CHAMP 1/12/21 Perturbing Ideak in Arbitrary Noetherian Local Rings and two meadic Continuity of HK-multiplicity Introduction Hilbert - Kunz Multiplicity Frobenius F. R. P. R. F(x) x (x+y)=x+y) (R,mR) - Noetherian local ring Char 7>0 (for e20) Fe : R - R , Fe(x) = x ? (F°(x)-x) Given ISR, $\mathbf{I}^{(p^c)} := \mathbf{F}^e(\mathbf{I}) \, \mathcal{R} = (\mathbf{x}^{p^c} \, | \, \mathbf{x} \in \mathbf{I})$ - (+,, ..., +c) where (f,...,f,) = I Ris F-finite if R FR is module finite. Frobenius (-{P-1=(+-1)})

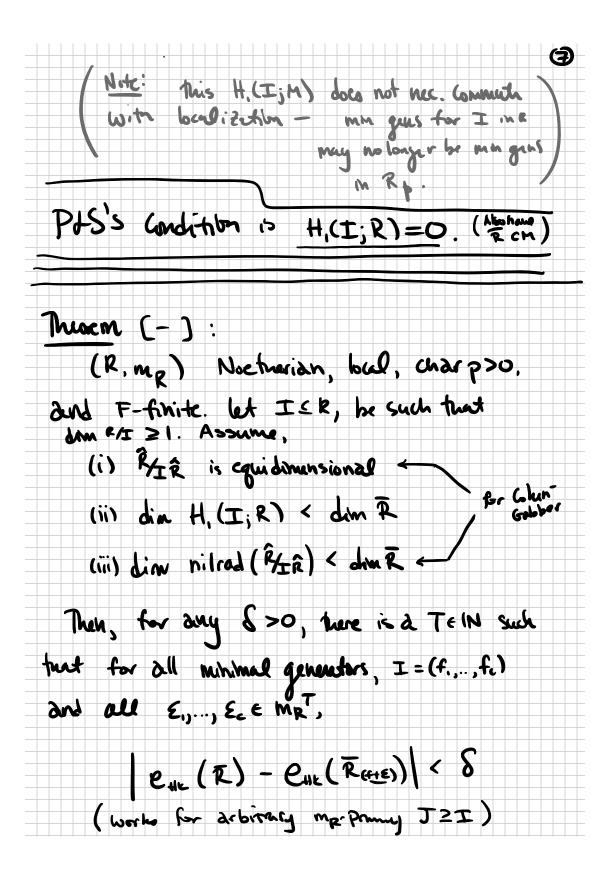
(i) ("Kunz's Thu	Noetherian, local, dim d. Charpon orem") far => F: R—R is flat
he = length	
May the value of	S to low to Regular
The limit dim CHK (R) :=	R) Noctresian, local, char pro, d. = lim pea \ (R/mcpeg) +ilbut-kunz multiplicity dR.

Sh.	eur (J, W) := line permin) / ([] ([] [] [] [] [] [] [] []
	(x)
(o+	s of interesting properties & en
_	Addithum S.e.S.
-	Associativity formula
-	Same de Hilbert - Samul mutt.
	if $dim(R) = 1$, or if $J = (x_1,, x_d)$
	is generated by a S.O.P.
	If R is unmixed, equidm.
	enc(R) = 1 (R is regular
	- ene(R) Can be imational
	ov tankendental.
An	of so hice property of HK-mult. 15
	is extremly hard to compute

[Notation] (R, mp) Noctor and book. If $I \subseteq R$ is generated by $f_1, ..., f_c$, will write $I = (f_1, ..., f_c) = (f_c)$. me-adic Peuturbantons: leals, form $(f,+\epsilon,\dots,f_c+\epsilon_c) = (f+\epsilon)$ Where I = (f,,..., f,) are minimal gens, and E,,.., E, € mg (T≥1), are mp-adic perturbations of I (parturbodism is "small" if T is "large") For M an R-module, and we are perturbing some fixed I, write M:= M, M(GLE):= (PLE)M where (f+E) is a pecturbation of I. (may refer to More, as "perturbations"

Therem (Polsta 1 Smi mov 18)	(5)
(R, mg) d-dimensional, F-fmile, (F)	
local ring of Mar p>0. Let I = (f,,,fc) be an idulin R generated by c>6 parameters.	
Assume P/I2 is reduced.	
Then, for any $\delta > 0$, there is a $T \in \mathbb{N}$ Such that for all $E_1, \dots, E_c \in \mathbb{M}_R^T$,	
eux (R) - enx (R(+15)) < 8	
Takind of me-adic continuity	
- T does not depend on the charle g	
- Thès works for JZI, mp-pomon)

Goal - drop the CM condition In the PHS result, Ris CM and I is generated by part of an S.O.P. => I gen'd by reg. 8eg. + R/I = R ... It is not hard to Show Some undition on I is necessary... being a parameter idul isn't enough butside the CM case (Example 5.2) Notation: Mafig. R-module, IER, H, (I; M) = first Kaszul Hounday) := H,((£),M) where (f,,...,fe) are min. gens. For I (in R) Fact: (R, mg) local of Noch =) different Choice of min. gent. give i=0 marphic H,(I;m)'s.

