Question 1. Perform the following calculations exactly. Show all your working. Unsupported answers will receive no credit.

a. $\lim_{x\to\infty} \frac{2x^3 - x^4}{1+3x^4}$

Answer:

$$\begin{aligned}\lim_{x\to\infty} \frac{2x^3 - x^4}{1+3x^4} &= \lim_{x\to\infty} \frac{\frac{2x^3}{x^4} - \frac{x^4}{x^4}}{\frac{1}{x^4} + \frac{3x^4}{x^4}} \\ &= \lim_{x\to\infty} \frac{\frac{2}{x} - 1}{\frac{1}{x^4} + 3} \\ &= \frac{0-1}{0+3} = -1/3\end{aligned}$$

b. $\frac{d}{dx}\left((x^2+1)\ln(x)\right)$

Answer:

$$\begin{aligned}\frac{d}{dx}\left((x^2+1)\ln(x)\right) &= \ln(x)\frac{d}{dx}(x^2+1) + (x^2+1)\frac{d}{dx}\ln(x) \\ &= 2x\ln(x) + (x^2+1)\frac{1}{x}\end{aligned}$$

c. $\lim_{x\to 0} \frac{e^x - x - 1}{x^2}$

Answer: By L'Hopital's rule (applied twice) we have (since the final limit exists)

$$\begin{aligned}\lim_{x\to 0} \frac{e^x - x - 1}{x^2} &= \lim_{x\to 0} \frac{e^x - 1}{2x} \\ &= \lim_{x\to 0} \frac{e^x}{2} \\ &= \frac{1}{2}.\end{aligned}$$

d. $\frac{d^2}{dx^2}\left(\arctan(x^2)\right)$

Answer:

$$\begin{aligned}\frac{d^2}{dx^2}\left(\arctan(x^2)\right) &= \frac{d}{dx}\left(2x \cdot \frac{1}{1+(x^2)^2}\right) \\ &= \frac{2}{1+x^4} - \frac{2x}{(1+x^4)^2} \cdot 4x^3 \\ &= \frac{2}{1+x^4} - \frac{8x^4}{(1+x^4)^2}\end{aligned}$$

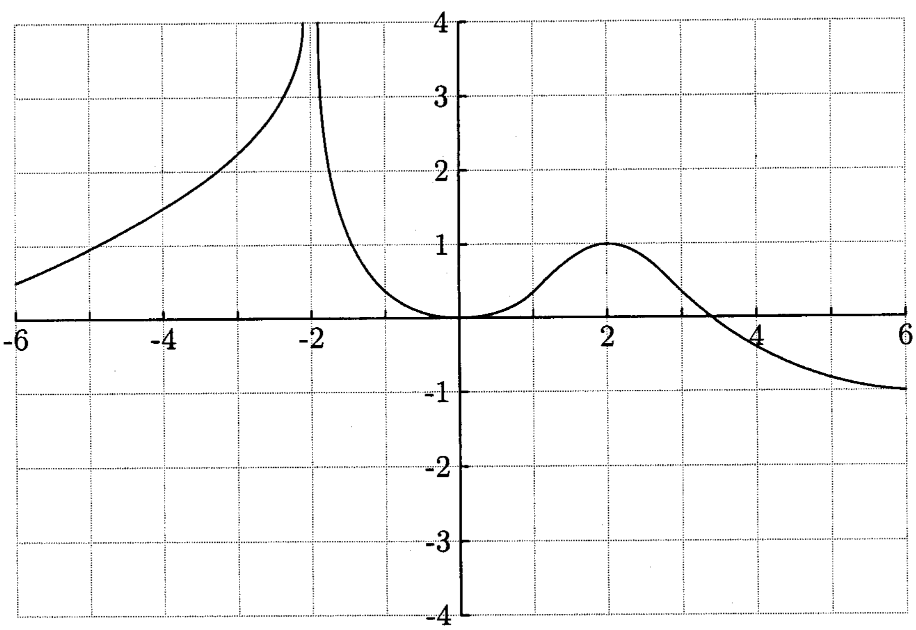
Question 2. Use the **definition of the derivative** to find the exact value of $f'(7)$ where $f(x) = \dfrac{2}{x+3}$. No credit will be given to solutions by other methods.

