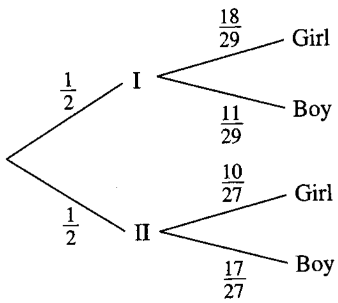
11. There are two fifth-grade classes in a school. Class I has 18 girls and 11 boys while class II has 10 girls and 17 boys. The principal will randomly select one fifth-grade class and one student from that class to represent the fifth grade on the student council. A probability tree diagram for this experiment is given next.



Find each of the following:

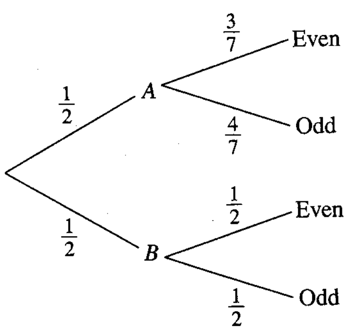
a. P(a girl is selected)

b. P(a boy is selected)

c. P(a girl is selected given that the student came from class II)

d. P(a boy is selected given that the student came from class I)

12. Box A contains 7 cards numbered 1 through 7, and box B contains 4 cards numbered 1 through 4. A box is chosen at random and a card is drawn. It is then noted whether the number on the card is even or odd. A probability tree diagram for this experiment is given next.



Find the following probabilities:

a. P(the number is even)

b. P(the number is odd)

c. P(the number is even given that it came from box A)

d. P(the number is odd given that it came from box B)

13. A triple test is a blood test, which can be offered to a woman who is in her 15th to 22nd week of pregnancy, to screen for such fetal abnormalities as Down syndrome. A positive triple test would indicate that the fetus might have a birth defect. The table below contains triple test results from a large sample of women. Assume that the results in the table are representative of the population as a whole.

Triple Test Results	Fetus with Down Syndrome	Fetus without Down Syndrome
Positive	5	208
Negative	1	2386

a. Given a positive test result, what is the probability that the fetus has Down syndrome?

b. Given a positive test result, what is the probability that the fetus does not have Down Syndrome?

c. Given that a fetus has Down syndrome, what is the probability that the test is negative?

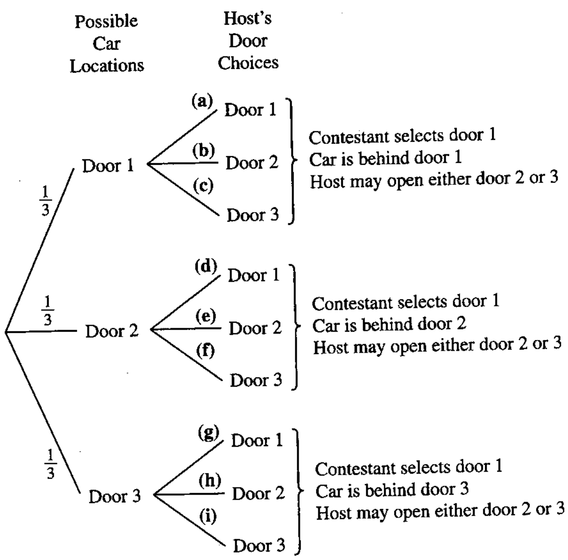
14. In a fuse factory, machines A, B, and C manufacture 20%, 45%, and 35%, respectively, of the total fuses. Of the outputs for each machine, A produces 5% defectives, B produces 2% defectives, and C produces 3% defectives. A fuse is drawn at random.

a. Given that a fuse is defective, what is the probability it came from machine A?

b. Given that a fuse is defective, what is the probability it came from machine B?

c. Given that a fuse is defective, what is the probability it came from machine C?

