Bursters and bifurcations

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Week's goal

- Get familiar with Krassy's neuron model
- Do some bifurcation analysis with it
- Use the neuron and its bifurcation analysis to write a comparison paper for continuation software

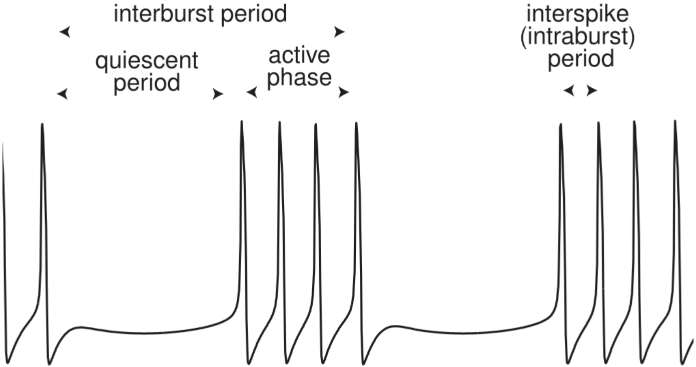
Krassy's neuron model

- Paper goal: classify the psuedo-plateau burster using the codimension burster classification
- Issue: I know nothing about burster dynamics!

Week’s activities

- Learned about burster dynamics
- Learned about the codimension classification system for bursters
- Used that to (sort of?) understand Krassy’s paper
- Found a paper that builds on it, and proposes a potentially very useful neuron model

What is bursting?



Rinzel's burster analysis

Consider the system

$$\dot{x} = f(x, y)\ FAST,$$
$$\dot{y} = \varepsilon g(x, y)\ SLOW,$$

where

$$|\varepsilon| \ll 1\ ,$$

and

$$f, g \in \mathcal{O}(1)\ .$$

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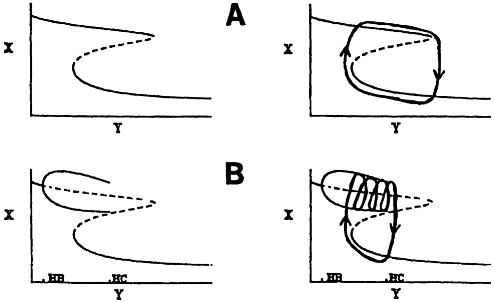
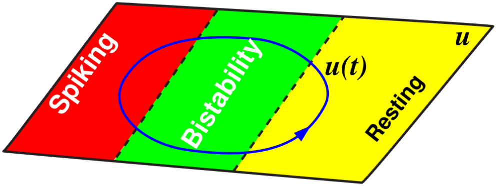
- Rinzel's approach: consider the bifurcations of the fast subsystem at the singular limit; take the slow subsystem state y to be a bifurcation parameter, and perform a bifurcation analysis of the fast subsystem with respect to y
- Bursting dynamics are then obtained when the slow subsystem dynamics drives the fast subsystem back and forth over one or more bifurcations.

Rinzel’s burster analysis





Krassy et al.'s paper

- Lots of work has been done to classify bursters
- Krassy's paper seeks to classify the (recently found) psuedo-plateau burster
- This is achieved by studying the unfolding of a codimension-4 singularity
- The singularity unfolding could (presumably?) also double up as a generic neuron model

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The paper builds on the work of Rinzel, Bertram, and Golubitsky (and other less relevant work), briefly recounted as follows.

Classifying bursters - background

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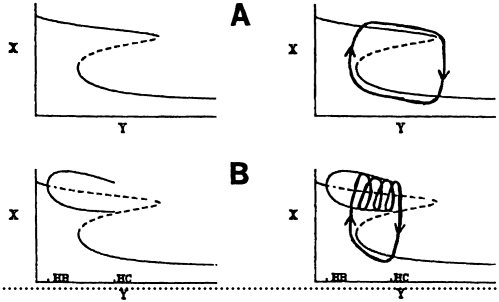

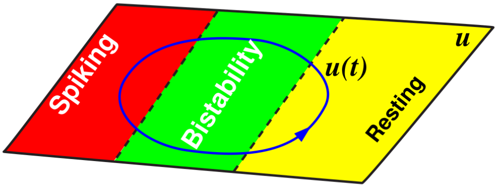

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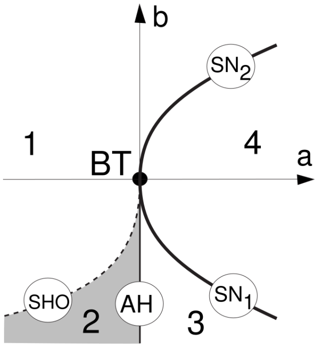
- Izhikevich notes that there are four bifurcations that can lead to the onset or termination of bursting, meaning 16 different bursters can exist for a planar fast subsystem
- Later work decided there's a better way of classifying bursters, in terms of unfoldings of high-codimension singularities

