

Neural Signal Processing Spike-Field Coherence

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<https://github.com/Mark-Kramer/MIT-Spike-Field-Coherence>

Outline

- Coherence in words / idea
- On ramp
- Coherence in equations
- Intuition
- Spike-field coherence
- Dependence on rate
- Next steps

Coherence: words

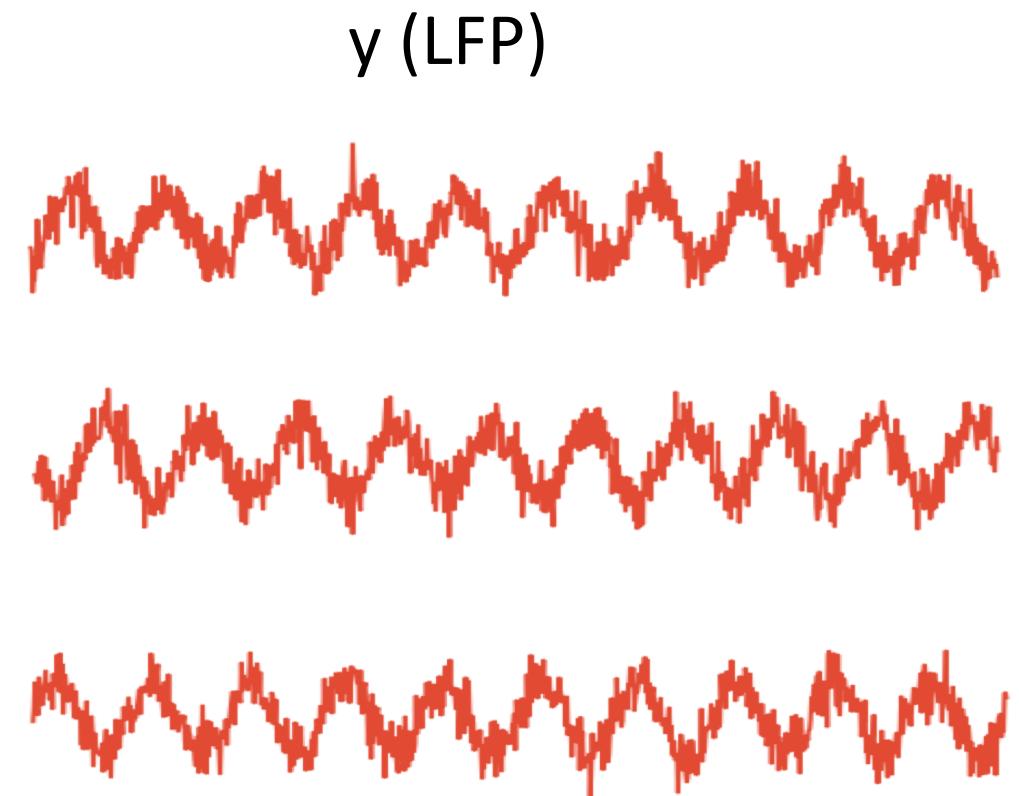
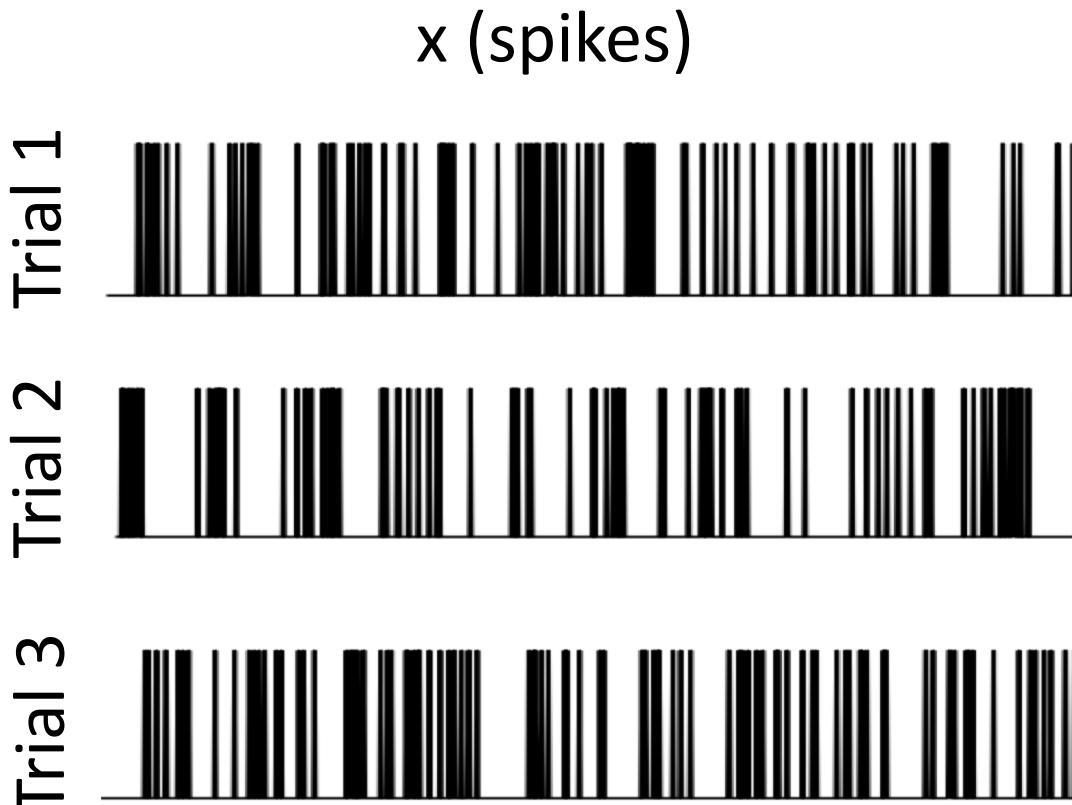
A constant phase relationship between two signals, at the same frequency, across trials.

Note

- “*same frequency*”
- “*across trials*”

