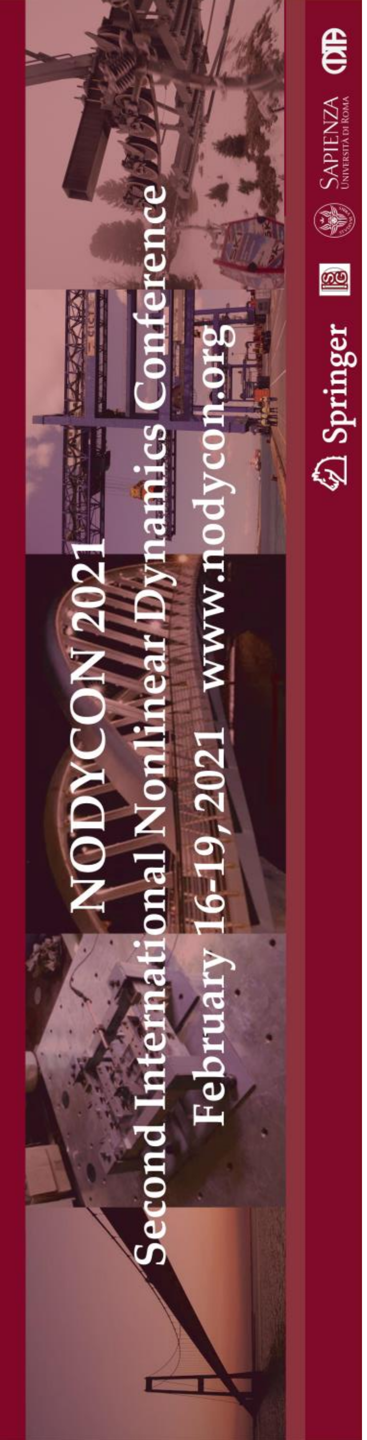

Bayesian local surrogate models for the control-based continuation of multiple-timescale systems

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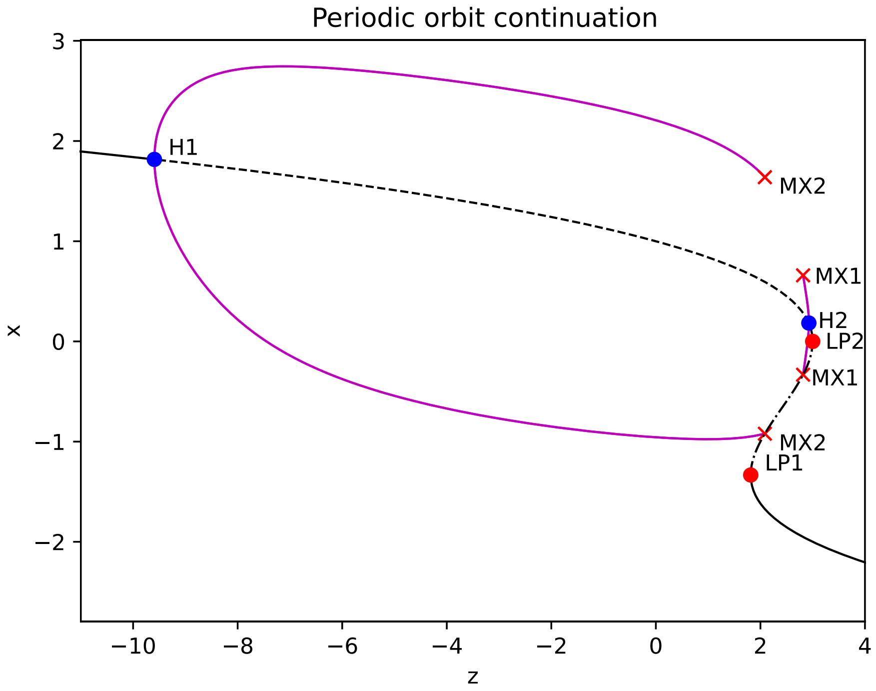
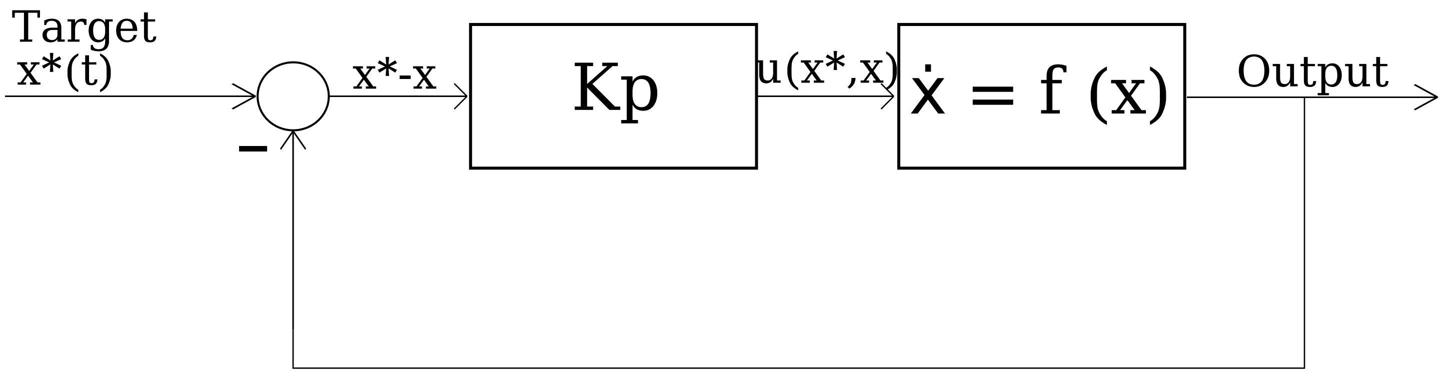
Numerical continuation needs models

