University of BRISTOL

[2020-07-15 Wed]

Bayesian methods for the control-based continuation of multiple-timescale systems

Mark Blyth

Plan de jour

- CBC maths
- Surrogate modelling
- Novel discretisations

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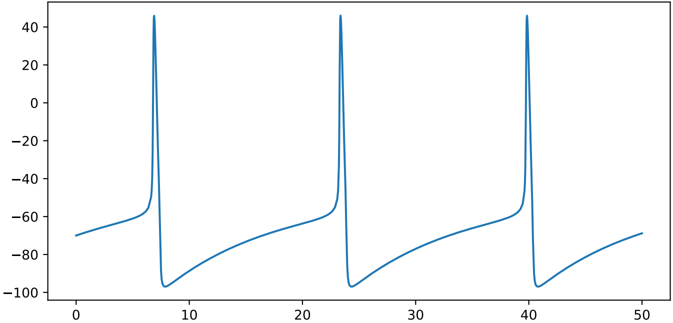
- Novel discretisations

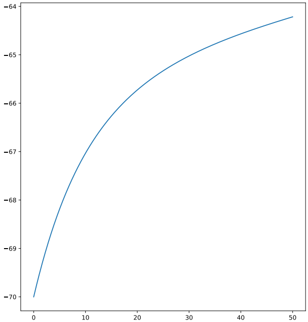
What is CBC?

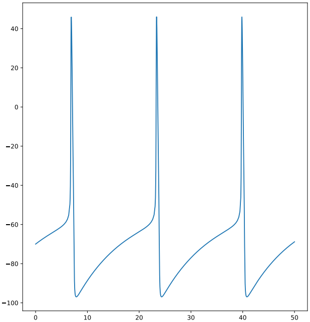
Dynamics are 'what something does'



What is CBC?

A bifurcation is a change in dynamics





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1. Find a feature

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4. Bifurcations occur when features change, appear, or disappear

What is CBC?

- Numerical continuation:
  - Features x defined given by $f(x, \lambda) = 0$
  - Change λ, see how x changes

George Box

All models are wrong, but some are useful

What is CBC?

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4. Find how noninvasive $u^*(t)$ changed
   - Tracks system features, bifurcations without ever needing a model

CBC

Control-based continuation

A model-free bifurcation analysis method. Uses a controller to stabilise a system, and continuation to track features.

My project: use CBC to analyse the bifurcations that make neurons fire

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  - *[interesting aside: optimal control developed some nice approaches for solving variational equations]*
- Discretisation lets us solve the problem by solving a finite set of equations

What is discretisation?

Goal: solve $I\left[u^*\right] = 0$

1. Translate problem to system of vector-valued equations
2. Solve system numerically
3. Translate solution back to a continuous function

Translation between continuous and vector-valued systems is discretisation

What is discretisation?

Definition (Discretisation)

The act of representing a continuous signal by a discrete counterpart

We want a discretisation that

- Has minimal discretisation error
- Is low-dimensional

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- The functional problem can be rewritten as $I\left(\mathbf{u}^*\right) = 0$
  - Finite-vector equation, solvable!

Issues with discretisation

- Solving the discretised system takes a long time when it is high-dimensional
- Neuron signals require lots of Fourier harmonics to discretise
- Higher-order harmonics are harder to get *[Nyquist cap]* and less accurate *[SNR]*

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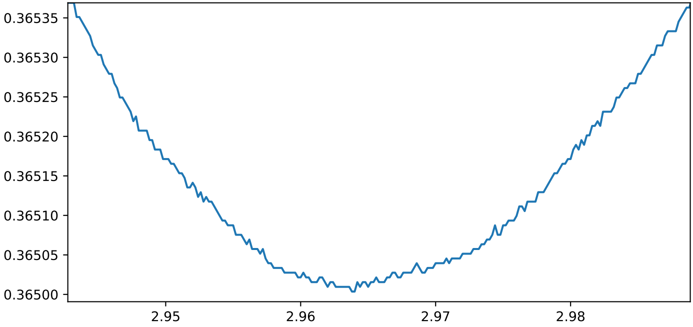
Definition (Surrogate models)

A local model for data, that can be used in place of experimental recordings

- Record experimental data
- Fit a surrogate model
- Perform analysis, eg. discretisation, on model instead of data

Why surrogates?

Real data are noisy



[Thanks to LR for the data]

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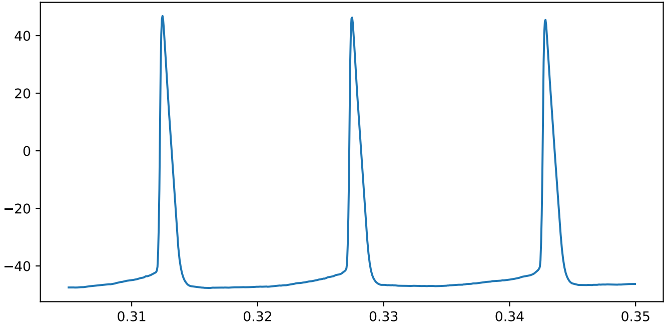

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  - Less noise = better discretisation

A primer on Bayes

The laws of probability, applied to beliefs instead of proportions-of-outcomes

- *[Frequentist]* probability:
  - How likely is something to happen?
  - An event is known to happen some proportion of the time; how can I reason about its outcomes?

