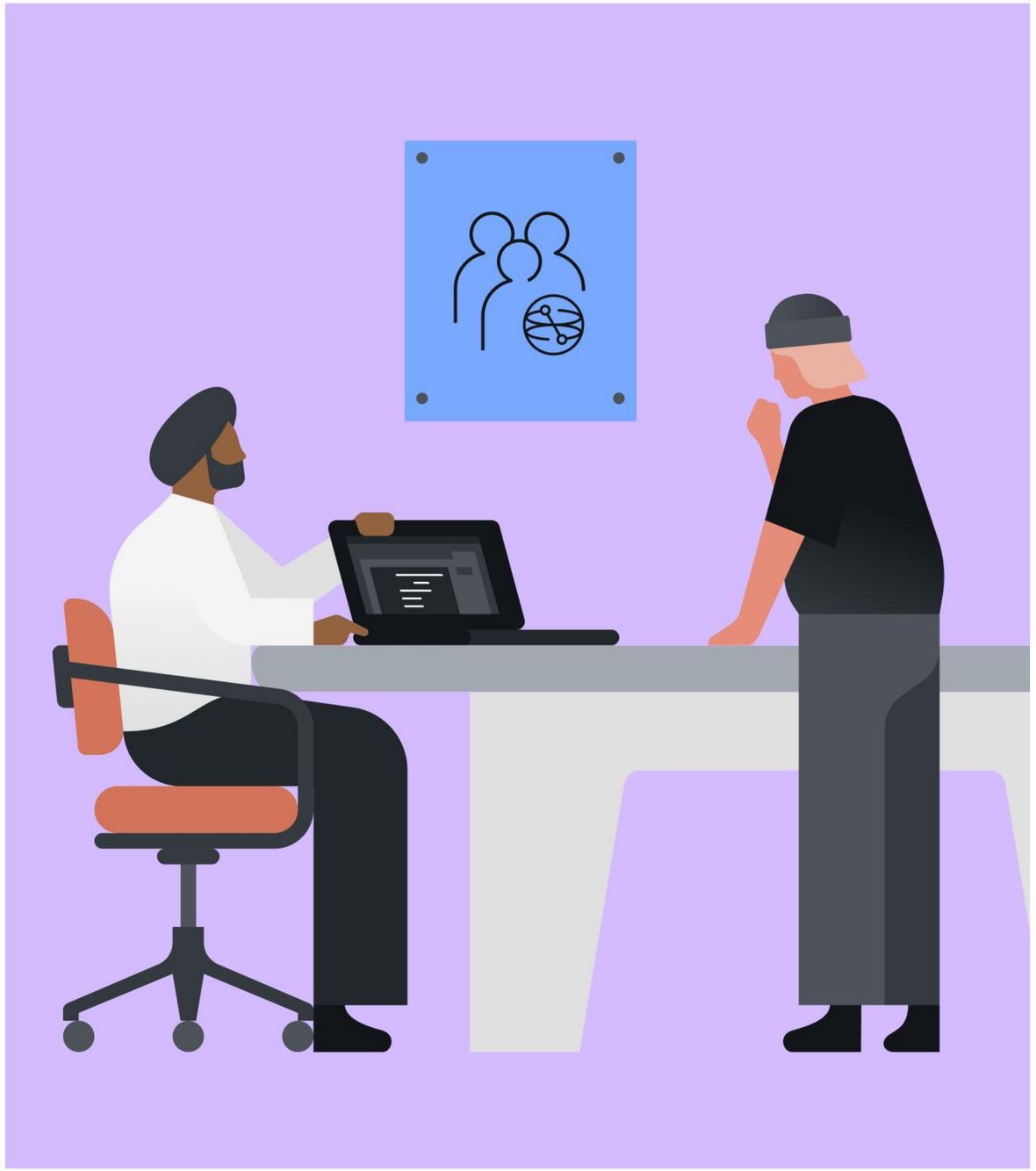
# QAMP 2025

# Qiskit Module on CSS-T qLDPC Codes

#41

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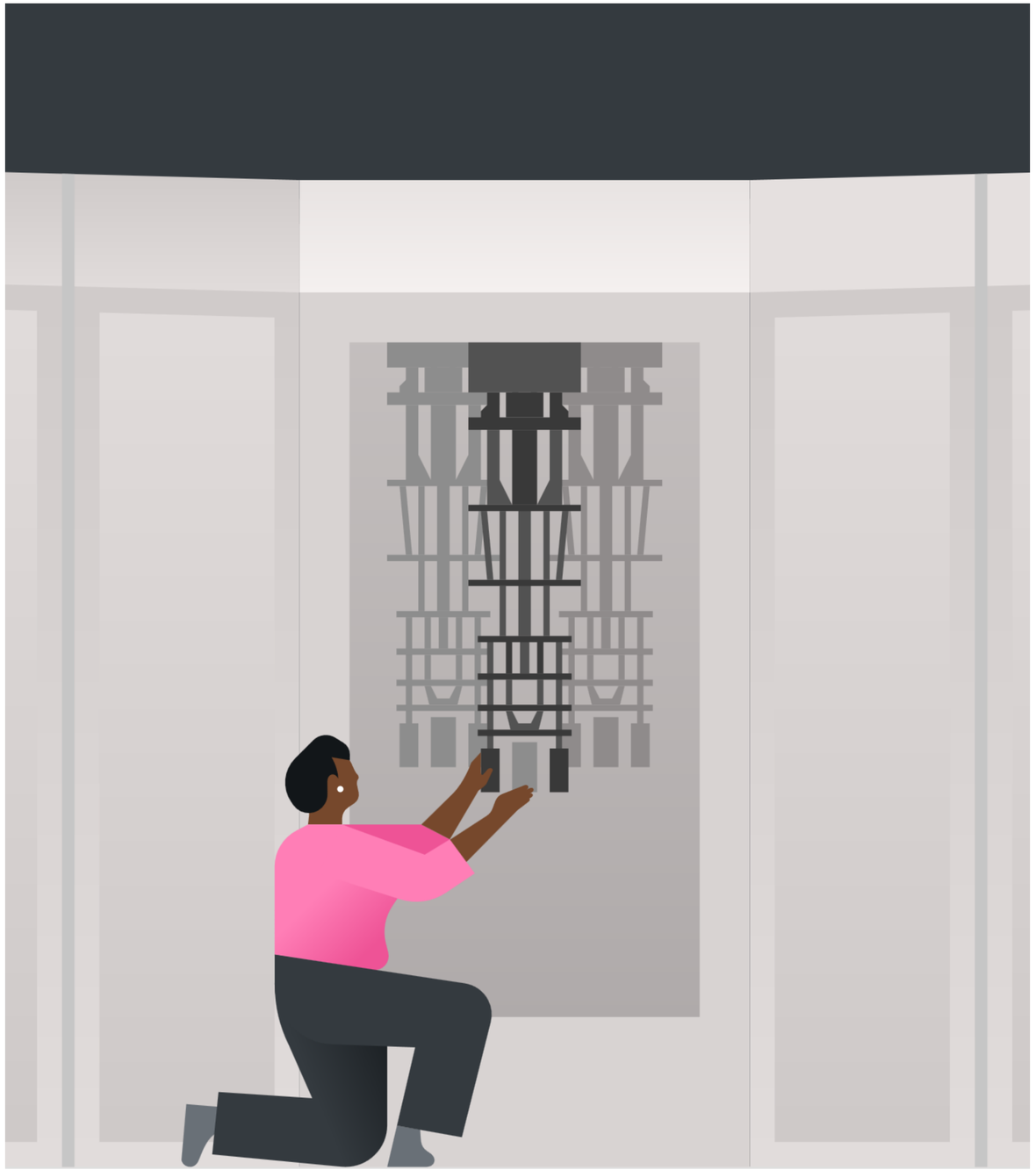
# Outline

- Codes for Clifford Gates and for T Gate
- Illustration of Construction
- Decoding Performance



# Codes for Transversal Clifford Gate

- Self-dual CSS codes
  - Construction: from classical self-dual codes
  - Support Clifford gates transversally

