Critical points
First derivative test
Second derivative test

The point p is a critical point if…

The point p is a critical point if…

- $f'(p) = 0,$

OR

- $f'(p)$ does not exist

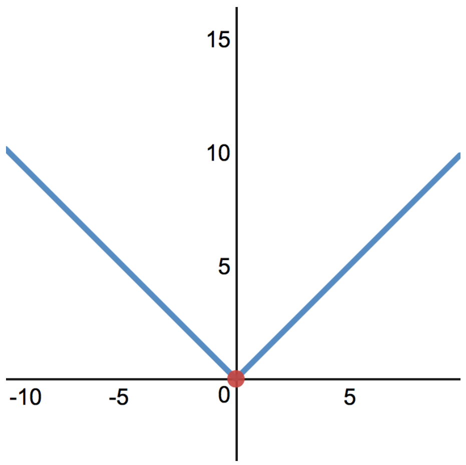
The point p is a critical point if…

- $f'(p) = 0$,

OR

- $f'(p)$ does not exist

Example: $f(x) = |x|$
0 is a critical point because $f'(0)$ DNE



A critical point might be…

A critical point might be…

- A local minimum
- A local maximum
- An inflection point
- Nothing!

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- A local maximum
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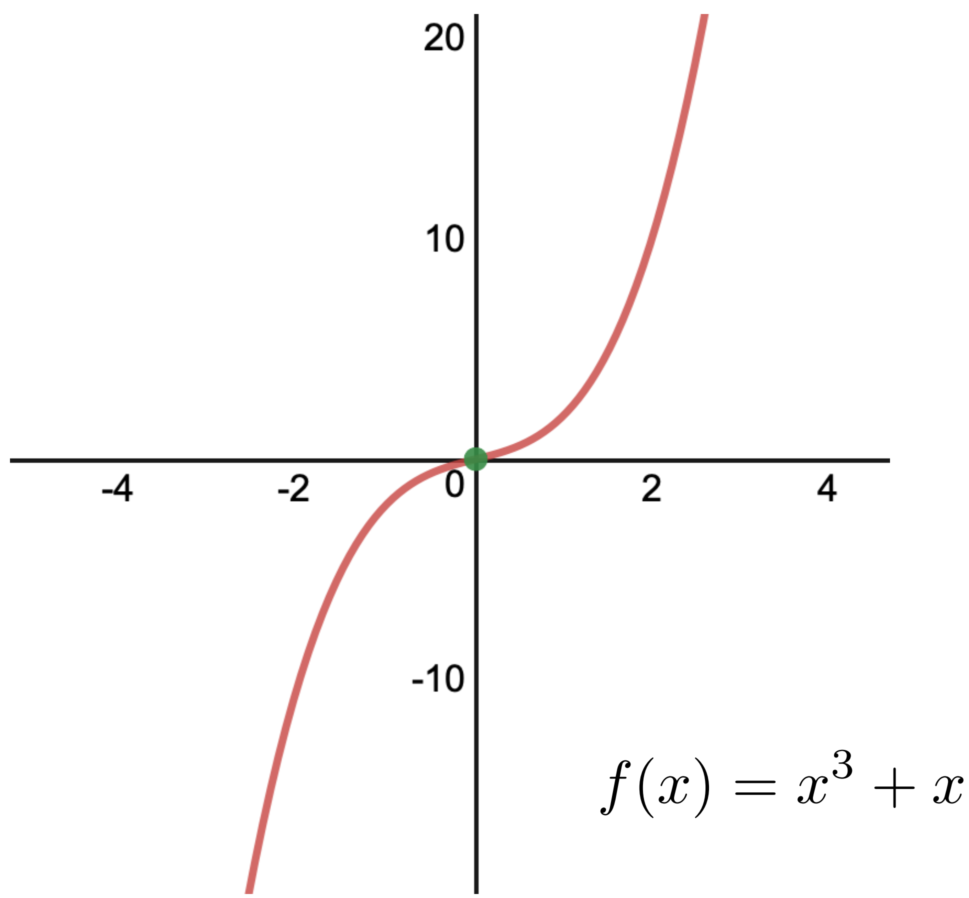
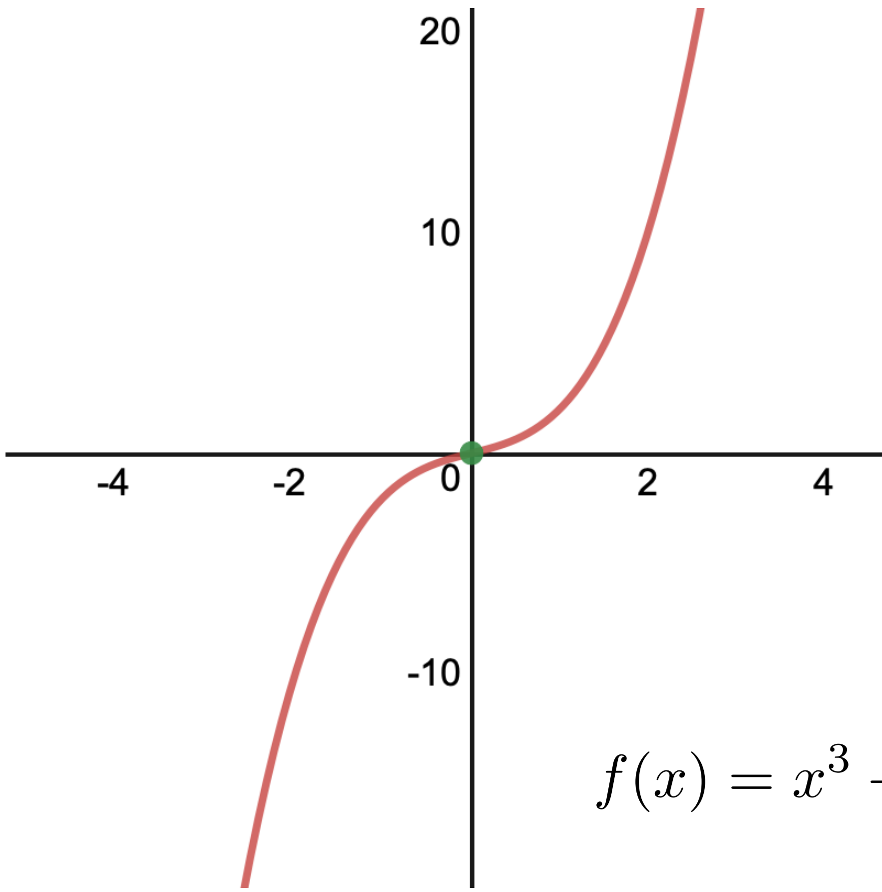
An inflection point is a point where the function changes concavity



