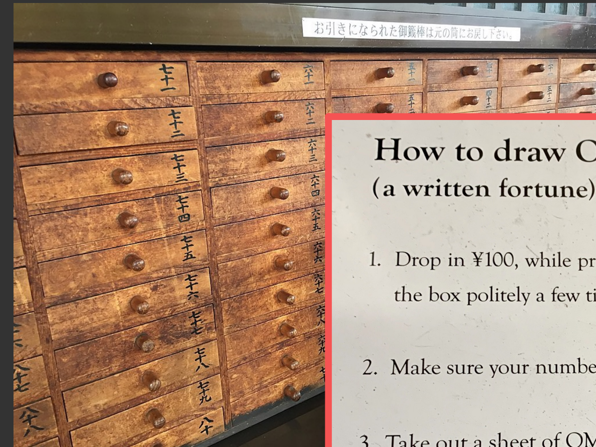
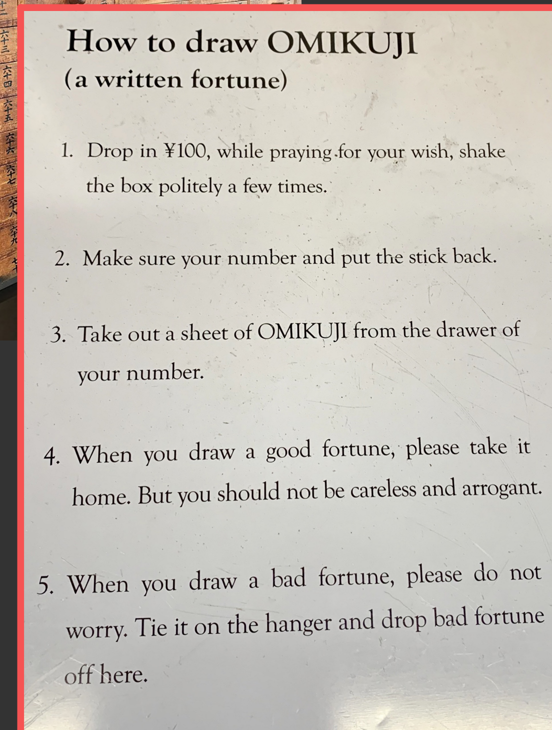
probability

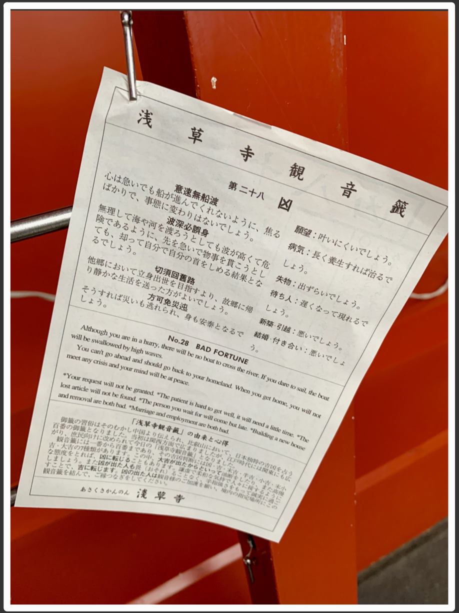
CSCI 373


みくじ 100円


お引きになられた御籤棒は元の筒にお戻し下さい。


How to draw OMIKUJI
(a written fortune)
1. Drop in ¥100, while praying for your wish, shake the box politely a few times.
2. Make sure your number and put the stick back.
3. Take out a sheet of OMIKUJI from the drawer of your number.
4. When you draw a good fortune, please take it home. But you should not be careless and arrogant.
5. When you draw a bad fortune, please do not worry. Tie it on the hanger and drop bad fortune off here.

omikuji

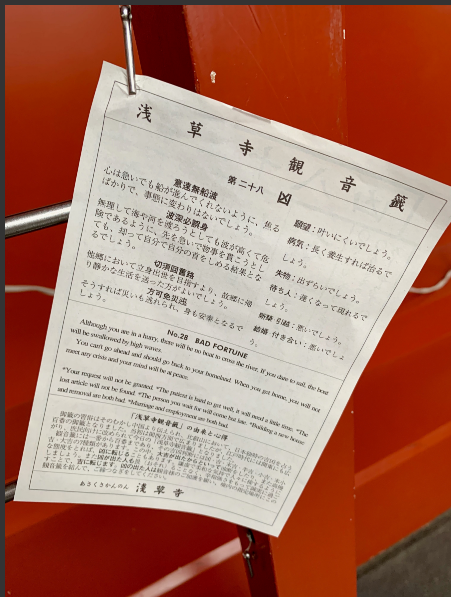


Fortunes [edit]

The standard *Ganzan Daishi Hyakusen* sequence contains the following fortunes (from best to worst):

