University of BRISTOL

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Homoclinics and continuations

Mark Blyth

Week's goal

- Reproduce some of the bifurcation diagrams from the literature
  - Use different continuation software packages, and add them to the comparison paper once I know them well enough to do so
- Learn how to find homoclinic bifurcations
- Use the bifurcation tools to learn more about Krassy's cubic Lienard model

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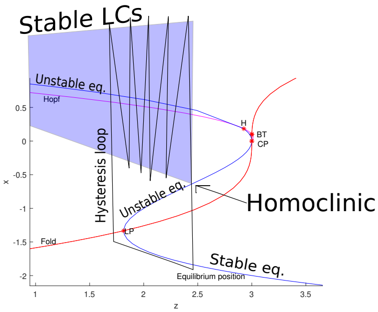
- Looked a bit into the maths and numerics of homoclinic bifurcations and continuation

Motivating problem



- Homoclinic bifurcations are fundamental to neuron function
- I can't seem to find them in my Hindmarsh-Rose bifurcation analysis

Motivating problem

- Homoclinic bifurcations are fundamental to neuron function, so they are a useful thing to understand
- Bogdanov-Takens points (intersection of Saddle-Node and Hopf bifurcations) necessarily produce a family of homoclinics
  - Krassy's model is an unfolding of a doubly-degenerate Bogdanov-Takens singularity
  - Since it's codim-4, it the parameter space contains three-dimensional subspaces of homoclinic bifurcations
  - The bifurcations are therefore unavoidable!
- CBC can't yet deal with homoclinic bifurcations, making it a particularly interesting area to study

Goals and results

- Goal: find and continue homoclinic bifurcations in the HR and Cubic Lienard models
- Results: *[none]*

The challenges of Homoclinic bifurcations (1)

The period of a limit cycle diverges to infinity at a homoclinic bifurcation, making numerical integration difficult:

- Homoclinic trajectories are the solution to a boundary value problem, where the boundaries (start, end state) are a saddle equilibrium
- The equilibrium is only reached as $t \to \pm\infty$, meaning our boundaries are not numerically tractable
- To fix this, we use projective boundary conditions, to truncate the problem onto a finite time domain
- These require some rather complicated maths

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- But, this becomes impossible near a BT point, since those trajectories become arbitrarily small
- Looking for homoclinc bifurcations at a BT point therefore becomes a problem of spotting nothing happening, where that nothing happens in an infinitely small region of phase space

The challenges of Homoclinic bifurcations (3)

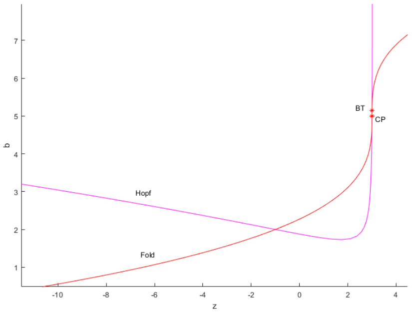
I flicked through a few different papers on the numerical aspects of homoclinic continuation. They build on maths that I don't know much about

- Reduction of the system to the center manifold
- Homeomorphism onto topological normal forms
- Everything to do with projective boundary conditions
- Homotopic shooting methods

Guckenheimer and Holmes (1983) contains most of the required maths, as well as lots of useful information on bifurcations. I've added it to my reading list!

Week's activities (week 2)