Coherence: idea

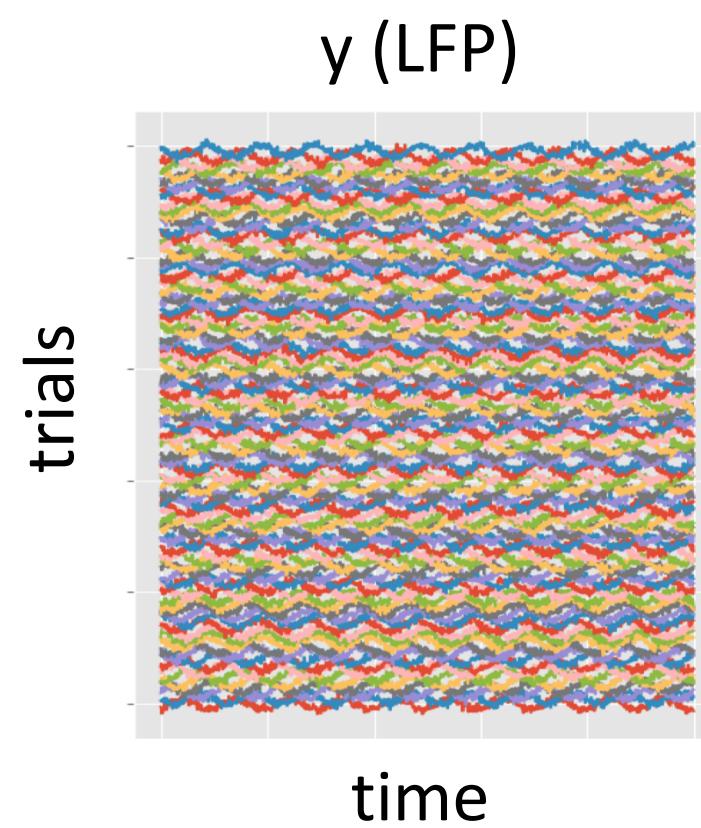
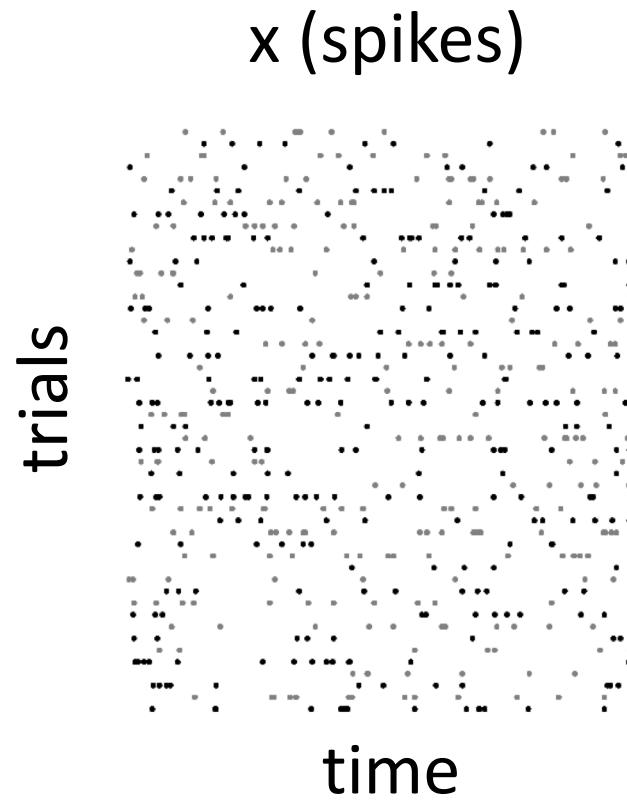
Example: Record data simultaneously from two sensors, across multiple trials



Is there a *constant phase relationship between x & y, at the same freq, across trials?*

Coherence: idea

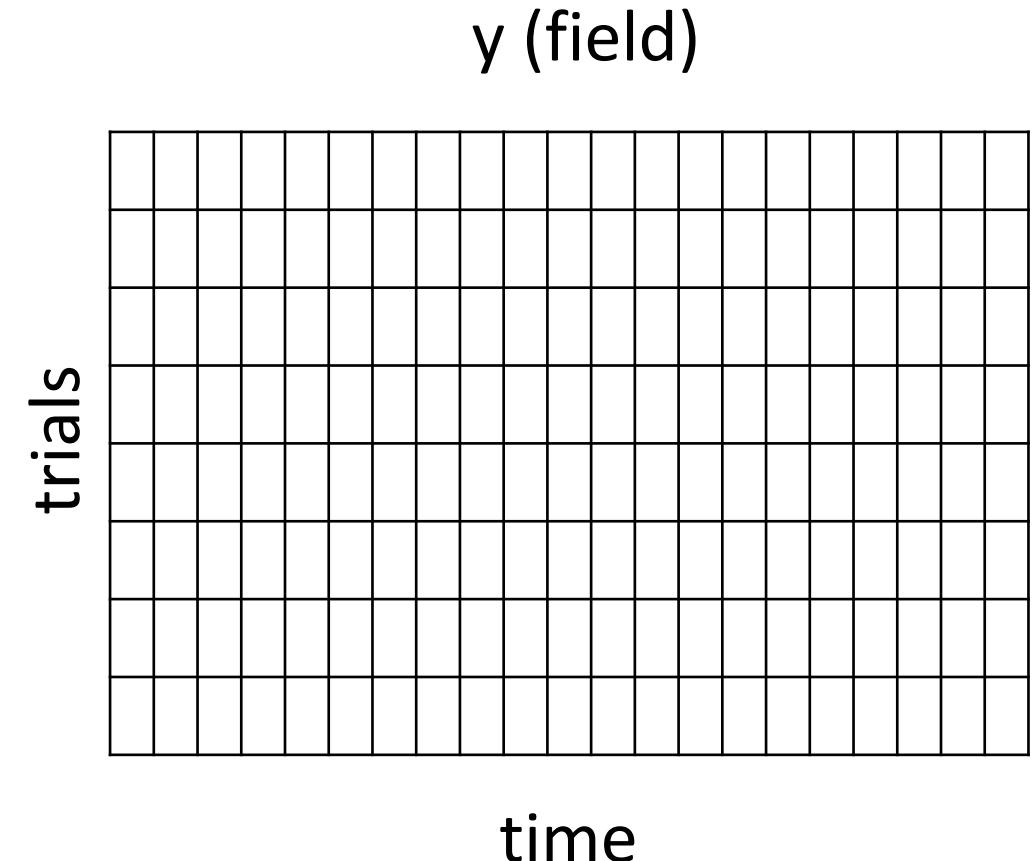
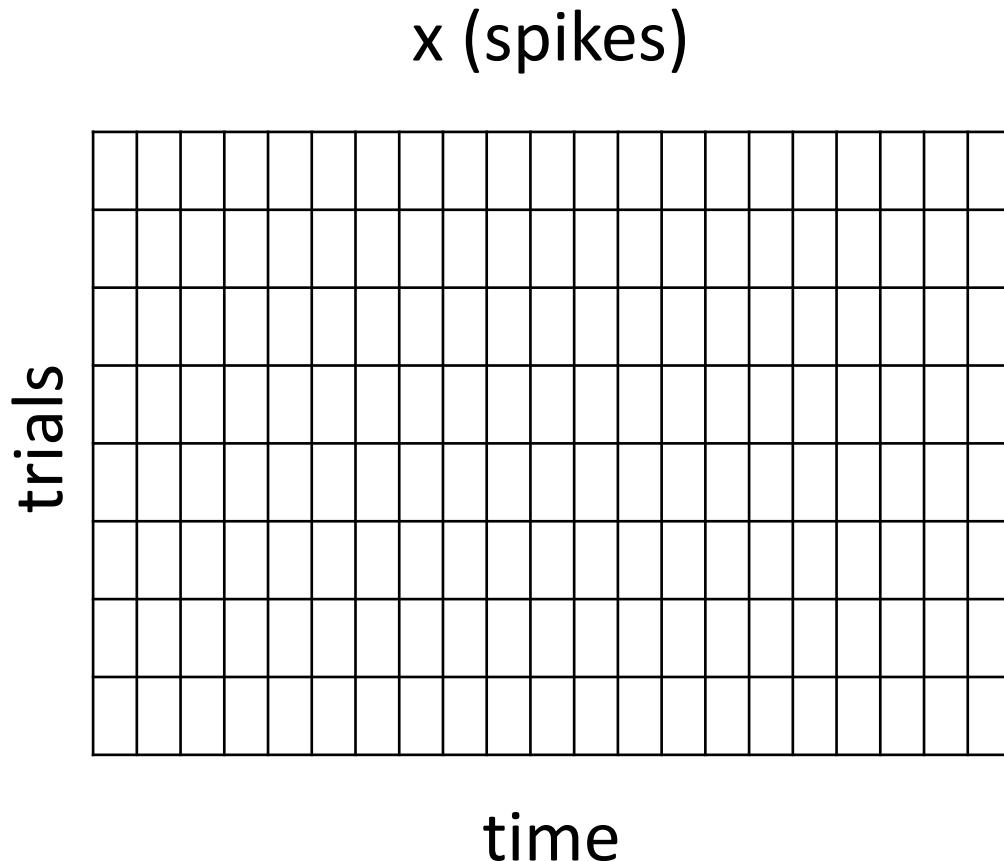
Example: Record data simultaneously from two sensors, across multiple trials



Is there a *constant phase relationship between x & y, at the same freq, across trials?*

Coherence: idea

Example: Record data simultaneously from two sensors, across multiple trials



Each row is a trial, each column is a time point, organize data in matrices.

