Performance Comparisons of the K-Means Algorithm

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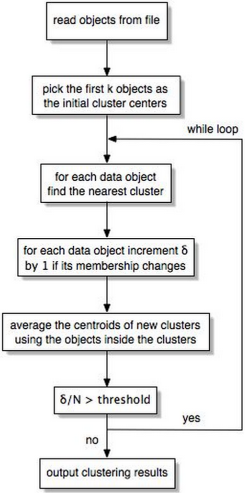
Outline

Outline

The K-Means Algorithm

- Algorithm that aims to partition n observations into k clusters where each observation belongs to the cluster nearest mean or centroid
- Vector quantization in signal processing
- Cluster analysis in data mining, how do you choose k?
- Feature learning in semi-supervised or unsupervised learning
- Clustering is NP-hard, but k-means is a heuristic that converges quickly to a local optimum
- Typical running time $O(nkdi)$, repeat for T trials
- For n points in $[0,1]^d$ with independent gaussian perturbations with 0 mean and σ^2 variance, upper bound expected running time by $O(n^{34}k^{34}d^8\log^4(n)/\sigma^6)$. [Arthur 2009]

The K-Means Algorithm



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    for j ← 0 to K-1
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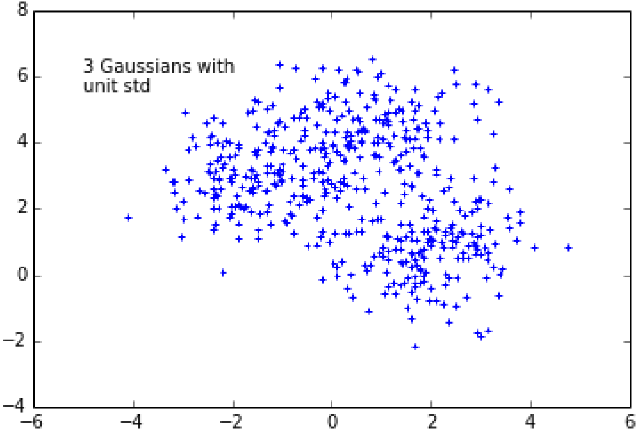
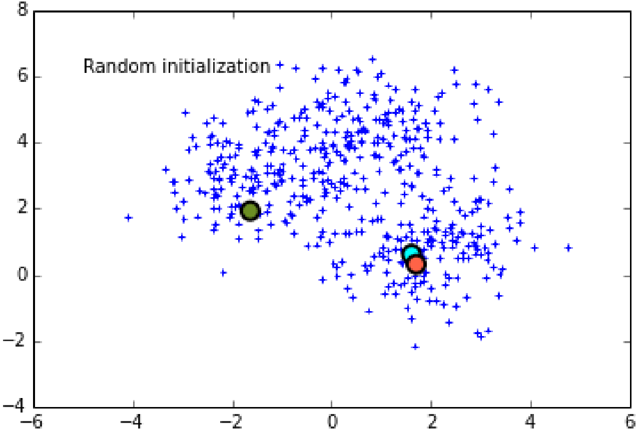
Examples



Examples



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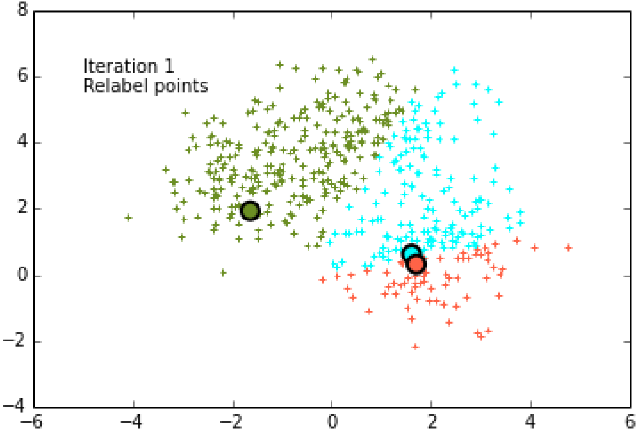

