q) = p$:
    - bestvalue = max (bestvalue, childvalue)
    - α = max (α, childvalue)
  - else:
    - bestvalue = min (bestvalue, childvalue)
    - β = min (β, childvalue)
  - return bestvalue



$\text{MINIMAX}(q, p, (M, U), \alpha, \beta)$: # $M = (Q, \Sigma, \Delta, q_0, F)$

- if $q \in F$: return $U(q, p)$
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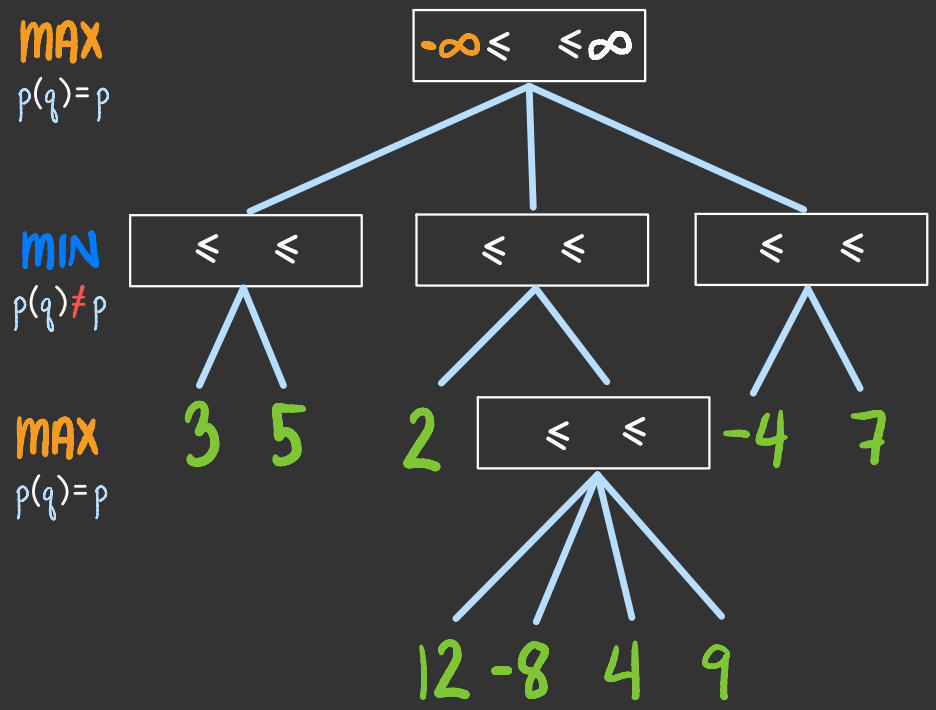
  - return bestvalue



