Knots, collocation, writing

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Week's activities

- ❧ Spline-Newton CBC with more knots
    - ▶ Goal: more numerical stability
    - ▶ Different results, but not really any better

- ❧ Looked into collocation references

- ❧ Started annual review report

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Newton iteration issues

- Converged solution doesn't actually solve continuation equations
    - ▶ Newton iterations should, but don't, give a vector that, when passed to the continuation equations, give a zero output
    - ▶ More iterations don't help
    - ▶ Different convergence criteria don't improve things

- Solution jumps
    - ▶ Jacobian is always well-conditioned
    - ▶ Probably a finite-differences issue?

- Idea: try more knots!
    - ▶ More knots = more attainable accuracy = perhaps better chance of finding a solution
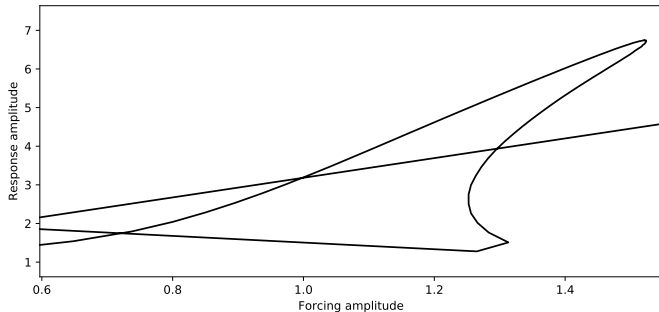
Baseline: 5 knots
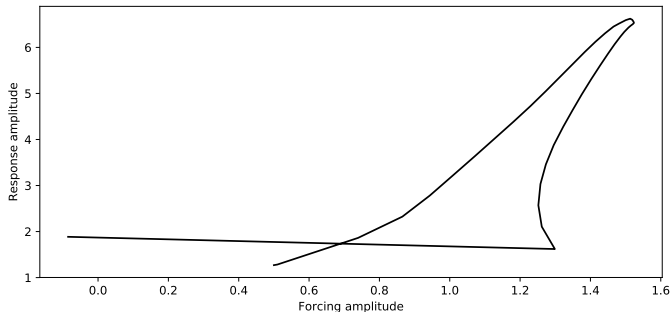
- Minimum 3 interior knots for a valid BSpline model
- Solution jumps
- Converged Newton-iteration vectors don't solve the continuation equations accurately

20 knots
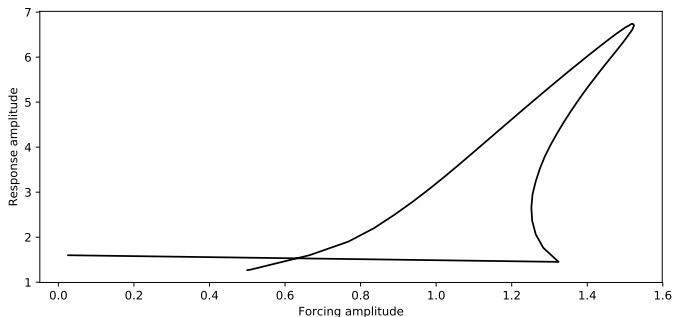
- Simulation is notably slower to run
- Solution still jumps
- Converged Newton-iteration vectors solve system to higher accuracy than before

30 knots

- Simulation is even slower to run
- Solution jumps at about the same place
- Converged Newton-iteration vectors again solve system to higher accuracy

Things to note

- SciPy solvers still get a solution with 5 knots
  - ▶ Means the equations can be solved, but not by the Newton solver
  - ▶ Presumably then, finite differences is unable to find an accurate gradient?