- Great blessing (大吉, *dai-kichi*)
- Blessing (吉, *kichi*)
- Small blessing (小吉, *shō-kichi*)
- Half-blessing (半吉, *han-kichi*)
- Future blessing (末吉, *sue-kichi*)
- Future small blessing (末小吉, *sue-shō-kichi*)
- Misfortune (凶, *kyō*)

from the wikipedia article "o-mikuji"



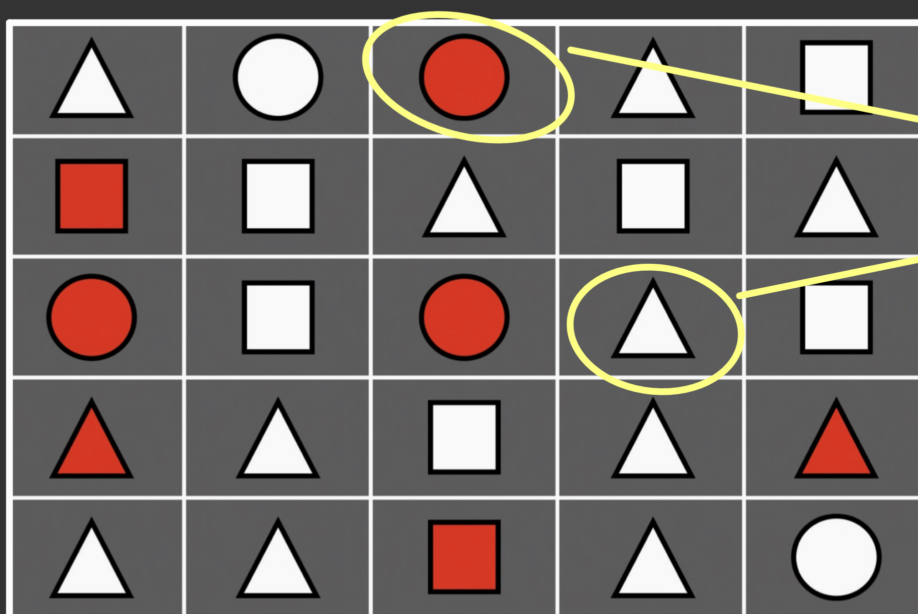
Fortunes [edit]

The standard *Ganzan Daishi Hyakusen* sequence contains the following fortunes (from best to worst):

- Great blessing (大吉, *dai-kichi*)
- Blessing (吉, *kichi*)
- Small blessing (小吉, *shō-kichi*)
- ~~Half blessing (半吉, *han-kichi*)~~ Great Misfortune (大凶, *dai-kyō*)
- ~~Future blessing (末吉, *sue-kichi*)~~ Small Misfortune (小凶, *shō-kyō*)
- ~~Future small blessing (末小吉, *sue-shō-kichi*)~~
- Misfortune (凶, *kyō*)

from the wikipedia article "o-mikuji"