MINIMAX$(q, p, (M, U), \alpha, \beta)$: # $M = (Q, \Sigma, \Delta, q_0, F)$

- if $q \in F$: return $U(q, p)$
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  - childvalue = MINIMAX$(q', p, (M, U), \alpha, \beta)$
  - if $p(q) = p$:
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    - α = max(α, childvalue)
  - else :
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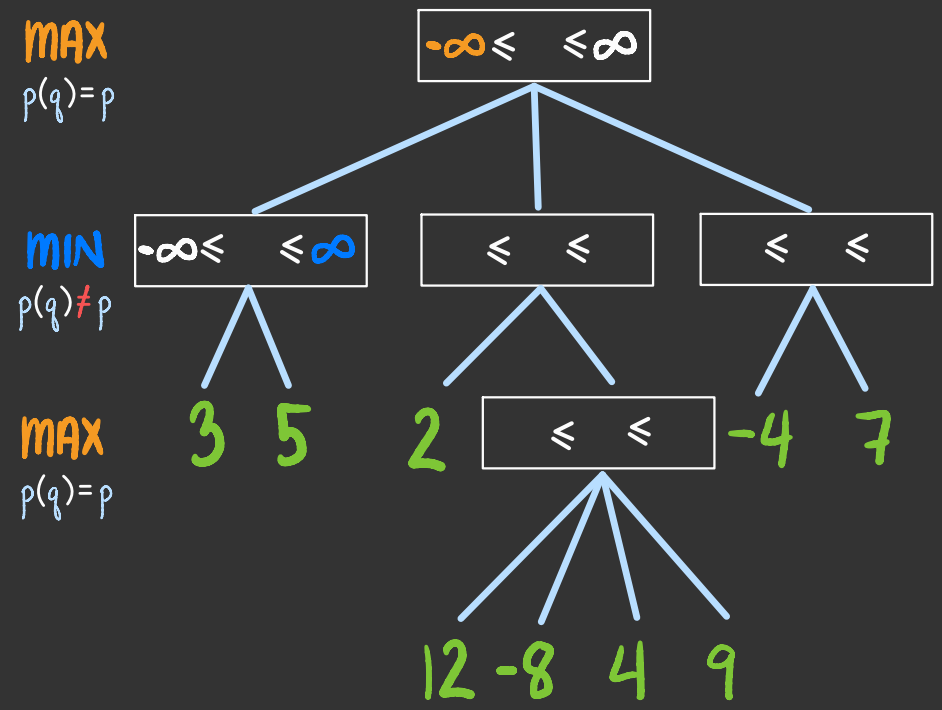
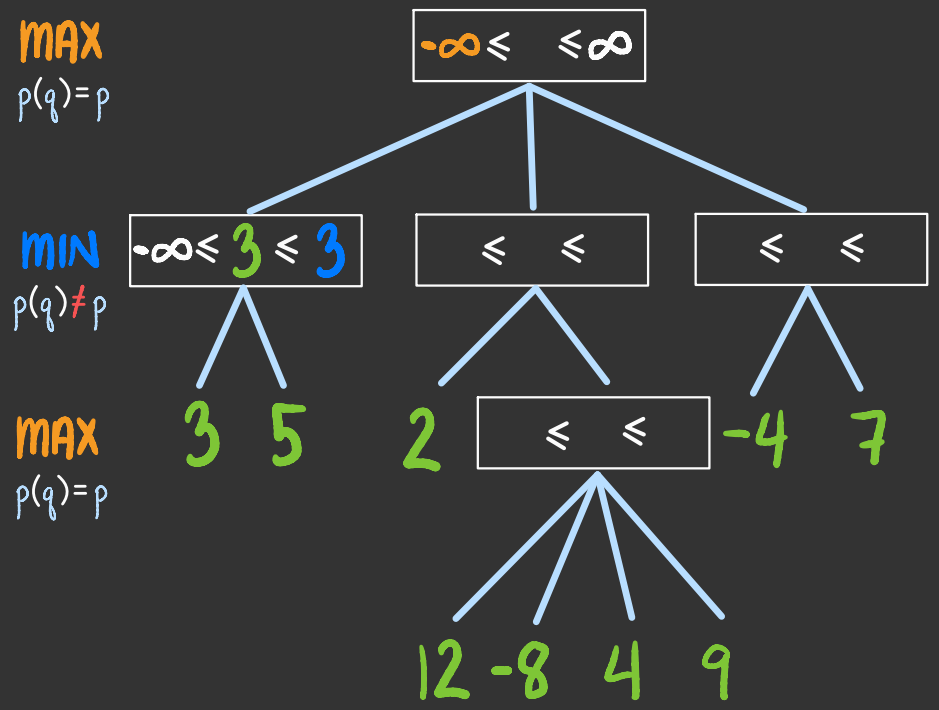
  - return bestvalue