- Numerical continuation is the standard tool for computational bifurcation analysis
- Continuation traces out implicitly defined manifolds, and as such, requires a model to define these manifolds
- Usable models are not always available
- Can we apply the same methods in cases where a usable model is unavailable?



Control-based continuation is model-free

- Control-based continuation is a reformulation of the traditional continuation paradigm



$\dot{x} = f\,(x) + u(x,\, x^*)$

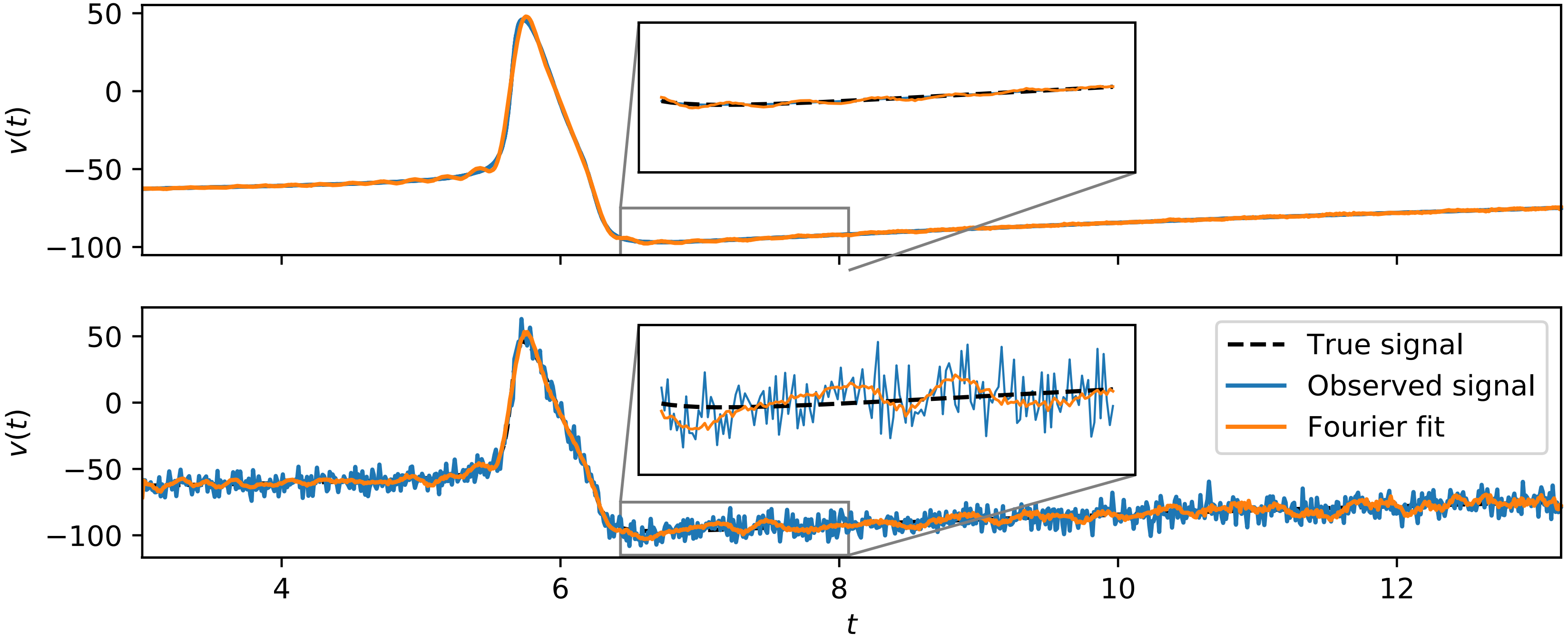
$u \equiv 0 \Rightarrow$ system operates under stabilised natural dynamics

Zero-problem: find x^* such that $u \equiv 0$

- Nonlinear oscillations require discretising before they can be studied

Discretisations are not noise-robust

- All continuation methods use nonlinear solvers for prediction-correction steps
- CBC applications use Fourier discretisation
- Multiple-timescale systems typically require many Fourier harmonics
- Larger numbers of Fourier harmonics cause less noise-robustness
- We can't filter the noise off using simple filters



Surrogate models can clean the data

- Noise reduces the discretisation accuracy, and it can't be filtered off easily; instead we propose a surrogates approach
- Surrogates are a statistical regression model

