Conjecture 3
The Stealth-Optimal Execution Distribution

Tomorrow Capital Research

December 2025

Problem Description

This conjecture arose from practical discussions with other systematic trading founders and concerns the interaction between *capital scaling*, *market impact* and *execution stealth*.

The informal observation is the following: given an algorithmic trading strategy with strong performance, it is almost inevitable that capital will, over time, accumulate behind it. As capital scales up, the typical order sizes and average position sizes increase, and firms routinely observe a deterioration of returns. A significant component of this decay is attributable to *adverse market impact* and the *loss of stealth* in execution. We are therefore concerned with two aspects of the same underlying effect:

- **Market impact:** how much our orders move prices against us.
- **Execution stealth:** how detectable our trading footprint and intent become to other market participants.

Large, obvious orders (e.g. a single multi-million buy sweep) tend to:

1. push the order book and short-term order imbalance in an unfavourable direction, and
2. leave a distinct footprint that other firms can learn from and exploit.

Over time, this leads to *alpha decay*: competitors infer the structure of the strategy, trade ahead of it, or "vulture" around it via sandwich-like behaviour.
The high-level goal of this conjecture is to characterise and, if possible, construct *stealth-optimal execution distributions*: a way to spread a given order across time, size and accounts so as to maximise stealth and minimise slippage/cost.

Problem set-up

Let an algorithmic trading signal (alpha) fire at time $t = 0$ and prescribe a net position change of notional size $Q > 0$ (for example $Q = 1{,}000{,}000$ USD) to be executed over an acceptable time horizon

$$T := [0, \tau],$$

from the signal time 0 to a hard time limit $\tau > 0$.
The notional Q corresponds to the total order associated with this particular signal, not the entire portfolio inventory. One may imagine a strategy that triggers many times, either to:

- build a position into a favourable opportunity, or
- gradually reduce a position into a profit-taking region.

Execution schedules. We model an execution schedule in two alternative ways:

1. **Discrete schedule:** a finite collection of trades

$$\mathcal{E}_{\text{disc}} := \{(t_k, q_k)\}_{k=1}^{K}, \qquad 0 \le t_k \le \tau, \quad \sum_{k=1}^{K} q_k = Q,$$

   where q_k denotes the signed notional of the k-th trade.

2. **Continuous schedule:** a measurable order-size density

$$\varphi : [0, \tau] \to \mathbb{R}, \qquad \int_0^{\tau} \varphi(t)\,\mathrm{d}t = Q.$$

In what follows we write $\mathcal{E}$ for a generic execution schedule, discrete or continuous.

Microstructure assumptions

We now list the basic modelling ingredients. The precise functional forms are deliberately left flexible.

1. **Impact / stealth trade-off (square-root visibility).** There exists a function $g : [0, \infty) \to [0, \infty)$, increasing and concave, such that the incremental loss of stealth or footprint associated with an order of size q scales as

$$\text{footprint}(q) \propto g(|q|),$$

   with $g(x) \approx c\sqrt{x}$ for some $c > 0$ on the relevant scale. Executing the full notional Q at time 0 is assumed to produce a near-maximal footprint, while executing very small slices (e.g. 10 USD) produces negligible incremental footprint.

2. **Account-splitting constraint.** The trader may split Q across at most $N_{\max}$ accounts,

$$Q = \sum_{i=1}^{N} Q_i, \qquad N \le N_{\max},$$

   where each account i follows its own execution schedule $\mathcal{E}^{(i)}$. For concreteness, in the informal description we may take $N_{\max} = 100$. This constraint rules out the degenerate solution of splitting Q across infinitely many infinitesimal accounts.

3. **Execution cost / slippage.** Each individual trade incurs both:

   - a base cost modelled via a proportional rate $k\%$ (covering spread, immediate slippage and explicit fees), and
   - an additional market impact component consistent with the footprint function g.

   Denote by $C(\mathcal{E})$ the resulting (random) total execution cost over horizon $[0, \tau]$, and by

$$J_{\text{cost}}(\mathcal{E}) := \mathbb{E}\big[C(\mathcal{E})\big]$$

   its expected value under the chosen price/microstructure model.

   Optionally, one may also assume that fee schedules produce a discount for larger tickets, so that proportional costs shrink by a factor $c\%$ as order size increases.

4. **Cross-impact and opposite flow.** The impact model may include cross-interactions between buys and sells. For example, buy and sell trades of similar magnitude in overlapping time windows can partially offset each other's impact. This can be encoded in the cost functional J_{cost} via a non-additive dependence on the path of order flow.

