Neural Signal Processing
Spike-Field Coherence

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https://github.com/Mark-Kramer/MIT-Spike-Field-Coherence

Outline

- Coherence in words / idea
- On ramp
- Coherence in equations
- Intuition
- Spike-field coherence
- Dependence on rate
- Next steps

Coherence: words

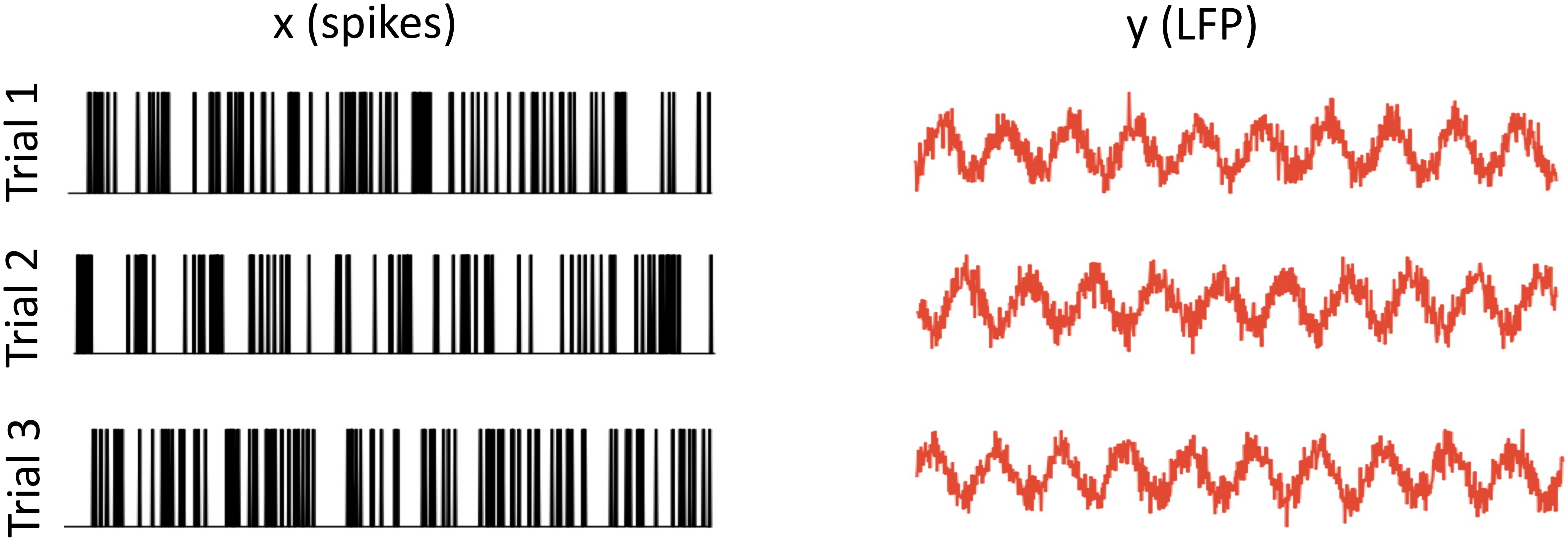
A constant phase relationship between two signals, at the same frequency, across trials.

Note

- “*same frequency*”
- “*across trials*”

Coherence: idea

<u>Example</u>: Record data simultaneously from two sensors, across multiple trials