- Homoclinic bifurcations are interesting, but would require a large time investment to make any progress
- Instead, I returned to numerical bifurcation analysis
  - Started learning about PyDSTool
  - Followed online tutorials
  - Managed to generate some bifurcation diagrams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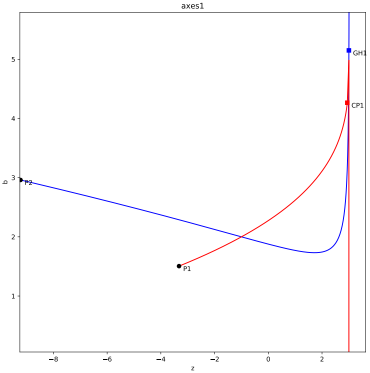
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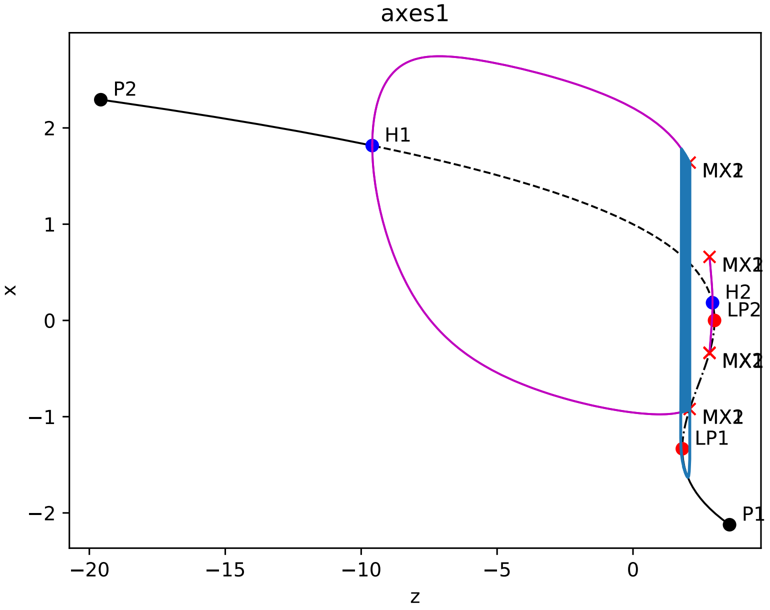
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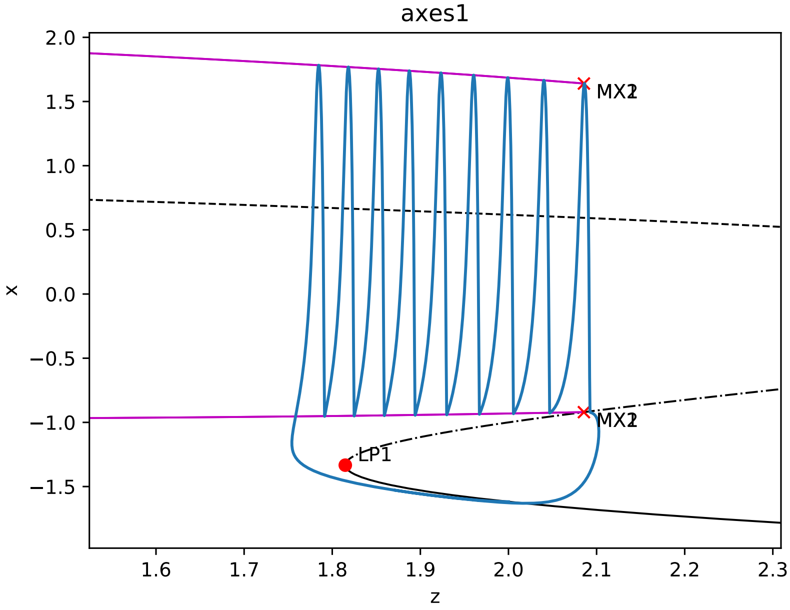
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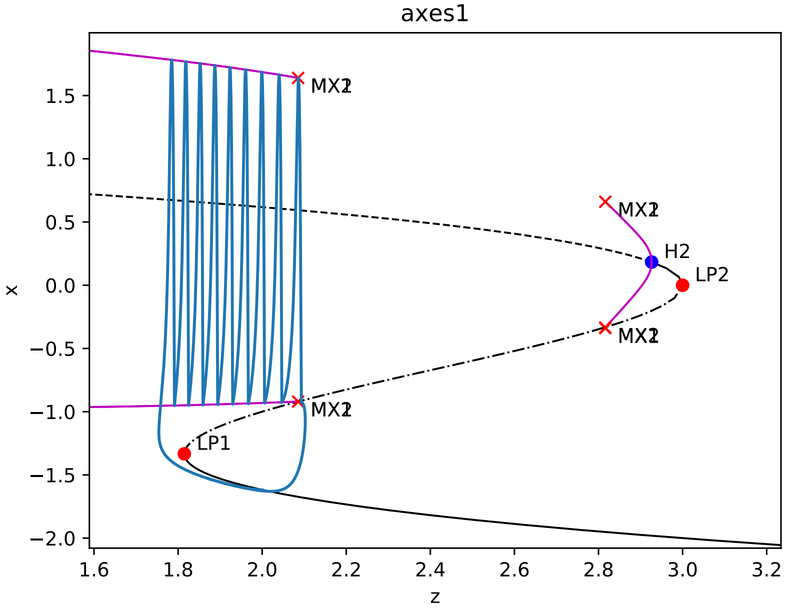
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Project ideas

1. Find a way of using CBC to track homoclinic bifurcations (challenge: CBC of global bifurcations)
   - Use the simplest possible system and the simplest possible controller
   - Use that knowledge to add homoclinic bifurcation analysis into PyDSTool, if it won't take too long to do so? Might be paper-worthy in itself?
2. Design a controller that'll work on neuron models; adapt the CBC approach to use Krassy's model and the new controller (challenge: discretising spiking signals, controlling neurons)
   - Use that for an in-silico neuron CBC simulation
   - See how things change when noise gets introduced
3. Use the newly developed CBC approach on a real live neuron (challenge: experiments)

Next steps

- Keep learning numerical continuation tools (next steps: CoCo, MATCONT CL)
- Start writing some notes about the ones I’ve used so far
  - QUESTION: what sort of things would be useful to discuss in the paper?
- Mix things up with some *Guckenheimer and Holmes*, when I get tired