

## Part 1.1

## 9.2.1)

$$\mathbf{a} - \cos(\theta) = (\mathbf{u} \cdot \mathbf{v}) / (||\mathbf{u}|| * ||\mathbf{v}||)$$

$$\text{A and B} \rightarrow \cos(\theta) = (3.06*2.68 + 500\alpha*320\alpha + 6\beta*4\beta) / (\sqrt{3.06^2 + (500\alpha)^2 + (6\beta)^2} * \sqrt{2.68^2 + (320\alpha)^2 + (4\beta)^2})$$

$$\text{A and C} \rightarrow \cos(\theta) = (3.06*2.92 + 500\alpha*640\alpha + 6\beta*6\beta) / (\sqrt{3.06^2 + (500\alpha)^2 + (6\beta)^2} * \sqrt{2.92^2 + (640\alpha)^2 + (6\beta)^2})$$

$$\text{B and C} \rightarrow \cos(\theta) = (2.68*2.92 + 320\alpha*640\alpha + 4\beta*6\beta) / (\sqrt{2.68^2 + (320\alpha)^2 + (4\beta)^2} * \sqrt{2.92^2 + (640\alpha)^2 + (6\beta)^2})$$

$$\mathbf{b} - \alpha = \beta = 1, \theta = \arccos(\cos(\theta))$$

$$\text{A and B} \rightarrow \theta = \arccos((3.06*2.68 + 500*320 + 6*4) / (\sqrt{3.06^2 + (500)^2 + (6)^2} * \sqrt{2.68^2 + (320)^2 + (4)^2})) = \mathbf{0.132 \text{ degrees}}$$

$$\text{A and C} \rightarrow \theta = \arccos((3.06*2.92 + 500*640 + 6*6) / (\sqrt{3.06^2 + (500)^2 + (6)^2} * \sqrt{2.92^2 + (640)^2 + (6)^2})) = \mathbf{0.175 \text{ degrees}}$$

$$\text{B and C} \rightarrow \theta = \arccos((2.68*2.92 + 320*640 + 4*6) / (\sqrt{2.68^2 + (320)^2 + (4)^2} * \sqrt{2.92^2 + (640)^2 + (6)^2})) = \mathbf{0.282 \text{ degrees}}$$

$$\mathbf{c} - \alpha = 0.01, \beta = 0.5$$

$$\text{A and B} \rightarrow \theta = \arccos((3.06*2.68 + 500*0.01*320*0.01 + 6*0.5*4*0.5) / (\sqrt{3.06^2 + (500*0.01)^2 + (6*0.5)^2} * \sqrt{2.68^2 + (320*0.01)^2 + (4*0.5)^2})) = \mathbf{7.743 \text{ degrees}}$$

$$\text{A and C} \rightarrow \theta = \arccos((3.06*2.92 + 500*0.01*640*0.01 + 6*0.5*6*0.5) / (\sqrt{3.06^2 + (500*0.01)^2 + (6*0.5)^2} * \sqrt{2.92^2 + (640*0.01)^2 + (6*0.5)^2})) = \mathbf{7.452 \text{ degrees}}$$

$$\text{B and C} \rightarrow \theta = \arccos((2.68*2.92 + 320*0.01*640*0.01 + 4*0.5*6*0.5) / (\sqrt{2.68^2 + (320*0.01)^2 + (4*0.5)^2} * \sqrt{2.92^2 + (640*0.01)^2 + (6*0.5)^2})) = \mathbf{14.26 \text{ degrees}}$$

$$\mathbf{d} - \alpha = 1/486, \beta = 1/5.33$$

$$\mathbf{A} = [3.06, 500 * 1/486, 6 * 1/5.33] = [3.06, 1.028, 1.125]$$

$$\mathbf{B} = [2.68, 0.658, 0.75]$$

$$\mathbf{C} = [2.92, 1.316, 1.125]$$

$$\text{A and B} \rightarrow \theta = \arccos((3.06*2.68 + 1.028*0.658 + 1.125*0.75) / (\sqrt{3.06^2 + (1.028)^2 + (1.125)^2} * \sqrt{2.68^2 + (0.658)^2 + (0.75)^2})) = \mathbf{6.072 \text{ degrees}}$$