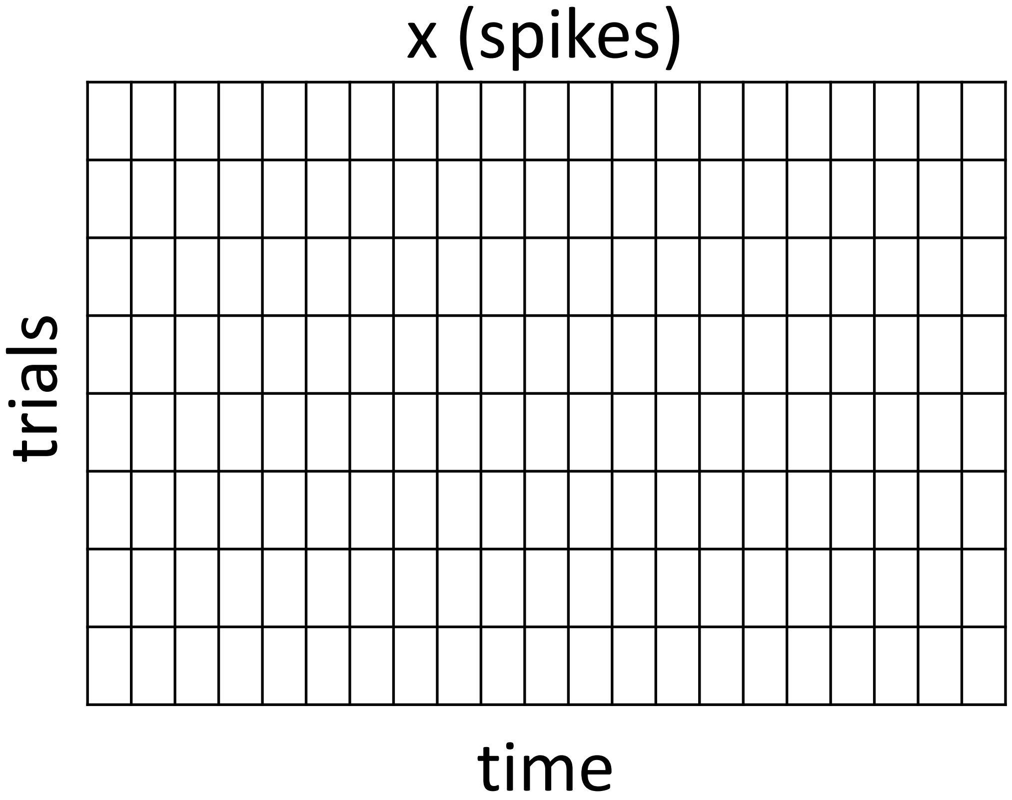


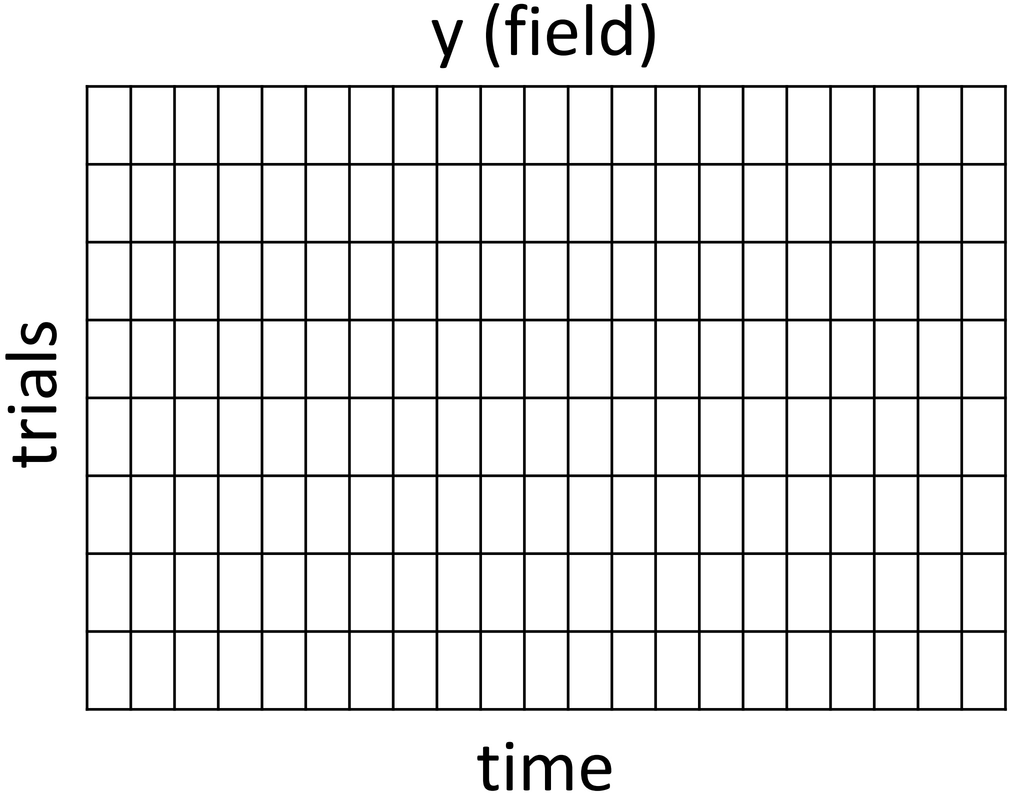
Is there a *constant phase relationship between x & y, at the same freq, across trials?*

Coherence: idea

Example: Record data simultaneously from two sensors, across multiple trials

Organize the data ...



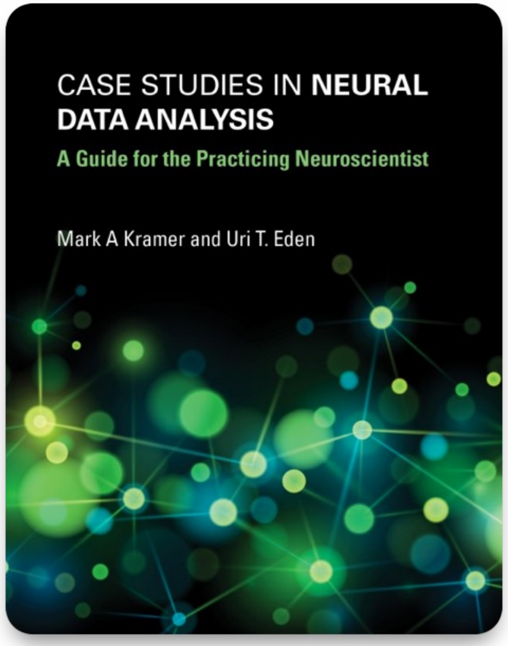


Each row is a trial, each column is a time point, organize data in matrices.

Coherence: on ramp

- Python
  https://github.com/Mark-Kramer/MIT-Spike-Field-Coherence

- MATLAB
  https://github.com/Mark-Kramer/Case-Studies-Kramer-Eden
  [Kramer & Eden, Ch 11]



Coherence: equations

This is what we'll compute:

$$\kappa_{xy,\,j} = \frac{|< S_{xy,\,j} >|}{\sqrt{<\boxed{S_{xx,\,j}}>}\sqrt{< S_{yy,\,j} >}}$$

$S_{xy,j}$ = Cross-spectrum at frequency index j

$S_{xx,j}, S_{yy,j}$ = Auto-spectra at frequency index j

$< S >$ = Average of S over trials

Define each piece ...

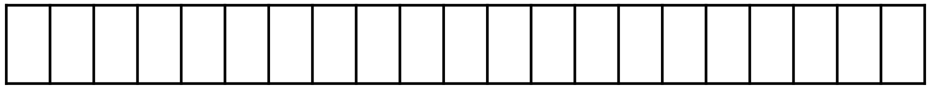
Coherence: equations

To start, imagine *x* is activity recorded from a single trial:

$$S_{xx,j} = \frac{2\Delta^2}{T}\boxed{X_j}X_j^*$$

(Auto-)spectrum of signal *x*

x (LFP)



