Q1a)

The set of equation that meets the requirement would be the following:

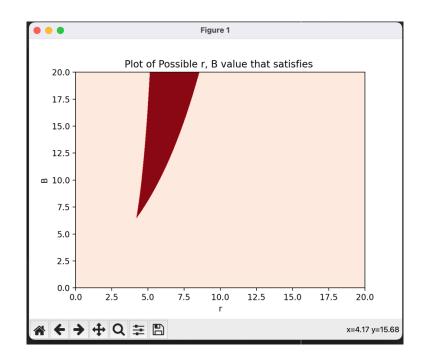
$$\begin{cases} |-(1-(1)^{r})^{\mathcal{B}}| & \leq \beta \\ |-(1-(1)^{r})^{\mathcal{B}}| & \leq \beta \end{cases}$$

Q1b)

Code:

b1) line 6 and line 9 of the code, same as the one in part 1a

b2) Graph:



b3) Overall satisfy. Since it is 96-dim signature, so r=6 and B=16 would be a good choice.

Q2a)

Approach:

First mapper:

Generate 10 random centroids and determine each vector's nearest centroid. Output the centroid id as the key and the vector as the value for each vector.

Reducer:

Recalculate the centroid vector for each key.

Second mapper:

Read the centroid and sort them by key (in case of multiple reduce and preserve ordering for better observation of convergence). Recalculate the centroid vector

```
q2a_2_mapper.py > ...
1 #!/usr/bin/env python3
2 """q2a_2_mapper.py"""
     import sys
import numpy as np
import ast
     centroid_ls = []
      count = 0
for line in sys.stdin:
           count += 1
vector_ls.append(np.fromstring(line.strip(), sep=",", dtype=int))
      with open("output_file" , 'r') as f:
            temp_ls = []
for line in f:
                  centroid_id, count ,ct_vector = line.strip().split('\t')
temp_ls.append((int(centroid_id), ct_vector))
            temp_ls = sorted(temp_ls, key=lambda x:x[0])
            for i in temp_ls:
                   centroid_ls.append(np.array(ast.literal_eval(i[1])))
     for vector in vector_ls:
           # compare
nearest_ct_key = -1
           mearest_ct_key = -I
min_distance = None
for key, ct in enumerate(centroid_ls):
    if min_distance == None:
        min_distance = np.sum(np.square(vector - ct))
        nearest_ct_key = key
                  cur_dis = np.sum(np.square(vector - ct))
if cur_dis < min_distance:</pre>
                        min_distance = cur_dis
nearest_ct_key = key
```

Reducer:

Recalculate the centroid vector for each key.

Using shell, loop the second mapper and reducer till there shows sign of converge. Copy the output file to a destination for chained Hadoop jobs

```
$ shell_command.sh
1  #!/bin/bash
2
3  for i in {1..100}; do
4   echo "Running iteration $i..."
5   hadoop jar /usr/lib/hadoop-mapreduce/hadoop-streaming-2.10.1.jar \
6   -files hdfs://dicvmc2.ie.cukh.edu.hk/user/s1155157657/hw3/output/output_file \
7   -D mapred.map.tasks=5 -D mapred.reduce.tasks=1 \
8   -file q2a_zmapper.py -mapper q2a_zmapper.py \
9   -file q2a_zmapper.py -reducer q2a_reducer.py \
10   -input hw3/rain_img -output hw3/output/output_$i >> log
11   hadoop fs -cp -f hw3/output/output_$i/part-00000 hw3/output/output_file
22   sync
13   done
```

After that, a beautifier program is used to meet the format required in the homework Command:

python3 q2a_beautify.py

```
$\qqa_\text{beautify,py} > ...
1 import sys
2
3 for line in sys.stdin:
4 id, count, vector = line.strip().split('\t')
5 print(f"Centroid {id}: {count}, {vector}")
```

Output:

```
0.0,\ 0.0,\ 0.0,\ 0.0,\ 0.0,\ 0.0,\ 0.0,\ 0.0,\ 0.0,\ 0.0,\ 0.0,\ 0.0,\ 0.0,\ 0.0,\ 0.0,
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 0.09287469287469287, \ 0.09418509418509419, \ 0.03865683865683866 \\
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```

Q2b)

Approach:

Similar to q2a for random seeds

First mapper of provided seeds:

Get the seed centroid by reading it

Reducer is the same as q2a

Second mapper

And the rest is similar to q2a

Shell script ran

After converging, download the output:

hadoop fs -get hw2/q2_output/random_1_output_74/part-00000 .

And

mv part-00000 output_2b_random_74

similar for other runs.

Then run the check_details.py and check_details_reducer.py Just like a MapReduce program but on local machine.