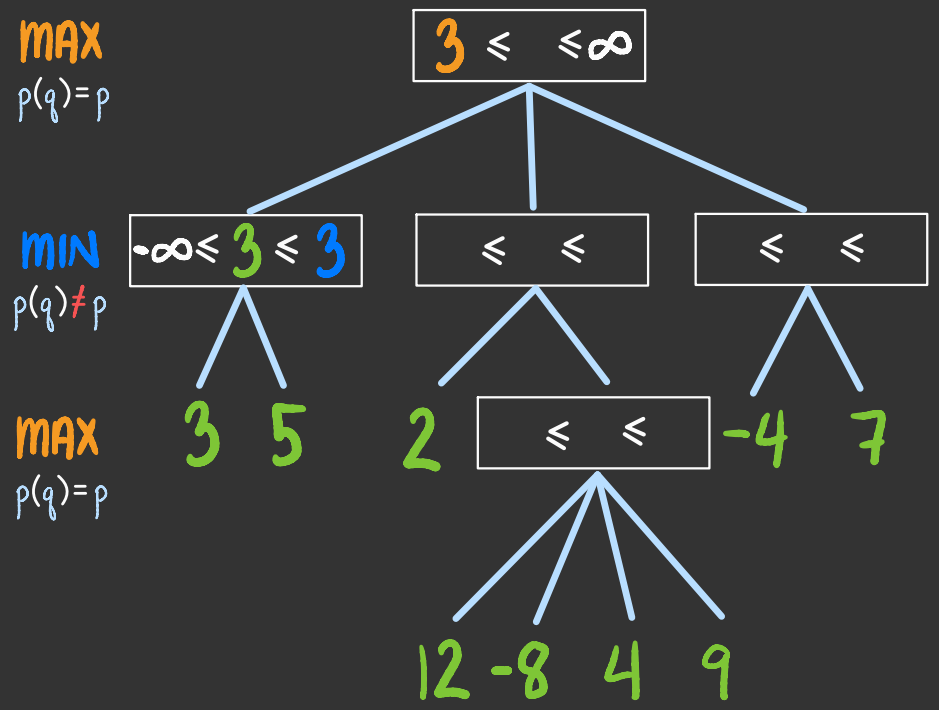
$\text{MINIMAX}(q, p, (M, U), \alpha, \beta)$: # $M = (Q, \Sigma, \Delta, q_0, F)$

- if $q \in F$: return $U(q, p)$
- bestvalue = $-\infty$ if $p(q) = p$ else ∞
- for $\langle q, \sigma, q' \rangle \in \Delta$:
  - if $\alpha \geq \beta$: return bestvalue
  - childvalue = $\text{MINIMAX}(q', p, (M, U), \alpha, \beta)$
  - if $p(q) = p$:
    - bestvalue = max (bestvalue, childvalue)
    - α = max (α, childvalue)
  - else :
    - bestvalue = min (bestvalue, childvalue)
    - β = min (β, childvalue)
  - return bestvalue



$\text{MINIMAX}(q, p, (M, U), \alpha, \beta)$: # $M = (Q, \Sigma, \Delta, q_0, F)$

- if $q \in F$: return $U(q, p)$
- bestvalue = $-\infty$ if $p(q) = p$ else ∞
- for $\langle q, \sigma, q' \rangle \in \Delta$:
  - if $\alpha \geq \beta$: return bestvalue
  - childvalue = $\text{MINIMAX}(q', p, (M, U), \alpha, \beta)$
  - if $p(q) = p$:
    - bestvalue = max (bestvalue, childvalue)
    - α = max (α, childvalue)
  - else :
    - bestvalue = min (bestvalue, childvalue)
    - β = min (β, childvalue)
  - return bestvalue