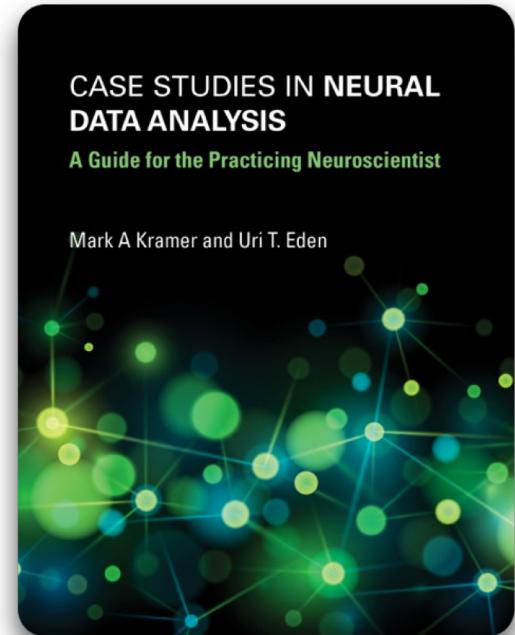
Coherence: on ramp

- Python

<https://github.com/Mark-Kramer/MIT-Spike-Field-Coherence>

- MATLAB

<https://github.com/Mark-Kramer/Case-Studies-Kramer-Eden>
[Kramer & Eden, Ch 11]



Coherence: equations

This is what we'll compute:

$$\kappa_{xy, j} = \frac{|\langle S_{xy, j} \rangle|}{\sqrt{\langle S_{xx, j} \rangle} \sqrt{\langle S_{yy, j} \rangle}}$$

$S_{xy, j}$ = Cross-spectrum at frequency index j

$S_{xx, j}, S_{yy, j}$ = Auto-spectra at frequency index j

$\langle S \rangle$ = Average of S over trials

Define each piece ...

Coherence: equations

To start, imagine x is activity recorded from a single trial:

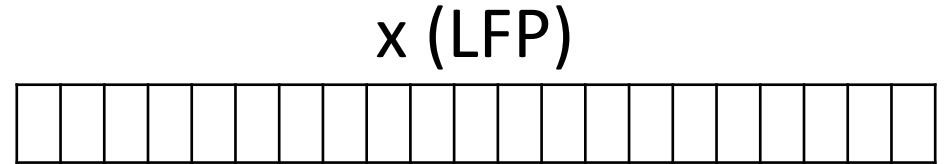
$$S_{xx,j} = \frac{2\Delta^2}{T} |X_j|^2$$

(Auto-)spectrum of signal x

Δ = sampling interval

T = total time of observation

X_j = Fourier transform of the data (x) at frequency j



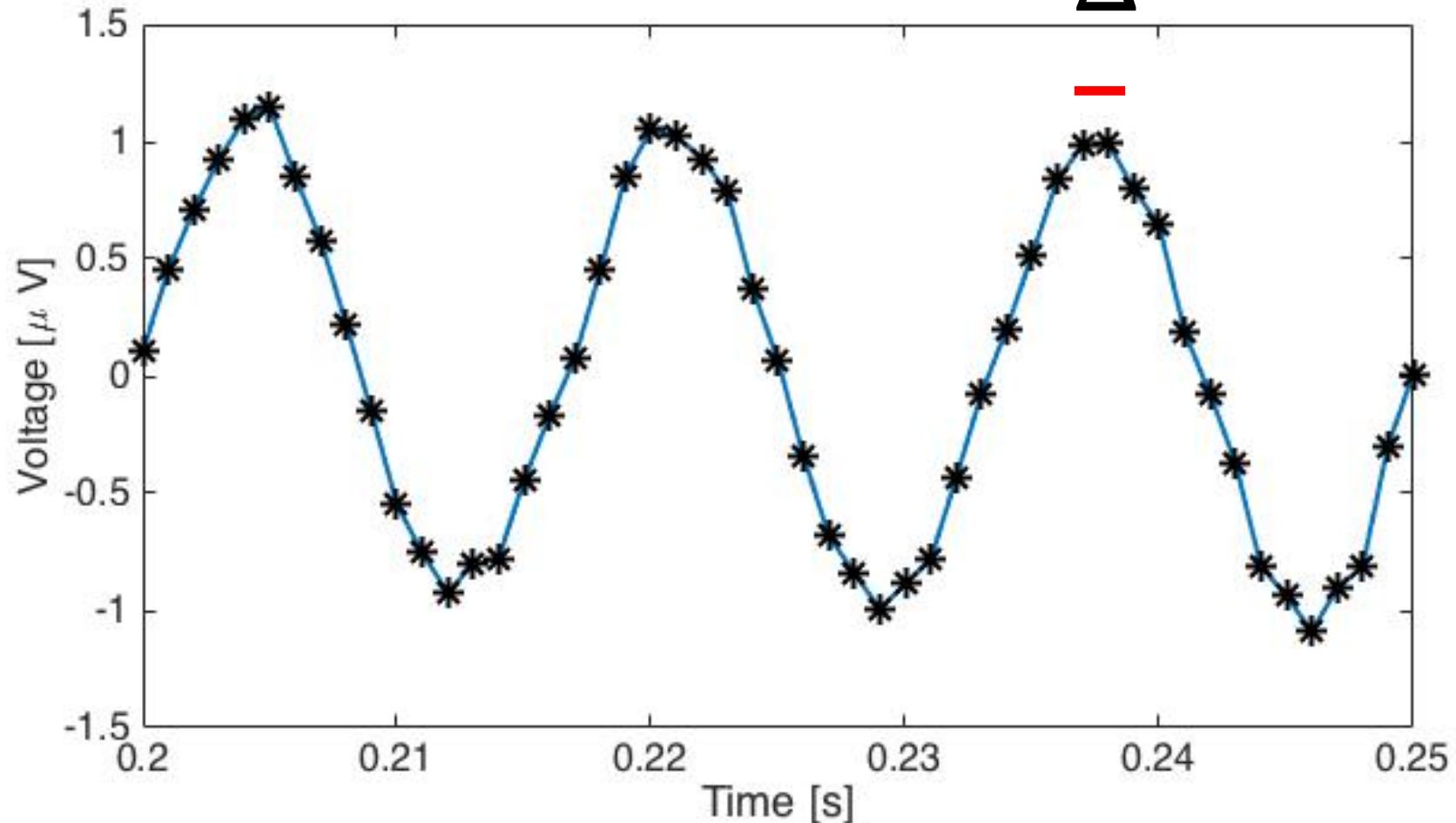
Note: Time is discrete

x_n = Data at index n

Coherence: equations (aside / quiz)

x_n = Data at index n

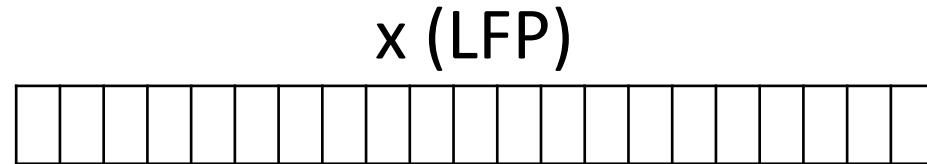
Δ time between samples



Coherence: equations

$$X_j = \sum_{n=1}^N x_n \exp(-2\pi i f_j t_n) . \quad \text{Fourier transform of the data } x.$$

x_n = Data at index n



$$i = \sqrt{-1}$$

t_n = Time at index n, $t_n = \Delta n$ where Δ = sampling interval

f_j = Frequency at index j, $f_j = j/T$ where T = total time of observation

Coherence: equations (aside / quiz)

