Heat-Loss/Hysteresis Models and Benchmarks

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Outline

Outline

Overburden and underburden rock temperatures

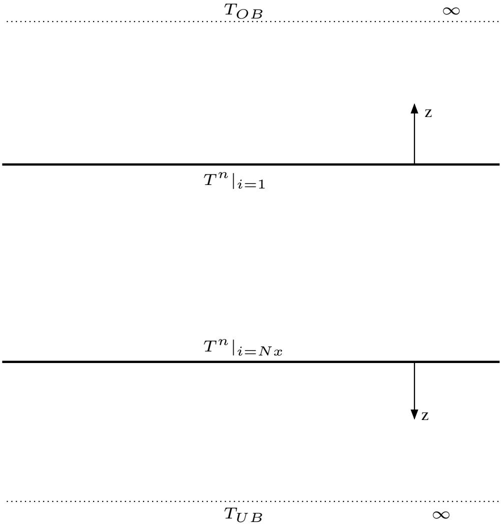


Figure: Overburden and underburden

Theory

Assumptions

- Conduction within cap rock rapidly wipes out any sharp temperature differences. Thus, temperature in cap rock varies smoothly.
- Longitudinal heat conduction in cap rock can be neglected due to high Peclet numbers.
- Temperature profile can be adequately approximated by any reasonably flexible function with a few parameters.

Outline

Equations

Fitting function:

$$T(t, z) = T_0 + (\theta + pz + qz^2)e^{-z/d}$$

- θ is temperature difference between temperature at interface between reservoir and cap/base rock and that at infinity (T_0), i.e., $\theta = T_r(t) - T_0$.
- p, q are model fit parameters to be determined.
- z is vertical distance from reservoir.
- d is the "diffusion length" given by $d = \frac{\sqrt{kt}}{2}$; k is thermal diffusivity of cap/base rock.

Determining model parameters

Heat flow into cap rock:

$$\frac{\partial T}{\partial t} = k\frac{\partial^2 T}{\partial z^2}$$

- At the interface, this yields $\frac{\partial \theta}{\partial t} = k\frac{\partial^2 T}{\partial z^2}\Big|_{z=0}$.
- Inserting the assumed function for T into this and finite differencing for time derivatives yields

$$\frac{\theta - \theta^n}{\Delta t} = k\left(\frac{\theta}{d^2} - \frac{2p}{d} + 2q\right)$$

where θ^n is interface temperature at beginning of time step.

Determining model parameters

Conservation of cap rock energy:

$$\frac{\partial}{\partial t}\int_0^{\infty} T \mathrm{dz} = k\frac{\partial T}{\partial z}\bigg|_0^{\infty}.$$

- Again inserting the assumed function for T, this yields

$$\frac{\theta d + pd^2 + 2qd^3 - I^n}{\Delta t} = k\left(\frac{\theta}{d} - p\right),$$

where $I^n = \theta^n d^n + p^n (d^n)^2 + 2q^n (d^n)^3$.

Determining model parameters

Eliminating q from heat flow & energy balance yields:

$$p = \frac{\frac{k\theta\Delta t}{d} + I^n - d^3\frac{(\theta-\theta^n)}{k\Delta t}}{3d^2 + k\Delta t}$$

$$q = \frac{(2pd - \theta + d^2\frac{(\theta-\theta^n)}{k\Delta t})}{2d^2}$$

The heat loss rate is:

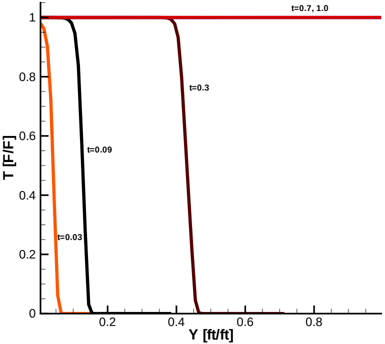
$$-\lambda\frac{\partial T}{\partial z}\bigg|_{z=0} = \lambda\left(\frac{\theta}{d} - p\right).$$

The energy stored in cap rock is given by:

$$E_c = \frac{\lambda}{k}\int_0^\infty T\mathrm{d}z = \frac{\lambda}{k}\left(\theta + pd + 2qd^2\right).$$

Outline

1-d hot water injection (Lauwerier's Test)



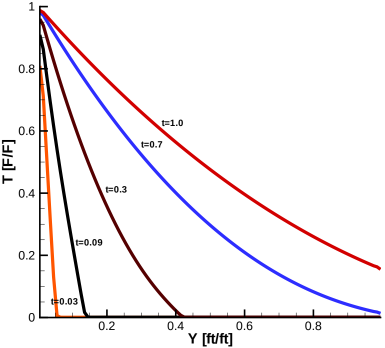


Figure: No heat loss (l); Heat loss (r)

