Name:

Math 417 Midterm Exam

Show all work! How you get your answer is just as important, if not more important, than the answer itself. If you think it, write it!

1. (25 pts.) Show that if G is a group and $a, g \in G$, then $|gag^{-1}| = |a|$.
   Use this to show that for every $a, b \in G$ we have $|ab| = |ba|$.

2. (25 pts.) Express the permutation

$$\alpha = (1, 3, 4, 5)(2, 3, 5)(1, 4, 5, 3)$$

as a product of transpositions.
Then use that expression to write α as a product of 3-cycles!

3. (25 pts.) Show that if G and H are groups, and $\varphi_1, \varphi_2 : G \to H$ are homomorphisms, then

$$K = \{g \in G \ : \ \varphi_1(g) = \varphi_2(g)\}$$

is a subgroup of G.
[FYI: K is called the *equalizer* of φ_1 and ϖ_2.]

4. (25 pts.) Suppose that H and K are (finite) subgroups of a group G and that $|H|$ and $|K|$ are relatively prime. Show that $H \cap K = \{e_G\}$.