Answer:

$$\begin{aligned}
f'(7) &= \lim_{h\to 0} \frac{f(7+h) - f(7)}{h} \\
&= \lim_{h\to 0} \frac{\frac{2}{7+h+3} - \frac{2}{7+3}}{h} \\
&= \lim_{h\to 0} \frac{\frac{2}{10+h} - \frac{2}{10}}{h} \\
&= \lim_{h\to 0} \frac{1}{h} \cdot \left(\frac{2}{10+h} - \frac{2}{10} \right) \\
&= \lim_{h\to 0} \frac{1}{h} \cdot \frac{20 - 2(10+h)}{(10+h)10} \\
&= \lim_{h\to 0} \frac{1}{h} \cdot \frac{-2h}{100+10h} \\
&= \lim_{h\to 0} \frac{-2}{100+10h} \\
&= \frac{-2}{100} = -\frac{1}{50}
\end{aligned}$$

Question 3. On the axes below sketch a graph of a function $f(x)$ that satisfies the following conditions:

(i) $\lim_{x\to -2^-} f(x) = \infty$, $\quad \lim_{x\to -2^+} f(x) = \infty$.
(ii) $f(0) = 0$.
(iii) $f'(0) = f'(2) = 0$, $f'(x) > 0$ on $(-\infty, -2)$ and $(0, 2)$ while $f'(x) < 0$ on $(-2, 0)$ and $(2, \infty)$.
(iv) $f''(x) > 0$ on $(-\infty, -2)$, $(-2, 1)$, and $(3, \infty)$, while $f''(x) < 0$ on $(1, 3)$.



Question 4. Consider the curve defined by $x^5y^3 + x^2y^8 = 0$. What is the equation of the tangent line to the curve at the point $(1, -1)$?

Answer: To find the slope at $(1, -1)$ we use implicit differentiation.

$$\begin{aligned}
\frac{d}{dx}\left(x^5y^3 + x^2y^8\right) &= \frac{d}{dx}0 \\
5x^4y^3 + x^5 3y^2\frac{dy}{dx} + 2xy^8 + x^2 8y^7\frac{dy}{dx} &= 0 \\
\frac{dy}{dx}\left(3x^5y^2 + 8x^2y^7\right) &= -5x^4y^3 - 2xy^8 \\
\frac{dy}{dx} &= -\frac{5x^4y^3 + 2xy^7}{3x^5y^2 + 8x^2y^7}.
\end{aligned}$$

At $x - 1, y = -1$ we have $\frac{dy}{dx} = -3/5$. The equation of the tangent line is thus (in point slope form)

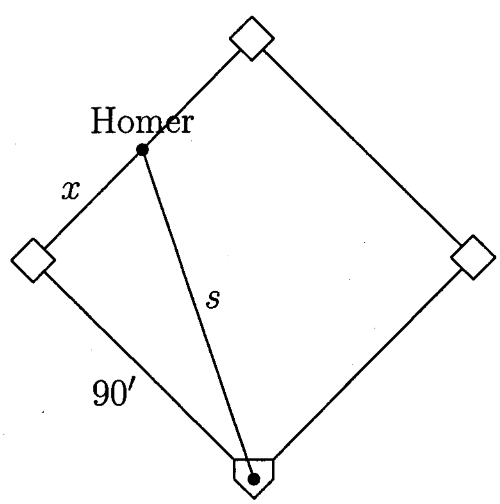
$$\frac{y+1}{x-1} = -\frac{3}{5},$$

or equivalently

$$y = -\frac{3}{5}(x-1) - 1 = -\frac{3}{5}x - \frac{2}{5}.$$

Question 5. On a baseball diamond Homer Runn is dashing from second base to third at a speed of 25 feet per second. How rapidly is the distance from Homer to home plate decreasing when he is exactly midway between second and third base? [A baseball diamond is a 90 feet by 90 feet square whose corners are named (in counter-clockwise order) first base, second base, third base, and home plate.]

Answer: In the diagram below we have, at the relevant time, $\frac{dx}{dt} = -25$, $x = 45$, and $s = \sqrt{90^2 + 45^2} = 45\sqrt{5}$.



Since $s^2 = 90^2 + x^2$ we have

$$\begin{aligned}
2s\frac{ds}{dt} &= 2x\frac{dx}{dt} \\
\frac{ds}{dt} &= \frac{x}{s}(-25) \\
&= \frac{-25}{\sqrt{5}} = -5\sqrt{5}
\end{aligned}$$

Question 6. Perform the following calculations exactly. Show all your work. Unsupported answers will receive no credit.

