

FINAL EXAM

Math 107, Spring Semester 2010

Name (Print): _____

Student ID Number: _____

TA Name: _____

Please Circle your professor's name:

M. Brittenham

M. Rammaha

B. Rogge

J. DeVries

Please Circle your class time:

8:30 M,W,F

9:30 T,R

10:30 M,W,F

12:30 M,W,F

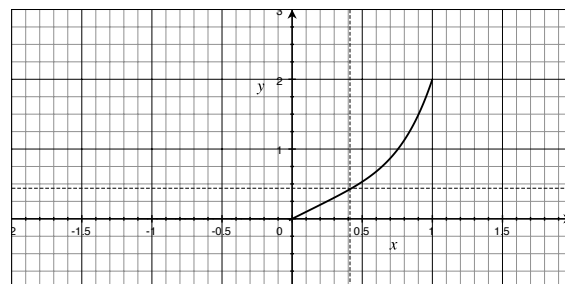
6:30 M,W

INSTRUCTIONS:

- *There are 9 pages of questions and this cover sheet.*
- *SHOW ALL YOUR WORK. Partial credit will be given only if your work is relevant and correct.*
- *This examination is closed book. Calculators that perform symbolic manipulations such as the TI-89, TI-92 or their equivalence, are **not permitted**. Other calculators may be used. Turn off and put away all cell phones.*

Question	Points	Score
1	12	
2	12	
3	24	
4	18	
5	12	
6	16	
7	12	
8	8	
9	24	
10	16	
11	26	
12	20	
Total	200	

1. [12 Points] Let **R** be the bounded region in the first quadrant enclosed by the graphs of $y = x^5 + x$, $y = 0$, and $x = 1$. Let **S** be the solid obtained by revolving **R** about the **y-axis**. Find (**but don't evaluate**) an integral whose value gives the volume of **S**.



2. [12 Points] For what values of x does the following series converge? For these values of x , what is the sum of the series?

$$\sum_{k=1}^{\infty} (-1)^{k+1} \left(\frac{1}{2}x\right)^k = \left(\frac{1}{2}x\right) - \left(\frac{1}{2}x\right)^2 + \left(\frac{1}{2}x\right)^3 - \dots \quad (1)$$

3. [24 Points] Evaluate the following integrals:

a) [12 Points] $\int 2x \arctan x \, dx$

b) [12 Points] $\int \frac{2x - 13}{(2x + 1)(x - 3)} \, dx$

4. [18 Points] Determine whether the following improper integrals are convergent or divergent. If the integral is convergent, find its exact value.

a) [9 Points] $\int_e^{\infty} \frac{1}{x (\ln x)^2} dx$

b) [9 Points] $\int_0^1 \frac{x^3}{\sqrt{1-x^4}} dx$

5. [12 Points] A tank has the shape of a right circular cone with its vertex on the ground. The height of the tank is 10 feet; the radius of its top is 6 feet. Assume that the tank is filled with water weighing 62.4 lb/ft^3 . Write down **but do not evaluate** an integral whose value is the work required to pump all of the water over the top of the tank.

6. [16 Points] Determine whether the following series converge absolutely, converge conditionally or diverge. You must show all details to receive credit.

a) [6 Points]
$$\sum_{k=1}^{\infty} \frac{k+4}{3k+1}$$

b) [10 Points]
$$\sum_{k=2}^{\infty} (-1)^k \frac{\sqrt{k}-1}{4k^2+k-1}$$

7. [12 Points] A particle is traveling in space from the point $P = (1, 4, 0)$ to the point $Q = (3, 2, 1)$ on a line segment with speed 0.5 cm/min , where the xyz -coordinate system is measured in centimeters.

a) [6 Points] Find the velocity vector of the particle.

b) [6 Points] Find the correct position function of the particle, and find and total time needed for the trip.

8. [8 Points] By using a suitable comparison Theorem, determine whether the following improper integral is convergent or divergent:

$$\int_1^{\infty} \frac{x}{4x^2 + x + 1} dx$$

9. [24 Points] Find the radius of convergence and the largest interval on which the following power series converges absolutely. At each endpoint of the interval, determine whether the series is absolutely convergent, conditionally convergent, or divergent.

$$\sum_{k=1}^{\infty} \frac{(-1)^k}{3k-1} (x-2)^k \quad (2)$$

10. [16 Points] The Taylor series of $\cos(x^2)$ about $x = 0$ is:

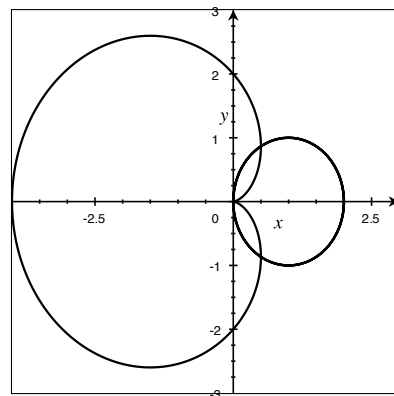
$$\cos(x^2) = \sum_{k=0}^{\infty} (-1)^k \frac{1}{(2k)!} x^{4k}, \quad x \in \mathbb{R}. \quad (3)$$

a) [6 Points] Find a series whose sum is $\int_0^1 \cos(x^2) dx$.

b) [10 Points] Use the series you found in part a) to approximate the integral $\int_0^1 \cos(x^2) dx$ with an error that does not exceed 0.01.

11. [26 Points] The graphs of the following polar curves are as shown:

$$\mathcal{C}_1 : r = 2(1 - \cos \theta); \quad 0 \leq \theta \leq 2\pi, \quad \mathcal{C}_2 : r = 2 \cos \theta; \quad 0 \leq \theta \leq \pi.$$



a) [10 Points] Find, but don't evaluate, an integral whose value is the area of the region **in the first quadrant** that lies inside \mathcal{C}_1 **and** inside \mathcal{C}_2 .

b) [8 Points] Find the slope of \mathcal{C}_2 at the point that corresponds to $\theta = \frac{\pi}{6}$.

c) [8 Points] Find, but don't evaluate, an integral whose value is the arc length of \mathcal{C}_1 .

12. [20 Points] Consider the vectors $\vec{v} = \langle 2, -1, -2 \rangle = 2\vec{i} - \vec{j} - 2\vec{k}$ and $\vec{w} = \langle 2, 2, -1 \rangle = 2\vec{i} + 2\vec{j} - \vec{k}$.

a) [5 Points] Find the vector $2\vec{v} + \vec{w}$.

b) [5 Points] Find the cosine of the angle between \vec{v} and \vec{w} .

c) [5 Points] Find $\text{proj}_{\vec{w}} \vec{v}$, i.e., the vector projection of \vec{v} onto \vec{w} .

d) [5 Points] Does there exist a **unit** vector \vec{u} that is parallel to the vector \vec{w} and orthogonal to \vec{v} ?
If “yes”, find it; and if “no” explain why not.