$$\text{A and C} \rightarrow \theta = \arccos((3.06*2.92 + 1.028*1.316 + 1.125*1.125) / (\sqrt{3.06^2 + (1.028)^2 + (1.125)^2} * \sqrt{2.92^2 + (1.316)^2 + (1.125)^2})) = \mathbf{5.373 \text{ degrees}}$$

B and C ->  $\theta = \arccos((2.68 \cdot 2.92 + 0.658 \cdot 1.316 + 0.75 \cdot 1.125) / (\sqrt{2.68^2 + (0.658)^2 + (0.75)^2} \cdot \sqrt{2.92^2 + (1.316)^2 + (1.125)^2})) = 10.82 \text{ degrees}$

### 9.2.3)

a – A: 4, B: 2, C: 5

Avg =  $11/3 \rightarrow \text{norm} = \text{comp} - \text{avg}$

Normalized: **A: 0.33, B: -1.66, C: 1.33**

b – I assume since it normalized the ratings, the components should be normalized as well:

Feature	A	B	C
Processor Speed	0.17	-0.21	0.03
Disk Size	14	-166	154
Main-Memory Size	0.67	-1.33	0.67

User Profile:

Processor Speed:  $(0.33 \cdot 0.17 + -1.66 \cdot -0.21 + 1.33 \cdot 0.03) / 3 = 0.1482$

Disk Size:  $(0.33 \cdot 14 + -1.66 \cdot -166 + 1.33 \cdot 153) / 3 = 161.223$

Memory-Size:  $(0.33 \cdot 0.67 + -1.66 \cdot -1.33 + 1.33 \cdot 0.67) / 3 = 1.106$

## Part 1.2

### 5.1.1)

$$M = \begin{bmatrix} \frac{1}{3} & \frac{1}{2} & 0 \\ \frac{1}{3} & 0 & \frac{1}{2} \\ \frac{1}{3} & \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

$$\therefore v_0, Mv_0, M^2v_0, \dots, M^kv_0$$

$$\begin{bmatrix} \frac{1}{3} \\ \frac{1}{3} \\ \frac{1}{3} \end{bmatrix}, \begin{bmatrix} \frac{5}{18} \\ \frac{5}{18} \\ \frac{4}{9} \end{bmatrix}, \begin{bmatrix} \frac{25}{108} \\ \frac{17}{54} \\ \frac{49}{108} \end{bmatrix}, \dots$$

$$\therefore \begin{bmatrix} \frac{895}{3888} \\ \frac{1201}{3888} \\ \frac{112}{243} \end{bmatrix} = \begin{bmatrix} 0.2302 \\ 0.3089 \\ 0.4609 \end{bmatrix}$$

PageRank of A, B, C are 0.2302, 0.3089, and 0.4609 respectively

### 5.1.2)

$$\beta = 0.8$$

$$v' = 0.8 M v + 0.2 e/n$$

$$M = \begin{bmatrix} 1/3 & 1/2 & 0 \\ 1/3 & 0 & 1/2 \\ 1/3 & 1/2 & 1/2 \end{bmatrix}$$