Vinsome-Westerveld vs Lauwerier's solution, 1980

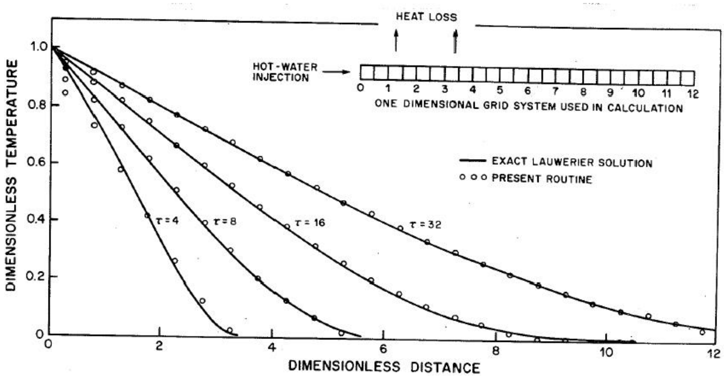


Figure: Vinsome-Westerveld vs Lauwerier's analytic solution

1-d cold water injection

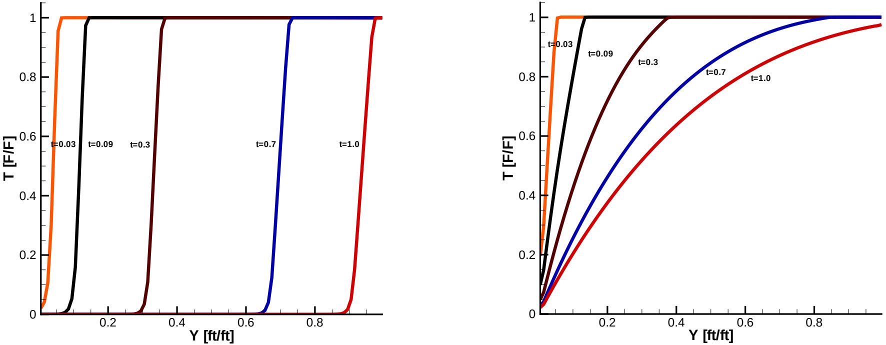


Figure: No heat loss (l); Heat loss (r)

Outline

1-d hot water injection in oil reservoir

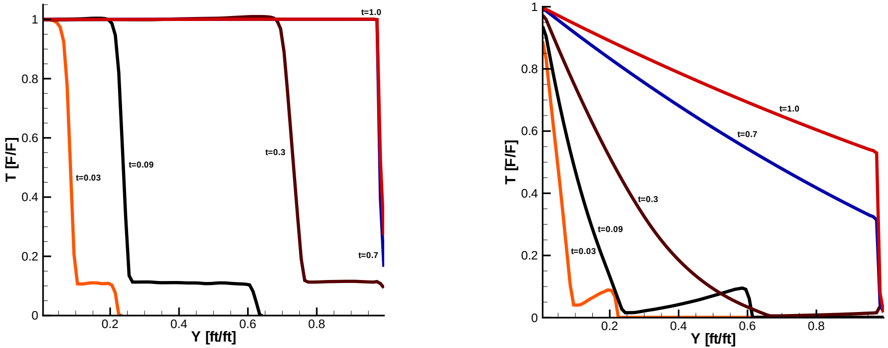


Figure: No heat loss (l); Heat loss (r)

Water saturation

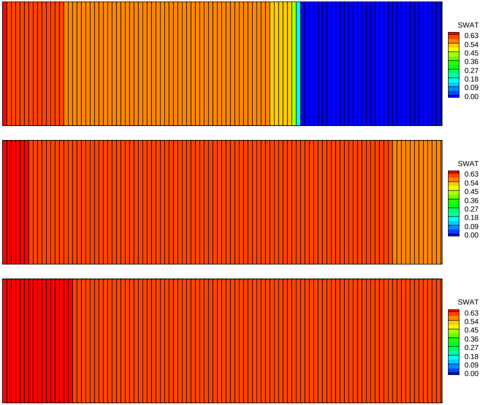


Figure: 100 days (top); 500 days (center); 1000 days (bottom)

Outline

2-d CO_2 injection in water: Solution profiles

CO_2 injection @553 mscf/d for 300 days. Simulation upto 1000.0 days.
Formation at critical depth so that phase change occurs as CO_2 floats up.

