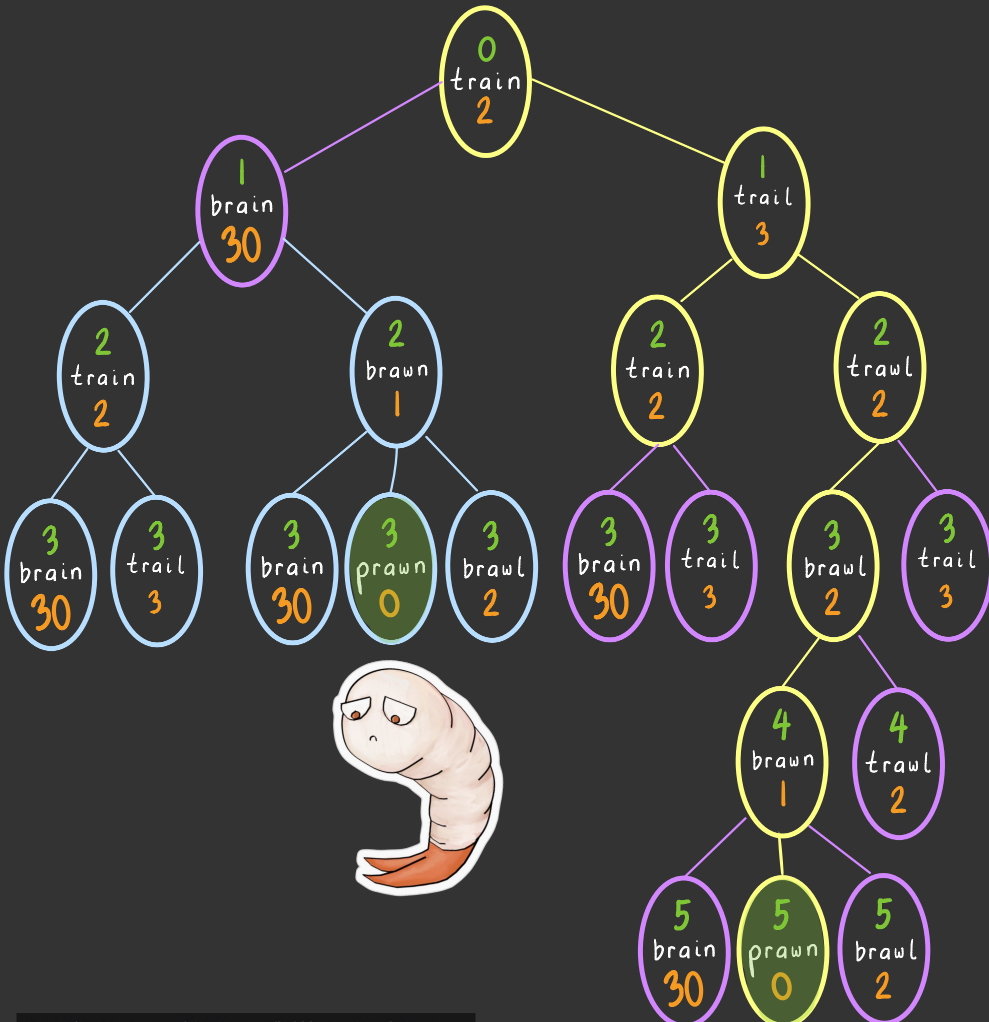
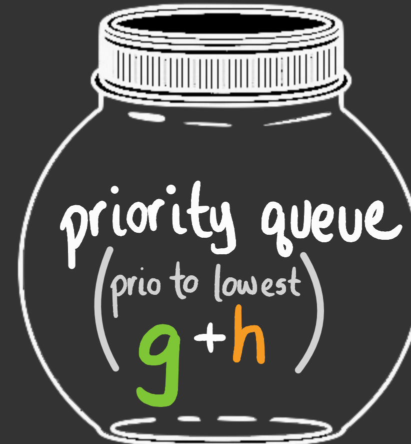
optimality of A* search

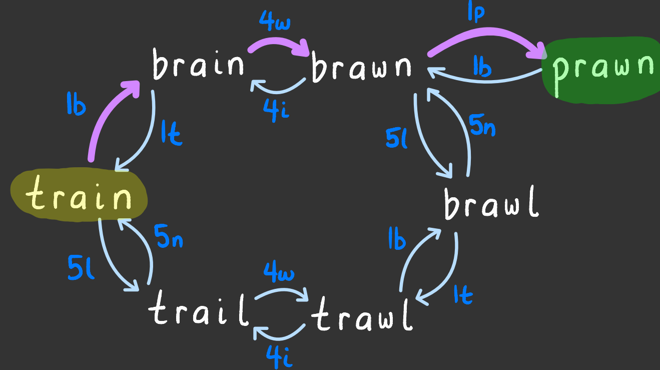
CSCI 373





under what conditions is A^* search optimal?

train
brain
brawn
prawn

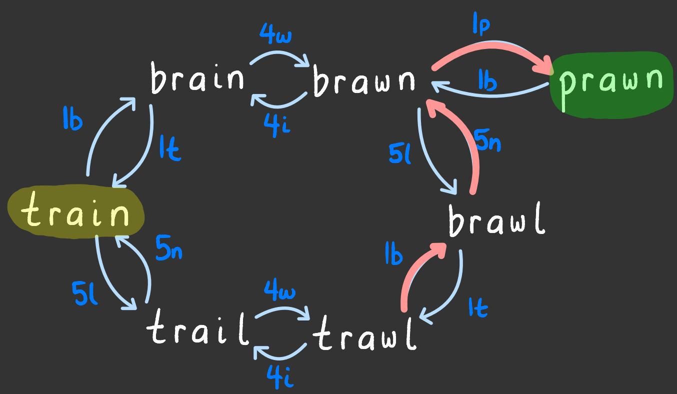


recall

formally: $\langle (\text{train}, 1b, \text{brain}), (\text{brain}, 4w, \text{brawn}), (\text{brawn}, 1p, \text{prawn}) \rangle$

states	$Q = \{\text{train}, \text{brain}, \text{trail}, \dots\}$
actions	$\Sigma = \{1a, \dots, 5z\}$
transitions	$\Delta = \{(\text{train}, 1b, \text{brain}), \dots\}$
initial state	$q_0 = \text{train}$
final states	$F = \{\text{prawn}\}$
weight function	$w = \{\delta \mapsto 1 \mid \delta \in \Delta\}$

a search path is a sequence $\langle \delta_0, \dots, \delta_k \rangle$ of transitions from Δ such that there exist states $q_1, \dots, q_{k+1} \in Q$ and actions $\sigma_0, \dots, \sigma_k \in \Sigma$ such that $\delta_i = (q_i, \sigma_i, q_{i+1}) \quad \forall i \in \{0, \dots, k\}$



states $Q = \{\text{train}, \text{brain}, \text{trail}, \dots\}$

actions $\Sigma = \{1a, \dots, 5z\}$

transitions $\Delta = \{(\text{train}, 1b, \text{brain}), \dots\}$

initial state $q_0 = \text{train}$

final states $F = \{\text{prawn}\}$

weight function $w = \{\delta \mapsto 1 \mid \delta \in \Delta\}$

a completion path from state q is a sequence $\langle \delta_1, \dots, \delta_k \rangle$ of transitions from Δ such that there exist states $q_1, \dots, q_{k+1} \in Q$ and actions $\sigma_1, \dots, \sigma_k \in \Sigma$

