Introduction to the Analysis of Neural Electrophysiology Data

NESS Short Course

Saturday, June 3, 2023

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Outline

Part 1 (Prof. Kramer)

- Rhythms & spectrum
- Coherence
- Spike-field coherence

Part 2 (Prof. Eden)

- Point processes
- Latent state models

https://github.com/Mark-Kramer/NESS-Short-Course-2023

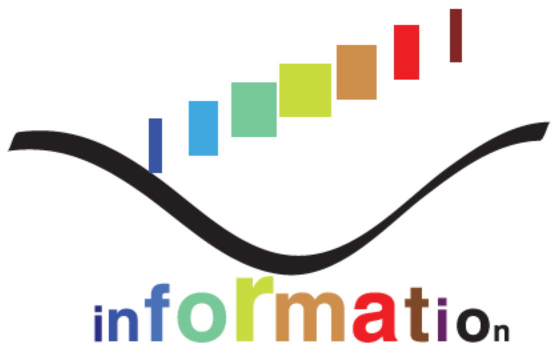
NESS-Short-Course-2023

This repository contains code discussed in the short course *Introduction to the analysis of neural electrophysiology data* at NESS 2023.

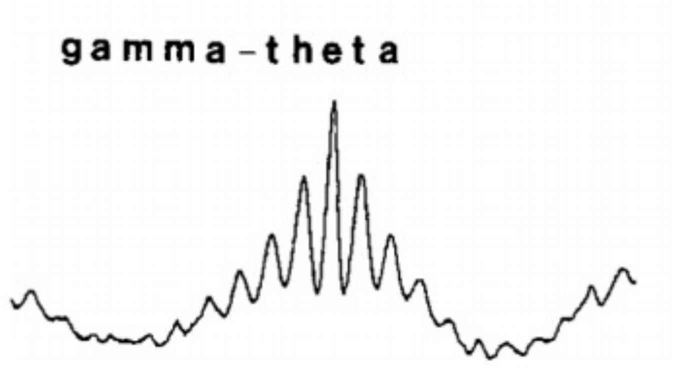
Lecture slides (Part 1)

Notebook	Run It
Spectrum	Open in Colab
Coherence	Open in Colab
Spike-field coherence	Open in Colab

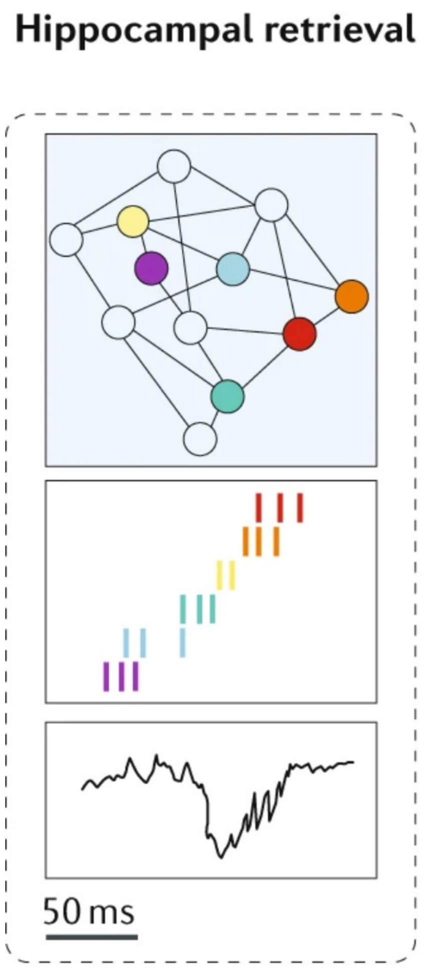
Rhythms do something in the brain ...



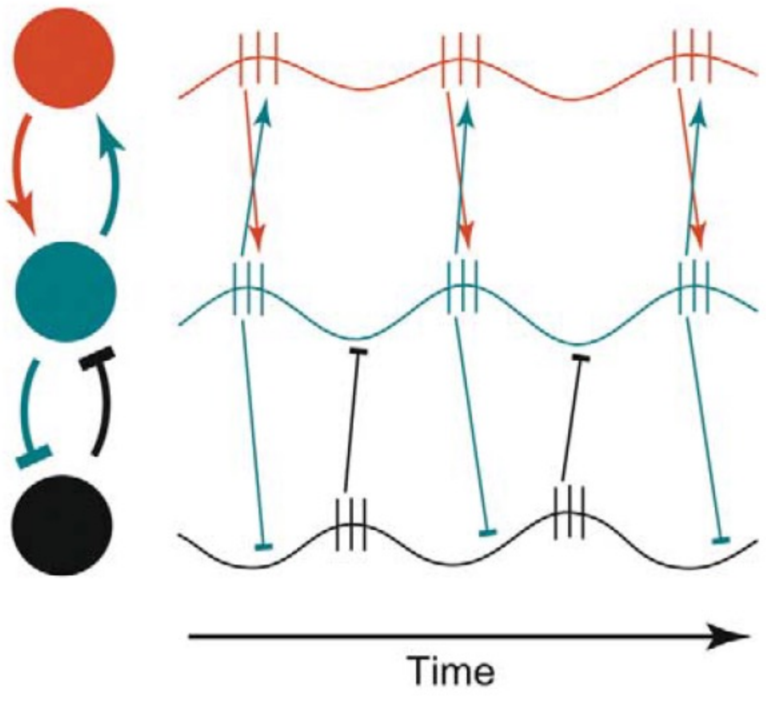
theta phase: [Buzsaki & Moser, 2013]



[Colgin J Neurosci, 2019]



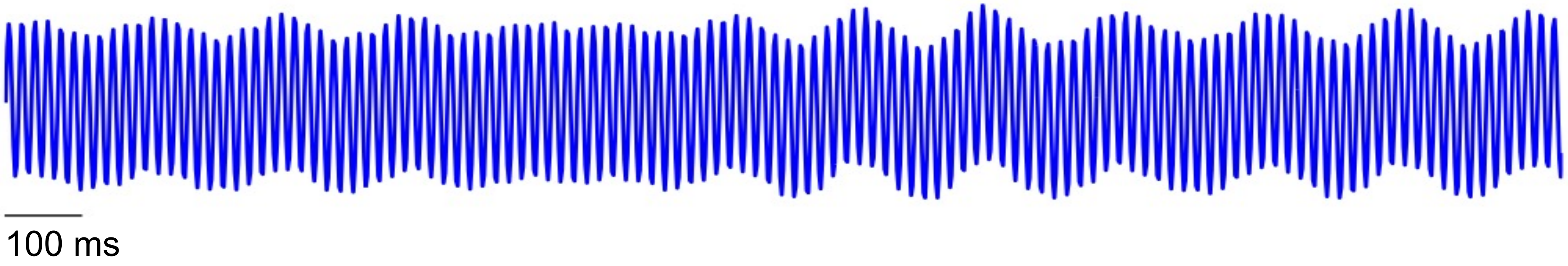
[Joo & Frank, 2018]



[Fries 2005]

Rhythms

Consider these (experimental data)

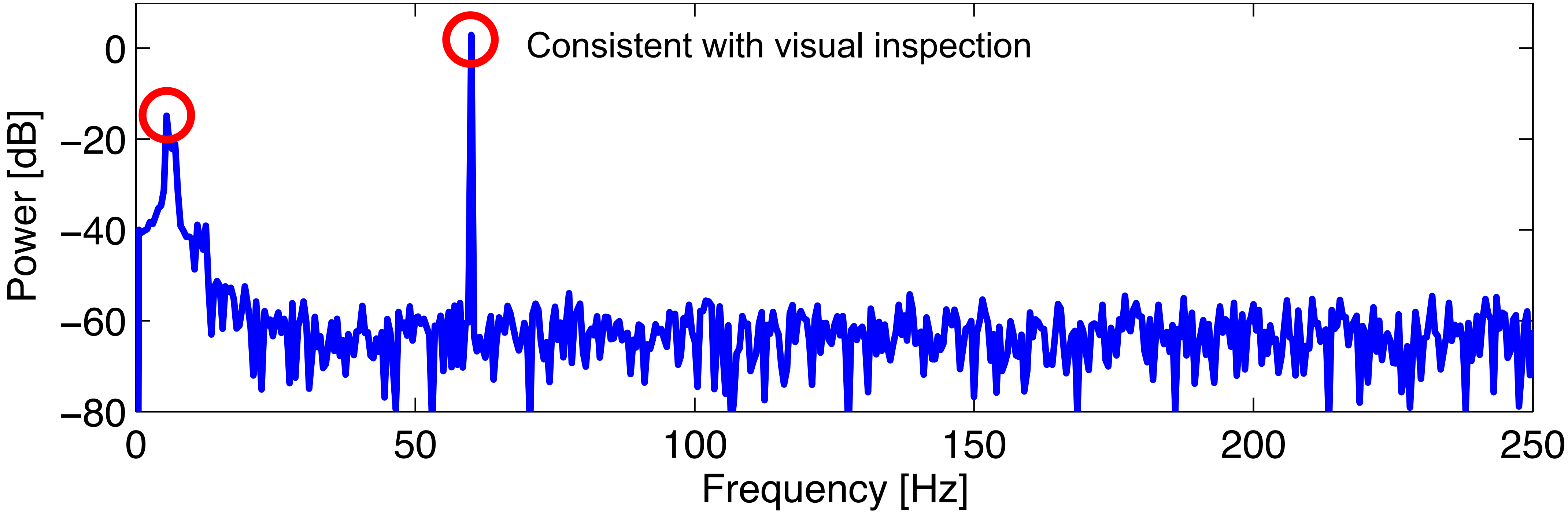


Visual inspection

- Rhythmic (dominant fast rhythm)
- It’s complicated
- Beyond visual inspection . . . quantitative characterization?

Spectrum

Beyond visual inspection . . .



<u>Axes</u>: Power [dB] vs Frequency [Hz]

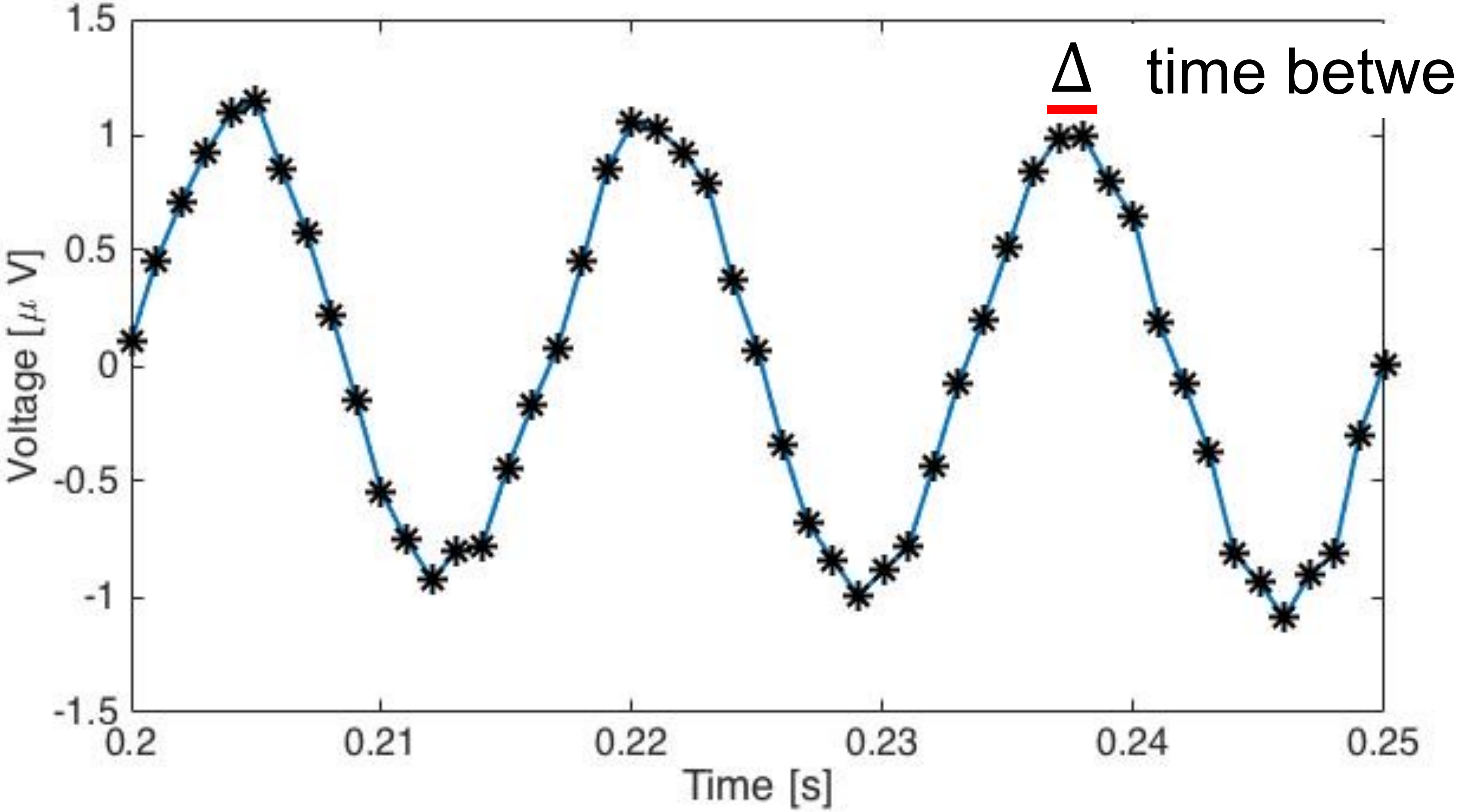
A simpler representation in frequency domain. Two peaks at ~5-8 Hz, 60 Hz

An improved understanding of rhythmic activity.

