Math 107H Section 3

Practice Problems for the Final Exam

Show all work. How you get your answer is just as important, if not more important, than the answer itself. If you think it, write it!

1. **Find the following integrals** (6 pts. each)

1-1: $\displaystyle\int \sec^3 x \tan^3 x \; dx$

1-2: $\displaystyle\int \frac{x^2}{\sqrt{3-x^2}} \; dx$

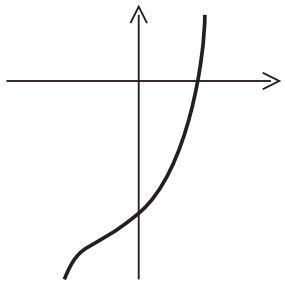
1-3: $\displaystyle\int x^2 e^{3x} \; dx$

1-4: $\displaystyle\int \frac{2x+3}{x^3+x^2-2} \; dx$

2. (15 pts.) Find the area of the region lying between the graphs of the functions

$$f(x) = 2x - 1 \quad \text{and} \quad g(x) = x^4 + x - 1$$

3. Find the volume of the region obtained by revolving the region lying between the graph of $f(x) = x^3 + 7x - 22$ and the x- and y-axes around the line $x = -2$.



4. Does the integral $\displaystyle\int_1^\infty \frac{1}{e^x - x} \, \mathrm{d}x$ converge or diverge?

5. Find the following limits (10 pts. each):

(a): $\displaystyle\lim_{x\to\infty} \frac{x^2 - 3x^3 + 9}{4x^2 - 6x + 1}$

(b): $\displaystyle\lim_{x\to\infty} \frac{(x^2+1)^x}{(x+1)^{2x}}$

6. **Determine the convergence or divergence of the following series** (5 pts. each)

6-1: $\displaystyle\sum_{n=1}^{\infty} \frac{(n+1)^{1/2}}{n^2}$

6-2: $\displaystyle\sum_{n=2}^{\infty} \frac{n!}{(n^2+n-3)^{3/2}}$

6-3: $\displaystyle\sum_{n=0}^{\infty} \left(\frac{n+3}{3n-5}\right)^n$

6-4: $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{n^{5/3}}$

7. (20 pts.) Find the degree three Taylor polynomial, centered at $c = 3$, for the function

$$f(x) = (x^2 - 5)^{5/2}$$

6. (15 pts.) Find the area inside of the graph of the polar curve

$$r = \sin(\theta) - \cos(\theta)$$

from $\theta = \dfrac{\pi}{4}$ to $\theta = \dfrac{5\pi}{4}$.

What does this curve look like? (Hint: multiply both sides by r.)

7. (16 pts.) A particle is moving around in space; at $t = 0$ its position is $(1, 2, 3)$ and its velocity is $(-1, 0, 2)$. At every time t, the acceleration of the particle is given by the vector

$$\vec{a}(t) = (\sin t, \sin(2t), 1) \ .$$

What is the particle's position at time $t = \pi$?

8. (20 pts.) Find the arclength of the parametrized curve

$$x = t^4 \quad , \quad y = t^6$$
$$0 \le t \le 2$$

Some basic integration formulas

$$\int x^n \ \mathrm{d}x = \frac{x^{n+1}}{n+1} + C \qquad \int \frac{1}{x} \ \mathrm{d}x = \ln|x| + C$$

$$\int a^x \ \mathrm{d}x = \frac{a^x}{\ln a} + C$$

$$\int \sin x \ \mathrm{d}x = -\cos x + C \qquad \int \cos x \ \mathrm{d}x = \sin x + C$$

$$\int \sec^2 x \ \mathrm{d}x = \tan x + C \qquad \int \sec x \tan x \ \mathrm{d}x = \sec x + C$$

$$\int \csc^2 x \ \mathrm{d}x = -\cot x + C \qquad \int \csc x \cot x \ \mathrm{d}x = -\csc x + C$$

$$\int \tan x \ \mathrm{d}x = \ln|\sec x| + C \qquad \int \cot x \ \mathrm{d}x = \ln|\sin x| + C$$

$$\int \sec x \ \mathrm{d}x = \ln|\sec x + \tan x| + C \qquad \int \csc x \ \mathrm{d}x = -\ln|\csc x + \cot x| + C$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} \ \mathrm{d}x = \mathrm{Arcsin}\ (\frac{x}{a}) + C$$

$$\int \frac{1}{x^2 + a^2} \ \mathrm{d}x = \frac{1}{a} \ \mathrm{Arctan}\ (\frac{x}{a}) + C$$

$$\int \frac{1}{x\sqrt{x^2 - a^2}} \ \mathrm{d}x = \frac{1}{a}\mathrm{Arcsec}\left|\frac{x}{a}\right| + C$$

$$\int \sec^n x \ dx = \frac{1}{n-1} \sec^{n-2} x \tan x + \frac{n-2}{n-1} \int \sec^{n-2} x \ dx$$