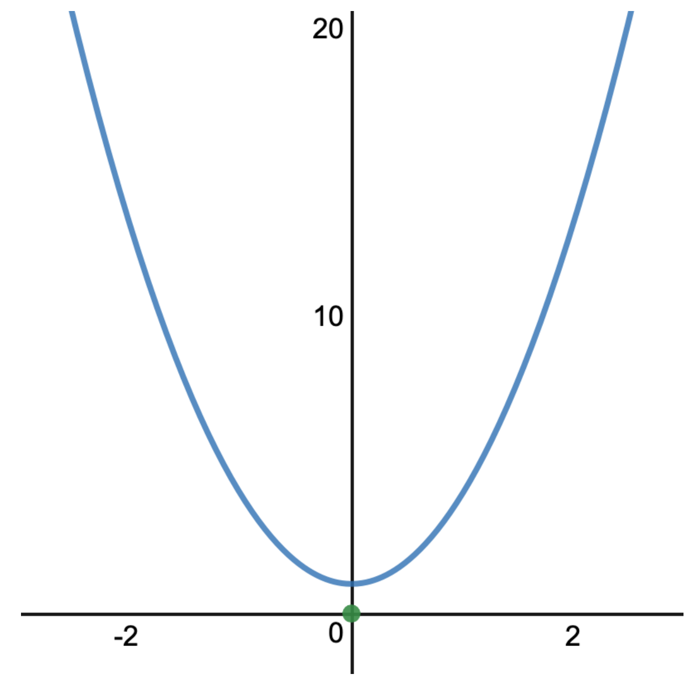
0 is an inflection point

Warning!