Note: Time is discrete

x_n = Data at index n

Δ = sampling interval

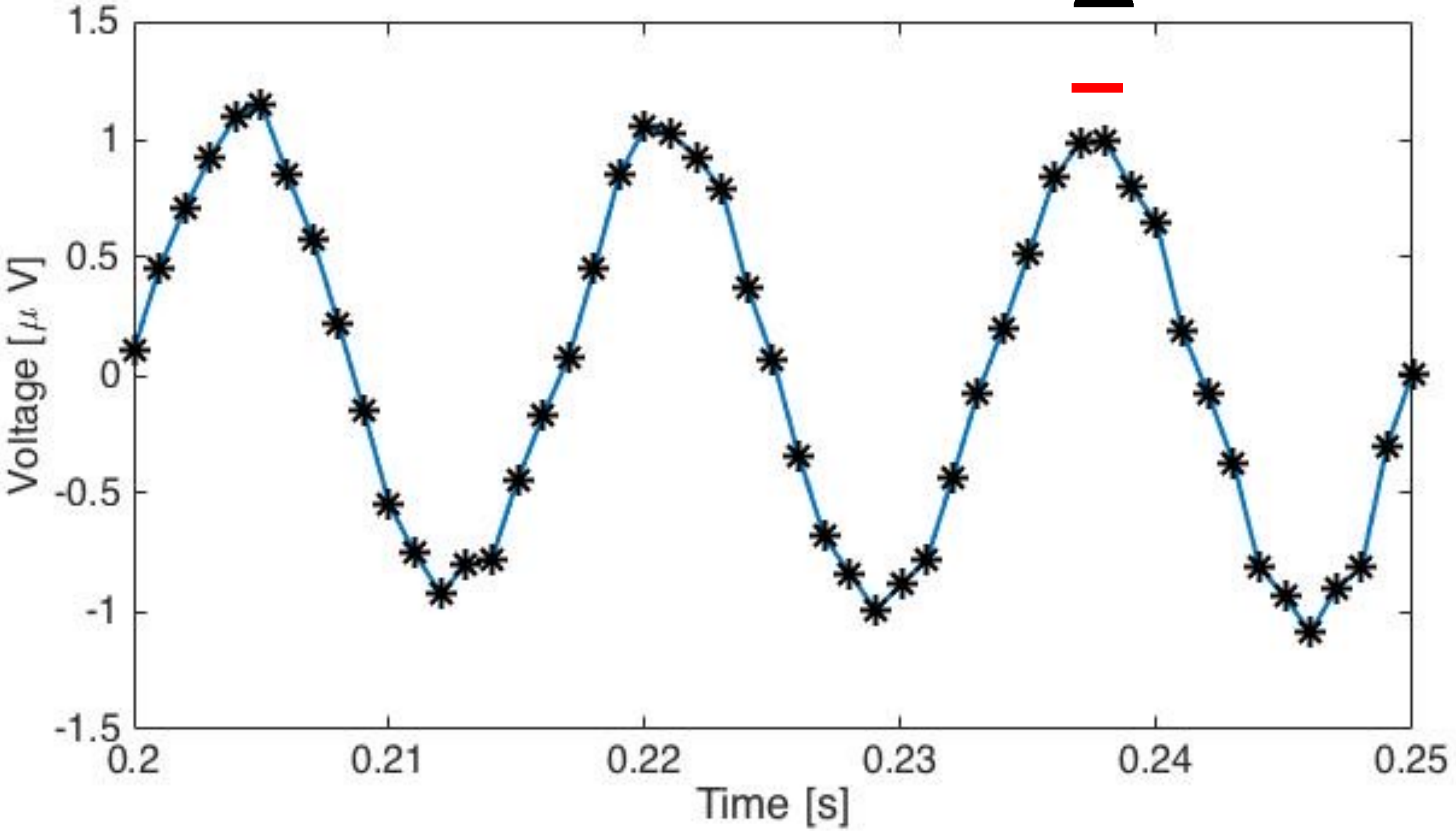
T = total time of observation

X_j = Fourier transform of the data (*x*) at frequency *j*

Coherence: equations (aside)

x_n = Data at index n

Δ time between samples

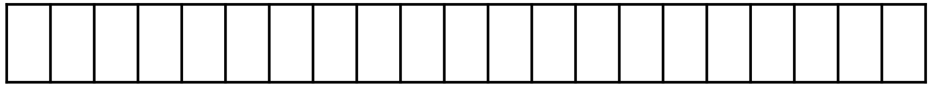


Coherence: equations

$$X_j = \sum_{n=1}^{N} x_n \exp(-2\pi i f_j t_n)\,.$$

Fourier transform of the data *x.*

x (LFP)



i = $\sqrt{-1}$

x_n = Data at index n

t_n = Time at index n, $t_n = \Delta n$ where Δ = sampling interval

f_j = Frequency at index j, $f_j = j/T$ where T = total time of observation

Coherence: equations (aside / quiz)

Collect 1 s of data sampled at 500 Hz:

Δ =

T =

Frequency resolution (df) = ... $f_j = j/T$ so, df = 1 / T

<u>Bonus</u>: Nyquist frequency (F_{NQ}) = ... sampling frequency / 2

In this example, df = F_{NQ} =

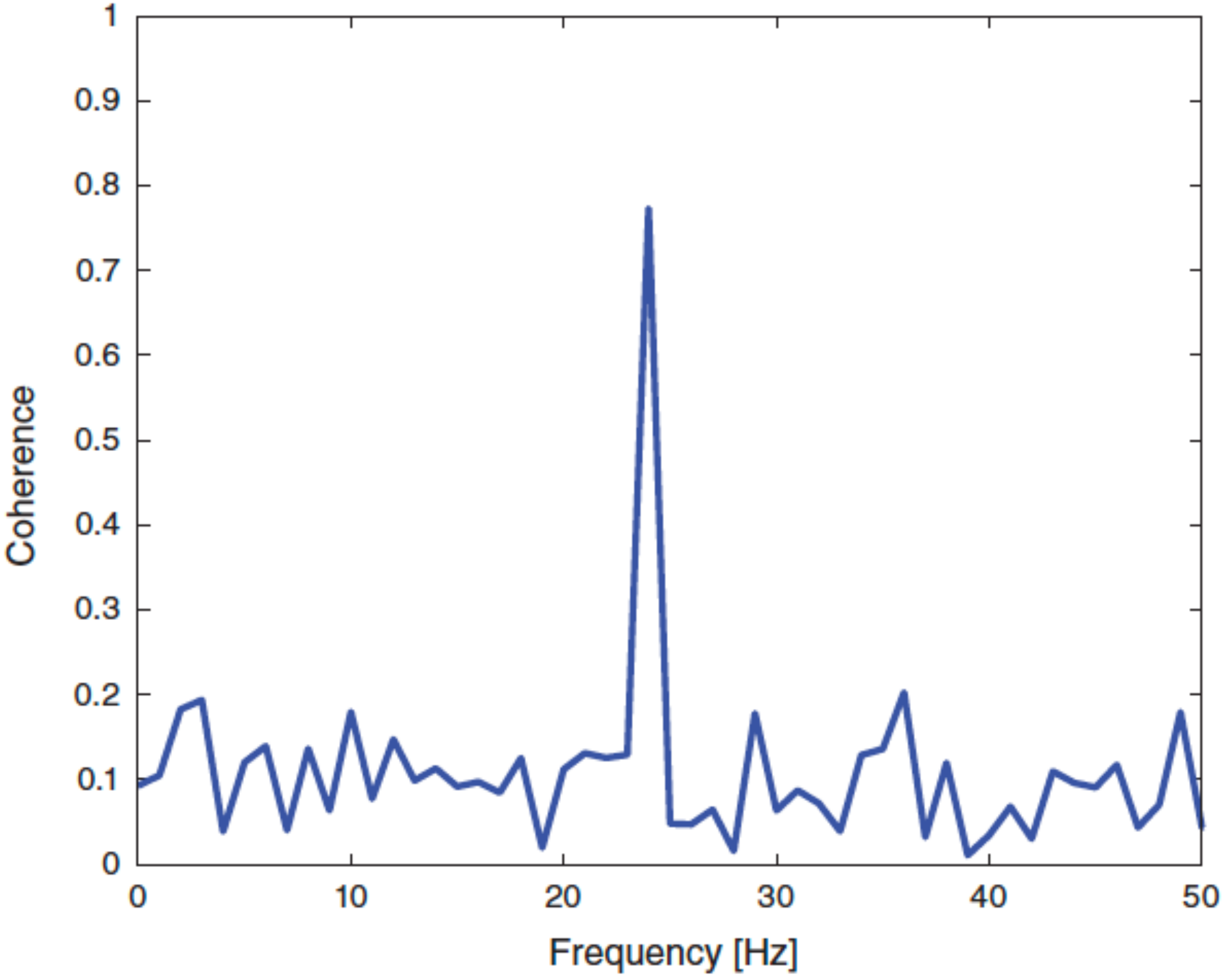
I have no idea what's going on ... see [Kramer & Eden, Case Studies, Ch 3 and 4]
https://mark-kramer.github.io/Case-Studies-Python/03.html and 04.html

Coherence: equations (aside / quiz)

Q: Why does this matter?

df = space between points on x-axis.

F_{NQ} = largest possible value on x-axis.



Coherence: equations

Fourier transform intuition:

Data as a function of time index n

Data as a function of frequency index j

$$X_j = \sum_{n=1}^{N} \boxed{x_n} \boxed{\exp(-2\pi i f_j t_n)}.$$

Sinusoids at frequency f_j

Euler's formula:

$$\exp(-2\pi i f_j t_n) = \cos(-2\pi f_j t_n) + i \sin(-2\pi f_j t_n).$$

So, at each time (index n) multiply data x_n by sinusoids at frequency f_j

Then sum up over all time.