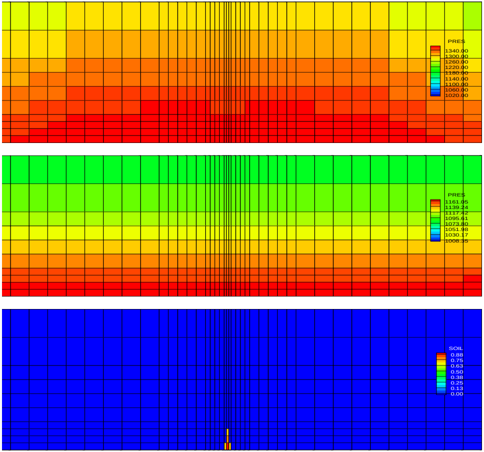


Figure: Pressure @ 300 d (t), 1000 d (c); “Oil” satn. @ 300 d (b)

2-d CO_2 injection in water: Solution profiles

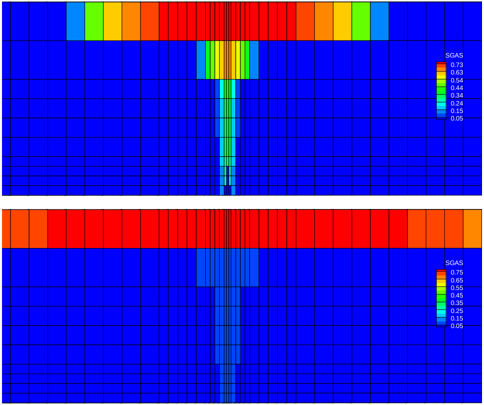


Figure: Gas saturation @ 300 days (t), 1000 days (b)

2-d CO_2 injection in water: Temperature

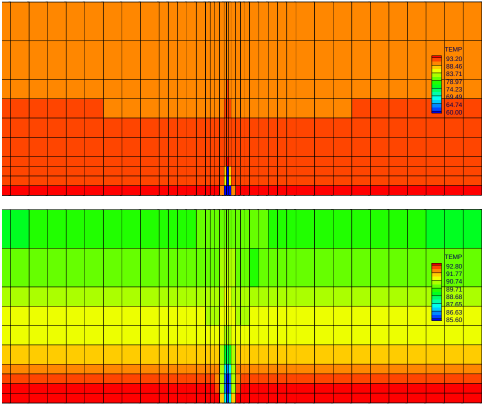


Figure: Temperature @ 300 days (t), 1000 days (b)

Outline

Theory

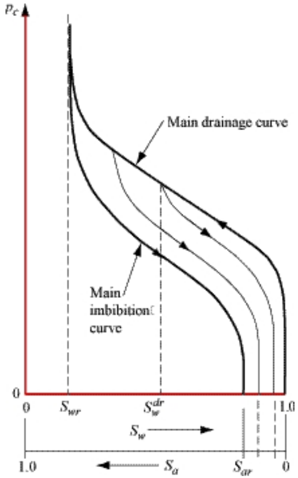
Defintion. Dependence of capillary pressures and relative permeabilities (in immiscible multiphase flow systems), on phase saturation is generally multivalued and based on wetting history. This phenomena is called "hysteresis".

- Commonly observed following fluid saturation path reversals.
- Attributed to effects such as contact angle, irregular pore geometry and non-wetting fluid entrapment.
- Several models exist for hysteresis [Snell 1962, Land 1968, Lenhard-Parker & Parker-Lenhard 1987, Delshad *et al* 1987-1989-2000].
- Can yield significantly different results in cyclic injection of CO_2 scenarios.

Schematic

Primary/Secondary Drainage/Imbibition curves

- Modeling hysteretic behavior requires accounting for non-wetting fluid entrapment.
- As wetting fluid moves into larger channels during imbibition, non-wetting fluid is entrapped.
- Entrapped non-wetting fluid is separated from continuous non-wetting fluid
- An apparent saturation is used to account for non-wetting fluid entrapment, $\bar{\bar{S}}_w = \bar{S}_w + \bar{S}_{nt}$.