20. In problem 19, the assumption has been made that the host will always open a door to reveal a goat. Suppose we allow the case in which, after the contestant selects a door, the host randomly selects a different door. In this case, the host might open a door to reveal the car. Fill in the missing probabilities in the following tree diagram.



21. The contestant selects door 1 in problem 19. Suppose the host opens door 2.

a. Find P(Car is behind door 1 | Host opens door 2).

b. Find P(Car is behind door 3 | Host opens door 2).

c. Considering the probabilities found in parts (a) and (b), is it to the contestant's advantage to switch doors?

22. Consider the tree diagram from problem 20. The contestant selects door 1. Suppose the host opens door 2.

a. Find P(Car is behind door 1 | Host opens door 2).

b. Find P(Car is behind door 3 | Host opens door 2).

c. Considering the probabilities found in parts (a) and (b), is it to the contestant's advantage to switch doors?

23. The contestant selects door 1 in problem 19. Suppose the host opens door 3.

a. Find P(Car is behind door 1 | Host opens door 3).

b. Find P(Car is behind door 2 | Host opens door 3).

c. Considering the probabilities found in parts (a) and (b), is it to the contestant's advantage to switch doors?

24. Consider the tree diagram from problem 20. The contestant selects door 1. Suppose the host opens door 3.

a. Find P(Car is behind door 1 | Host opens door 3).

b. Find P(Car is behind door 2 | Host opens door 3).

c. Considering the probabilities found in parts (a) and (b), is it to the contestant's advantage to switch doors?

25. Two classes at a university are studying modern Latin-American fiction. Twenty of the 25 students in the first class speak Spanish, and 12 of the 18 students in the second class speak Spanish. If a student is selected at random from each of the 2 classes, what is the probability that both students speak Spanish? Show how you calculate your answer. What probability property did you use? Explain.

26. Two assembly lines are producing ink cartridges for a desktop printer. Five percent of the cartridges produced by the first assembly line are defective, while 10% of those produced from the second assembly line are defective. If a cartridge is selected randomly from each line, what is the probability that neither cartridge will be defective? Show how you calculate your answer. What probability property did you use? Explain.

27. Suppose you set your compact disc player to randomly play the 11 tracks on a CD. Tracks 1, 4, and 5 are your favorites. You listen to two songs.

a. Find the probability that the second song is one of your favorites.

b. Find the probability that the second song is one of your favorites, given that the first song was one of your favorites.

c. Are these events independent? Explain your reasoning.

28. Suppose the random-track-selection feature on your CD player is malfunctioning so that once a track is played, the same track is twice as likely to be selected next. After setting your compact disc player to randomly play the five tracks on a CD, you listen to two songs.

a. Find the probability track 3 plays first.

b. Find the probability track 3 plays second, given that track 3 played first.

c. Are these events independent? Explain your reasoning.

35. For visiting a resort (and listening to a sales presentation), you will receive one gift. The probability of receiving each gift and the manufacturer's suggested retail value are as follows:

gift A, 1 in 52,000 ($9272.00)
gift B, 25,736 in 52,000 ($44.95)
gift C, 1 in 52,000 ($2500.00)
gift D, 3 in 52,000 ($729.95)
gift E, 25,736 in 52,000 ($26.99)
gift F, 3 in 52,000 ($1000.00)
gift G, 180 in 52,000 ($44.99)
gift H, 180 in 52,000 ($63.98)
gift I, 160 in 52,000 ($25.00)

Find the expected value of your gift. Round to the nearest cent.

36. According to a publisher's records, 20% of the children's books published break even, 30% lose $1000, 25% lose $10,000, and 25% earn $20,000. When a book is published, what is the expected income for the book?

37. Suppose you and a friend play a game. Two standard dice are rolled and the numbers showing on each die are multiplied. If the product is even, your friend gives you a quarter, but if the product is odd, you must give your friend one dollar.

a. What is the expected value of the game for you? Round to the nearest cent.

b. What is the expected value of the game for your friend? Round to the nearest cent.

c. How could you change the amount you pay your friend so that the expected value of the game for you is $0.05?

