Problem Set 5

Instructions: You are encouraged to work together on these problems, but each student should hand in their own final draft, written in a way that indicates their individual understanding of the solutions. Never submit something for grading that you do not completely understand. You cannot use any resources besides me, your classmates, our course notes, and the textbook.

I will post the .tex code for these problems for you to use if you wish to type your homework. If you prefer not to type, please *write neatly.* As a matter of good proof writing style, please use complete sentences and correct grammar. You may use any result stated or proven in class or in a homework problem, provided you reference it appropriately by either stating the result or stating its name (e.g. the definition of ring or Lagrange's Theorem). Do not refer to theorems by their number in the course notes or textbook.

Problem 1. Let R be a commutative ring. Show that R is noetherian if and only if every ideal of R is a finitely generated R-module.

Problem 2. Let R be a commutative ring and I an ideal of R. Show that if R is noetherian then R/I is also noetherian.

Problem 3. Let R be a commutative ring with $1 \neq 0$. Show that

$$\operatorname{ann}_R(M \oplus N) = \operatorname{ann}_R(M) \cap \operatorname{ann}_R(N)$$

Problem 4. Consider the matrix

$$A = \begin{bmatrix} 1 & 6 & 5 & 2 \\ 2 & 1 & -1 & 0 \\ 3 & 0 & 3 & 0 \end{bmatrix} \in \mathrm{M}_{3,4}(\mathbb{Z}).$$

Determine the simplest representative in the isomorphism class of the $\mathbb{Z}$-module presented by A.