○ : great □ : regular △ : small

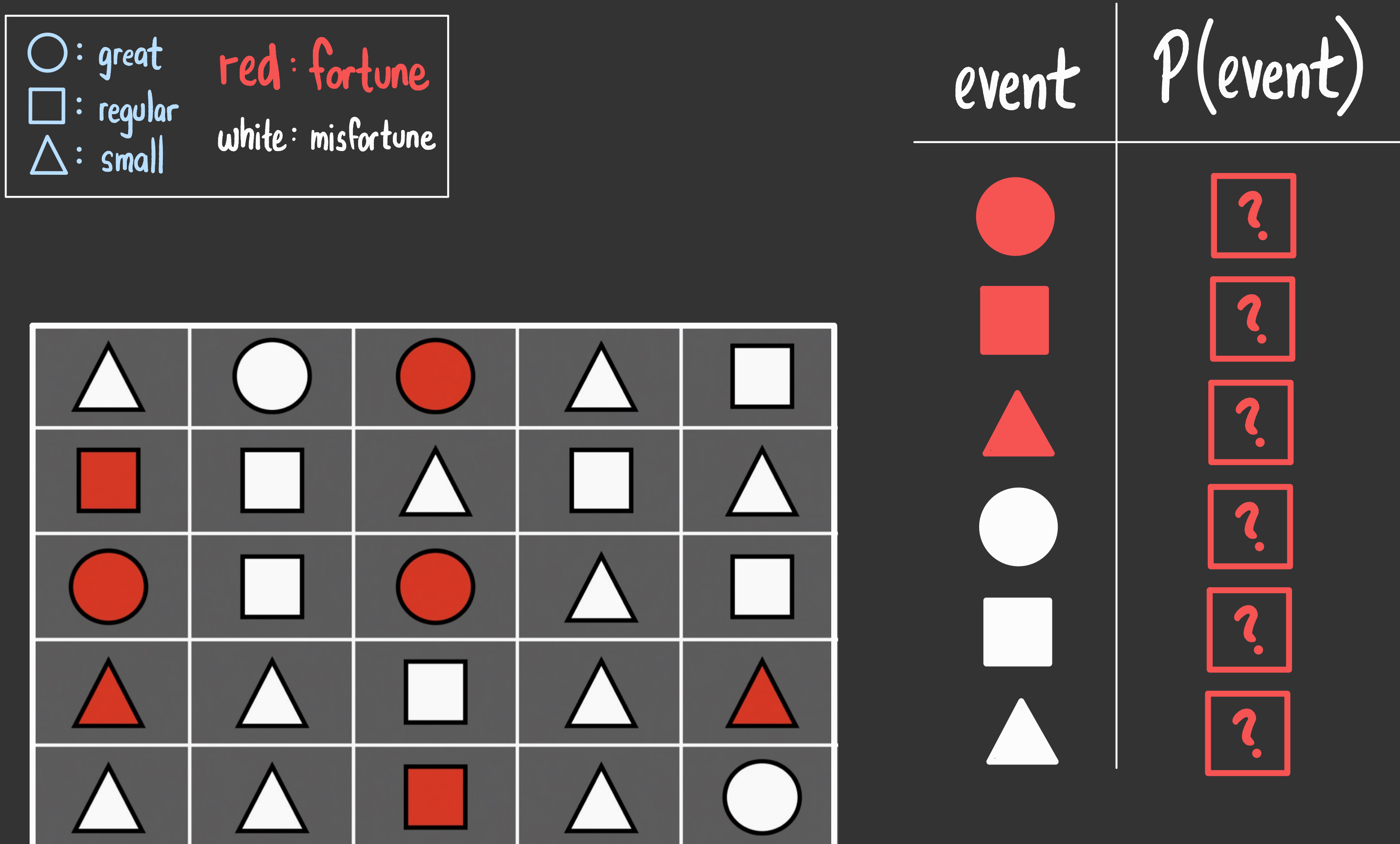


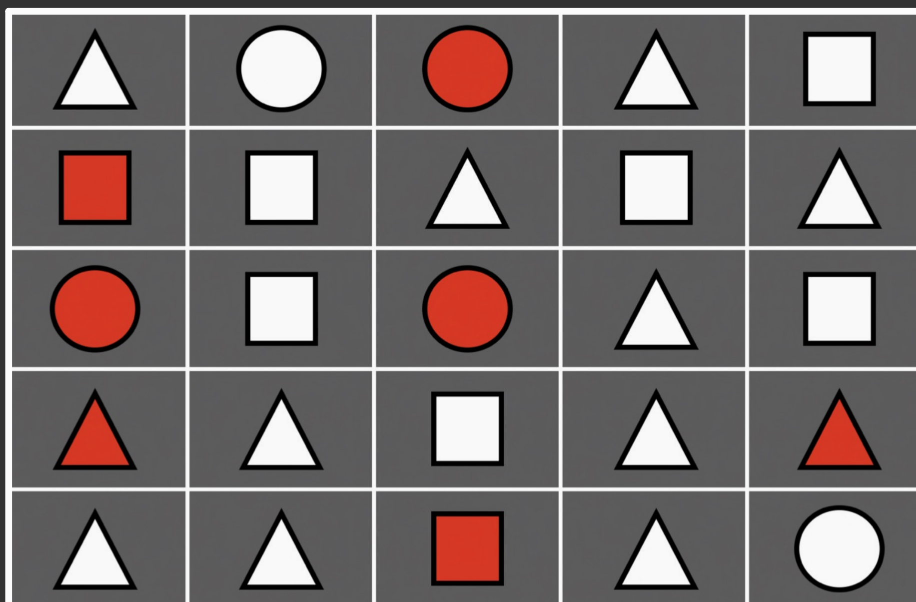
great fortune

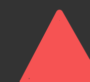
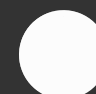
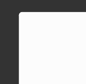
small misfortune