How?

Spectrum

To start, consider activity recorded from a single trial:



Δ time between samples

Notation: x_n = Data at index n

Spectrum

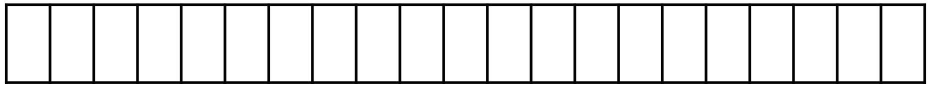
So, x is activity recorded from a single trial:

$$S_{xx,j} = \frac{2\Delta^2}{T} \boxed{X_j} X_j^*$$

(Auto-)spectrum of signal *x*

complex conjugate

x (LFP, EEG, ...)



Note: Time is discrete

x_n = Data at index n

Δ = sampling interval

T = total time of observation

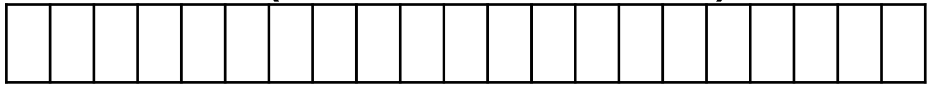
X_j = Fourier transform of the data (*x*) at frequency *j*

Spectrum

$$X_j = \sum_{n=1}^{N} x_n \exp(-2\pi i f_j t_n) \, .$$

Fourier transform of the data *x*.

x (LFP, EEG, ...)



x_n = Data at index n

i = $\sqrt{-1}$

t_n = Time at index n $\quad t_n = \Delta n \quad$ where Δ = sampling interval

f_j = Frequency at index j $\quad f_j = j/T \quad$ where T = total time of observation

Spectrum: idea

Fourier transform intuition:

Data as a function of time index n

Data as a function of frequency index j

$$X_j = \sum_{n=1}^{N} \boxed{x_n} \boxed{\exp(-2\pi i f_j t_n)} \,.$$

Sinusoids at frequency f_j

Euler's formula:

$$\exp(-2\pi\, i f_j\, t_n) = \cos(-2\pi\, f_j\, t_n) + i \sin(-2\pi\, f_j\, t_n).$$

So, at each time (index n) multiply data x_n by sinusoids at frequency f_j
Then sum up over all time.

Spectrum: idea

Fourier transform intuition:

Data as a function of frequency index j

$$X_j = \sum_{n=1}^{N} \boxed{x_n} \boxed{\exp(-2\pi i f_j t_n)}.$$

Data as a function of time index n

Sinusoids at frequency f_j

Idea: compare our data x_n to sinusoids at frequency f_j and see how well they “match”.

Good match: X_j = big   Bad match: X_j = small

X_j reveals the frequencies f_j that match our data.

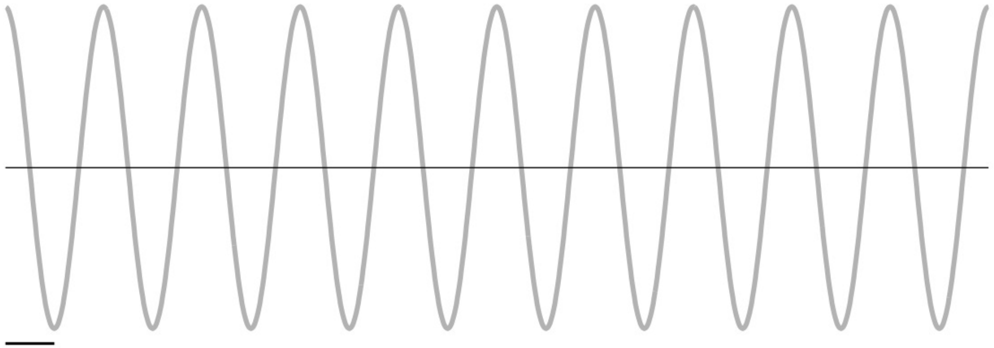
Spectrum: idea

Fourier transform <u>intuition</u>: "Compare" data to sinusoids at different frequencies

Match: X_j at frequency f_j is <u>large</u>

Mismatch: X_j at frequency f_j is <u>small</u>

<u>Example:</u>



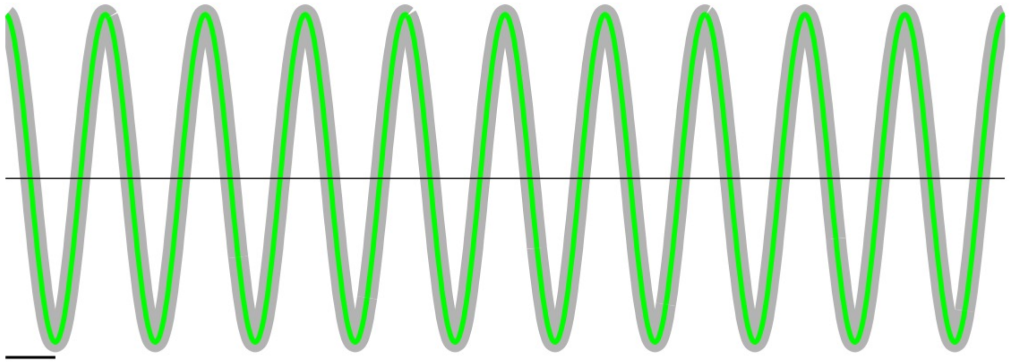
"Data"
10 Hz cosine

4 Hz

Multiply (+,-,+,-,...) & add

... <u>small</u> value

4 Hz does not match data



10 Hz

Multiple (+,+,+,+,...) & add

... <u>large</u> value

10 Hz matches data

Spectrum: idea

$$S_{xx,j} = \frac{2\Delta^2}{T} X_j X_j^*$$

More spectrum intuition ...

Fourier transform of the data *x.*

$$X_j = \sum_{n=1}^{N} x_n \exp(-2\pi i f_j t_n)\,.$$

X_j can be complex

- the Fourier transform of x_n can have both real and imaginary parts.

So, X_j lives in the complex-plane ...

Spectrum: idea (part 2)

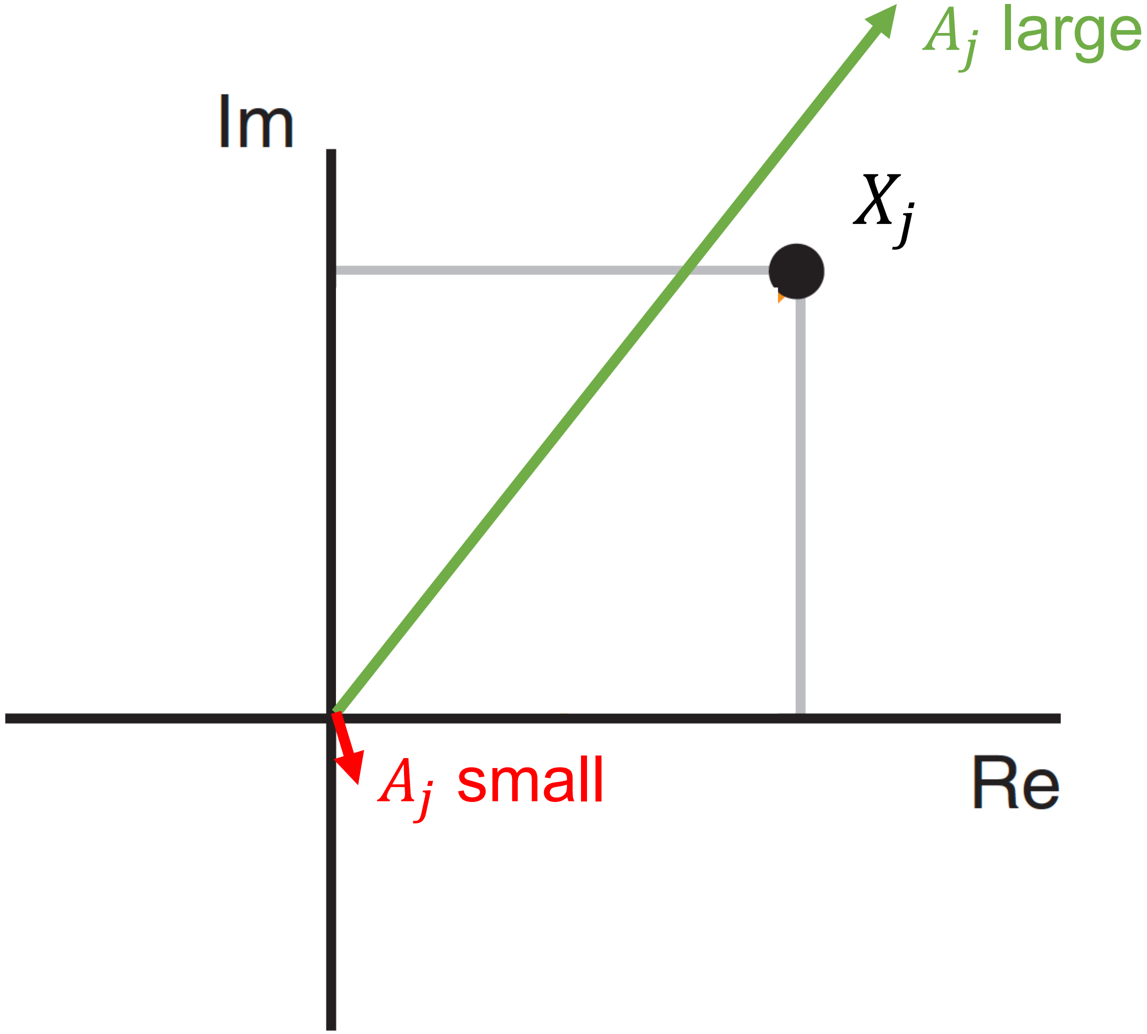
X_j lives in the complex-plane:

Express X_j in polar coordinates:

$$X_j = A_j \exp(i\phi_j)$$

A_j = Amplitude at frequency index j

ϕ_j = Phase at frequency index j



Match: X_j at frequency f_j is large

Mismatch: X_j at frequency f_j is small

Spectrum: idea (part 2)

Consider the spectrum of x_n:

Express X_j in polar coordinates:

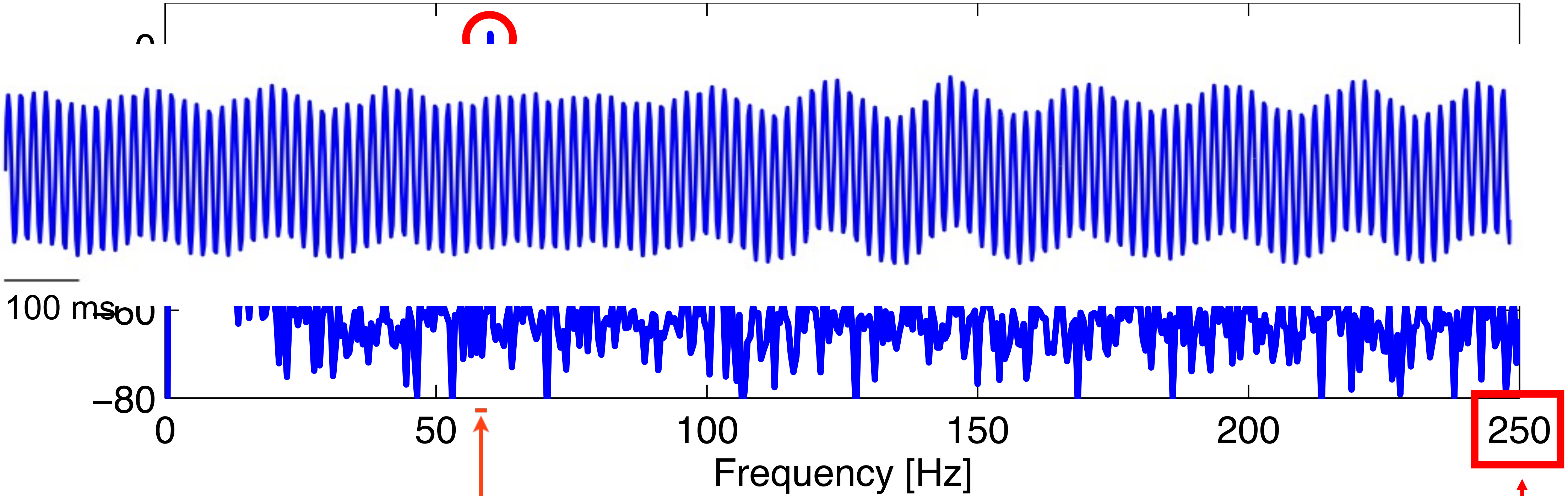
$$S_{xx,j} = \frac{2\Delta^2}{T} X_j X_j^* = \frac{2\Delta^2}{T} \left(A_j \exp(i\phi_j)\right)\left(A_j \exp(-i\phi_j)\right)$$

$$= \frac{2\Delta^2}{T} A_j^2 \exp(i\phi_j - i\phi_j) \boxed{= \frac{2\Delta^2}{T} A_j^2 .}$$

More direct interpretation of the spectrum at frequency f_j :
better match → larger amplitude of X_j in the complex plane → more power

Spectrum: idea (summary)

... reveals the dominant frequencies that "match" the data.



Important quantities

Frequency resolution: df

Nyquist frequency: F_{NQ}

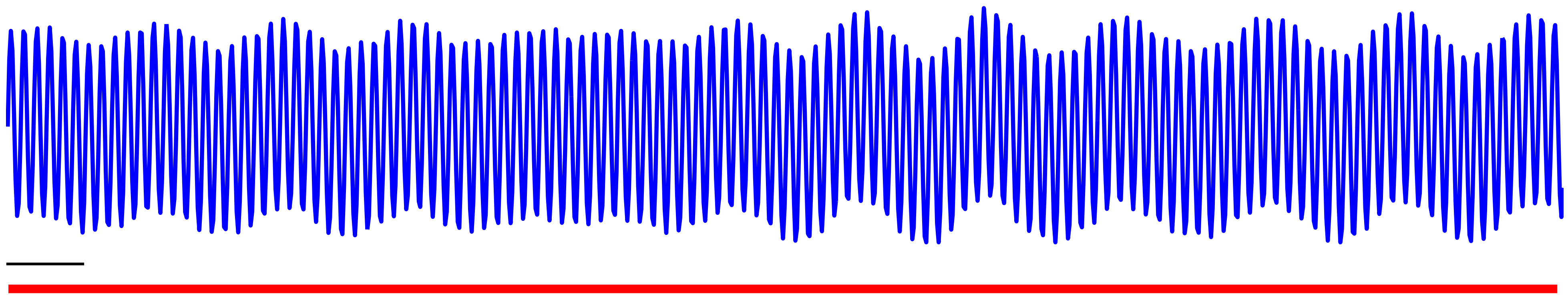
Define these two quantities.

Spectrum: df

- What is df? $\boxed{df = \frac{1}{T}}$ frequency resolution

where T = Total duration of recordings.

Ex.



T = 2 s so df = 0.5 Hz

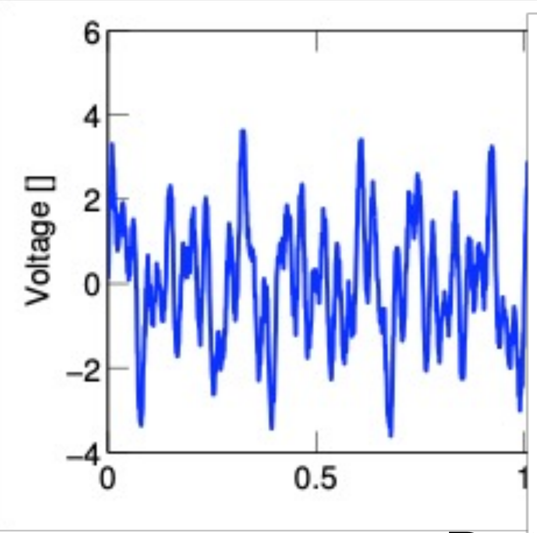
Q: How do we improve frequency resolution?

A: Increase T or record for longer time.

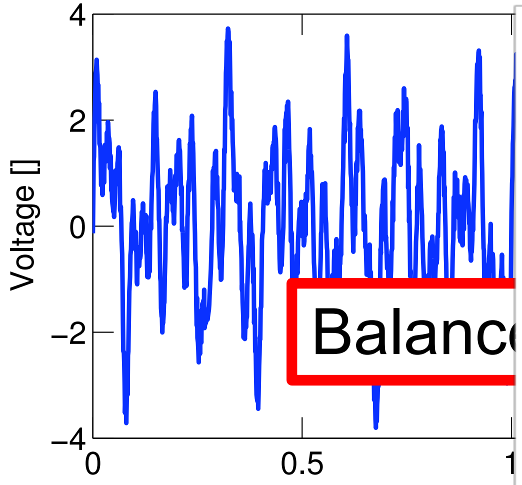
Spectrum: df

- Demand 0.2 Hz frequency resolution

df = 0.2 Hz = 1/T , so T = 5 s



But, data may change during longer recordings . . .



Spectrum: F_{NQ}

- What is F_{NQ}?

$$\boxed{F_{NQ} = f_0/2}$$

Nyquist frequency

where f_0 = sampling frequency.

The **highest** frequency we can observe.

True signal: f_s

Sample:

$f_0 >> 2\ f_s$

Accurate reconstruction

$f_0 = 2\ \boxed{f_s}$

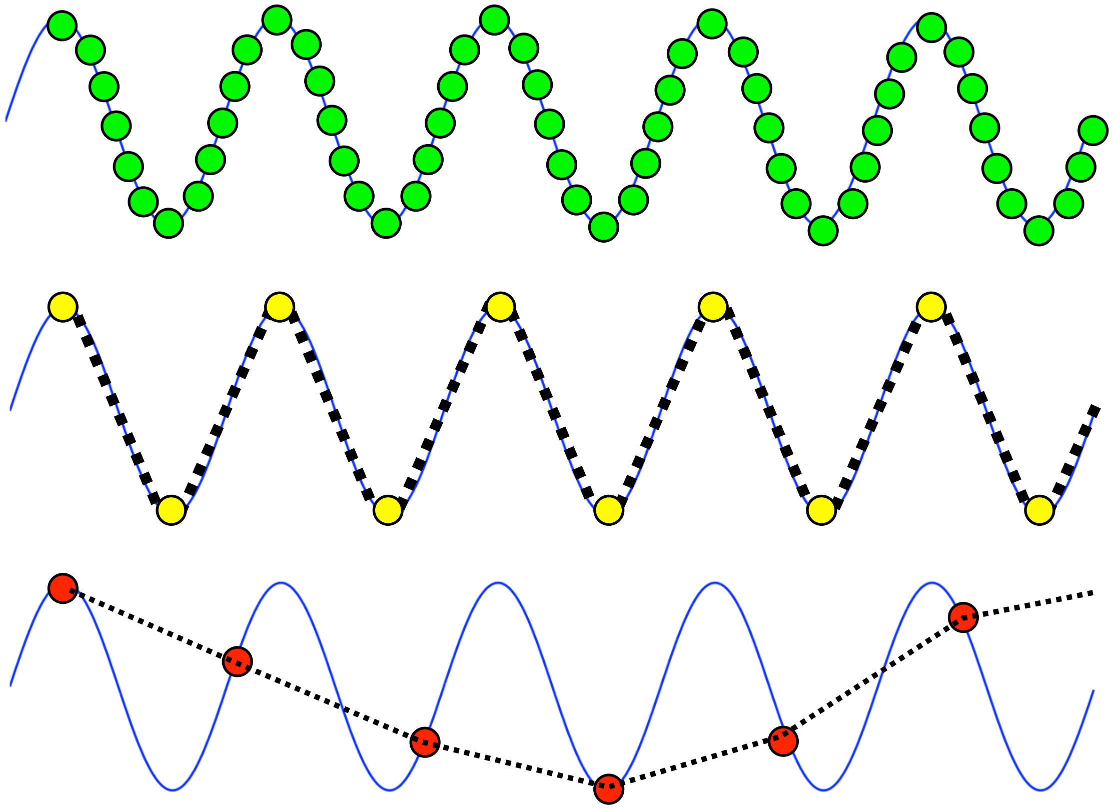
Max freq we can observe at this sample rate!

2 samples/cycle

Enough to reconstruct signal, but just barely.

$f_0 < 2\ f_s$

High frequency (in data) mapped to low frequency (**aliased**).



All hope lost! Indistinguishable from true low frequency signals.

Spectrum: $df,\ F_{NQ}$

Summary

Frequency resolution $$df = \frac{1}{T}$$ ← Duration of recording

Nyquist frequency $$f_{\mathrm{NQ}} = \frac{f_0}{2}$$ ← Sampling frequency

For finer frequency resolution: record more data.

To observe higher frequencies: increase sampling rate.