5. **Market memory / recycle rate.** The market and counterparties are assumed to have a finite *memory horizon* for order flow: beyond some time scale, or after appropriate account recycling, older activity becomes less informative about current intent.

   One toy model is as follows: in each calendar month one may open up to $N_{\max}$ new accounts, each endowed with a "stealth bonus" that decays as a function of its cumulative traded volume. This can be encoded in a *stealth functional* $J_{\text{stealth}}(\mathcal{E})$, where higher values correspond to greater detectability (lower stealth).

Stealth and cost functionals

Abstractly, we consider two functionals defined on the space of admissible execution policies Π (i.e. families $\pi = \{\mathcal{E}^{(i)}\}_{i=1}^{N}$ satisfying the constraints above):

$$\begin{aligned} J_{\text{stealth}}(\pi) &\geq 0 && \text{(higher } J_{\text{stealth}} = \text{more detectable / less stealthy)}, \\ J_{\text{cost}}(\pi) &:= \mathbb{E}\big[C(\pi)\big] && \text{(expected total slippage + execution cost)}. \end{aligned}$$

A simple example would be

$$J_{\text{stealth}}(\pi) = \mathbb{E}\left[\int_0^\tau \Psi\big(q^{\text{tot}}(t)\big)\,\mathrm{d}t\right], \qquad \Psi(x) \approx c\sqrt{|x|},$$

where $q^{\text{tot}}(t)$ is the instantaneous net order flow across all accounts at time t.

One may then study:

- a purely constrained formulation

  $$\text{minimise } J_{\text{stealth}}(\pi) \quad \text{subject to } J_{\text{cost}}(\pi) \leq \kappa,$$

  or

- a scalarised objective

  $$J_\lambda(\pi) := \lambda J_{\text{cost}}(\pi) + (1-\lambda) J_{\text{stealth}}(\pi), \qquad \lambda \in (0,1),$$

  and seek minimisers of J_λ.

Conjecture (Stealth-Optimal Execution Distributions)

Conjecture 1 (Stealth-Optimal Execution Distributions)**.** *Under a reasonable microstructure model satisfying the assumptions above, and for any choice of admissible* stealth *and* cost *functionals* J_{stealth}, J_{cost} *of the type described, there exists at least one execution policy*

$$\pi^\star \in \Pi$$

which is stealth-optimal *in the following sense:*

1. *For a given tolerance κ on expected costs, $\pi^\star$ minimises J_{stealth} over all policies with $J_{\text{cost}} \leq \kappa$; or*
2. *For a given trade-off parameter $\lambda \in (0,1)$, $\pi^\star$ minimises the combined objective $J_\lambda(\pi)$ over Π.*

In particular, $\pi^\star$ induces an optimal joint distribution *of:*

- *execution times $\{t_k^{(i)}\}$,*
- *trade sizes $\{q_k^{(i)}\}$, and*
- *account splits $\{Q_i\}_{i=1}^{N}$ with $N \leq N_{\max}$,*

that:

1. *maximises execution stealth, interpreted as minimal information leakage or detectability of the underlying strategy; and*
2. *simultaneously minimises expected slippage and total execution cost, relative to the alpha's target price path,*

subject to the constraints $t \in [0,\tau]$, $\sum_k q_k^{(i)} = Q_i$ for each account i, and $\sum_{i=1}^{N} Q_i = Q$, $N \leq N_{\max}$.

Open problems and challenges

Natural questions arising from this conjecture include:

1. **Well-posedness.** Under what regularity and compactness assumptions on the functionals J_{stealth} and J_{cost} (and on the admissible policy class Π) can one guarantee the existence of a minimiser $\pi^\star$? When can uniqueness (up to natural symmetries) be established?

2. **Structural properties.** In specific parametric models (e.g. square-root impact with linear fees), does an optimal policy exhibit qualitatively interpretable structure, such as:

   - time-smoothing of order flow,
   - bounded per-trade sizes,
   - a non-trivial optimal number of active accounts $N^\star \in \{1, \ldots, N_{\max}\}$?

3. **Constructive algorithms.** Can one design explicit algorithms which, given:

   - an alpha signal,
   - a total notional Q,
   - a horizon τ and constraints $(N_{\max}, k, c, \ldots)$,

   produce an approximately stealth-optimal execution policy $\hat{\pi}$? How do such algorithms compare in practice to standard TWAP/VWAP and Almgren–Chriss-type schedules when evaluated on realistic data?

4. **Robustness.** How sensitive is $\pi^\star$ to mis-specification of impact, memory, and fee parameters? Can one characterise policies that are robust across a family of plausible microstructure models?