MINIMAX$(q, p, (M, U), \alpha, \beta)$: # $M = (Q, \Sigma, \Delta, q_0, F)$

- if $q \in F$: return $U(q, p)$
- bestvalue = $-\infty$ if $p(q) = p$ else ∞
- for $\langle q, \sigma, q' \rangle \in \Delta$:
  - if $\alpha \geq \beta$: return bestvalue
  - childvalue = MINIMAX$(q', p, (M, U), \alpha, \beta)$
  - if $p(q) = p$:
    - bestvalue = max(bestvalue, childvalue)
    - α = max(α, childvalue)
  - else :
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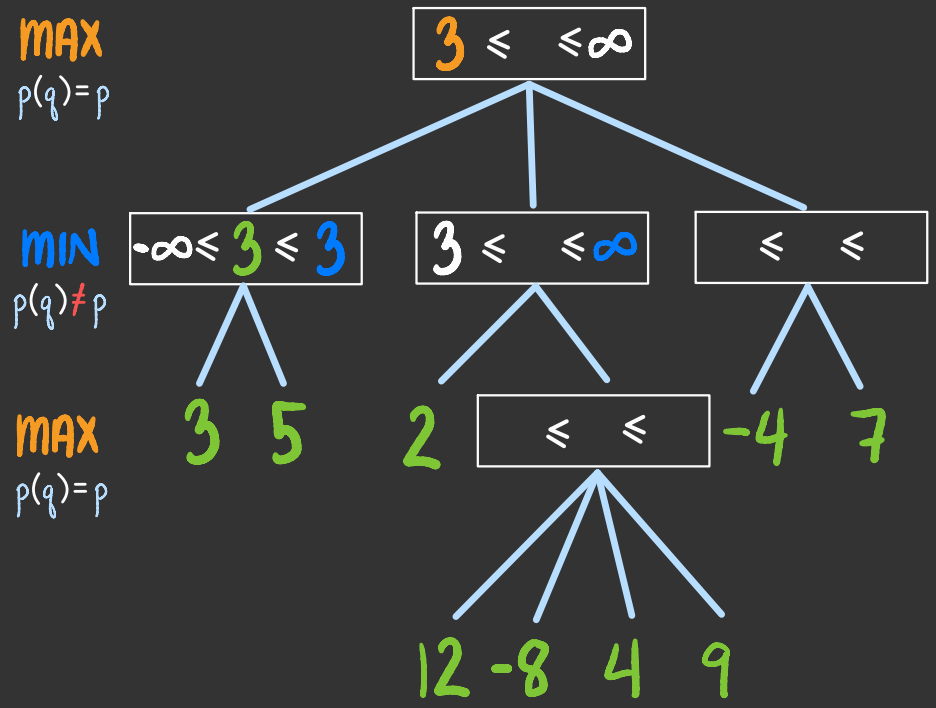
  - return bestvalue



$\text{MINIMAX}(q, p, (M, U), \alpha, \beta)$: # $M = (Q, \Sigma, \Delta, q_0, F)$

- if $q \in F$: return $U(q, p)$
- bestvalue = $-\infty$ if $p(q) = p$ else ∞
- for $\langle q, \sigma, q' \rangle \in \Delta$:
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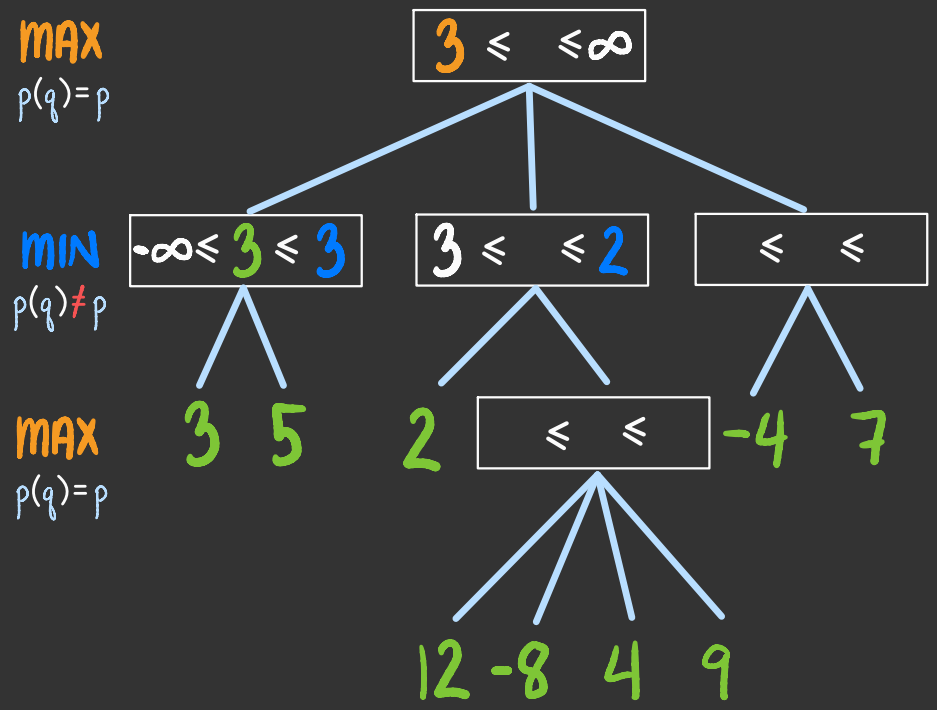