Coherence: equations

Fourier transform intuition:

Data as a function of time index n

Data as a function of frequency index j

$$X_j = \sum_{n=1}^{N} \boxed{x_n} \boxed{\exp(-2\pi i f_j t_n)}.$$

Sinusoids at frequency f_j

Idea: compare our data x_n to sinusoids at frequency f_j and see how well they "match".

Good match: X_j = big

Bad match: X_j = small

X_j reveals the frequencies f_j that match our data.

Coherence: equations (reminder)

$$\kappa_{xy,\,j} = \frac{|< S_{xy,\,j} >|}{\sqrt{< \boxed{S_{xx,\,j}} >}\sqrt{< S_{yy,\,j} >}}$$

coherence between *x* and *y* at frequency *j*

$$S_{xx,j} = \frac{2\Delta^2}{T}\boxed{X_j}X_j^*$$

(Auto-)spectrum of signal *x*

$$X_j = \sum_{n=1}^{N} x_n \exp(-2\pi i f_j t_n)\,.$$

Fourier transform of the data *x*.

Coherence: equations

Fourier transform of the data *x.*

$$X_j = \sum_{n=1}^{N} x_n \exp(-2\pi i f_j t_n) \,.$$

X_j can be complex

- the Fourier transform of x_n can have both real and imaginary parts.

So, X_j lives in the complex-plane ...

Coherence: equations

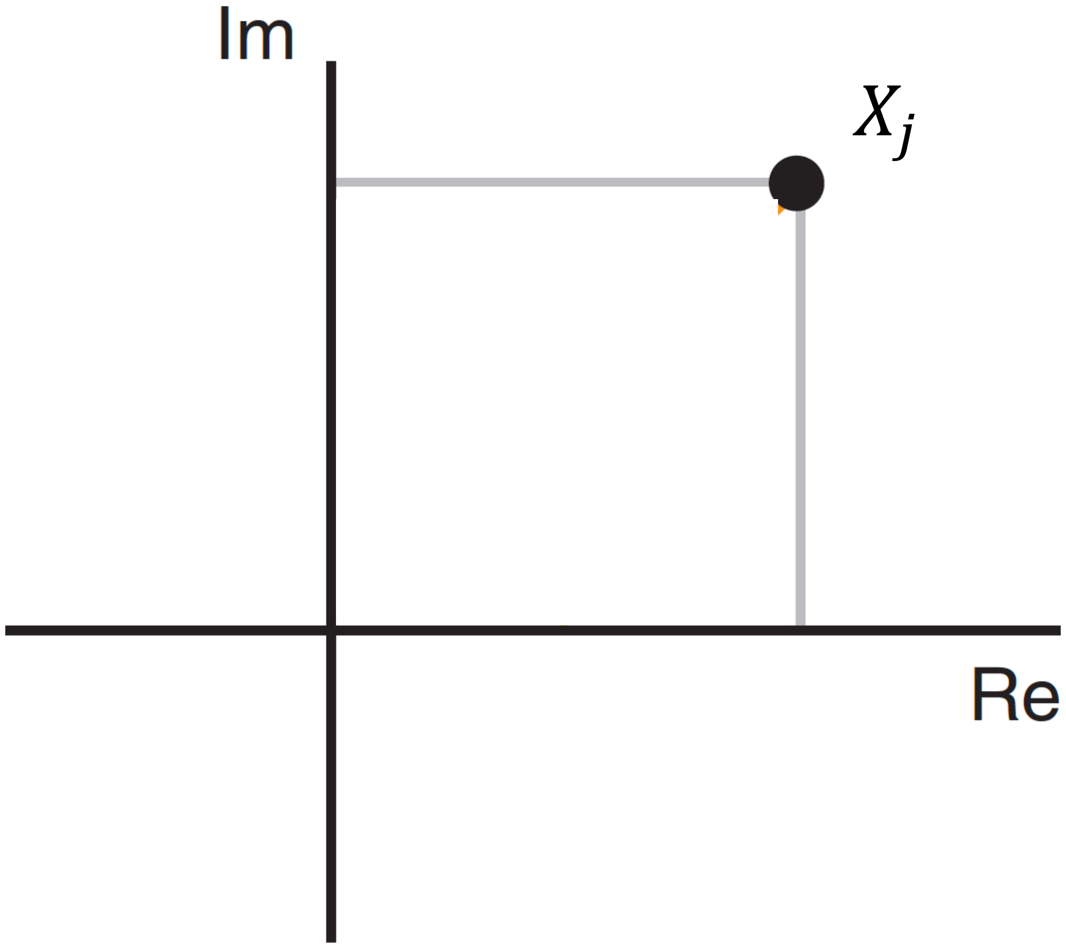
X_j lives in the complex-plane:

Express X_j in polar coordinates:

$$X_j = A_j \exp(i\phi_j)$$

A_j = Amplitude at frequency index j

ϕ_j = Phase at frequency index j



Coherence: equations

Consider the spectrum of x_n:

Express X_j in polar coordinates:

$$S_{xx,j} = \frac{2\Delta^2}{T} X_j X_j^* = \frac{2\Delta^2}{T} \left(A_j \exp(i\phi_j)\right)\left(A_j \exp(-i\phi_j)\right)$$

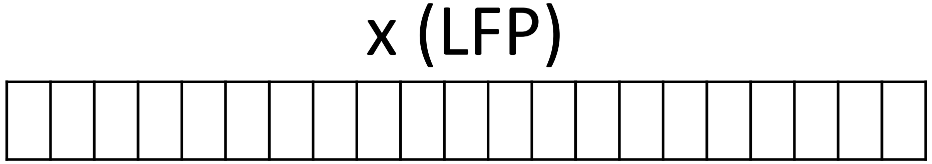
$$= \frac{2\Delta^2}{T} A_j^2 \exp(i\phi_j - i\phi_j) \boxed{= \frac{2\Delta^2}{T} A_j^2 \, .}$$

More direct interpretation of the spectrum at frequency f_j :
proportional to the squared amplitude of the point X_j in the complex plane.

