Problem Set 11

25. sample of 50 persons ($n=50$); $\sigma = \$600$; $\bar{x} = \$1599$

a) Find margin of error (MOE) for 95% confidence interval (CI)

CI for μ when σ known $= \left[\bar{x} + z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}\right]$

MOE

Therefore, $MOE = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$;

$1 - \alpha = .95 \rightarrow \alpha = .05 \rightarrow \frac{\alpha}{2} = .025$

$P(Z \geq z_{\alpha/2}) = .025 \rightarrow 1 - P(Z \leq z_{\alpha/2}) = .025$

$P(Z \leq z_{\alpha/2}) = .975$

$z_{\alpha/2} = 1.96$

$MOE = 1.96 \cdot \frac{600}{\sqrt{50}} = 196 \cdot 84.85 = 166.31$

MOE = \$166.31

b) Find sample size (n) for MOE ≤ \$150.

$150 = 1.96 \cdot \frac{600}{\sqrt{n}}$ from $MOE = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$

$76.53 = \frac{600}{\sqrt{n}}$

$\sqrt{n}\ 76.53 = 600$

$\sqrt{n} = 7.84$

$n = 61.47$

The sample size must be at least 62 persons to make the margin of error be no larger than \$150.

Problem Set 11

26. $\sigma = 25$ pounds; MOE = 3 pounds; Find sample size (n) for 90% CI with MOE = 3 pounds.

→ Confidence Interval

CI for μ when σ known $= \left[\bar{x} + z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}\right]$

($z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$ = MOE)

Therefore, $MOE = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$:

Finding $z_{\alpha/2}$: $1 - \alpha = .90 \rightarrow \alpha = .1 \rightarrow \frac{\alpha}{2} = .05$

$P(Z \geq z_{\alpha/2}) = .05 \rightarrow 1 - P(Z \leq z_{\alpha/2}) = .05$

$P(Z \leq z_{\alpha/2}) = .95$

$z_{\alpha/2} = 1.64$

$MOE = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}} \rightarrow 3 = 1.64 \cdot \frac{25}{\sqrt{n}}$

$1.83 = \frac{25}{\sqrt{n}}$

$1.83\sqrt{n} = 25$

$\sqrt{n} = 13.66$

$n = 186.63$

The required sample size to construct a 90% Confidence Interval is 187 persons.