Thermal effects on action potentials in squid giant axon

Simulations using the Hodgkin-Huxley model

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Research questions

1. What is the relationship between the temperature and the duration of an action potential?
2. How does temperature affect the shape of the membrane potential curve during an AP?

Hypothesis

Biological processes are generally known to speed up as temperature increases. Therefore we hypothesize that the duration of APs shall decrease as temperature increases. We expect that all membrane potential curves resemble the theoretical curve (fig. 2) during an action potential.

Model description

The different phases of an action potential are represented in the following voltage-time graph.

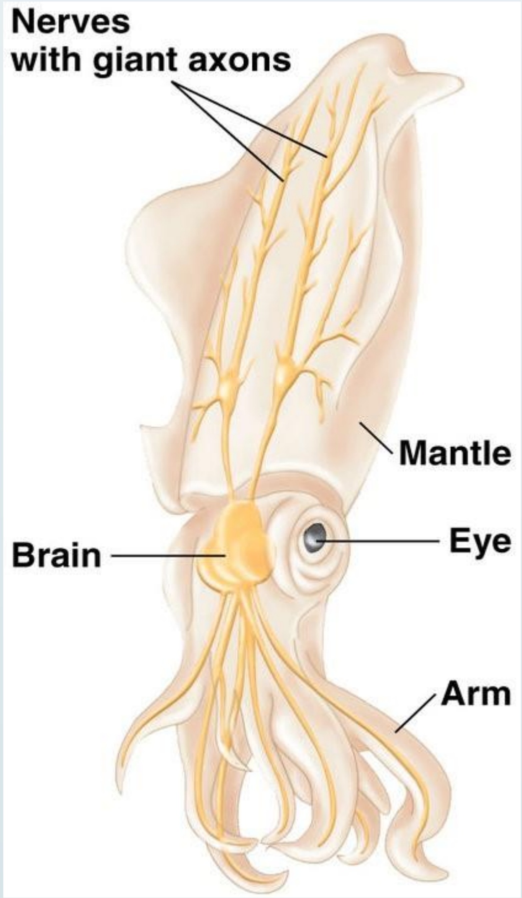

Fig. 1: Squid giant axon

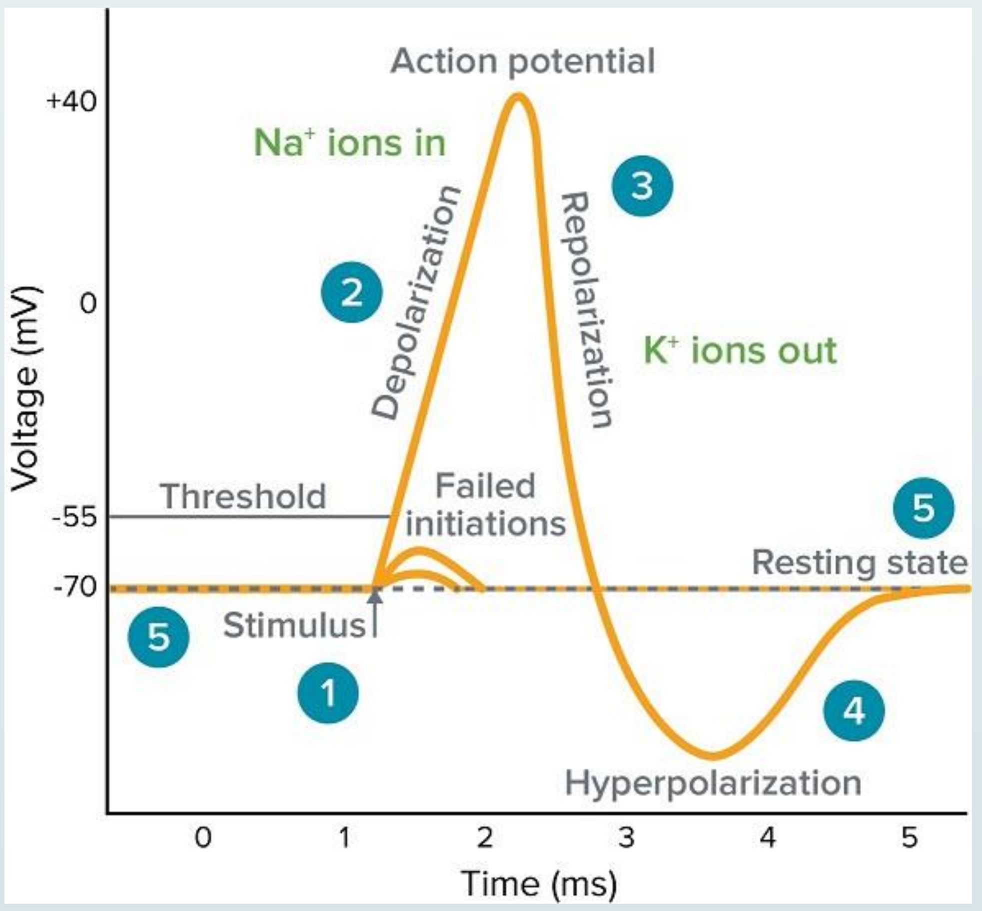

Fig. 2: Typical action potential

It is known that a cell membrane can be modelled as a capacitor, which yields the following equation:

$$C_m \frac{dV}{dt} = -I_{ion}(V,t).$$

Hodgkin & Huxley split I_{ion} into four components: the (externally) applied current, the Na^+ current, the K^+ current, and the remaining currents.

Method

The splitting of I_{ion} into four components transforms the membrane equation above into a system of four coupled ODEs. We approximate the solution by using the Runge-Kutta 4 method for vector-valued functions. This solves equations of the form $\dot{\boldsymbol{x}} = f(\boldsymbol{x})$ with $\boldsymbol{x} \in \mathbb{R}^n, f: \mathbb{R}^n \to \mathbb{R}^n$.

Results

In fig. 3 we show a least-squares fit of degrees 2 and 3 through the first 48 data points. The MSE of the approximation is ± 0.38 for degree 2 and ± 0.0066 for degree 3.