a. $\displaystyle\int \left(3x^{2.1} + \sin(2x)\right)\, dx$

Answer:

$$\int \left(3x^{2.1} + \sin(2x)\right)\, dx = \frac{3}{3.1}x^{3.1} - \frac{1}{2}\cos(2x) + c.$$

b. $\displaystyle\frac{d}{dx}\left(\int_0^{2x} \cos(t^2)\, dt\right)$

Answer:

$$\begin{aligned}\frac{d}{dx}\left(\int_0^{2x} \cos(t^2)\, dt\right) &= \cos\left((2x)^2\right) \cdot \frac{d}{dx}(2x) \\ &= 2\cos(4x^2).\end{aligned}$$

c. $\displaystyle\int_0^{\sqrt[3]{\pi/2}} \theta^2 \sin\left(\theta^3\right)\, d\theta$

Answer: Making the substitution $u = \theta^3$, $du = 3\theta^2 d\theta$, we have

$$\begin{aligned}\int_0^{\sqrt[3]{\pi/2}} \theta^2 \sin\left(\theta^3\right)\, d\theta &= \frac{1}{3}\int_{u=0}^{u=\pi/2} \sin(u)\, du \\ &= -\frac{1}{3}\cos(u)\Big|_{u=0}^{u=\pi/2} \\ &= 0 - \left(-\frac{1}{3}\right) = 1/3.\end{aligned}$$

Question 7. Using calculus, find the maximum and minimum values of $f(x) = x^3 + 3x^2 - 9x + 4$ on the interval $-2 \leq x \leq 2$. [You should be sure to justify your answer.]

Answer: The maximum and minimum values occur either at critical points of f or at the endpoints of the interval. To find the critical points we need to find places where $f'(x)$ is either 0 or undefined. We have $f'(x) = 3x^2 + 6x - 9$, which is clearly always defined. It is 0 exactly when we have

$$\begin{aligned}3x^2 + 6x - 9 &= 0 \\ 3(x+3)(x-1) &= 0 \\ x &= 1, -3.\end{aligned}$$

The only critical point in our interval is $x = 1$. That candidates are thus $x = -2, 1, 2$, and since $f(-2) = 26$, $f(1) = -1$, and $f(2) = 2$, the maximum value is 26 and the minimum value is -1.

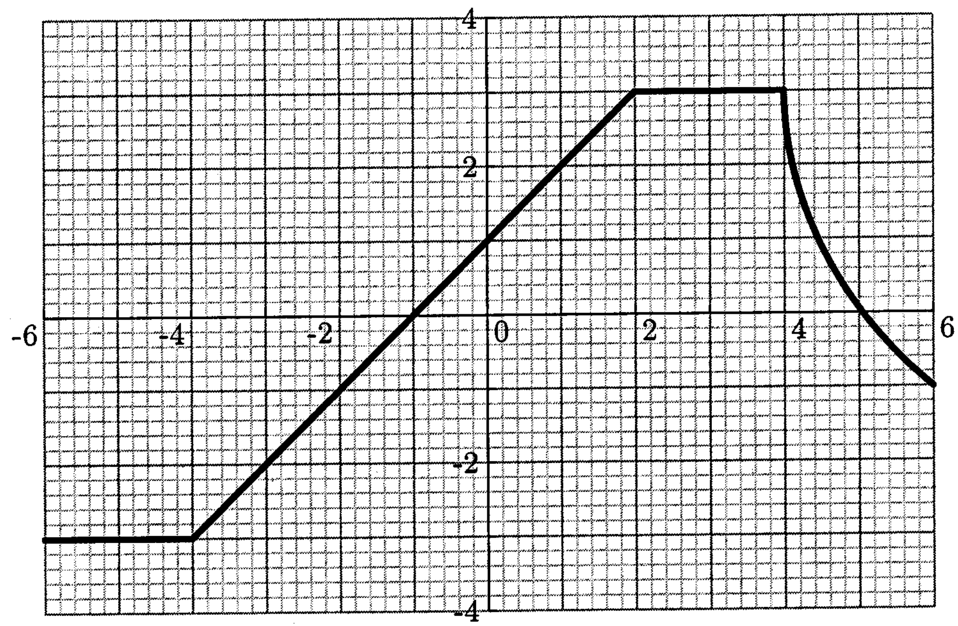
Question 8. The sum of two positive numbers is 80. What is the minimum value of the sum of their cubes? Justify your answer by showing your work.

Answer: Call the numbers x and y. We have $x, y \geq 0$ and $x + y = 80$. We are trying to minimize $S = x^3 + y^3$. We have

$$\frac{dS}{dx} = \frac{d}{dx}(x^3 + y^3) = 3x^2 + 3y^2\frac{dy}{dx}.$$

Implicitly differentiating the constraint we have $1+\frac{dy}{dx}=0$, i.e., $\frac{dy}{dx}=-1$. Thus $\frac{dS}{dx}=0$ when $3y^2=3x^2$, which happens (for positive x, y) only when $x=y$. Thus the only critical point is $x=y=40$. At the endpoints of the domain we have $S=80^3$ and at the critical point we have $S=40^3+40^3<80^3$. Therefore the minimum is $2\times 40^3=128{,}000$ and occurs when $x=y=40$.

