

## Problem Set 4

To solve these problems, you are not allowed to use any additional Macaulay2 packages besides the `Complexes` package and the ones that are automatically loaded with Macaulay2.

**Problem 1.** Let  $k$  be any field and consider the prime ideal in  $k[x, y, z]$

$$P = (x^3 - yz, y^2 - xz, z^2 - x^2y)$$

defining the curve parametrized by  $(t^3, t^4, t^5)$ . Give (with proof!) two different ideals  $J$  such that  $P^{(n)} = (P^n : J^\infty)$  for all  $n \geq 1$ , and test your proposed ideals  $J$  in Macaulay2 with your own choice of  $k$  and  $n$ .

**Problem 2.** Let  $R$  be a finitely generated  $k$ -algebra, and  $P$  a prime ideal in  $R$ . Show that  $P^{(n)}$  is  $P$ -primary for all  $n \geq 1$ .

**Problem 3.** Let  $k$  be any field,  $d \geq 1$ , and  $R = k[x_1, \dots, x_d]$ . Let

$$I = \left( \prod_{i \neq j} x_i \mid 1 \leq j \leq d \right)$$

be the monomial ideal generated by all the squarefree monomials of degree  $d - 1$ .

- a) Find an irredundant primary decomposition of  $I$ . What is the height of  $I$ ?
- b) Find a set of generators for  $I^{(n)}$  for each  $n \geq 1$ .
- c) Show that  $I^{(2n-2)} \not\subseteq I^n$  for all  $n < d$ .