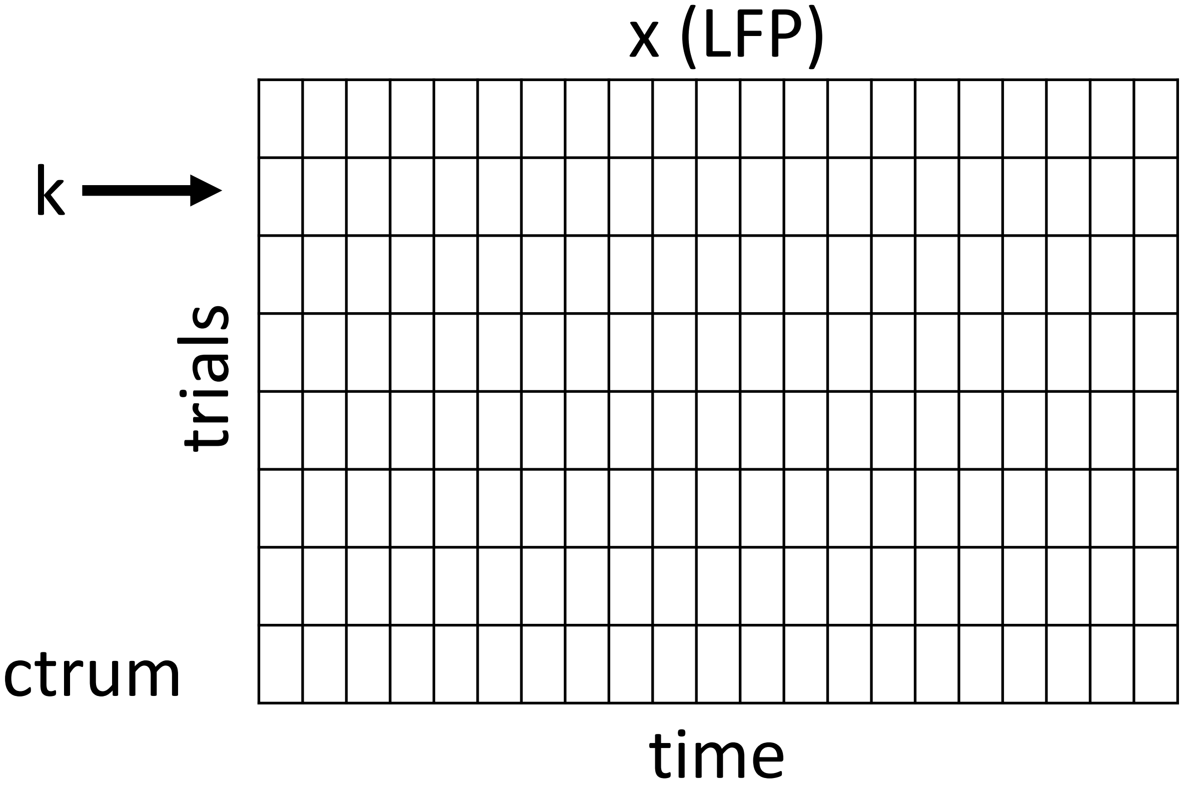
Coherence: equations

To compute coherence, we need the trial-averaged spectrum:

To keep notation simple, we started with one trial:



Now, consider all trials:



$$S_{xx,j} \quad \text{to} \quad < S_{xx,j} >$$

single trial spectrum

trial-averaged spectrum

Coherence: equations

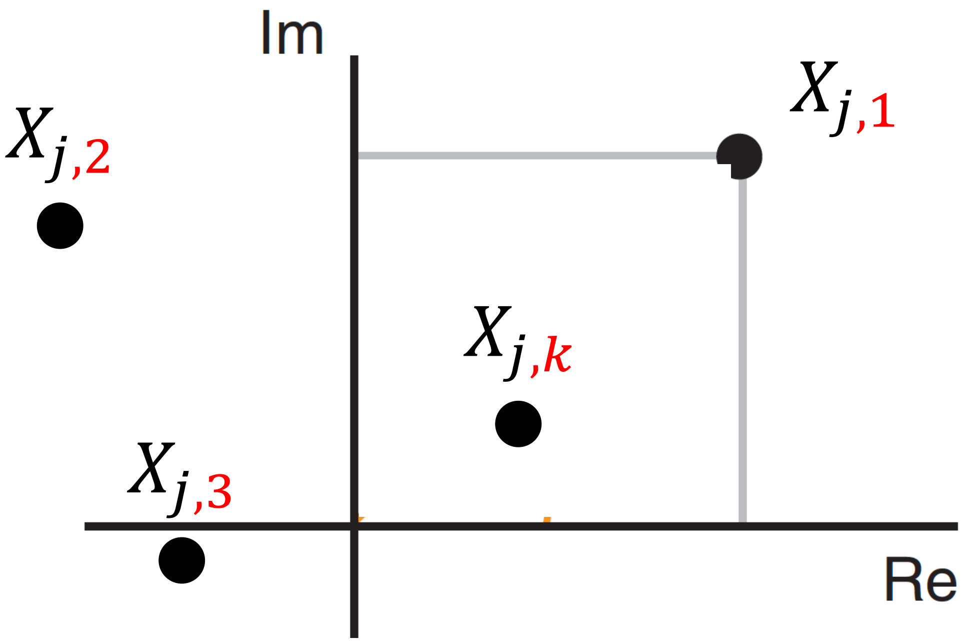
To compute coherence, we need the <u>trial-averaged</u> spectrum:

Fourier transform of x at frequency j: a point <u>for each trial</u> in the complex-plane:

In polar coordinates:

$A_{j,k}$ = Amplitude at frequency index j and trial index k

$\phi_{j,k}$ = Phase at frequency index j and trial index k



Coherence: equations

To compute coherence, we need the trial-averaged spectrum:

Single trial spectrum

Average squared amplitude over trials

$$< S_{xx,j} = \frac{2\Delta^2}{T} A_j^2 \cdot \boxed{\sum_{k=1}^{K} A_{j,k}^2}$$

$A_{j,k}$ = the amplitude of the signal x, at frequency index j, and trial index k .

K = total number of trials

Coherence: equations

Similarly, for signal y_n. Fourier transform of y at frequency j, and trial k:

$$Y_{j,k} = B_{j,k} \exp(i\,\theta_{j,k})$$

$B_{j,k}$ = the amplitude of the signal y at frequency index j and trial index k .

$\theta_{j,k}$ = the phase of the signal y at frequency index j and trial index k .

The trial-averaged spectrum of y at frequency index j

$$< S_{yy,j} > = \frac{2\Delta^2}{T} \frac{1}{K} \sum_{k=1}^{K} B_{j,k}^{2}$$

Coherence: equations (reminder)

$$\kappa_{xy,\,j} = \frac{\left|\boxed{< S_{xy,\,j} >}\right|}{\sqrt{< S_{xx,\,j} >}\sqrt{< S_{yy,\,j} >}}$$

$$< S_{xx,j} > = \frac{2\Delta^2}{T}\frac{1}{K}\sum_{k=1}^{K} A_{j,k}^2 \quad \checkmark$$

$$< S_{yy,j} > = \frac{2\Delta^2}{T}\frac{1}{K}\sum_{k=1}^{K} B_{j,k}^2 \quad \checkmark$$

Consider the trial averaged cross-spectrum ...

Coherence: equations

The trial averaged cross-spectrum at frequency index j:

Like the auto-spectrum, but use X and Y.

$$< S_{xy,j} >= \frac{2\Delta^2}{T}\frac{1}{K}\sum_{k=1}^{K} X_{j,k}Y_{j,k}^*$$

In polar coordinates:

$$< S_{xy,j} >= \frac{2\Delta^2}{T}\frac{1}{K}\sum_{k=1}^{K} A_{j,k}B_{j,k}\exp\left(i\Phi_{j,k}\right)$$

where $\Phi_{j,k} = \underbrace{\phi_{j,k}}_{\text{Phase of x}} - \underbrace{\theta_{j,k}}_{\text{Phase of y}}$ is the phase difference between the two signals, at frequency index j and trial k.