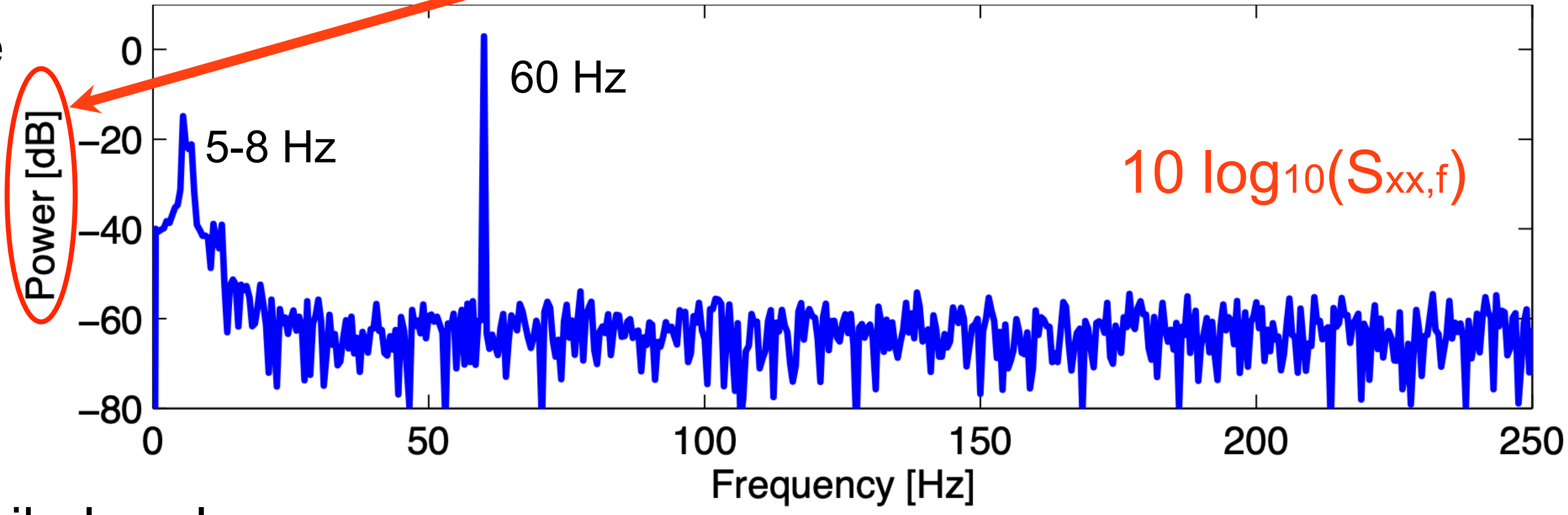
Spectrum: three (important) asides

- Scale
- Tapers
- Multiple trials

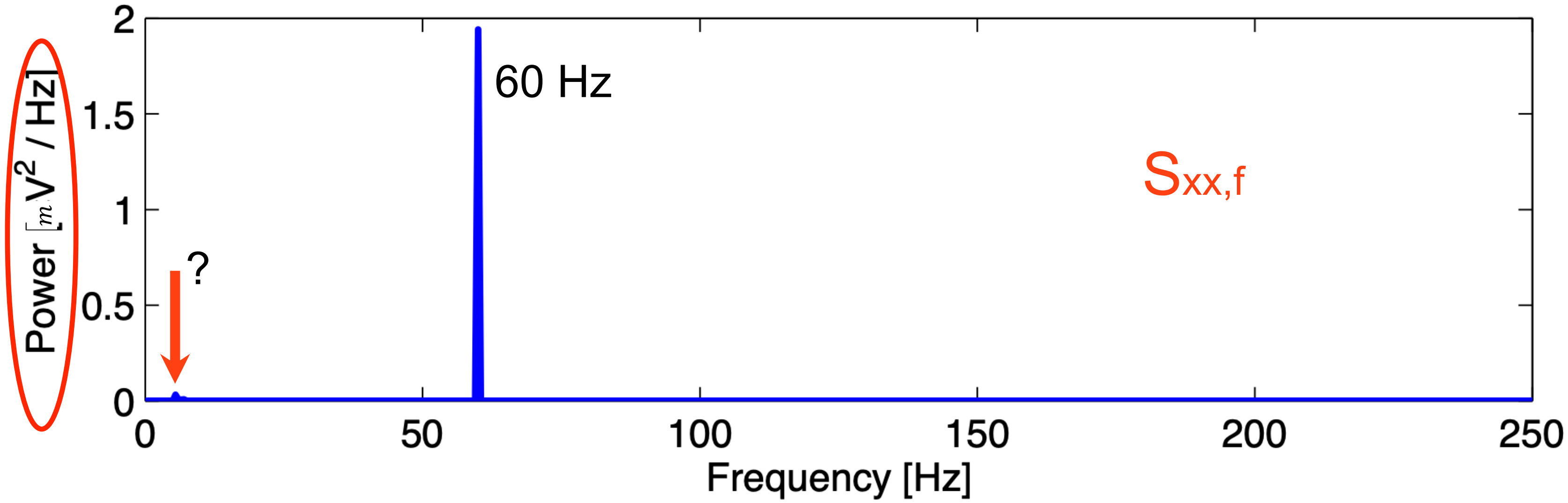
Spectrum: scale

Emphasize lower amplitude signals

A note on scale



Without the decibel scale . . .



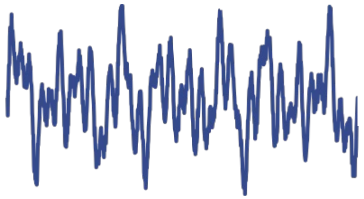
Spectrum: tapers

Doing nothing, we make an implicit taper choice . . .

. . . Data goes on forever . . .

What we're observing:

Signal



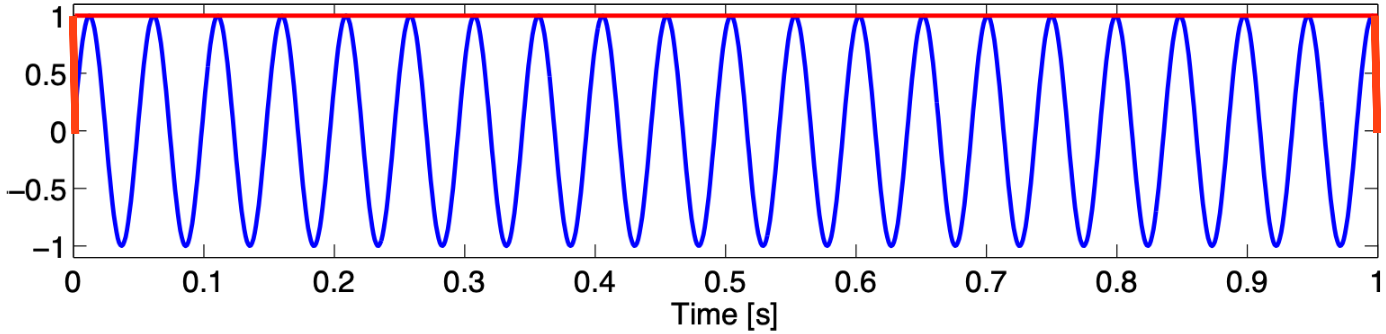
x

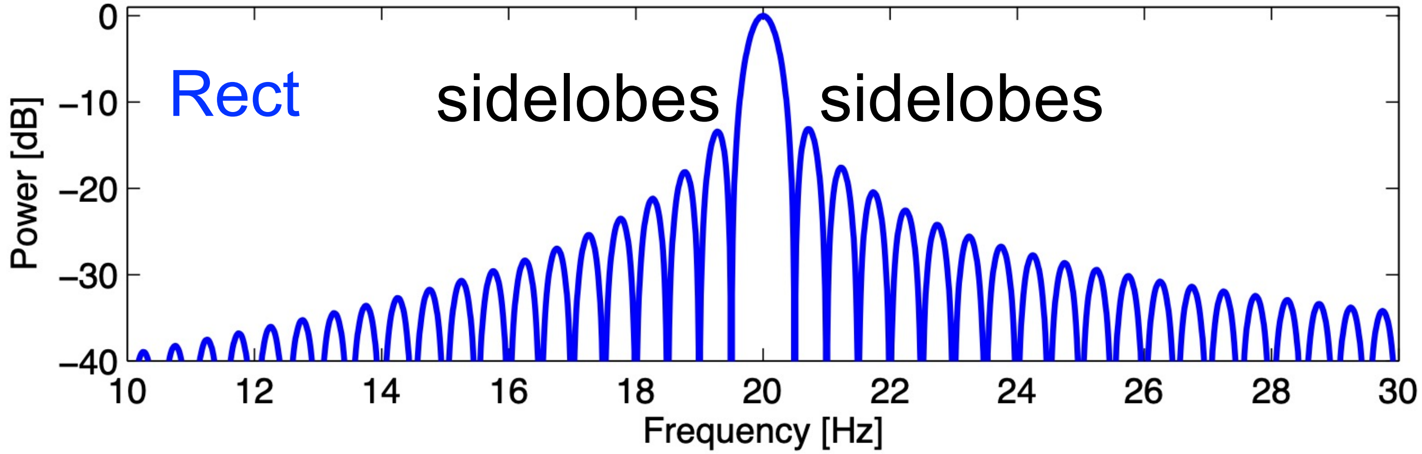
Rectangle taper

Spectrum: tapers

The rectangle taper impacts the power spectrum.

Pure
sinusoid
at 20 Hz

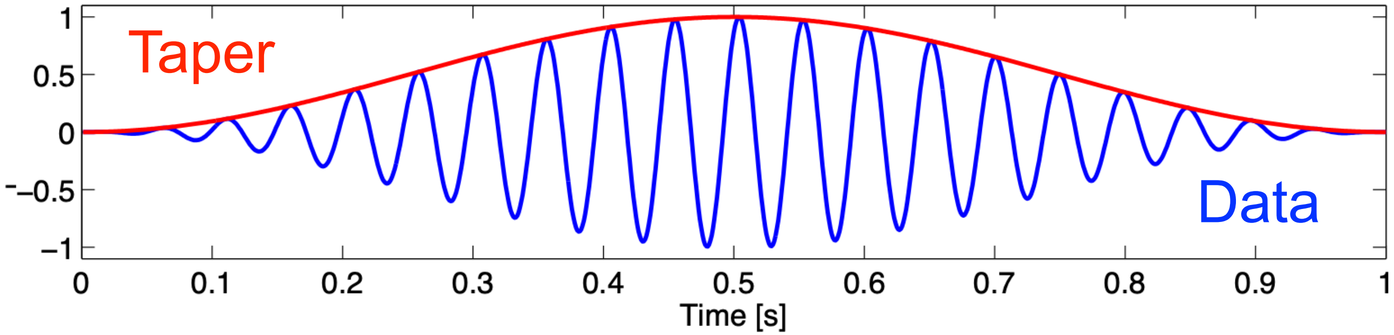




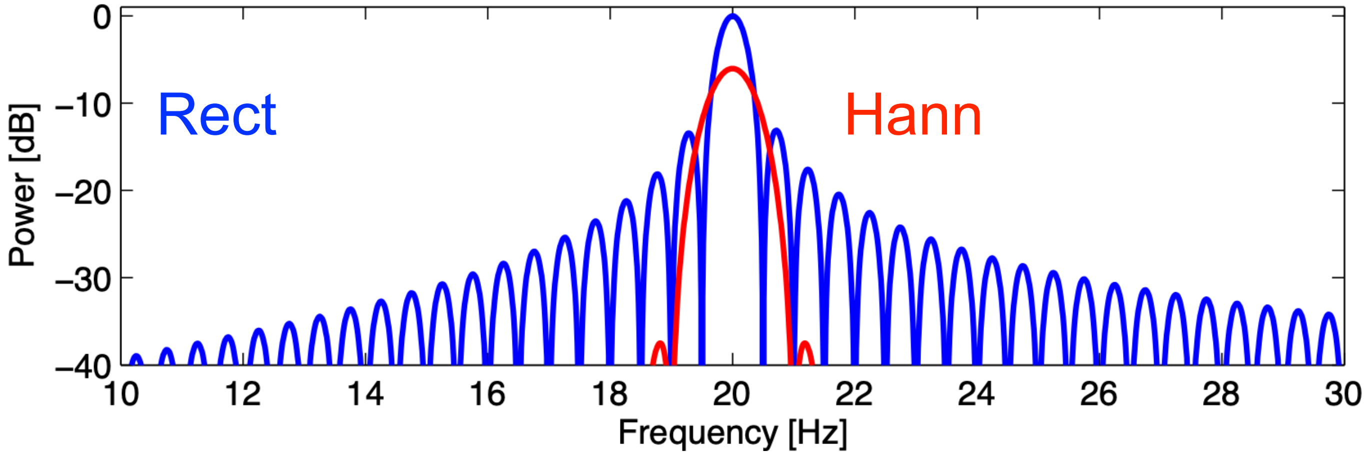
Sharp peak is “smeared out” . . .

Spectrum: tapers

Idea: smooth the sharp edges of rectangle taper.



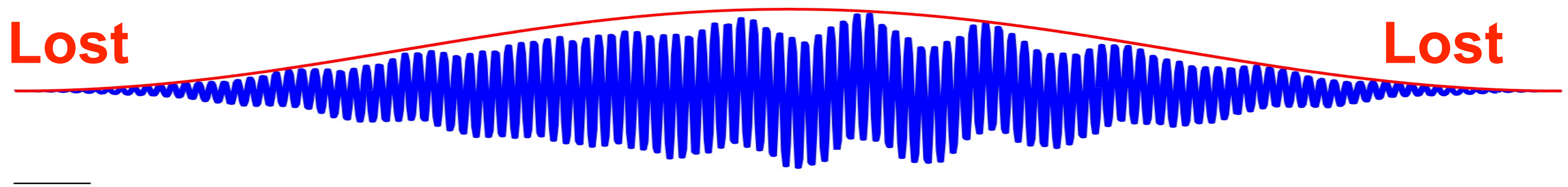
Compute spectrum of tapered data.