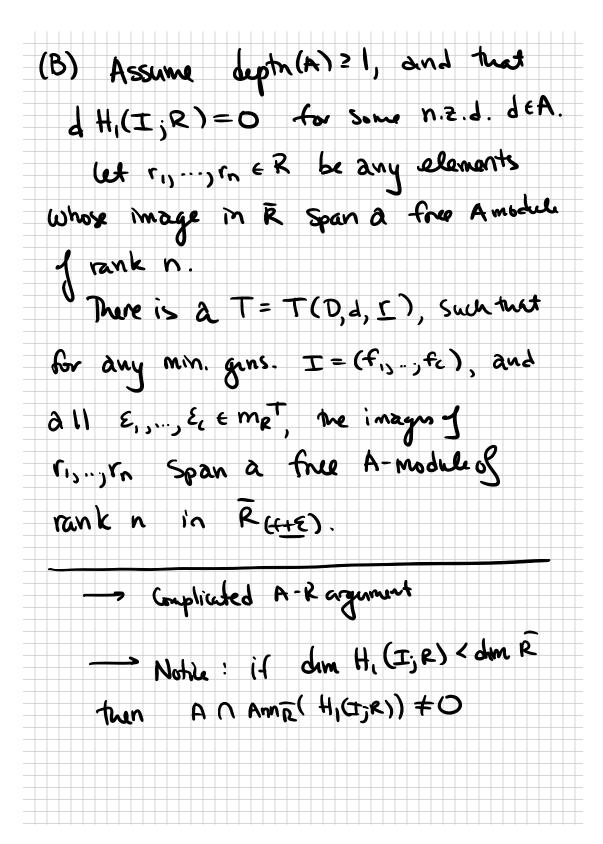


Corollary (-): Suppose	the conditions of the theorem (2) A R is reduced and complete.	Ď
are surs fiel, and the	A R is reduct and complete.	
let (f,, fc)=I be n	nin gens. Then,	
1im Euc (R[f,,	f= (R/I)	
1 Proof)		
The proof of P	its's theorem begins by and envoking Cohen-Coubber	
reducing to $R = \hat{R}$,	, and envoking when-babber	
	the diagram of local rings:	
<u>D:</u>	R A an RIR, with A com R fride tymerically the	
	I frate to generically & the	4
A finise	(Reduced Car)	
S in their setup,		
P = 1/reg. &	y. is CM, so it is	
free as an A-	module They show,	
for T>>0,	(f+E) are also parameters.	
Go the R(498)	(f+E) are also parameters, are also CM and two	
free over A.		

This allows them to show the discriminants of R and Rome) are ma-adically cox m A (for T>>0). MM> Gives unitarm Control and Convergence rates of the ene(Rose)). Disciminant: A, Slocal, Noeth. with A a domain A -> S module finite, and let $s_1, \dots, s_n = s \in s$, be elements that map to a QF(A) - basis for SOAQF(A) = QF(A) The discriminant of Sour A 15, DA(S) := det (trace(S; S;));;

- if A is normal Da(s). A
- if $S'_{1,}$, $S'_{n} = \underline{S'} \in S$ gives another
ban's for Son GF(A), then the
discriminants differ by a unit in A.
DA(S) +0 (A C>S is generically étale
and if no s one F-fruite, and A is an PLR,
Da(s) can be used to explicitly (noted) for invergence rate of line 1 hs (Singles)
The distribution of the part o
Plan: relate two Da (R) and D(R(42))
for E's E MRT, TOO
A COR
AC-SK
we can find ri,, on c R that map to a QFCA)-bask
AC-SK

This ghe Dack) + Dack (+20) MA- whically lose. Suppose (R, ME) and (A, MA) are Complete Nochorian local rings, that ISR is an ideal and there is a commuting diagram of local ings (nok: I+MAR
i's MR-Prinary) (A) Let TEIN be such that mr = mr (I+mnr) let ris..., rm eR be elements which map to a minimal geneating set for R as an A-module. Then, for any (f.,.., fe) = I, and any Ei,..., Ec & mR, the images of Tis..., rm are minimal generators for P(++E) de an A-module (R = RCHE))



Along the way ... get a result about Hilbert - Samuel multiplicity: Thrown (-): (R, mR) equicharacteristic Noetherian local ring. I = R con i dul Such that din \$\overline{R} \ge 1. let \$1 be a finite R-module. Assume dim H, (I; M) < dim R. Then, there is a TEIN, such that for all minimal gens (f.,..,fc) = I, and all E, ..., Ecempt, (e(m, R) = e(m(+1E), R(+1E)) (ATTO WOTHS for day mx-pomey J = I) Ex: $f = xy \in \mathcal{K}(x,y,z)$, $4 \in \mathcal{K}(z,y,z)$, $4 \in \mathcal{K}(z,y,z)$, $2 \in \mathcal{K}(z,y,z)$, 2

Thank You!

Archive: M-adic Perturbations in Nocharian Local Rings

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