Experimental constructions of self-dual codes

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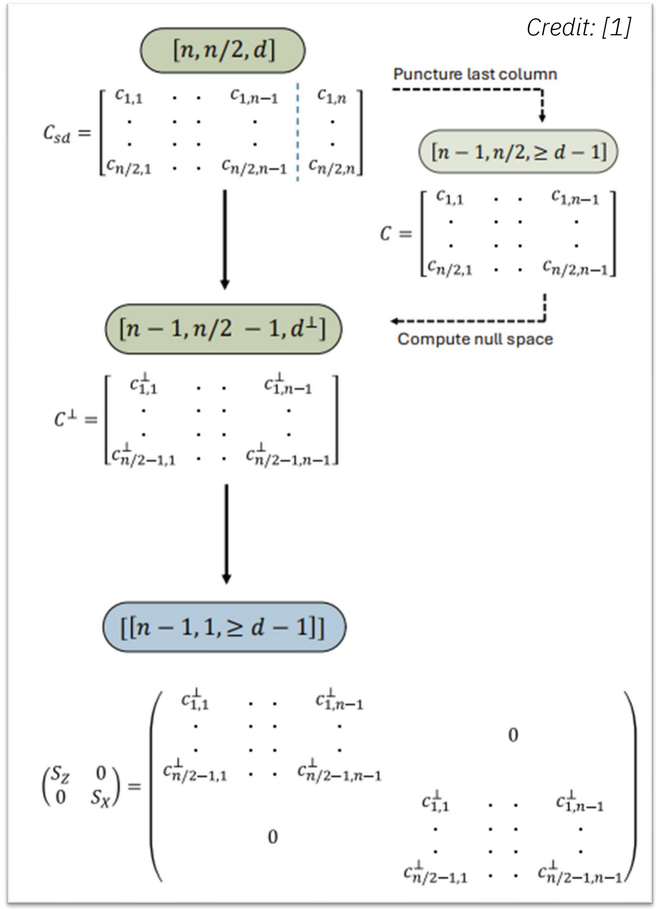



GO03_self_dual

| | |
|---|---|
| n4_d2.alist | n34_d6.alist |
| n6_d2.alist | n36_d8.alist |
| n8_d2.alist | n38_d8.alist |
| n10_d2.alist | n40_d8.alist |
| n12_d4.alist | n42_d8.alist |
| n14_d4.alist | n44_d8.alist |
| n16_d4.alist | n46_d8.alist |
| n18_d4.alist | n48_d8.alist |
| n20_d4.alist | n50_d8.alist |
| n22_d6.alist | n52_d10.alist |
| n24_d6.alist | n54_d8.alist |
| n24_d8.alist | n56_d10.alist |
| n26_d6.alist | n58_d10.alist |
| n28_d6.alist | n60_d12.alist |
| n30_d6.alist | n62_d10.alist |
| n32_d8.alist | n64_d10.alist |

[1] S. P. Jain and V. V. Albert, “Transversal Clifford and T-gate codes of short length and high distance,” IEEE Journal on Selected Areas in Information Theory, 2025.

# Codes for Transversal Clifford Gate

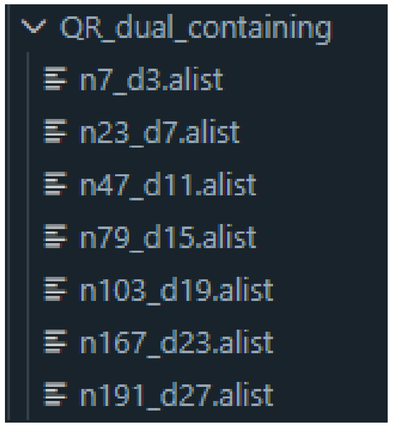
- Self-dual CSS codes
  - Construction: from classical self-dual codes
  - Support Clifford gates transversally

Transversal Clifford and $T$-gate codes of short length and high distance

Shubham P. Jain and Victor V. Albert

| QUADRATIC-RESIDUE BASED | |
|---|---|
| extended QR | doubly even |
| [8, 4, 4] | [[7, 1, 3]] [1] |
| | [[17, 1, 5]] [3] |
| [24, 12, 8] | [[23, 1, 7]] [4] |
| [48, 24, 12] | [[47, 1, 11]] [7] |
| [80, 40, 16] | [[79, 1, 15]] |
| [104, 52, 20] | [[103, 1, 19]] |
| [168, 84, 24] | [[167, 1, 23]] |
| [192, 96, 28] | [[191, 1, 27]] |

*Credit: [1]*

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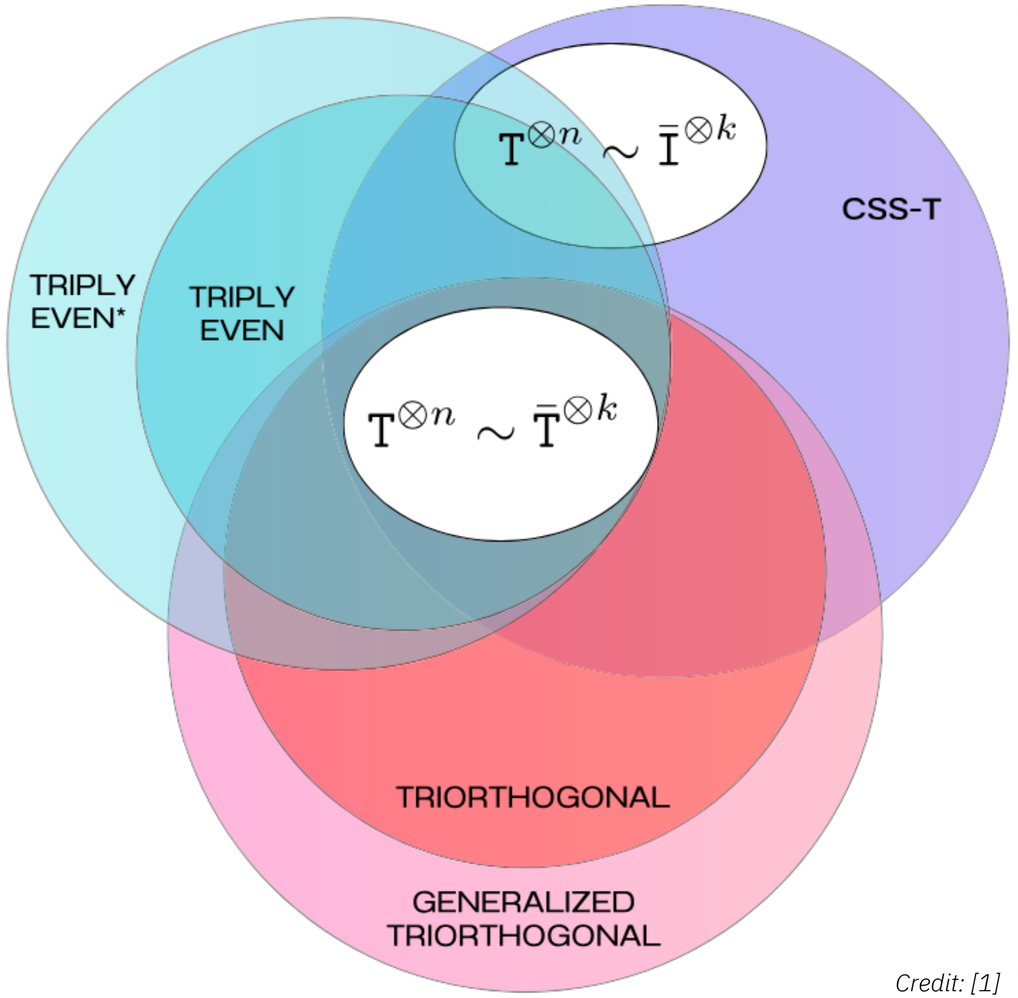
# Codes for T Gate

- Self-dual CSS codes
- **Universal gate set**: Clifford + T = {H, S, CNOT, T}
- **Triorthogonal / Triply Even Codes**
  - Support T gate transversally




*Credit: [1]*

[1] S. P. Jain and V. V. Albert, “Transversal clifford and t-gate codes of short length and high distance,” IEEE Journal on Selected Areas in InformationTheory, 2025.

# Codes for T Gate

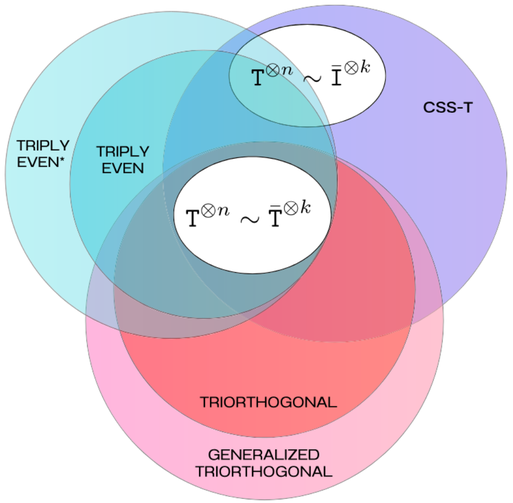
- Self-dual CSS codes
- Universal gate set: Clifford + T = {H, S, CNOT, T}
- Triorthogonal / Triply Even Codes
  - Support T gate transversally
  - Doubling construction