Question 9. Using the graph of $f(t)$ below, answer the following questions. In each case indicate whether your value is exact or approximate.



a. $f'(3)$

Answer: The slope of the curve is clearly exactly 0 when $t=3$, so $f'(3)=0$ exactly.

b. The average rate of change of f between $t=-1$ and $t=6$.

Answer: The average rate of change between $t=-1$ and $t=6$ is defined to be

$$\frac{f(6)-f(-1)}{6-(-1)}=\frac{-1-0}{7}=-1/7.$$

This is exact (assuming that $f(6)=-1$ exactly and $f(-1)=0$ exactly).

c. $\int_{-6}^{0} f(t)\,dt$.

Answer: This integral is, since the relevant area is just made up of rectangles and triangles, exactly

$$\int_{-6}^{0} f(t)\,dt=-(2\times 3)-\frac{1}{2}(3\times 3)+\frac{1}{2}(1\times 1)=-10.$$

d. The average value of f on the interval $0\le t\le 4$.

Answer: This average value is defined to by $\frac{1}{4-0}\int_0^4 f(t)\,dt$. Again the integral can be computed exactly, so we have

$$(\text{Average value of } f \text{ on } 0\le t\le 4)=\frac{1}{4}\left(\frac{1}{2}(1+3)(2)+2\times 3\right)=\frac{5}{2}.$$

As it happens, $f(t)$ is the rate of change of $W(t)$, where $W(t)$ is the amount of water (measured in gallons) in a certain tank of water as a function of time (t, measured in seconds). Thus $f(t) = W'(t)$.

e. What does it mean about W that $f(1) = 2$?

Answer: It says that the rate of change of the amount of water in the tank is 2 gallons per second at time $t = 1$. Thus the amount of water in the tank is (instantaneously) increasing by 2 gallons per second then.

f. If $W(-6) = 15$ what is $W(0)$?

Answer: We know by the Fundamental Theorem of Calculus that

$$W(0) - W(-6) = \int_{-6}^{0} W'(t)\,dt = \int_{-6}^{0} f(t)\,dt,$$

so, looking back at part c,

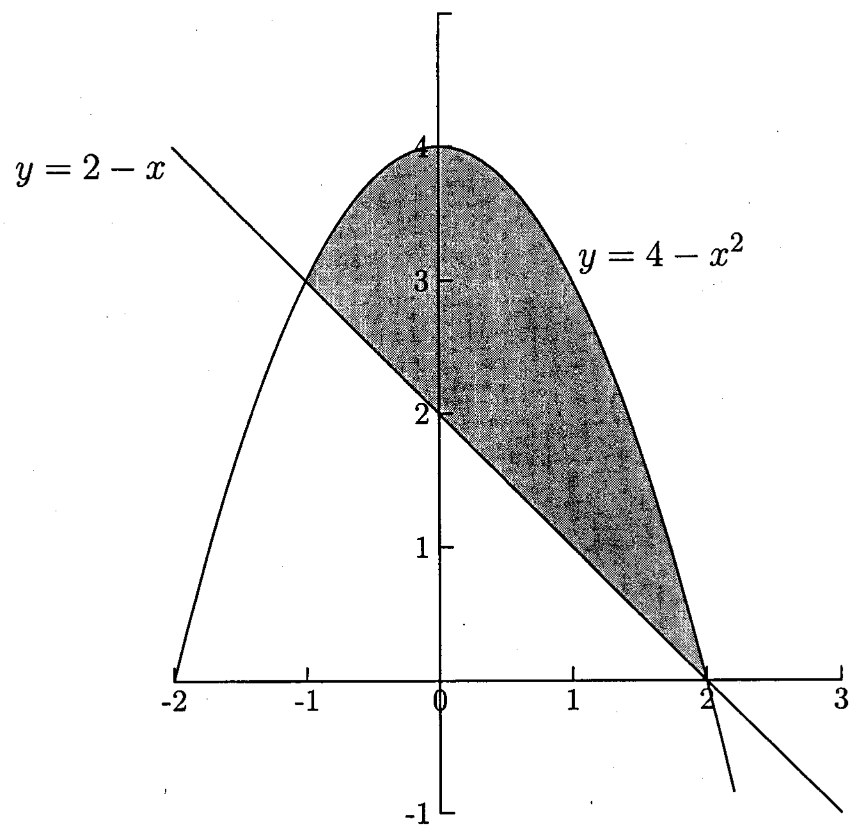
$$W(0) = W(-6) + \int_{-6}^{0} f(t)\,dt = 15 + (-10) = 5.$$

At $t = 0$ there are 10 fewer gallons of water in the tank (since W has been mostly decreasing over that time, so there are 5 gallons in the tank when $t = 0$.

