Hands-on workshop on brain criticality

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CNA2023 W3 workshop session

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UNIVERSITAS JAGELLONICA CRACOVIENSIS

Basic org info

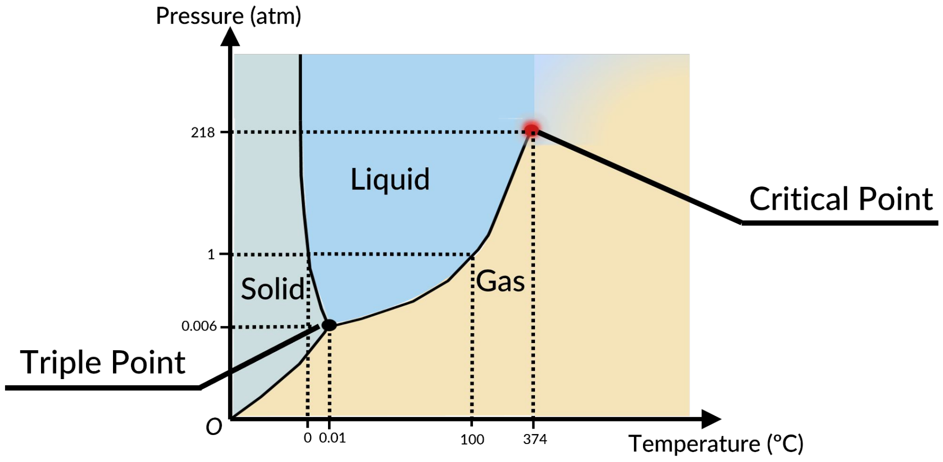
- (15:30 - 18:30) 3 hours with two-three 15 min breaks
- Venue: room G-01-09 or on-line
- Workshop language: **python**
  a crash-course https://bit.ly/python-crash is provided

- The working notebook
  - open http://bit.ly/critical-brain and use **google colab**
  - OR git repo https://bit.ly/critical-brain-github, open **exercises.ipynb**

- Outline
  - The concept of criticality
  - Towards a brain model
  - Criticality in the Haimovici model
  - Playing around

The concept of criticality

- a fascinating statistical physics concept connected with phases of matter



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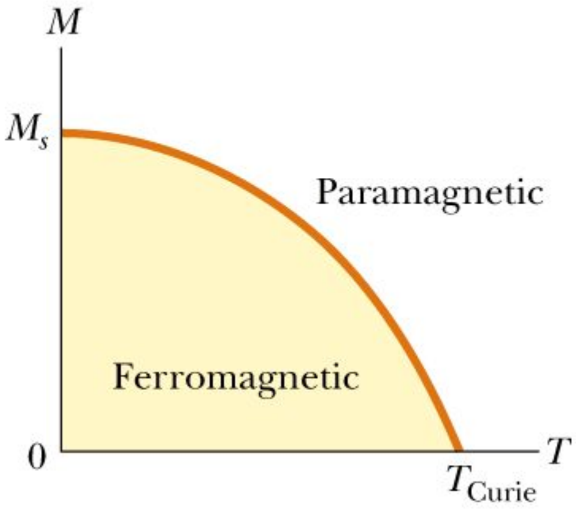
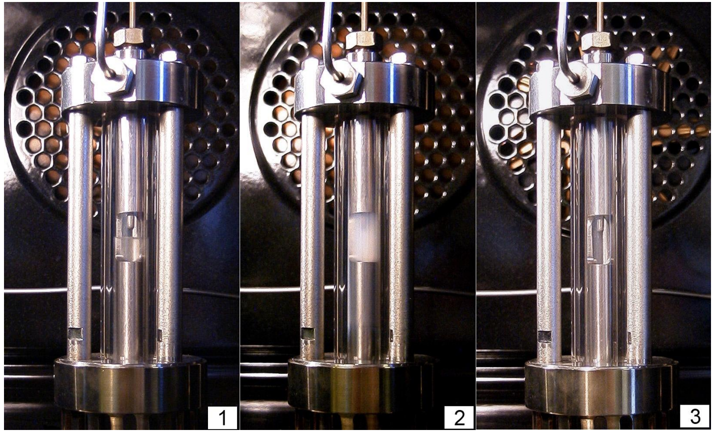
- a fascinating statistical physics concept connected with phases of matter
- also present in magnetism, liquid-gas mixtures; **the critical state** (long-range corr, rich dynamics)