Collect 1 s of data sampled at 500 Hz:

$$\Delta =$$

$$T =$$

Frequency resolution (df) = ... $f_j = j/T$ so, $\text{df} = 1/T$

Bonus: Nyquist frequency (F_{NQ}) = ... sampling frequency / 2

In this example,

$$\text{df} =$$

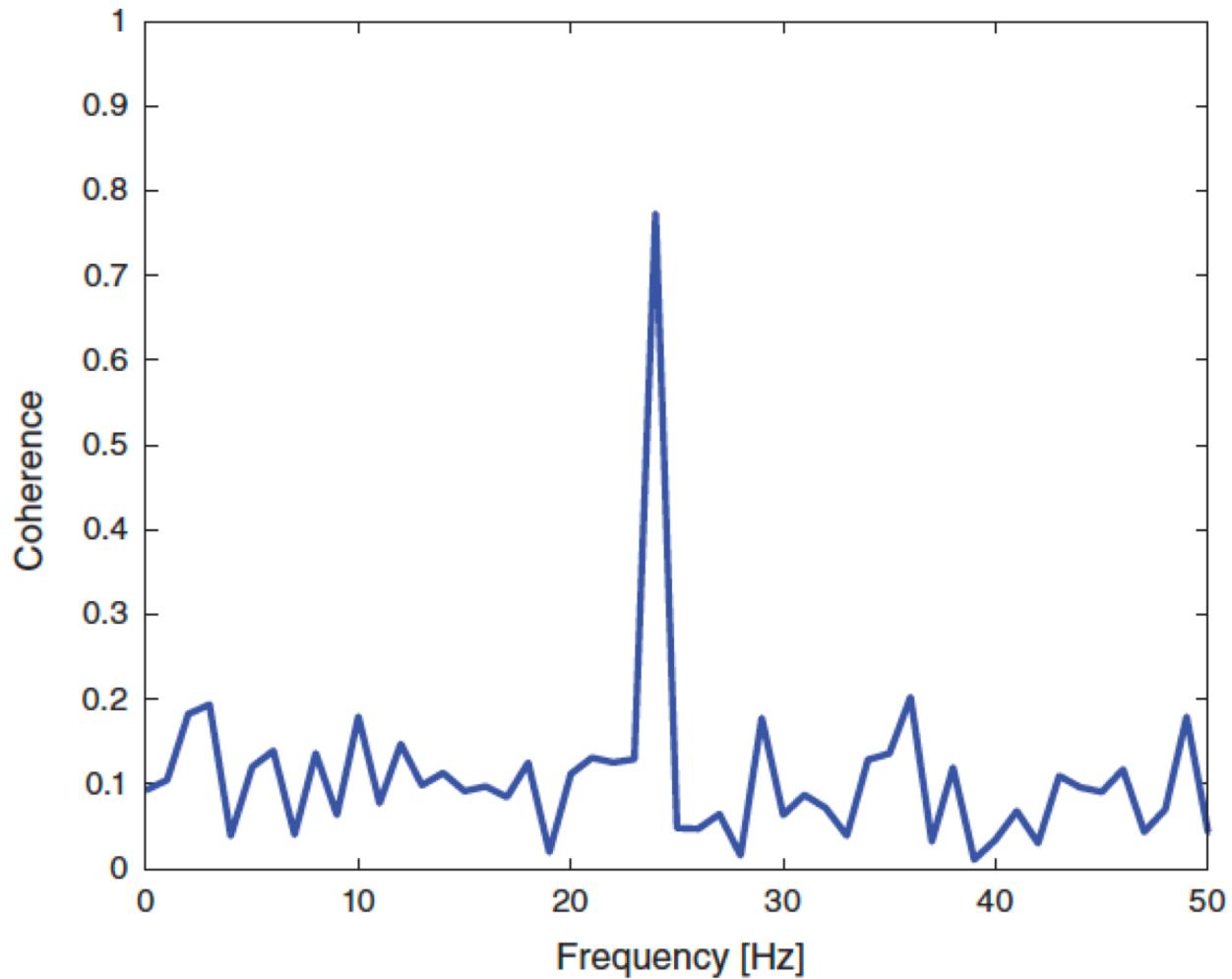
$$F_{NQ} =$$

Coherence: equations (aside / quiz)

Q: Why does this matter?

df = space between points on x-axis.

F_{NQ} = largest possible value on x-axis.



Coherence: equations

Fourier transform intuition:

Data as a function of frequency index j

$$X_j = \sum_{n=1}^N x_n \exp(-2\pi i f_j t_n).$$

Data as a function of time index n
Sinusoids at frequency f_j

Euler's formula:

$$\exp(-2\pi i f_j t_n) = \cos(-2\pi f_j t_n) + i \sin(-2\pi f_j t_n).$$

So, at each time (index n) multiply data x_n by sinusoids at frequency f_j
Then sum up over all time.

Coherence: equations

Fourier transform intuition:

Data as a function of
frequency index j

$$X_j = \sum_{n=1}^N x_n \exp(-2\pi i f_j t_n).$$

Data as a function of time index n
Sinusoids at frequency f_j

Idea: compare our data x_n to sinusoids at frequency f_j and see how well they “match”.

Good match: $X_j = \text{big}$

Bad match: $X_j = \text{small}$

X_j reveals the frequencies f_j that match our data.

Coherence: equations (reminder)

$$\kappa_{xy, j} = \frac{|\langle S_{xy, j} \rangle|}{\sqrt{\langle S_{xx, j} \rangle} \sqrt{\langle S_{yy, j} \rangle}}$$

coherence between x and y at frequency j

$$S_{xx, j} = \frac{2\Delta^2}{T} |X_j X_j^*|$$

(Auto-)spectrum of signal x

$$X_j = \sum_{n=1}^N x_n \exp(-2\pi i f_j t_n).$$

Fourier transform of the data x .

Coherence: equations

$$X_j = \sum_{n=1}^N x_n \exp(-2\pi i f_j t_n).$$

Fourier transform of the data x .

X_j can be complex

- the Fourier transform of x_n can have both real and imaginary parts.

So, X_j lives in the complex-plane ...