Question 10. Consider the region R bounded by the curves $y = 2 - x$ and $y = 4 - x^2$.

a. Find the points where the two curves intersect.

Answer: The curves intersect when $2 - x = 4 - x^2$, i.e., $x^2 - x - 2 = 0$, or $(x-2)(x+1) = 0$. Thus the curves intersect at $(-1, 3)$ and $(2, 0)$.



b. Compute the area of the region R.

Answer:

$$\begin{aligned}
\text{Area} &= \int_{-1}^{2} (4 - x^2) - (2 - x)\,dx \\
&= \int_{-1}^{2} 2 + x - x^2\,dx \\
&= 2x + \frac{x^2}{2} - \frac{x^3}{3}\bigg|_{-1}^{2} \\
&= \left(4 + 2 - \frac{8}{3}\right) - \left(-2 + \frac{1}{2} + \frac{1}{3}\right) \\
&= \frac{9}{2}.
\end{aligned}$$

c. Write down an integral (**which you need not evaluate**) whose value is the volume of the solid obtained by revolving the region R about the line $x = 5$.

Answer: Since vertical cross-sections of the region are easy to work with, in this case the method of shells is easiest:

$$\text{Volume} = \int_{-1}^{2} 2\pi(5 - x)\left((4 - x^2) - (2 - x)\right)dx.$$

Question 11. Consider the curve $y = 3x^{3/2}$.

a. Write down an integral whose value is the length of this curve between $x = 1$ and $x = 9$.

Answer: The general arclength integral looks like $\int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx$. Since

$$\frac{dy}{dx} = \frac{9}{2}x^{1/2}$$

we get

$$\text{Arclength} = \int_1^9 \sqrt{1 + \frac{81}{4}x}\,dx.$$

b. Evaluate the integral in the previous part. [Show your work; unsupported answers will receive no credit.]

Answer: Substituting $u = 1 + 81x/4$, $du = \frac{81}{4}dx$, we get

$$\begin{aligned}
\int_1^9 \sqrt{1 + \frac{81}{4}x}\,dx &= \frac{4}{81}\int_{u=85/4}^{u=733/4} \sqrt{u}\,du \\
&= \frac{4}{81}\cdot\frac{2}{3}u^{3/2}\bigg|_{u=85/4}^{u=733/4} \\
&= \frac{1}{243}\left(733\sqrt{733} - 85\sqrt{85}\right).
\end{aligned}$$

FINAL EXAM Solutions

Math 106, Spring Semester 2007

Name (Print):________________________

Student ID Numer:________________________

TA Name:________________________

Please Circle your professor's name:
J. Campbell   M. Rammaha   B. Thomas

Please Circle your class time:
9:30   11:30   1:30   6:30 p.m.

INSTRUCTIONS:

- *There are 7 pages of questions and this cover sheet.*
- *SHOW ALL YOUR WORK. Partial credit will be given only if your work is relevant and correct.*
- *This examination is closed book. Calculators that perform symbolic manipulations such as the TI-89, TI-92 or their equivalence, are* **not permitted**. *Other calculators may be used. Turn off and put away all cell phones.*

Question	Points	Score
1	12	
2	24	
3	14	
4	14	
5	12	
6	16	
7	10	
8	18	
9	14	
10	12	
11	10	
12	16	
13	28	
Total	200	

1. [12 Points] By using the limit definition of the derivative, find $f'(3)$ if $f(x) = \dfrac{1}{x^2}$. Other methods for finding the derivative will not receive credit. Show all your work.

$$f'(3) = \lim_{h \to 0} \left[\frac{f(3+h) - f(3)}{h}\right] = \lim_{h \to 0} \left[\frac{\frac{1}{(3+h)^2} - \frac{1}{9}}{h}\right]$$

$$= \lim_{h \to 0} \left(\frac{9 - (3+h)^2}{h(9)(3+h)^2}\right) = \lim_{h \to 0} \left[\frac{h(-6+h)}{h(9)(9+6h+h^2)}\right]$$

$$= \lim_{h \to 0} \left[\frac{-6+h}{9(9+6h+h^2)}\right] = -\frac{6}{81} = -\frac{2}{27}$$