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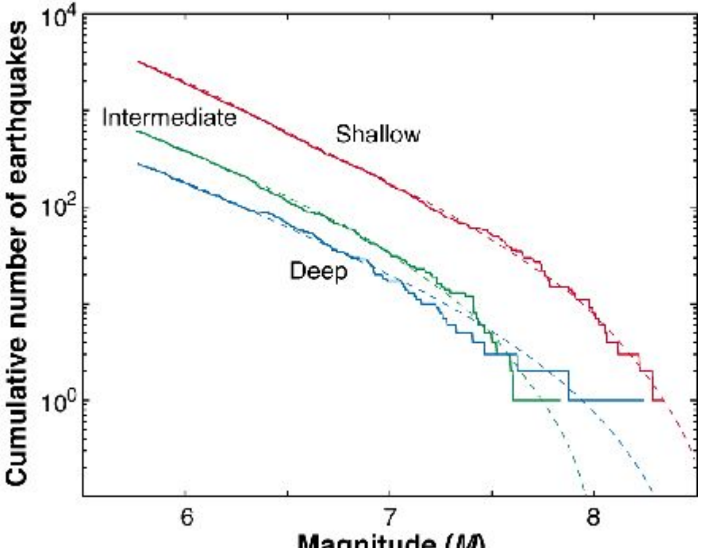

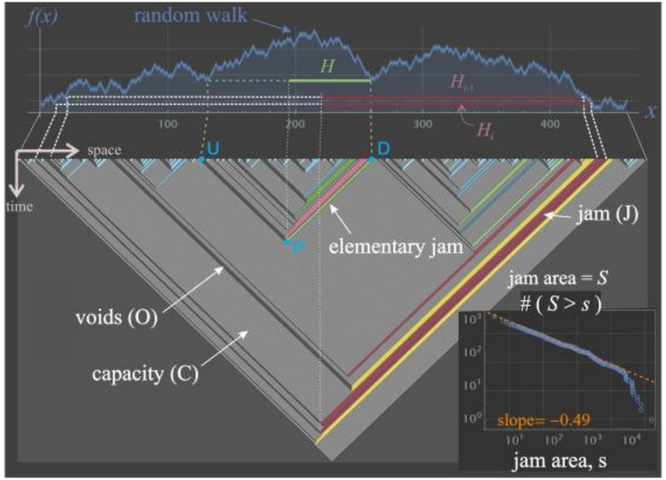

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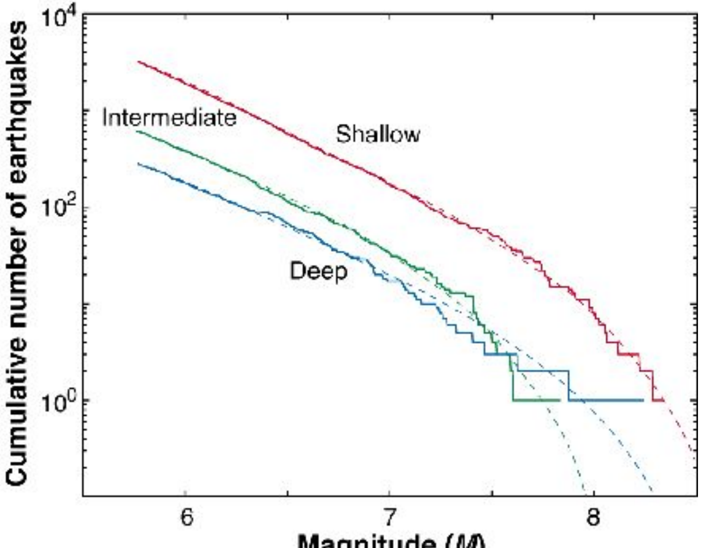
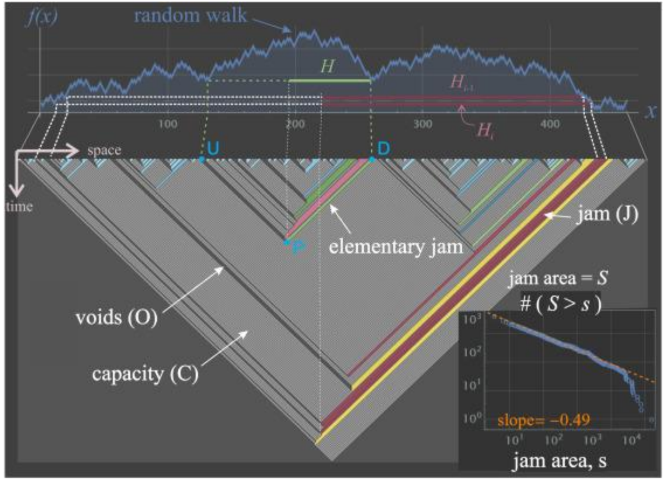
- beyond typical physics - earthquakes, traffic jams (power laws)
- complexity, self-organization (Saturday lecture L17 by prof. Chialvo for more!)