Coherence: equations

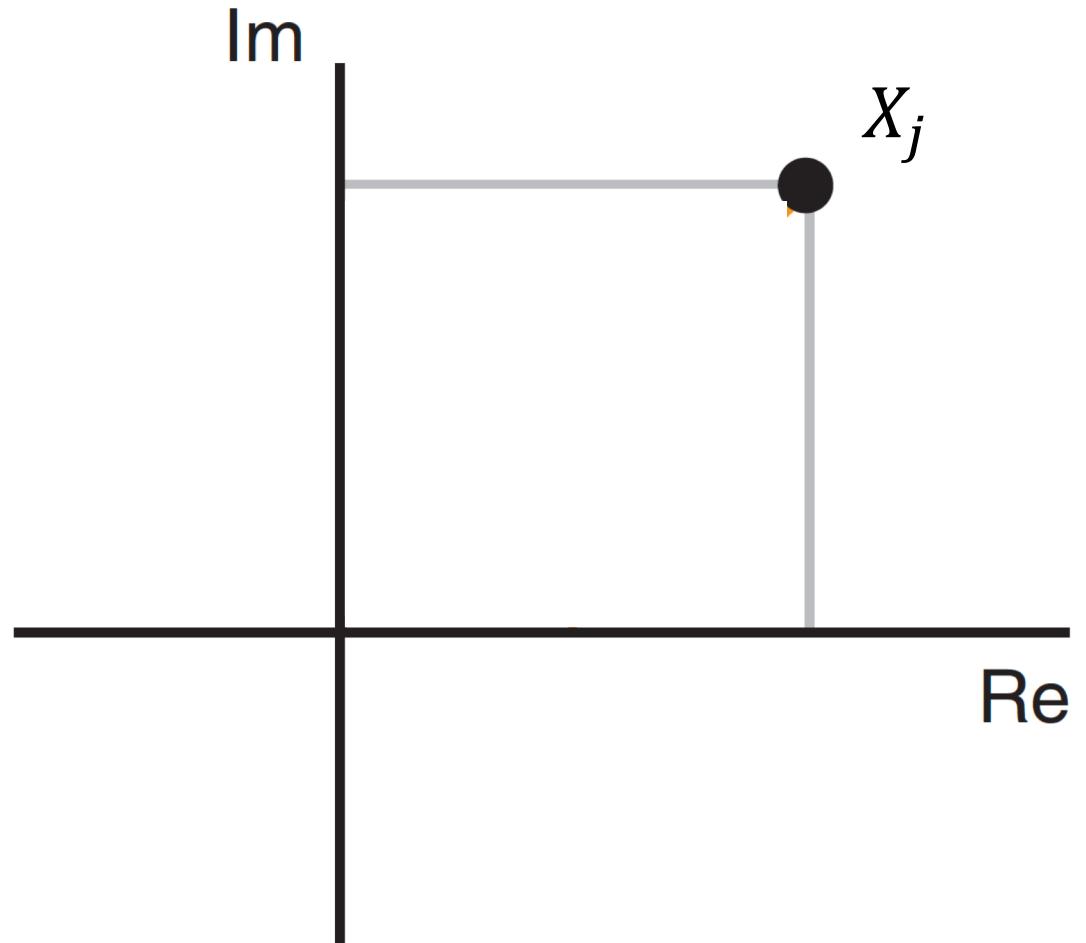
X_j lives in the complex-plane:

Express X_j in polar coordinates:

$$X_j = A_j \exp(i\phi_j)$$

A_j = Amplitude at frequency index j

ϕ_j = Phase at frequency index j



Coherence: equations

Consider the spectrum of x_n :

$$S_{xx,j} = \frac{2\Delta^2}{T} X_j X_j^* = \frac{2\Delta^2}{T} (A_j \exp(i\phi_j)) (A_j \exp(-i\phi_j))$$

Express X_j in polar coordinates:

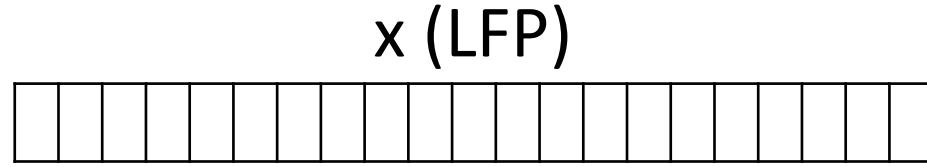
$$= \frac{2\Delta^2}{T} A_j^2 \exp(i\phi_j - i\phi_j) = \boxed{\frac{2\Delta^2}{T} A_j^2}.$$

More direct, interpretation of the spectrum at frequency f_j :
proportional to the squared amplitude of the point X_j in the complex plane.

Coherence: equations

To compute coherence, we need the trial-averaged spectrum:

To keep notation simple, we started with one trial:

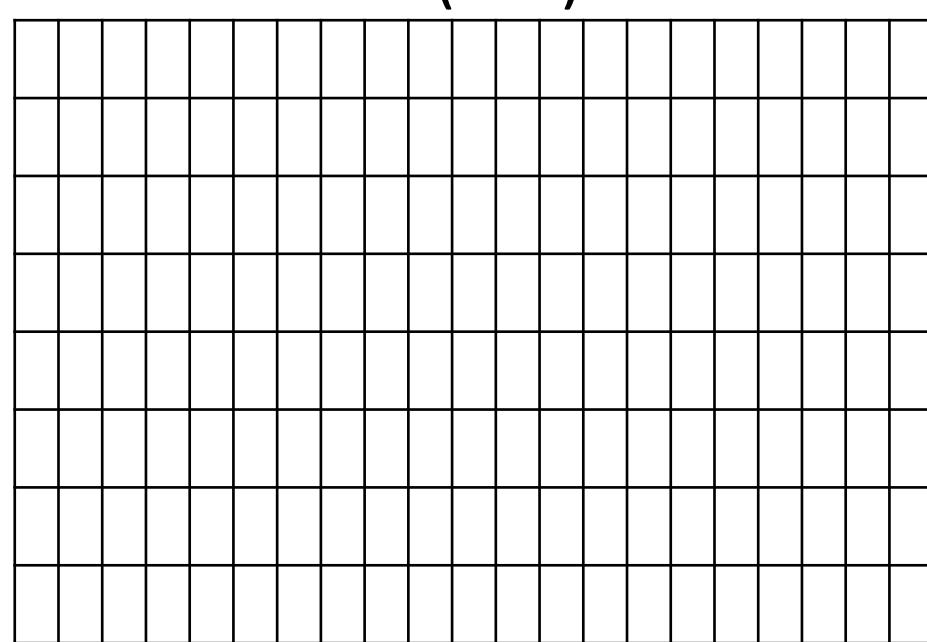


Now, consider all trials:

$S_{xx,j}$ to $\langle S_{xx,j} \rangle$
single trial spectrum trial-averaged spectrum

$k \rightarrow$

trials



time

Coherence: equations

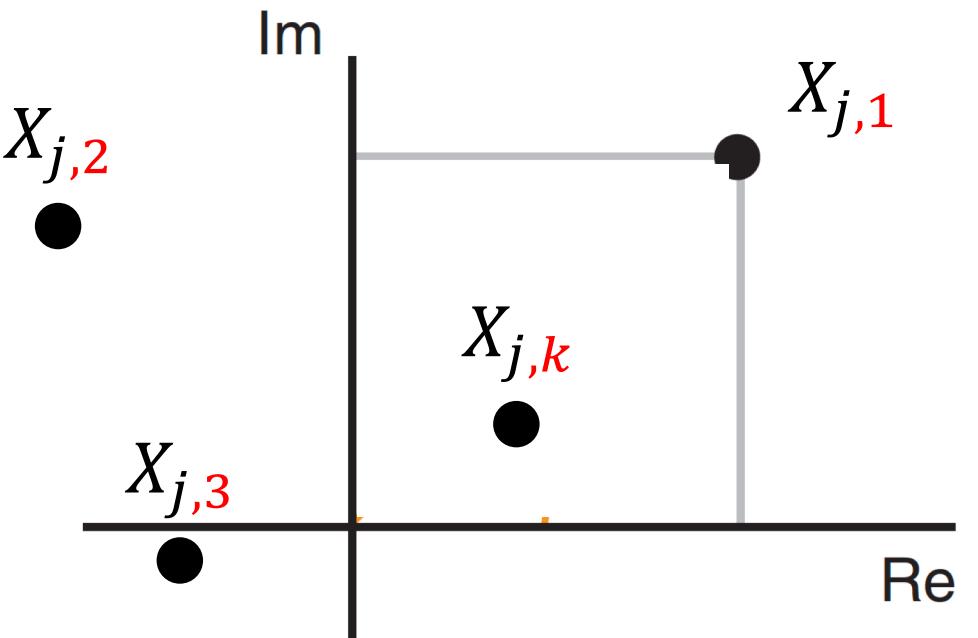
To compute coherence, we need the trial-averaged spectrum:

Fourier transform of x at frequency j : a point for each trial in the complex-plane:

In polar coordinates:

$A_{j,k}$ = Amplitude at frequency index j
and trial index k

$\phi_{j,k}$ = Phase at frequency index j
and trial index k



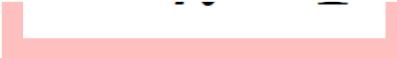
Coherence: equations

To compute coherence, we need the trial-averaged spectrum:

Single trial spectrum 

Average squared amplitude
over trials

$$\langle S_{xx,j} \rangle = \frac{2\Delta^2}{T} A_j^2.$$



$A_{j,k}$ = the amplitude of the signal x , at frequency index j , and trial index k .

K = total number of trials

Coherence: equations

Similarly, for signal y_n . Fourier transform of y at frequency j, and trial k:

$$Y_{j,k} = B_{j,k} \exp(i \theta_{j,k})$$

$B_{j,k}$ = the amplitude of the signal y at frequency index j and trial index k .

$\theta_{j,k}$ = the phase of the signal y at frequency index j and trial index k .

The trial-averaged
spectrum of y at
frequency index j

$$\langle S_{yy,j} \rangle = \frac{2\Delta^2}{T} \frac{1}{K} \sum_{k=1}^K B_{j,k}^2$$

Coherence: equations (reminder)

$$\kappa_{xy, j} = \frac{|\langle S_{xy, j} \rangle|}{\sqrt{\langle S_{xx, j} \rangle} \sqrt{\langle S_{yy, j} \rangle}}$$
$$\langle S_{xx, j} \rangle = \frac{2\Delta^2}{T} \frac{1}{K} \sum_{k=1}^K A_{j,k}^2$$
$$\langle S_{yy, j} \rangle = \frac{2\Delta^2}{T} \frac{1}{K} \sum_{k=1}^K B_{j,k}^2$$

Consider the trial averaged cross-spectrum ...

Coherence: equations

The trial averaged cross-spectrum at frequency index j:

$$\langle S_{xy,j} \rangle = \frac{2\Delta^2}{T} \frac{1}{K} \sum_{k=1}^K X_{j,k} Y_{j,k}^*$$

Like the auto-spectrum,
but use X and Y.

In polar coordinates:

$$\langle S_{xy,j} \rangle = \frac{2\Delta^2}{T} \frac{1}{K} \sum_{k=1}^K A_{j,k} B_{j,k} \exp(i\Phi_{j,k})$$

Phase of x Phase of y

where $\Phi_{j,k} = \phi_{j,k} - \theta_{j,k}$ is the phase difference between the two signals,
at frequency index j and trial k.

Coherence: equations

Put it all together ...

$$\kappa_{xy, j} = \frac{|\langle S_{xy, j} \rangle|}{\sqrt{\langle S_{xx, j} \rangle} \sqrt{\langle S_{yy, j} \rangle}}$$

In polar coordinates ...

cross-spectrum of x & y,
depends on trial averaged
amplitudes, phase difference.

x trial averaged amplitude,
at frequency index j

$$= \frac{\left| \sum_{k=1}^K A_{j,k} B_{j,k} \exp(i\Phi_{j,k}) \right|}{\sqrt{\sum_{k=1}^K A_{j,k}^2} \sqrt{\sum_{m=1}^K B_{j,m}^2}}$$

y trial averaged amplitude,
at frequency index j

Coherence: intuition

To build intuition, assume: the amplitude is identical for both signals and all trials.

$$A_{j,k} = B_{j,k} = C_j$$

Note: no trial dependence

then

$$\mathcal{K}_{xy,j} = \left| \frac{\sum_{k=1}^K A_{j,k} B_{j,k} \exp(i\Phi_{j,k})}{\sqrt{\sum_{k=1}^K A_{j,k}^2} \sqrt{\sum_{m=1}^K B_{j,m}^2}} \right|$$

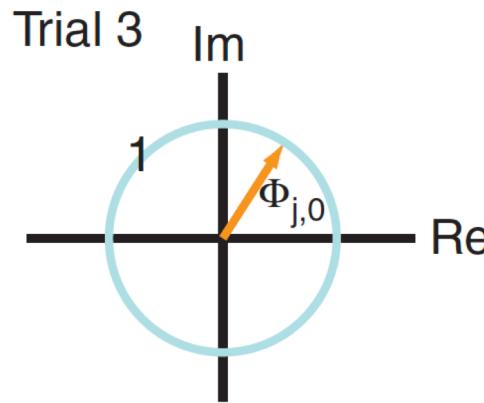
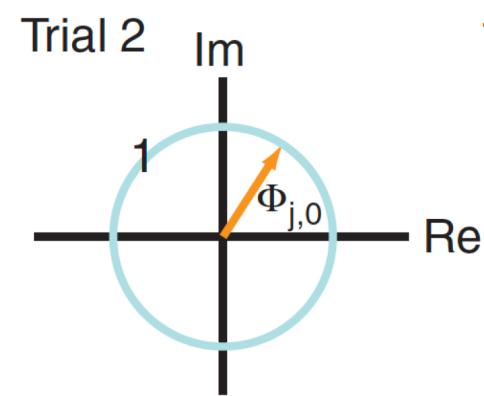
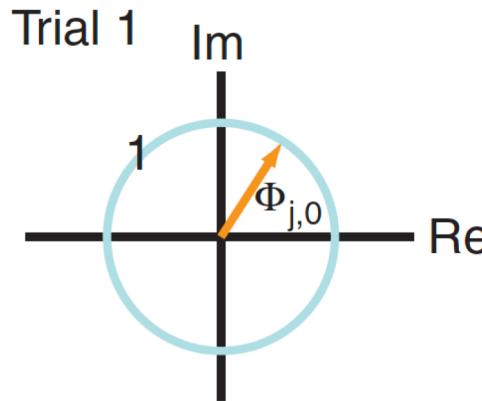
only involves the phase difference between the two signals averaged across trials.

Coherence: intuition

Case 1: Phases align across trials.

$$\Phi_{j,k} = \Phi_{j,0}$$

Plot $\exp(i\Phi_{j,k})$ in the complex plane.