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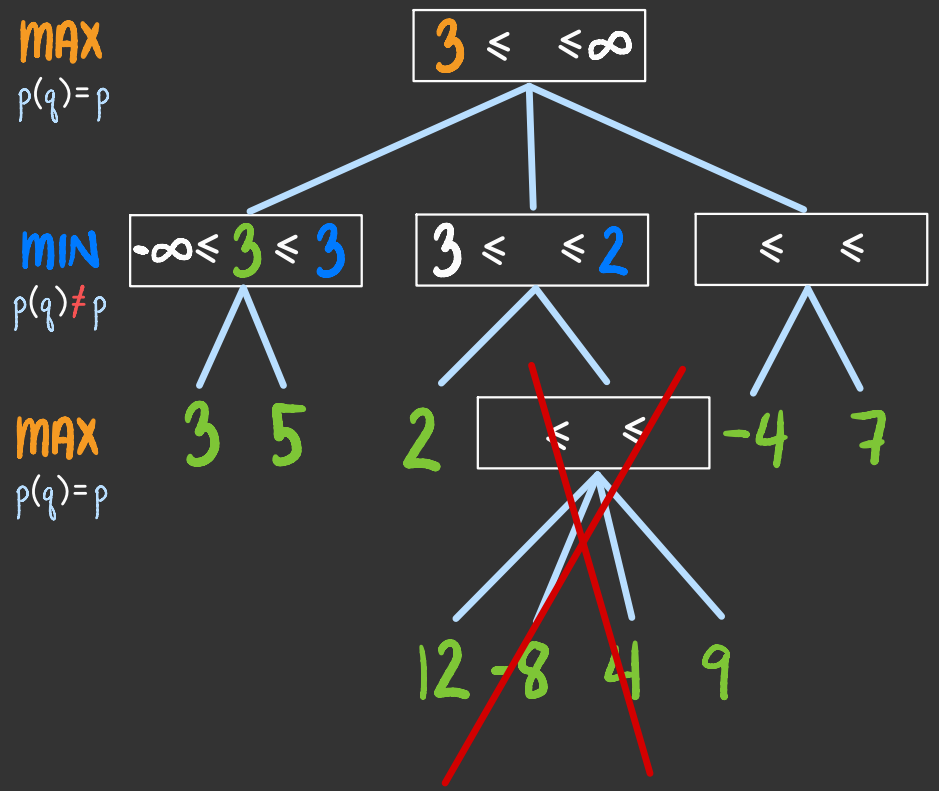

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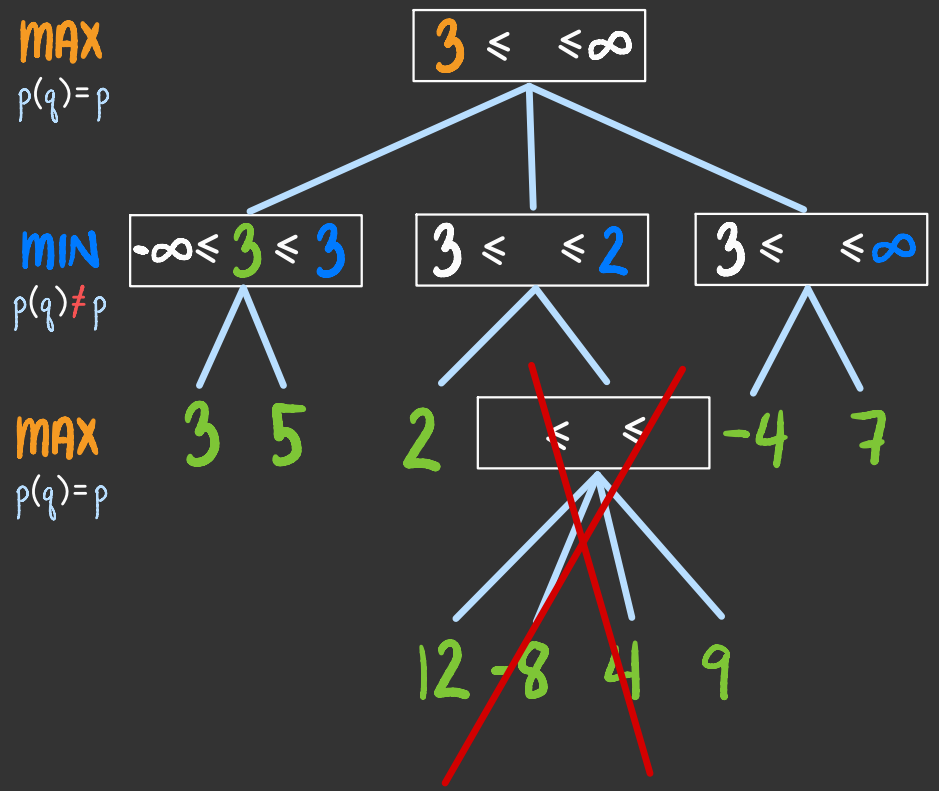