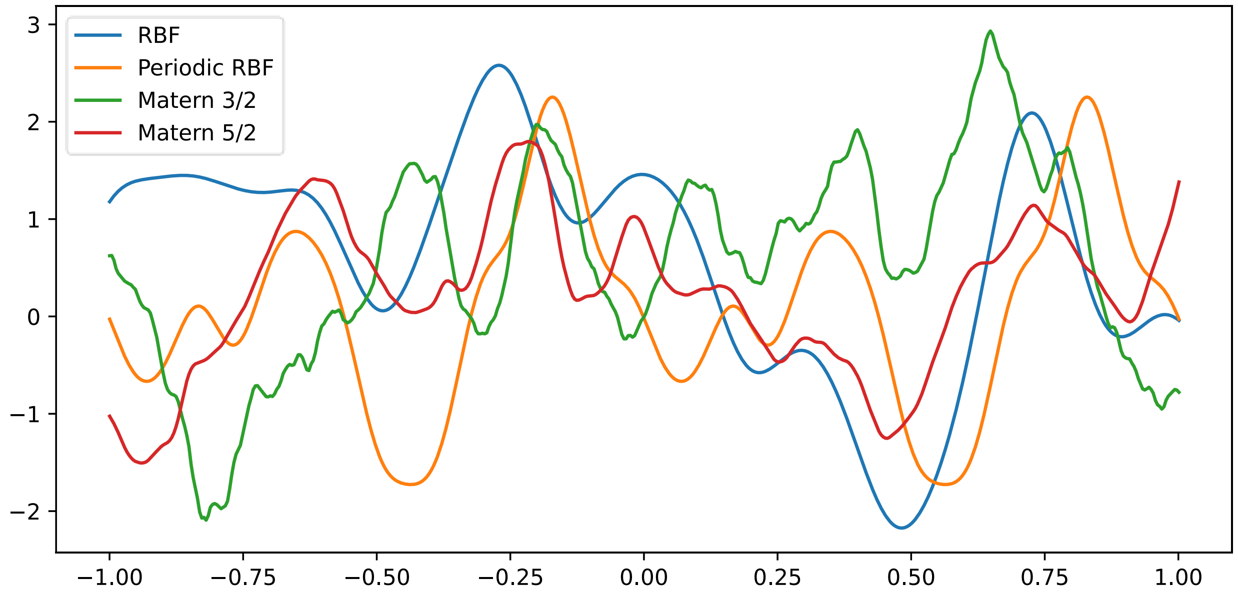
Assume $x_i = f(t_i) + \varepsilon_i$,

$$\varepsilon_i \sim N(0, \sigma^2).$$

- A well-fitted model allows us to separate samples into signal and noise
- Challenge: find some function f(t) that is sufficiently general to describe the signals of interest

