Spring project summary

Mark Blyth

Last meeting

Discussion about single-cell and multi-cell approaches

Single-cell:

- Strong literature precedent for what to expect

Last meeting

Discussion about single-cell and multi-cell approaches

Single-cell:

- Strong literature precedent for what to expect
- Lots of accepted models to test *in-silico*

Last meeting

Discussion about single-cell and multi-cell approaches

Single-cell:

- Strong literature precedent for what to expect
- Lots of accepted models to test *in-silico*
- Easy-to-spot bifurcations

Last meeting

Discussion about single-cell and multi-cell approaches

Single-cell:

- Strong literature precedent for what to expect
- Lots of accepted models to test *in-silico*
- Easy-to-spot bifurcations
  - Hopf, fold both easily detectable with CBC

Last meeting

Discussion about single-cell and multi-cell approaches

Single-cell:

- Strong literature precedent for what to expect
- Lots of accepted models to test *in-silico*
- Easy-to-spot bifurcations
  - Hopf, fold both easily detectable with CBC
- Reuse Bath single-cell microfluidics device

Last meeting

Discussion about single-cell and multi-cell approaches

Multi-cell:

- Assume there's an arbitrarily large number of cells

Last meeting

Discussion about single-cell and multi-cell approaches

Multi-cell:

- Assume there's an arbitrarily large number of cells
  - Neural continuum fields (limited descriptive ability)

Last meeting

Discussion about single-cell and multi-cell approaches

Multi-cell:

- Assume there's an arbitrarily large number of cells
  - Neural continuum fields (limited descriptive ability)
  - Spatially extended cubic Lienard system

Last meeting

Discussion about single-cell and multi-cell approaches

Multi-cell:

- Assume there's an arbitrarily large number of cells
  - Neural continuum fields (limited descriptive ability)
  - Spatially extended cubic Lienard system
- Search experimentally and numerically for PDE bifurcations

Last meeting

Discussion about single-cell and multi-cell approaches

Multi-cell:

- Assume there's an arbitrarily large number of cells
  - Neural continuum fields (limited descriptive ability)
  - Spatially extended cubic Lienard system
- Search experimentally and numerically for PDE bifurcations
  - Not an area I know much about (yet...)

Last meeting

Discussion about single-cell and multi-cell approaches

Multi-cell:

- Assume there's an arbitrarily large number of cells
  - Neural continuum fields (limited descriptive ability)
  - Spatially extended cubic Lienard system
- Search experimentally and numerically for PDE bifurcations
  - Not an area I know much about (yet. . .)
- Can build on work by Krasi's Munich collaborators

Last meeting

Discussion about single-cell and multi-cell approaches

Multi-cell:

- Assume there's an arbitrarily large number of cells
  - Neural continuum fields (limited descriptive ability)
  - Spatially extended cubic Lienard system
- Search experimentally and numerically for PDE bifurcations
  - Not an area I know much about (yet. . .)
- Can build on work by Krasi's Munich collaborators
- Or could reuse Bath microfluidic device

Last meeting

Discussion about single-cell and multi-cell approaches

Multi-cell:

- Assume there's an arbitrarily large number of cells
  - Neural continuum fields (limited descriptive ability)
  - Spatially extended cubic Lienard system
- Search experimentally and numerically for PDE bifurcations
  - Not an area I know much about (yet. . .)
- Can build on work by Krasi's Munich collaborators
- Or could reuse Bath microfluidic device
  - Would require minor alterations to increase spatial resolution

Single- vs multi-cell

Deciding factors:

- No lab access for the forseeable future
  - Work can be guided less by experiments
- Single-cell easier than multi-cell
  - I know enough about single-cell CBC to start working on it

Conclusion: work on single-cell case

Current goals

- Single-cell *in-silico* CBC
- Tutorial-review paper for numerical continuation

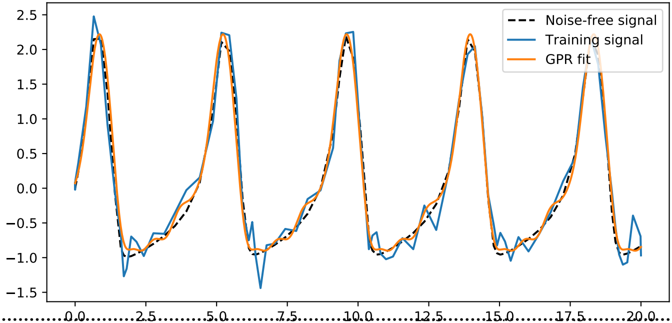
Challenges of *in-silico* CBC

Data aren't ideal to work with:

- Real signals are noise-corrupted
  - Difficult to filter off, since spikes contain lots of high-frequency components
  - Hard to run continuation on stochastic and noisy signals
  - Current work
- Neurons are fast-spiking
  - Fourier discretisation won't work
  - Discretisations need to be very high-dimensional, making Jacobian very slow to find
  - Next work

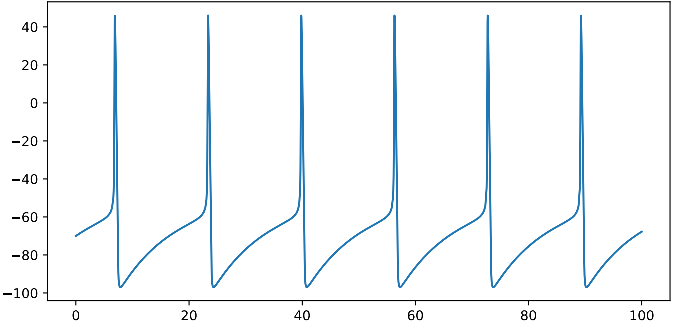
Issue 1: noise corruption

Instead of running continuation on noisy signal measurements, let's run it on a surrogate data source