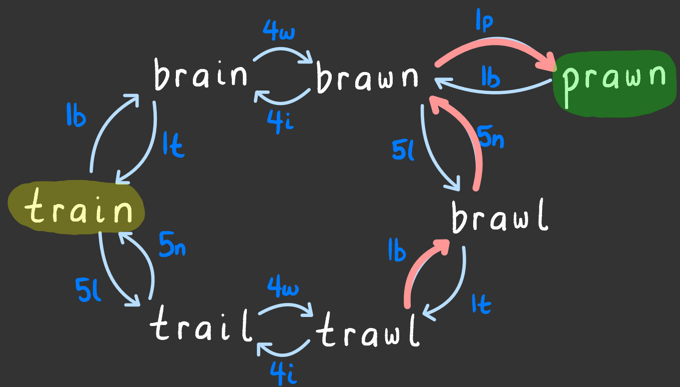
such that

- $\delta_i = (q_i, \sigma_i, q_{i+1}) \quad \forall i \in \{0, \dots, k\}$
- $q_1 = q$
- $q_{k+1} \in F$

in other words:

search path = path from initial state

completion path = path to a final state



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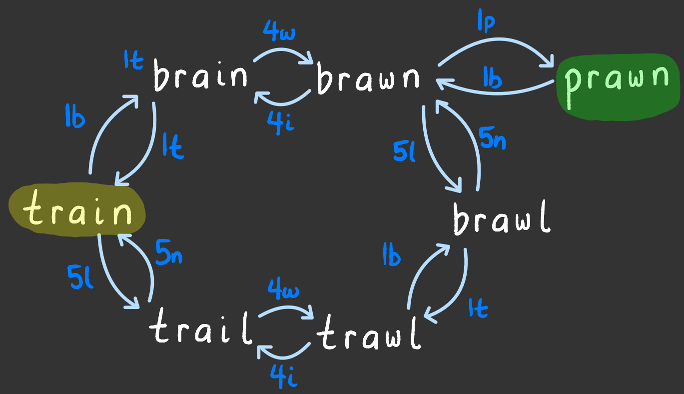
the cost of a completion path is the sum of the weights of the transitions

an optimal completion path from state q is the completion path of minimum cost



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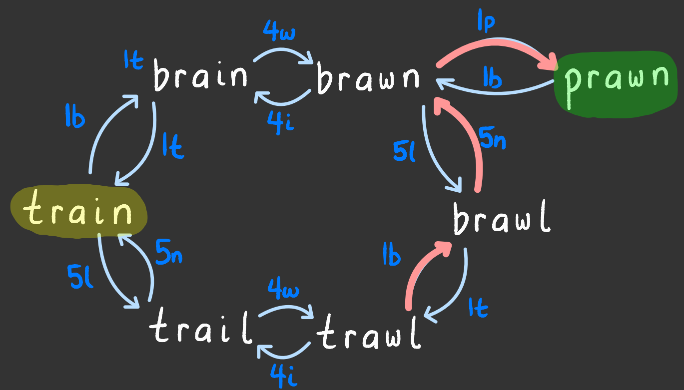
what is the optimal completion path from state trawl ?



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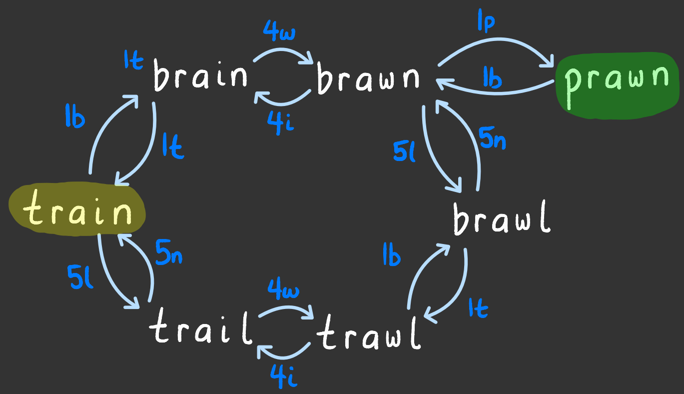
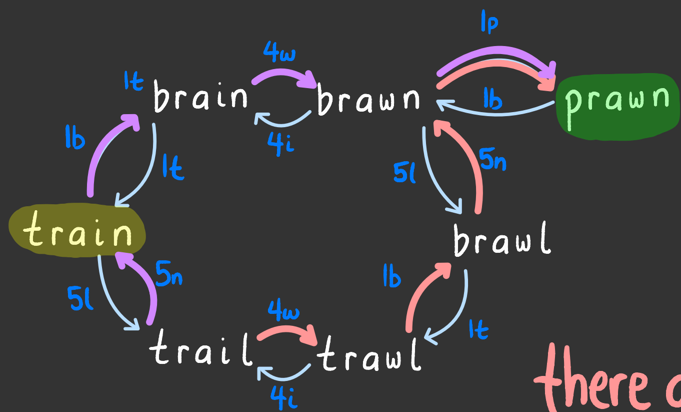


the cost of a completion path is the sum of the weights of the transitions

an optimal completion path from state q is the completion path of minimum cost

what is the optimal completion path

from state trail?



there are

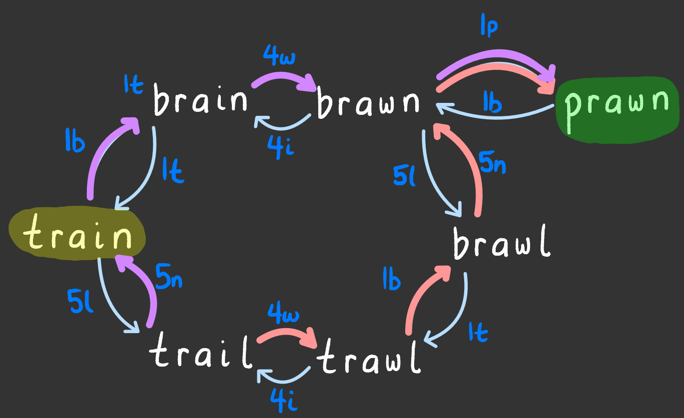
two

the cost of a completion path is the sum of the weights of the transitions

an optimal completion path from state q is the completion path of minimum cost

what is the optimal completion path

from state trail ?