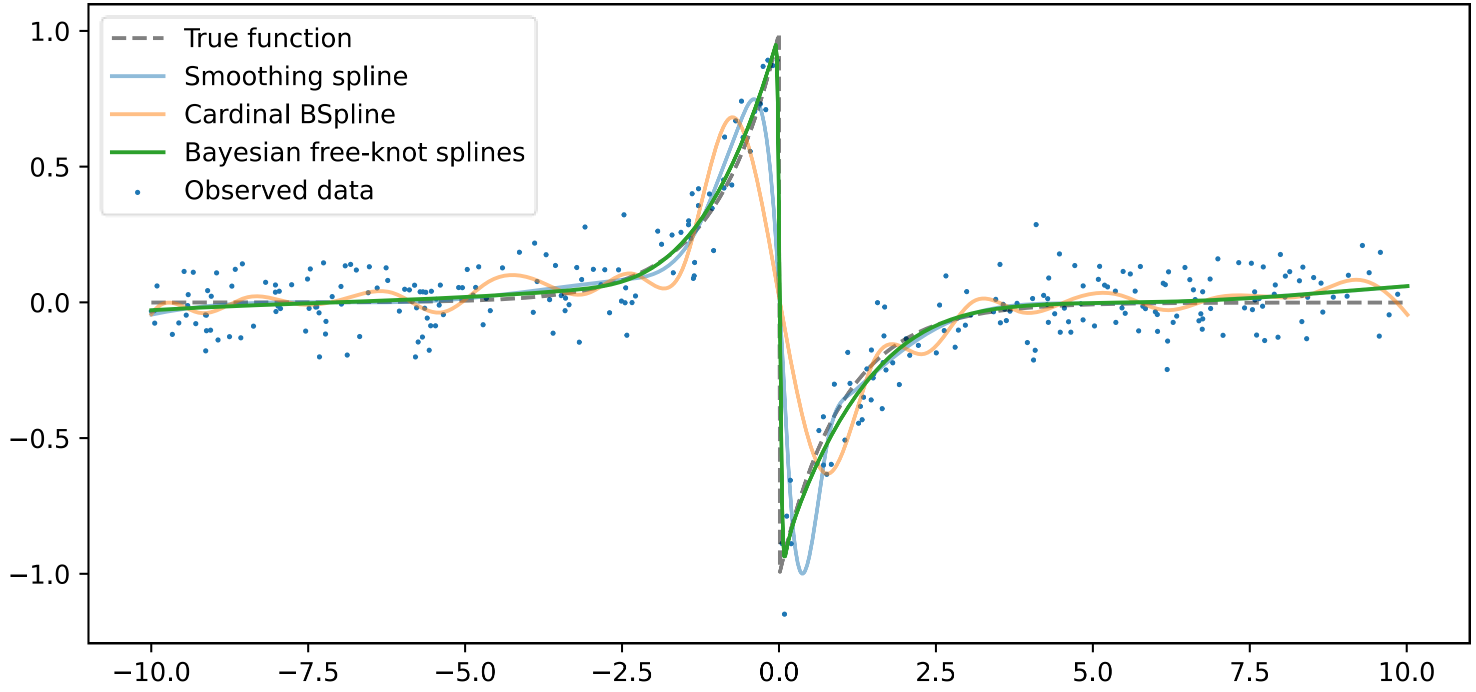
Gaussian process regression

- Gaussian processes generalise the normal distribution to infinite dimensions
- Gaussian process regression is a nonparametric function-space regression method
- Bayesian methods require priors; GP priors are covariance functions
- We use periodic and non-periodic RBF and Matern 3/2, and Matern 5/2 kernels