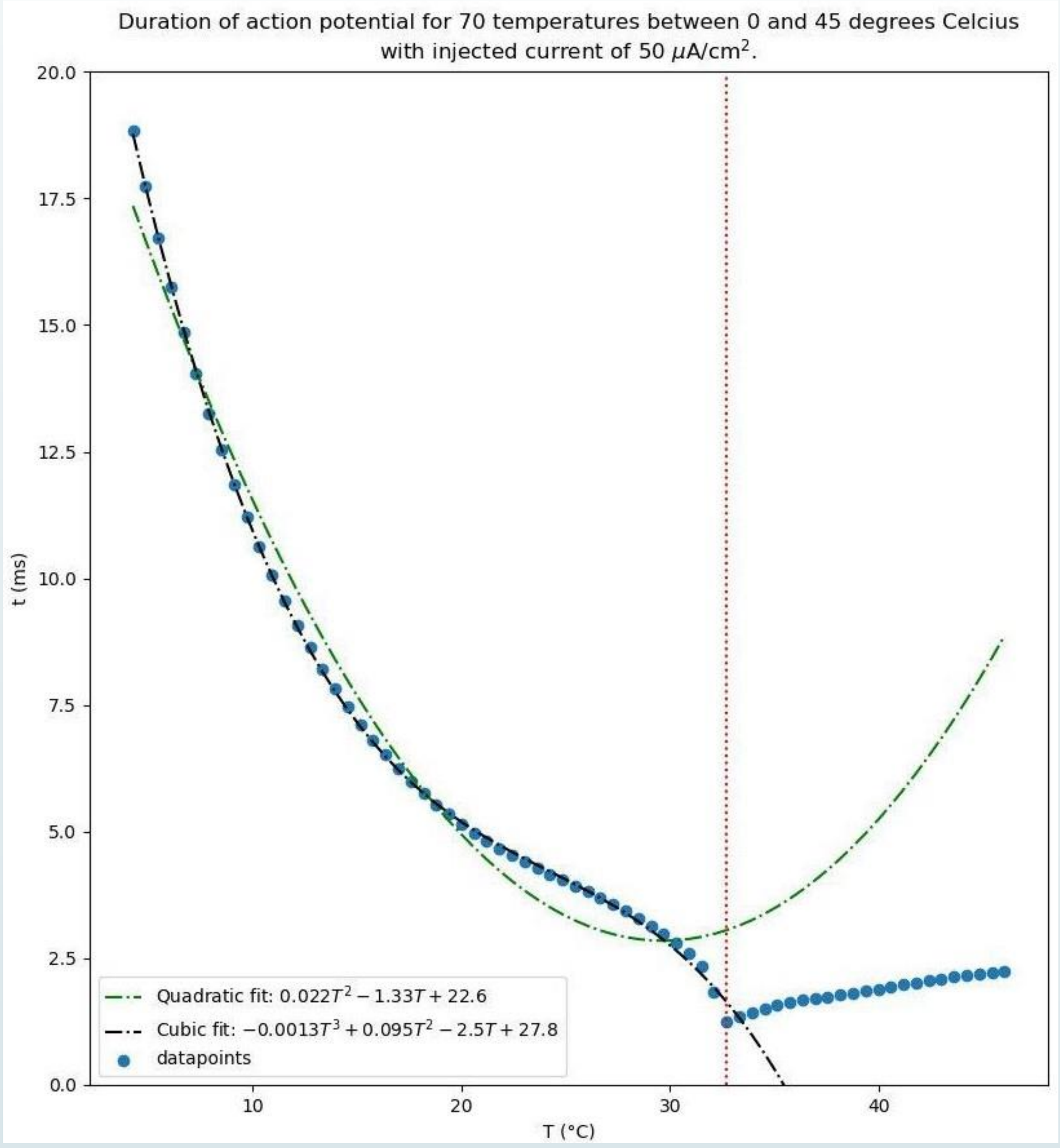

Fig. 3: Duration of action potential plotted against temperature with polynomial fitting

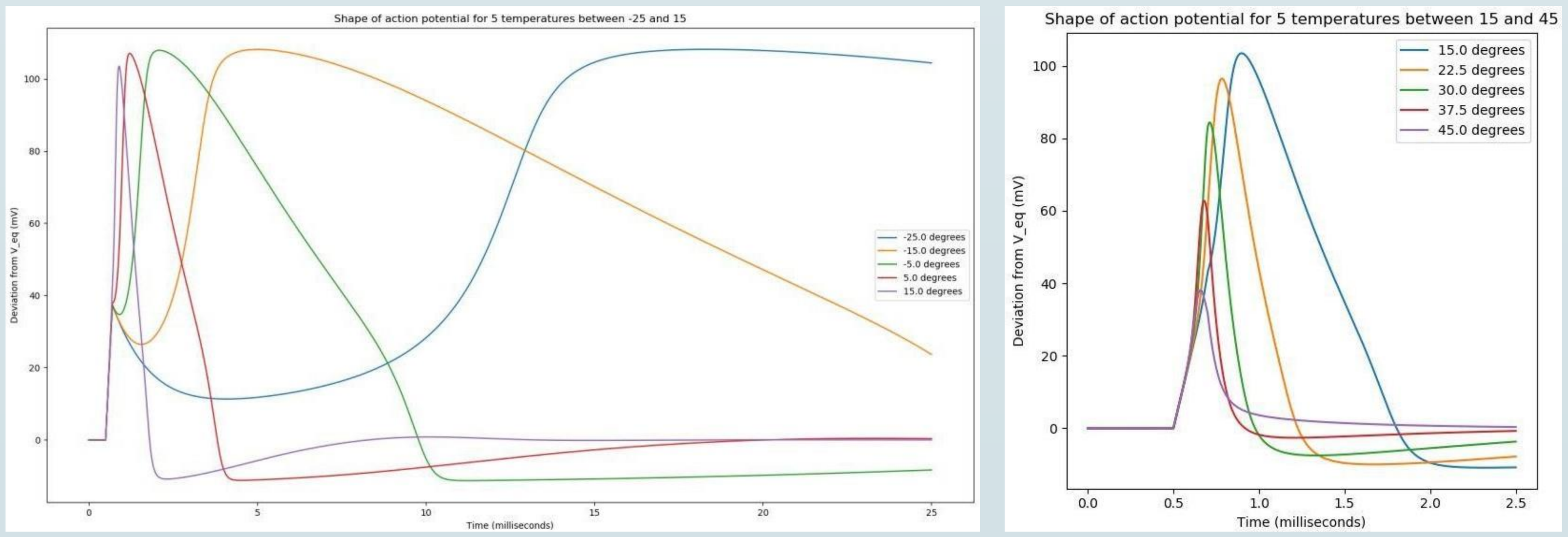

Fig. 4: Membrane potential during an action potential for low (left) and high (right) temperatures

Conclusion

A clear conclusion we can draw from fig. 3 is that the duration of an action potential is shortened as temperature increases, agreeing with our hypothesis. The polynomial fits strongly suggest that this relationship is cubic. The sudden increase after the red dashed line is due to the fact that no action potential occurs for high temperatures with this current strength. This high threshold corresponds with our model verification in fig. 6.