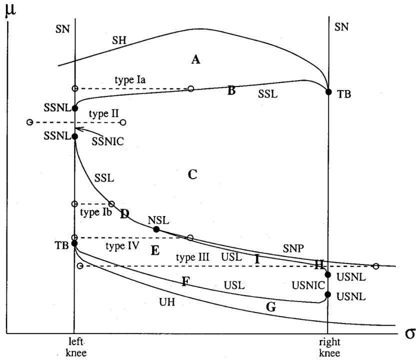
Classifying bursters - Bertram

- Observation: hysteresis-loop bursters require two bifurcations - one to start spiking, and one to stop it
- Instead of considering them as isolated bifurcations, consider them as part of the unfolding of a higher-codimension singularity



Classifying bursters - Bertram

- Bursting behaviours are defined by their paths across fast-subsystem bifurcations
- This is represented as horizontal paths on (here) a two-parameter bifurcation diagram
- These cuts represent the paths in parameter space that the slow subsystem drives the fast system through
- Not only does this allow us to classify bursters, but it also allows the prediction of new ones



Classifying bursters - Golubitsky

- Golubitsky et al. produced a more rigorous version of Bertram's classification
- The classification is extended to the codimension-3 degenerate Bogdanov-Takens singularity
- Bursting behaviour later appeared that couldn't be explained as an unfolding of a codim-2 singularity, but could be explained in codim-3
- The complexity of a burster is defined as the codimension of the singularity in whose unfolding the bursting behaviour first appears; the codim-3 burster would therefore be considered more complex than the codim-2 ones

Classifying bursters - Krassy et al.

- Psuedo-plateau bursting is a type of bursting where there's no sustained oscillations in the active phase
- As far as we know, it can't be explained in terms of codim-3 unfoldings
- Krassy's paper expands the existing burster classification to include psuedo-plateau bursters
- A codim-4 doubly-degenerate Bogdanov Takens singularity is shown to include the burster in its unfoldings
- It is thought to be codim-4, as no codim-3 unfolding is yet known to contain the bursting dynamics

Towards a generic neuron model

- The codim-4 unfolding will contain all known bursters (I think?)
- By ignoring the slow subsystem, we can instead let injected current drive the system across a bifurcation (not necessarily in a biologically plausible way)
- The model will therefore be able to demonstrate all the bifurcations a non-bursting neuron can undergo
- This makes it a potential candidate for a generic model

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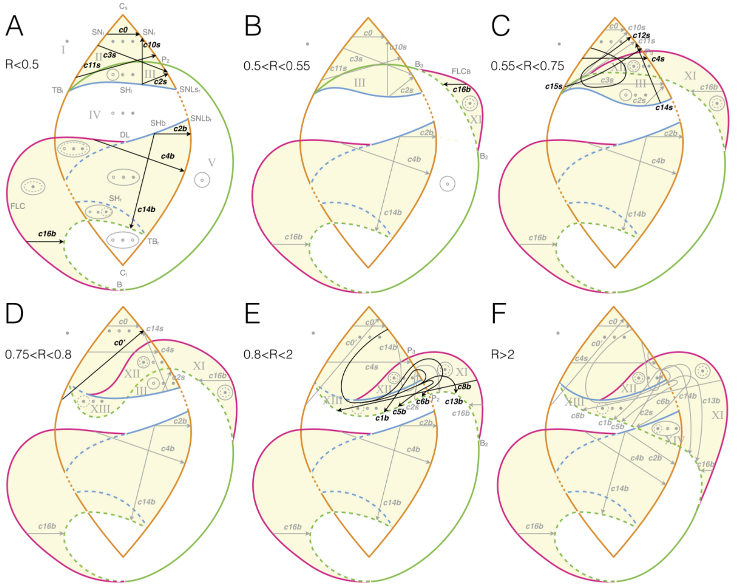
- To model all neuron types (inc. hysteresis- and slow-wave bursters), we need a different slow subsystem model
- I've found a paper (ref below) that builds extensively on Krassy's paper to develop such a model
- It is designed to model just about every single neuron that's likely to exist, making it another good generic neuron model

Next steps

- I don’t really understand the bifurcations of Krassy’s neuron model, so work on achieving that
- Read paper about the generic neuron model, and its bifurcations
- Decide which bifurcations to test myself
- Use XPP etc. to do a bifurcation analysis on the model
- Use those analyses to produce a software comparison paper
- Also, look at networks of neurons and their models, dynamics, bifurcations, etc.
- Then, start learning about control strategies

Saggio, Maria Luisa, et al. "Fast–Slow Bursters in the Unfolding of a High Codimension Singularity and the Ultra-slow Transitions of Classes." The Journal of Mathematical Neuroscience 7.1 (2017): 7.



A
R<0.5
B
0.5<R<0.55
C
0.55<R<0.75
D
0.75<R<0.8
E
0.8<R<2
F
R>2
Cs
SNl
SNr
c0
c10s
c3s
c11s
P2
c2s
TBl
SHl
SNLsr
IV
SHb
SNLbr
c2b
DL
c4b
V
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c14b
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Ci
B
B3
FLCB
c16b
B2
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c4s
c15s
c14s
XI
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XIII
c0'
c14s
P3
c6b
c5b
c1b
c8b
c13b
XIV