the cost of a completion path is the sum of the weights of the transitions

an optimal completion path from state q is the completion path of minimum cost

$H^*(\text{brain}) = \boxed{?}$

$H^*(\text{trail}) = \boxed{?}$

$H^*(\text{train}) = \boxed{?}$

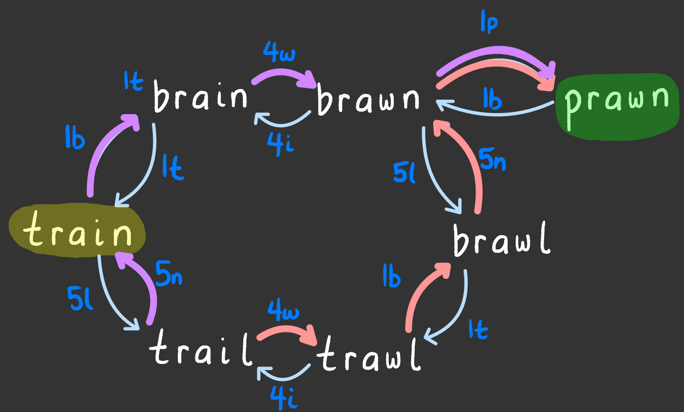
$H^*(\text{brawn}) = \boxed{?}$

$H^*(\text{trawl}) = \boxed{?}$

$H^*(\text{brawl}) = \boxed{?}$

$H^*(\text{prawn}) = \boxed{?}$

let $H^*(q)$ be the cost of the optimal completion path from state q



the cost of a completion path is the sum of the weights of the transitions

an optimal completion path from state q is the completion path of minimum cost

$H^*(\text{brain}) = 2$

$H^*(\text{trail}) = 4$

$H^*(\text{train}) = 3$

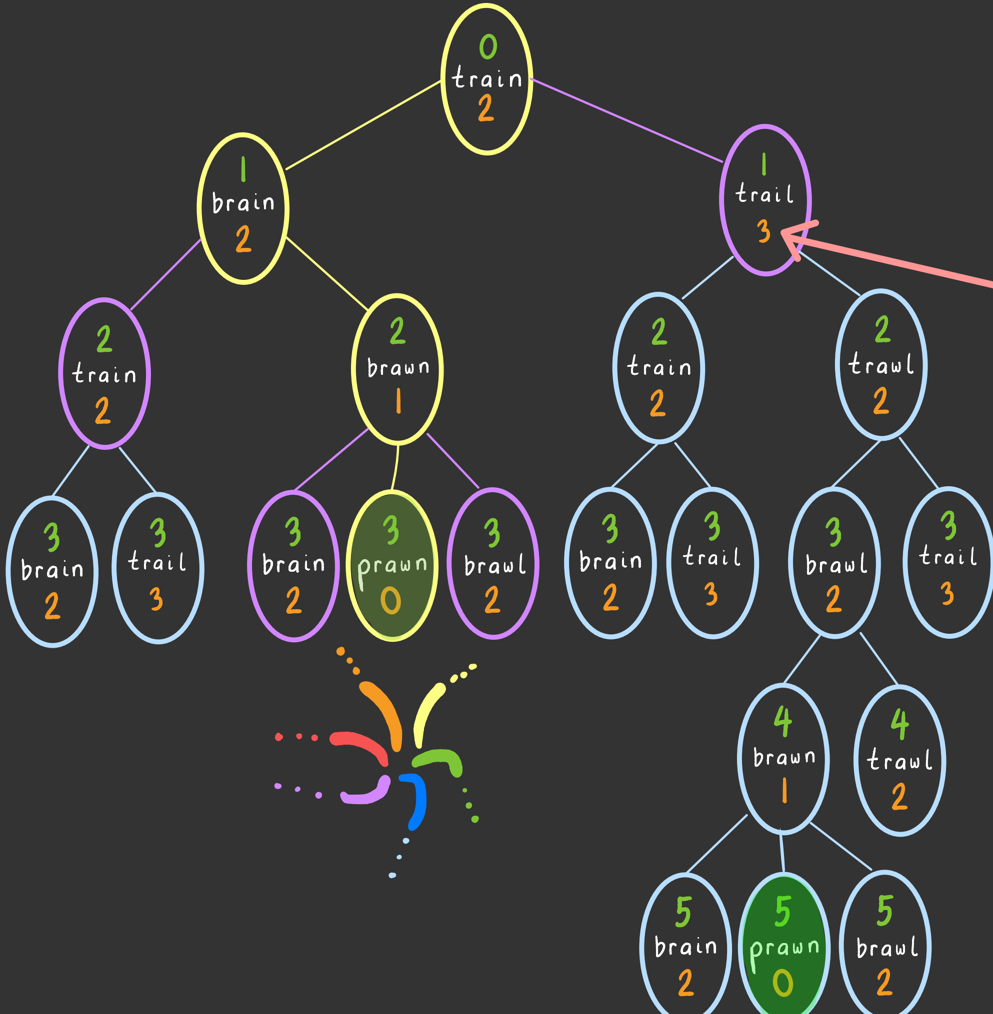
$H^*(\text{brawn}) = 1$

$H^*(\text{trawl}) = 3$

$H^*(\text{brawl}) = 2$

$H^*(\text{prawn}) = 0$

let $H^*(q)$ be the cost of the optimal completion path from state q

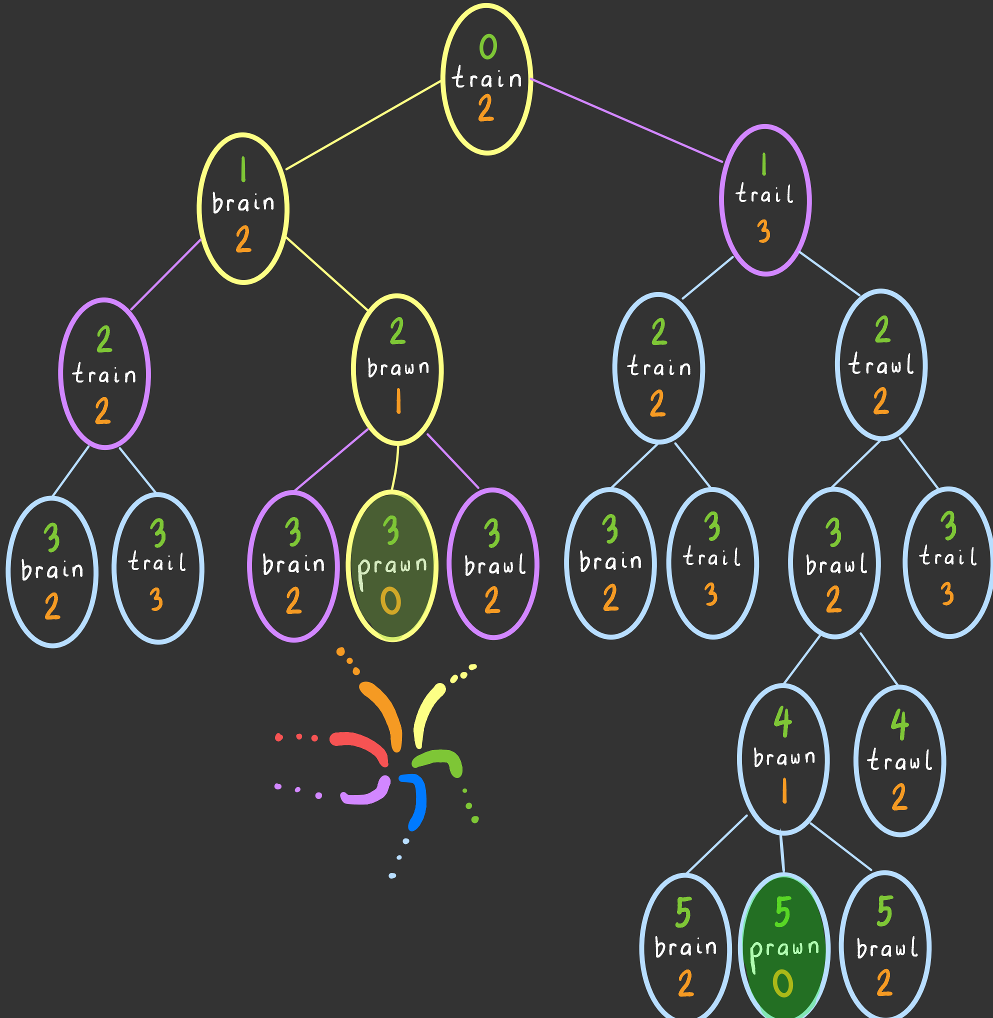


consider the heuristic we've been using for **word ladder**

trail

prawn

at least 3 steps to a final state



for every state q