red : fortune

white : misfortune

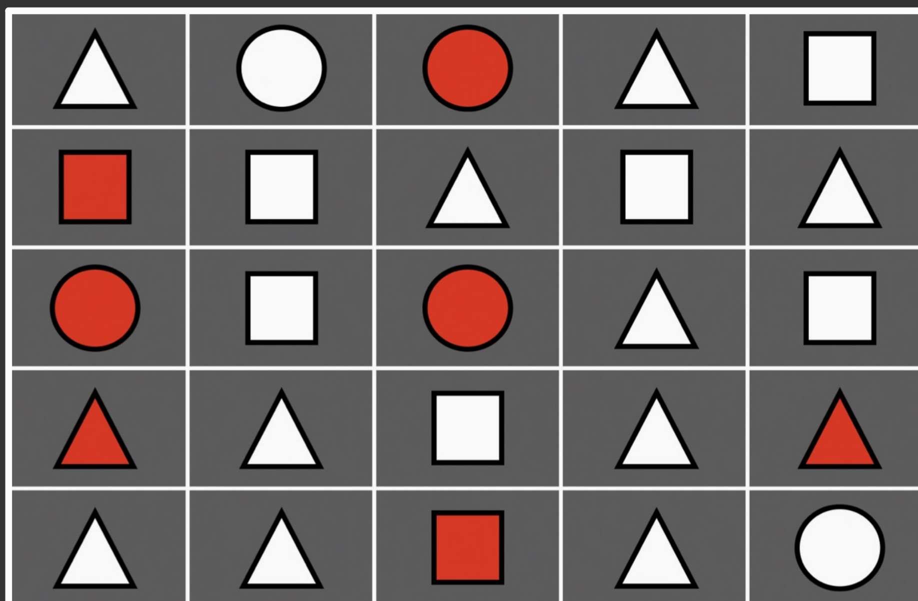

: great
: regular
: small
red : fortune
white : misfortune
event
P(event)
?
?
?
?
?
?



event	P(event)
	$\frac{3}{25}$
	$\frac{2}{25}$
	$\frac{2}{25}$
	$\frac{2}{25}$
	$\frac{6}{25}$
	$\frac{10}{25}$

○ : great
□ : regular
△ : small

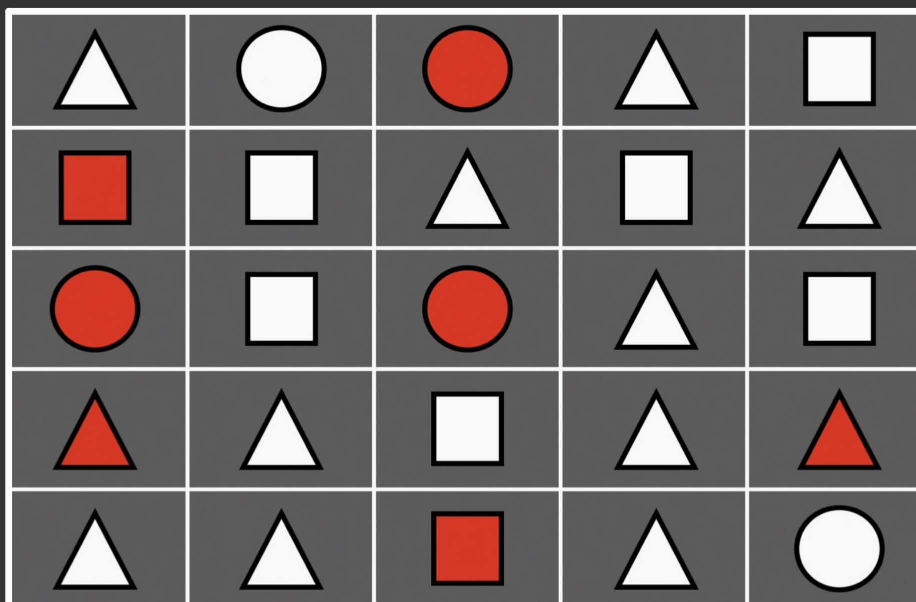
red : fortune
white : misfortune



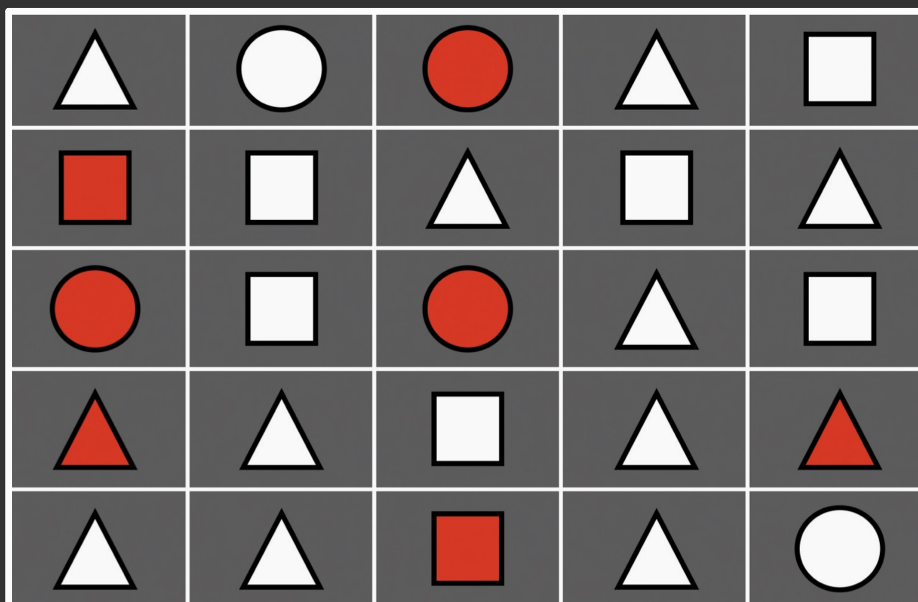
C	S	P(c, s)
red	●	$\frac{3}{25}$
red	■	$\frac{2}{25}$
red	▲	$\frac{2}{25}$
white	●	$\frac{2}{25}$
white	■	$\frac{6}{25}$
white	▲	$\frac{10}{25}$