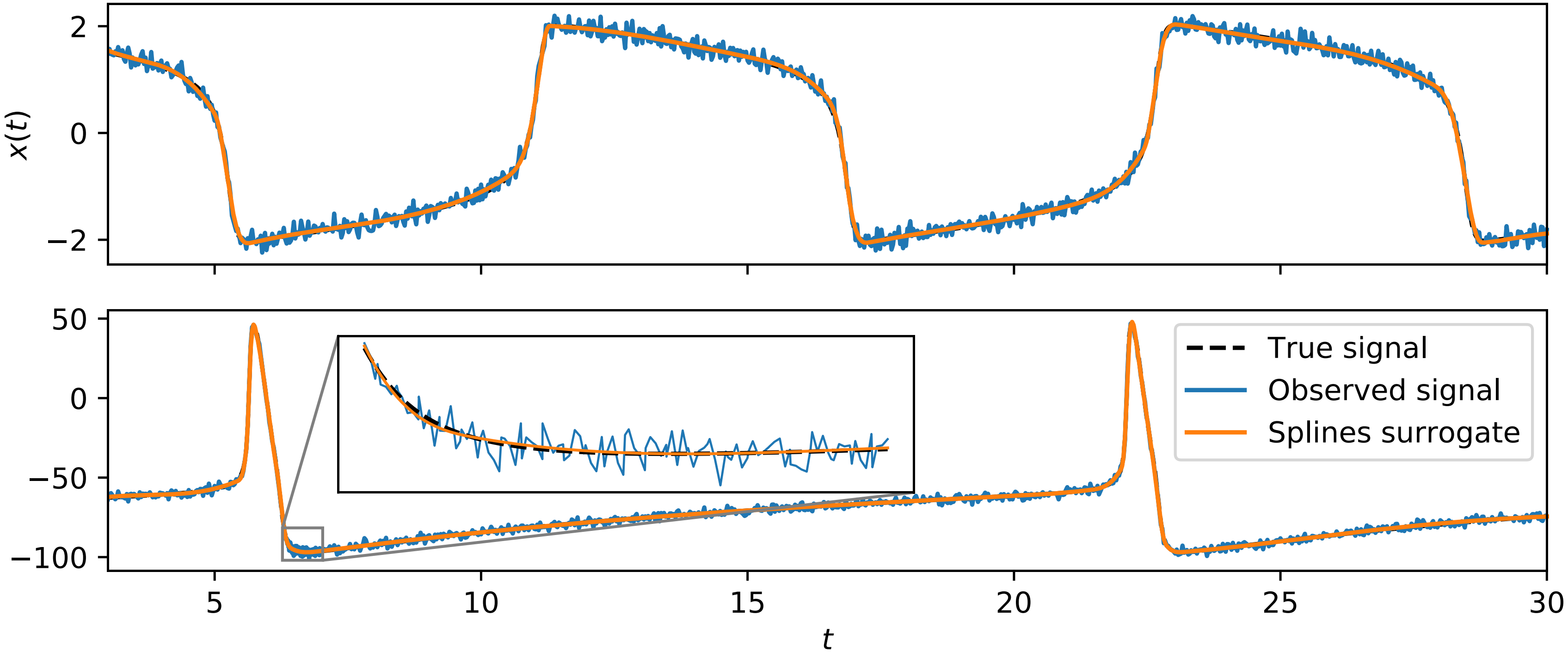
Bayesian free-knot splines

- Spline regressors are maximally smooth piecewise-polynomial curves
- BSplines are a set of basis functions over an associated set of spline curves
- Choosing good BSpline knots can be hard
- Bayesian inference can be used to maintain a distribution over spline curves



No single surrogate model is always best

- Surrogates are tested on noise-corrupted outputs from simulations of two multiple-timescale models
- Goodness-of-fit is quantified by fitting the models to noisy data, then comparing the actual and predicted values at unseen datapoints



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	$\sigma = 0$	$\sigma = 1$	$\sigma = 2$
SNR (dB)	-	37.4	31.4
Bayesian splines	8.00×10^{-2}	0.185	0.597
GPR: Periodic Matern 3/2	1.80×10^{-4}	0.213	0.641
GPR: Periodic RBF	3.36×10^{-2}	0.406	0.806