- Solution is jumping after the second fold
  - ▶ I'd have expected this to be one of the more numerically stable places

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Other things to try

- ☙ Adaptive stepsize
  - ▶ Should allow greater accuracy around difficult regions (eg. folds)

- ☙ Adaptive knots
  - ▶ Essential for 'harder' (eg. neuronal) signals
  - ▶ (Presumably) unimportant here

- ☙ Idea: Jacobian checking
  - ▶ Use a secant predictor to estimate the next Jacobian
  - ▶ If the finite-differences Jacobian differs much from the secant prediction, try FD again with a new stepsize
  - ▶ Extension: adaptive-stepsize finite differences

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Effects of control gain

Another thing to try: increasing the control gain

- 🙾 Was originally using $K_p = 1$
    - ▶ This worked fine for Duffing Fourier
    - ▶ Keeping K_p as low as possible seems to give the best-possible accuracy with Fourier
- 🙾 Intuitively, increasing K_p would make it *harder* to find a correct solution, not easier
    - ▶ Smaller gains mean bigger proportional errors mean more difference between invasive and noninvasive targets, between solutions and non-solutions
    - ▶ In limit, large K_p means every control target solves the continuation equations, whether or not they're noninvasive
    - ▶ Intuition: smaller K_p gives a larger gradient at the fixed-point, and therefore a more accurate solution can be found
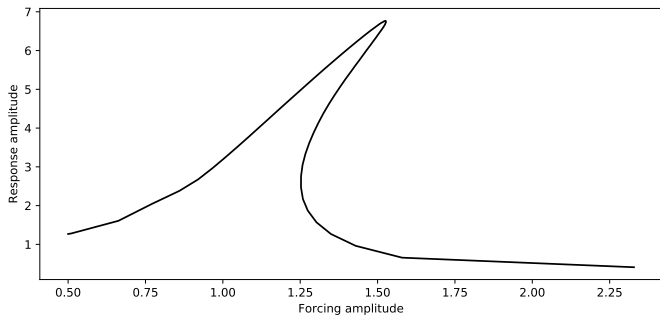
5 knots, $K_p = 2$
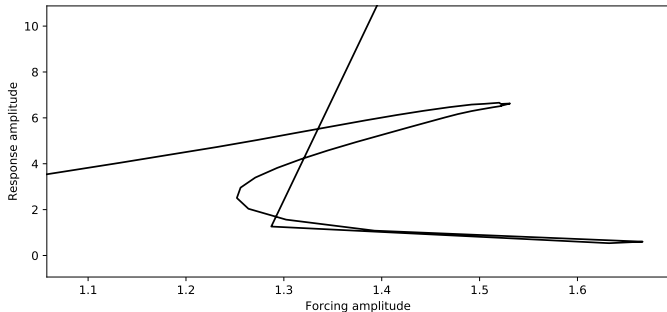
- Unexpected: slight improvement in results
- Using $K_p = 2$ delayed the 'jump'
    - Jump region is controllable with $Kp = 1$ for Fourier, but not splines
- Still doesn't explain why non-Newton solvers could find a solution at $K_p = 1$!
    - If the SciPy solver can find a solution at $K_p = 1$, why can't a Newton solver?

Idea: what's the solution basin of attraction?

20 knots, $K_p = 2$

- Solution takes a huge leap at the end, but it's a correct leap
- It works, but doesn't seem like it should; opposite result to what was expected
- Still doesn't explain what was going wrong with $K_p = 1$

Standard continuation

Other work: considering a 'standard' (non-control-based) continuation of the Duffing oscillator

- Removes any issue from controllers being weird

- Simplifies down to just a discretisation and predictor/corrector problem

- Plan of action:

  1. Learn about collocation and periodic-orbit continuation *[in progress]*
  2. Learn about BSpline collocation for BVPs *[in progress]*
  3. Combine them
  4. Add in the extras (BSpline periodicity structure, choice of knots, choice of collocation meshpoints, if any)
  5. Code up and test
  6. Make the step 4 extras adaptive

Next steps

- Annual review report

- Then...
  - More collocation
  - 'Standard' continuation
  - Investigate solution basin of attraction?
  - Adaptive CBC algos