Bursters and bifurcations

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Some misc. ideas

- Barton's electronic neurons could be a nice quick and easy test experiment
- Stochastic behaviour introduces a new class of bifurcation, with weird behaviours such as
  - coherence resonance;
  - stochastic resonance;
  - noisy bifurcation precursors.

It could be interesting to try investigating these using CBC

Week’s goal

- Get familiar with Krassy’s neuron model
- Do some bifurcation analysis with it
- Use the neuron and its bifurcation analysis to write a comparison paper for continuation software

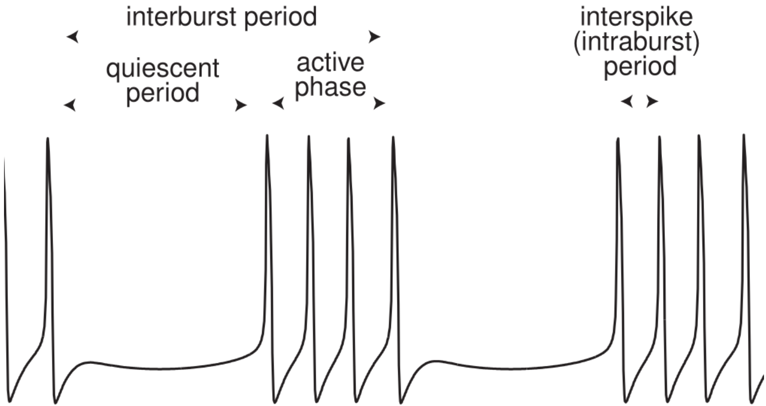
Krassy’s neuron model

- Paper goal: classify the psuedo-plateau burster using the codimension burster classification
- Issue: I know nothing about burster dynamics!

Week's activities

- Learned about burster dynamics
- Learned about the codimension classification system for bursters
- Used that to (sort of?) understand Krassy's paper
- Found a paper that builds on it, and proposes a potentially very useful neuron model

What is bursting?



Rinzel's burster analysis

Consider the system

$$\dot{x} = f(x, y)\ FAST,$$
$$\dot{y} = \varepsilon g(x, y)\ SLOW,$$

where

$$|\varepsilon| \ll 1\ ,$$

and

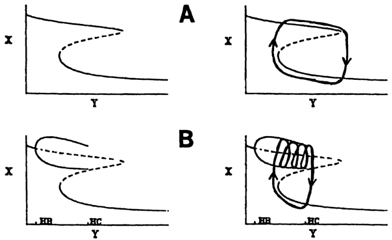
$$f, g \in \mathcal{O}(1)\ .$$

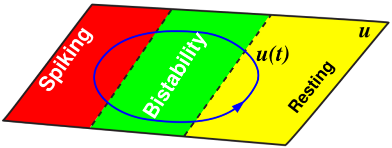
Rinzel's burster analysis

- Consider the singular limit $\varepsilon \to 0$
- The change in y drops to zero, so y becomes a constant
- As y is now a constant vector, it can be considered as a parameter vector to the fast subsystem
- Rinzel's approach: consider the bifurcations of the fast subsystem at the singular limit; take the slow subsystem state y to be a bifurcation parameter, and perform a bifurcation analysis of the fast subsystem with respect to y
- Bursting dynamics are then obtained when the slow subsystem dynamics drives the fast subsystem back and forth over one or more bifurcations.

Ref: *Rinzel, John. "Bursting oscillations in an excitable membrane model." Ordinary and partial differential equations. Springer, Berlin, Heidelberg, 1985. 304-316.*

Rinzel's burster analysis





Krassy et al.'s paper

- Lots of work has been done to classify bursters
- Krassy's paper seeks to classify the (recently found) psuedo-plateau burster
- This is achieved by studying the unfolding of a codimension-4 singularity
- The singularity unfolding could (presumably?) also double up as a generic neuron model

Ref: *Osinga, H. M., A. Sherman, and K. Tsaneva-Atanasova. "Cross-currents between biology and mathematics on models of bursting." Bristol Centre for Applied Nonlinear Mathematics preprint 1737 (2011).*