Transversal Clifford and $T$-gate codes of short length and high distance

Shubham P. Jain and Victor V. Albert

| self dual CSS | triorthogonal |
|---|---|
| [[7, 1, 3]] [1] | [[15, 1, 3]] [2] |
| [[17, 1, 5]] [3] | [[49, 1, 5]] [2] |
| [[23, 1, 7]] [4] | [[95, 1, 7]] [5] |
| [[45, 1, 9]] | [[185, 1, 9]] |
| [[47, 1, 11]] [7] | [[279, 1, 11]] |
| [[69, 1, 13]] | [[417, 1, 13]] |
| [[79, 1, 15]] | [[575, 1, 15]] |
| [[101, 1, 17]] | [[777, 1, 17]] |
| [[103, 1, 19]] | [[983, 1, 19]] |

| doubly even | triply even* |
|---|---|
| [[7, 1, 3]] [1] | [[15, 1, 3]] [2] |
| [[17, 1, 5]] [3] | [[49, 1, 5]] [2] |
| [[23, 1, 7]] [4] | [[95, 1, 7]] [5] |
| [[47, 1, 11]] [7] | [[189, 1, 9]]<br>[[283, 1, 11]] |
| [[79, 1, 15]] | [[441, 1, 13]]<br>[[599, 1, 15]] |
| [[103, 1, 19]] | [[805, 1, 17]]<br>[[1011, 1, 19]] |
| [[167, 1, 23]] | [[1345, 1, 21]]<br>[[1679, 1, 23]] |




Qiskit-CSS-T / doubling-CSST / alistMats / **JA25_triorthogonal** /

Name

- ..
- n15_d3_Hx.alist
- n15_d3_Hz.alist
- n49_d5_Hx.alist
- n49_d5_Hz.alist
- n95_d7_Hx.alist
- n95_d7_Hz.alist

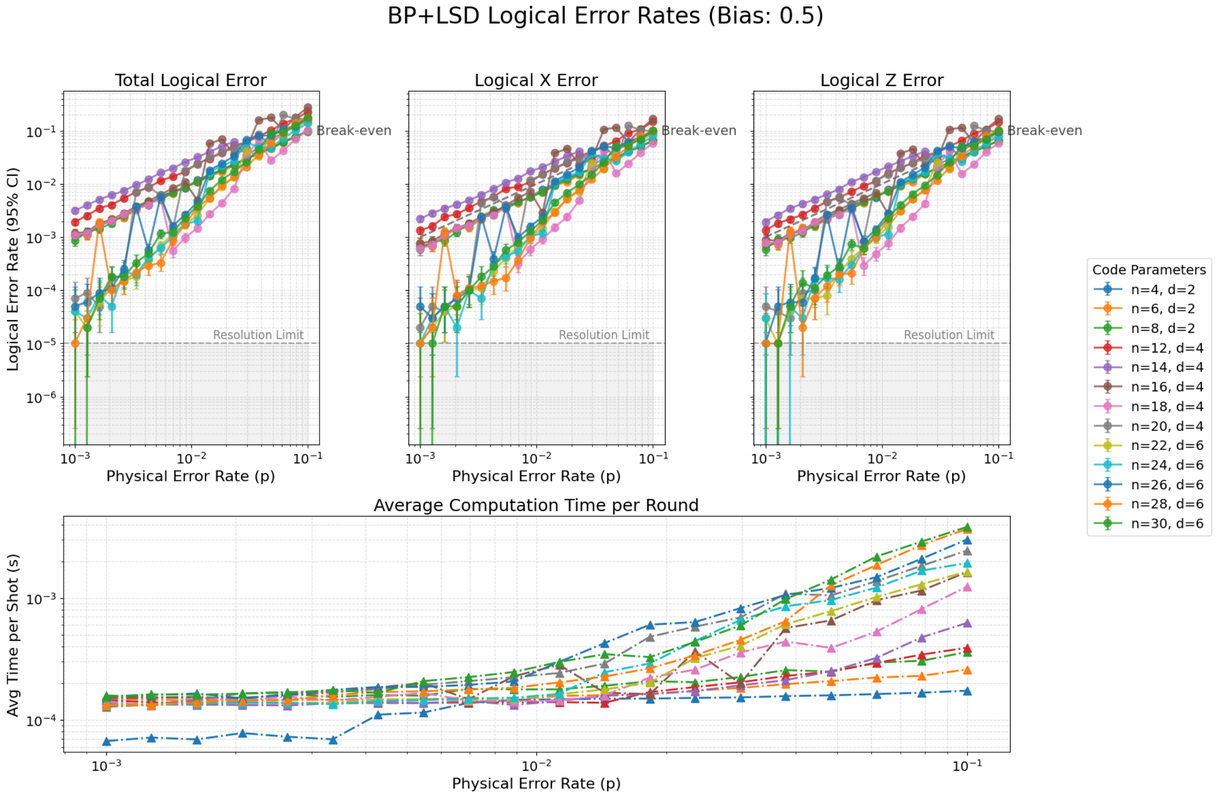
# Comparison of various decoder for self-dual CSS/CSS-T codes

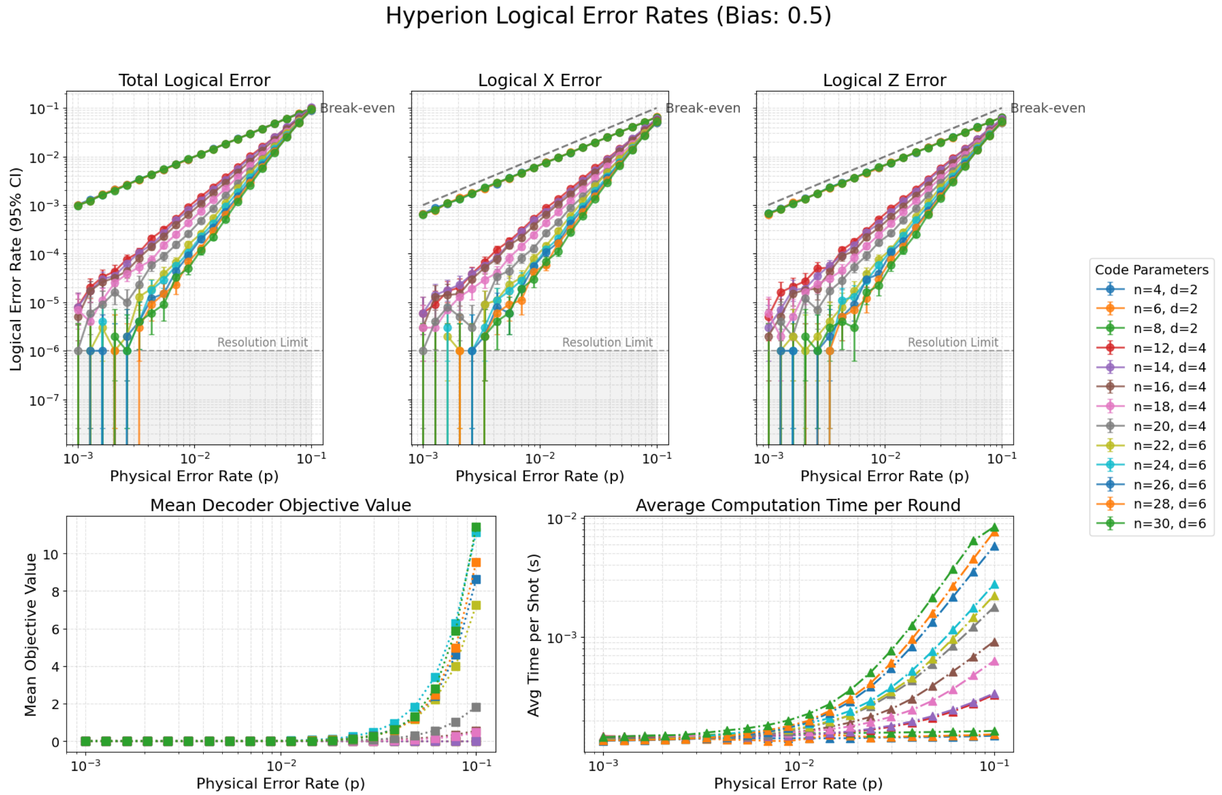
## Objective

Finding a suitable decoding algorithm for target self-dual CSS or CSS-T codes codes.

## Observation and Discussion

- Conducted Monte Carlo simulations for self-dual CSS codes by directly decoding sampled Pauli error strings.
- Results from Hyperion showed one-level higher accuracy than BP+LSD decoders with comparable computation time.
- Since Hyperion works for finding a parity factor for a general Hypergraph (Tanner graph), it was more suitable for our self-dual CSS or CSS-T codes that doesn't start from geometric structures.
- Hyperion also provides rigorous optimality, and proximity bounds through the objective value (primal-dual gap).