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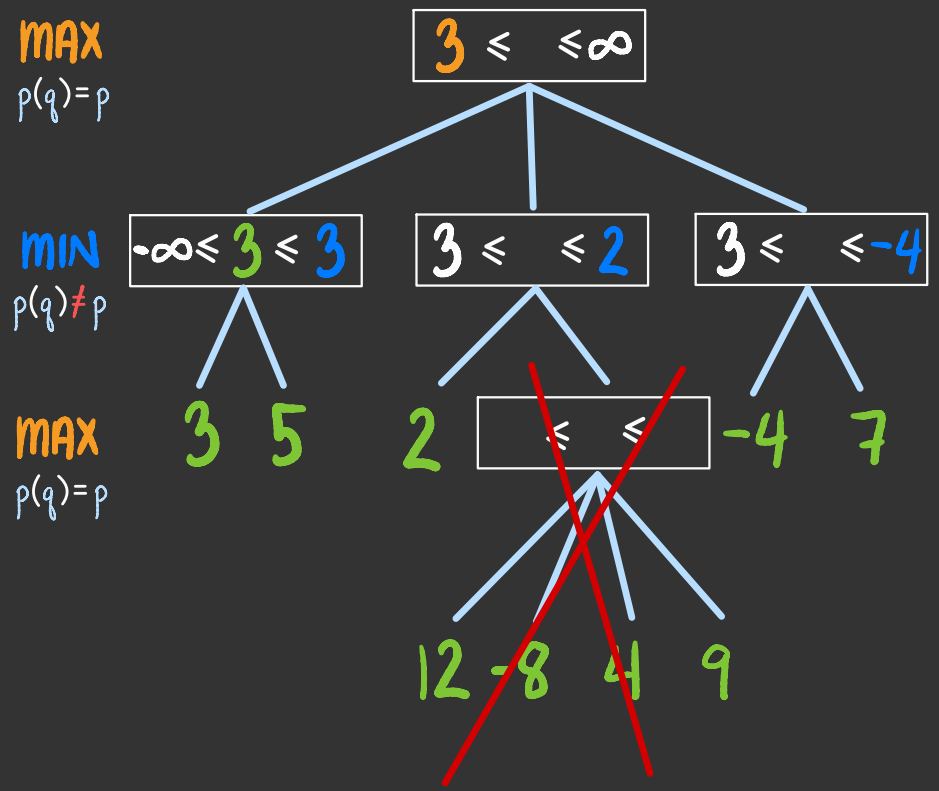