The concept of criticality

- Ising model: a toy model of magnetism

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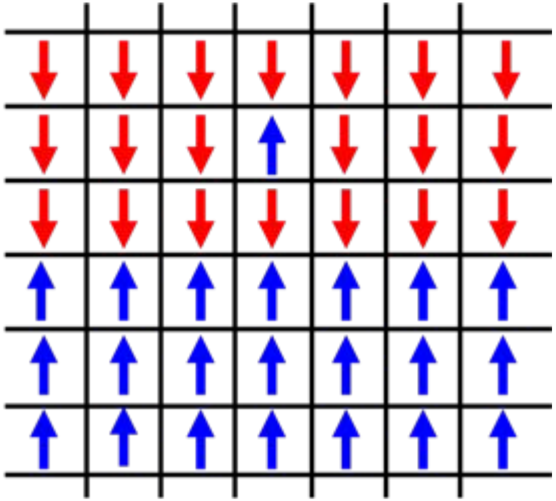
  - spins on a 2D grid, each with 4 neighbours

$$\sigma_k \in \{+1, -1\}$$



The concept of criticality

- Ising model: a toy model of magnetism
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- energy of the system

$$E(\sigma; J) = -J \sum_{\langle ij \rangle} \sigma_i \sigma_j$$

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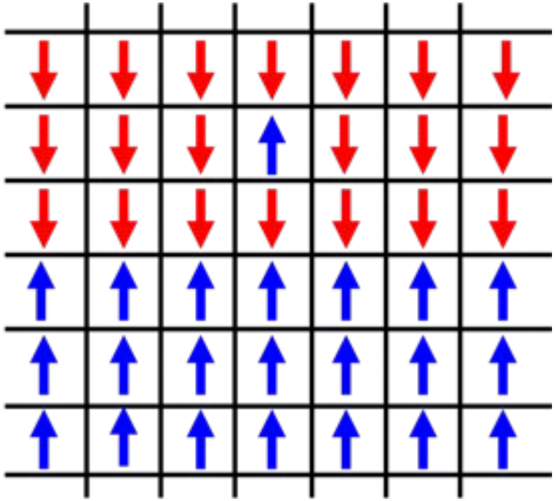

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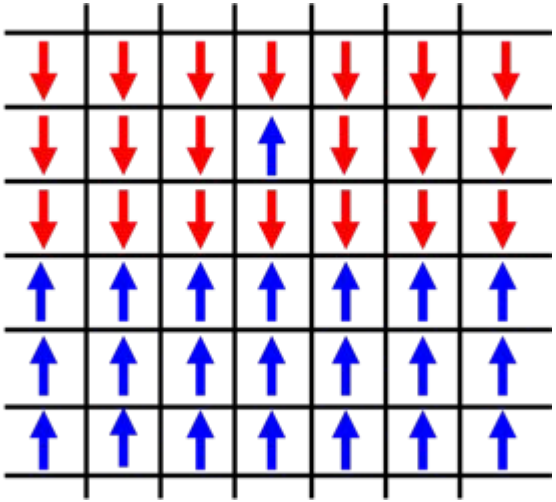

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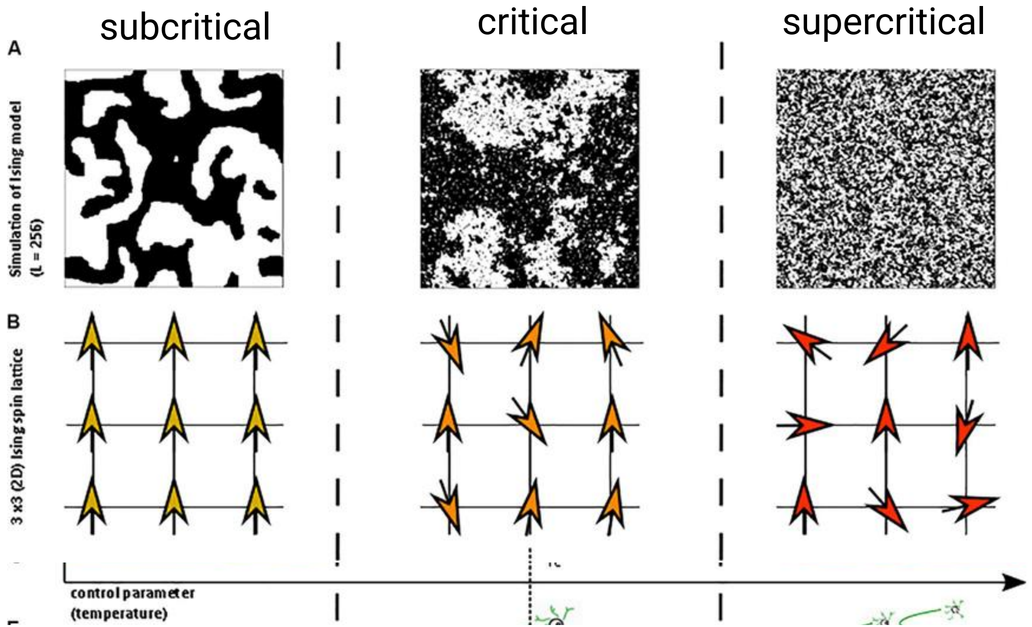
sum over neighbours

- statistical model -> temperature T as a control parameter

$$P(\sigma; T) \sim \exp\left(-\frac{E(\sigma; J)}{T}\right)$$

The concept of criticality

- control parameter T is changed; three regimes!