```
python3 check_details.py > out_2b_rand_1
cat out_2b_rand_1 | sort -k1,1 | python3 check_details_reducer.py
```

Change the input file name for different run for the 4 result

python3 check_details.py

```
check_details.py >
        #!/usr/bin/env python3
"""check_details.py"""
        import ast
       vector_ls = []
label_ls = []
count = 0
centroid_ls = []
       with open('MNIST/train_img', 'r') as f:
              for line in f:
    count += 1
    vector_ls.append(np.fromstring(line.strip(), sep=",", dtype=int))
                   label_ls.append(int(line.strip()))
        with open('output_2b_provided_126', 'r') as f:
             for line in f:
                  i, each_count,vt = line.strip().split('\t')
centroid_ls.append(np.array(ast.literal_eval(vt)))
        for i, vector in enumerate(vector_ls):
             nearest_ct_key = -1
             min_distance = None
for key, ct in enumerate(centroid_ls):
    if min_distance == None:
                   min_distance = np.sum(np.square(vector - ct))
nearest_ct_key = key
                  cur_dis = np.sum(np.square(vector - ct))
if cur_dis < min_distance:</pre>
                       min_distance = cur_dis
nearest_ct_key = key
```

python3 check details reducer.py

```
check_details_reducer.py > ...

    #!/usr/bin/env python3

    """check_details_reducer.py"""

    import sys
    import numpy as np

    current_centroid_key = None
    cur_count = 0
    centroid_key = None

for line in sys.stdin:
    centroid_key, count = line.strip().split('\t')
    count = int(count)

if centroid_key != current_centroid_key:
    if current_centroid_key != None:
        print(f"(current_centroid_key) t{cur_count}")
        cur_count = 1
        cur_count += 1

if current_centroid_key = centroid_key:
        if current_centroid_key = centroid_key
        if current_centroid_key = centroid_key:
        if current_centroid_key = centroid_key:
        print(f"(current_centroid_key)\t(cur_count)")
```

Output follow this format: (Cluster id),(ground-truth label) (count)

Random Seed 1:

```
hadoop fs -get hw2/q2_output/random_1_output_74/part-00000 .
mv part-00000 output_2b_random_74
```

change file name in check_details.py

```
python3 check_details.py > out_2b_rand_1
cat out_2b_rand_1 | sort -k1,1 | python3 check_details_reducer.py
```

Result:

- 0,0 32
- 0,1 2964
- 0,2 366
- 0,3 87
- 0,4 245
- 0,5 731
- 0,6 193
- 0,7 238
- 0,8 396
- 0,9 79
- 1,1 3726
- 1,2 328 1,3 372
- 1,4 99
- 1,5 116
- 1,6 232
- 1,7 229
- 1,8 280
- 1,9 140
- 2,0 4677
- 2,2 56
- 2,3 33
- 2,4 4
- 2,5 65
- 2,6 74 2,7 13
- 2,8 28
- 2,9 29
- 3,0 14
- 3,1 9
- 3,2 37
- 3,3 163
- 3,4 1634
- 3,5 319
- 3,6 1
- 3,7 2372
- 3,8 211
- 3,9 2499
- 4,0 27
- 4,1 7
- 4,2 61

