Math 208H, Section 1

Quiz number 4 Solutions

Show all work. How you get your answer is just as important, if not more important, than the answer itself. If you think it, write it!

In which direction is the function

$$z = f(x,y) = \frac{xy^2}{x^2+y}$$

increasing the fastest, at the point $(a,b) = (1,2)$?
What is the equation for the tangent plane to the graph of f, at this same point?

The gradient points in the direction of factest increase, and so we compute:

$f_x = \dfrac{(x^2+y)(y^2)-(xy^2)(2x)}{(x^2+y)^2}$, and $f_y = \dfrac{(x^2+y)(2xy)-(xy^2)(1)}{(x^2+y)^2}$.

At $(a,b) = (1,2)$, we then have

$f_x(1,2) = \dfrac{(1^2+2)(2^2)-(1\cdot 2^2)(2\cdot 1)}{(1^2+2)^2} = \dfrac{12-8}{3^2} = \dfrac{4}{9}$,

and $f_y(1,2) = \dfrac{(1^2+2)(2\cdot 1\cdot 2)-(1\cdot 2^2)(1)}{(1^2+2)^2} = \dfrac{12-4}{3^2} = \dfrac{8}{9}$.

So $\nabla f(1,2) = (\frac{4}{9}, \frac{8}{9})$ points in the direction of fastest increase.

We also use the partial derivatives to give an equation for the tangent plane:

$f(1,2) = \dfrac{1\cdot 2^2}{1^2+2} = \dfrac{4}{3}$, and so the equation for the tangent plane is

$$\begin{aligned} L(x,y) &= \frac{4}{3} + \frac{4}{9}(x-1) + \frac{8}{9}(y-2) \\ &= \frac{4}{9}x + \frac{8}{9}y + \left(\frac{12}{9} - \frac{4}{9} - \frac{16}{9}\right) \\ &= \frac{4}{9}x + \frac{8}{9}y - \frac{8}{9}\ . \end{aligned}$$

[The first version is usually the one we will find it most convenient to work with...]