The concept of criticality

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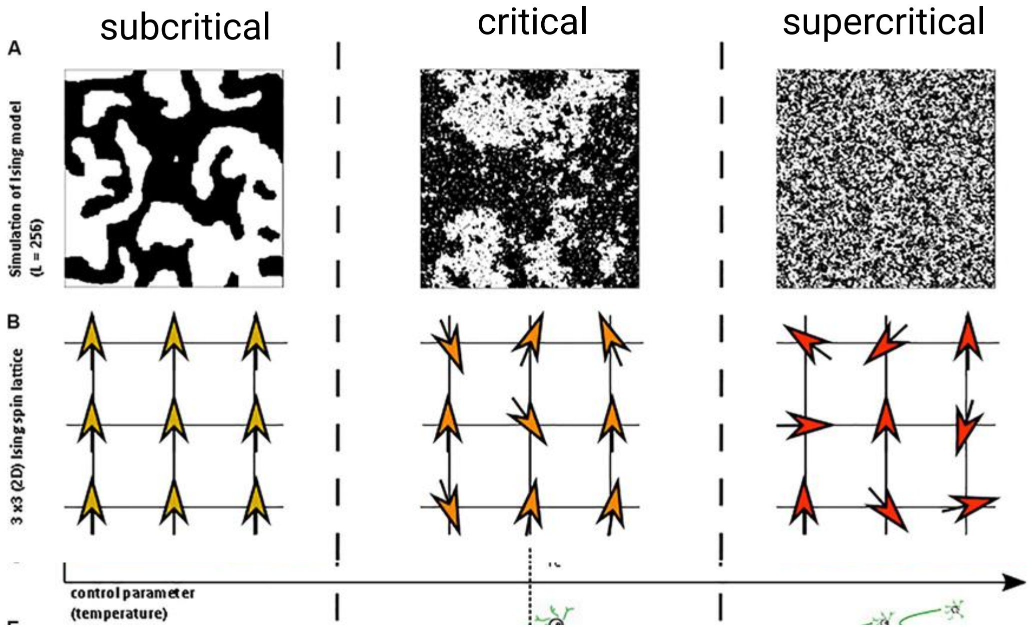
$$T_c = \frac{2J}{\ln\left(\sqrt{2}+1\right)}$$



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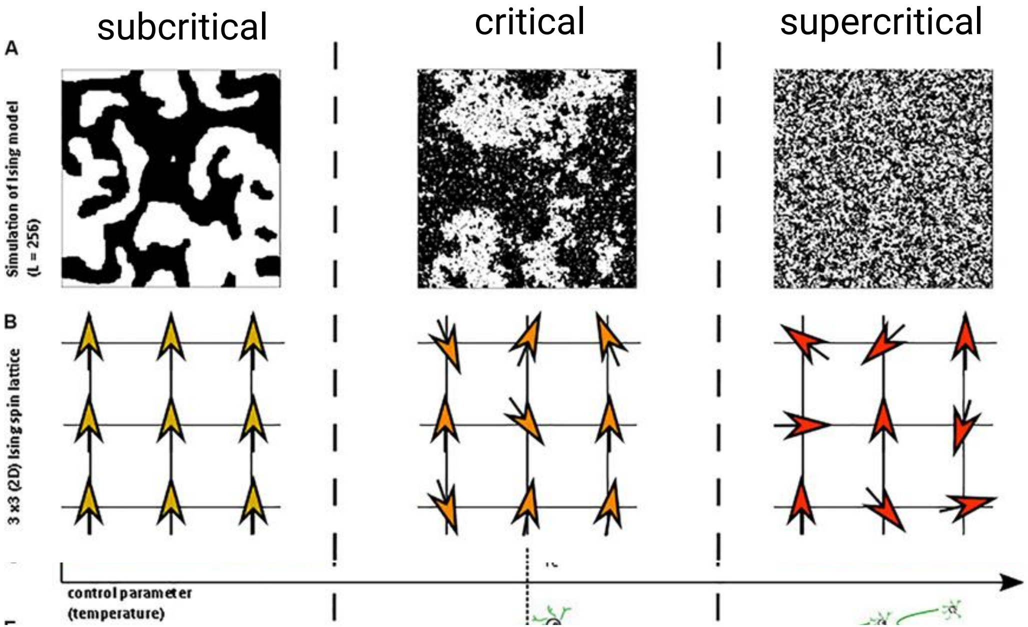
- but what really changes near Tc?