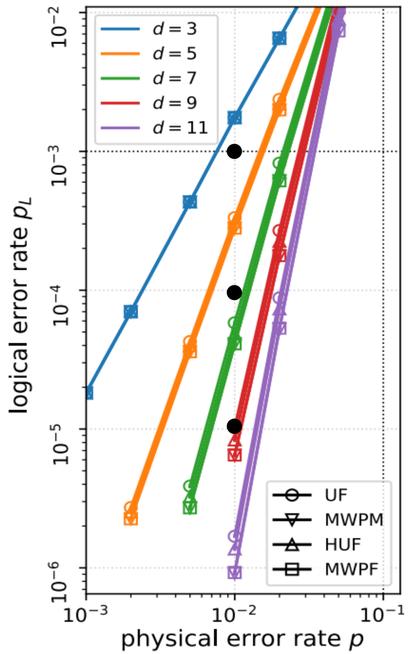
# Recreating the results from “Minimum-Weight Parity Factor Decoder for Quantum Error Correction”

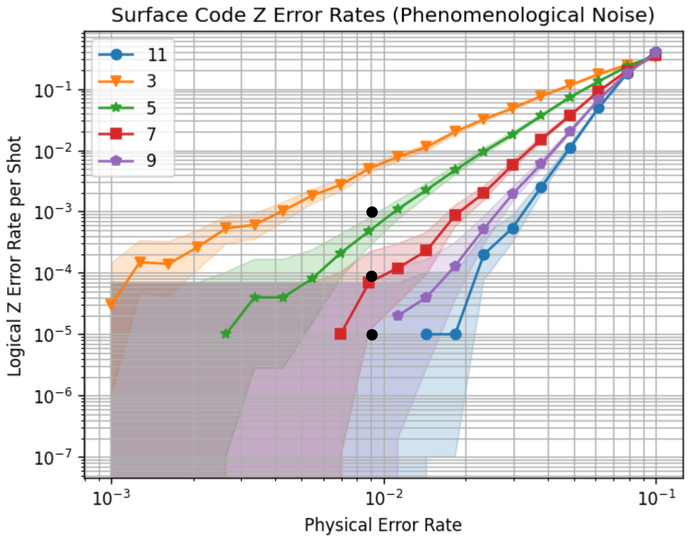
## Objective

Verifying the circuit-level noise simulation by comparing the results in the reference.

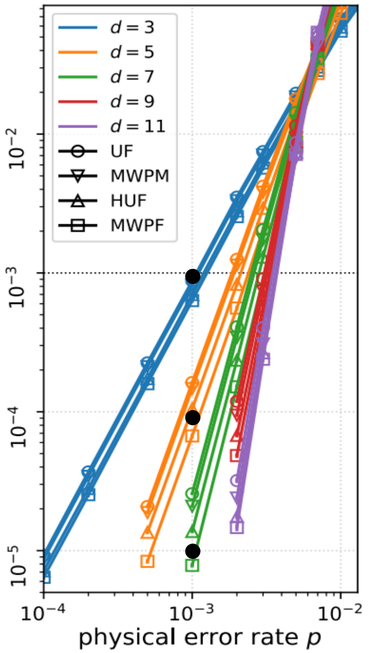
## Progress

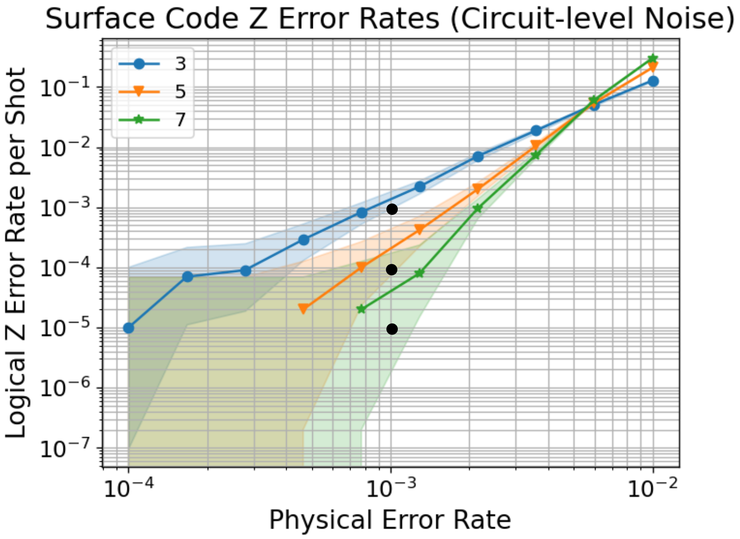
- Conducted circuit-level noise simulation for Rotated Surface codes and Triangular Color codes provided in Stim package.
- Compared to the results in the reference, Our results were almost identical for the Surface codes and little-bit better for Color codes of same size.




(a) bit-flip noise




(d) circuit-level noise

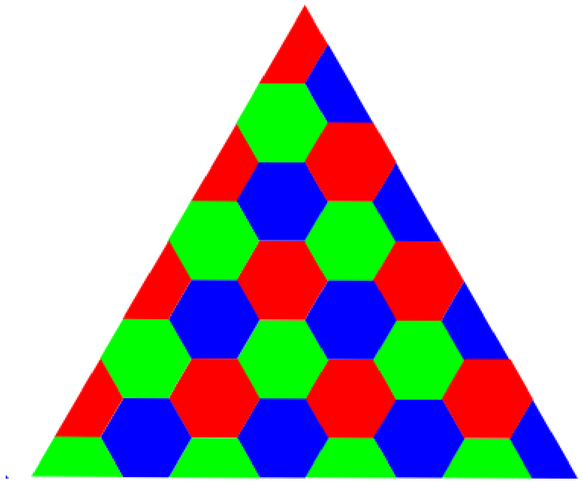
# Recreating the results from “Minimum-Weight Parity Factor Decoder for Quantum Error Correction”

## Objective

Verifying the circuit-level noise simulation by comparing the results in the reference.

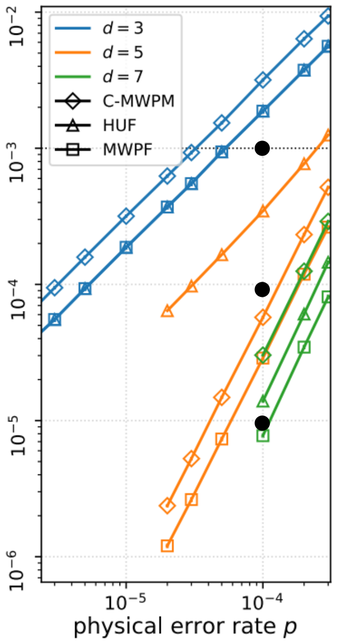
## Progress

- Conducted circuit-level noise simulation for Rotated Surface codes and Triangular Color codes provided in Stim package.
- Compared to the results in the reference, Our results were almost identical for the Surface codes and little-bit better for Color codes of same size.