```
4,3 32
```

- 4,4 1519
- 4,5 264
- 4,6 1
- 4,7 2848
- 4,8 229
- 4,9 1721
- 5,0 567
- 5,1 6
- 5,2 140
- 5,3 1220
- 5,4 12
- 5,5 1811
- 5,6 113
- 5,7 3 5,8 3124
- 5,9 44
- 6,0 41
- 6,1 9
- 6,2 4190
- 6,3 212
- 6,4 15
- 6,5 13
- 6,6 77 6,7 31
- 6,8 44
- 6,9 4

- 7,0 98 7,1 4 7,2 235
- 7,3 65 7,4 2183
- 7,5 184 7,6 621

- 7,7 526 7,8 133
- 7,9 1340
- 8,0 238
- 8,1 11
- 8,2 158
- 8,3 60
- 8,4 131
- 8,5 145
- 8,6 4576
- 8,7 2
- 8,8 55
- 8,9 8
- 9,0 229
- 9,1 6
- 9,2 387
- 9,3 3887
- 9,5 1773
- 9,6 30
- 9,7 3

Cluster index	Major Label of	# Train images	# Correctly	Classification
	the cluster	belonging to	clustered	Accuracy(%)
		the cluster	images	
0	1	5331	2964	55.60
1	1	5522	3726	67.48
2	0	4979	4677	93.93
3	9	7259	2499	34.43
4	7	6709	2848	42.45
5	8	7040	3124	44.38
6	2	4636	4190	90.38
7	4	5389	2183	40.51
8	6	5384	4576	84.99
9	3	7751	3887	50.15
Total Set	N/A	60000	34674	57.79

Random Seed 2:

```
hadoop fs -get hw3/q2_output/random_2_output_102/part-00000 .
mv part-00000 output_2b_random_102
modify check_details.py
python3 check_details.py > out_2b_rand_2
cat out_2b_rand_2 | sort -k1,1 | python3 check_details_reducer.py
```

Result:

```
0,0 50
0,1 9
0,2 55
0,3 37
0,4 1599
0,5 554
0,6 4
0,7 1900
0,8 321
0,9 1666
1,0 20
1,1 6606
1,2 698
1,3 393
1,4 147
1,5 668
1,6 361
1,7 384
1,8 595
1,9 156
2,0 547
2,1 20
2,2 141
2,3 1211
2,48
2,5 1614
2,6 101
2,7 6
2,8 2992
2,9 36
3,0 12
3,1 37
3,2 48
3,3 207
3,4 2145
3,5 457
3,6 7
3,7 425
3,8 380
3,9 2763
```

4,0 45 4,1 41 4,2 4179

```
4,3 223
```

- 4,4 16
- 4,5 14 4,6 44
- 4,7 38

- 4,8 54 4,9 3
- 5,0 254
- 5,1 14
- 5,2 140 5,3 70
- 5,4 103
- 5,5 154 5,6 4364
- 5,7 1 5,8 45
- 5,9 7
- 6,0 3
- 6,1 3
- 6,2 60 6,3 30
- 6,4 4
- 6,5 10
- 6,7 3170 6,8 6

- 6,9 122 7,0 217

- 7,1 11 7,2 379 7,3 3892
- 7,4 1 7,5 1773
- 7,6 28
- 7,7 5
- 7,8 1307
- 7,9 86
- 8,0 130
- 8,1 1 8,2 202
- 8,3 37
- 8,4 1817
- 8,5 113 8,6 940
- 8,7 327
- 8,8 124
- 8,9 1085
- 9,0 4645
- 9,2 56
- 9,3 31
- 9,4 2
- 9,5 64
- 9,6 69
- 9,7 9
- 9,8 27

Cluster index	Major Label of	# Train images	# Correctly	Classification
	the cluster	belonging to	clustered	Accuracy(%)
		the cluster	images	
0	7	6195	1900	30.67
1	1	10028	6606	65.88
2	8	6676	2992	44.82
3	9	6481	2763	42.63
4	2	4657	4179	89.74
5	6	5152	4364	84.70
6	7	3408	3170	93.02
7	3	7699	3892	50.55
8	4	4776	1817	38.04
9	0	4928	4645	94.26
Total Set	N/A	60000	36328	60.55

Provided seed 1:

4,3 164