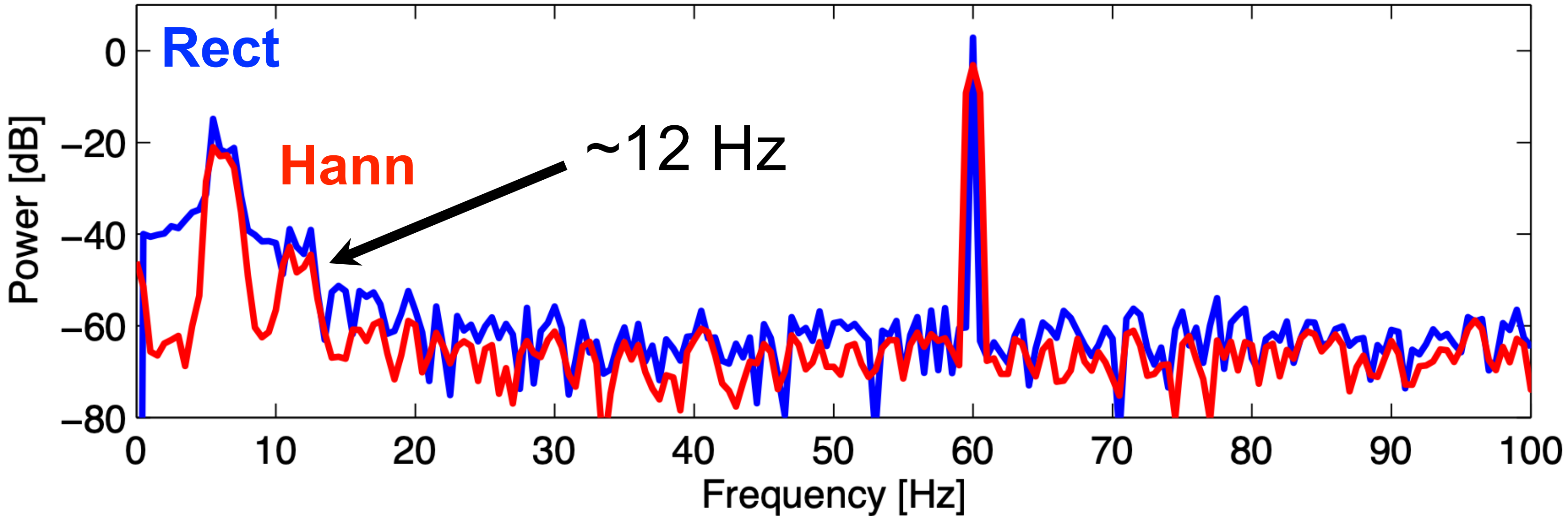


Taper reduces the sidelobes.

Spectrum: tapers

Ex:





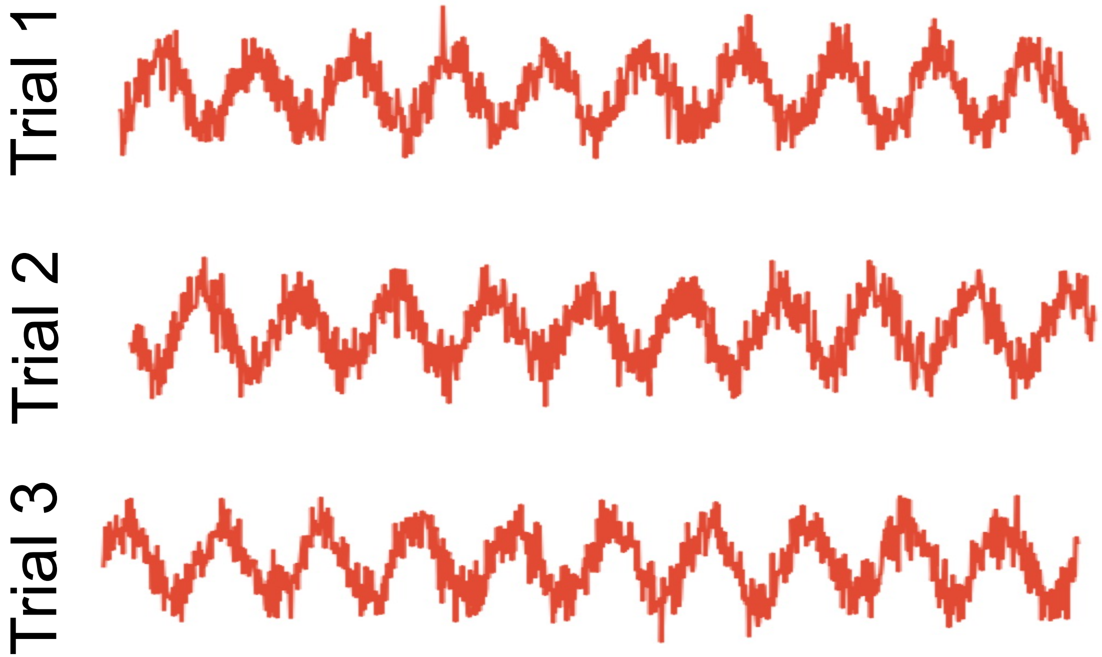
Good: Reduced sidelobes reveals a new peak.

Bad: Broader peaks & lose data at edges.

"*More lives have been lost looking at the [rectangular tapered spectrum] than by any other action involving time series.*" [Tukey 1980]

Spectrum: multiple trials

Ex: Record data across multiple trials

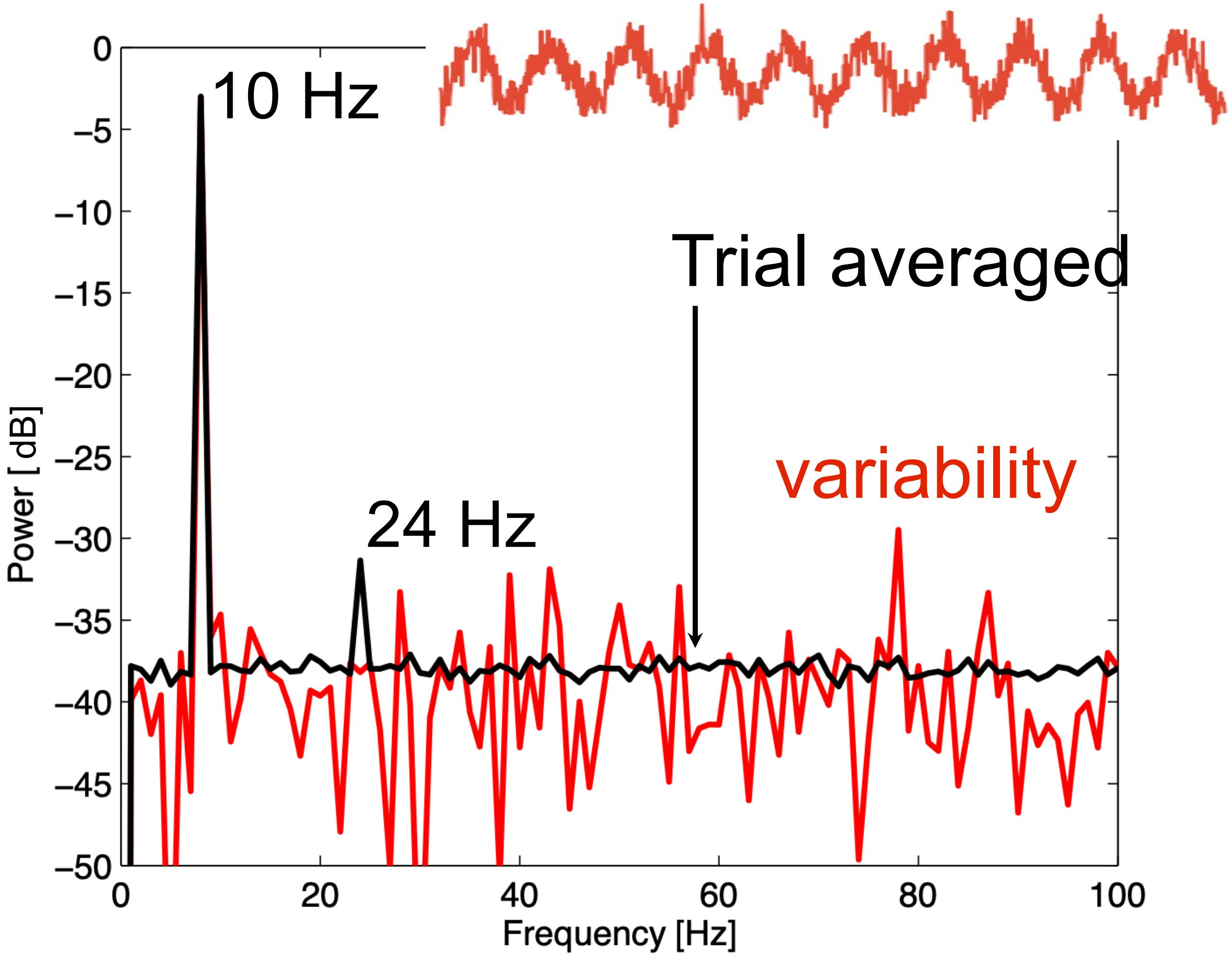


K total trials

Q. Spectrum?

Spectrum: multiple trials

Single trial:



Trial averaged spectrum:

reduced variability.

reveals another peak . . .

Spectrum: summary

$$S_{xx,j} = \frac{2\Delta^2}{T} X_j X_j^*$$

Data as a function of time index n

$$X_j = \sum_{n=1}^{N} \boxed{x_n} \boxed{\exp(-2\pi i f_j t_n)}.$$

Sinusoids at frequency f_j

Frequency resolution $df = \frac{1}{T}$

Nyquist frequency $f_{\mathrm{NQ}} = \frac{f_0}{2}$

Decibel scale, tapers, multiple trials, ...

Spectrum: example

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Outline

Part 1 (Prof. Kramer)

- Rhythms & spectrum
- Coherence
- Spike-field coherence

Part 2 (Prof. Eden)

- Point processes
- Latent state models

Coherence: words

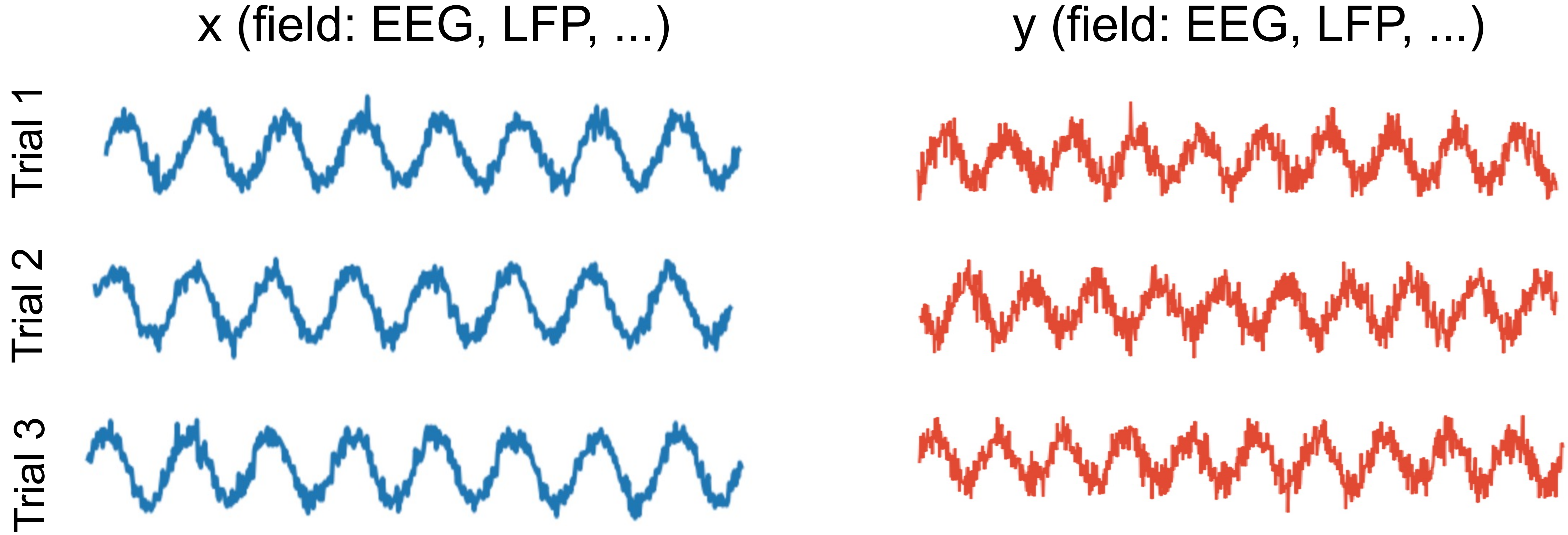
A constant phase relationship between two signals, at the same frequency, across trials.