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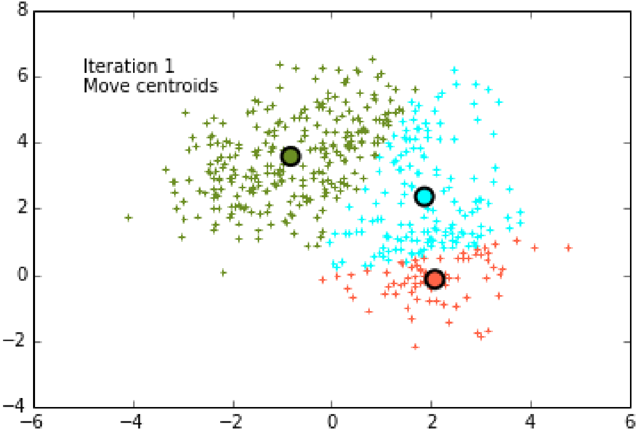

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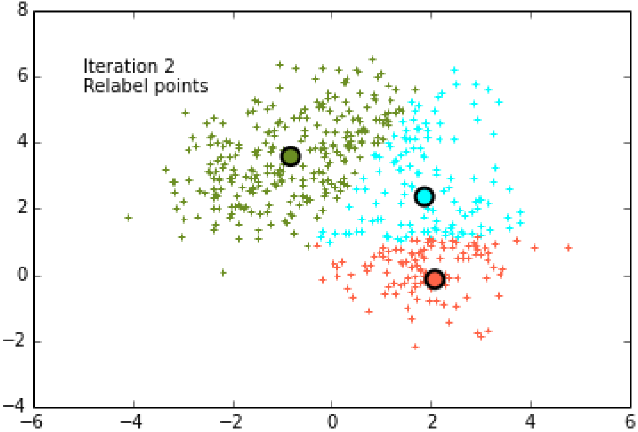

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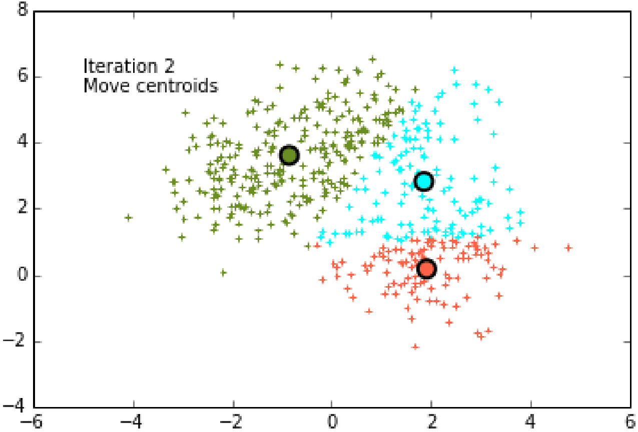

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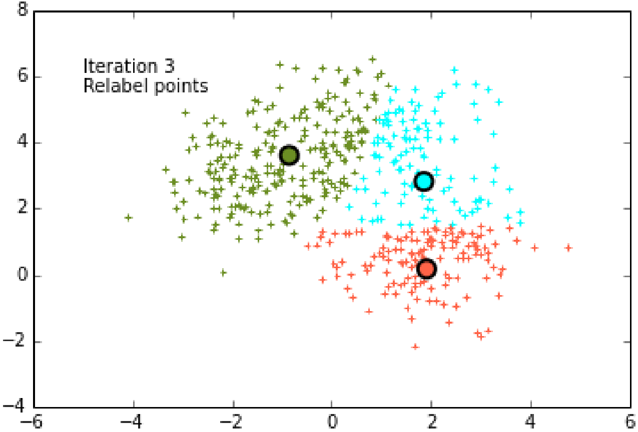

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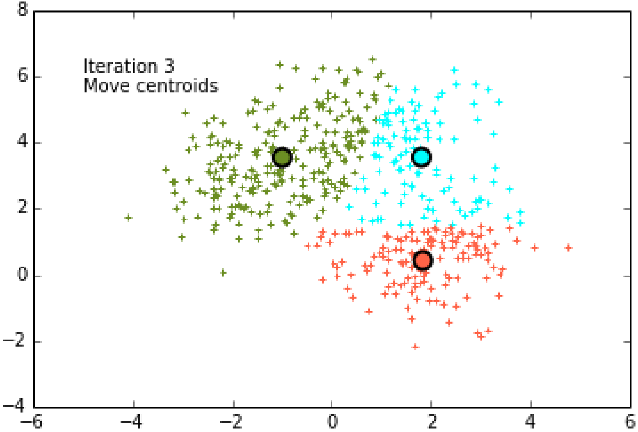

