Quiz number 4 Solutions

Use the superaugmented matrix of the matrix

$$A = \begin{pmatrix} 1 & 5 & 2 \\ 1 & 4 & 3 \\ 3 & 5 & 14 \end{pmatrix}, \quad \text{to find solutions to the equation } A\vec{x} = \vec{b}, \text{ for } \vec{b} = \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix} \text{ and } \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix}.$$

Building the superaugmented matrix and row reducing:

$$\begin{pmatrix} 1 & 5 & 2 & | & 1 & 0 & 0 \\ 1 & 4 & 3 & | & 0 & 1 & 0 \\ 3 & 5 & 14 & | & 0 & 0 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 5 & 2 & | & 1 & 0 & 0 \\ 0 & -1 & 1 & | & -1 & 1 & 0 \\ 0 & -10 & 8 & | & -3 & 0 & 1 \end{pmatrix} \rightarrow$$

$$\begin{pmatrix} 1 & 5 & 2 & | & 1 & 0 & 0 \\ 0 & 1 & -1 & | & 1 & -1 & 0 \\ 0 & -10 & 8 & | & -3 & 0 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 7 & | & -4 & 5 & 0 \\ 0 & 1 & -1 & | & 1 & -1 & 0 \\ 0 & 0 & -2 & | & 7 & -10 & 1 \end{pmatrix} \rightarrow$$

$$\begin{pmatrix} 1 & 0 & 7 & | & -4 & 5 & 0 \\ 0 & 1 & -1 & | & 1 & -1 & 0 \\ 0 & 0 & 1 & | & -7/2 & 5 & -1/2 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & 0 & | & 41/2 & -30 & 7/2 \\ 0 & 1 & 0 & | & -5/2 & 4 & -1/2 \\ 0 & 0 & 1 & | & -7/2 & 5 & -1/2 \end{pmatrix}$$

So setting $B = \begin{pmatrix} 41/2 & -30 & 7/2 \\ -5/2 & 4 & -1/2 \\ -7/2 & 5 & -1/2 \end{pmatrix}$, $BA = I_3$, so $A\vec{x} = \vec{b}$ for $\vec{x} = B\vec{b}$, so

$$A\vec{x} = \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix} \text{ has solution } \quad \vec{x} = \begin{pmatrix} 41/2 & -30 & 7/2 \\ -5/2 & 4 & -1/2 \\ -7/2 & 5 & -1/2 \end{pmatrix} \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix} =$$

$$\begin{bmatrix} (41/2)(1) + (-30)(3) + (7/2)(2) \\ (-5/2)(1) + (4)(3) + (-1/2)(2) \\ (-7/2)(1) + (5)(3) + (-1/2)(2) \end{bmatrix} = \begin{bmatrix} -125/2 \\ 17/2 \\ 21/2 \end{bmatrix}, \text{ and}$$

$$A\vec{x} = \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix} \text{ has solution } \quad \vec{x} = \begin{pmatrix} 41/2 & -30 & 7/2 \\ -5/2 & 4 & -1/2 \\ -7/2 & 5 & -1/2 \end{pmatrix} \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix} =$$

$$\begin{bmatrix} (41/2)(0) + (-30)(1) + (7/2)(3) \\ (-5/2)(0) + (4)(1) + (-1/2)(3) \\ (-7/2)(0) + (5)(1) + (-1/2)(3) \end{bmatrix} = \begin{bmatrix} -39/2 \\ 5/2 \\ 7/2 \end{bmatrix}.$$

Quiz number 4 Solutions

Use the superaugmented matrix of the matrix

$$A = \begin{pmatrix} 3 & 4 & 2 \\ 1 & 2 & 3 \\ -1 & 5 & 21 \end{pmatrix}, \quad \text{to find solutions to the equation } A\vec{x} = \vec{b}, \text{ for } \vec{b} = \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix} \text{ and } \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix}.$$

Building the superaugmented matrix and row reducing:

$$\begin{pmatrix} 3 & 4 & 2 & | & 1 & 0 & 0 \\ 1 & 2 & 3 & | & 0 & 1 & 0 \\ -1 & 5 & 21 & | & 0 & 0 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 2 & 3 & | & 0 & 1 & 0 \\ 3 & 4 & 2 & | & 1 & 0 & 0 \\ -1 & 5 & 21 & | & 0 & 0 & 1 \end{pmatrix} \rightarrow$$

$$\begin{pmatrix} 1 & 2 & 3 & | & 0 & 1 & 0 \\ 0 & -2 & -7 & | & 1 & -3 & 0 \\ 0 & 7 & 24 & | & 0 & 1 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 2 & 3 & | & 0 & 1 & 0 \\ 0 & 1 & 7/2 & | & -1/2 & 3/2 & 0 \\ 0 & 7 & 24 & | & 0 & 1 & 1 \end{pmatrix} \rightarrow$$

$$\begin{pmatrix} 1 & 0 & -4 & | & 1 & -2 & 0 \\ 0 & 1 & 7/2 & | & -1/2 & 3/2 & 0 \\ 0 & 0 & -1/2 & | & 7/2 & -19/2 & 1 \end{pmatrix} \rightarrow \begin{pmatrix} 1 & 0 & -4 & | & 1 & -2 & 0 \\ 0 & 1 & 7/2 & | & -1/2 & 3/2 & 0 \\ 0 & 0 & 1 & | & -7 & 19 & -2 \end{pmatrix} \rightarrow$$

$$\begin{pmatrix} 1 & 0 & 0 & | & -27 & 74 & -8 \\ 0 & 1 & 0 & | & 24 & -65 & 7 \\ 0 & 0 & 1 & | & -7 & 19 & -2 \end{pmatrix}$$

So setting $B = \begin{pmatrix} -27 & 74 & -8 \\ 24 & -65 & 7 \\ -7 & 19 & -2 \end{pmatrix}$, $BA = I_3$, so $A\vec{x} = \vec{b}$ for $\vec{x} = B\vec{b}$, so

$$A\vec{x} = \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix} \text{ has solution } \quad \vec{x} = \begin{pmatrix} -27 & 74 & -8 \\ 24 & -65 & 7 \\ -7 & 19 & -2 \end{pmatrix} \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix} =$$

$$\begin{bmatrix} (-27)(1) + (74)(3) + (-8)(2) \\ (24)(1) + (-65)(3) + (7)(2) \\ (-7)(1) + (19)(3) + (-2)(2) \end{bmatrix} = \begin{bmatrix} 179 \\ -157 \\ 46 \end{bmatrix}, \text{ and}$$

$$A\vec{x} = \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix} \text{ has solution } \quad \vec{x} = \begin{pmatrix} -27 & 74 & -8 \\ 24 & -65 & 7 \\ -7 & 19 & -2 \end{pmatrix} \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix} =$$

$$\begin{bmatrix} (-27)(0) + (74)(1) + (-8)(3) \\ (24)(0) + (-65)(1) + (7)(3) \\ (-7)(0) + (19)(1) + (-2)(3) \end{bmatrix} = \begin{bmatrix} 50 \\ -44 \\ 13 \end{bmatrix}.$$