38. At a carnival, you play a dice game in which you roll two standard dice. If you roll a total of 7, then you win $1. If you roll double 6s, you lose $5. If you roll any other combination, you win $0.25.

a. What is the expected value of the game?

b. If the carnival wants to make sure that the player loses $0.10 on average, how should the payoff for rolling a total of 7 be adjusted?

39. Suppose that the probability of an event is $\frac{1}{5}$.

a. What are the odds in favor of the event?

b. What are the odds against the event?

40. Suppose the probability of an event is $\frac{7}{19}$.

a. What are the odds in favor of the event?

b. What are the odds against the event?

41. If the odds against an event are 2 to 1, what is the probability of the event?

42. If the odds in favor of an event are 13:11, what is the probability of the complement of the event?

43. Suppose three coins are tossed.

a. What are the odds in favor of getting all heads?

b. What are the odds against getting only one head?

c. If event T is defined as getting exactly two tails, then what are the odds for $\overline{T}$?

44. Suppose two standard dice are rolled.

a. What are the odds in favor of getting a sum of 6?

b. What are the odds against getting a 3 on the second die?

c. If event L is defined as getting a total of at least 9, what are the odds in favor of the complement of L?

Extended Problems

45. In an effort to fight an apparent growth in the use of illegal drugs, many companies, professional sports teams, and schools have established drug-testing programs. Research the most common forms of drug testing: urinalysis, blood testing, saliva testing, and hair testing. For each test, what is the probability of getting a false positive result or a false negative result?

46. Two major trials in the latter half of the 1990s focused the attention of the nation on DNA testing: the 1995 murder trial of O. J. Simpson and the 1998–1999 impeachment trial of President Bill Clinton. Research DNA testing by searching keywords "DNA testing" on the Internet. How accurate are these tests in general, and what circumstances could lead to a false match? Describe what DNA is, and list some of the purposes for which DNA testing has been used, along with the probabilities involved. Write a report summarizing your findings.

Key Ideas and Questions

The following questions review the main ideas of this chapter. Write your answers to the questions and then refer to the pages listed by number to make certain that you have mastered these ideas.

1. What is the difference between an outcome and an event? pg. 634 How can all the elements in a sample space be listed? pgs. 634–635
2. How are experimental probabilities determined? pgs. 635–636 How are theoretical probabilities of events with equally likely outcomes calculated? pgs. 636–637
3. What is meant by the union of two events and the intersection of two events? pg. 639 What does it mean to say that two events are mutually exclusive? pg. 639 How is the probability of mutually exclusive events calculated? pg. 639
4. What is the complement of an event? pg. 640 How are the probabilities of an event and the complement of an event related? pg. 640
5. How do you compute the probability the union of two events that are not mutually exclusive? pg. 642 What are the properties of probability? pg. 643
6. What types of experiments can be represented by one-stage tree diagrams? pg. 652 What types of experiments can be represented by two-stage tree diagrams? pg. 652
7. What is the Fundamental Counting Principal? pg. 654 Under what condition can the probabilities of events in a probability tree diagram be added? pg. 657
8. For the experiment of drawing two marbles from a jar, how do drawing with replacement and drawing without replacement affect probability? pgs. 658–661
9. For what reason would you multiply probabilities along a series of branches in a probability tree diagram? pg. 660
10. How do you describe the meaning of conditional probability? pg. 674 How do you compute conditional probability? pg. 674
11. What does it mean to say two events are independent? pg. 680 If A and B are independent, then what is the conditional probability of A given B? pg. 681
12. What is the interpretation of the expected value of an experiment? pg. 682 How do you calculate the expected value of an experiment? pg. 682
13. What is the difference between the odds in favor of an event and the odds against an event? pg. 684 How can the probability of an event be determined given the odds in favor of the event? pg. 684

Vocabulary

Following is a list of key vocabulary for this chapter. Mentally review each of these terms, write down the meaning of each one in your own words, and use it in a sentence. Then refer to the page number following each term to review any material that you are unsure of before solving the Chapter 10 Review Problems.

