Math 445

Exam 1

Show all work. How you get your answer is just as important, if not more important, than the answer itself. If you think it, write it!

1. (20 pts.) Show that $3|n^3 + 5n$ for every $n \geq 1$.

2. (25 pts.) Use the facts that $\text{ord}_{23}(2) = 11$ and $\text{ord}_{23}(5) = 22$ to find the period of the repeating decimal expansion of $\frac{1}{23}$.

3. (25 pts.) Show that if p is prime, $(a,p) = (b,p) = 1$, and *neither* of the equations

$$x^2 \equiv a \pmod{p} \qquad \text{or} \qquad x^2 \equiv b \pmod{p}$$

have a solution, then the equation $x^2 \equiv ab \pmod{p}$ *does* have a solution.

4. (15 pts. each) For each of the following equations, determine if it has a solution, and if so, how many (modulo 49):

(a): $x^5 \equiv 10 \pmod{49}$

(b): $x^7 \equiv 10 \pmod{49}$