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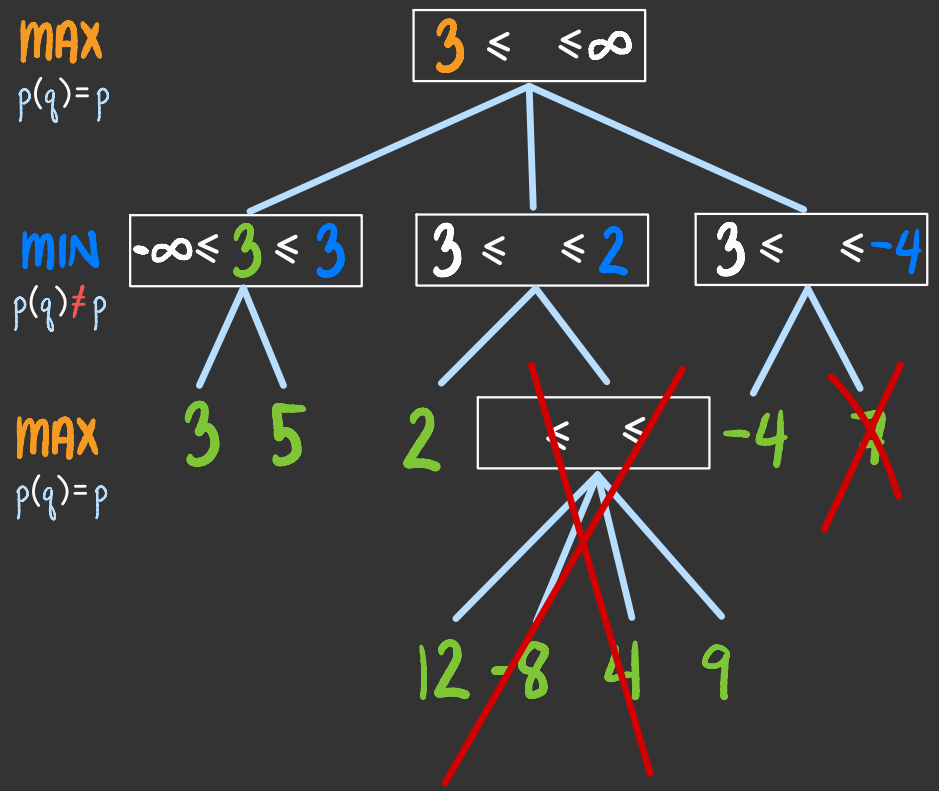