2. [24 Points] Evaluate each of the following: (Credit will be given only if you show work that justifies your answer.)

a) [8 Points] $\displaystyle\int_1^2 \left(e^{3x} + \frac{1}{x}\right) dx$. (Decimal approximations such as 2.1234 will not get credit.)

$$\left.\frac{1}{3} e^{3x} + \ln|x|\right]_1^2 = \frac{1}{3} e^6 + \ln 2 - \left(\frac{1}{3} e^3 + \ln 1\right)$$

$$= \frac{1}{3} e^6 + \ln 2 - \frac{1}{3} e^3$$

b) [8 Points] $\displaystyle\int (10x^4 + \sin(4x))\, dx$. $= 2x^5 - \frac{1}{4}\cos 4x + C$

c) [8 Points] $\displaystyle\int x\sqrt{1+2x^2}\, dx$. $u = 1+2x^2, \quad du = 4x\,dx$

$$\frac{1}{4}\int u^{1/2}\, du = \frac{1}{4}\left(\frac{2}{3}\right) u^{3/2} + C$$

$$\frac{1}{6}(1+2x^2)^{3/2} + C$$

3. [14 Points] Find the equation of the tangent line to the curve $xe^{5y} + x^3 = 3y$ at the point $(0,0)$.

$$x(5)e^{5y}\frac{dy}{dx} + e^{5y} + 3x^2 = 3\frac{dy}{dx}$$

$$(5xe^{5y} - 3)\frac{dy}{dx} = -3x^2 - e^{5y}$$

$$\frac{dy}{dx} = \left(\frac{-3x^2 - e^{5y}}{5xe^{5y} - 3}\right)\Bigg|_{(0,0)} = \frac{-1}{-3} = \frac{1}{3}$$

$$y - 0 = \frac{1}{3}(x - 0)$$

$$y = \frac{1}{3}x$$

4. [14 Points] Find:

a) [6 Points] $\frac{d}{dx}F(x)$, where $F(x) = \int_4^{3x} e^{\cos t}\,dt$.

$$3e^{\cos 3x}$$

b) [8 Points] $f(x)$, if $f'(x) = 6\sqrt{x} + \frac{1}{1+x^2}$ and $f(0) = 4$.

$$\int\left(6\sqrt{x} + \frac{1}{1+x^2}\right)dx = 4x^{3/2} + \tan^{-1}x + C$$

$$f(0) = 0 + 0 + C = 4 \Rightarrow C = 4$$

$$f(x) = 4x^{3/2} + \tan^{-1}x + 4$$

5. [12 Points] Find the **exact** value of the following limit: $\lim_{x\to 0} \frac{x \sin x}{1-\cos x}$. (Show work that justifies your answer. Numerical and/or graphical reasoning is not sufficient.)

$$\lim_{x\to 0}\left(\frac{x \sin x}{1-\cos x}\right) \to 0/0, \quad \text{L'Hopitals Rule}$$

$$= \lim_{x\to 0}\left[\frac{x\cos x + \sin x}{\sin x}\right] \to 0/0$$

$$\lim_{x\to 0}\left[\frac{x(-\sin x)+\cos x + \cos x}{\cos x}\right] = \frac{2}{1} = 2$$

6. [16 Points] Find $\frac{dy}{dx}$ for each of the following (Show work that justifies your answer. DO NOT SIMPLIFY):

a) [8 Points] $y = \frac{\ln(x^3+1)}{2^x + \tan x}$.

$$\frac{dy}{dx} = \frac{(2^x+\tan x)\left(\frac{3x^2}{x^3+1}\right) - (\ln(x^3+1))((\ln 2)2^x + \sec^2 x)}{(2^x+\tan x)^2}$$

b) [8 Points] $y = x\sin^{-1}(e^x)$.

$$\frac{dy}{dx} = x\left(\frac{e^x}{\sqrt{1-e^{2x}}}\right) + \sin^{-1}(e^x)$$

7. [10 Points] A stone thrown into a pond creates a circular ripple. If the area within the largest circle is increasing at the rate of $24\, ft^2/sec$, at what rate is the radius of the largest circle increasing when the radius is 9 feet?