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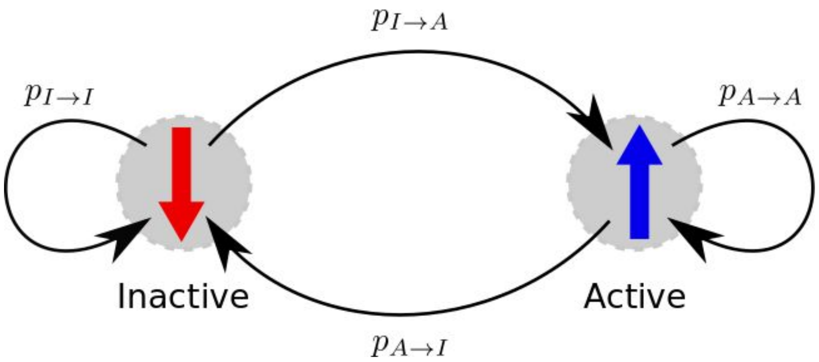
  - repeat
- dynamics are given by a transition graph

$$p_{I\to A} = 1 + \left(e^{-\Delta E/T} - 1\right)\theta(\Delta E)$$



The concept of criticality

Your tasks are:

- **1.1 Magnetization**
  Run the Ising model simulation for a set of temps T; find magnetization as a function of T

- **1.2 Snapshots**
  Plot snapshots in three regimes: subcritical, critical and supercritical.

- **1.3 Binomial model** (*)
  Run a “binomial model” simulation. Plot again the magnetization and snapshots of the dynamics. What is the main difference between the Ising and this model?

Towards a brain model

- …but where's the brain? Consider the Ising energy…

$$E(\sigma; J) = -J \sum_{\langle ij \rangle} \sigma_i \sigma_j$$

Towards a brain model

- …but where’s the brain? Consider the Ising energy…
- with a weighted grid

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Towards a brain model

- …but where’s the brain? Consider the Ising energy…
- with a weighted grid
- grid -> a general graph G
- … but need some neuron-like behavior

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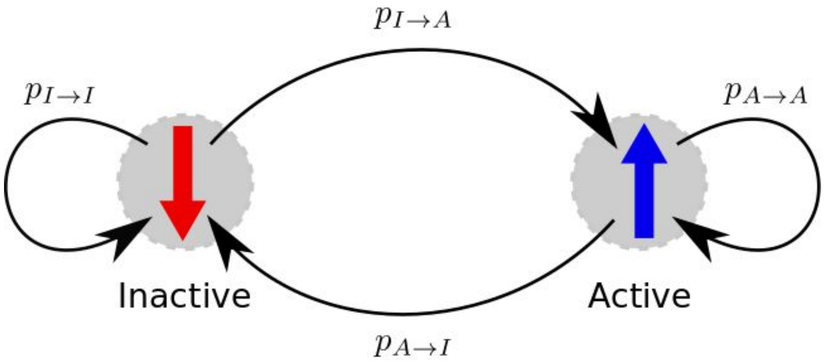
Towards a brain model

- transition graph is modified accordingly:



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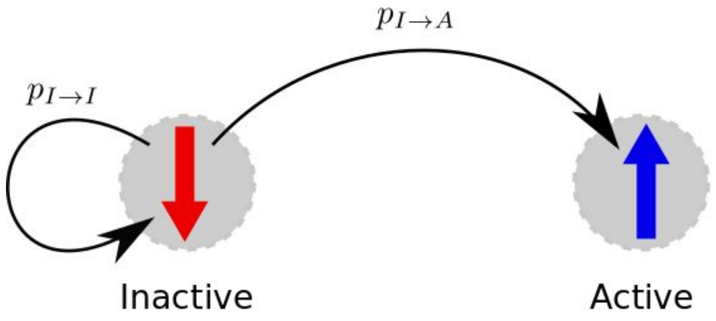

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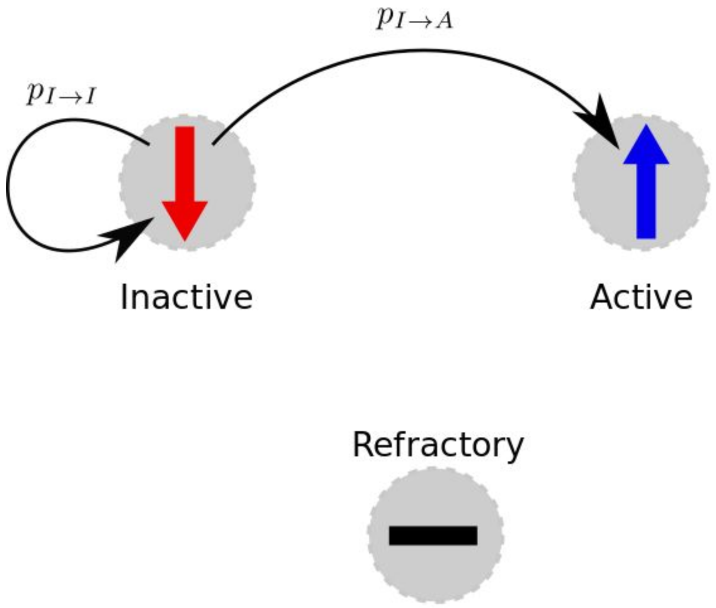