While critical points might be inflection points, inflection points might not be critical points!



$$f(x) = x^3 + x$$



$$f'(x) = 3x^2 + 1$$

0 is an inflection point, but not a critical point

The first derivative test

When do we apply it?

The first derivative test

When do we apply it?

If f is continuous

and

f has a critical point at p

The First-Derivative Test for Local Maxima and Minima

Suppose p is a critical point of a continuous function f. Moving from left to right:

- If f' changes from negative to positive at p, then f has a local minimum at p.
- If f' changes from positive to negative at p, then f has a local maximum at p.

If p is on the interval (a,b),

- f is differentiable on (a,b)

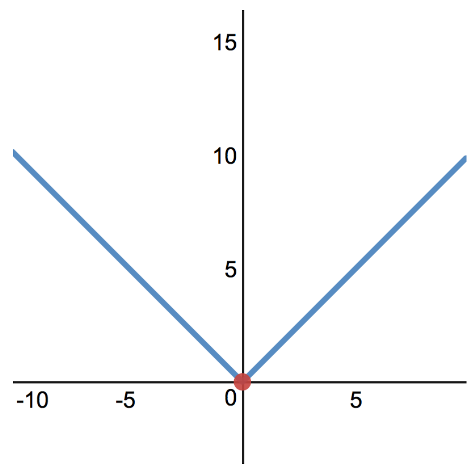
and

- p is a local maximum or a local minimum for

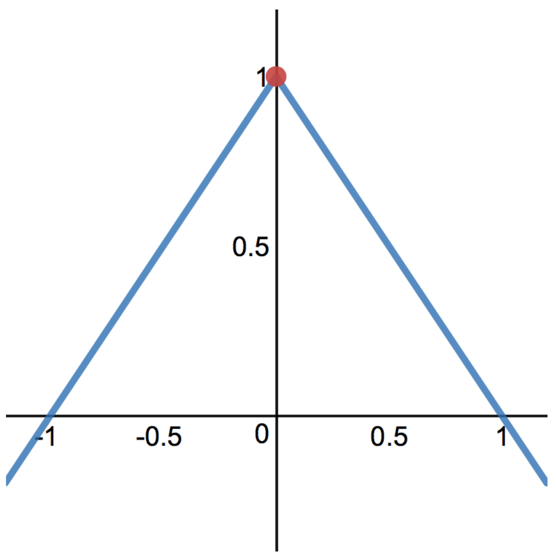
then $f'(p) = 0$

If the derivative is undefined, our point can be:

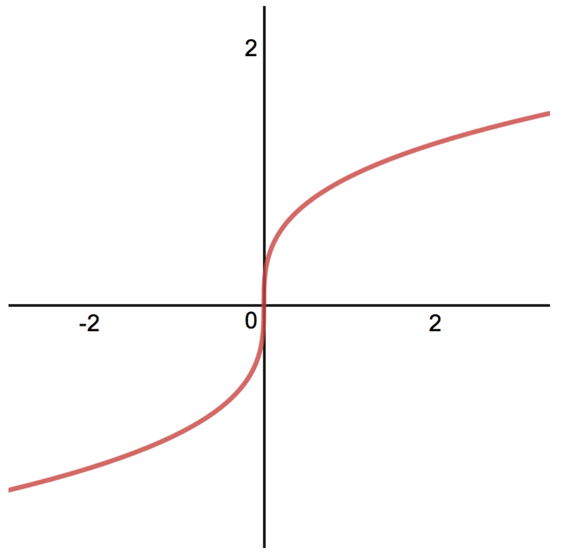
- A local minimum



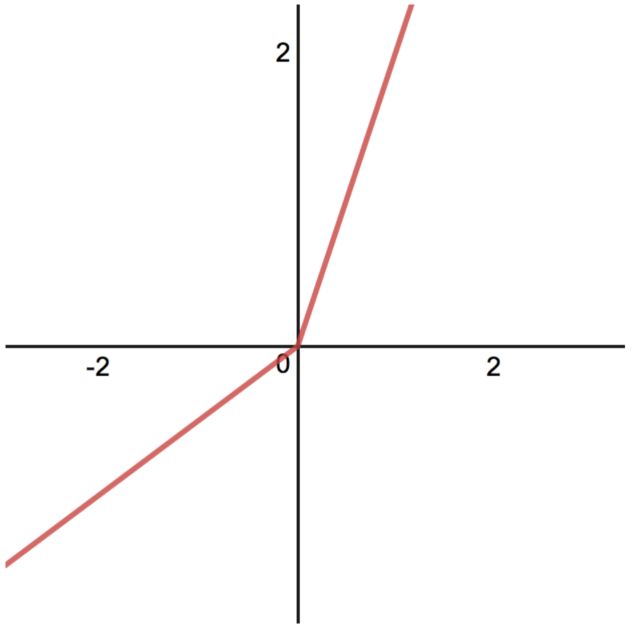
- A local maximum



- An inflection point

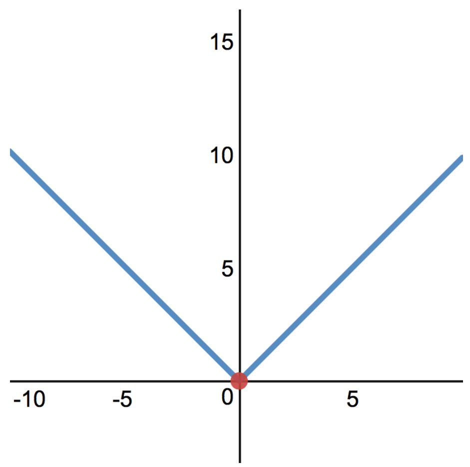


- Nothing!

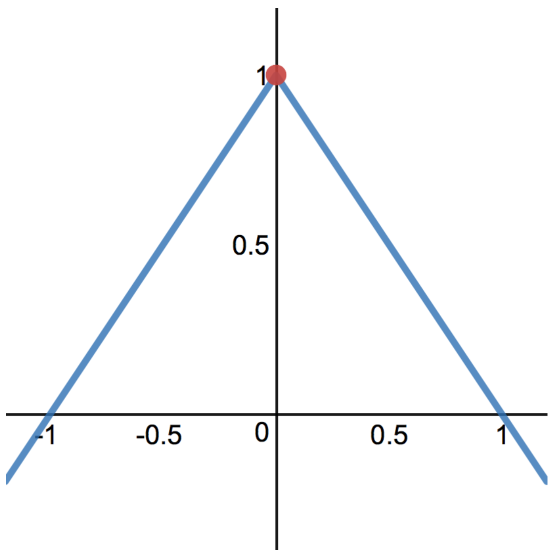


If the derivative is undefined, our point can be:

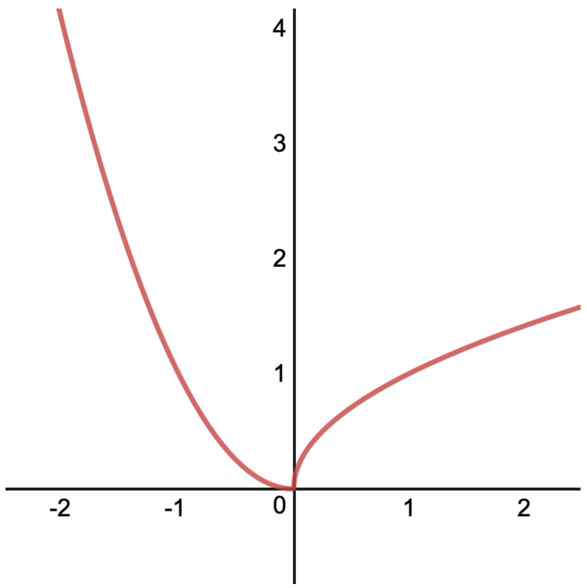
- A local minimum



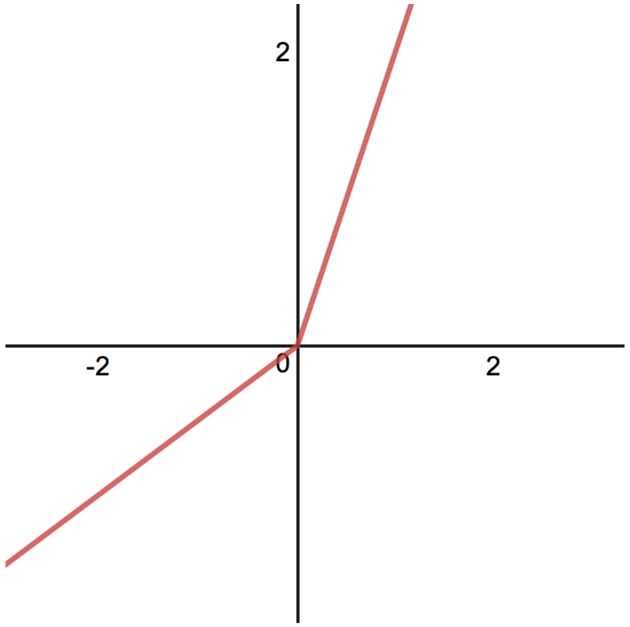
- A local maximum



- An inflection point

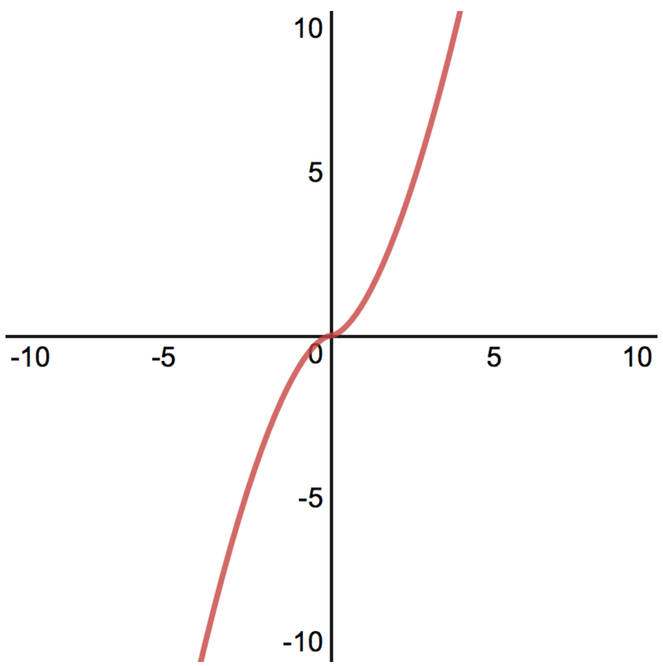


- Nothing!