a joint distribution over variables $\{C, S\}$ →

C	S	$P(c, s)$
red	●	$\frac{3}{25}$
red	■	$\frac{2}{25}$
red	▲	$\frac{2}{25}$
white	●	$\frac{2}{25}$
white	■	$\frac{6}{25}$
white	▲	$\frac{10}{25}$



what is the probability of misfortune?



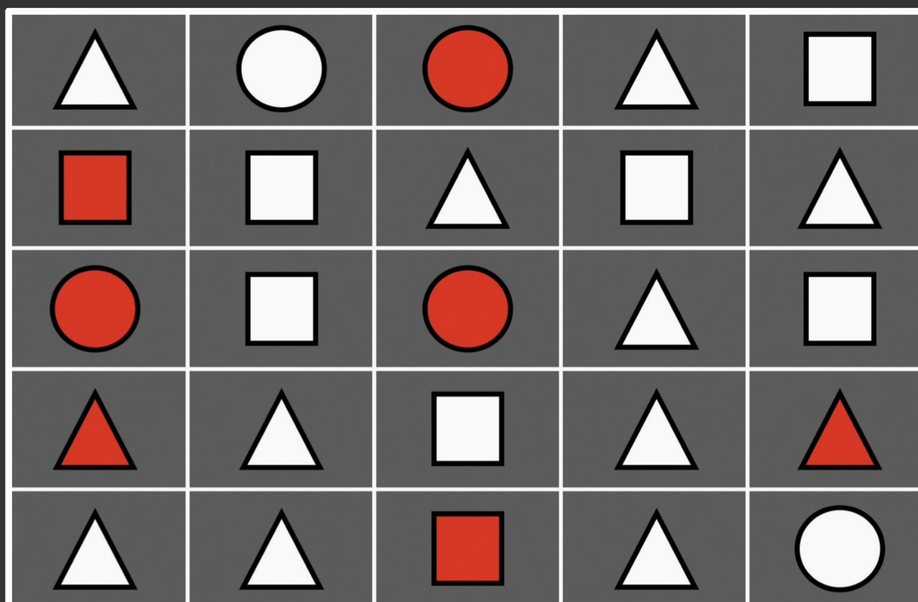
C	S	$P(c,s)$
red	○	$\frac{3}{25}$
red	□	$\frac{2}{25}$
red	△	$\frac{2}{25}$
white	○	$\frac{2}{25}$
white	□	$\frac{6}{25}$
white	△	$\frac{10}{25}$

○ : great
□ : regular
△ : small

red : fortune
white : misfortune

what is the probability of misfortune?

what is $P(C = \text{white})$?



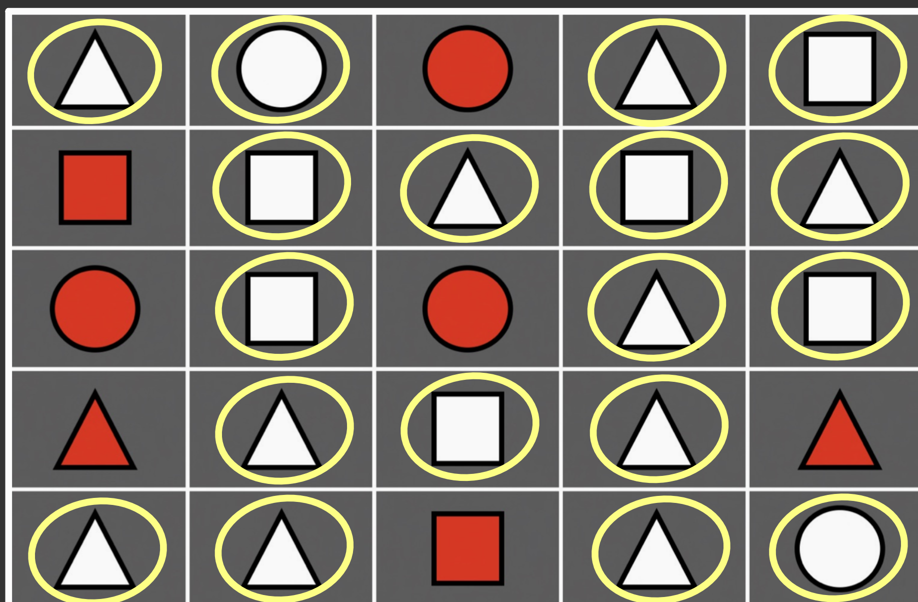
C	S	$P(c,s)$
red	○	$\frac{3}{25}$
red	□	$\frac{2}{25}$
red	△	$\frac{2}{25}$
white	○	$\frac{2}{25}$
white	□	$\frac{6}{25}$
white	△	$\frac{10}{25}$