Example of hysteresis model - Land's method

Predicts amount of non-wetting fluid trapped in pore spaces by an empirical-graphical method

- Effective phase saturations are given by $\bar{S}_w = \frac{S_w - S_m}{1 - S_m}$; $\bar{S}_n = \frac{S_n}{1 - S_m}$.
- Land's algorithm is given by the empirical formula for the effective residual non-wetting phase saturation $\bar{S}_{nr} = \frac{1 - \Delta \bar{S}_w}{1 + R_{nw}(1 - \Delta \bar{S}_w)}$, $R_{nw} = \frac{1}{S^0_{nr}} - 1$.
- Effective entrapped non-wetting fluid saturation is then linearly interpolated ($\bar{S}_{nt} = 0,\ \bar{S}_{nr}$ @ $\bar{\bar{S}}_w = \Delta \bar{S}_w,\ 1$ resply.) by $\bar{S}_{nt} = \bar{S}_{nr} \left(\frac{\bar{\bar{S}}_w - \Delta \bar{S}_w}{1 - \Delta \bar{S}_w} \right)$, $\bar{\bar{S}}_w \geq \Delta \bar{S}_w$.

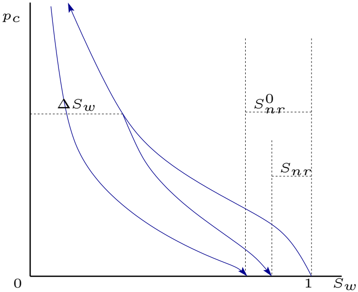


Figure: Land's method

Land's method

Implications

- In modeling hysteretic fluid flow, when $\bar{\bar{S}}_w < \Delta\bar{S}_w$, main drainage is followed and $\Delta\bar{S}_w$ is updated to $\bar{\bar{S}}_w$.
- As interfaces between non-wetting and wetting fluids advance into larger-sized pores, non-wetting fluid is entrapped (discontinuous).
- As the interfaces advance into smaller-sized pores, non-wetting fluid is released (into continuous phase) in identical amounts.
- In this way, hysteresis is ignored in dependence between $\bar{S}_{nt}$ and $\bar{\bar{S}}_w$

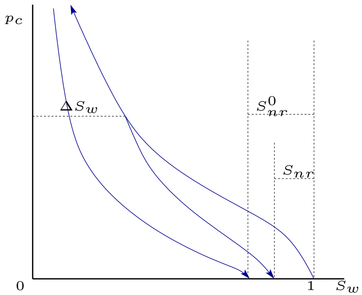


Figure: Land's method

Relative permeability, capillary pressure expressions

Aq. phase relative permeability (BCB)

$$k_{rw} = \bar{S}_w^{\frac{2+3\lambda}{\lambda}}$$

Non-aq. phase relative permeability (BCB)

$$k_{rn} = k_{rn}^0 (1 - \bar{\bar{S}}_w)^2 (1 - \bar{\bar{S}}_w^{\frac{2+\lambda}{\lambda}})$$

Capillary pressure (BCB)

$$p_c = p_d \bar{\bar{S}}_w^{\frac{-1}{\lambda}}$$

Outline

Cyclic injection of CO_2 in aquifer

- 2500 m long aquifer at depth of 9000 ft, saturated with water.
- 1-d approximation, with 100 elements.
- CO_2 injected at 250 mscf/d at one end. BHP specified producer.
- Gas injection turned off every 100 days and water injected in place. (WAG with 100 day cycle).
- 1000 day simulation end time.

CO_2 gas saturation - no hysteresis

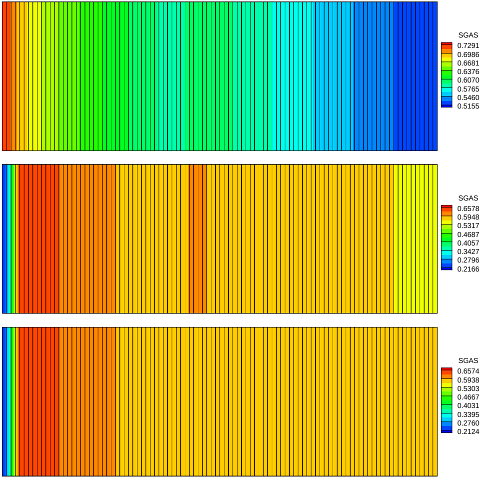


Figure: Gas saturation @ 300 d (t), 600 d (c), 1000 d (b)

CO_2 gas saturation - with hysteresis

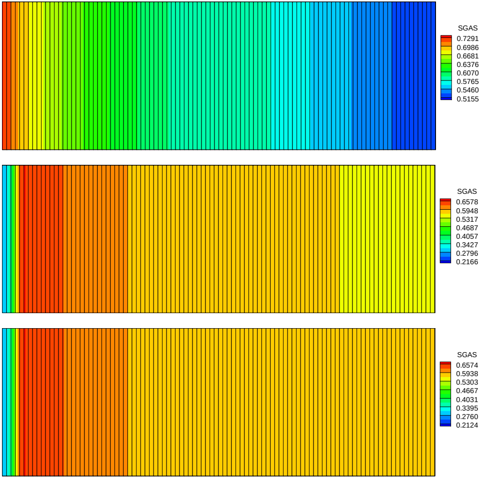


Figure: Gas saturation @ 300 d (t), 600 d (c), 1000 d (b)