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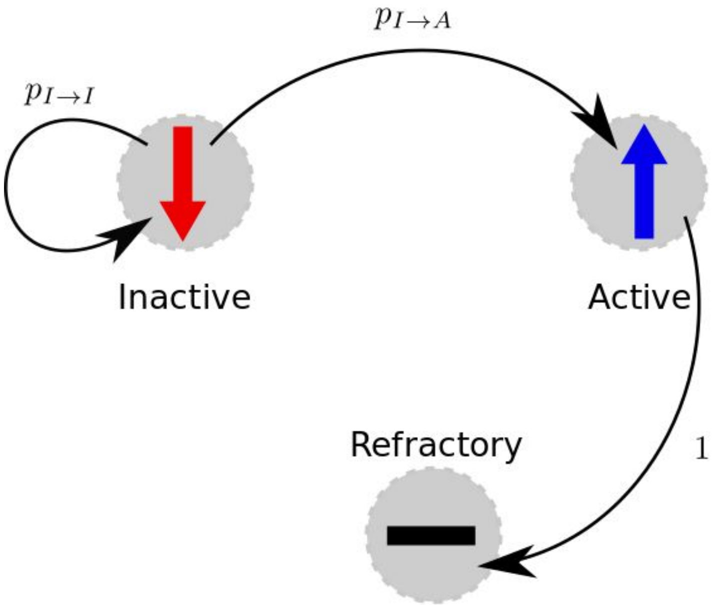

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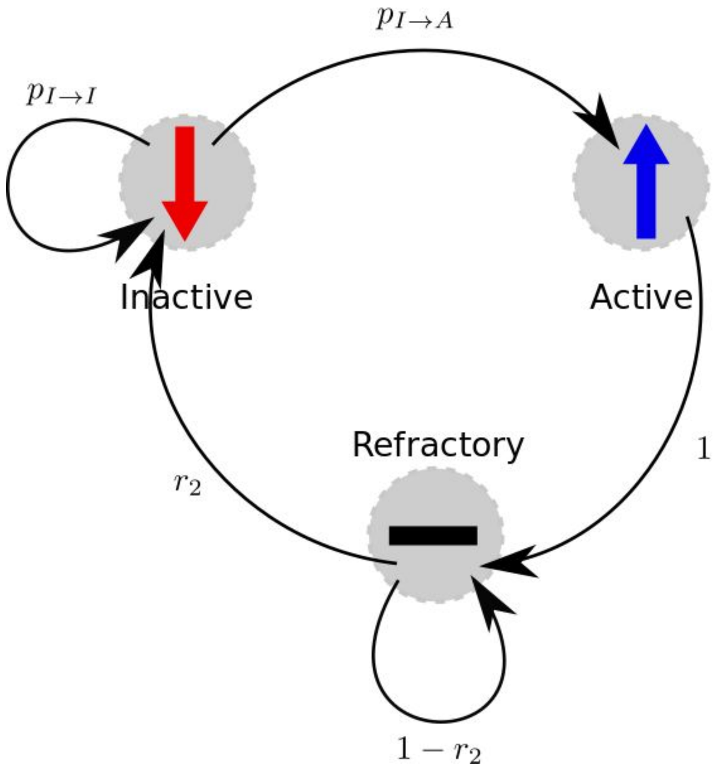

