Problem Set 10

1. $X_1, X_2, \ldots, X_n$ are independent and identically distributed random variables w/ mean value μ and standard deviation σ

sample total: $T = X_1 + X_2 + \ldots + X_n = \overline{X}n$

a) Prove $E(T) = n\mu$: $\boxed{E(T) = E(\overline{X}n) = nE(\overline{X}) = n\mu}$

constant

sample total

$\frac{T}{n} = \overline{X}$ → sample mean ; Therefore $T = \overline{X}n$

n → # of elements in sample

$E(\overline{X}) = \mu$

expectation of sample mean = population mean

b) Prove $Std(T) = \sqrt{n}\,\sigma$: $Var(T) = Var(\overline{X}n) = n^2 Var(\overline{X}) = n^2 \cdot \frac{\sigma^2}{n} = n\sigma^2$

constant

$Var(\overline{X}) = \frac{\sigma^2}{n}$ because $Std(\overline{X}) = \frac{\sigma}{\sqrt{n}}$

$\boxed{Std(T) = \sqrt{n\sigma^2} = \sqrt{n}\,\sigma}$

2. 40 students ($n = 40 \geq 30$ → sample total has approximately a normal distribution)

$E(\overline{X}) = 6$ $Std(\overline{X}) = 6$

a) From 6:50 pm to 11 pm → 250 minutes ; Finding $P(T \leq 250)$

Expected value: $E(T) = n\overset{E(\overline{X})}{\mu} = 40 \cdot 6 = 240$ → Therefore $T \sim N(240, 37.947)$

Standard Deviation: $Std(T) = \sqrt{n}\,\overset{Std(\overline{X})}{\sigma} = \sqrt{40} \cdot 6 = 37.947$

$z = \frac{250 - 240}{37.947} = .264$ → $P(z \leq .264) = \boxed{0.6026}$

missing sports report

b) Finding $P(T > 260)$: $P(T > 260) = 1 - P(T \leq 260)$ → $1 - P(z \leq .527)$

.53

$z = \frac{260 - 240}{37.947} = .527$

$1 - .7019 = \boxed{0.2981}$