Solution – Homework 8: Lectures 15 & 16

CS 440: Programming Languages and Translators, Spring 2020

Lecture 15: Typechecking and Substitution

1. (Compatible types?)

   a. (α, `int`, δ, α, β) and (`int`, δ, `char`, β, δ) are incompatible; the second components require δ = `int`, but the third components require δ = `char`.

   b. (β, (γ, η), (β, α)) and ((α, γ), ζ, (β, α)) are compatible using ζ = (γ, η), β = (α, γ). The result is ((α, γ), (γ, η), ((α, γ), α)).

Lecture 16: Unification

2. (Any mgus?)

   a. [`A` ↦ `E`, `D` ↦ `E` → `E`] is more general; [`E` ↦ `B*C`] takes it to [`A` ↦ `B*C`, `D` ↦ `B*C` → `B*C`] and [`E` ↦ `B*B`] takes it to [`A` ↦ `B*B`, `D` ↦ `B*B` → `B*B`].

   b. None of [`A` ↦ `t(bb, C, D)`], [`A` ↦ `t(bb, cc, D)`], and [`A` ↦ `t(B, C, dd)`] are more general than the other two. The first unifier is more general than the second (as proved by [`C` ↦ `cc`], which takes the first to the second), but no substitution takes the first unifier to the third. In addition, no substitution takes the second unifier to the third and vice versa.

Lecture 16: Unification Algorithm

3. (Solve unification problems)

   a. {`X = d(Y, Z)`, `X = d(Z, Y)`} is solved by [`Y` ↦ `Z`, `X = d(Z, Z)`]:

Substitution	**Problem**
[]	{X = d(Y, Z), X = d(Z, Y)}
[X = d(Y, Z)]	{d(Y, Z) = d(Z, Y)}
[X = d(Y, Z)]	{Y = Z, Z = Y}
[Y ↦ Z, X = d(Z, Z)]	{Z = Z}
[Y ↦ Z, X = d(Z, Z)]	{ }

   b. {`d(a, b) = d(A, B)`, `f(A) = f(B)`} is not solvable:

Substitution	**Problem**
[]	{d(a, b) = d(A, B), f(A) = f(B)}
[]	{a = A, b = B, f(A) = f(B)}
[A ↦ a]	{b = B, f(a) = f(B)}
[B ↦ b, A ↦ a]	{f(a) = f(b)}
[B ↦ b, A ↦ a]	{a = b}
	fails

c. {p(X, Y) = p(X, p(X, Z)), Y = p(A, y), X = p(x, Z), W = p(X, Y)} is solved by [Y ↦ p(p(x, y), y), X ↦ p(x, y), Z ↦ y, A ↦ p(x, y), W ↦ p(p(x, y), p(p(x, y), y))}

Substitution	**Problem**
[]	{p(X, Y) = p(X, p(X, Z)), Y = p(A, y), X = p(x, Z), W = p(X, Y)}
[]	{X = X, Y = p(X, Z), Y = p(A, y), X = p(x, Z), W = p(X, Y)}
[]	{Y = p(X, Z), Y = p(A, y), X = p(x, Z), W = p(X, Y)}
[Y ↦ p(X, Z)]	{p(X, Z) = p(A, y), X = p(x, Z), W = p(X, p(X, Z))}
[Y ↦ p(X, Z)]	{X = A, Z = y, X = p(x, Z), W = p(X, p(X, Z))}
[X ↦ A, Y ↦ p(A, Z)]	{Z = y, A = p(x, Z), W = p(A, p(A, Z))}
[Z ↦ y, X ↦ A, Y ↦ p(A, Z)]	{A = p(x, y), W = p(A, p(A, y))}
[A ↦ p(x, y), Z ↦ y, X ↦ A, Y ↦ p(A, Z)]	{W = p(p(x, y), p(p(x, y), y))}
[Y ↦ p(p(x, y), y), X ↦ p(x, y), Z ↦ y, A ↦ p(x, y), W ↦ p(p(x, y), p(p(x, y), y))}	

d. {t(A, B, d(a, c)) = t(p(a, E), B, C), p(d(E, c), d(a, F)) = p(d(b, F), C)} is solved by [F ↦ c, E ↦ b, C ↦ d(a, c), A ↦ p(a, b)].

Substitution	**Problem**
[]	{t(A, B, d(a, c)) = t(p(a, E), B, C), p(d(E, c), d(a, F)) = p(d(b, F), C)}
[]	A = p(a, E), B = B, d(a, c) = C, p(d(E, c), d(a, F)) = p(d(b, F), C)}
[A ↦ p(a, E)]	{B = B, d(a, c) = C, p(d(E, c), d(a, F)) = p(d(b, F), C)}
[A ↦ p(a, E)]	{d(a, c) = C, p(d(E, c), d(a, F)) = p(d(b, F), C)}
[C ↦ d(a, c), A ↦ p(a, E)]	{p(d(E, c), d(a, F)) = p(d(b, F), d(a, c))}
[C ↦ d(a, c), A ↦ p(a, E)]	{d(E, c) = d(b, F), d(a, F)) = d(a, c))}
[C ↦ d(a, c), A ↦ p(a, E)]	{E = b, c = F, d(a, F)) = d(a, c))}
[E ↦ b, C ↦ d(a, c), A ↦ p(a, b)]	{c = F, d(a, F)) = d(a, c))}
[F ↦ c, E ↦ b, C ↦ d(a, c), A ↦ p(a, b)]	{d(a, c)) = d(a, c))}
[F ↦ c, E ↦ b, C ↦ d(a, c), A ↦ p(a, b)]	{a = a, c = c}
[F ↦ c, E ↦ b, C ↦ d(a, c), A ↦ p(a, b)]	{c = c}
[F ↦ c, E ↦ b, C ↦ d(a, c), A ↦ p(a, b)]	{ }

