

Name:

Math 221, Section 3

## Quiz number 3

Show all work. How you get your answer is just as important, if not more important, than the answer itself. If you think it, write it!

1. Find the solution to the initial value problem

Standard form:

$$y' - \frac{2}{x}y = \ln x$$

Homogeneous:

$$y' = \frac{2}{x}y, \quad \frac{dy}{y} = \frac{2}{x}dx$$

$$\ln y = 2 \ln x + C = \ln(x^2) + C$$

$$y = e^{\ln(x^2) + C} = Ax^2$$

$$y = A(x) \cdot x^2 = x^2 A(x)$$

$$y' = 2xA(x) + x^2 A'(x)$$

$$y' - \frac{2}{x}y = 2xA' + x^2 A' - \frac{2}{x}x^2 A' = x^2 A' = \ln x$$

$$A'(x) = \frac{1}{x^2} \ln x = x^{-2} \ln x$$

$$A(x) = \int x^{-2} \ln x dx \quad u = \ln x \quad du = \frac{1}{x} dx \quad dv = x^{-2} dx \quad v = -x^{-1}$$

$$= -x^{-1} \ln x - \int -x^{-1} \frac{1}{x} dx$$

$$= -x^{-1} \ln x + \int x^{-2} dx$$

$$= -x^{-1} \ln x - x^{-1} + C$$

$$y = x^2 (-x^{-1} \ln x - x^{-1} + C)$$

$$= (x^2 - x) \ln x - x$$

$$2 = y(1) = C - 1 \cdot 0 - 1 = C - 1; C = 3$$

$$\boxed{y = 3x^2 - x \ln x - x}$$

$$\frac{dy}{dx} = \frac{2}{x}y + \ln x$$

$$y(1) = 2.$$

-OR-

Standard form:

$$y' - \frac{2}{x}y = \ln x$$

$$y' + p(x)y = q(x)$$

$$p(x) = -\frac{2}{x}, \quad q(x) = \ln x$$

$$\int p(x) dx = \int -\frac{2}{x} dx = -2 \ln x$$

$$e^{\int p(x) dx} = e^{-2 \ln x} = e^{\ln(x^{-2})} = x^{-2}$$

$$\int e^{\int p(x) dx} q(x) dx = \int x^{-2} \ln x dx$$

$$= (\text{from left side!}) = -x^{-1} \ln x - x^{-1} + C$$

$$y = e^{-\int p(x) dx} \left( \int e^{\int p(x) dx} q(x) dx + C \right)$$

$$= e^{2 \ln x} (-x^{-1} \ln x - x^{-1} + C)$$

$$= x^2 (-x^{-1} \ln x - x^{-1} + C)$$

$$= -x \ln x - x + Cx^2$$

$$2 = y(1) = -1 \cdot 0 - 1 + C \cdot 1^2 = C - 1$$

$$C = 2 + 1 = 3$$

$$\boxed{y = 3x^2 - x \ln x - x}$$