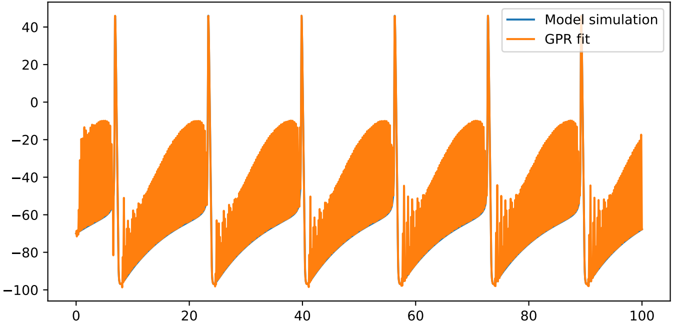
Surrogate models

Surrogate models don’t always work well!



Surrogate models

Surrogate models don't always work well!



Machine learning for dynamical systems

- Current approach: Gaussian process regression
  - Predict new points as an intelligently weighted sum of example points
- Bayesian kernel method
  - Kernel specifies a distribution over basis functions
  - Good kernel choice = good data fit
- Most kernels are stationary, and can't handle the spiking behaviours of neurons

Current goal: find an ML approach to fitting a surrogate model

Next questions

- Predictor-corrector design
- Stochastic models

Continuation issues

- Discretisation is required to make predictor-corrector methods work
- It has issues for fast-spiking data
  - Slow to find a Jacobian
  - High noise-sensitivity
- Discretisation-free predictor-correctors might overcome these

Alternative continuation approach

Predictor-corrector design:

- We could try discretisation-free predictor steps, using a surrogate model
  - Let $f_i(t)$ be the surrogate model for system behaviours at parameter λ_i
  - Given periodic orbits $f_{i-1},\ f_i$, predict $f_{i+1} = f_i + h\left[f_i - f_{i-1}\right]$
- Corrector step would be harder

An idea for discretisation-free correction

Main goal of CBC: find $x^*(t)$ such that $\forall t, u(x, x^*) = 0$.

Alternative formulation:

- Let $S[x^*] = \int_0^T u^2(x, x^*)\mathrm{d}t$ measure control invasiveness

An idea for discretisation-free correction

Main goal of CBC: find $x^*(t)$ such that $\forall t, u(x, x^*) = 0$.

Alternative formulation:

- Let $S[x^*] = \int_0^T u^2(x, x^*)\mathrm{d}t$ measure control invasiveness
- $S : \mathcal{H} \to \mathbb{R}$ is a functional on control actions x^*

An idea for discretisation-free correction

Main goal of CBC: find $x^*(t)$ such that $\forall t, u(x, x^*) = 0$.

Alternative formulation:

- Let $S[x^*] = \int_0^T u^2(x, x^*)\mathrm{d}t$ measure control invasiveness
- $S : \mathcal{H} \to \mathbb{R}$ is a functional on control actions x^*
- CBC becomes a calculus of variations problem; find $x^*(t)$ that minimises S

An idea for discretisation-free correction

Main goal of CBC: find $x^*(t)$ such that $\forall t, u(x, x^*) = 0$.

Alternative formulation:

- Let $S[x^*] = \int_0^T u^2(x, x^*)\mathrm{d}t$ measure control invasiveness
- $S : \mathcal{H} \to \mathbb{R}$ is a functional on control actions x^*
- CBC becomes a calculus of variations problem; find $x^*(t)$ that minimises S
- $S = 0$ if and only if $x^*(t)$ is an invariant set of the open-loop system

Calculus of variations

Alternative formulation: find $x^*(t)$ that minimises $S[x^*]$

- Calculus of variations provides a framework for finding minimising functions
- Might be possible to define an iteration scheme on functions, rather than discretisations

Calculus of variations

- Well-studied in control theory; lots of precedent to build on
- Shifts the noninvasiveness requirement away from the continuation scheme, and onto the controller

Variational noninvasiveness

Ideally, corrector would find some iteration sequence $f_1,\ f_2,\ \ldots$, such that $S[f_i] > S[f_{i-1}]$

- Then we've found a function-space iteration scheme to reach noninvasive control
- Works on functions at every step, so we avoid the issues of discretisation

Might be a dead-end.

Variational noninvasiveness

Overall idea:

- Set up CBC as a calculus of variations problem
- Reach noninvasiveness by minimising functional S
- Find a numerical method to do this though iterations on control target $x^*(t)$
- Use the variational equations to reformulate Newton iterations onto functions, rather than vectors
  - Main question: is this even possible?

Stochastic models

Real neurons are stochastic

- Stochasticity introduces new challenges
  - Coherence and stochastic resonance
  - Random attractors
  - Stochastic calculus
- Current work: CBC on noise-corrupted simulations
- Next work: CBC on true stochastic models

Goals

Actions:

- Find a surrogate modelling method for neural data
- Attempt a discretisation-free corrector?
- Run CBC on deterministic models, then stochastic

Results:

- Write up surrogate modelling into a conference abstract *[July]*
  - Maybe a conference paper *[September]*
- Use surrogate modelling for an *in-silico* CBC paper *[next year?]*