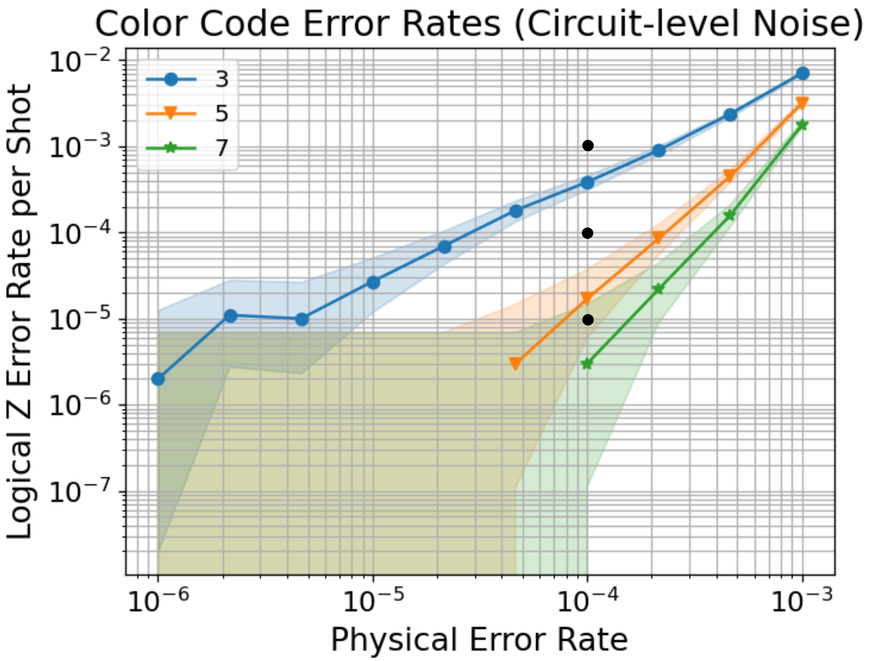


6.6.6 code

$[[(3d^2+1)/4, 1, d]]$




(b) circuit-level noise



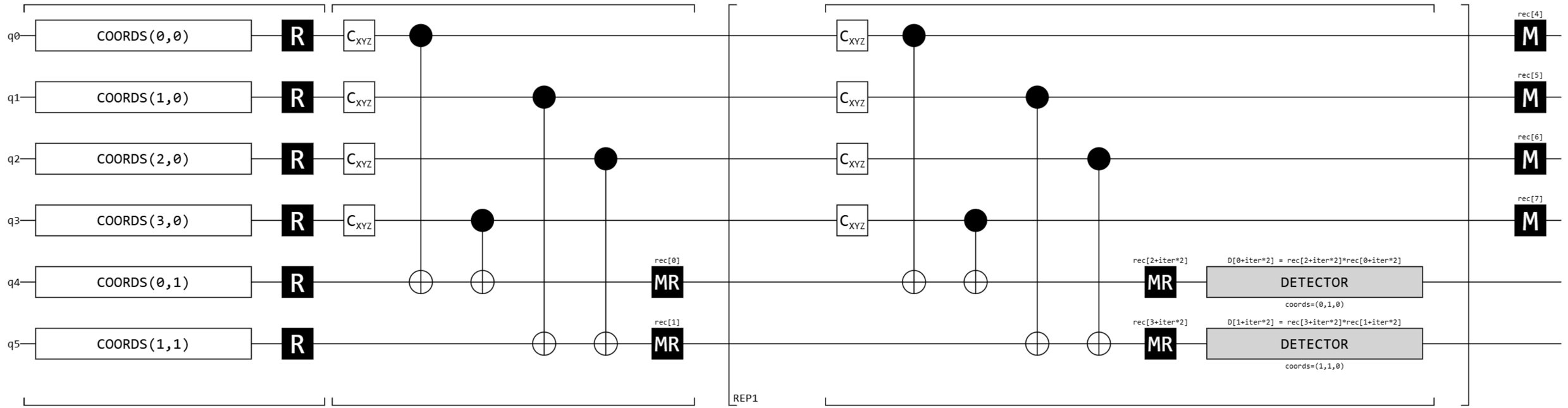
# Circuit-level noise simulation for self-dual CSS codes via Stim

## Objective

Studying the error capacity of our self-dual CSS/CSS-T codes.

## Progress

- Designed measurement circuit for syndrome extraction.



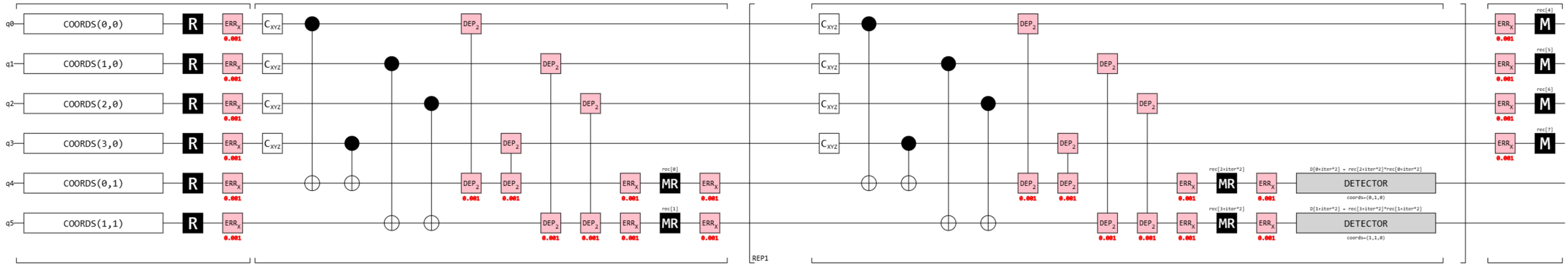
# Circuit-level noise simulation for self-dual CSS codes via Stim

## Objective

Studying the error capacity of our self-dual CSS/CSS-T codes.

## Progress

- Added noise for various noise models(Standard circuit-level Noise model, SI1000 Noise model).



# Circuit-level noise simulation for self-dual CSS codes via Stim

## Objective

Studying the error capacity of our self-dual CSS/CSS-T codes.

## Progress

- Added noise for various noise models(Standard circuit-level Noise model, SI1000 Noise model).

```python
def standard_depolarizing_noise_model(
        circuit: stim.Circuit,
        data_qubits: list[int],
        after_clifford_depolarization: float,
        after_reset_flip_probability: float,
        before_measure_flip_probability: float,
        before_round_data_depolarization: float
) -> stim.Circuit:
    result = stim.Circuit()
    first_reset_seen = False
    tick_count = 0

    for instruction in circuit:
        if isinstance(instruction, stim.CircuitRepeatBlock):
            result.append(stim.CircuitRepeatBlock(
                repeat_count=instruction.repeat_count,
                body=standard_depolarizing_noise_model(
                    instruction.body_copy(),
                    data_qubits,
                    after_clifford_depolarization,
                    after_reset_flip_probability,
                    before_measure_flip_probability,
                    before_round_data_depolarization
                )))
        elif instruction.name == 'R' and not first_reset_seen:
            result.append(instruction)
            result.append('X_ERROR', instruction.targets_copy(), after_reset_flip_probability)
            first_reset_seen = True
        elif instruction.name in ['CNOT', 'CX', 'CZ']:
            result.append(instruction)
            result.append('DEPOLARIZE2', instruction.targets_copy(), after_clifford_depolarization)
        elif instruction.name == 'MR':
            result.append('X_ERROR', instruction.targets_copy(), before_measure_flip_probability)
            result.append(instruction)
            result.append('X_ERROR', instruction.targets_copy(), after_reset_flip_probability)
        elif instruction.name == 'M':
            result.append('X_ERROR', instruction.targets_copy(), before_measure_flip_probability)
            result.append(instruction)
        elif instruction.name == 'TICK':
            result.append(instruction)
            tick_count += 1
            # Assuming a standard surface code schedule where a round is ~9 ticks?
            # Note: Your specific circuit might not follow the 9-tick structure exactly.
            # This logic adds idle noise at start of rounds.
            if first_reset_seen and before_round_data_depolarization > 0:
                # Simple heuristic: Apply if it looks like the start of a round block
                # or simply apply it if your schedule relies on TICKs.
                # Original code logic: if tick_count >= 2 and (tick_count - 1) % 9 == 1:
                # Adjusting to apply generally for this example:
                pass
        else:
            result.append(instruction)
```

```python
def si1000_noise_model(
        circuit: stim.Circuit,
        data_qubits: list[int],
        probability: float
) -> stim.Circuit:
    # 1. Pre-scan to find all used qubits in the entire circuit
    all_qubits_in_circuit = set()
    for op in circuit.flattened():
        for t in op.targets_copy():
            if t.is_qubit_target:
                all_qubits_in_circuit.add(t.value)

    all_qubits_list = list(all_qubits_in_circuit)
    result = stim.Circuit()
    first_reset_seen = False
    tick_count = 0

    for instruction in circuit:
        # Handle repeat blocks recursively
        if isinstance(instruction, stim.CircuitRepeatBlock):
            result.append(stim.CircuitRepeatBlock(
                repeat_count=instruction.repeat_count,
                body=si1000_noise_model(instruction.body_copy(), data_qubits, probability)
            ))
            continue

        # Extract targeted qubits for the current instruction
        targets = [t.value for t in instruction.targets_copy() if t.is_qubit_target]

        # --- NOISE LOGIC ---

        # 1. Initial Reset (R)
        if instruction.name == 'R' and not first_reset_seen:
            result.append(instruction)
            result.append('X_ERROR', targets, 2 * probability)
            # Idle noise on qubits not being reset
            idle_qubits = list(all_qubits_in_circuit - set(targets))
            if idle_qubits:
                result.append('DEPOLARIZE1', idle_qubits, 2 * probability)
            first_reset_seen = True

        # 2. Single Qubit Gates
        elif instruction.name in ['H', 'S', 'X', 'Y', 'Z', 'S_DAG', 'H_DAG']:
            result.append(instruction)
            result.append('DEPOLARIZE1', targets, probability / 10)

        # 3. Two-Qubit Gates (CNOT)
        elif instruction.name in ['CNOT', 'CX']:
```

# Circuit-level noise simulation for self-dual CSS codes via Stim

## Objective

Studying the error capacity of our self-dual CSS/CSS-T codes.

## Observation and Discussion

- Implanted the self-dual CSS codes and got the standard depolarizing circuit-level noise simulation results.