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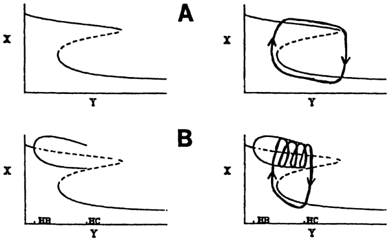
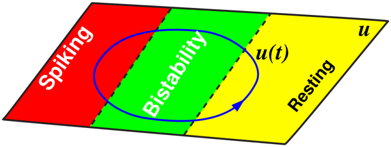
The paper builds on the work of Rinzel, Bertram, and Golubitsky (and other less relevant work), briefly recounted as follows.

Classifying bursters - background

- Rinzel's work allows for the classification of bursters, according to the bifurcations at either end of the hysteresis loop





Classifying bursters - background

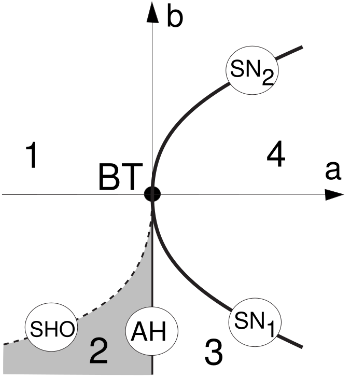
- Rinzel's work allows for the classification of bursters, according to the bifurcations at either end of the hysteresis loop [1]
- Izhikevich notes that there are four bifurcations that can lead to the onset or termination of bursting, meaning 16 different bursters can exist for a planar fast subsystem [2]
- Later work decided there's a better way of classifying bursters, in terms of unfoldings of high-codimension singularities [3][4]

Refs

- [1] *Rinzel, John. "A formal classification of bursting mechanisms in excitable systems." Mathematical topics in population biology, morphogenesis and neurosciences. Springer, Berlin, Heidelberg, 1987. 267-281.*
- [2] *Izhikevich, Eugene M., and Frank Hoppensteadt. "Classification of bursting mappings." International Journal of Bifurcation and Chaos 14.11 (2004): 3847-3854.*
- [3] *Bertram, Richard, et al. "Topological and phenomenological classification of bursting oscillations." Bulletin of mathematical biology 57.3 (1995): 413-439.*
- [4] *Golubitsky, Martin, Kresimir Josic, and Tasso J. Kaper. "An unfolding theory approach to bursting in fast-slow systems." Global analysis of dynamical systems (2001): 277-308.*

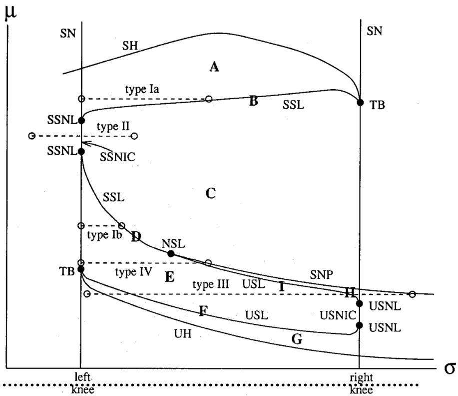
Classifying bursters - Bertram [3]

- Observation: hysteresis-loop bursters require two bifurcations - one to start spiking, and one to stop it
- Instead of considering them as isolated bifurcations, consider them as part of the unfolding of a higher-codimension singularity



Classifying bursters - Bertram [3]

- Bursting behaviours are defined by their paths across fast-subsystem bifurcations
- This is represented as horizontal paths on (here) a two-parameter bifurcation diagram
- These cuts represent the paths in parameter space that the slow subsystem drives the fast system through
- Allows for both discovery and classification



Classifying bursters - Golubitsky [4]

- Golubitsky et al. produced a more rigorous version of Bertram's classification
- The classification is extended to the codimension-3 degenerate Bogdanov-Takens singularity
- Bursting behaviour later appeared that couldn't be explained as an unfolding of a codim-2 singularity, but could be explained in codim-3
- The complexity of a burster is defined as the codimension of the singularity in whose unfolding the bursting behaviour first appears; the codim-3 burster would therefore be considered more complex than the codim-2 ones

Classifying bursters - Krassy et al.