Coherence: equations

Put it all together ...

$$\kappa_{xy,\,j} = \frac{|< S_{xy,\,j} >|}{\sqrt{< S_{xx,\,j} >}\sqrt{< S_{yy,\,j} >}}$$

In polar coordinates ...

cross-spectrum of x & y, depends on trial averaged amplitudes, phase difference.

$$= \frac{\left|\sum_{k=1}^{K} A_{j,k} B_{j,k} \exp\left(i\Phi_{j,k}\right)\right|}{\sqrt{\sum_{k=1}^{K} A_{j,k}^{2}}\sqrt{\sum_{m=1}^{K} B_{j,m}^{2}}}$$

x trial averaged spectrum, at frequency index j

y trial averaged spectrum, at frequency index j

Coherence: intuition

To build intuition, assume: the amplitude is identical for both signals and all trials.

$$A_{j,k} = B_{j,k} = C_j$$

Note: no trial dependence

then

$$\kappa_{xy,j} = \frac{\left| \sum_{k=1}^{K} A_{j,k} B_{j,k} \exp\left(i\Phi_{j,k}\right) \right|}{\sqrt{\sum_{k=1}^{K} A_{j,k}^2} \sqrt{\sum_{m=1}^{K} B_{j,m}^2}}$$

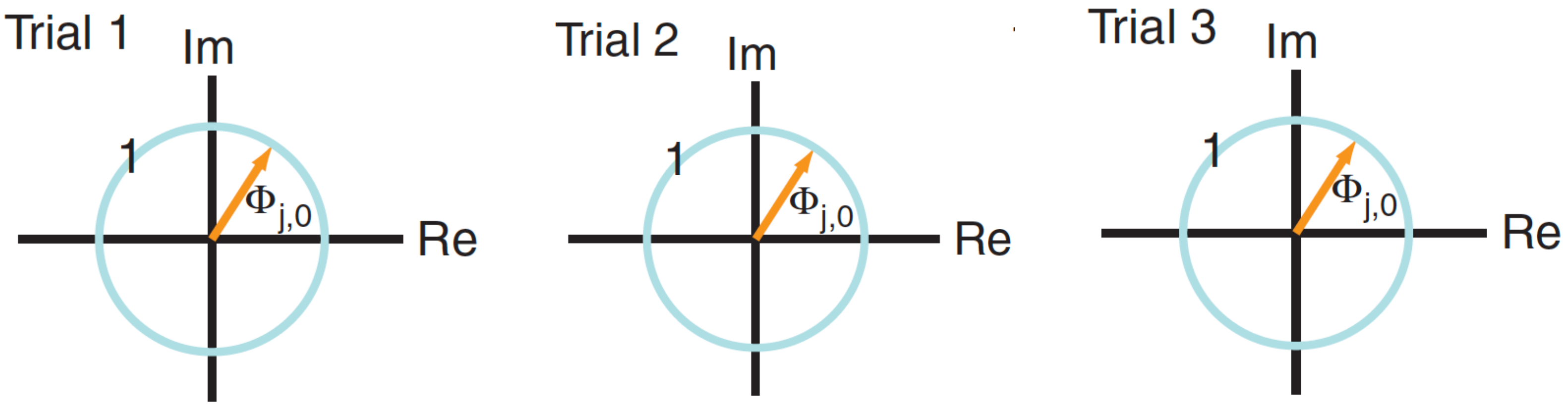
only involves the phase difference between the two signals averaged across trials.

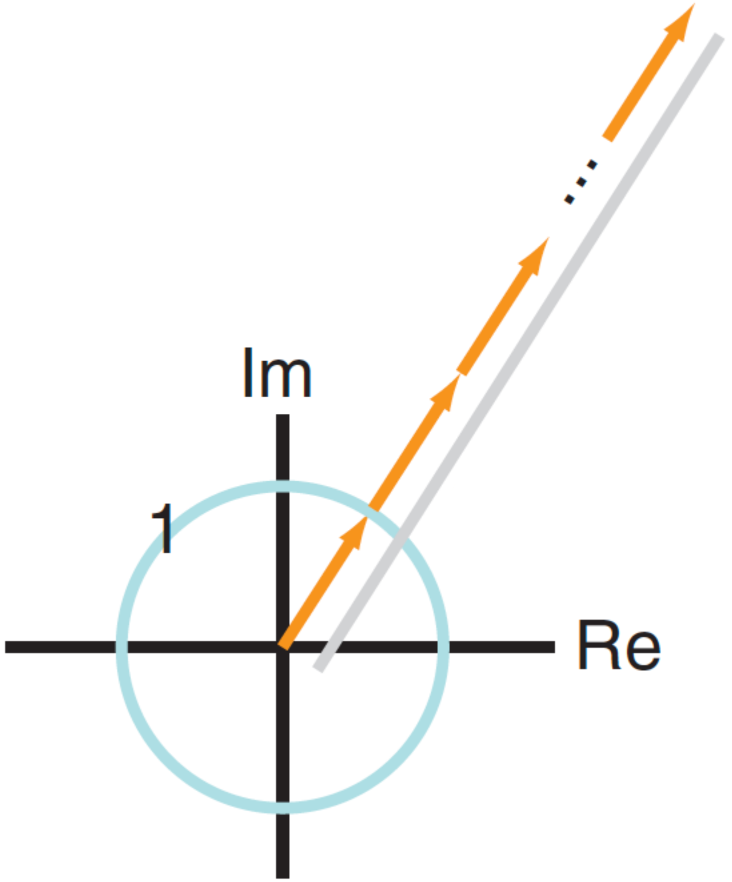
Coherence: intuition

$$\kappa_{xy,j} = \frac{1}{K}\left|\sum_{k=1}^{K} \exp\left(i\Phi_{j,k}\right)\right|$$

Case 1: Phases align across trials. $\boxed{\Phi_{j,k} = \Phi_{j,0}}$

Plot $\exp\left(i\Phi_{j,k}\right)$ in the complex plane.





sum these vectors end to end across trials

divide by K

$$\kappa_{xy,j} \approx 1$$

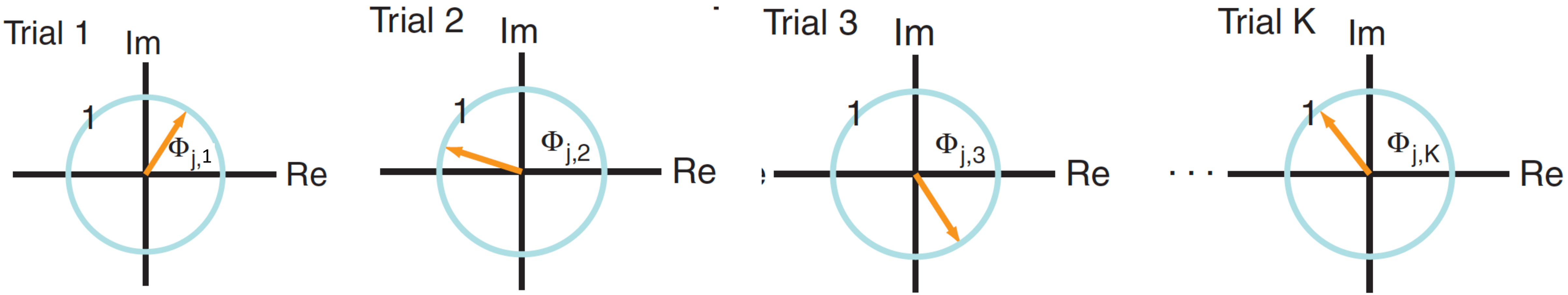
<u>strong coherence</u> - constant phase relation between the two signals across trials at frequency index j.