Qiskit-CSS-T / doubling-CSST / alistMats / GO03_self_dual /

LeaCat added n24 d8 self-dual code

Name

..

n10_d2.alist

n12_d4.alist

n14_d4.alist

n16_d4.alist

n18_d4.alist

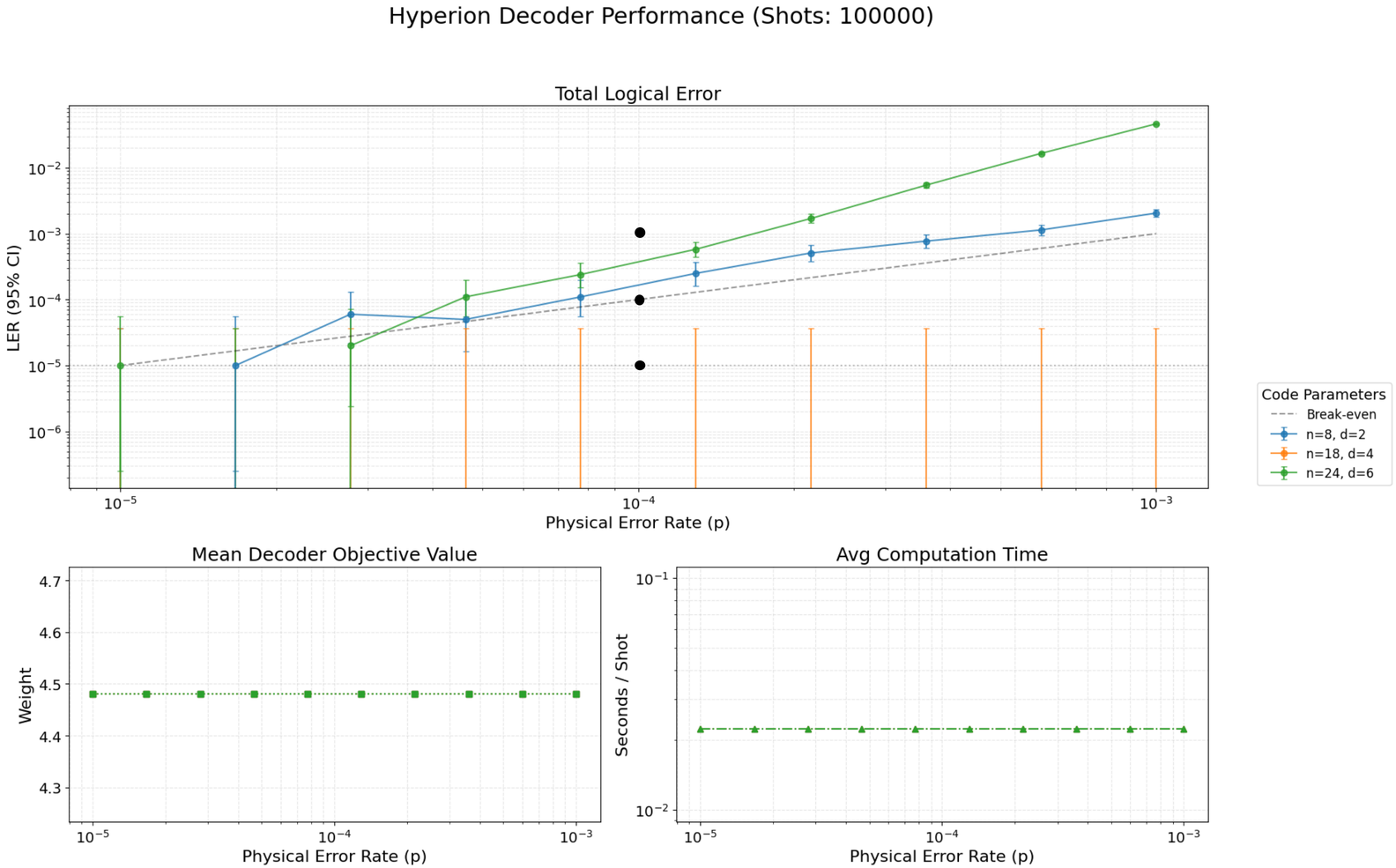
n20_d4.alist

n22_d6.alist

n24_d6.alist

n24_d8.alist

n26_d6.alist



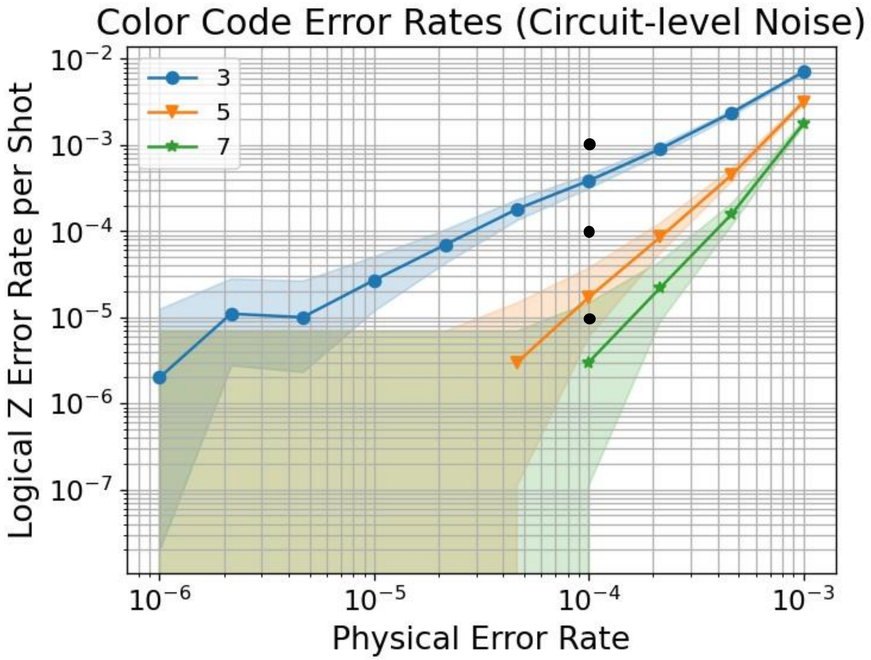
# Circuit-level noise simulation for self-dual CSS codes via Stim

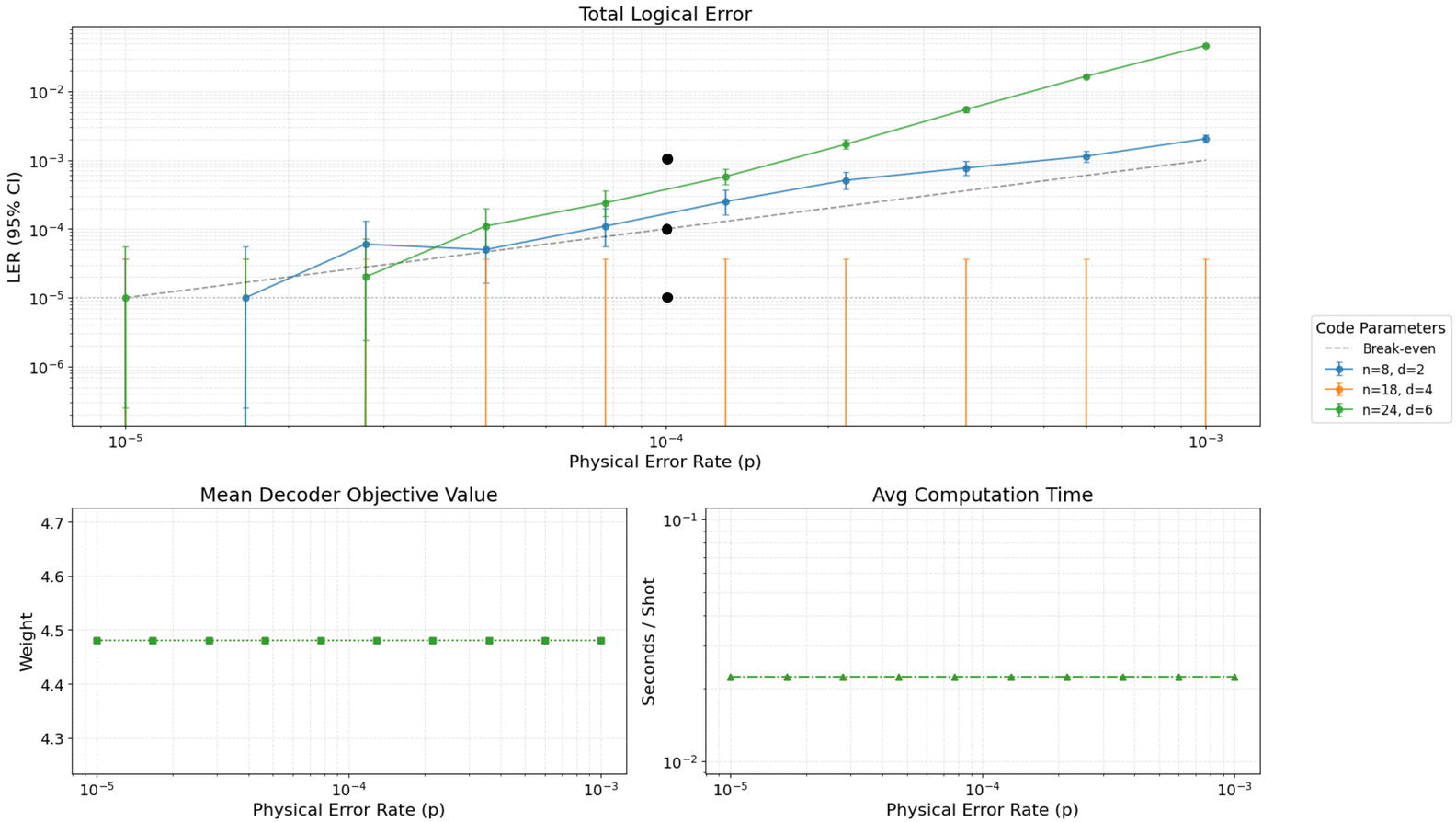
## Objective

Studying the error capacity of our self-dual CSS/CSS-T codes.

