Problem Set 1

Problem 1. Consider the category R-**mod**.

a) Show that a homomorphism of R-modules is injective if and only if it is a mono in R-**mod**, and surjective if and only if it is an epi in R-**mod**.

b) Show that the homomorphism of $\mathbb{Z}$-modules $\mathbb{Z} \xrightarrow{2} \mathbb{Z}$ is monic but has no left inverse in $\mathbb{Z}$-**mod**.

c) Show that the canonical homomorphism of $\mathbb{Z}$-modules $\mathbb{Z} \twoheadrightarrow \mathbb{Z}/2\mathbb{Z}$ is epic but has no right inverse in $\mathbb{Z}$-**mod**.

Problem 2.

a) Show that in any category, every isomorphism is both an epi and a mono.

b) Show that the usual inclusion $\mathbb{Z} \hookrightarrow \mathbb{Q}$ is an epi in the category **Ring**.

This *should* feel weird: it says being epi and being surjective are *not* the same thing.

c) Show that the canonical projection $\mathbb{Q} \longrightarrow \mathbb{Q}/\mathbb{Z}$ is a mono in the category of divisible abelian groups.[1]

Again, this is very strange: it says being monic and being injective are *not* the same thing.

[1] An abelian group A is divisible if for every $a \in A$ and every positive integer n there exists $b \in A$ such that $nb = a$.