Numerical continuation in computational biology

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What is computational biology?

- Goal: use maths to understand the mechanisms behind living processes
- Differential equations are used to explain lots of these processes
  - Hodgkin-Huxley: neural dynamics
  - Lotka-Volterra: population dynamics
  - SIR model: epidemic dynamics

Differential equations for biology

Ordinary differential equation

Description of how a system state changes in time

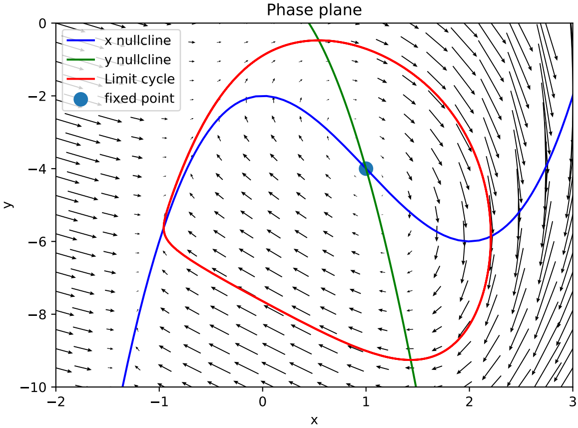
System state

Minimal amount of information to describe something's behaviour

Nonlinear system

A set of ordinary differential equations, where the change in state doesn't follow a simple proportional relationship

Drawing pictures



The role of parameters

Every equation has parameters:

- Some of these are fixed
- Some of these we can play with

The dynamics of a system necessarily depend on these parameters

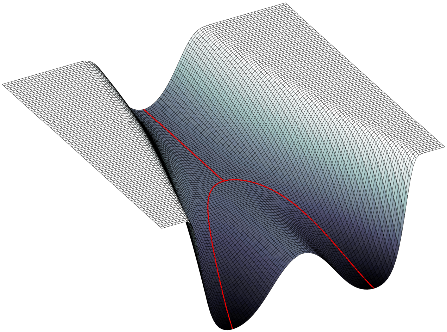
Bifurcations

Bifurcation

If the dynamics of a system change at some parameter value, a bifurcation is said to have occurred

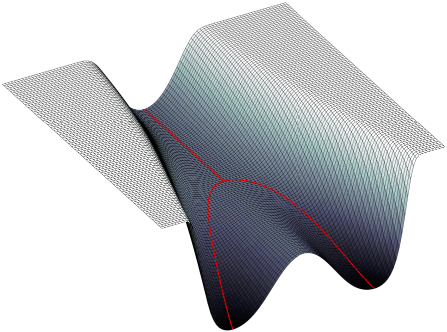
This usually means equilibria or periodic orbits appearing and disappearing – but not always!

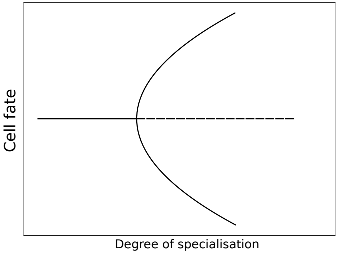
Biological bifurcations



- Waddington describes cell specialisation like marbles rolling down a valley
- When the valley splits, two cell fates emerge
- This is a nice example of a bifurcation!

Biological bifurcations





The role of bifurcation analysis in biology

- Bifurcations can explain seisures, heart attacks, Parkinson's, and many other diseases
- Bifurcations can be used to explain the functionality of biological systems
- Bifurcations can be used to design biological systems

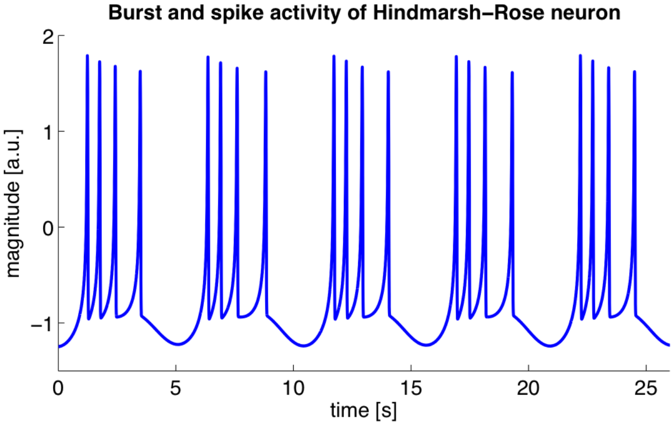
Methods for bifurcation analysis

- Analytical calculations
- Brute force computation
- Numerical continuation

Numerical continuation

- We use numerical continuation to track ‘interesting’ points
  - We vary a parameter
  - Continuation tells us how the point changes
- Test functions indentify bifurcations

