Goal: Develop methods for finding solutions to systems of linear equations, and reasons for wanting to do so.

System of equations: $\qquad a_{11}x_1 + \cdots + a1nx_n = b_1$

$$\vdots$$

$$a_{m1}x_1 + \cdots + amnx_n = b_m$$

Method of successive substitution (MoSS):

Write the first equation as $x_1 = \dfrac{1}{a_{11}}(b_1 - a_{12}x_2 - \cdots a_{1n}x_n)$, and substitute into the remaining equations. The remaining equations now constitute $m-1$ equations in $n-1$ unknowns, i.e., a "simpler" system. Now repeat the process, solving for x_2, and continue.
Need: lead coefficient $a_{11} \neq 0$

Simplify the notation: **matrices**. Decide what the variables are, and what order they appear in. Then the coefficients are all that we need, in order to unambiguously describe the system of equations. (A variable missing in an equation has coefficient 0.)

$$
\begin{array}{l}
8x - 6y + z = 1 \\
5x + 5y - 9z = 2 \\
9x + 2z = 3
\end{array}
\quad \Leftrightarrow \quad
\left(
\begin{array}{ccc|c}
8 & -6 & 1 & 1 \\
5 & 5 & -9 & 2 \\
9 & 0 & 2 & 3
\end{array}
\right)
$$

The left side of the expression on the right is the *coefficient matrix*; the whole thing is an *augmented matrix*.

The idea: translate our solution method MoSS into one that manipulates the rows of the augmented matrix. Each MoSS step can be broken down into 3 basic steps.

(1) Swap rows.

(In MoSS, this allows us to ensure that our lead coefficient a_{11} is non-zero.)

The substitution can be split into 2 parts.

(2) Multiply a row by a non-zero constant.

(In MoSS, this lets us make the coefficient of x_1 equal to 1, by dividing by a_{11}.)

(3) Add a multiple of one row to another.

(In MoSS, we thought of this as replacing x_1 by some expression *blah*; but this is the same as adding $a_{i1}(blah- x_1)$ to the i-th equation.)

These three kinds of steps are enough to carry out MoSS. And we can see that for each of these smaller steps, we get a new set of equations but **have not changed the set of solutions**. This is because anything solving the equations before the step solves the equation after the step, and *each basic step is reversible*: swapping back, multiplying by the inverse, and subtracting the multiple of the one row undoes each step.

The point is, we shall see, that with these three steps, we can simplify the equations we need to solve to the point where the solution becomes immediate. The other point is that do not have to do the exact steps which implement MoSS; we can do any combination of steps we wish to, that leads to a "simpler" set of equations.