Spring project summary

Mark Blyth

Last meeting

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- Reuse Bath single-cell microfluidics device

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- Or could reuse Bath microfluidic device
  - Would require minor alterations to increase spatial resolution

Single- vs multi-cell

Deciding factors:

- No lab access for the forseeable future
  - Work can be guided less by experiments
- Single-cell easier than multi-cell
  - I know enough about single-cell CBC to start working on it

Conclusion: work on single-cell case

Current goals

- Single-cell *in-silico* CBC
- Tutorial-review paper for numerical continuation

Challenges of *in-silico* CBC

Data aren't ideal to work with:

- Real signals are noise-corrupted
  - Difficult to filter off, since spikes contain lots of high-frequency components
  - Hard to run continuation on stochastic and noisy signals
- Neurons are fast-spiking
  - Fourier discretisation won't work
  - Discretisations need to be very high-dimensional, making Jacobian very slow to find

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  - Accurate derivatives, of arbitrary order
  - Allows for accurate delay embeddings, collocation discretisation – conventional continuation can be used on it

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  - Statistically optimal, when noise is Gaussian

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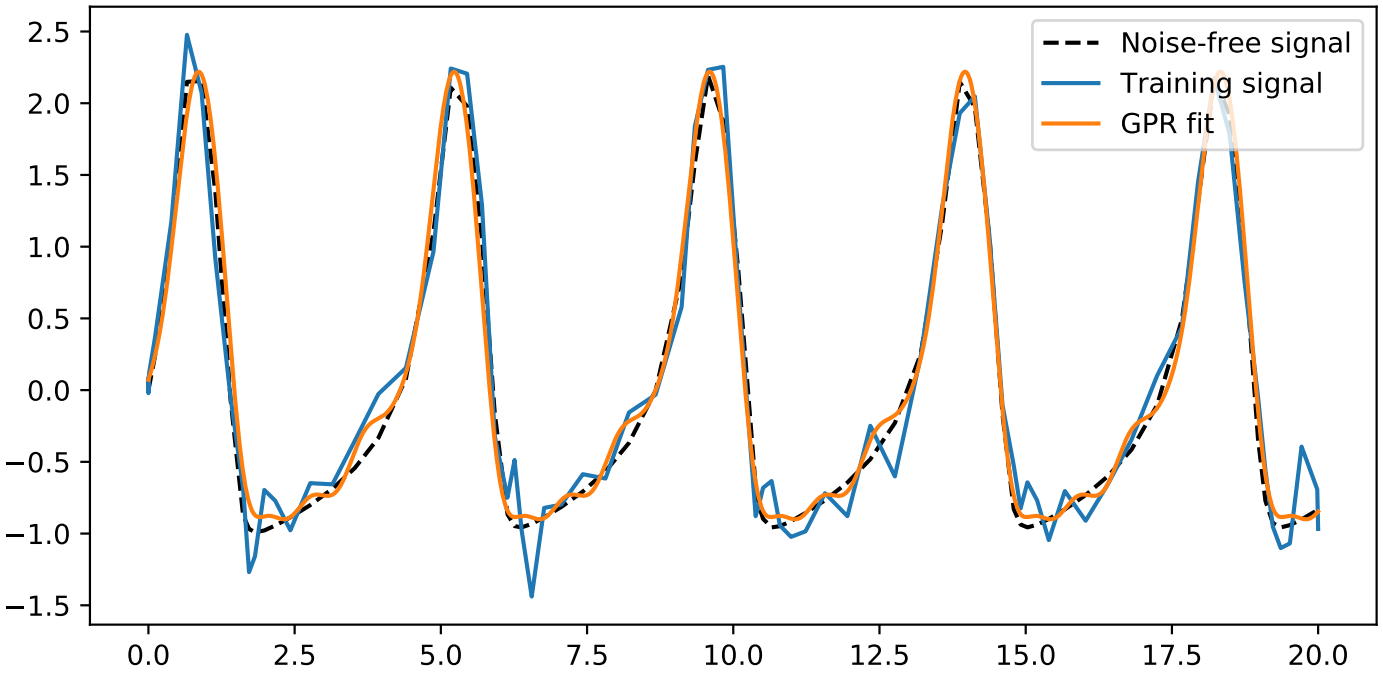
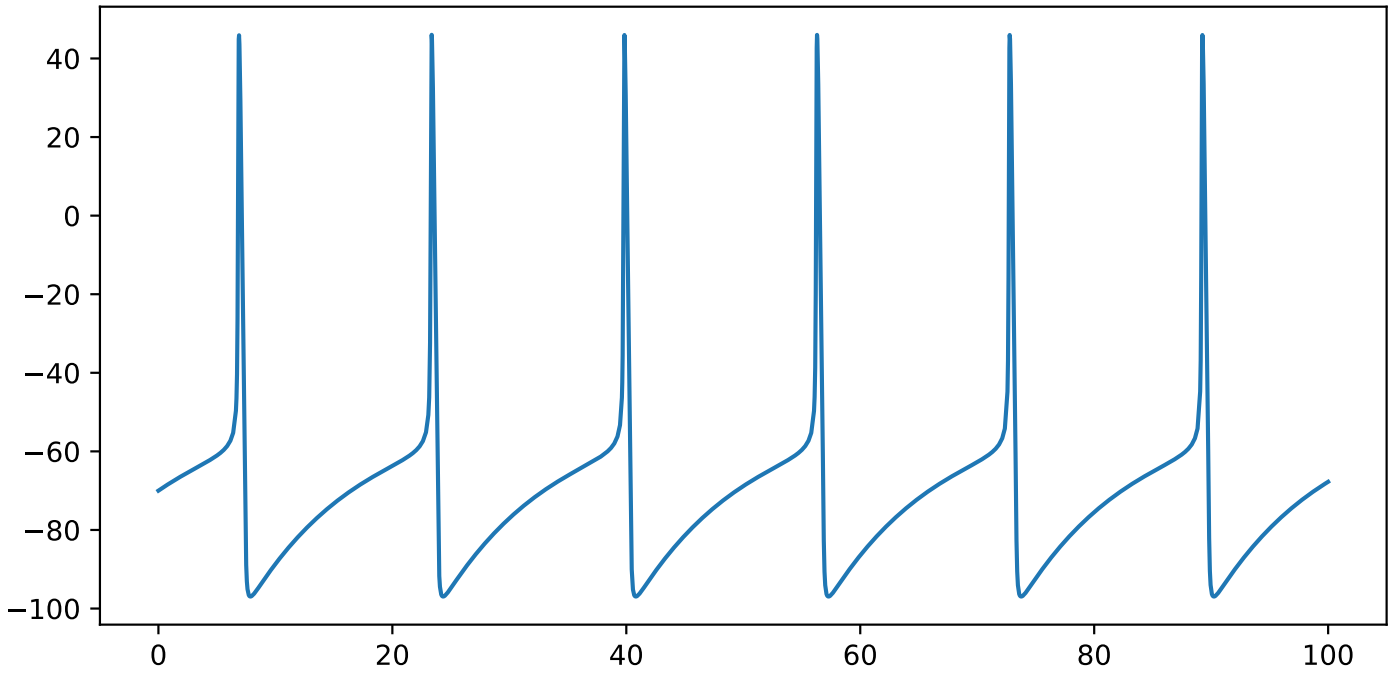
Gaussian process regression