○ : great
□ : regular
△ : small

red : fortune
white : misfortune

what is the probability of misfortune?

what is $P(C = white)$?



C	S	$P(c,s)$
red	●	$\frac{3}{25}$
red	■	$\frac{2}{25}$
red	▲	$\frac{2}{25}$
white	●	$\frac{2}{25}$
white	■	$\frac{6}{25}$
white	▲	$\frac{10}{25}$

$$\frac{2}{25} + \frac{6}{25} + \frac{10}{25} = \frac{18}{25}$$

○ : great
□ : regular
△ : small

red : fortune
white : misfortune

C	S	P(c, s)	
red	●	$\frac{3}{25}$	
red	■	$\frac{2}{25}$	$= \frac{7}{25}$
red	▲	$\frac{2}{25}$	
white	●	$\frac{2}{25}$ +	
white	■	$\frac{6}{25}$ +	$= \frac{18}{25}$
white	▲	$\frac{10}{25}$	

joint probability

C	P(c)
red	$\frac{7}{25}$
white	$\frac{18}{25}$

marginal probability

the law of total probability

C	S	P(c, s)
red	●	$\frac{3}{25}$
red	■	$\frac{2}{25}$
red	▲	$\frac{2}{25}$
white	●	$\frac{2}{25}$
white	■	$\frac{6}{25}$
white	▲	$\frac{10}{25}$

$\frac{3}{25}+\frac{2}{25}+\frac{2}{25} = \frac{7}{25}$

$\frac{2}{25}+\frac{6}{25}+\frac{10}{25} = \frac{18}{25}$

joint probability

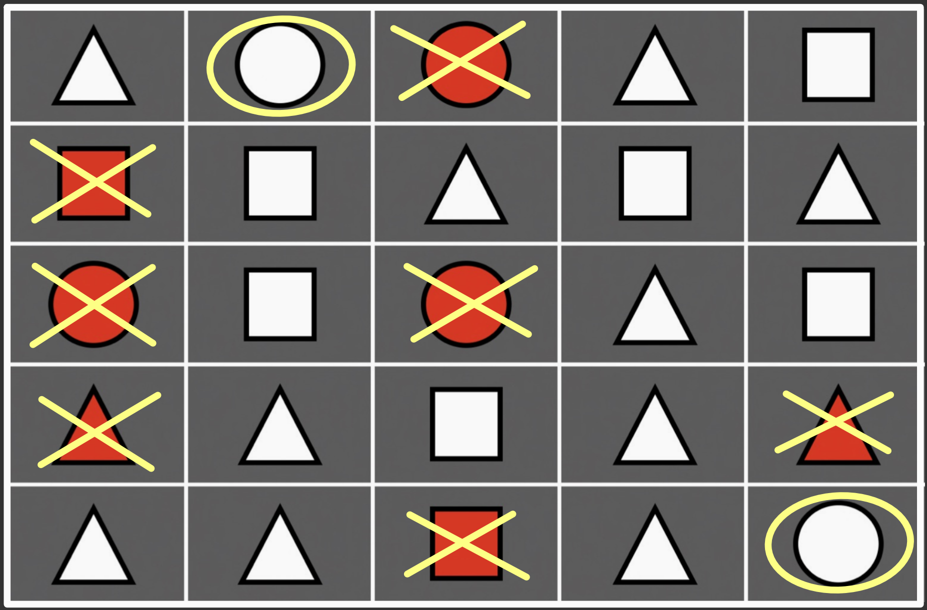
C	P(c)
red	$\frac{7}{25}$
white	$\frac{18}{25}$

marginal probability

$$\underbrace{P(x_1, \dots, x_m)}_{\text{marginal probability}} = \sum_{x_{m+1} \in D(X_{m+1})} \cdots \sum_{x_n \in D(X_n)} \underbrace{P(x_1, \dots, x_n)}_{\text{joint probability}}$$