Bifurcation analysis of a bursting neuron

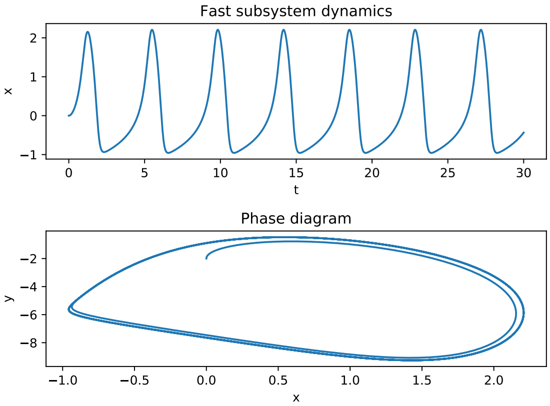


The Hindmarsh Rose model

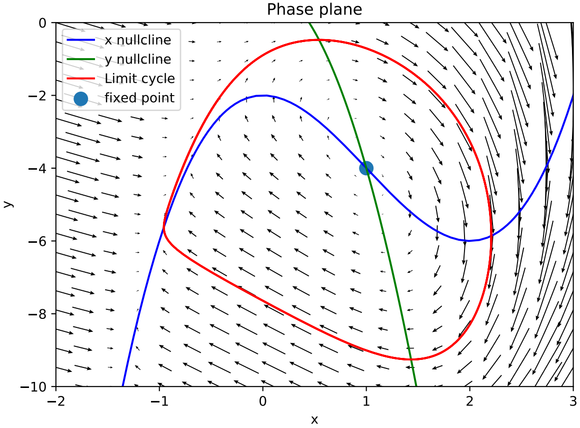
$$
\begin{aligned}
\frac{\mathrm{d}x}{\mathrm{d}t} &= y - ax^3 + bx^2 - z + I \, , \\
\frac{\mathrm{d}y}{\mathrm{d}t} &= c - dx^2 - y \, , \\
\frac{\mathrm{d}z}{\mathrm{d}t} &= r\left[s(x - x_R) - z\right] \, .
\end{aligned} \tag{1}
$$

$$|r| \ll 1$$

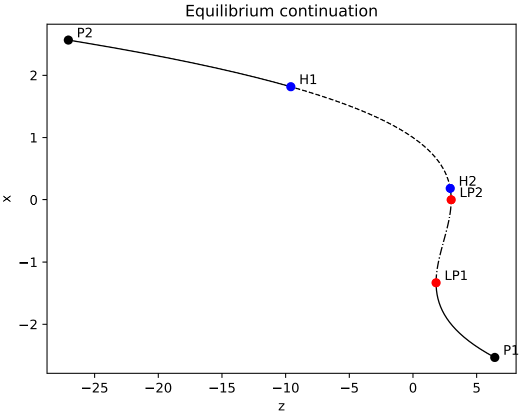
Exploratory step



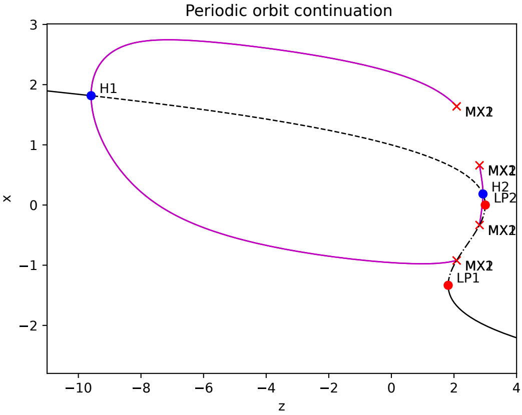
Initialisation step



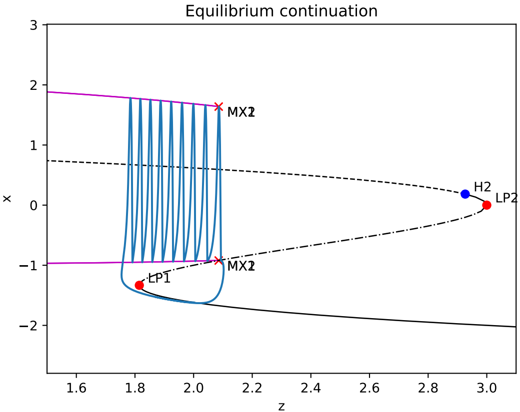
Equilibrium point curve



Periodic orbit continuation



Full system dynamics



Software tools

There's lots of software to do these sorts of calculations!

Questions? Feedback?