GPR can recover the underlying signal from noise-corrupted observations

Surrogate models



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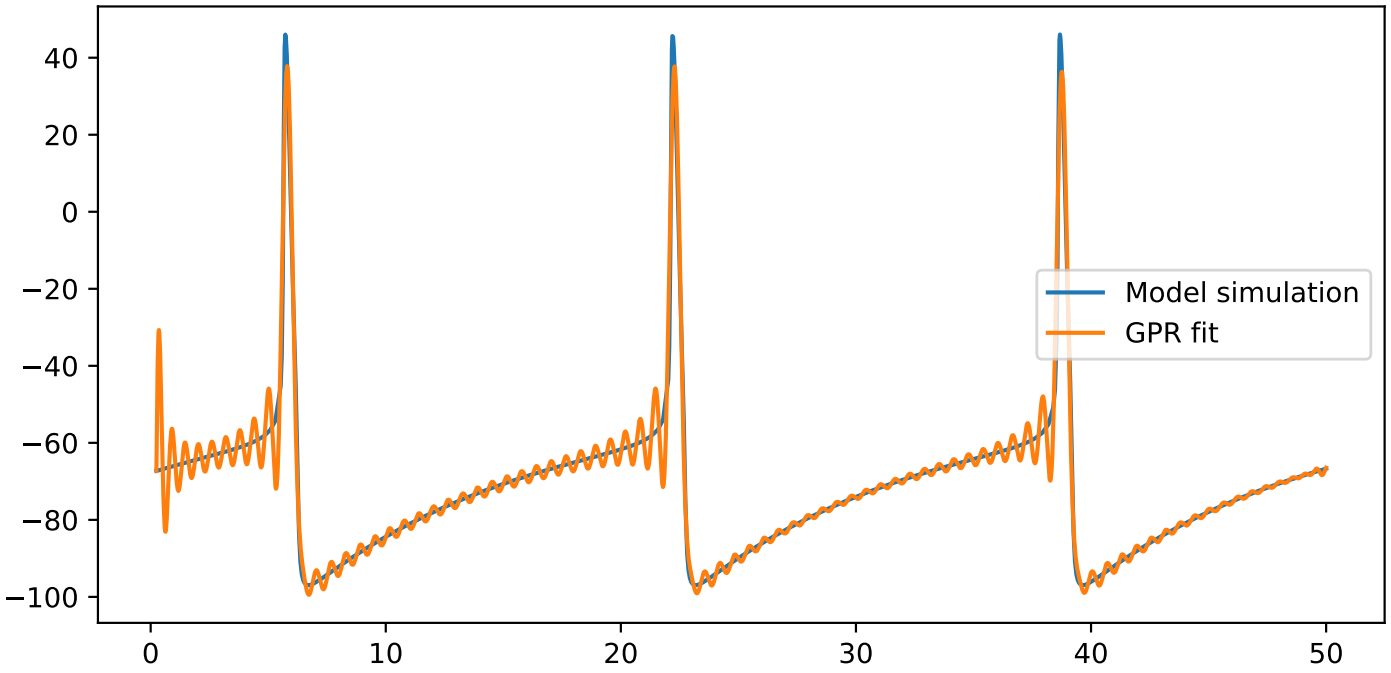

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  - Can't handle the spiking behaviours of neurons

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Next questions

- Surrogate models on real data
- Predictor-corrector design
- Stochastic models

Continuation issues

- Discretisation is required to make predictor-corrector methods work
  - Can't run continuation on a function; must discretise it into a vector
- Discretisation has issues when used on fast-spiking data
  - Requires lots of datapoints
  - Slow to find a Jacobian for Newton-iterations
  - High noise-sensitivity
- Surrogate models and discretisation-free predictor-correctors might help overcome these

Alternative continuation approach

Predictor-corrector design:

- We could try discretisation-free predictor steps, using a surrogate model
  - Let $f_i(t)$ be the surrogate model for system behaviours at parameter λ_i
  - Given periodic orbits $f_{i-1},\ f_i$, predict $f_{i+1} = f_i + h\big[f_i - f_{i-1}\big]$
- Corrector step would be harder

Stochastic models

Another challenge: real neurons are stochastic

- Stochasticity introduces new challenges
  - Coherence and stochastic resonance
  - Random attractors
  - Stochastic calculus
  - Not an area I know much about *[yet. . .]*
- Current work: CBC on noise-corrupted simulations
- Next work: CBC on truly stochastic models

Big question: how different would truly stochastic models be?