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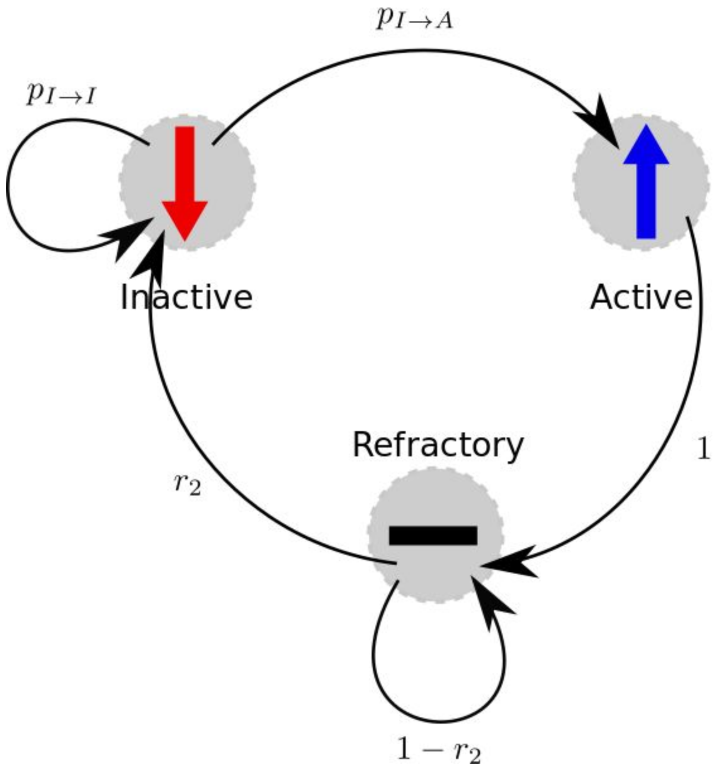
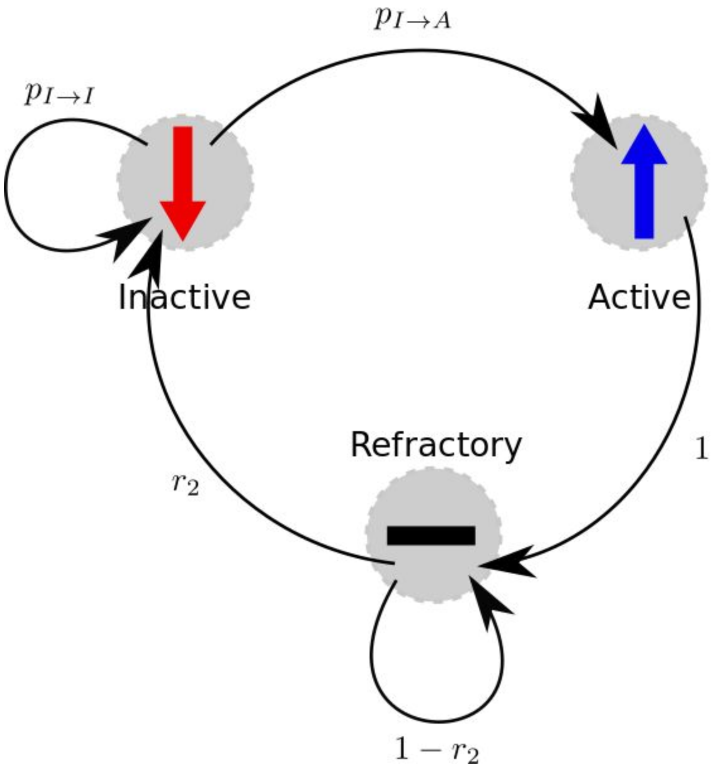


$$p_{I\to A} = 1 + (r_1 - 1)\,\theta(\Delta E)$$

$$\Delta E = \mathcal{T} - \sum_{j\ active} w_{ij}$$

Towards a brain model

- **Haimovici model** of the brain
  - defined on a graph (connectome)
  - 3-state system (active - inactive - refractory)
  - out-of-equilibrium (no energy func)



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Towards a brain model

Your tasks are:

- **2.1 Haimovici model**
  Run the Haimovici model simulation with Hagmann connectome for a set of thresholds T. Find "magnetizations" for each neuron sub-population ("active" = "excited", "refractory", "inactive" = "susceptible")

- **2.2 Temperature or threshold?**
  Investigate how the threshold parameter differ from the temperature of the Ising model?
  Tip: How do the sub-critical-super regimes behave? Inspect temporal dynamics or "magnetizations".

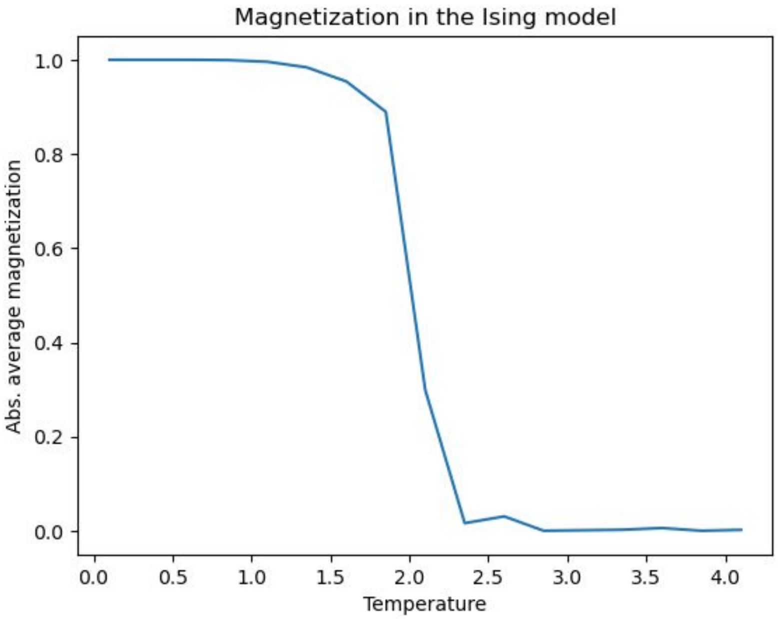
- **2.3 Artificial connectomes**
  Investigate artificial connectomes, try Watts-Strogatz or others. Look at magnetizations and different parameter regimes.