## Observation and Discussion

- Implanted the self-dual CSS codes and got the standard depolarizing circuit-level noise simulation results.
- Distance 2 results are comparable and distance 6 worked worse than the Color codes, but distance 4 results went under the resolution limit which is quite exciting!



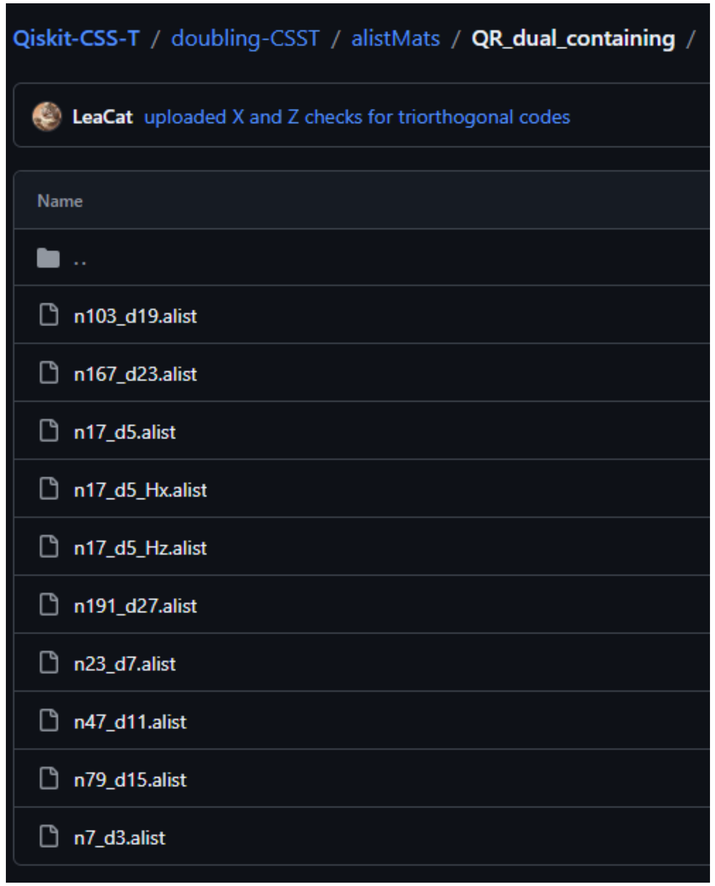
# Circuit-level noise simulation for self-dual CSS codes via Stim

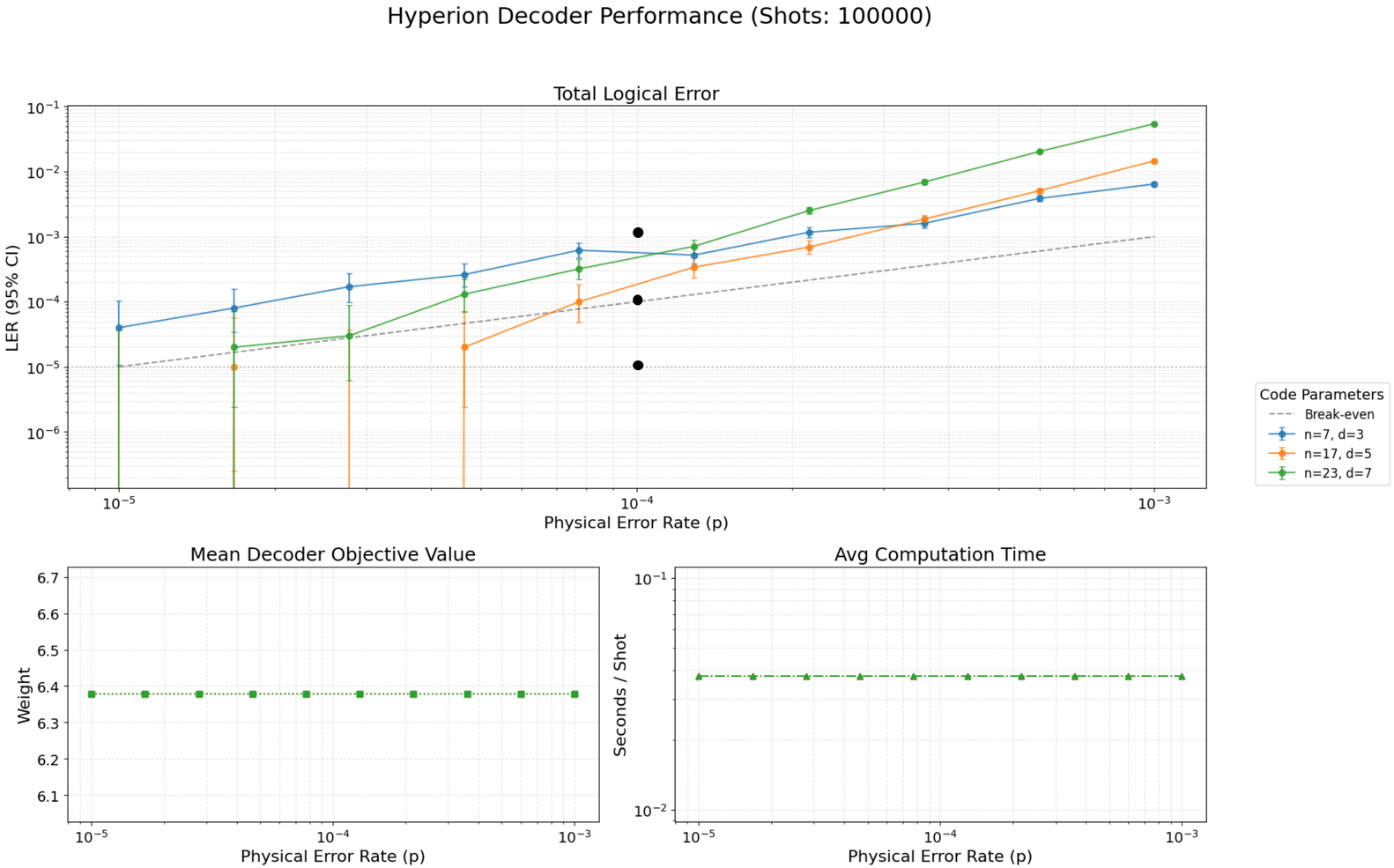
## Objective

Studying the error capacity of our self-dual CSS/CSS-T codes.

## Observation and Discussion

- Implanted the dual-containing CSS codes and got the standard depolarizing circuit-level noise simulation results.



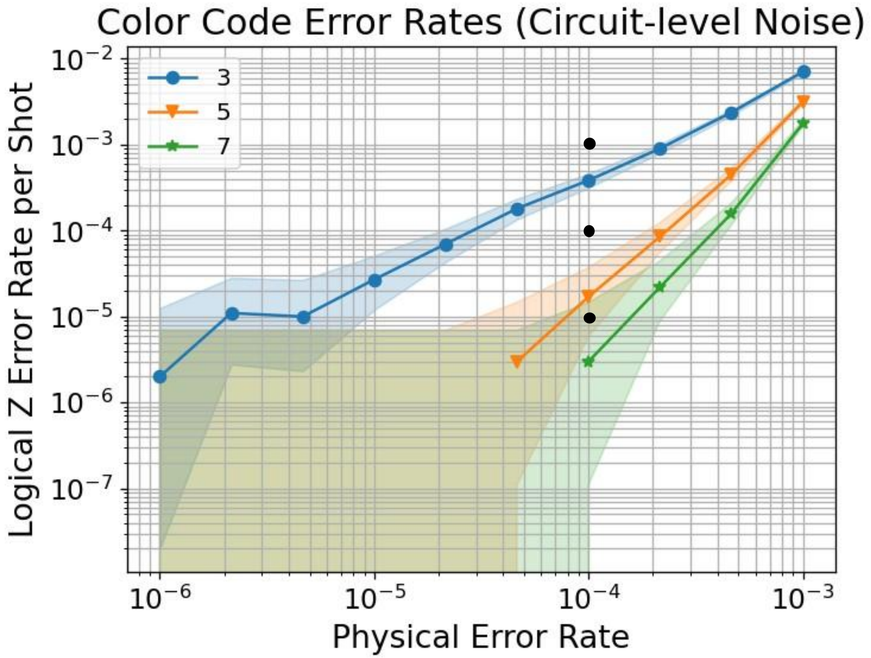
# Circuit-level noise simulation for self-dual CSS codes via Stim