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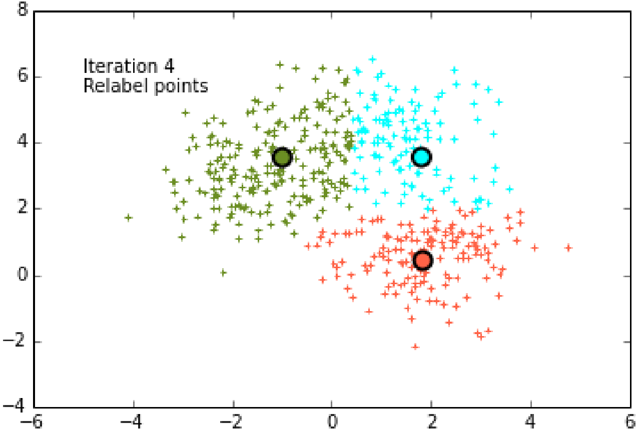

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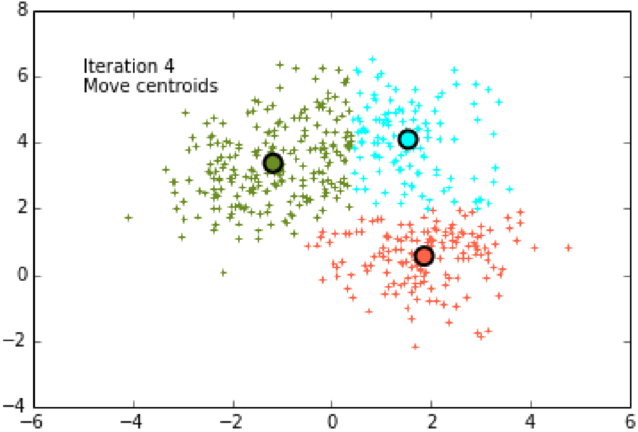

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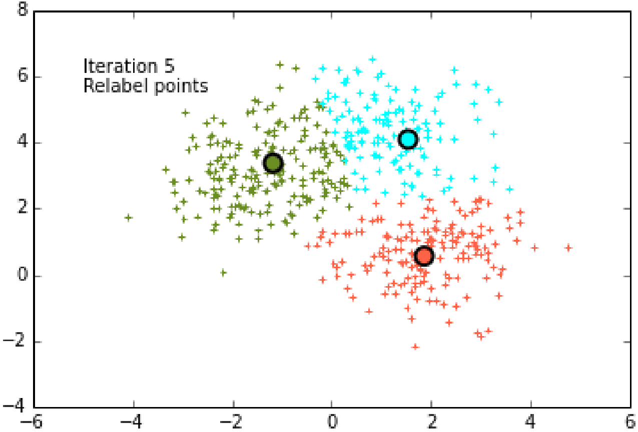

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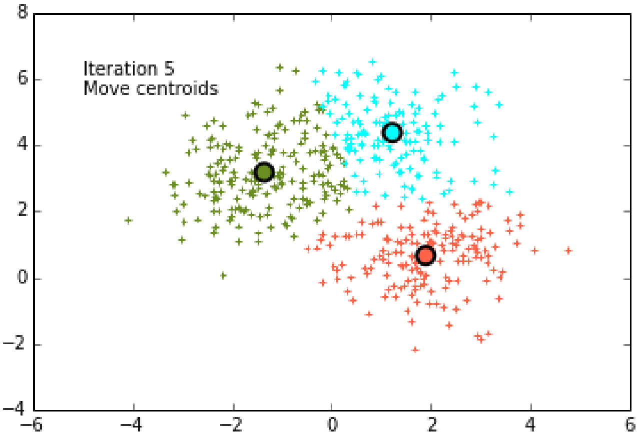