$$H(q) \; \boxed{<?>} \; H^*(q)$$

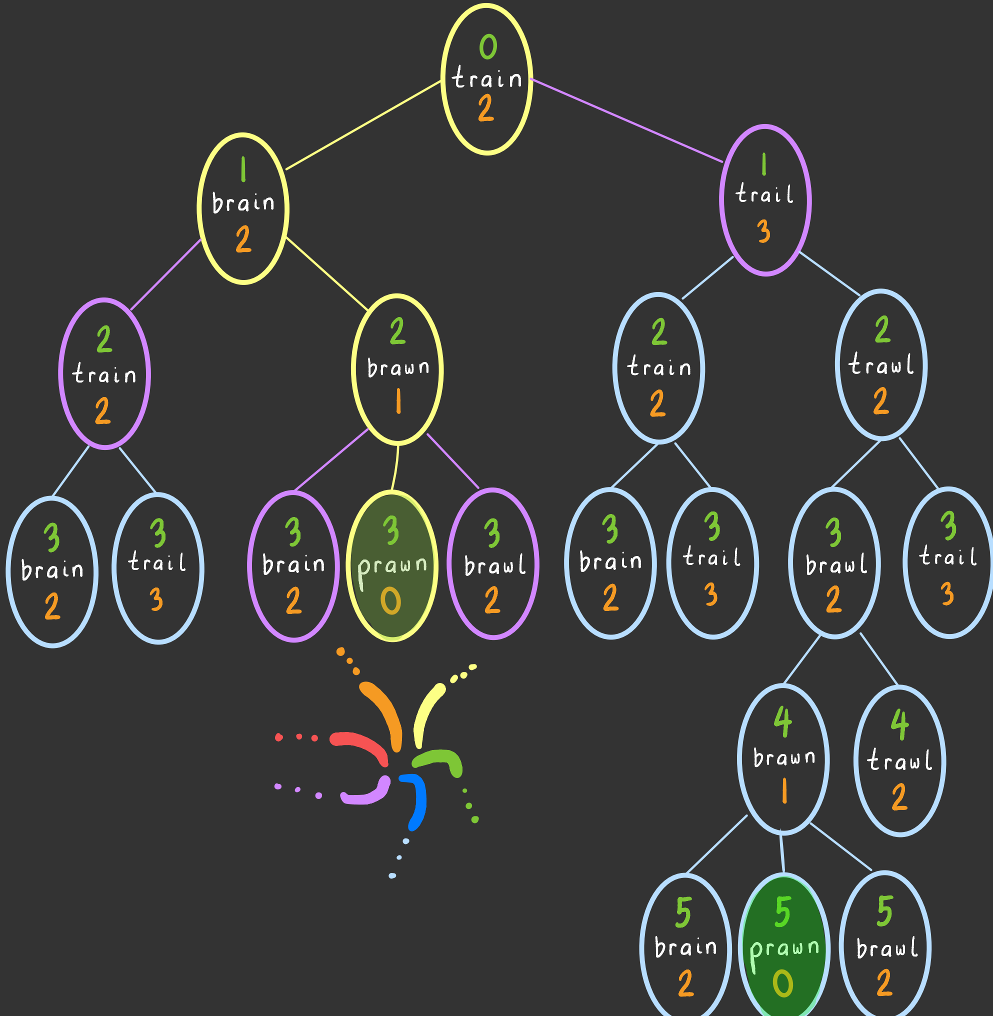
heuristic function

optimal completion cost

trail

prawn

at least 3 steps to a final state



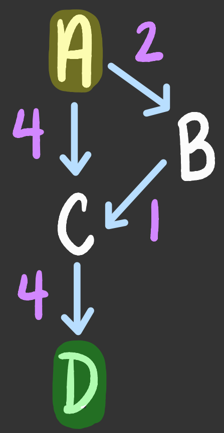
for every state q

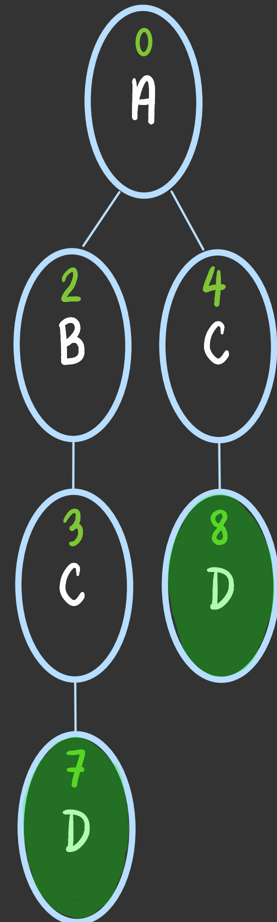
$$H(q) \leq H^*(q)$$

heuristic function

optimal completion cost

a heuristic function H that satisfies this condition is called **admissible** is A^* optimal if we use an admissible heuristic function?





heuristic function H is **admissible** if for every state q:

$$H(q) \leqslant H^*(q)$$

heuristic function

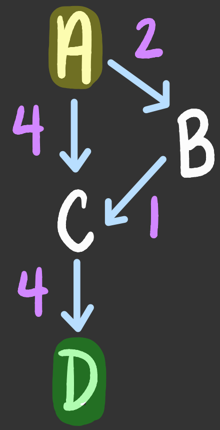
optimal completion cost

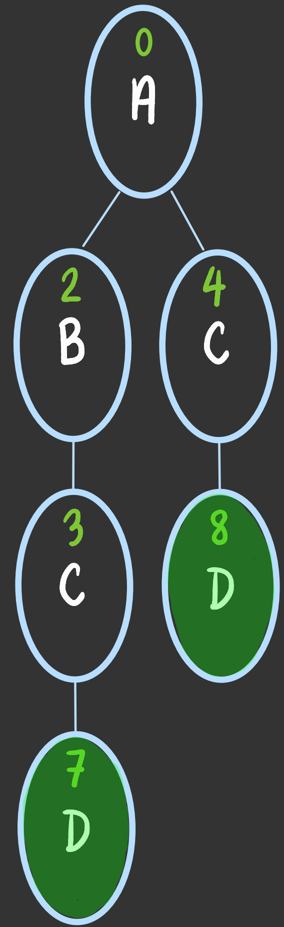
7 $H^*(A)=\boxed{?}$

5 $H^*(B)=\boxed{?}$

4 $H^*(C)=\boxed{?}$

0 $H^*(D)=\boxed{?}$





heuristic function H is **admissible** if for every state q:

$$H(q) \leq H^*(q)$$

$H(q)$ ← heuristic function

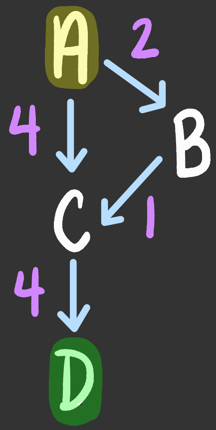
$H^*(q)$ ← optimal completion cost

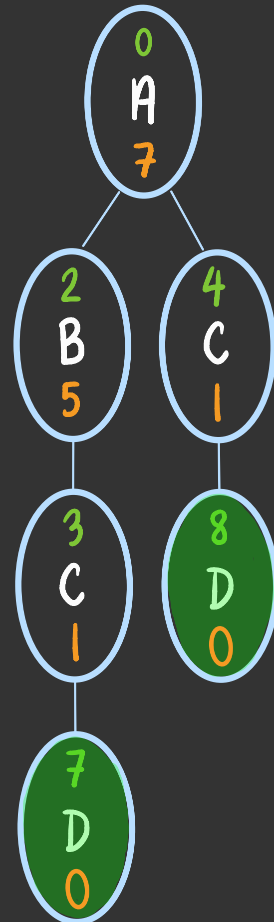
$H^*(A) = 7$

$H^*(B) = 5$

$H^*(C) = 4$

$H^*(D) = 0$





heuristic function H is **admissible** if for every state q:

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heuristic function

optimal completion cost

$$H(A) = 7 \leqslant H^*(A) = 7$$

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$$H(C) = 1 \leqslant H^*(C) = 4$$

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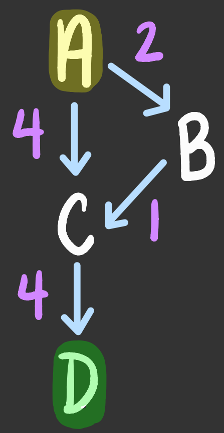
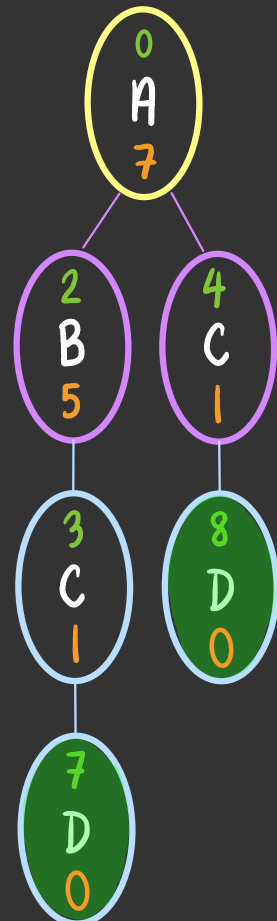
admissible!





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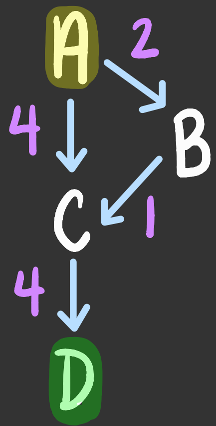

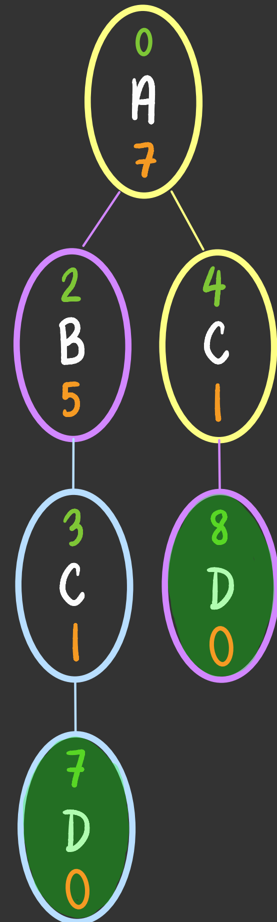

heuristic function H

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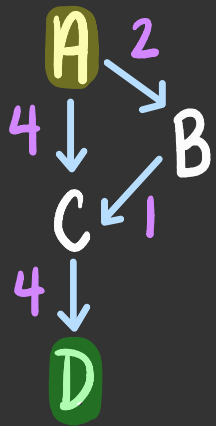

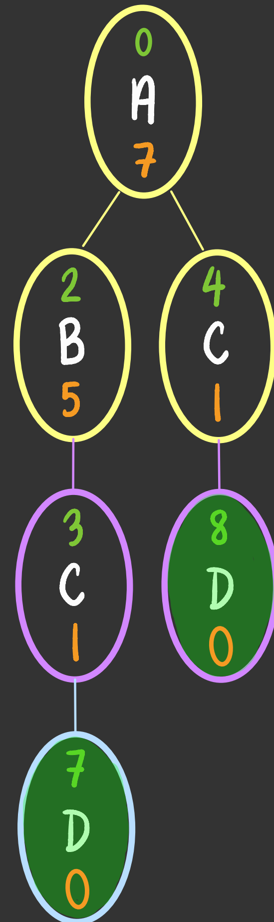

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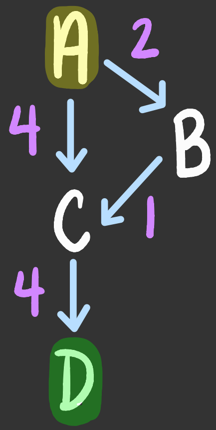

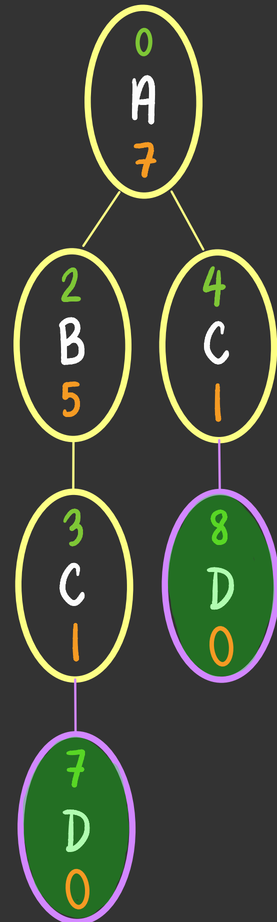

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heuristic function

optimal completion cost

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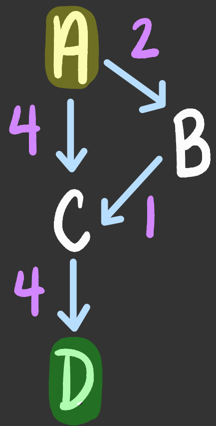
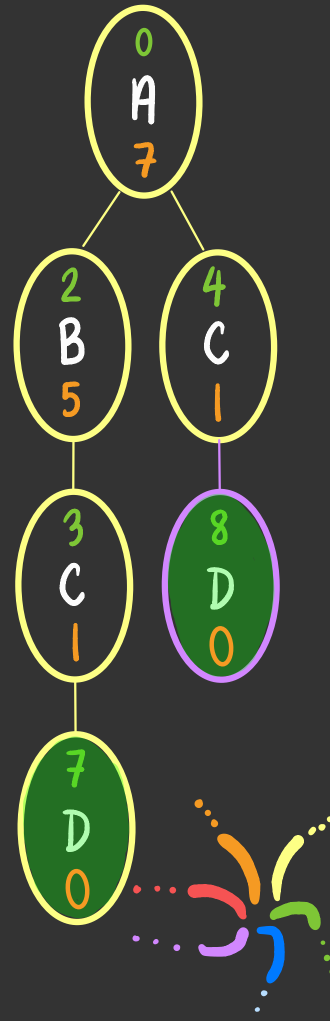
admissible!





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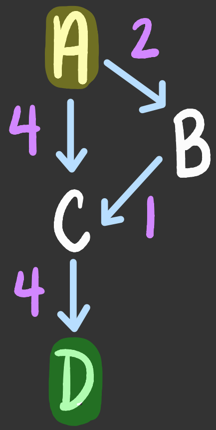