- *[Bayesian]* beliefs:
  - Encoding uncertain beliefs; reasoning in the face of ignorance
  - I have some beliefs about an event; how can I update my beliefs after seeing some evidence?
  - Let's us combine beliefs and evidence to make better decisions

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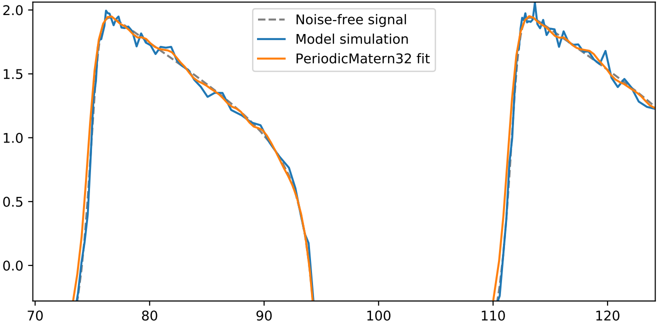
- This is Gaussian process regression!

Gaussian process regression surrogates

Build a statistically optimal regression model from noisy observations



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  - Bayes with bad priors = bad results!

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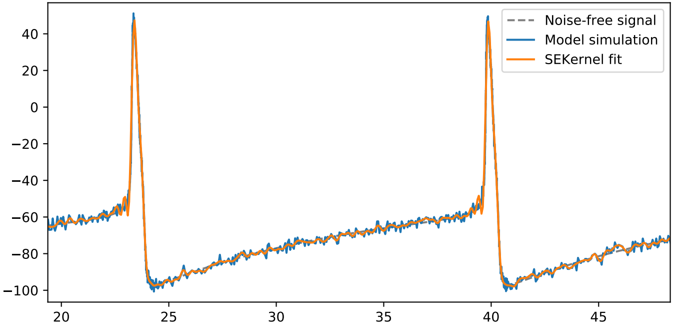
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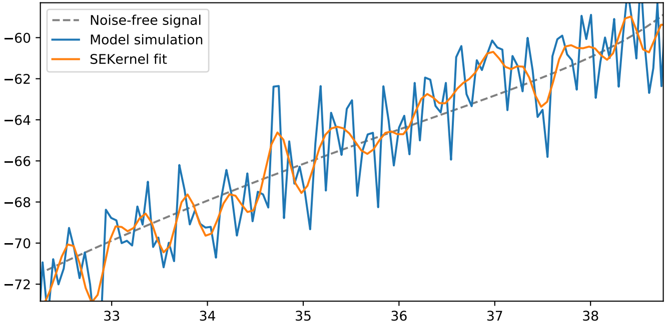
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- Non-stationary GPR is hard!

Splines

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- Choose polynomials so that the function is smooth
- Finite, low degree-of-freedom, forcibly averages out noise

Bayesian splines

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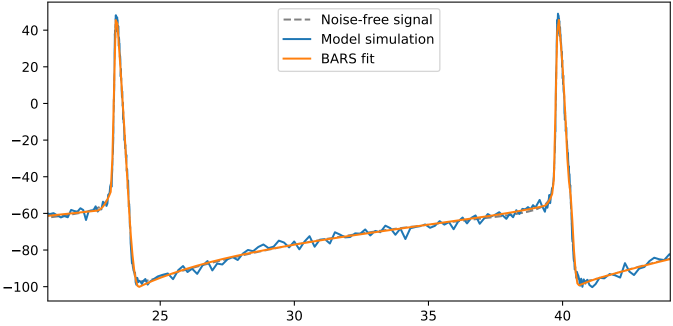
- This is Bayesian free-knot splines

Splines as a surrogate

Result 1: splines outperform stationary GPR as neuronal data surrogate



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- Issue: too many coefficients are needed to discretise the signal

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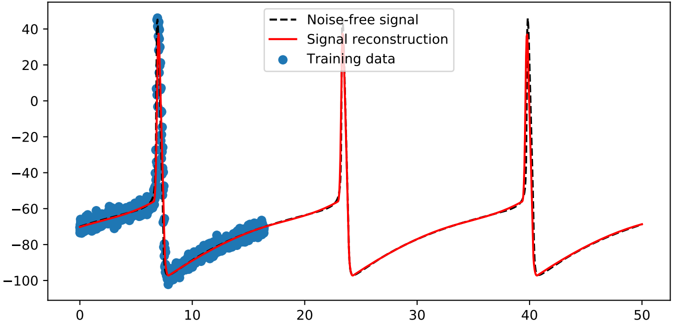
Spline discretisation

8-dimensional discretisation; but does it work with continuation?



Where next?

- Compare Fourier vs splines discretisation
  - What error for what discretisation-size?
- See if the discretisation breaks down with stochastic models
  - It probably will
- Test the discretisation with continuation