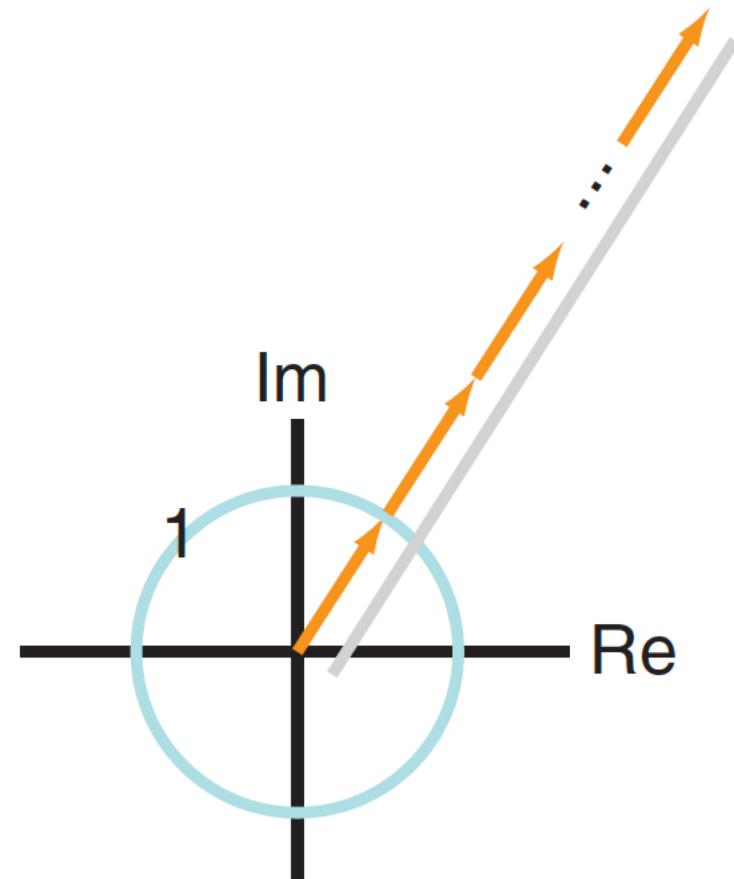
sum these vectors end to end across trials

divide by K

$$\kappa_{xy,j} \approx 1$$

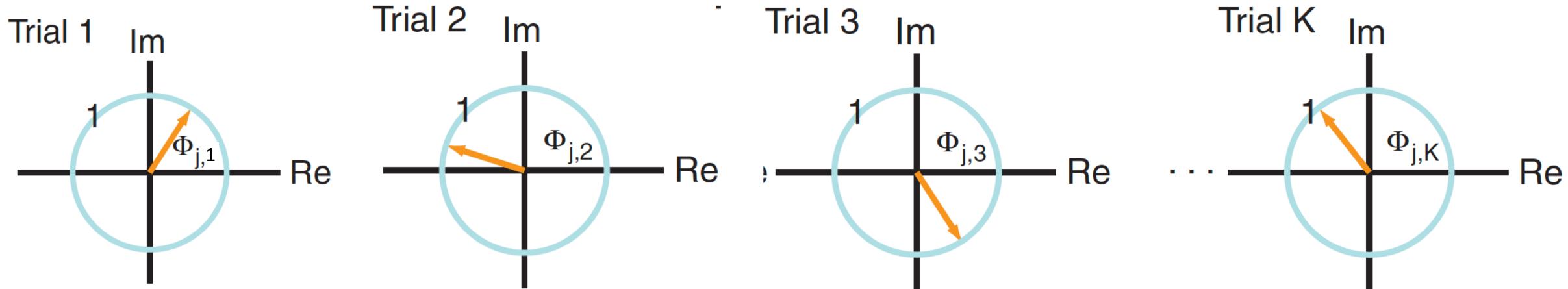
strong coherence - constant phase relation between
the two signals across trials at frequency index j.

$$\kappa_{xy,j} = \frac{1}{K} \left| \sum_{k=1}^K \exp(i\Phi_{j,k}) \right|$$



Coherence: intuition

Case 2: Random phase differences across trials.
Plot $\exp(i\Phi_{j,k})$ in the complex plane.

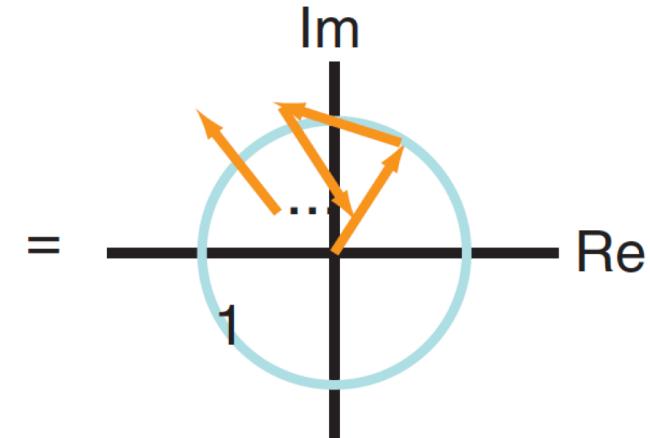


sum these vectors end to end across trials

divide by K

$$\kappa_{xy,j} \approx 0$$

weak coherence - random phase relation between
the two signals across trials at frequency index j.



Coherence: summary

$$0 \leq \kappa_{xy,j} \leq 1$$

0: no coherence between signals x and y at frequency index j

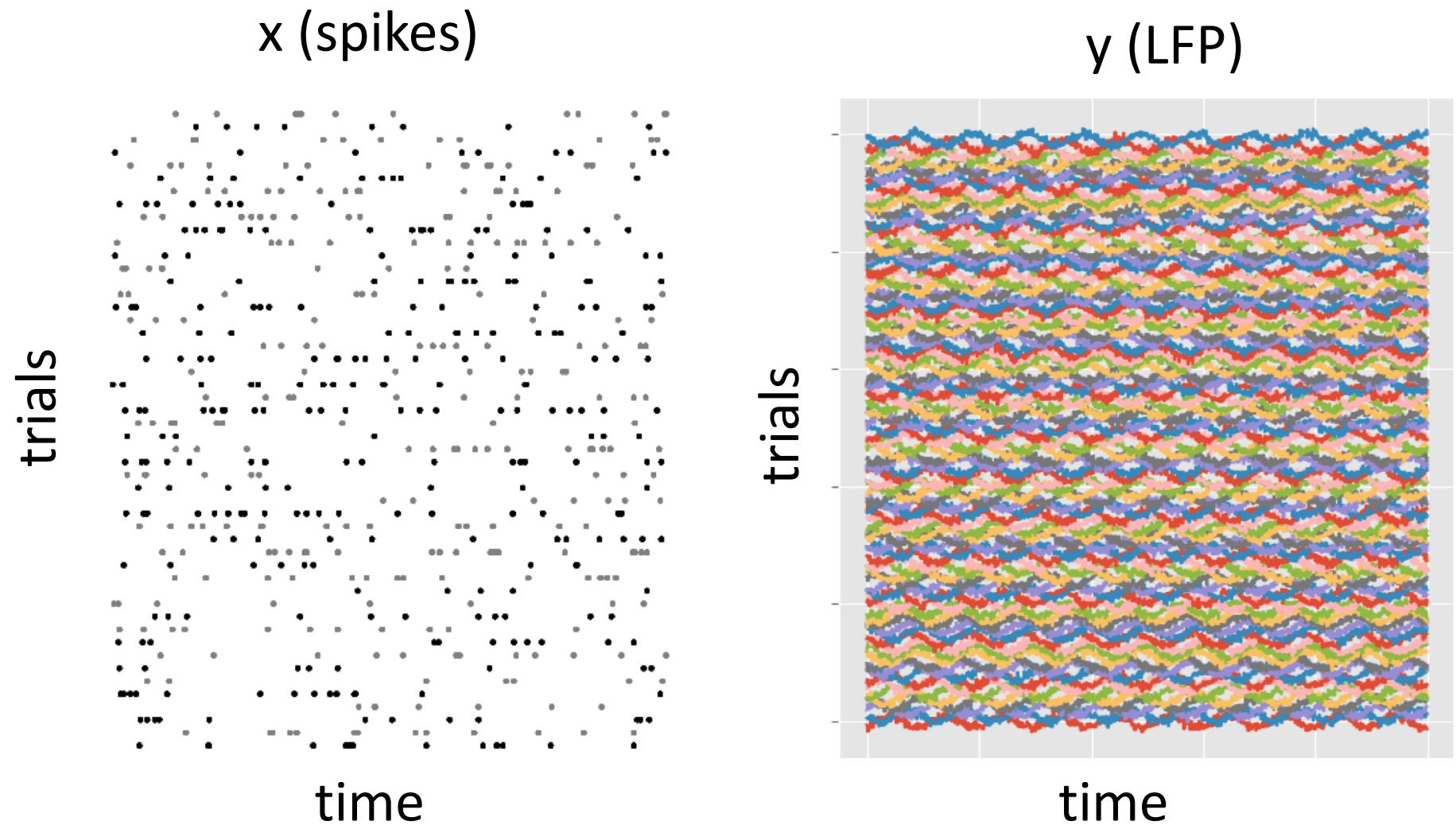
1: strong coherence between signals x and y at frequency index j .