Goals

Actions:

- Find a surrogate modelling method for neural data
- Attempt a discretisation-free corrector?
- Run CBC on deterministic models, then stochastic

Results:

- Write up surrogate modelling into a conference abstract *[July]*
  - Maybe a conference paper *[September]*
- Use surrogate modelling for an *in-silico* CBC paper *[next year?]*

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  - Gradient boosting: combine several weak learners to make a single strong learner

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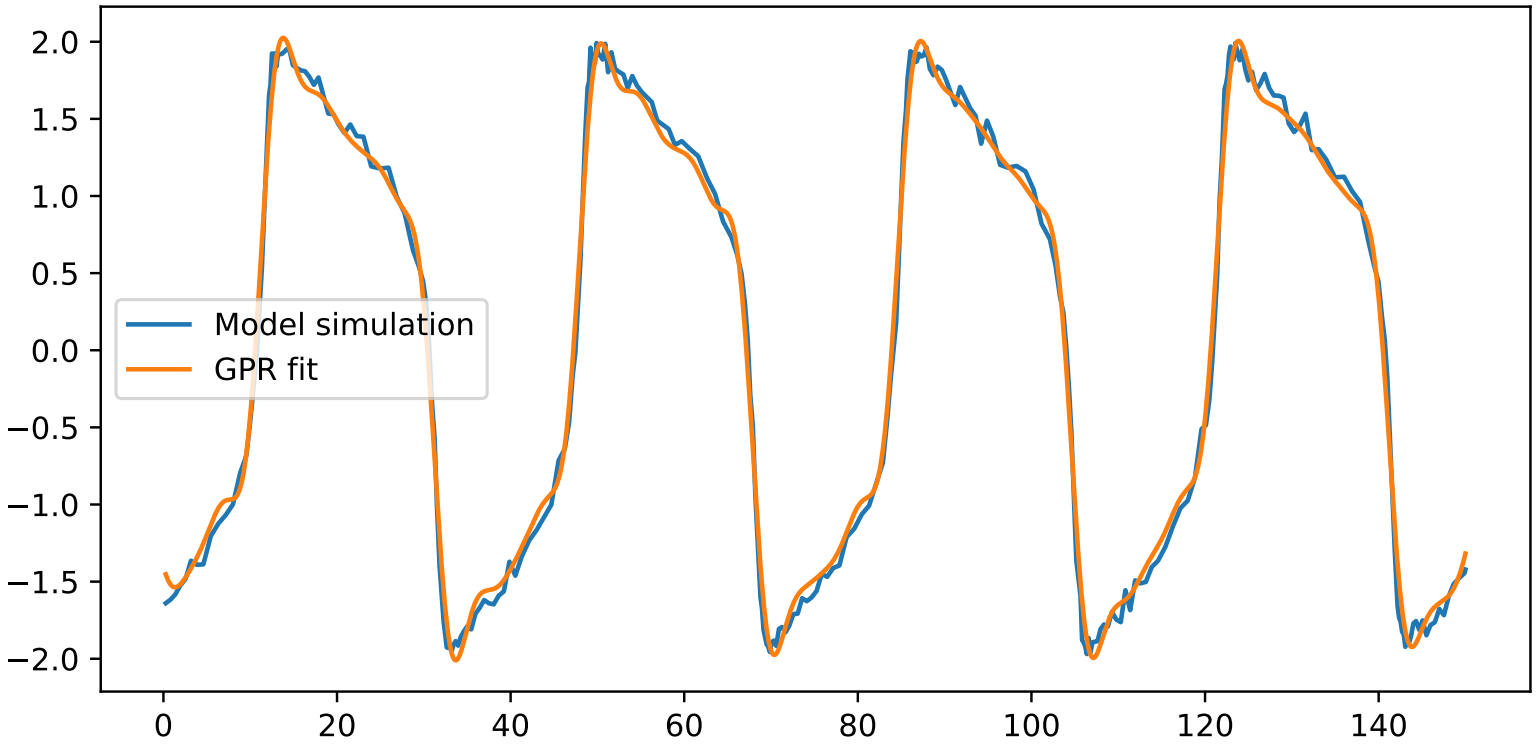
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140
Model simulation
GPR fit