$$\frac{dA}{dt} = 24\ ft^2/sec,$$

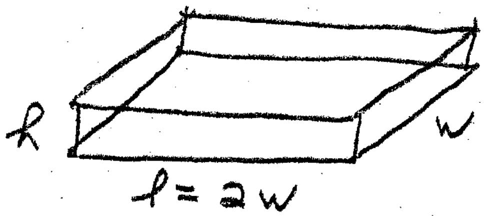
$$A(t) = \pi\,(r(t))^2 \Rightarrow$$

$$\frac{dA}{dt} = 2\pi\,(r)\,\frac{dr}{dt}$$

$$24 = 2\pi\,(9)\,\frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{24}{2\pi(9)} = \frac{4}{3\pi}\ ft/sec$$

8. [18 Points] A manufacturer intends to construct a storage box having a volume of $30\ ft^3$. He insists on having the length of its base twice as much as its width. What are the dimensions of the box that requires the least amount of material to build?



$$\text{Volume} = (2w)(w)(h) = 30\ ft^3$$

$$h = \frac{30}{2w^2} = \frac{15}{w^2}\ ft$$

$$\text{material} = M = 2(2w^2) + 2(hw) + 2(2wh)$$

$$\Rightarrow\quad M = 4w^2 + 6wh$$

$$M = 4w^2 + 6w\left(\frac{15}{w^2}\right) = 4w^2 + \frac{90}{w}$$

$$M' = 8w - \frac{90}{w^2} = 0 \Rightarrow 8w^3 = 90$$

$$w = \sqrt[3]{\frac{90}{8}} = \frac{\sqrt[3]{90}}{2}\ ft$$

$$M'' = 8 + \frac{180}{w^3} > 0$$

$\therefore$ Concave up

$\therefore$ minimum

$$l = \sqrt[3]{90}\ ft$$

$$h = \frac{15}{\frac{(90)^{2/3}}{4}} = \frac{60}{(90)^{2/3}}\ ft$$

9. [14 Points] Let $f(x) = x^5 + x - 6$.

a) [7 Points] Show that f has an inverse. **Do not find a formula for the inverse.**

$f'(x) = 5x^4 + 1 > 0$ for all x

$\therefore$ always inc.

$\therefore$ has an inverse

b) [7 Points] Find the derivative of $f^{-1}(x)$ at the point (28,2).

$\frac{d}{dx}\left(f^{-1}(28)\right) = \frac{1}{f'(2)} = \frac{1}{5x^4+1}\Big]_{x=2} = \frac{1}{81}$

10. [12 Points] This problem has two independent parts:

a) [6 Points] Let $f(x)$ be a differentiable function whose tangent line at $x = 2$ is given by $3x + y = 4$. What are $f(2)$ and $f'(2)$?

$3x+y=4 \Rightarrow y = -3x+4 \Rightarrow f'(2) = -3$

$f(2) = y = -3(2)+4 = -2$

$f'(2) = -3$ & $f(2) = -2$

b) [6 Points] Let $g(x)$ be a differentiable function with $g(4) = -1$ and $g'(4) = 3$. If $G(x) = g(\sqrt{x^2+15})$, find $G'(1)$.

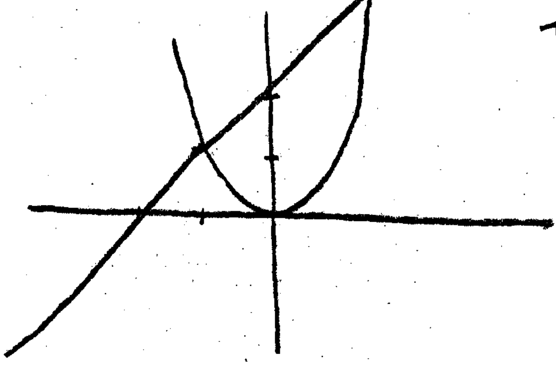
$G'(x) = g'(\sqrt{x^2+15}) \cdot \frac{d}{dx}\left((x^2+15)^{1/2}\right)$

$G'(x) = g'(\sqrt{x^2+15}) \cdot \left(\frac{x}{\sqrt{x^2+15}}\right)$

$G'(1) = g'(4) \cdot \frac{1}{4}$

$3/4$

11. [10 Points] Find, **but don't evaluate**, a definite integral whose value gives the area of the bounded region enclosed by the graphs of $y = x + 2$ and $y = x^2$.