Outline

CO_2 injection in slanted aquifer

Features

- Slanted reservoir @1% dip.
- Dimensions of 100 km $\times$ 100 km $\times$ 50 m
- Horizontal well at bottom of reservoir injecting CO_2 @1 Mt/y.
- Injection period is 20 years...monitored to infinity.
- Interested in CO_2 distribution in phases with time and time of immobility.

CO_2 gas saturation

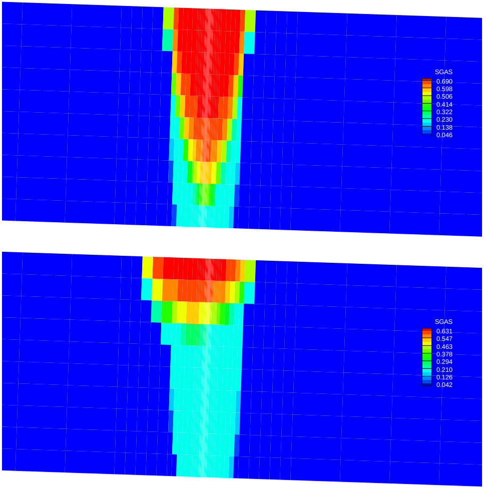


Figure: Gas saturation @ 100 years (t), 1000 years (b)

CO_2 gas saturation

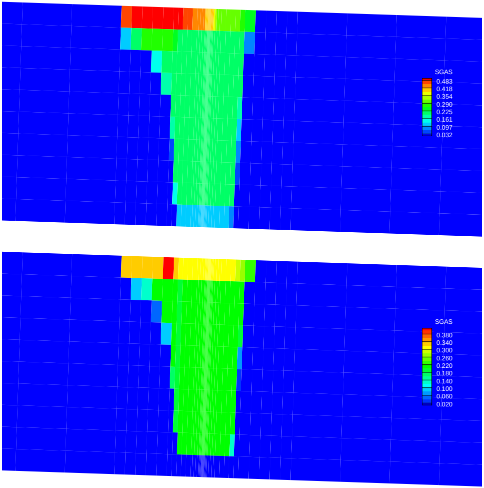


Figure: Gas saturation @ 100 years (t), 1000 years (b)

CO_2 distribution in phase

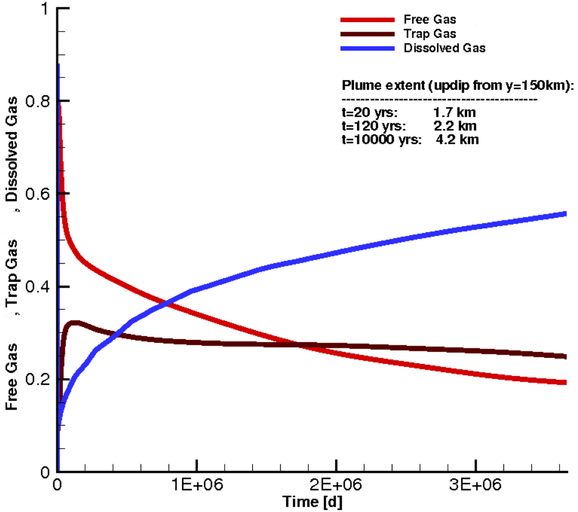


Figure: CO_2 distribution in phase

Current and Future Work

- Implementation of hysteresis model.
- Testing on more benchmarks.

References

Y. B. Chang
Development and application of an EOS simulator.
Ph.D. dissertation, U.T. Austin, 1990.

R. H. Dean, J. A. Wheeler
IPARSv3 documentation on compositional model.
CSM, U.T. Austin, 2000.

M. Delshad
Energy balance equation in UTChem v5.2.
PGE, U.T. Austin, 1993.

F. M. Orr
Thermodynamics.
Basic Concepts in EOR, 159–185.

C. N. Dawson
Godunov-mixed methods for advection-diffusion equations in higher dimensions.
SIAM J. Num. Anal., 1993.

A. Ebigbo, R. Helmig, H. Class
CO_2 leakage through an abandoned well: problem oriented benchmarks.
Comp. Geo., 2007.