domain of X_n

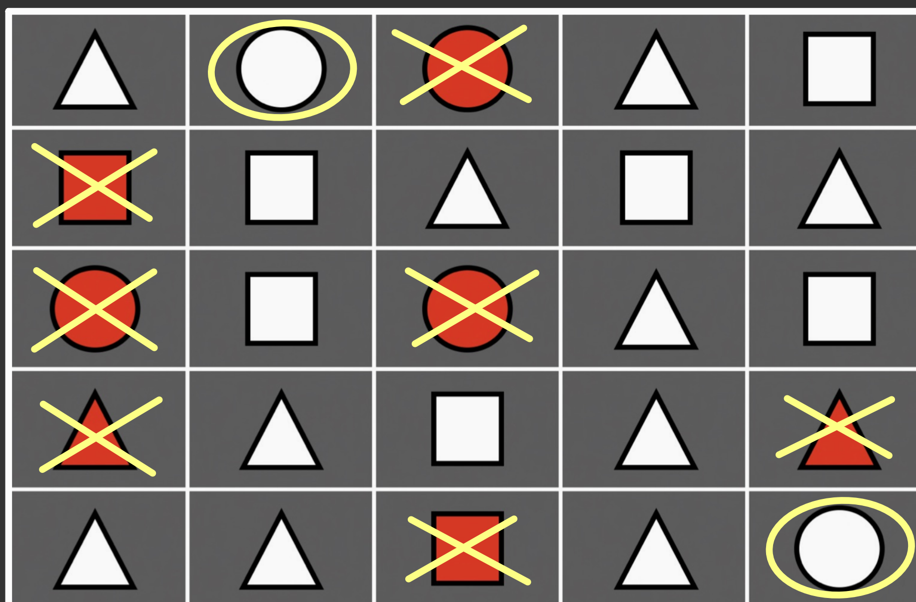
what is the probability of great misfortune, given we draw some kind of misfortune?



what is the probability of great misfortune, given we draw some kind of misfortune?

what is $P(S = \bullet \mid C = \text{white})$?

$$\frac{2\ \bullet}{18\ \text{white}} = \frac{1}{9}$$



○ : great
□ : regular
△ : small

red : fortune
white : misfortune

what is the probability of great misfortune, given we draw some kind of misfortune?

what is $P(S=\bullet \mid C=\text{white})$?

$$\frac{2\ \bullet}{18\ \text{white}} = \frac{1}{9}$$

C	S	P(c, s)
red	●	$\frac{3}{25}$
red	■	$\frac{2}{25}$
red	▲	$\frac{2}{25}$
white	●	$\frac{2}{25}$
white	■	$\frac{6}{25}$
white	▲	$\frac{10}{25}$

$$\frac{2}{25} + \frac{6}{25} + \frac{10}{25} = \frac{18}{25}$$

or:

$$\frac{P(C=\text{white}, S=\bullet)}{P(C=\text{white})} = \frac{\frac{2}{25}}{\frac{18}{25}} = \frac{1}{9}$$

○ : great
□ : regular
△ : small

red : fortune
white : misfortune

C	S	$P(c,s)$	
red	●	$\frac{3}{25}$	
red	■	$\frac{2}{25}$	$= \frac{7}{25}$
red	▲	$\frac{2}{25}$	
white	●	$\frac{2}{25}$	
white	■	$+\ \frac{6}{25}$	$= \frac{18}{25}$
white	▲	$+\ \frac{10}{25}$	

joint probability

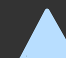
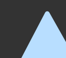
C	$P(c)$
red	$\frac{7}{25}$
white	$\frac{18}{25}$

marginal probability

C	S	$P(s \mid c) = \frac{P(c,s)}{P(c)}$
red	●	$\frac{3}{7}$
red	■	$\frac{2}{7}$
red	▲	$\frac{2}{7}$
white	●	$\frac{2}{18}$
white	■	$\frac{6}{18}$
white	▲	$\frac{10}{18}$

conditional probability

$$P(x_1, \ldots, x_m \mid x_{m+1}, \ldots, x_n) = \frac{P(x_1, \ldots, x_n)}{P(x_{m+1}, \ldots, x_n)}$$

C	S	$P(s \mid c) = \frac{P(c,s)}{P(c)}$
red		$\frac{3}{7}$
red		$\frac{2}{7}$
red		$\frac{2}{7}$
white		$\frac{2}{18}$
white		$\frac{6}{18}$
white		$\frac{10}{18}$

conditional probability

given the definition of conditional probability

$$P(s|c) = \frac{P(s)\,P(c|s)}{P(c)}$$

show that:

$$P(s|c) = \frac{P(c|s)\,P(s)}{P(c)}$$

observe:

$$P(s|c) = \frac{P(c,s)}{P(c)} \quad \text{and} \quad P(c|s) = \frac{P(c,s)}{P(s)}$$

so: $P(s|c)P(c) = P(c,s)$ and $P(c|s)P(s) = P(c,s)$

therefore: $P(s|c)P(c) = P(c|s)P(s)$

which means: $P(s|c) = \dfrac{P(c|s)P(s)}{P(c)}$

C	S	$P(s\|c) = \frac{P(c,s)}{P(c)}$
red	●	$\frac{3}{7}$
red	■	$\frac{2}{7}$
red	▲	$\frac{2}{7}$
white	●	$\frac{2}{18}$
white	■	$\frac{6}{18}$
white	▲	$\frac{10}{18}$

conditional probability

observe:

$$P(s|c) = \frac{P(c,s)}{P(c)} \quad \text{and} \quad P(c|s) = \frac{P(c,s)}{P(s)}$$