```
hadoop fs -get hw3/q2_output/provided_1_output_39/part-00000
mv part-00000 output_2b_provided_39
modify check_details.py
python3 check_details.py > out_2b_prov_1
cat out_2b_prov_1 | sort -k1,1 | python3 check_details_reducer.py
result:
0,0 3906
0,2 35
0,3 21
0,46
0,5 48
0,6 68
0,7 12
0,8 31
0,9 35
1,0 14
1,1 6628
1,2 696
1,3 403
1,4 193
1,5 589
1,6 433
1,7 473
1,8 576
1,9 235
2,0 7
2,1 39
2,2 4146
2,3 143
2,4 26
2,5 5
2,6 69
2,7 50
2,8 46
2,9 13
3,0 218
3,1 9
3,2 275
3,3 3954
3,5 1830
3,6 26
3,7 4
3,8 1036
3,9 94
4,0 41
4,1 12
4,2 155
```

- 4,4 2614
- 4,5 319
- 4,6 86
- 4,7 1148 4,8 180
- 4,9 2394
- 5,0 1508
- 5,1 6
- 5,2 178
- 5,3 524
- 5,4 45
- 5,5 1370
- 5,6 788 5,7 14
- 5,8 225
- 5,9 17
- 6,0 157
- 6,1 11
- 6,2 164
- 6,3 40
- 6,4 96
- 6,5 82
- 6,6 4409
- 6,7 4 6,8 41
- 6,9 6
- 7,0 3
- 7,1 5 7,2 96 7,3 40

- 7,4 7 7,5 20 7,7 4255
- 7,8 20
- 7,9 192
- 8,0 62
- 8,1 23
- 8,2 188 8,3 820
- 8,4 11
- 8,5 835
- 8,6 30
- 8,7 47
- 8,8 3505
- 8,9 62
- 9,0 7
- 9,1 9
- 9,2 25
- 9,3 22
- 9,4 2844
- 9,5 323
- 9,6 9
- 9,7 258
- $9,8 \ 191$

9,9 2901

Cluster index	Major Label of	# Train images	# Correctly	Classification
	the cluster	belonging to	clustered	Accuracy(%)
		the cluster	images	
0	0	4162	3906	93.85
1	1	10240	6628	64.73
2	2	4544	4146	91.24
3	3	7446	3954	53.10
4	4	7113	2614	36.75
5	0	4675	1508	32.26
6	6	5010	4409	88.0
7	7	4638	4255	91.74
8	8	5583	3505	62.78
9	9	6589	2901	44.03
Total Set	N/A	60000	37826	63.04

Provided seed 2:

4,2 194 4,3 67

```
hadoop fs -get hw3/q2_output/provided_2_output_126/part-00000
mv part-00000 output_2b_provided_126
modify check_details.py
python3 check_details.py > out_2b_prov_2
cat out_2b_prov_2 | sort -k1,1 | python3 check_details_reducer.py
Result:
0,0 2771
0,2 10
0,3 18
0,4 2
0,5 62
0,6 106
0,7 7
0,8 34
0,9 21
1,0 19
1,1 6605
1,2 687
1,3 395
1,4 239
1,5 864
1,6 436
1,7 371
1,8 576
1,9 173
2,0 22
2,1 38
2,2 4201
2,3 238
2,4 16
2,5 17
2,6 94
2,7 38
2,8 55
2,9 5
3,0 163
3,1 10
3,2 333
3,3 4017
3,5 1804
3,6 30
3,7 3
3,8 1192
3,9 93
4,0 52
4,18
```

- 4,4 2259
- 4,5 196
- 4,6 418
- 4,7 585
- 4,8 130
- 4,9 1461
- 5,0 2500
- 5,2 95
- 5,3 118
- 5,4 7
- 5,5 229
- 5,6 101
- 5,7 14
- 5,8 29
- 5,9 17
- 6,0 193
- 6,1 14
- 6,2 170
- 6,3 59
- 6,4 126
- 6,5 137
- 6,6 4642
- 6,7 2
- 6,8 56
- 6,98
- 7,0 19
- 7,1 6 7,2 59

- 7,3 43 7,4 1609
- 7,5 352 7,6 1
- 7,7 2752
- 7,8 212
- 7,9 1672
- 8,0 164
- 8,1 28
- 8,2 162 8,3 1007
- 8,4 13
- 8,5 1450
- 8,6 87
- 8,7 15
- 8,8 3390
- 8,9 38
- 9,0 20
- 9,1 33
- 9,2 47
- 9,3 169
- 9,4 1571 9,5 310
- 9,6 3
- 9,7 2478
- 9,8 177

9,9 2461

Cluster index	Major Label of	# Train images	# Correctly	Classification
	the cluster	belonging to	clustered	Accuracy(%)
		the cluster	images	
0	0	3031	2771	91.42
1	1	10365	6605	63.72
2	2	4724	4201	88.92
3	3	7645	4017	52.54
4	4	5370	2259	42.06
5	0	3110	2500	80.38
6	6	5407	4642	85.85
7	7	6725	2752	40.92
8	8	6354	3390	53.35
9	7	7269	2478	34.09
Total Set	N/A	60000	35615	59.36

To conclude, provided seed 1 has the best accuracy. So it is the best random seed.