$$e = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \quad n = 3$$

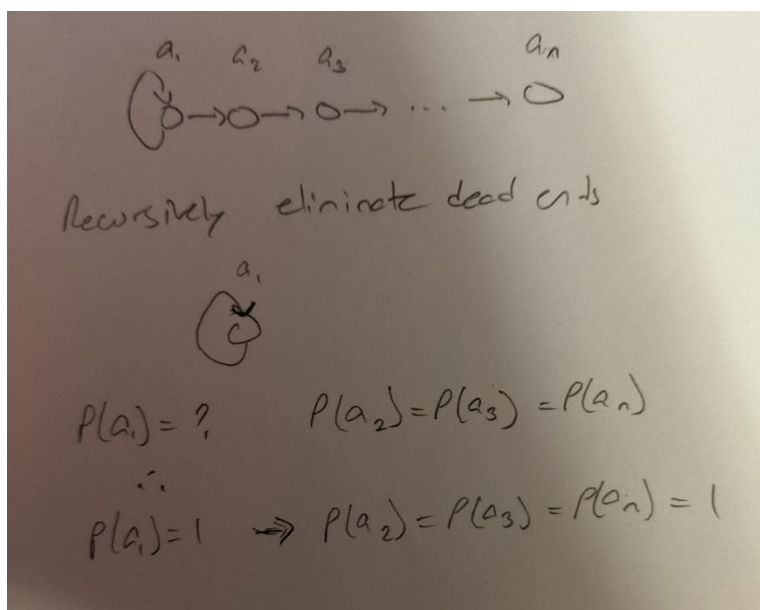
$$\therefore v' = \begin{bmatrix} 4/15 & 2/5 & 0 \\ 4/15 & 0 & 2/5 \\ 4/15 & 2/5 & 2/5 \end{bmatrix} v + \begin{bmatrix} 1/15 \\ 1/15 \\ 1/15 \end{bmatrix}$$

$$\therefore \begin{bmatrix} 1/3 \\ 1/3 \\ 1/3 \end{bmatrix}, \begin{bmatrix} 13/45 \\ 13/45 \\ 19/45 \end{bmatrix}, \begin{bmatrix} 7/27 \\ 211/675 \\ 289/675 \end{bmatrix}, \dots$$

$$\therefore \begin{bmatrix} 2641/10125 \\ 3109/10125 \\ 35/81 \end{bmatrix} = \begin{bmatrix} 0.2608 \\ 0.307 \\ 0.4322 \end{bmatrix}$$

PageRank of A, B, C are 0.2608, 0.307, and 0.4322 respectively.

### 5.1.6)



The PageRank for each node would be 1.

### Part 1.3

#### a) Normalized degree centrality

For graph a:

Node 1:  $1/4$

Node 2:  $3/4$

Node 3:  $2/4$

Node 4:  $4/(5-1) = 4/4 = 1$

Node 5:  $2/4$

For graph b:

Node 1:  $2/4$

Node 2:  $3/4$

Node 3:  $2/4$

Node 4:  $2/4$

Node 5:  $3/4$

#### b) Normalized closeness centrality

Graph a:

Node 1:  $4/(1+2+2+2) = 4/7$

Node 2:  $4/(1+1+1+2) = 4/5$

Node 3:  $4/(1+1+2+2) = 4/6$

Node 4:  $4/(1+1+1+1) = 4/4 = 1$

Node 5:  $4/(1+1+2+2) = 4/6$

Graph b:

Node 1:  $4/(1+1+2+2) = 4/6$

Node 2:  $4/(1+1+1+2) = 4/5$

Node 3:  $4/(1+1+2+2) = 4/6$

Node 4:  $4/(1+1+2+2) = 4/6$

Node 5:  $4/(1+1+1+2) = 4/5$

#### c) Normalized betweenness centrality

Graph a:

Node 1: **No shortest path go through node 1, thus:  $0 \cdot 2(\text{undirected}) / (2 \cdot (4 \text{ choose } 2)) = 0/12 = 0$**

Node 2:  **$(1) \cdot 2/12 = 2/12$**

Node 3:  **$0/12$**

Node 4:  **$(3+1) \cdot 2/12 = 8/12$**

Node 5:  **$0/12$**

Graph b:

Node 1:  **$(1) \cdot 2/12 = 2/12$**

Node 2:  **$(1+1) \cdot 2/12 = 4/12$**

Node 3:  **$0/12$**

Node 4:  **$(1) \cdot 2/12 = 2/12$**

Node 5:  **$(2) \cdot 2/12 = 4/12$**