Problems for the midterm

In the midterm, there will be some new problems you have never seen before, together with at least one problem from the problem sets, and at least one problem from the list below.

Problem 1 (January 2021)**.** Let R be a domain and M an R-module. We say that a subset S of M is **maximally linearly independent** if it is linearly independent and every subset T of M properly containing S is not linearly independent. Recall that we say a module M is **torsion** if for every $m \in M$ there exists a nonzero $r \in R$ such that $rm = 0$.

a) Let S be a linearly independent set of M and let N be the submodule generated by S. Show that S is maximally linearly independent if and only if M/N is torsion.

b) Suppose that for every module M, every maximally linearly independent set of M generates M. Show that R must be a field.

Problem 2 (May 2019)**.** Consider the $\mathbb{Z}$-module $M = \mathbb{Z} \oplus \mathbb{Z}/\mathbb{Z}(7, 11)$. Show that M is torsion-free.