Note

- “*same frequency*”
- “*across trials*”

Coherence: idea

Ex: Record data simultaneously from two sensors, across multiple trials



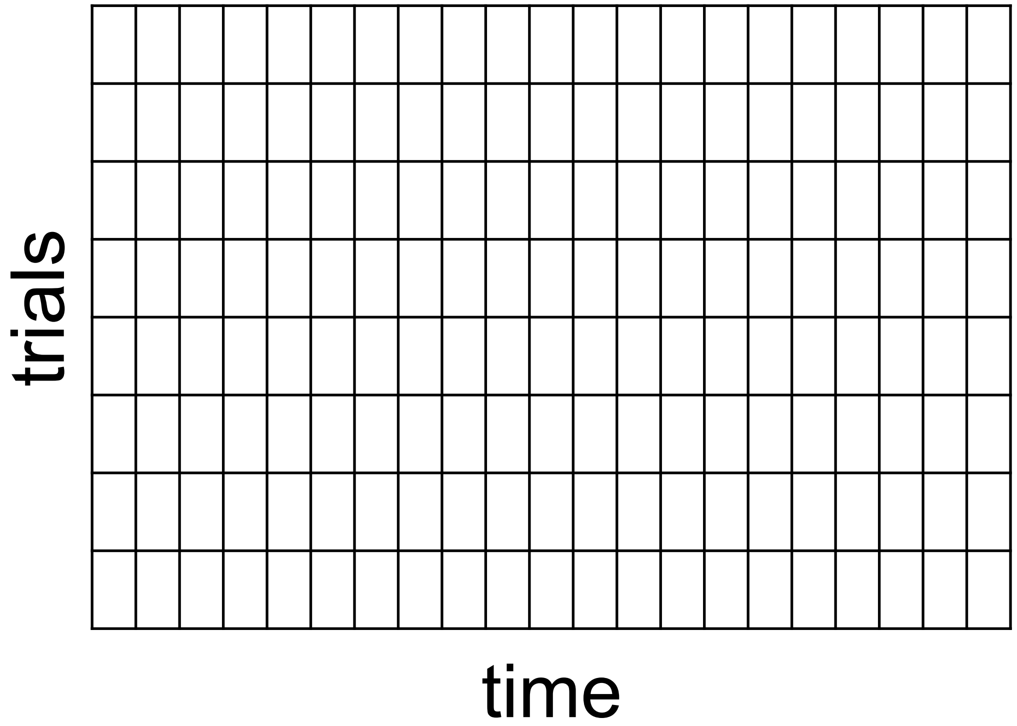
Is there a *constant phase relationship between x & y, at the same f, across trials?*

Coherence: idea

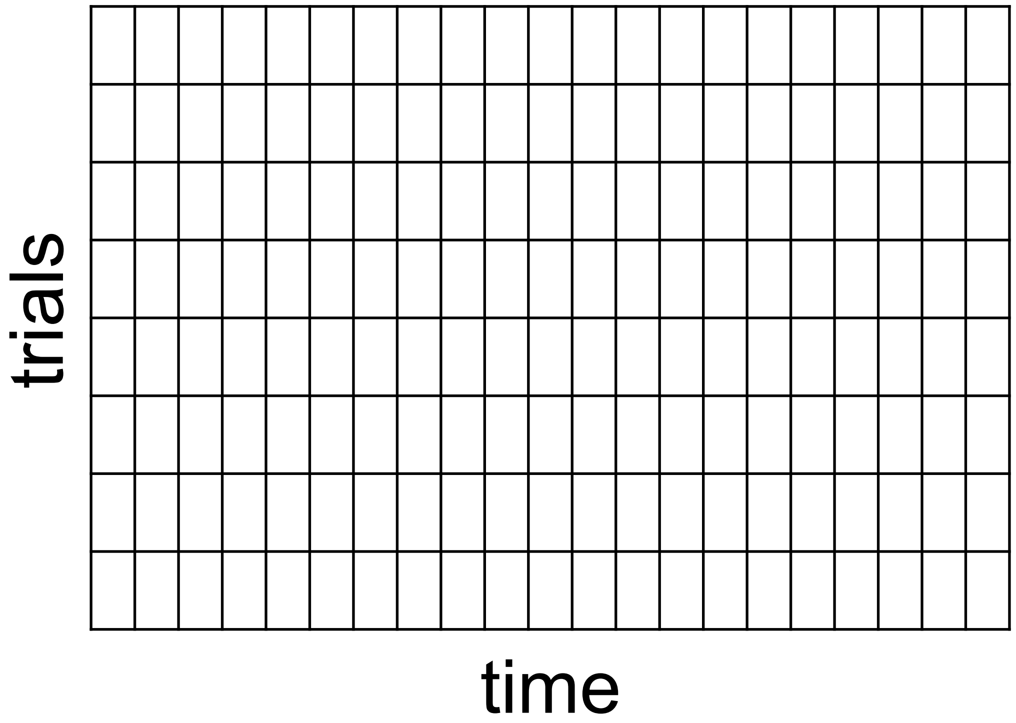
Ex: Record data simultaneously from two sensors, across multiple trials

Organize the data ...

x (field: EEG, LFP, ...)



y (field: EEG, LFP, ...)



Each row is a trial, each column is a time point, organize data in matrices.

Coherence: equations

This is what we'll compute:

$$\kappa_{xy,\,j} = \frac{|< S_{xy,\,j} >|}{\sqrt{< \boxed{S_{xx,\,j}} >}\sqrt{< S_{yy,\,j} >}}$$

$S_{xy,j}$ = Cross-spectrum at frequency index j

$S_{xx,j}, S_{yy,j}$ = Auto-spectra at frequency index j ✓

$< S >$ = Average of S over trials

Define each piece ...

Coherence: equations (reminder)

$$\kappa_{xy,\,j} = \frac{|< S_{xy,\,j} >|}{\sqrt{<\boxed{S_{xx,\,j}}>}\sqrt{< S_{yy,\,j} >}}$$

$$S_{xx,j} = \frac{2\Delta^2}{T}\boxed{X_j}X_j^*$$

(Auto-)spectrum of signal *x*

$$X_j = \sum_{n=1}^{N} x_n \exp(-2\pi i f_j t_n)\,.$$

Fourier transform of the data *x*.

Coherence: equations

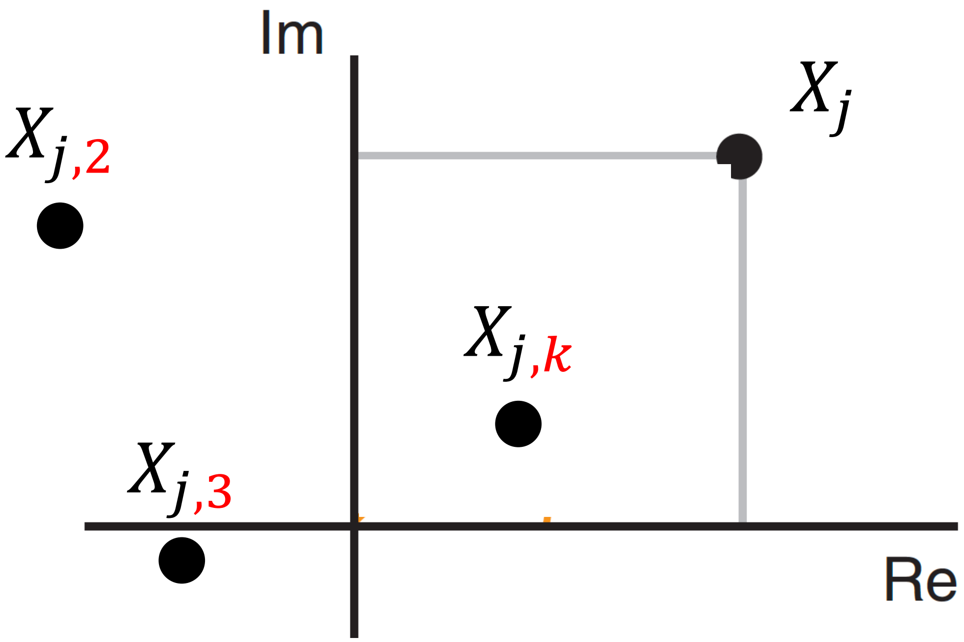
Remember, X_j lives in the complex-plane:

Fourier transform <u>for each trial</u> lives in the complex-plane:

In polar coordinates:

$A_{j,k}$ = Amplitude at frequency index j
and trial index k

$\phi_{j,k}$ = Phase at frequency index j
and trial index k



Coherence: equations

To compute coherence, we need the <u>trial-averaged</u> spectrum:

Single trial spectrum

Average squared amplitude over trials

$$< S_{xx,j} = \frac{2\Delta^2}{T} A_j^2 \cdot \sum_{k=1}^{K} A_{j,k}^2$$

$A_{j,k}$ = the amplitude of the signal x, at frequency index j, and trial index k .

K = total number of trials

Coherence: equations

Similarly, for signal y_n. Fourier transform of y at frequency j, and trial k:

$$Y_{j,k} = B_{j,k} \exp(i\,\theta_{j,k})$$

$B_{j,k}$ = the amplitude of the signal y at frequency index j and trial index k .

$\theta_{j,k}$ = the phase of the signal y at frequency index j and trial index k .

The trial-averaged spectrum of y at frequency index j

$$< S_{yy,j} > = \frac{2\Delta^2}{T} \frac{1}{K} \sum_{k=1}^{K} B_{j,k}^2$$

Coherence: equations

$$\kappa_{xy,\,j} = \frac{\left|\boxed{< S_{xy,\,j} >}\right|}{\sqrt{< S_{xx,\,j} >}\sqrt{< S_{yy,\,j} >}}$$

$\checkmark$ $\checkmark$

$$< S_{xx,j} > = \frac{2\Delta^2}{T}\frac{1}{K}\sum_{k=1}^{K} A_{j,k}^2 \qquad < S_{yy,j} > = \frac{2\Delta^2}{T}\frac{1}{K}\sum_{k=1}^{K} B_{j,k}^2$$

Consider the trial averaged cross-spectrum ...

Coherence: equations

The trial averaged cross-spectrum at frequency index j:

Like the auto-spectrum, but use X and Y.

$$< S_{xy,j} >= \frac{2\Delta^2}{T}\frac{1}{K}\sum_{k=1}^{K} X_{j,k} Y_{j,k}^*$$

In polar coordinates:

$$< S_{xy,j} >= \frac{2\Delta^2}{T}\frac{1}{K}\sum_{k=1}^{K} A_{j,k} B_{j,k} \exp\left(i\Phi_{j,k}\right)$$

Phase of x Phase of y

where $\Phi_{j,k} = \phi_{j,k} - \theta_{j,k}$ is the phase difference between the two signals, at frequency index j and trial k.

Coherence: equations

Put it all together ...

$$\kappa_{xy,\,j} = \frac{|< S_{xy,\,j} >|}{\sqrt{< S_{xx,\,j} >}\sqrt{< S_{yy,\,j} >}}$$

In polar coordinates

cross-spectrum of x & y,
depends on trial averaged
amplitudes, phase differences.

$$= \frac{\left| \sum_{k=1}^{K} A_{j,k} B_{j,k} \exp\left(i\Phi_{j,k}\right) \right|}{\sqrt{\sum_{k=1}^{K} A_{j,k}^2}\sqrt{\sum_{m=1}^{K} B_{j,m}^2}}$$

x trial averaged spectrum,
at frequency index j

y trial averaged spectrum,
at frequency index j

Coherence: intuition

To build intuition, assume: the amplitude is <u>identical</u> for both signals and all trials.

$$A_{j,k} = B_{j,k} = C_j$$

<u>Note</u>: no trial dependence

then

$$\kappa_{xy,j} = \frac{\left| \sum_{k=1}^{K} A_{j,k} B_{j,k} \exp\left(i\Phi_{j,k}\right) \right|}{\sqrt{\sum_{k=1}^{K} A_{j,k}^2} \sqrt{\sum_{m=1}^{K} B_{j,m}^2}}$$

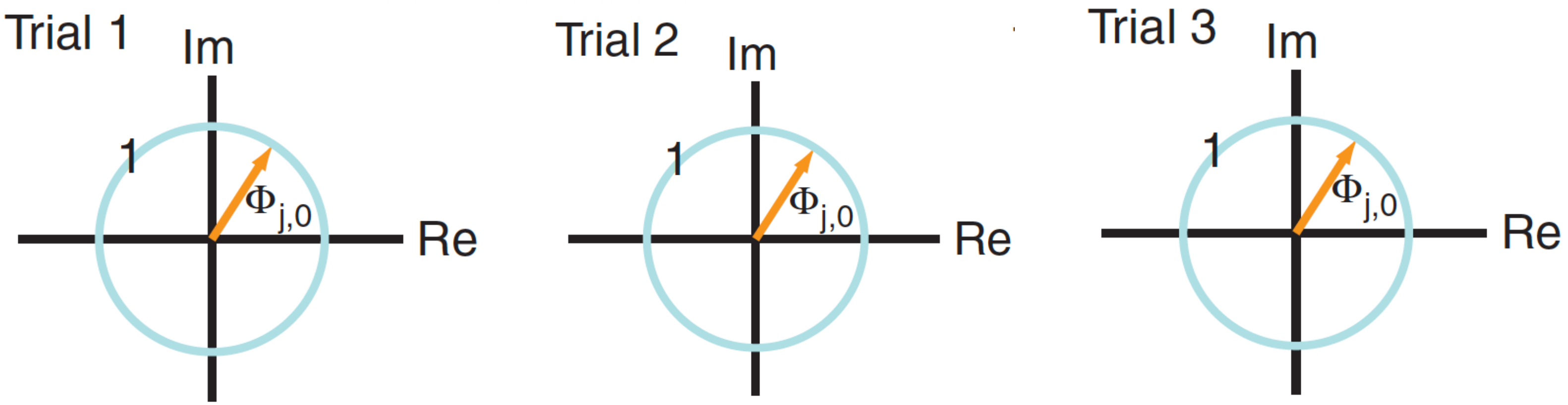
only involves the <u>phase difference</u> between the two signals averaged across trials.