- Psuedo-plateau bursting is a type of bursting where there's no sustained oscillations in the active phase
- As far as we know, it can't be explained in terms of codim-3 unfoldings
- Krassy's paper expands the existing burster classification to include psuedo-plateau bursters
- A codim-4 doubly-degenerate Bogdanov Takens singularity is shown to include the burster in its unfoldings
- It is thought to be codim-4, as no codim-3 unfolding is yet known to contain the bursting dynamics

Towards a generic neuron model

- The codim-4 unfolding will contain all known bursters (I think?)
- By ignoring the slow subsystem, we can instead let injected current drive the system across a bifurcation (not necessarily in a biologically plausible way)
- The model will therefore be able to demonstrate all the bifurcations a non-bursting neuron can undergo
- This makes it a potential candidate for a generic model

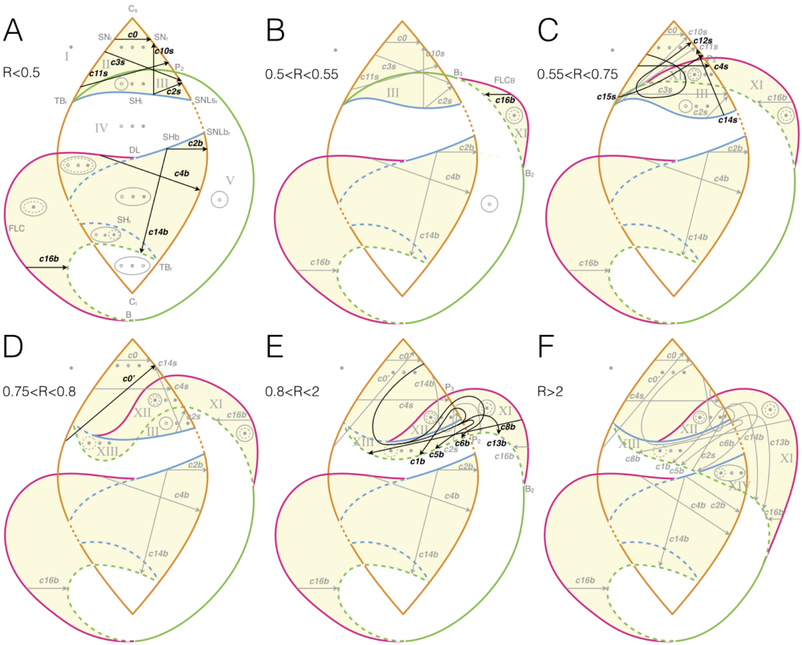
Towards a generic neuron model

- Bursters in Krassy's paper are driven by a sinusoidal forcing term
- This means the slow subsystem must be self-oscillating (called a slow-wave burster)
- We can also have resonant slow subsystems, which don't oscillate on their own (hysteresis-loop bursters, acting in similar ways to Fitzhugh-Nagumo)
- To model all neuron types (inc. hysteresis- and slow-wave bursters), we need a different slow subsystem model
- I've found a paper (ref below) that builds extensively on Krassy's paper to develop such a model
- It is designed to model just about every single neuron that's likely to exist, making it another good generic neuron model

Saggio, Maria Luisa, et al. "Fast–Slow Bursters in the Unfolding of a High Codimension Singularity and the Ultra-slow Transitions of Classes." The Journal of Mathematical Neuroscience 7.1 (2017): 7.

Next steps

- I don't really understand the bifurcations of Krassy's neuron model, so work on achieving that
- Read paper about the generic neuron model, and its bifurcations
- Decide which bifurcations to test myself
- Use XPP etc. to do a bifurcation analysis on the model
- Use those analyses to produce a software comparison paper
- Also, look at networks of neurons and their models, dynamics, bifurcations, etc.
- Then, start learning about control strategies



A
R<0.5
B
0.5<R<0.55
C
0.55<R<0.75
D
0.75<R<0.8
E
0.8<R<2
F
R>2