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C	S	$P(s\|c) = \frac{P(c,s)}{P(c)}$
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red	■	$\frac{2}{7}$
red	▲	$\frac{2}{7}$
white	●	$\frac{2}{18}$
white	■	$\frac{6}{18}$
white	▲	$\frac{10}{18}$

conditional probability

observe:

$$P(s|c) = \frac{P(c,s)}{P(c)} \quad \text{and} \quad P(c|s) = \frac{P(c,s)}{P(s)}$$

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C	S	$P(s\|c) = \frac{P(c,s)}{P(c)}$
red	●	$\frac{3}{7}$
red	■	$\frac{2}{7}$
red	▲	$\frac{2}{7}$
white	●	$\frac{2}{18}$
white	■	$\frac{6}{18}$
white	▲	$\frac{10}{18}$

conditional probability

observe:

$$P(s|c) = \frac{P(c,s)}{P(c)} \quad \text{and} \quad P(c|s) = \frac{P(c,s)}{P(s)}$$

so: $P(s|c)P(c) = P(c,s)$ and $P(c|s)P(s) = P(c,s)$

therefore: $P(s|c)P(c) = P(c|s)P(s)$

which means: $$P(s|c) = \frac{P(c|s)P(s)}{P(c)}$$

C	S	$P(s\|c) = \frac{P(c,s)}{P(c)}$
red	●	$\frac{3}{7}$
red	■	$\frac{2}{7}$
red	▲	$\frac{2}{7}$
white	●	$\frac{2}{18}$
white	■	$\frac{6}{18}$
white	▲	$\frac{10}{18}$

conditional probability

observe:

$P(s|c) = \frac{P(c,s)}{P(c)}$ and $P(c|s) = \frac{P(c,s)}{P(s)}$

this is called

bayes rule

so: $P(s|c)P(c) = P(c,s)$ and $P(c|s)P(s) = P(c,s)$

therefore: $P(s|c)P(c) = P(c|s)P(s)$

which means: $\boxed{P(s|c) = \frac{P(c|s)P(s)}{P(c)}}$

given variables X, Y with domains $D(X)$ and $D(Y)$
and joint probability $P(x, y)$

$$P(x) = \sum_{y \in D(Y)} P(x, y)$$

marginal probability of X

$$P(y \mid x) = \frac{P(x, y)}{P(x)}$$

conditional probability of Y given X

$$P(y \mid x) = \frac{P(x \mid y)\, P(y)}{P(x)}$$

bayes rule

given variables $X_1, \ldots, X_n$ with domains $D(X_i)$

and joint probability $P(x_1, \ldots, x_n)$

$$P(x_1, \ldots, x_m) = \sum_{x_{m+1} \in D(X_{m+1})} \cdots \sum_{x_n \in D(X_n)} P(x_1, \ldots, x_n)$$

marginal probability

$$P(x_{m+1}, \ldots, x_n \mid x_1, \ldots, x_m) = \frac{P(x_1, \ldots, x_n)}{P(x_1, \ldots, x_m)}$$

conditional probability

$$P(x_{m+1}, \ldots, x_n \mid x_1, \ldots, x_m) = \frac{P(x_1, \ldots, x_m \mid x_{m+1}, \ldots, x_n)\, P(x_1, \ldots, x_m)}{P(x_{m+1}, \ldots, x_n)}$$

bayes rule

given variables $X_1, \dots, X_n$ with domains $D(X_i)$

and joint probability $P(x_1, \dots, x_n)$

$$P(x_1, \dots, x_m) = \sum_{x_{m+1} \in D(X_{m+1})} \cdots \sum_{x_n \in D(X_n)} P(x_1, \dots, x_n)$$

$$P(x_{m+1}, \dots, x_n \mid x_1, \dots, x_m) = \frac{P(x_1, \dots, x_n)}{P(x_1, \dots, x_m)}$$

$$P(x_{m+1}, \dots, x_n \mid x_1, \dots, x_m) = \frac{P(x_1, \dots, x_m \mid x_{m+1}, \dots, x_n)\, P(x_1, \dots, x_m)}{P(x_{m+1}, \dots, x_n)}$$