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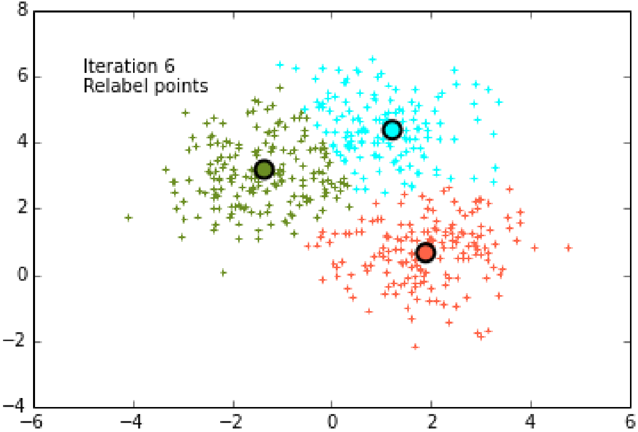

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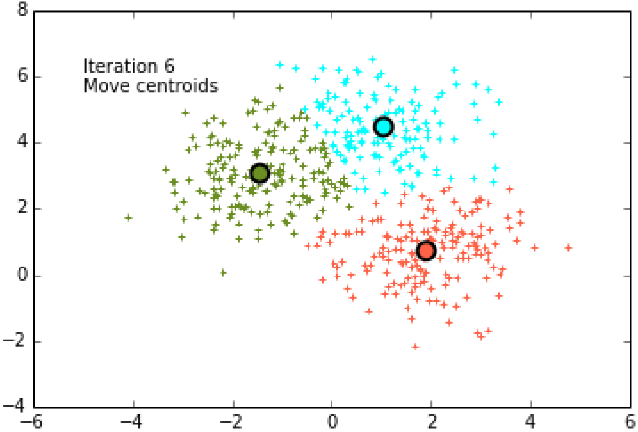


Examples

229,931 colors

Examples

256 colors

Examples

128 colors

Examples

64 colors

Examples

32 colors

Examples

16 colors

Examples

10 colors

Examples

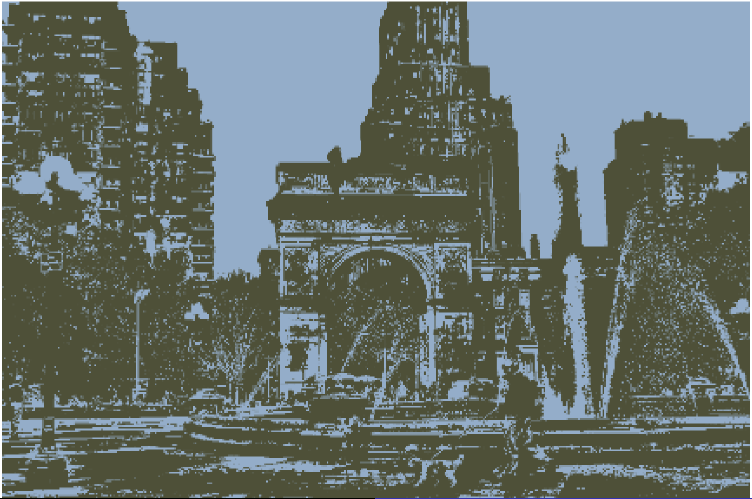
5 colors

Examples

3 colors

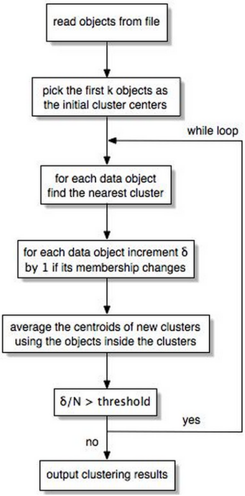
Examples

2 colors



Outline

MPI Communication Overview



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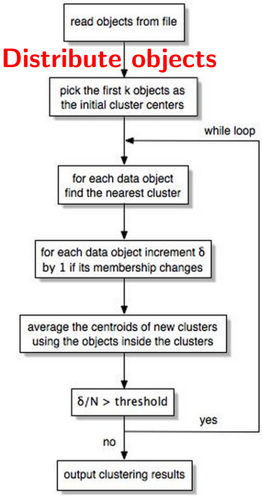

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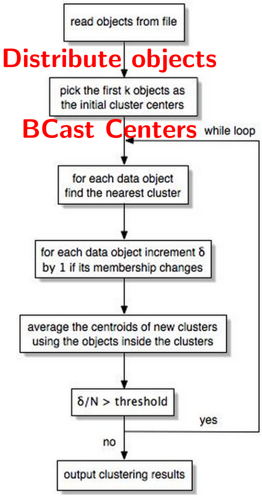