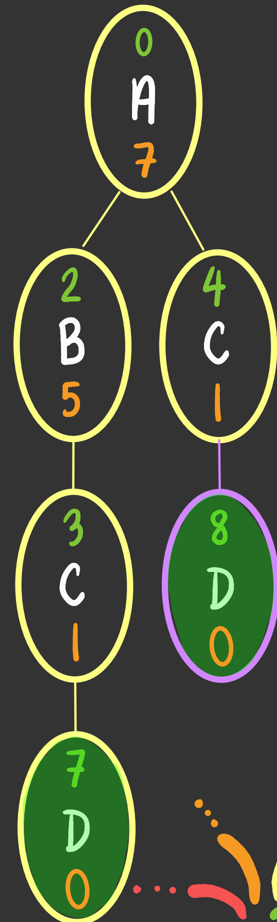

heuristic function H is **admissible** if for every state q:

$$H(q) \leq H^*(q)$$

heuristic function

optimal completion cost

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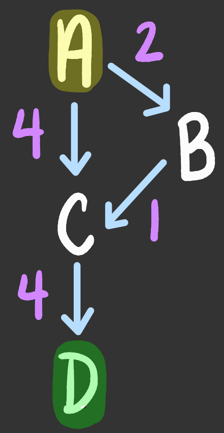
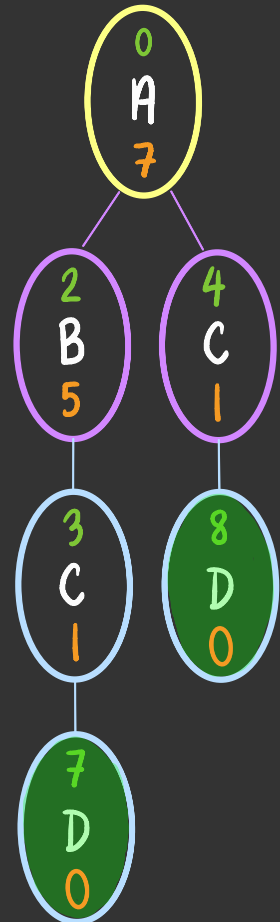
admissible!

but what if we **memoize?**





visited

A

heuristic function H is **admissible** if for every state q:

$$H(q) \leq H^*(q)$$

H ← heuristic function

H^* ← optimal completion cost

$$H(A) = 7 \leq H^*(A) = 7$$

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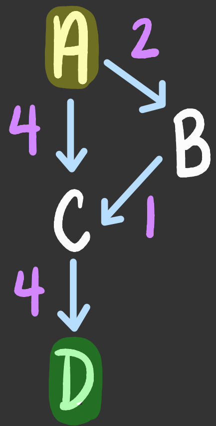
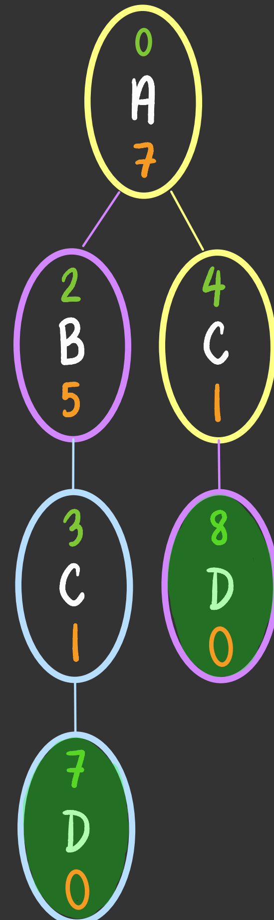
admissible!





heuristic function H is **admissible** if for every state q:

$$H(q) \leq H^*(q)$$

heuristic function

optimal completion cost

visited

A C

$H(A) = 7 \leq H^*(A) = 7$

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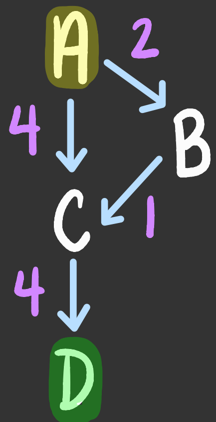
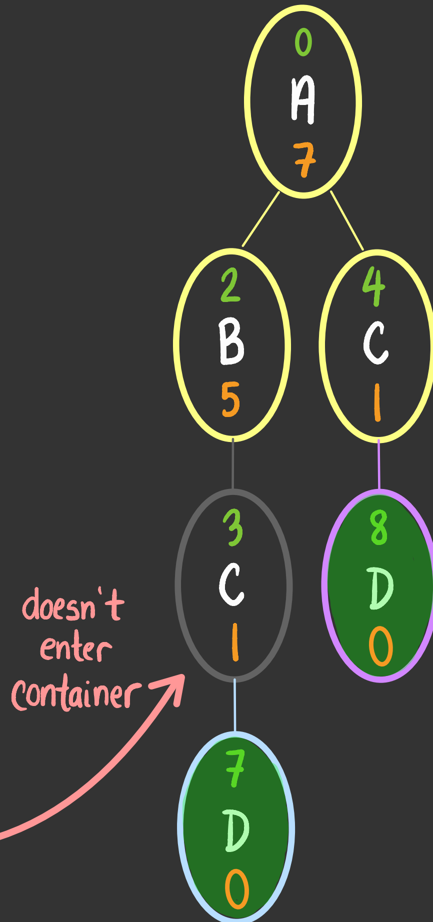
admissible!





visited

A C B

heuristic function H is **admissible** if for every state q:

$$H(q) \leqslant H^*(q)$$

H: heuristic function

H^*: optimal completion cost

$$H(A) = 7 \leqslant H^*(A) = 7$$

$$H(B) = 5 \leqslant H^*(B) = 5$$

$$H(C) = 1 \leqslant H^*(C) = 4$$

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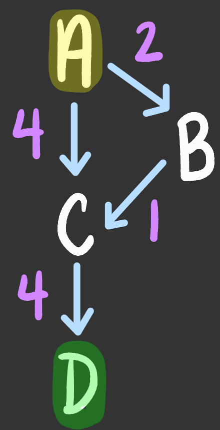
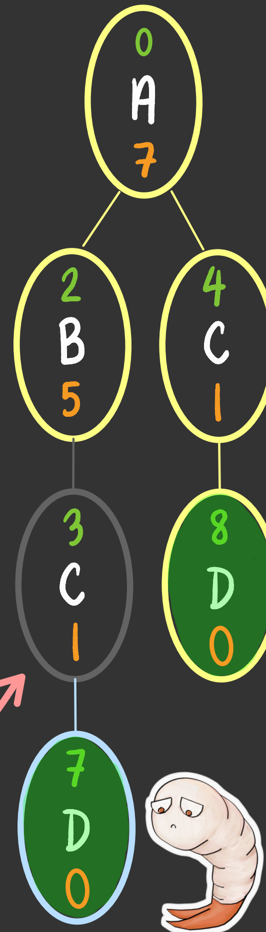
admissible!



visited

A C B

doesn't enter container



heuristic function H is **admissible** if for every state q:

$$H(q) \leqslant H^*(q)$$

heuristic function

optimal completion cost

$H(A) = 7 \leqslant H^*(A) = 7$

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admissible!

is A* optimal if we use an admissible heuristic function?

yes, if we don't memoize

is A* optimal if we use an admissible heuristic function?

yes, if we don't memoize

but what if we want to memoize?

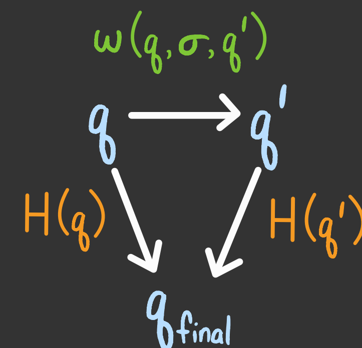
heuristic function H is **admissible** if for every state q:

$$\underbrace{H(q)}_{\text{heuristic function}} \leq \underbrace{H^*(q)}_{\text{optimal completion cost}}$$

heuristic function H is **consistent** if

- $H(q) = 0$ for all final states $q \in F$
- $H(q) \leq \omega(q, \sigma, q') + H(q')$

  for all transitions $\langle q, \sigma, q' \rangle \in \Delta$



heuristic function H is **consistent** if

- $H(q) = 0$ for all final states $q \in F$
- $H(q) \leq \omega(q, \sigma, q') + H(q')$ for all transitions $\langle q, \sigma, q' \rangle \in \Delta$