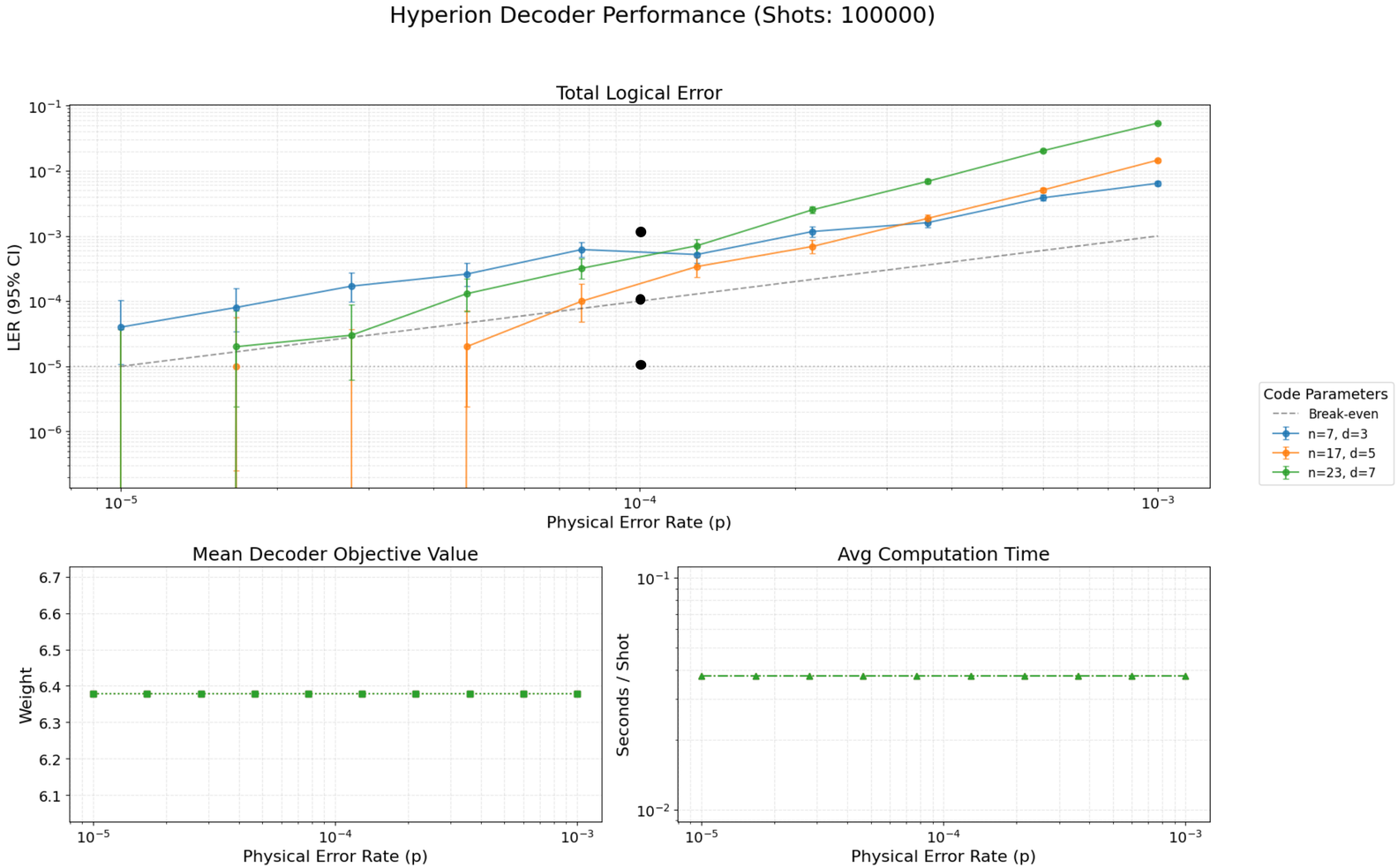
## Objective

Studying the error capacity of our self-dual CSS/CSS-T codes.

## Observation and Discussion

- Implanted the dual-containing CSS codes and got the standard depolarizing circuit-level noise simulation results.
- The dual containing codes results were worse from the examples tested, barely touching the break-even.

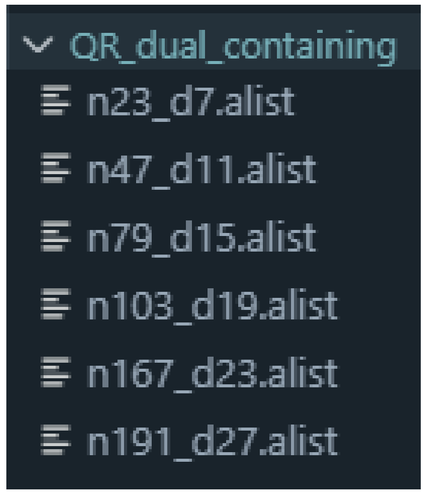


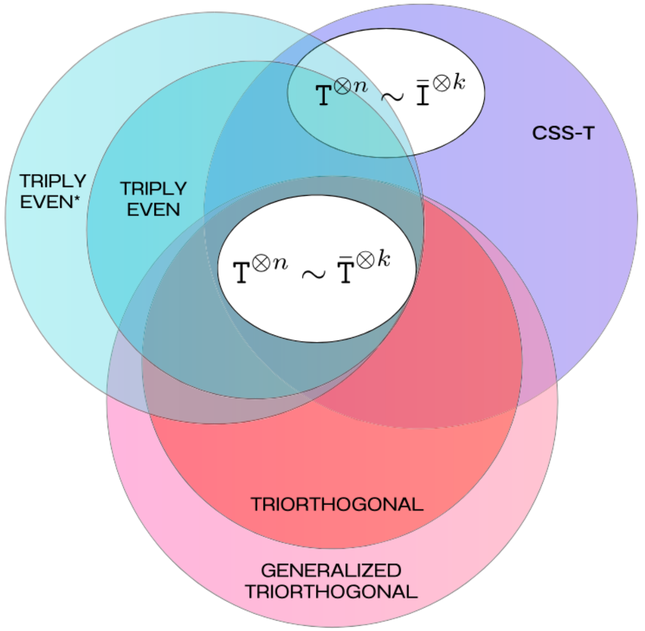



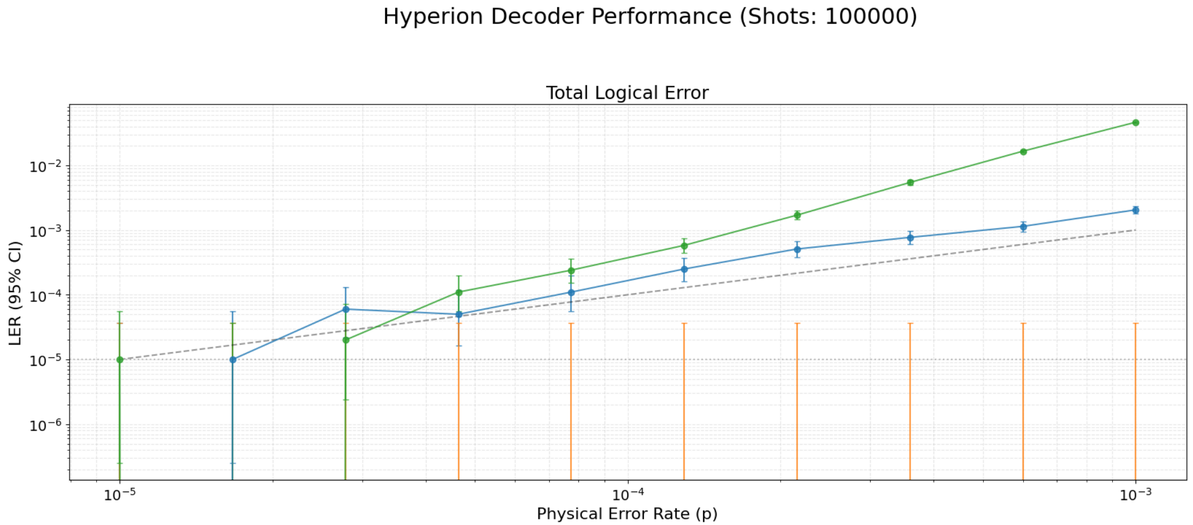
# Future work

- Looking for self-dual codes bridging the distance gap

- Going to test decoding of Tri-orthogonal codes which has Transversal T gates.



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