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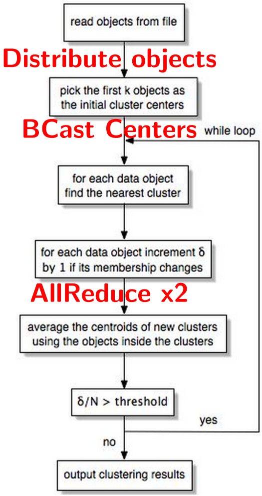

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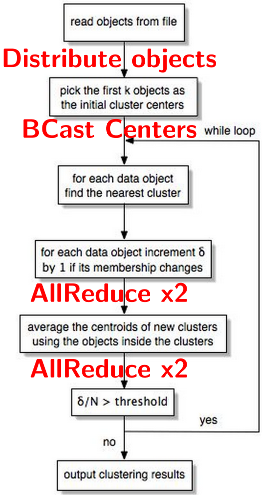

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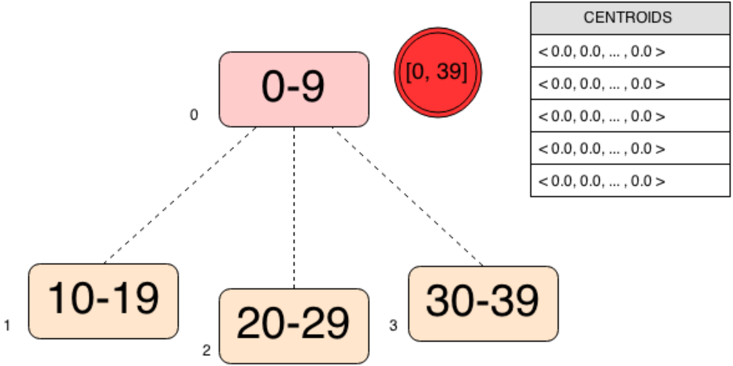

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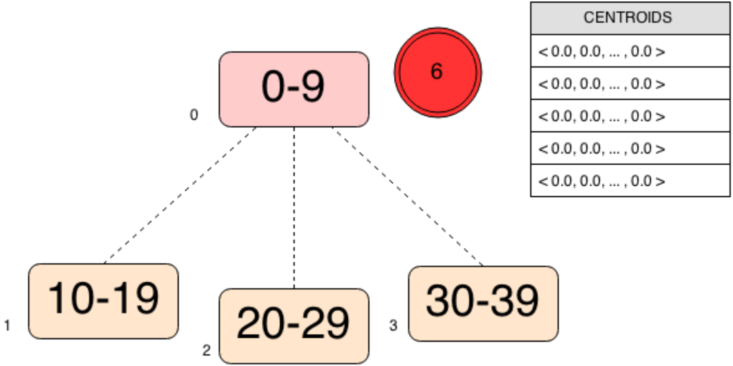

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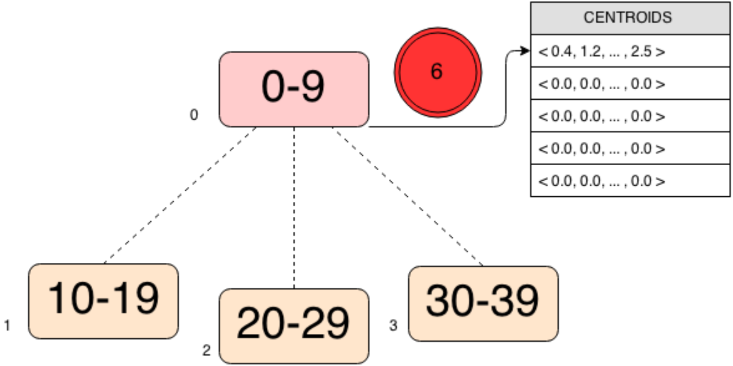