```
Q2c)
Code:
```

Same as check_details.py

But this time we already know the label so the format would be:

(cluster_id),(truth_label) (count)

Result:

0,0 656

0,29

0,3 1

0,5 6

0,6 17

0,7 1

0,8 7

0,9 6

1,0 2

1,1 1110

1,2 139

1,3 54 1,4 27

1,5 87

1,6 44

1,7 84

1,8 74

1,9 28

2,0 1

2,1 13

2,2 702 2,3 27

2,4 3

2,5 2

2,6 15

2,7 15 2,8 8

2,9 2

3,0 49

3,1 3

3,2 52

3,3 707

3,5 312 3,6 3

3,8 185

3,98

4,0 3

4,2 21

4,3 17

4,4 454

4,5 51

4,6 20

4,7 174

4,8 24

4,9 442

- 5,0 225 5,2 27 5,3 76 5,4 12 5,5 218 5,6 110 5,7 2 5,8 32 5,9 37 6,1 3 6,2 2 6,3 4 6,6 12 6,9 2 7,0 1 7,2 15 7,3 5 7,7 700 7,8 13 7,9 5 8,1 6 8,2 40 8,3 117 8,5 5 8,7 8 8,8 13 8,6 5 8,7 8 8,8 13 8,6 5 8,7 8 8,8 13 8,9 1 9,0 1 9,1 1 9,2 2 9,4 6 9,6 2 9,6 2 9,7 6 9,7 6 9,7 6 9,7 6 9,7 6 9,7 6 9,7 6 9,7 6 9,7 7 9,7 7 9,7 7 9,7 8 9,

- 9,7 44 9,8 36 9,9 475

Cluster index	Major Label of	# Test images	# Correctly	Classification
	the cluster	belonging to	clustered	Accuracy(%)
		the cluster	images	
0	0	703	656	93.31
1	1	1649	1110	67.31
2	2	788	702	89.09
3	3	1319	707	53.60
4	4	1206	454	37.65
5	0	708	225	31.78
6	6	854	742	86.89
7	7	766	700	91.38
8	8	918	583	63.51
9	9	1089	475	43.62
Total Set	N/A	10000	6354	63.54

Pink part is the comment

```
上午12:33 11月18日 週六
                          💋 🔗 🔗 🔷 Tr 📿 🖴 🖟
        Procedure Caltrob(n, k)
             prob ← [
            for i = 1 to d(m(X[n])):

prob \in prob^*((q[n][i])^{**} \times [n][i])^*(1 - q[n][i])^{**}(1 - X[n][i]))
                                           // To qix (1-qi) (1-xi)
         Procedure Cal Variable (n, k):
             local_sum < 0
             for i = 1 to K:
                   (ocal_sum + total_sum + toti) * (al Prob(h, k)
             return Tij * Calprob(n,k) / local_Sum // Topp(xnlqk)

Etajp(xnlqk))
        global \chi \in \text{olata points}

global \pi \in [\pi_1, \pi_2, \pi_3 ... \pi_k]

global q \in [q_1, q_1, q_3 ... q_k]
        global K < number of cluster
global N < number of vectors
          while not converge:
             for k = 0 to K
                 count = 0
                 sum-all = 0
                 for n=0 to N
                      output_param = (a) Variable (n, lc) // y(tak)

sum_all += output_param // 5 y(tak)
                                                                  11 ($ y(2,6) xh
                      Sum-all_X += output_param * X [n]
                   d[k] = sum-all-x/sum-all
                   7C(k) = sum-all / N
```