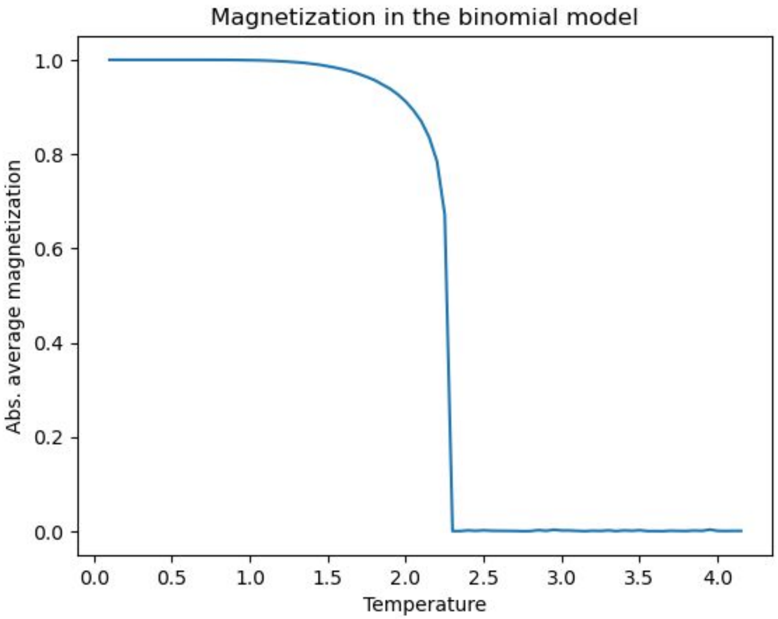
Criticality in the Haimovici model

- But what about criticality?

Criticality in the Haimovici model

- But what about criticality? Magnetization is not enough…





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- Other indicators:
  - Variance of magnetization
  - Autocorrelation
  - Sizes of the largest clusters

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Maximal sets of **connected nodes** sharing the same type of activity.
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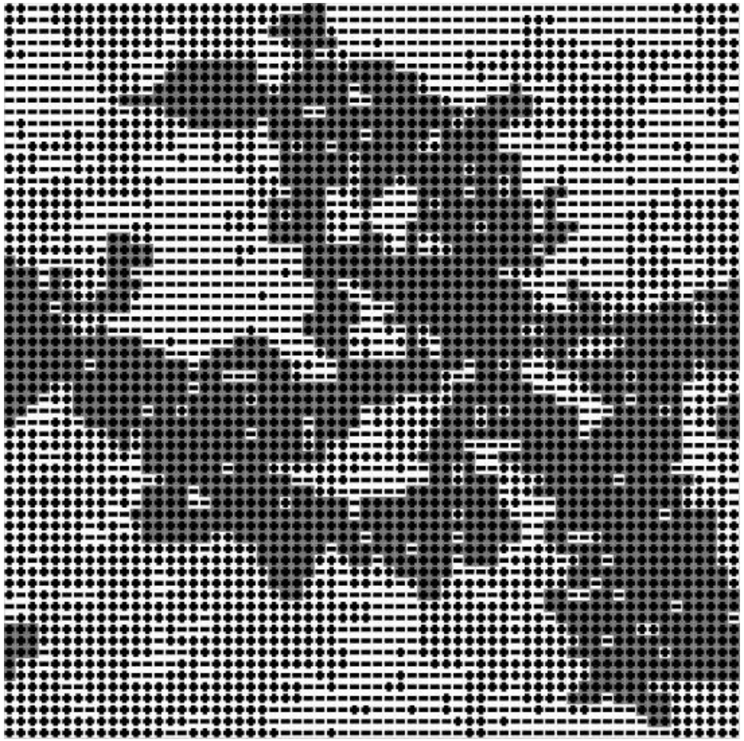

Towards a brain model

Your tasks are:

- **3.1 Clusters in the Ising model**
  Use an Ising model snapshot near Tc and plot the largest and the second-largest cluster. What is different near T = 0?

- **3.2 Criticality indicators in the Haimovici model**
  Find cluster sizes in the Haimovici model and investigate other indicators as well (st. dev. of activity and autocorrelation).

- **3.3 Detective work**
  We give you simulated data and a set of derived criticality indicators. Is the data taken from a system poised at criticality?

Playing around

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- So far: the healthy brain is posed at criticality (Hagmann)
- What about changes in the brain?
  - strokes
  - epileptic seizures
  - drugs
  - etc

Playing around

Your tasks are:

- **4.1 Lobotomy**
  Take a healthy brain (Hagmann connectome) and create an artificial lobotomy-like procedure. Show cluster sizes.

- **4.2 Stroke**
  What happens if you disconnect a single RSN from the brain? Show cluster sizes.

- **4.3 Epilepsy**
  Model epilepsy by rescaling/translating connectome weights. What happens then?

Some references

- Janarek et al. http://bit.ly/cool-paper (to appear in Nat. Sci. Rep.)
- Rocha et al., Nat. Comm. 13 (1), 3683 649 (2022)
- Haimovici et al., Phys. Rev. Lett. 110, 178101 (2013)
- Hagmann et al., PLoS Biology 6 (7) (2008)