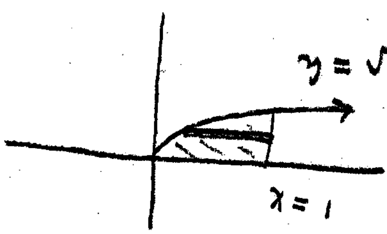
$$\left.\begin{array}{l} y = x+2 \\ y = x^2 \end{array}\right\} \; x^2 = x+2 \Rightarrow x = -1, \; 2$$

$$\text{Area} = \int_{-1}^{2} \left((x+2) - x^2\right) dx$$

12. [16 Points] Let **R** be the region in the first quadrant bounded by the graphs of $y = \sqrt{x}$, $x = 1$ and $y = 0$. Let **S** be the solid obtained by revolving **R** about the **y-axis**

a) [8 Points] By using the method of slicing, find, **but don't evaluate**, a definite integral whose value gives the volume of **S**.

disk



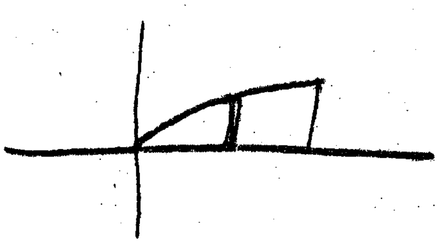
$$V = \pi \left[(O.R)^2 - (I.R)^2\right] \text{thick.}$$

$$V = \pi \int_0^1 \left(1^2 - (y^2)^2\right) dy$$

$$\pi \int_0^1 (1 - y^4)\, dy$$

b) [8 Points] By using the method of cylindrical shells, find, **but don't evaluate**, a definite integral whose value gives the volume of **S**.

shell

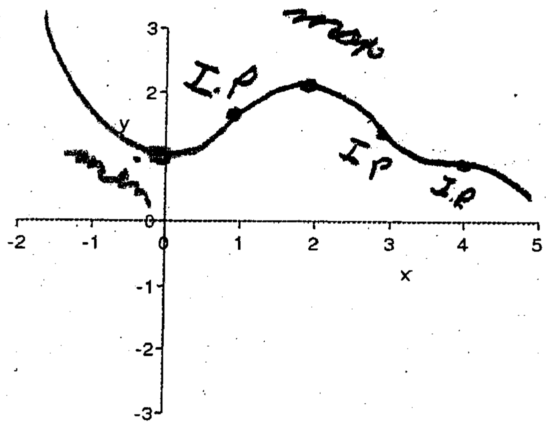


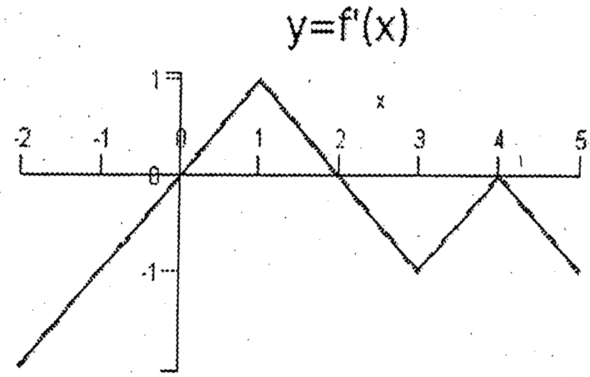
$$V = 2\pi r (ht)(tck)$$

$$V = 2\pi \int_0^1 x \sqrt{x}\, dx$$

$$2\pi \int_0^1 x^{3/2} dx$$

13. [28 Points] Let $f(x)$ be a continuous function on $[-2, 5]$ with $f(0) = 1$, and whose **derivative** $f'(x)$ is as shown below.





a) [8 Points] Find the x-coordinates of all critical points of f in the interval $[-2, 5]$ and classify them as local maximum, local minimum, or neither.

$f'(x) = 0 \Rightarrow x = 0, 2, \text{ or } 4$

$x < 0 \to$ dec
$x > 0 \to$ inc

$x < 2 \to$ inc
$x > 2 \to$ dec.

$x < 4 \to$ dec
$x > 4 \to$ dec

$\therefore$ $x = 0$ local min, $\therefore$ $x = 2$ local max, $\therefore$ $x = 4$ neither

b) [8 Points] List all inflection points of f and all intervals on which f is concave up and concave down.

$f'(x)$ changes at $x = 4, 3, \& 1 \Rightarrow$ infl. pt

concave up : $-2 < x < 1$, $3 < x < 4$

concave down: $1 < x < 3$, $4 < x < 5$

c) [5 Points] Find $f(5)$.

$$\int_0^5 f'(x)\,dx = f(5) - f(0)$$

$$-\tfrac{1}{2} = f(5) - 1 \Rightarrow f(5) = \tfrac{1}{2}$$

d) [7 Points] Sketch the graph of $y = f(x)$ in the empty plot next to the graph of $f'(x)$. Make sure to highlight all important features of the graph of $y = f(x)$.