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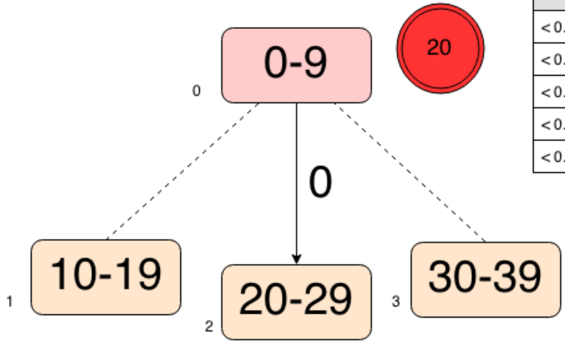

CENTROIDS
< 0.4, 1.2, ... , 2.5 >
< 0.0, 0.0, ... , 0.0 >
< 0.0, 0.0, ... , 0.0 >
< 0.0, 0.0, ... , 0.0 >
< 0.0, 0.0, ... , 0.0 >

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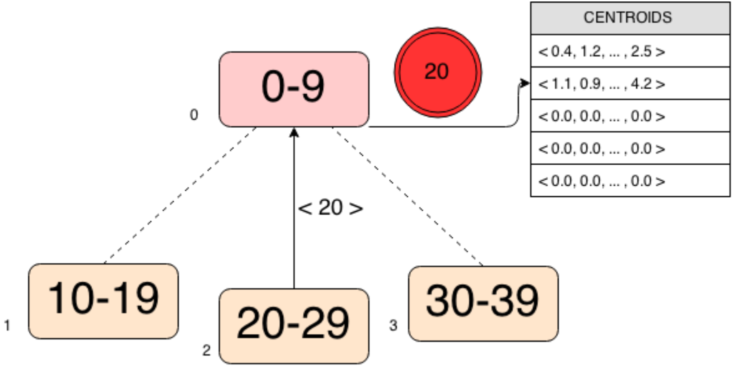

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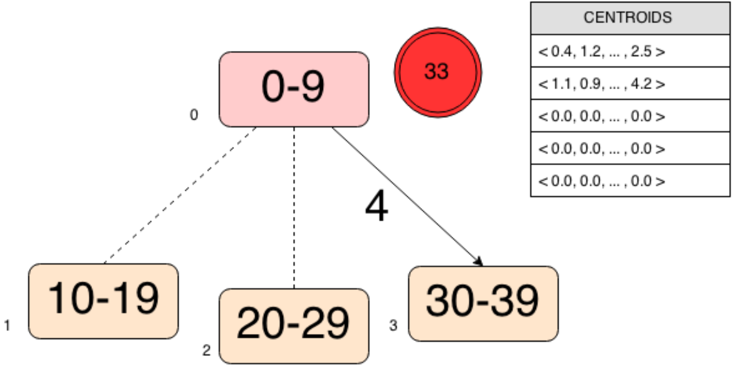

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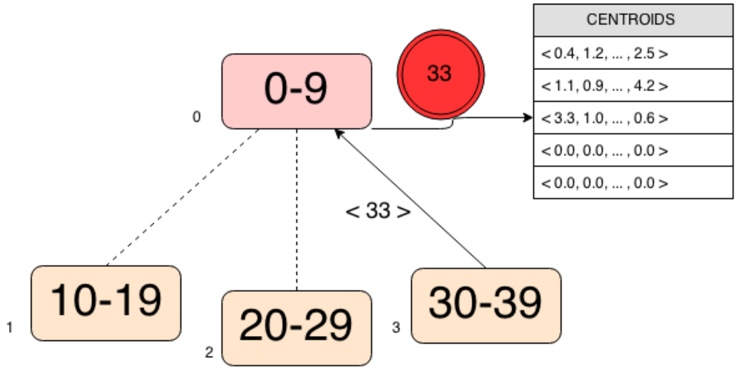

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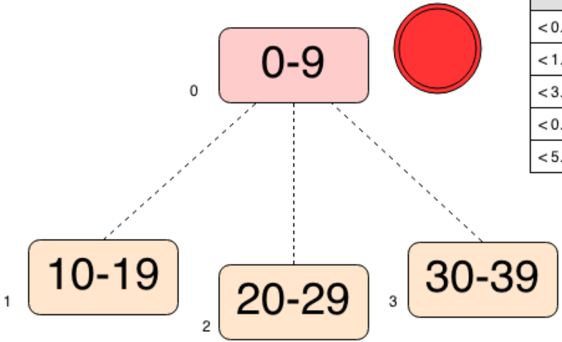

