

Problem Set 1

Problem 1. Consider the category $R\text{-mod}$.

- a) Show that a homomorphism of R -modules is injective if and only if it is a mono in $R\text{-mod}$, and surjective if and only if it is an epi in $R\text{-mod}$.
- b) Show that the homomorphism of \mathbb{Z} -modules $\mathbb{Z} \xrightarrow{2} \mathbb{Z}$ is monic but has no left inverse in $\mathbb{Z}\text{-mod}$.
- c) Show that the canonical homomorphism of \mathbb{Z} -modules $\mathbb{Z} \twoheadrightarrow \mathbb{Z}/2\mathbb{Z}$ is epic but has no right inverse in $\mathbb{Z}\text{-mod}$.

Problem 2.

- a) Show that in any category, every isomorphism is both an epi and a mono.
- b) Show that the usual inclusion $\mathbb{Z} \hookrightarrow \mathbb{Q}$ is an epi in the category **Ring**.
This *should* feel weird: it says being epi and being surjective are *not* the same thing.
- c) Show that the canonical projection $\mathbb{Q} \twoheadrightarrow \mathbb{Q}/\mathbb{Z}$ is a mono in the category of divisible abelian groups.¹

Again, this is very strange: it says being monic and being injective are *not* the same thing.

¹An abelian group A is divisible if for every $a \in A$ and every positive integer n there exists $b \in A$ such that $nb = a$.