Coherence: intuition

$$\kappa_{xy,j} = \frac{1}{K}\left|\sum_{k=1}^{K}\exp\left(i\Phi_{j,k}\right)\right|$$

Case 1: Phases align across trials. $\boxed{\Phi_{j,k} = \Phi_{j,0}}$

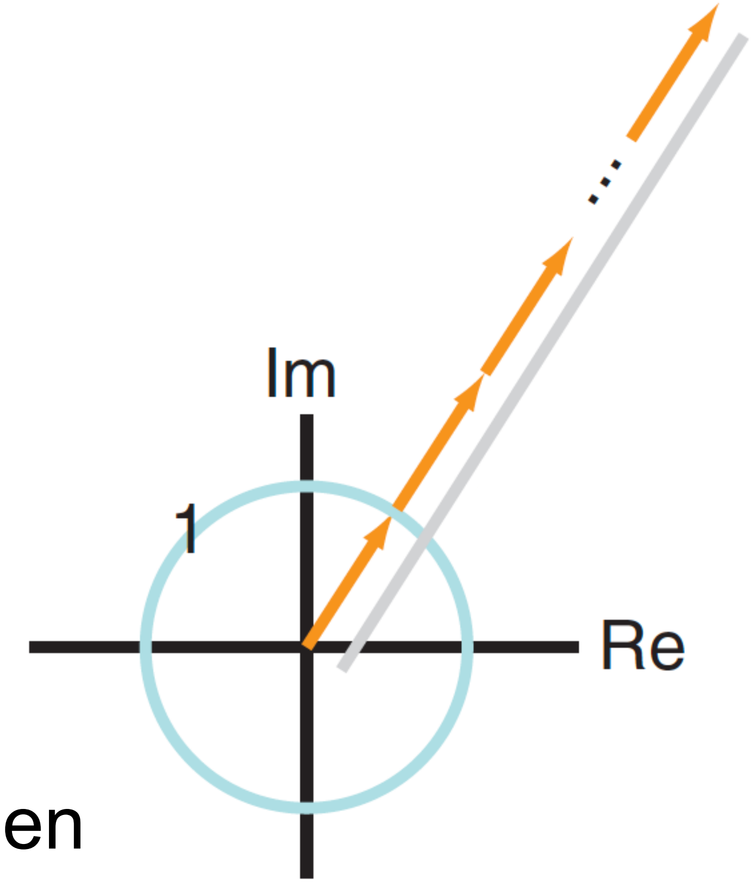
Plot $\exp\left(i\Phi_{j,k}\right)$ in the complex plane.



sum these vectors end to end across trials

divide by K

$$\kappa_{xy,j} \approx 1$$



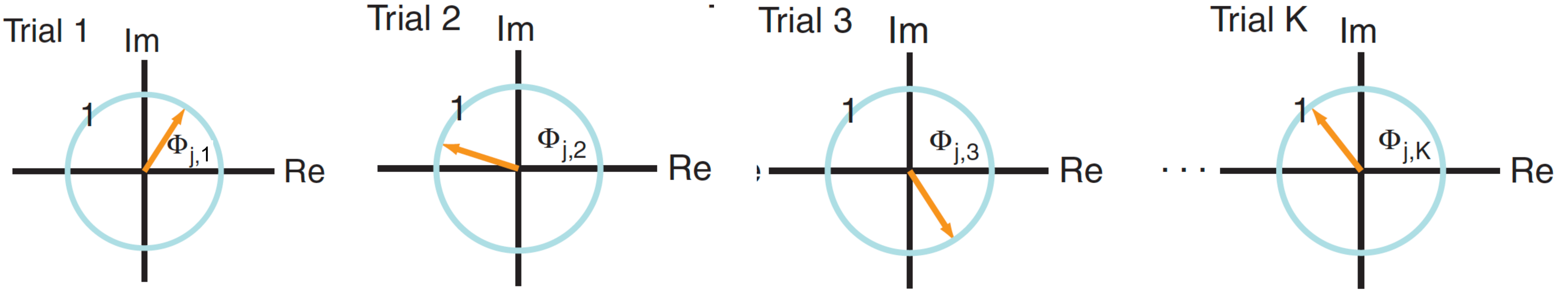
<u>strong coherence</u> - constant phase relation between the two signals across trials at frequency index j.

Coherence: intuition

$$\kappa_{xy,j} = \frac{1}{K}\left|\sum_{k=1}^{K}\exp\left(i\Phi_{j,k}\right)\right|$$

Case 2: Random phase differences across trials.

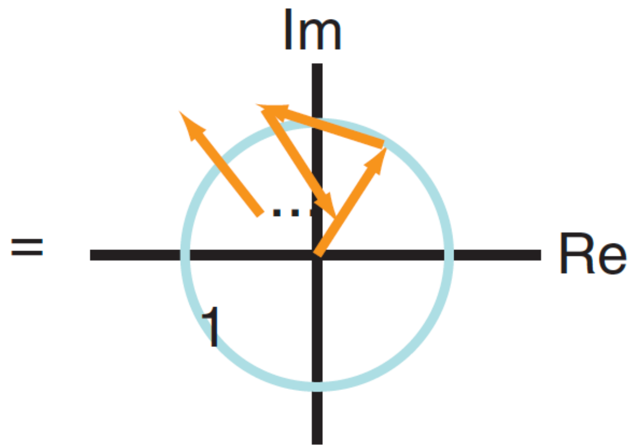
Plot $\exp\left(i\Phi_{j,k}\right)$ in the complex plane.



sum these vectors end to end across trials

divide by K

$$\kappa_{xy,j} \approx 0$$



<u>weak coherence</u> - random phase relation between the two signals across trials at frequency index j.

Coherence: summary

$$0 \leq \kappa_{xy,j} \leq 1$$

0: no coherence between signals x and y at frequency index j

1: strong coherence between signals x and y at frequency index j .

The coherence is a measure of the phase consistency between two signals at frequency index j across trials.

Coherence: example

https://github.com/Mark-Kramer/NESS-Short-Course-2023

NESS-Short-Course-2023

This repository contains code discussed in the short course *Introduction to the analysis of neural electrophysiology data* at NESS 2023.

Lecture slides (Part 1)

Notebook	Run It
Spectrum	Open in Colab
Coherence	Open in Colab
Spike-field coherence	Open in Colab

Outline

Part 1 (Prof. Kramer)

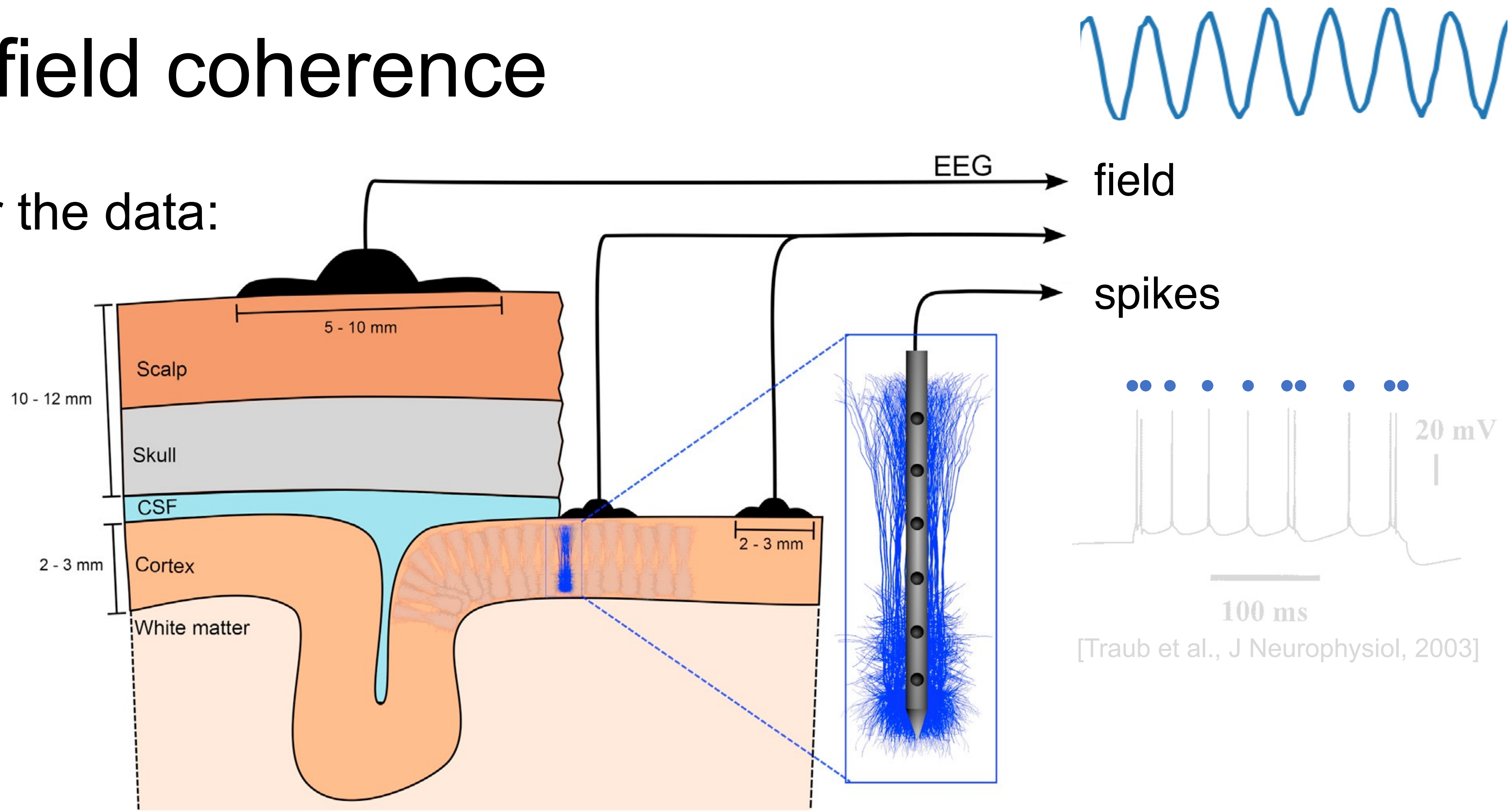
- Rhythms & spectrum
- Coherence
- $\boxed{\text{Spike-field coherence}}$ ... (a riddle)

Part 2 (Prof. Eden)

- Point processes
- Latent state models

Spike-field coherence

Consider the data:

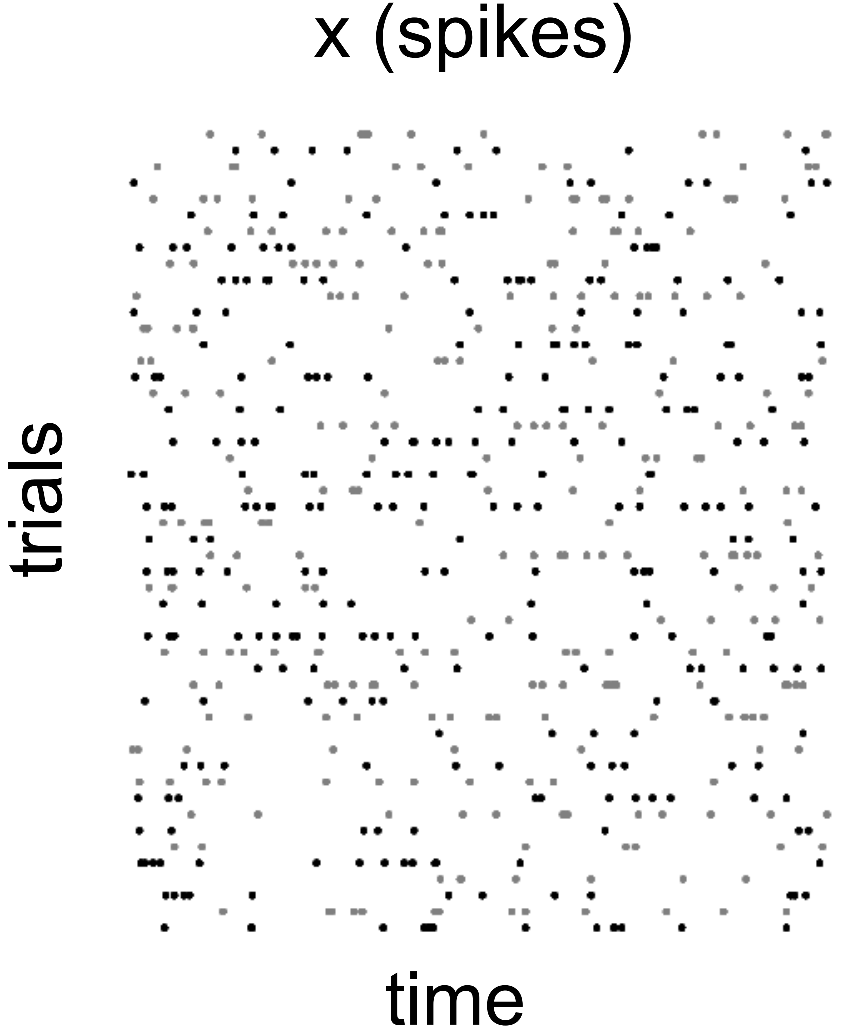


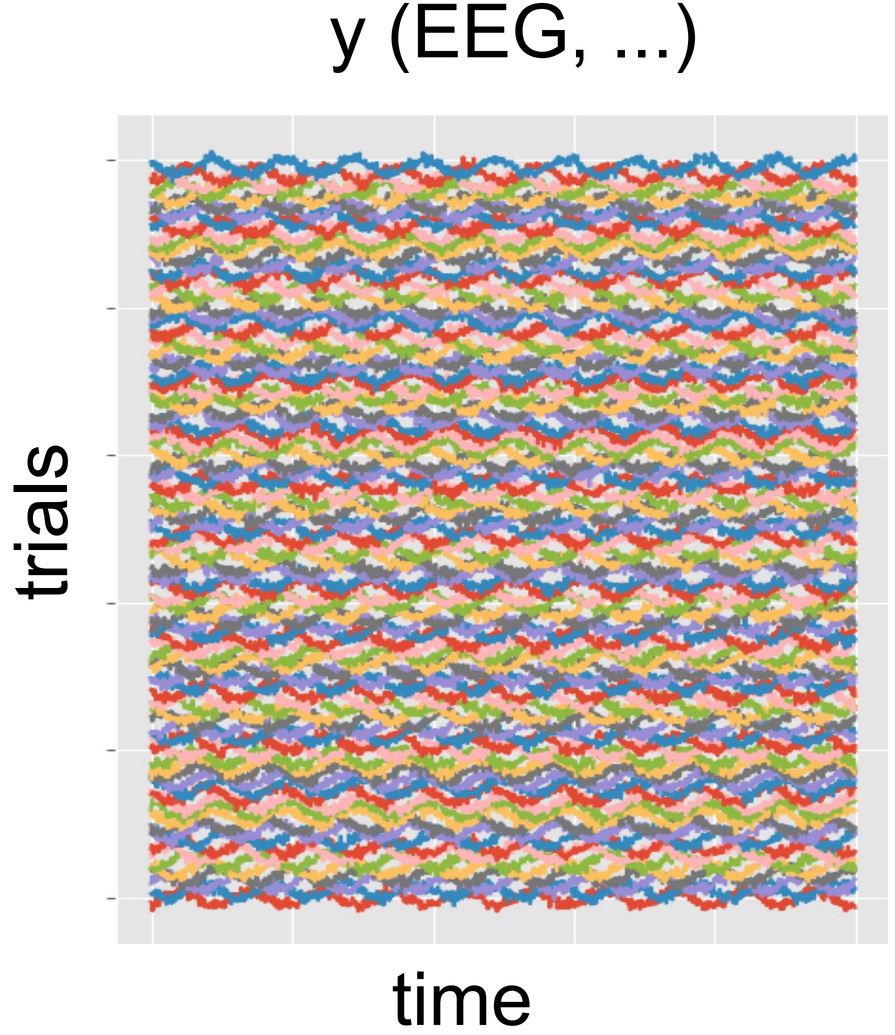
[Einevoll et al, Neuron, 2019]

Q: Is there consistent neural spiking at a specific phase of the field rhythms?

Spike-field coherence

Consider the data:





Q: Is there consistent neural spiking at a specific phase of the field rhythms?

Spike-field coherence

trial averaged cross spectrum

$$\kappa_{ny,\,j} = \frac{|< S_{ny,\,j} >|}{\sqrt{< S_{nn,\,j} >}\sqrt{< S_{yy,\,j} >}}$$

trial averaged spike spectrum

trial averaged field spectrum

n = spike train (e.g., [0 0 0 0 0 0 1 0 0 0 1 0 0 0 0 ...])

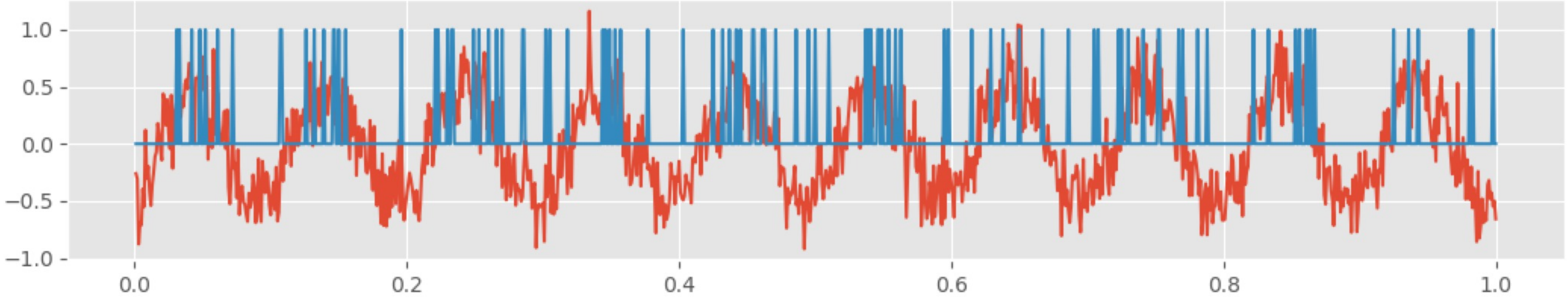
y = field signal (e.g., EEG, ...) 

Same equations ... but new problems

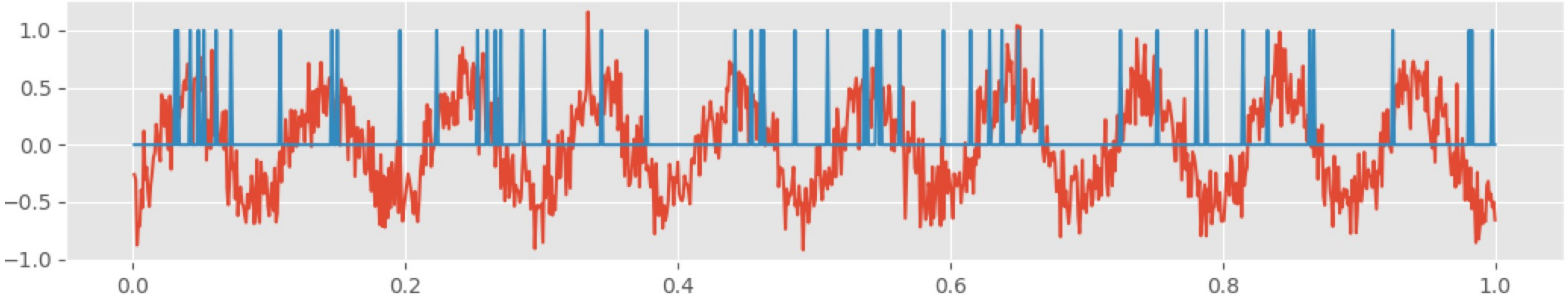
Spike-field coherence: dependence on rate

Q: Does the spike-field coherence depend on the firing rate of the neuron?

Original spike & field data for 1 trial



"Thinned" spike & field (remove 50% of spikes, chosen at random)



Spike-field coherence: dependence on rate

Q: Does the spike-field coherence depend on the firing rate of the neuron?

Here, rate: expected number of spikes in a given duration

Try it ... https://github.com/Mark-Kramer/NESS-Short-Course-2023

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Lecture slides (Part 1)

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Spike-field coherence: dependence on rate

Q: Does the spike-field coherence depend on the <u>firing rate</u> of the neuron?

<u>Observations:</u>

greater thinning → fewer spikes → lower coherence

as the rate tends to 0, so does the spike-field coherence

The spike-field coherence reflects

(1) the relationship between spiking activity and the phase of field, and
(2) the <u>mean firing rate</u>.

[Lepage et al., Neural Comp, 2011]

Spike-field coherence: dependence on rate

Q: So what next?

Q: If, in your experiment, the overall spike rate differs between two neurons, then how do you compare the spike-field coherence?

- include a rate adjustment factor in the coherence measure to account for rate dependence.
- build a generalized linear model to separate overall neural activity from spike train-LFP oscillatory coupling.

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Computational Neuroscience

Rate-adjusted spike–LFP coherence comparisons from spike-train statistics

Mikio C. Aoi*, Kyle Q. Lepage, Mark A. Kramer, Uri T. Eden

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Journal of Neuroscience Methods 213 (2013) 43–62

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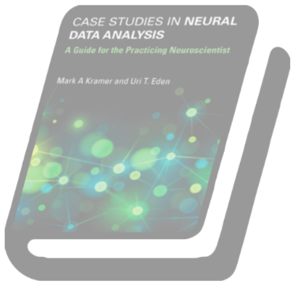
Computational Neuroscience

A procedure for testing across-condition rhythmic spike-field association change

Kyle Q. Lepage[a,*], Georgia G. Gregoriou[b,c], Mark A. Kramer[a], Mikio Aoi[a], Stephen J. Gotts[d], Uri T. Eden[a], Robert Desimone[e]

Additional resources

https://mark-kramer.github.io/Case-Studies-Python/intro.html



Case Studies in Neural Data Analysis

Search this book...

Case-Studies-Python

This repository is a companion to the textbook Case Studies in Neural Data Analysis, by Mark Kramer and Uri Eden. That textbook uses MATLAB to analyze examples of neuronal data. The material here is similar, except that we use Python.

The intended audience is the *practicing neuroscientist* - e.g., the students, researchers, and clinicians collecting neuronal data in the hospital or lab. The material can get pretty math-heavy, but we've tried to outline the main concepts as directly as possible, with hands-on implementations of all concepts. We focus on only two main types of data: spike trains and electric fields (such as the local field potential [LFP], or electroencephalogram [EEG]). If you're interested in other data (e.g., calcium imaging, or BOLD), you may still find the examples indirectly useful (for example, demonstrations of how to compute and interpret a power spectrum of a signal).

This repository was created by Emily Schlafly and Mark Kramer, with important contributions from Dr. Anthea Cheung.

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- MIT Press for publishing the MATLAB version of this material.
- NIH NIGMS R25GM114827 and NSF DMS #1451384 for support.

Quick start to learning Python for neural data analysis:

- Visit the web-formatted version of the book.
- Watch this 2 minute video.
- Read and interact with the Python code in your web browser.

On this page

Up next:

Prof. Uri Eden