

Solutions

Name:

Math 221, Section 3

Quiz number 9

Show all work. How you get your answer is just as important, if not more important, than the answer itself. If you think it, write it!

1. Find the general solution to the differential equation

$$y''' + 4y' - 16y = 0$$

Auxiliary eqn: $r^3 + 4r - 16 = 0$

Try: $r = \pm 1, \pm 2, \pm 4, \pm 8, \pm 16$

$r=1$: $1 + 4 - 16 = 0$? No

$r=-1$: $-1 - 4 - 16 = 0$? No

$r=2$: $8 + 8 - 16 = 0$? Yes $\boxed{r=2}$

$$r^3 + 4r - 16 = (r-2)(r^2 + 2r + 8)$$

roots $r=2$, $r = \frac{-2 \pm \sqrt{4-32}}{2} = \frac{-2 \pm \sqrt{-28}}{2} = -1 \pm \sqrt{7}i$

General solution:

$$y = c_1 e^{2x} + c_2 e^{-x} \cos(\sqrt{7}x) + c_3 e^{-x} \sin(\sqrt{7}x)$$

Fund. solns: $e^{2x}, e^{-x} \cos(\sqrt{7}x), e^{-x} \sin(\sqrt{7}x)$