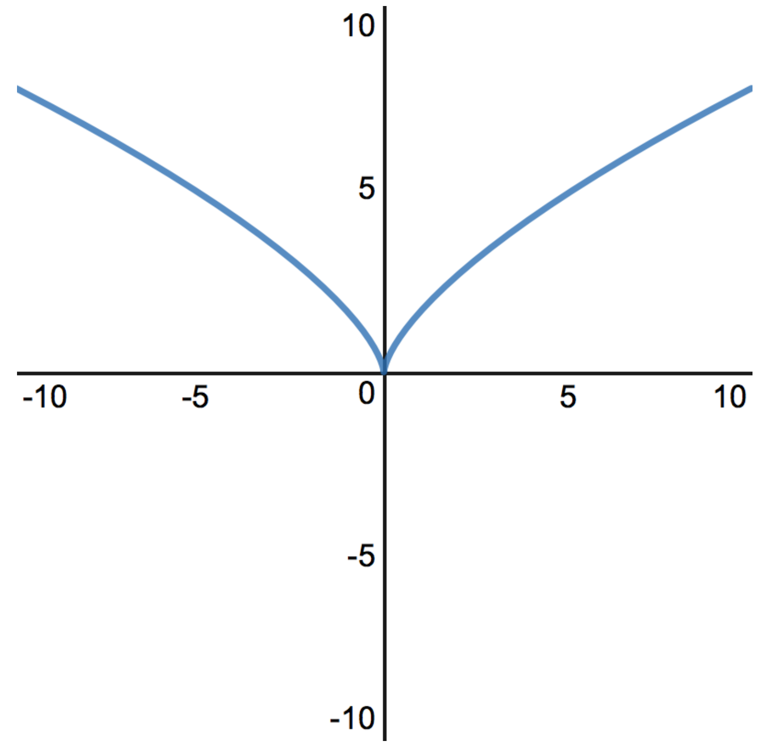


Yet another example of

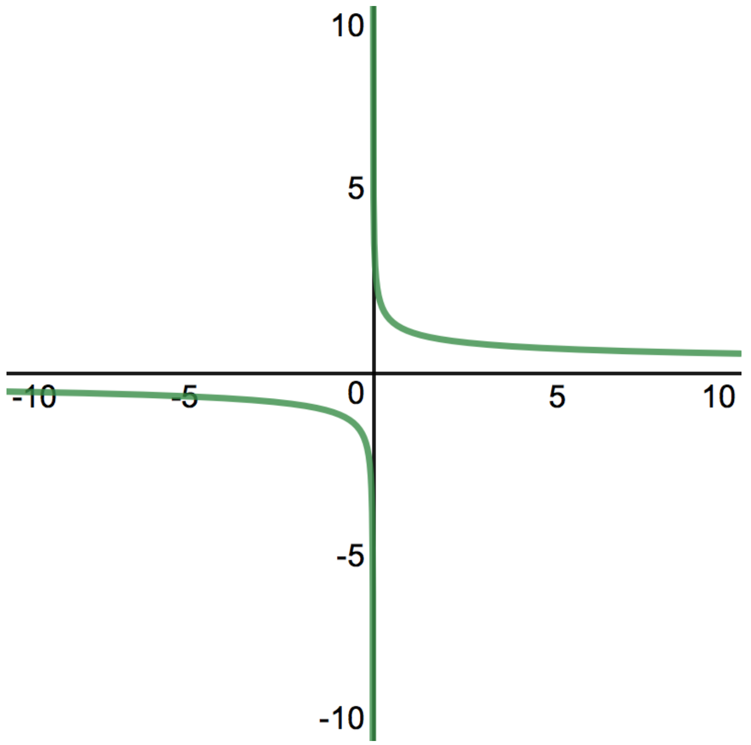
an inflection point where the second derivative is undefined



$$f(x) = x^{5/3}$$



$$f'(x) = \frac{5}{3}x^{2/3}$$



$$f'(x) = \frac{10}{9}x^{-1/3}$$

How would we check that 0 is an inflection point?

We would make a table for the second derivative, identifying all the intervals where $f'' > 0$ **and** $f'' < 0$ **(separated by points x where** $f''(x) = 0$ **or** $f''(x)$ **dne)**

The Second-Derivative Test for Local Maxima and Minima

- If $f'(p) = 0$ and $f''(p) > 0$ then f has a local minimum at p.
- If $f'(p) = 0$ and $f''(p) < 0$ then f has a local maximum at p.
- If $f'(p) = 0$ and $f''(p) = 0$ then the test tells us nothing.

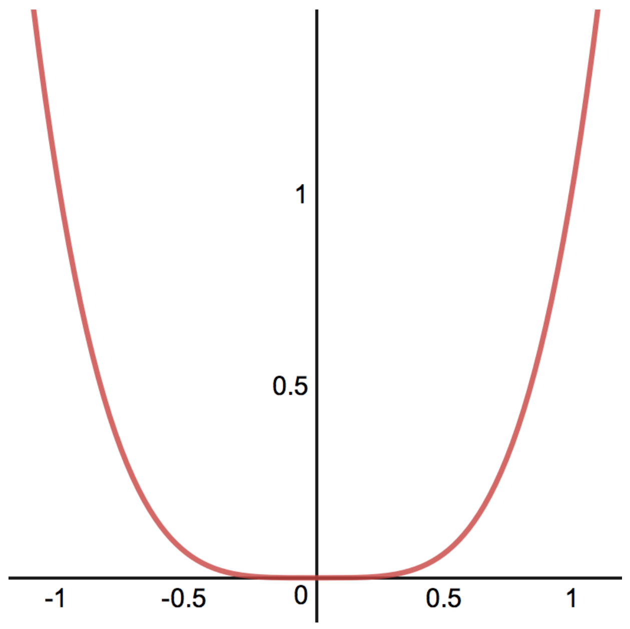
If $f'(p) = f''(p) = 0$

the Second Derivative test is useless!

Example: $f(x) = x^4$

$x = 0$ is local minimum

Cannot apply the
second derivative test



But we can apply the first derivative test!