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< 0.7, 2.8, ... , 0.4 >
< 5.6, 0.8, ... , 0.9 >

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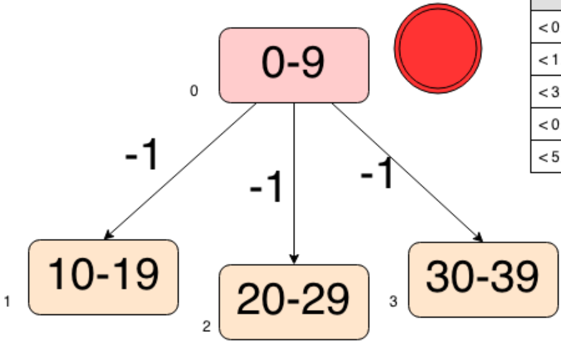

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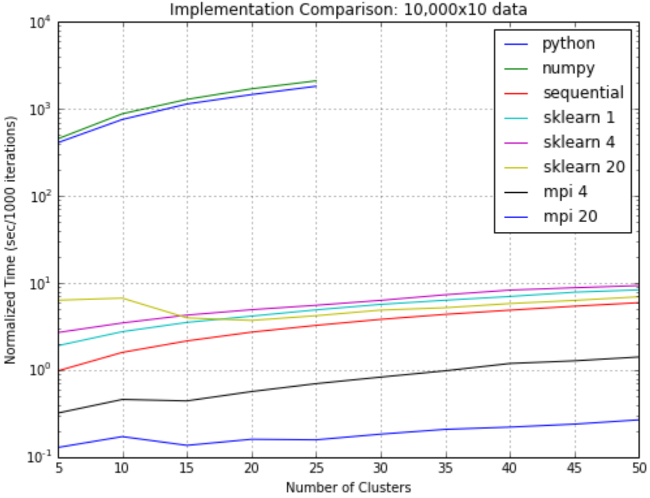
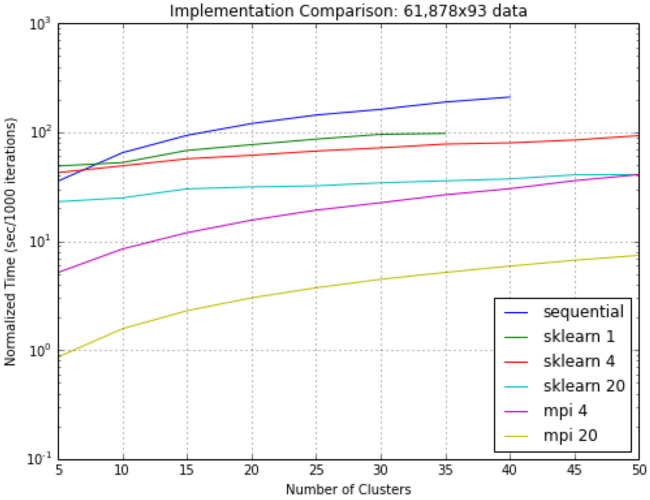
Outline