Coherence: intuition

$$\kappa_{xy,j} = \frac{1}{K}\left|\sum_{k=1}^{K} \exp\left(i\Phi_{j,k}\right)\right|$$

Case 2: Random phase differences across trials.

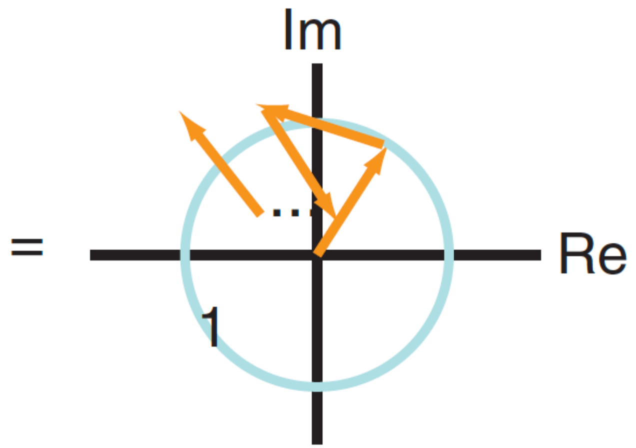
Plot $\exp\left(i\Phi_{j,k}\right)$ in the complex plane.



sum these vectors end to end across trials

divide by K

$$\kappa_{xy,j} \approx 0$$



<u>weak coherence</u> - random phase relation between the two signals across trials at frequency index j.

Coherence: summary

$$0 \leq \kappa_{xy,j} \leq 1$$

0: no coherence between signals x and y at frequency index j

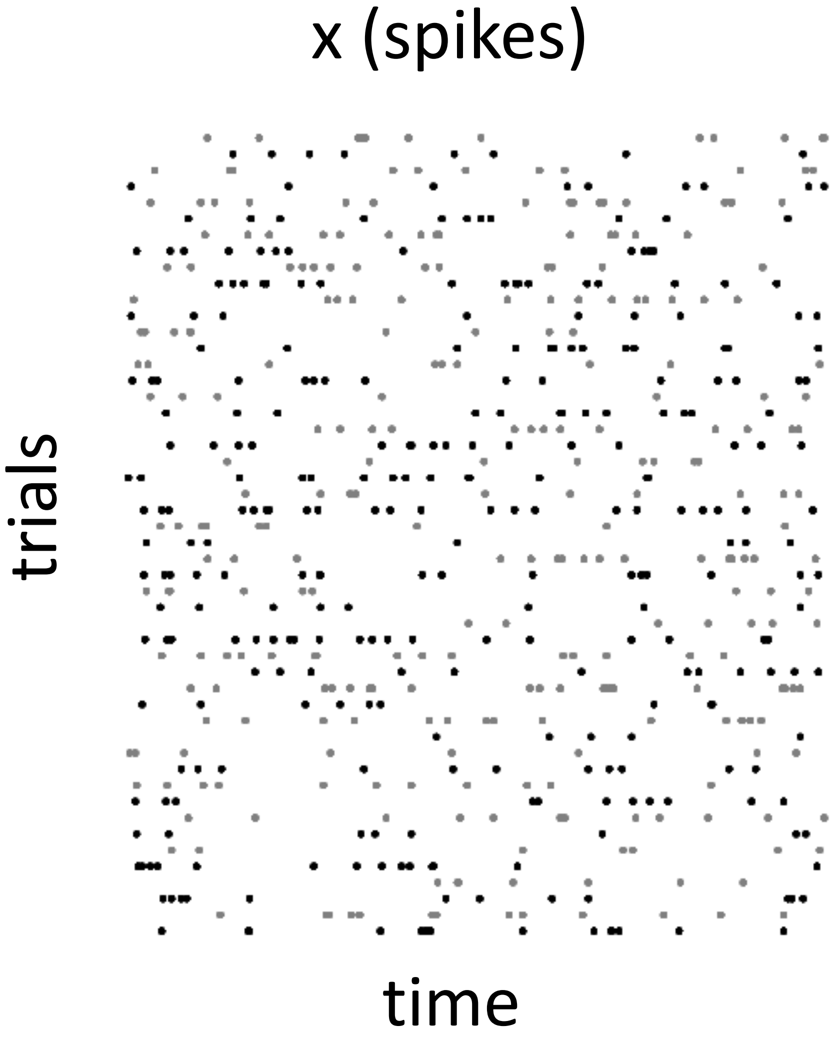
1: strong coherence between signals x and y at frequency index j .

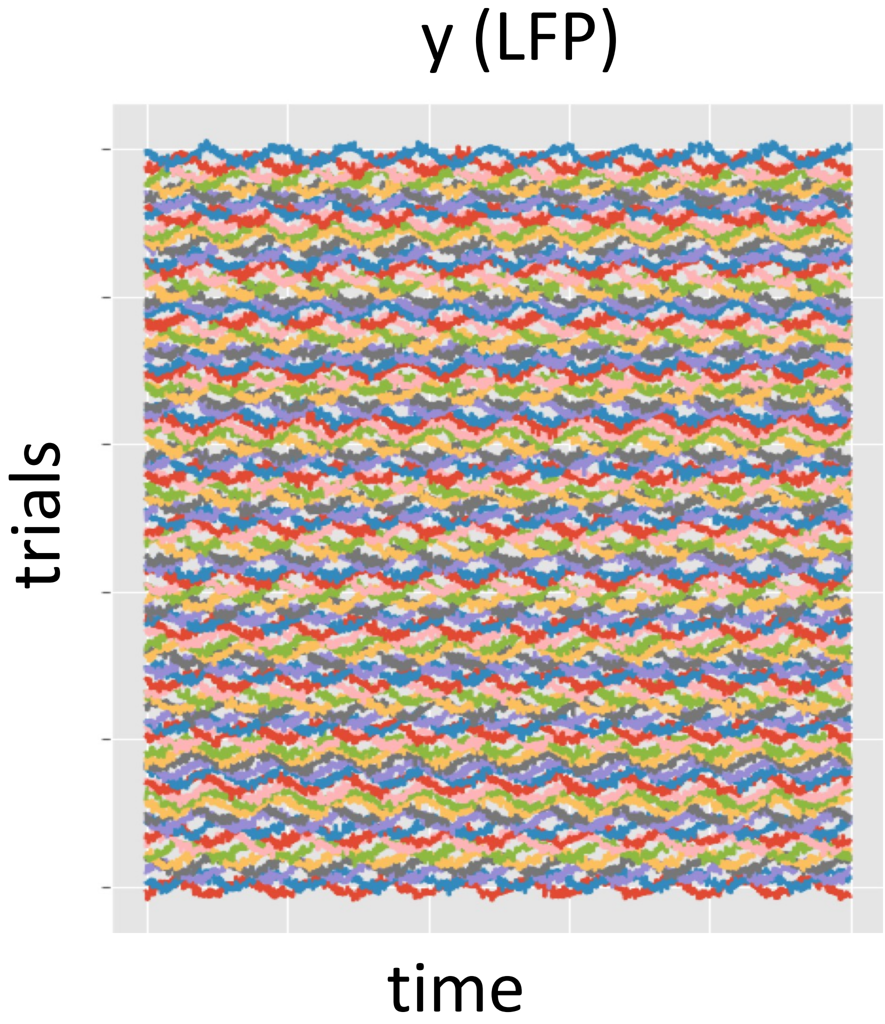
The coherence is a measure of the phase consistency between two signals at frequency index j across trials.