"it takes 3 hours to drive to Boston"

$$\begin{tikzcd}
q \arrow[rr, "\omega(q,\sigma,q')"] \arrow[dr, "H(q)"'] & & q' \arrow[dl, "H(q')"] \\
& q_{final} &
\end{tikzcd}$$

"from Boston I think it takes 2 hours to drive to Providence"

heuristic function H is **consistent** if

- $H(q) = 0$ for all final states $q \in F$
- $H(q) \leq \omega(q, \sigma, q') + H(q')$ for all transitions $\langle q, \sigma, q' \rangle \in \Delta$

"it takes 3 hours to drive to Boston"

$$
\begin{tikzcd}
q \arrow[r, "\omega(q,\sigma,q')"] \arrow[dr, "H(q)"'] & q' \arrow[d, "H(q')"] \\
 & q_{final}
\end{tikzcd}
$$

"I think it takes 6 hours to drive to Providence"

"from Boston I think it takes 2 hours to drive to Providence"

heuristic function H is **consistent** if

- $H(q) = 0$ for all final states $q \in F$
- $H(q) \leq \omega(q, \sigma, q') + H(q')$

  for all transitions $\langle q, \sigma, q' \rangle \in \Delta$

"it takes **3 hours** to drive to Boston"

$$\begin{tikzcd}
q \arrow[rr, "\omega(q,\sigma,q')"] \arrow[dr, "H(q)"'] & & q' \arrow[dl, "H(q')"] \\
& q_{final} &
\end{tikzcd}$$

"I think it takes ~~6 hours~~ to drive to Providence"

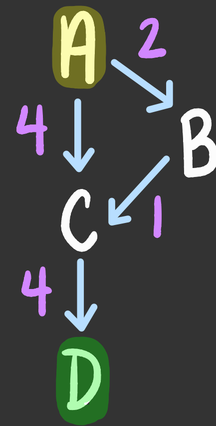
"from Boston I think it takes **2 hours** to drive to Providence"

well no... you've already described a route that you think takes 5 hours... $H(q)$ should be ≤ 5

heuristic function H is consistent if

- $H(q) = 0$ for all final states $q \in F$
- $H(q) \leq \omega(q, \sigma, q') + H(q')$

  for all transitions $\langle q, \sigma, q' \rangle \in \Delta$



$H(A) = 7 \leq H^*(A) = 7$

$H(B) = 5 \leq H^*(B) = 5$

$H(C) = 1 \leq H^*(C) = 4$

$H(D) = 0 \leq H^*(D) = 0$

was this heuristic function consistent?

your answer here

heuristic function H is **consistent** if

- $H(q) = 0$ for all final states $q \in F$
- $H(q) \leq \omega(q, \sigma, q') + H(q')$ for all transitions $\langle q, \sigma, q' \rangle \in \Delta$

$$\begin{tikzcd} A \arrow[r, "2"] \arrow[d, "4"'] & B \arrow[dl, "1"] \\ C \arrow[d, "4"'] & \\ D & \end{tikzcd}$$

$H(A) = 7 \leq H^*(A) = 7$

$H(B) = 5 \leq H^*(B) = 5$

$H(C) = 1 \leq H^*(C) = 4$

$H(D) = 0 \leq H^*(D) = 0$

was this heuristic function consistent?

no. $\underbrace{H(A)}_{7} > \underbrace{\omega(A, \cdot, C)}_{4} + \underbrace{H(C)}_{1}$

heuristic function H is consistent if

- $H(q) = 0$ for all final states $q \in F$
- $H(q) \leq \omega(q, \sigma, q') + H(q')$ for all transitions $\langle q, \sigma, q' \rangle \in \Delta$

if H is consistent, then A^* is optimal

heuristic function H is consistent if

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no successor of a node can have a better priority than the node it succeeded $\rightarrow$ if two nodes are visited by A^*, the first node visited never has a worse priority than the second $\rightarrow$ if H is consistent, then A^* is optimal

heuristic function H is consistent if

- $H(q) = 0$ for all final states $q \in F$
- $H(q) \leq \omega(q, \sigma, q') + H(q')$ for all transitions $\langle q, \sigma, q' \rangle \in \Delta$

if node $n' \in \text{successors}_{m,H}(n)$ and H is consistent, then:

$$g(n') + h(n') \geq g(n) + h(n)$$

no successor of a node can have a better priority than the node it succeeded → if two nodes are visited by A^*, the first node visited never has a worse priority than the second → if H is consistent, then A^* is optimal

heuristic function H is consistent if

- $H(q) = 0$ for all final states $q \in F$
- $H(q) \leq w(q, \sigma, q') + H(q')$ for all transitions $\langle q, \sigma, q' \rangle \in \Delta$

if node $n' \in successors_{m,H}(n)$ and H is consistent, then:

$$g(n') + h(n') \geq g(n) + h(n)$$

$$\begin{aligned}
g(n') + h(n') &= \Big(g(n) + w\big(q(n), \sigma, q(n')\big)\Big) + h(n') \\
&= g(n) + \Big(w\big(q(n), \sigma, q(n')\big) + H(q(n'))\Big) \\
&\geq g(n) + H(q(n))
\end{aligned}$$

heuristic function H is consistent if

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if node $n' \in \text{successors}_{m,H}(n)$ and H is consistent, then:

$$g(n') + h(n') \geq g(n) + h(n)$$

$$g(n') + h(n') = \Big(g(n) + w\big(q(n), \sigma, q(n')\big)\Big) + h(n')$$

$$= g(n) + \Big(w\big(q(n), \sigma, q(n')\big) + H(q(n'))\Big)$$

$$\geq g(n) + H(q(n))$$

heuristic function H is consistent if

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$$\begin{aligned} g(n') + h(n') &= \Big(g(n) + w\big(q(n), \sigma, q(n')\big)\Big) + h(n') \\ &= g(n) + \Big(w\big(q(n), \sigma, q(n')\big) + H(q(n'))\Big) \\ &\geq g(n) + H(q(n)) \end{aligned}$$

heuristic function H is consistent if

- $H(q) = 0$ for all final states $q \in F$
- $H(q) \leqslant w(q, \sigma, q') + H(q')$ for all transitions $\langle q, \sigma, q' \rangle \in \Delta$

if node $n' \in \text{successors}_{m,H}(n)$ and H is consistent, then:

$$g(n') + h(n') \geqslant g(n) + h(n)$$

$$
\begin{aligned}
g(n') + h(n') &= \Big(g(n) + w\big(q(n), \sigma, q(n')\big)\Big) + h(n') \\
&= g(n) + \Big(w\big(q(n), \sigma, q(n')\big) + H(q(n'))\Big) \\
&\geqslant g(n) + H(q(n)) \qquad \text{why?}
\end{aligned}
$$

heuristic function H is consistent if

- $H(q) = 0$ for all final states $q \in F$
- $H(q) \leq w(q, \sigma, q') + H(q')$ for all transitions $\langle q, \sigma, q' \rangle \in \Delta$

if node $n' \in \text{successors}_{m,H}(n)$ and H is consistent, then:

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g(n') + h(n') &= \Big(g(n) + w\big(q(n), \sigma, q(n')\big)\Big) + h(n') \\
&= g(n) + \Big(w\big(q(n), \sigma, q(n')\big) + H(q(n'))\Big) \\
&\geq g(n) + H(q(n))
\end{aligned}$$

because $H(q) \leq w(q, \sigma, q') + H(q')$ where $q = q(n)$, $q' = q(n')$

heuristic function H is **consistent** if

- $H(q) = 0$ for all final states $q \in F$
- $H(q) \leq w(q, \sigma, q') + H(q')$ for all transitions $\langle q, \sigma, q' \rangle \in \Delta$

if node $n' \in \text{successors}_{m,H}(n)$ and H is **consistent**, then:

$$g(n') + h(n') \geq g(n) + h(n)$$

$$\begin{aligned}
g(n') + h(n') &= \Big(g(n) + w\big(q(n), \sigma, q(n')\big)\Big) + h(n') \\
&= g(n) + \Big(w\big(q(n), \sigma, q(n')\big) + H(q(n'))\Big) \\
&\geq g(n) + H(q(n)) \\
&= g(n) + h(n)
\end{aligned}$$

heuristic function H is consistent if

- $H(q) = 0$ for all final states $q \in F$
- $H(q) \leqslant \omega(q, \sigma, q') + H(q')$ for all transitions $\langle q, \sigma, q' \rangle \in \Delta$

if node $n' \in \text{successors}_{m,H}(n)$ and H is consistent, then:

$$g(n') + h(n') \geqslant g(n) + h(n)$$

if node n is visited by A^* before node n', then:

$$g(n') + h(n') \geqslant g(n) + h(n)$$

no successor of a node can have a better priority than the node it succeeded → if two nodes are visited by A^*, the first node visited never has a worse priority than the second → if H is consistent, then A^* is optimal

heuristic function H is consistent if

- $H(q) = 0$ for all final states $q \in F$
- $H(q) \leq \omega(q, \sigma, q') + H(q')$ for all transitions $\langle q, \sigma, q' \rangle \in \Delta$

if node $n' \in \text{successors}_{m,H}(n)$ and H is consistent, then:

$$g(n') + h(n') \geq g(n) + h(n)$$

if node n is visited by A* before node n', then:

$$g(n') + h(n') \geq g(n) + h(n)$$

- assume $g(n') + h(n') < g(n) + h(n)$.
- when we visit node n, either node n' or one of its ancestors are in the container. call this n^*.
- $g(n^*) + h(n^*) \leq g(n') + h(n') < g(n) + h(n)$

heuristic function H is consistent if

- $H(q) = 0$ for all final states $q \in F$
- $H(q) \leq \omega(q, \sigma, q') + H(q')$ for all transitions $\langle q, \sigma, q' \rangle \in \Delta$

if node $n' \in \text{successors}_{m,H}(n)$ and H is consistent, then:

$$g(n') + h(n') \geq g(n) + h(n)$$

if node n is visited by A^* before node n', then:

$$g(n') + h(n') \geq g(n) + h(n)$$

- assume $g(n') + h(n') < g(n) + h(n)$.
- when we visit node n, either node n' or one of its ancestors are in the container. call this n^*.
- $g(n^*) + h(n^*) \leq g(n') + h(n') < g(n) + h(n)$

heuristic function H is consistent if

- $H(q) = 0$ for all final states $q \in F$
- $H(q) \leq \omega(q, \sigma, q') + H(q')$ for all transitions $\langle q, \sigma, q' \rangle \in \Delta$

if node $n' \in \text{successors}_{m,H}(n)$ and H is consistent, then:

$$g(n') + h(n') \geq g(n) + h(n)$$

if node n is visited by A^* before node n', then:

$$g(n') + h(n') \geq g(n) + h(n)$$

inductively

- assume $g(n') + h(n') < g(n) + h(n)$.
- when we visit node n, either node n' or one of its ancestors are in the container. call this n^*.
- $g(n^*) + h(n^*) \leq g(n') + h(n') < g(n) + h(n)$

heuristic function H is consistent if

- $H(q) = 0$ for all final states $q \in F$
- $H(q) \leq \omega(q, \sigma, q') + H(q')$ for all transitions $\langle q, \sigma, q' \rangle \in \Delta$

if node $n' \in \text{successors}_{m,H}(n)$ and H is consistent, then:

$$g(n') + h(n') \geq g(n) + h(n)$$

if node n is visited by A^* before node n', then:

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- $g(n^*) + h(n^*) \leq g(n') + h(n') < g(n) + h(n)$

heuristic function H is consistent if

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if node $n' \in \text{successors}_{m,H}(n)$ and H is consistent, then:

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$$g(n') + h(n') \geq g(n) + h(n)$$

- assume $g(n') + h(n') < g(n) + h(n)$.
- when we visit node n, either node n' or one of its ancestors are in the container. call this n^*.
- $g(n^*) + h(n^*) \quad < \quad g(n) + h(n)$

heuristic function H is consistent if

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- assume $g(n') + h(n') < g(n) + h(n)$.
- when we visit node n, either node n' or one of its ancestors are in the container. call this n^*.
- $g(n^*) + h(n^*)$ < $g(n) + h(n)$

contradiction

heuristic function H is consistent if

- $H(q) = 0$ for all final states $q \in F$
- $H(q) \leq w(q, \sigma, q') + H(q')$ for all transitions $\langle q, \sigma, q' \rangle \in \Delta$

if node n is visited by A^* before node n', then:

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no successor of a node can have a better priority than the node it succeeded → if two nodes are visited by A^*, the first node visited never has a worse priority than the second → if H is consistent, then A^* is optimal

heuristic function H is consistent if

- $H(q) = 0$ for all final states $q \in F$
- $H(q) \leqslant \omega(q, \sigma, q') + H(q')$ for all transitions $\langle q, \sigma, q' \rangle \in \Delta$

if node n is visited by A^* before node n', then:

$g(n') + h(n') \geqslant g(n) + h(n)$

consider goal nodes n and n'.

suppose A^* visits n before n'.

$$g(n') + h(n') \geqslant g(n) + h(n)$$

thus: $g(n') \geqslant g(n)$

so node n' is no better than n

no successor of a node can have a better priority than the node it succeeded $\rightarrow$ if two nodes are visited by A^*, the first node visited never has a worse priority than the second $\rightarrow$ if H is consistent, then A^* is optimal

heuristic function H is consistent if

- $H(q) = 0$ for all final states $q \in F$
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no successor of a node can have a better priority than the node it succeeded $\rightarrow$ if two nodes are visited by A^*, the first node visited never has a worse priority than the second $\rightarrow$ if H is consistent, then A^* is optimal

heuristic function H is consistent if

- $H(q) = 0$ for all final states $q \in F$
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if node n is visited by A^* before node n', then:

$$g(n') + h(n') \geq g(n) + h(n)$$

Consider goal nodes n and n'.

suppose A^* visits n before n'.

thus: $g(n') + \underline{\underline{h(n')}} \geq g(n) + \underline{\underline{h(n)}}$ these are zero

$$g(n') \geq g(n)$$

so node n' is no better than n

no successor of a node can have a better priority than the node it succeeded $\rightarrow$ if two nodes are visited by A^*, the first node visited never has a worse priority than the second $\rightarrow$ if H is consistent, then A^* is optimal

heuristic function H is consistent if

- $H(q) = 0$ for all final states $q \in F$
- $H(q) \leq w(q, \sigma, q') + H(q')$ for all transitions $\langle q, \sigma, q' \rangle \in \Delta$

if node n is visited by A^* before node n', then:

$$g(n') + h(n') \geq g(n) + h(n)$$

consider goal nodes n and n'.

suppose A^* visits n before n'.

$$g(n') + h(n') \geq g(n) + h(n)$$

thus: $$g(n') \geq g(n)$$

so node n' is no better than n

no successor of a node can have a better priority than the node it succeeded

if two nodes are visited by A^*, the first node visited never has a worse priority than the second

if H is consistent, then A^* is optimal if H is consistent, then A* is optimal

if H is admissible, then non-memoized A* is optimal