The coherence is a measure of the phase consistency between two signals at frequency index j across trials.

Quiz: What are the units of coherence?

Spike-field coherence

Consider the data:



We want a measure of consistent neural spiking at a specific phase of the field ...

Spike-field coherence

$$\kappa_{ny, j} = \frac{|\langle S_{ny, j} \rangle|}{\sqrt{\langle S_{nn, j} \rangle} \sqrt{\langle S_{yy, j} \rangle}}$$

trial averaged cross spectrum

I averaged spike spectrum
trial averaged field spectrum

y = field signal (e.g., EEG, MEG, LFP, ...)

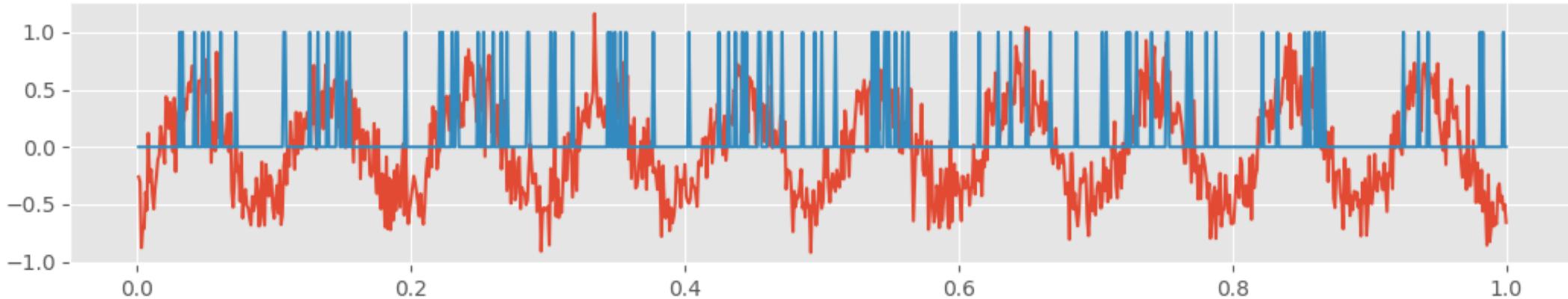
n = spike train (e.g., [0 0 0 0 0 1 0 0 0 0 0 0 0 ...])

Same equations ... but new problems ...

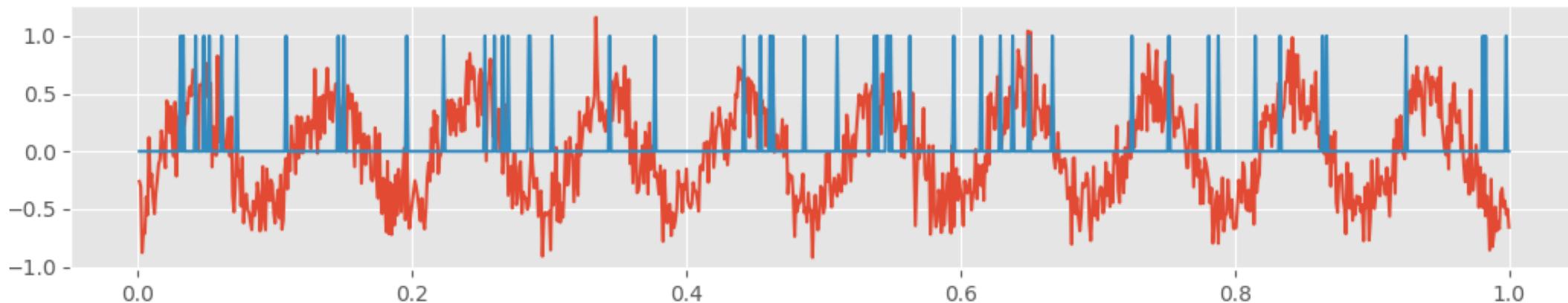
Spike-field coherence: dependence on rate

Q: Does the spike-field coherence depend on the firing rate of the neuron?

Original spike & field



“Thinned” spike & field (remove 50% of spikes, chosen at random)



Spike-field coherence: dependence on rate

Q: Does the spike-field coherence depend on the firing rate of the neuron?

Here, rate: expected number of spikes in a given duration

Try it ...

<https://github.com/Mark-Kramer/MIT-Spike-Field-Coherence>

Spike-field coherence: dependence on rate

Q: Does the spike-field coherence depend on the firing rate of the neuron?

Observations:

greater thinning → fewer spikes → lower coherence

as the rate tends to 0, so does the spike-field coherence

The spike-field coherence reflects

- (1) the relationship between spiking activity and the phase of field, and
- (2) the mean firing rate.

Spike-field coherence: dependence on rate

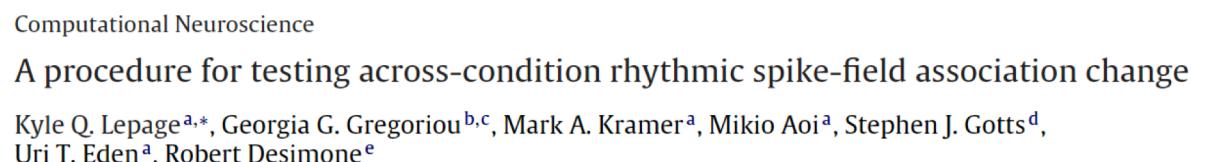
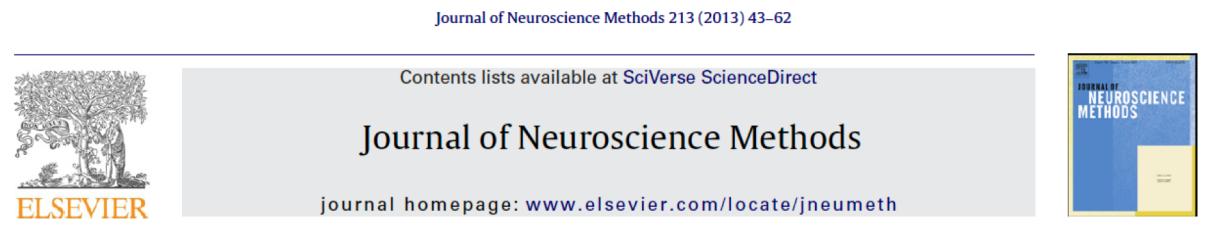
Q: So what next?

Q: If, in your experiment, the overall spike rate differs between two animals, then how do you compare the spike-field coherence?

- include a rate adjustment factor in the coherence measure to account for rate dependence.



- build a generalized linear model to separate overall neural activity from spike train-LFP oscillatory coupling.



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