Papers, splines, and other ideas

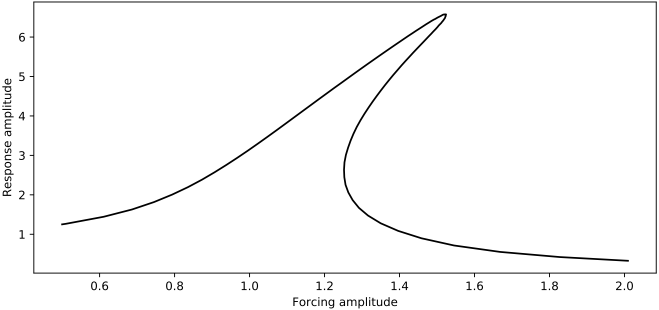
Mark Blyth

Presentation overview

- Splines discretisation progress
- Some ideas: projects that will enhance CBC, and make it more powerful for studying neurons
  - Lots of exciting ideas; these ones are limited to the project-relevant ones

CBC code progress

- Rewritten for a new solver
- Works for Fourier



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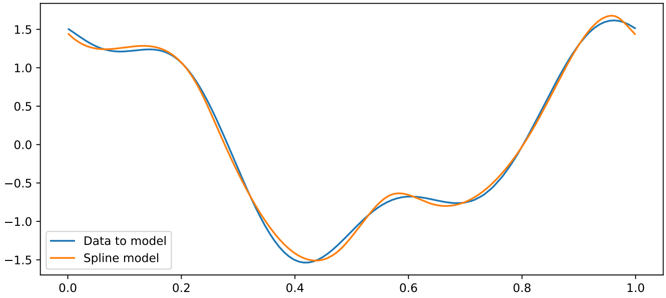
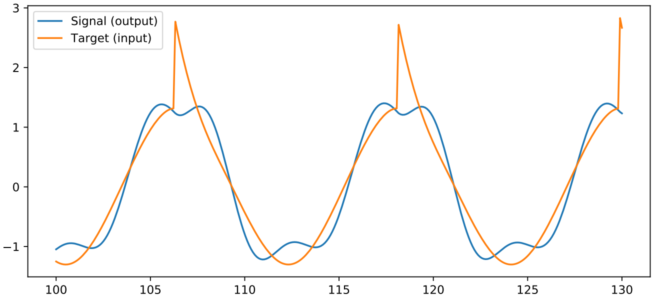


Spline model describes data fairly accurately

Splines discretisation progress



Control-target input is discontinuous during Jacobian estimation

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  - Finite differences evaluates gradients by using small perturbations
  - Finite differences perturbations lead to discontinuous changes in the model
- Currently a best-guess conjecture
  - Need to play about more to see if this is actually the case
  - Need to test different finite-differences stepsizes

Novel discretisors

Ideal: journal paper comparing…

- Splines
  - Knot selection methods?
- Wavelets
- Collocation
- Fourier

Compare for noise-robustness, dimensionality, computational speed

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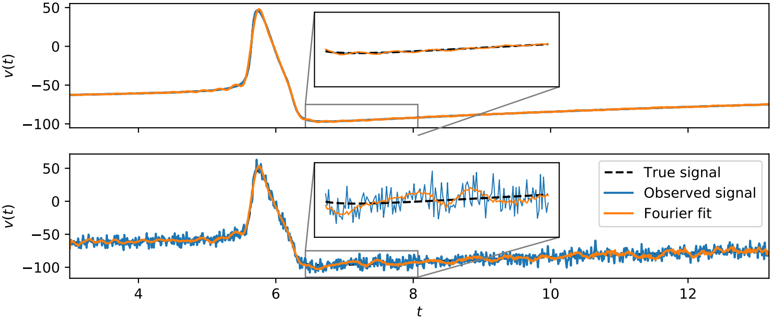
  - Optimally balances goodness-of-fit against model complexity; ensures simplest, most generalizable discretisations; statistically optimal

The sweetspot problem



- Sweetspot problem: too few harmonics don't fit; too many harmonics overfit noise
- Bayesian Fourier model selection would find the sweet spot by automatically trading goodness-of-fit against model complexity

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- Further extension: something resembling a Picard iteration

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- Research output: comparison of convergence time, noise-robustness for

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- Aim main paper at a numerical methods audience; possibly additional journal paper comparing CBC speeds, noise-robustnesses for various numerical solvers

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  - Solve the CBC equations accurately, even faced with measurement noise *[and stochasticity?]*

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Testing the new methods

Any CBC rigs I could demonstrate these methods on?

Next steps

- Edit NODYCON paper
- Edit continuation paper
- Keep working on splines discretisation code
- Test other discretisor methods