SECTION 10.1

Experiment 634
Outcome 634
Sample Space 634
Event 634
Experimental Probability 635
Probability of an Event 636
Theoretical Probability 637
Fair Coin/Fair Die 637
Union 639
Intersection 639
Mutually Exclusive 639
Complement of the Event E 640

SECTION 10.2

Tree Diagram 652
One-Stage Tree Diagram 652
Two-Stage Tree Diagram 652–653
Primary Branches 653
Secondary Branches 653
Fundamental Counting Principle 654
Probability Tree Diagram 656
Additive Property of Probability Tree Diagrams 657
Drawing with Replacement 658
Drawing without Replacement 658
Multiplicative Property of Probability Tree Diagrams 660

SECTION 10.3

Conditional Sample Space 674
Conditional Probability 674
Independent Events 680
Expected Value 682
Fair Game 682
Odds in Favor of an Event 684
Odds Against an Event 684

9. A fast-food restaurant requires that employees wear a uniform. Employees have three color options for shirts (red, yellow, white) and four color options for slacks (black, gray, navy, khaki). Suppose a shirt and a pair of slacks are each selected at random.

a. How many uniform combinations are possible?

b. Construct the probability tree diagram for the experiment of selecting a uniform.

c. Find the probability the employee is wearing black slacks and a shirt that is not white.

10. **a.** What is the probability of drawing three aces in a row from a thoroughly shuffled deck if the cards are drawn with replacement?

b. What is the probability of drawing three aces in a row from a thoroughly shuffled deck if the cards are drawn without replacement?

11. A pediatric dental assistant randomly sampled 200 patients and classified them according to whether or not they had at least one cavity on their last checkup and according to the type of tooth decay preventive measures they used. The information is presented in the following table.

	At Least One Cavity	No Cavities
Brush only	69	2
Brush and floss only	34	11
Brush and tooth sealants only	22	13
Brush, floss, and tooth sealants	3	46

If a patient is picked at random from this group, find the probability that

a. the patient had at least one cavity.

b. the patient brushes only.

c. the patient had no cavities, given he or she brushes, flosses, and has tooth sealants.

d. the patient brushes only, given that he or she had at least one cavity.

12. A jar contains four marbles: two red and two blue. Marbles will be drawn one at a time, without replacement, until two marbles of the same color have been drawn.

a. Draw a probability tree diagram to represent this experiment.

b. What is the probability that the first marble drawn is blue?

c. What is the probability that the second marble drawn is blue?

d. What is the probability that three drawings are needed and the final marble drawn is blue?

e. What is the probability that only two drawings are necessary?

13. Suppose there are three urns, each containing six balls. The first urn has three red and three white balls. The second urn has four red and two white. The third urn has five red and one white. Two of the urns are selected randomly and their contents are mixed. Then four balls are drawn without replacement. What is the probability that all four balls are red?

14. Suppose two standard dice are rolled. Let F be the event of getting a 4 on the first die. Let O be the event of getting an odd number on the second die. Find and interpret $P(F \cup O)$ and $P(F \cap O)$. Are events F and O independent? Explain.

15. A small college has two psychology classes. The first class has 25 students, 15 of whom are female, and the other class has 18 students, 8 of whom are female. One of the classes is selected at random, and two students are randomly selected from that class for an interview. If both of the students are female, what is the probability they both came from the first class?

16. The probability is 0.6 that a student will study for a true/false test. If the student studies, she has a 0.8 probability of getting an A. If she does not study, she has a 0.3 probability of getting an A. Make a probability tree diagram for this experiment. What is the probability that she gets an A? If she gets an A, what is the conditional probability that she studied?