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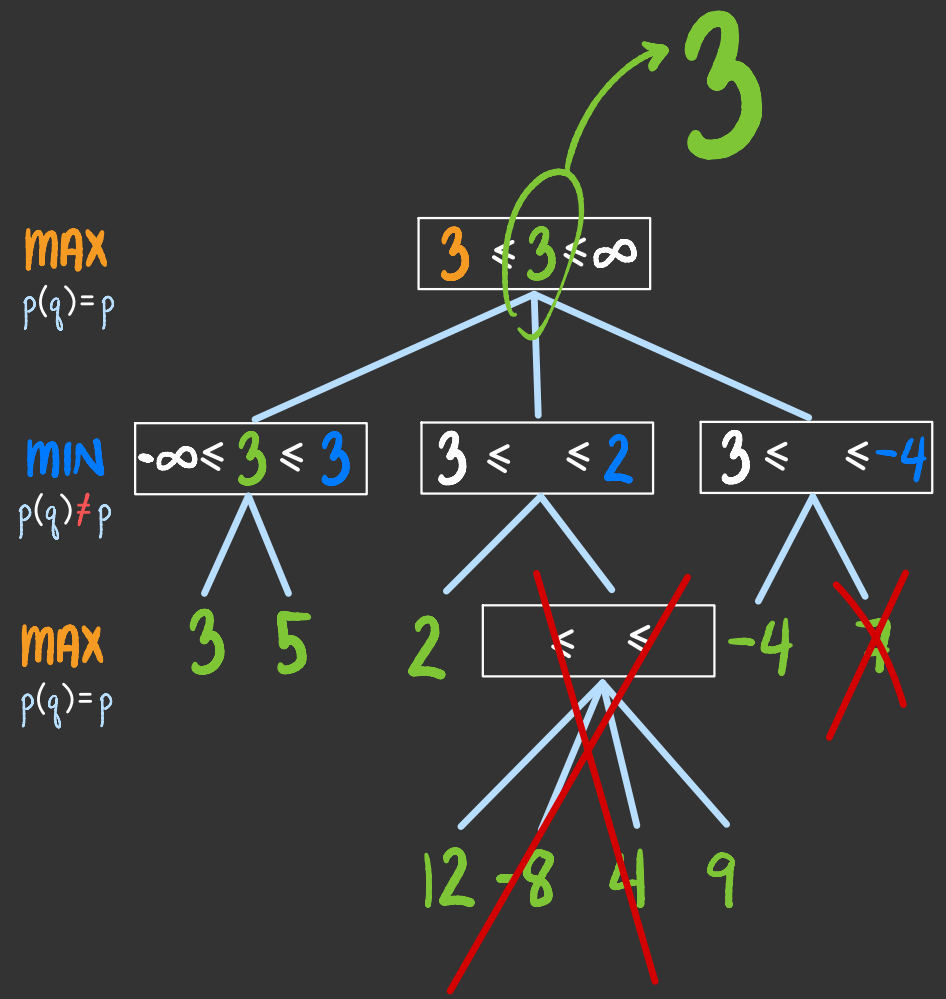
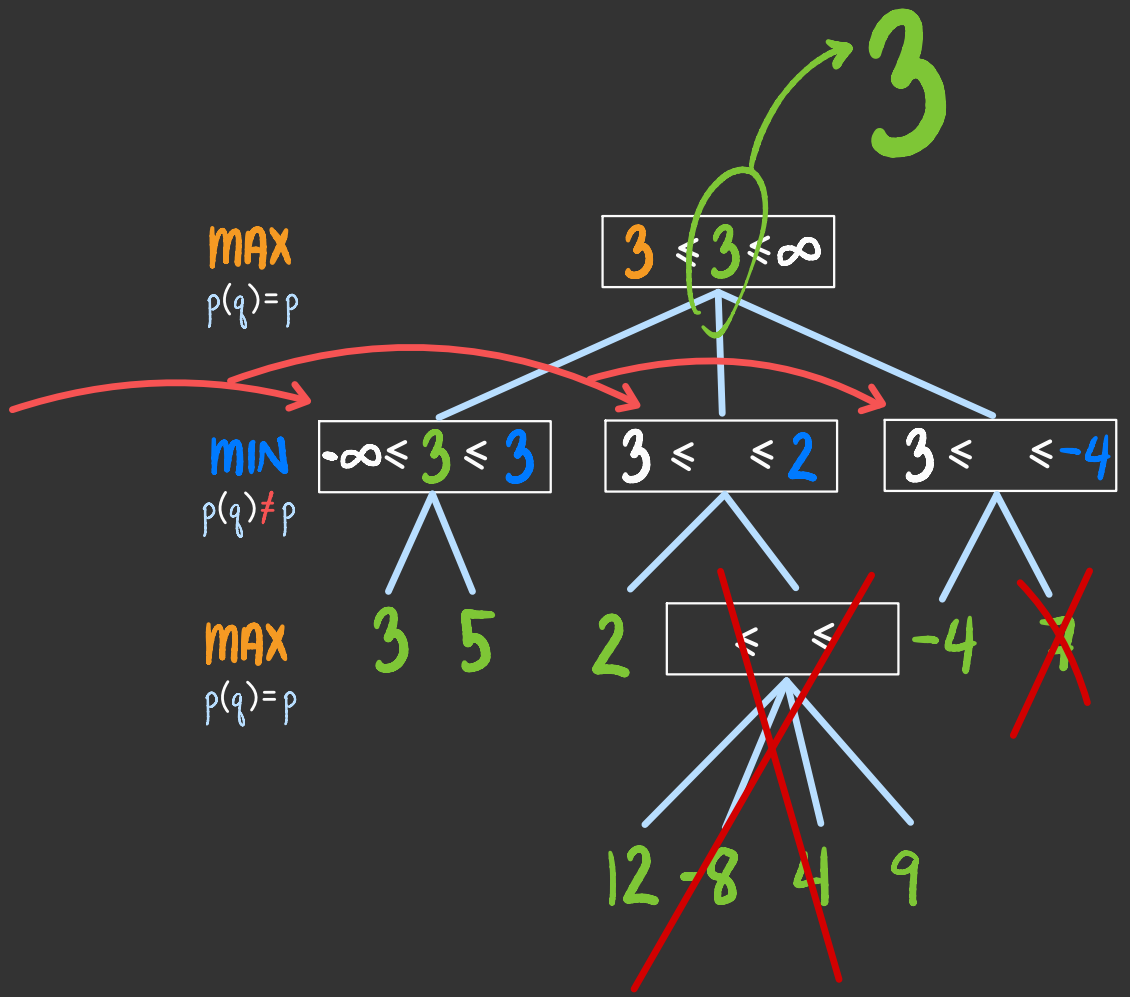


what do these **bounds** describe?

your answer **here**



what do these **bounds** describe?

conditions under which this subtree can affect the value of its parent