In figures 4 and 5 we miss the hyperpolarization phase for high temperatures and we see a potential decrease after injection for low temperatures. It is unclear if this has biological reasons or is due to a flaw in the model. Another unanticipated result is that the peak voltage decreases as temperature increases.

Model verification

To verify the model, we test whether it satisfies the *all-or-nothing principle:* for a fixed temperature, the peak voltage should not depend on the injected current after some threshold.

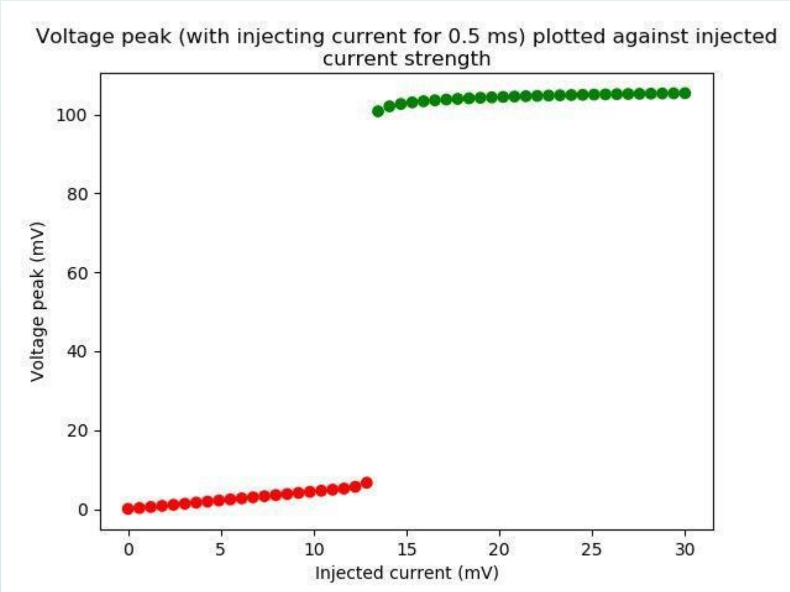

Fig. 5: Model verification
T = 6.3 degrees celsius

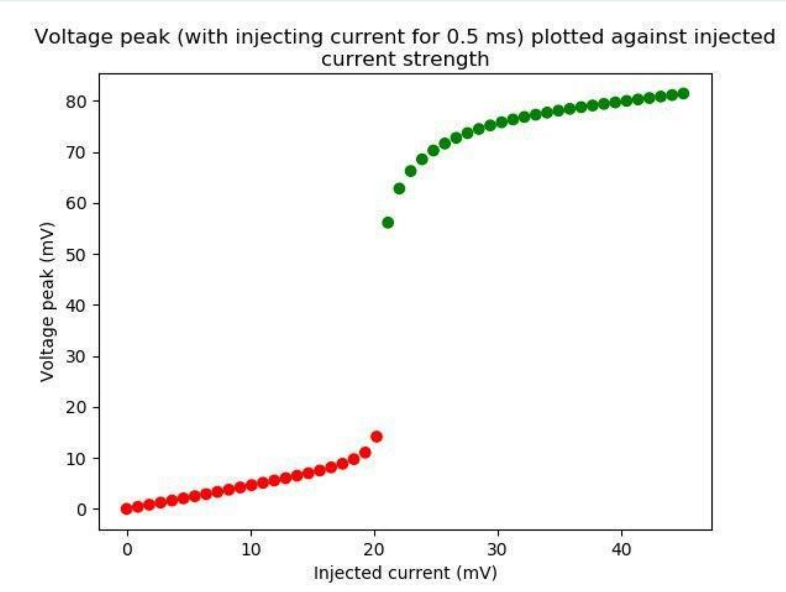

Fig. 6: Model verification
T = 26.3 degrees celsius

Our tests show that the model indeed satisfies this principle at T = 6.3. For T = 26.3 we also see it satisfying the principle, however the jump is less sharp and the threshold value is higher. This calls into question whether the model is correct for higher temperatures.

References

[1] Hodgkin, A., & Huxley, A. (1952). A quantitative description of membrane current and its application to conduction and excitation in nerve. *Journal of Physiology*, 117(4), 500–544.

[2] Keener, J., & Sneyd, J. (2009). *Mathematical Physiology I, Cellular Physiology* (2nd ed.).

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