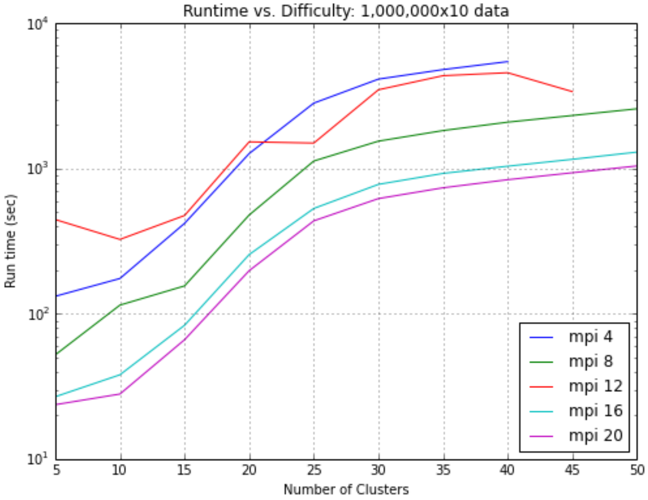
Multiple Implementations



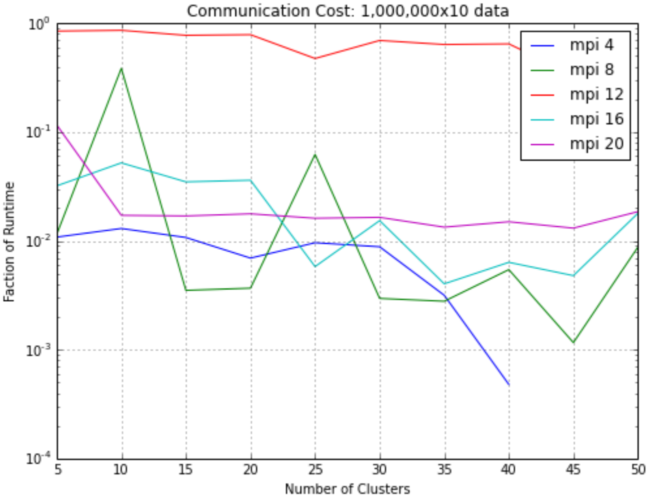
Larger Dataset



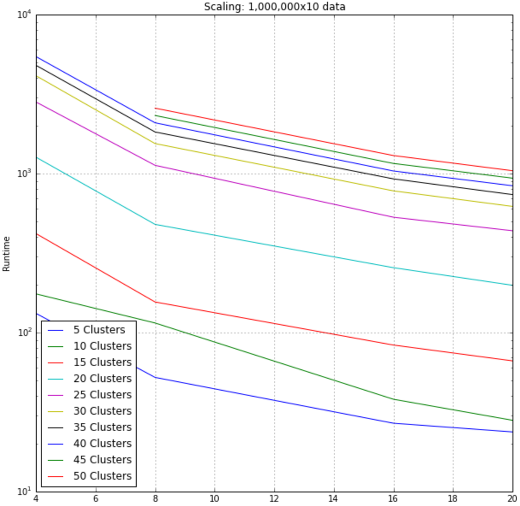
Affect of the Number of Clusters



Communication Cost



Strong Scaling



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Comments

- The best method for parallelization will depend on your data
- There are much better methods for initializing the centroids to converge faster, kmeans++
- Reading plain text files with MPI is annoying
- Every now and then communication would hang (locally) for a long time before
- Saw some errors/warnings when running on Mercer for larger data that I didn't see previously

huhh??

```
Computation time: 10412.692955s
Communication time: 68.016616s
--------------------------------------------------------------------------
_orterun noticed that process rank 6 with PID 10686 on node compute-15-9 exited on signal 11 (Segmentation fault).
--------------------------------------------------------------------------
--------------------------------------------------------------------------
WARNING: a request was made to bind a process. While the system
supports binding the process itself, at least one node does NOT
support binding memory to the process location.

  Node:  compute-15-9

This usually is due to not having the required NUMA support installed
on the node. In some Linux distributions, the required support is
contained in the libnumactl and libnumactl-devel packages.
This is a warning only; your job will continue, though performance may be degraded.
--------------------------------------------------------------------------
--------------------------------------------------------------------------
The library attempted to open the following supporting CUDA libraries,
but each of them failed.  CUDA-aware support is disabled.
libcuda.so.1: cannot open shared object file: No such file or directory
/usr/lib64/libcuda.so.1: cannot open shared object file: No such file or directory
If you are not interested in CUDA-aware support, then run with
--mca mpi_cuda_support 0 to suppress this message.  If you are interested
in CUDA-aware support, then try setting LD_LIBRARY_PATH to the location
of libcuda.so.1 to get passed this issue.
--------------------------------------------------------------------------
(END)
```

Further Reading I

Arthur, David and Manthey, Bodo and Roglin, H
k-Means has polynomial smoothed complexity
Foundations of Computer Science, 2009. FOCS'09. 50th Annual IEEE Symposium on. (2009) 405-414

MacQueen, James and others
Some methods for classification and analysis of multivariate observations
Proceedings of the fifth Berkeley symposium on mathematical statistics and probability. (1967) 281-297

Arthur, David and Vassilvitskii, Sergei
k-means++: The advantages of careful seeding
Proceedings of the eighteenth annual ACM-SIAM symposium on Discrete algorithms. (2007) 1027-1035