<u>Quiz</u>: What are the units of coherence?

Spike-field coherence

Consider the data:





We want a measure of consistent neural spiking at a specific phase of the field ...

Spike-field coherence

trial averaged <u>cross</u> spectrum

$$\kappa_{ny,\,j} = \frac{|< S_{ny,\,j} >|}{\sqrt{< S_{nn,\,j} >}\sqrt{< S_{yy,\,j} >}}$$

trial averaged <u>spike</u> spectrum

trial averaged <u>field</u> spectrum

y = field signal (e.g., EEG, MEG, LFP, ...)

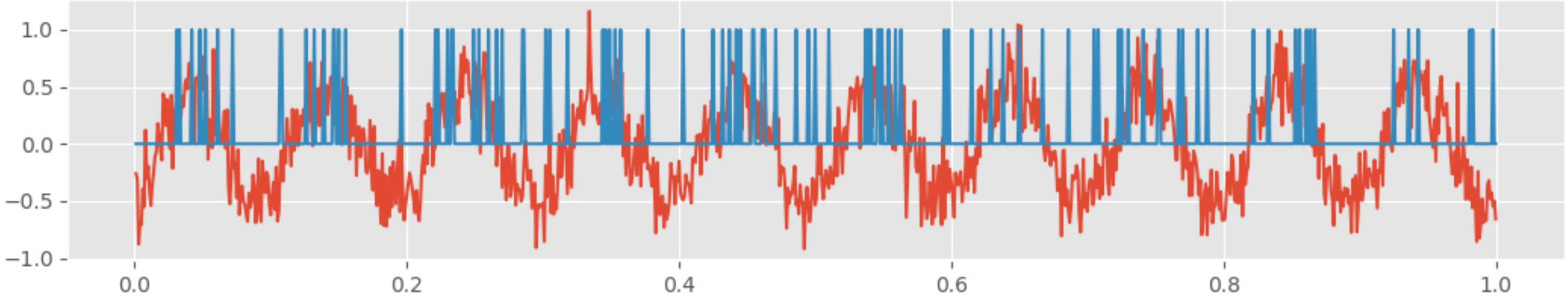
n = spike train (e.g., [0 0 0 0 0 0 1 0 0 0 0 0 0 0 0 ...])

Same equations ... but new problems ...

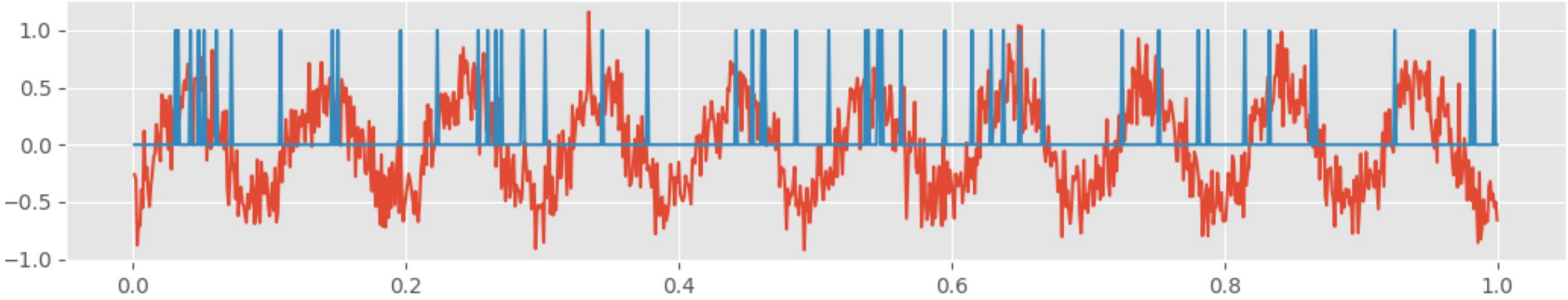
Spike-field coherence: dependence on rate

Q: Does the spike-field coherence depend on the firing rate of the neuron?

Original spike & field



"Thinned" spike & field (remove 50% of spikes, chosen at random)



Spike-field coherence: dependence on rate

Q: Does the spike-field coherence depend on the firing rate of the neuron?

Here, rate: expected number of spikes in a given duration

Try it ...

https://github.com/Mark-Kramer/MIT-Spike-Field-Coherence

Spike-field coherence: dependence on rate

Q: Does the spike-field coherence depend on the firing rate of the neuron?

Observations:

greater thinning → fewer spikes → lower coherence

as the rate tends to 0, so does the spike-field coherence

The spike-field coherence reflects
(1) the relationship between spiking activity and the phase of field, and
(2) the mean firing rate.

Spike-field coherence: dependence on rate

Q: So what next?

Q: If, in your experiment, the overall spike rate differs between two neurons, then how do you compare the spike-field coherence?

Try it ...

https://github.com/Mark-Kramer/MIT-Spike-Field-Coherence

Spike-field coherence: dependence on rate

Q: So what next?

Q: If, in your experiment, the overall spike rate differs between two neurons, then how do you compare the spike-field coherence?

- include a rate adjustment factor in the coherence measure to account for rate dependence.

Journal of Neuroscience Methods 240 (2015) 141–153

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Computational Neuroscience

Rate-adjusted spike–LFP coherence comparisons from spike-train statistics

Mikio C. Aoi*, Kyle Q. Lepage, Mark A. Kramer, Uri T. Eden

Department of Mathematics & Statistics, Boston University, Boston, MA 02215, USA

- build a generalized linear model to separate overall neural activity from spike train-LFP oscillatory coupling.

Journal of Neuroscience Methods 213 (2013) 43–62

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Computational Neuroscience

A procedure for testing across-condition rhythmic spike-field association change

Kyle Q. Lepage[a,*], Georgia G. Gregoriou[b,c], Mark A. Kramer[a], Mikio Aoi[a], Stephen J. Gotts[d], Uri T. Eden[a], Robert Desimone[e]

Summary

- Coherence in words / idea
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- Intuition
- Spike-field coherence
- Dependence on rate
- Next steps

https://github.com/Mark-Kramer/MIT-Spike-Field-Coherence
https://mark-kramer.github.io/Case-Studies-Python/