GPR: Matern 3/2	3.24×10^{-2}	0.759	1.82
GPR: Matern 5/2	1.72×10^{-2}	0.915	1.91
GPR: RBF	5.61×10^{-1}	1.78	2.60

Hodgkin-Huxley MSPE

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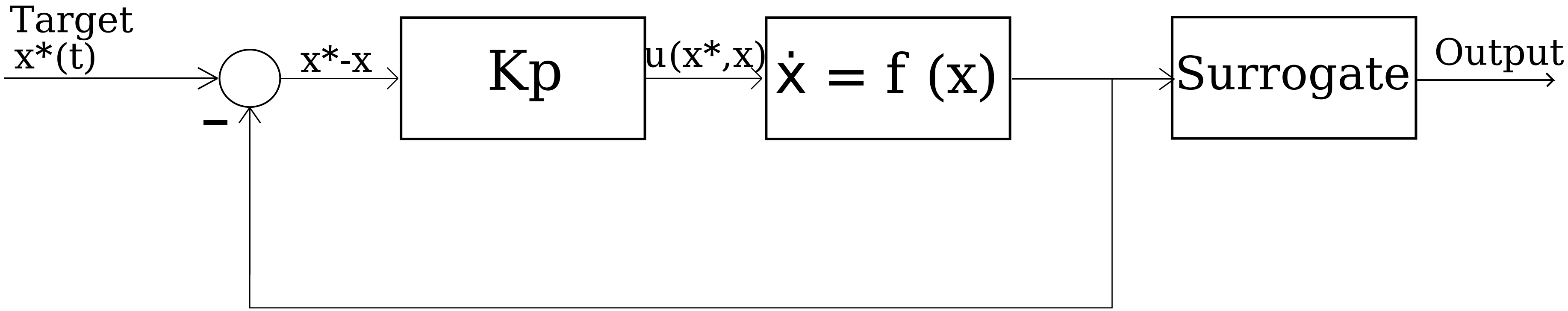
Surrogates fit easily into CBC

Surrogates appear before the numerical solver to pre-process the system output



The discretisor-solver step now receives a much less noisy system output

Conclusion

- CBC is a method for analysing the bifurcation structure of black-box and physical systems
- Oscillatory dynamics require discretising to be tracked
- It is difficult to accurately Fourier-discretise noisy multiple-timescale signals
- Bayesian regression models can be used instead to average out the noise
- Future work includes alternatives to Fourier discretisation, however these are often even less noise-robust