PCS Report

Lucas Riedstra

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1 Introduction

The Hodgkin-Huxley equations are given by the following system of ODE's:

$$\begin{cases} C_m \frac{\mathrm{d}V}{\mathrm{d}t} & = I_{\mathrm{app}}(t) - I_{\mathrm{K}}(V) - I_{\mathrm{Na}}(V) - I_L(V); \\ \frac{\mathrm{d}\star}{\mathrm{d}t} & = (1-\star)\alpha_\star(V) - \star\beta_\star(V), \end{cases} \tag{1}$$

with $\star \in \{n, m, h\}$.

When we have a propagated action potential, it is the case that

$$I_{\mathrm{app}} = \frac{a}{2R}\frac{\partial^2 V}{\partial x^2}, \tag{2}$$

which is an equation derived from *cable theory*.

We can rewrite the equations in terms of the *traveling wave varibale* $\xi = x - ct$, where c is the traveling speed of the wave. The idea is that if the wave travels with a fixed speed c, then it must be the case that $x = ct$ (if $x = 0$ for the starting position), and therefore, V must be a function of only the variable $x - ct$ (that is, if $x_1 - ct_1 = x_2 - ct_2$, then $V(x_1, t_1) = V(x_2, t_2)$).

For a function $u(x)$ we have $\frac{\mathrm{d}u}{\mathrm{d}x} = \frac{\partial u}{\partial \xi}$, while for a function $u(t)$ we have $\frac{\mathrm{d}u}{\mathrm{d}t} = -c\frac{\partial u}{\partial \xi}$. Therefore we can combine (1) and (2) to obtain

$$\begin{cases} -cC_m \frac{\mathrm{d}V}{\mathrm{d}\xi} & = \frac{a}{2R}\frac{\mathrm{d}V}{\mathrm{d}\xi^2} - I_{\mathrm{K}}(V) - I_{\mathrm{Na}}(V) - I_L(V); \\ -c\frac{\mathrm{d}\star}{\mathrm{d}\xi} & = (1-\star)\alpha_\star(V) - \star\beta_\star(V). \end{cases} \tag{3}$$

Now all we must do is set $W = \frac{\mathrm{d}V}{\mathrm{d}\xi}$, and we get the system of equations

$$\begin{aligned} \frac{\mathrm{d}V}{\mathrm{d}\xi} &= W; \\ \frac{\mathrm{d}W}{\mathrm{d}\xi} &= \frac{2R}{a}(I_K(V) + I_{\mathrm{Na}}(V) + I_L(V) - cC_m W) \\ \frac{\mathrm{d}\star}{\mathrm{d}\xi} &= \frac{-1}{c}((1-\star)\alpha_\star(V) - \star\beta_\star(V)). \end{aligned}$$