

Problem 1

$f\ x\ y\ z = x : ([\ y\] : [\ z\])$ What is the type of f ?

f has a type of $[a] \rightarrow a \rightarrow [a] \rightarrow [[a]]$. So, the function takes in a list, an element, and another list, which are all of the same type. The function then returns a list of a list of the same type.

Problem 2

- (a) The test $False < True$ is allowed because $<$ is provided by a typeclass that $Bool$ is an instance of. What is the typeclass and what is the type $(<)$ (including the typeclass)? $<$ has a type of $Ord\ a \Rightarrow a \rightarrow a \rightarrow Bool$. So, $(<)$ takes any two elements of the same type and returns a boolean. The type of those two elements must be of the Ord typeclass. Ord are for types that have an ordering and it also must be of the Eq typeclass ($:info\ (<)$ outputs *class Eq a => Ord a where ...* $(<) :: a \rightarrow a \rightarrow Bool$).
- (b) What are the functions that give the *ASCII* code for a character and give the *ASCII* character for an integer (if you use a type notation $:: Char$)? (i.e., $fnc1\ 'a'$ yields 97, $fnc2\ 97 :: Char$ yields 'a'.) Also, what are their types (including the typeclass)? *fromEnum*, *toEnum* are functions that give ASCII code for a character and give the ASCII character for an integer respectively. *fromEnum* has a type of $Enum\ a \Rightarrow a \rightarrow Int$. It takes in an element of a type from $Enum$ typeclass and returns an Int . *toEnum* has a type of $Enum\ a \Rightarrow Int \rightarrow a$. It takes in an Int and returns an element of a type from the $Enum$ typeclass.
- (c) The functions in part (b) are provided by a typeclass that $Char$ is an instance of. What is the typeclass?
The typeclass is $Enum$.

Problem 3

The function *twice list* should return true iff some values occurs twice in the list. E.g.,

$$filter\ twice\ [[],[1],[1,2],[2,2],[1,2,3],[1,2,1],[1,1,2],[1,2,2]] \\ [[2,2],[1,2,1],[1,1,2],[1,2,2]]$$

- (a) What is the type of *twice*?
twice has a type of $Eq\ a \Rightarrow [[a]] \rightarrow Bool$. It takes in a list of lists and returns a boolean whether or not some values occur twice. The typeclass is Eq .
- (b) Briefly describe the syntactic and semantic bugs in the program below.
: $\{$

```

twice [] = False
twice [-] = False
twice [x,x] = False
twice ( _ ++ [x] ++ _ ++ [y] ++ _ ) = x == y
twice (h1 : h2 : t) == (h1 == h2 || twice h1 t)
:}

```

twice [x,x] = False, results in an error because *x* was used twice which resulted in conflicting definitions for *x*, because this defines *x* twice.

twice (_ ++ [x] ++ _ ++ [y] ++ _) = x == y, results in an error because it uses *++* and the arguments are *_* and a list. It should instead use *:* and remove the brackets.

twice (h1 : h2 : t) == (h1 == h2 || twice h1 t) results in an error because of the *==* and *twice h1 t* (wrong type). To make it compile, it should instead be ***twice (h1 : h2 : t) = (h1 == h2 || twice (h1 : t))***.

- (c) Rewrite *twice* to make it work. Keep using definition by cases; feel free to add/change/delete cases as you see fit.

```

:{
twice [] = False
twice [-] = False
twice [x,y] = x == y
twice (h1 : t) = (h1 'elem' t || twice t)
:}

```

- (d) Write a definition by cases for *twice* that only has two cases(one recursive, one not).

```

:{
twice [] = False
twice (h1 : t) = (h1 'elem' t || twice t)
:}

```

- (e) Rewrite your definition from part (c) using cases and guards; break up the 3-clause logical or test to use a sequence of guards. (Don't leave any *||* in the definition)

```

twice x pattern
  | guard1 = result1
  | guard2 = result2
  (omitted)

```

```

:{
twice x | length x <= 1 = False ; otherwise = twice (h1 : t) = (h1 'elem' t
|| twice t)
:}

```

- (f) Rewrite your definition from part (c) to be of the form `twice x = case x of ...`. You can add guards to a case clause using the syntax

```
case expr of pattern | guard1 -> result1
              | guard2 -> result2
              (omitted)
```

```
:{
twice x = case length x of
0 -> False
1 -> False
_ -> let x' = x in twice (x' : xs) = (x' 'elem' xs || twice xs)
:}
```

Problem 4

Consider the following claim: "A Haskell function is higher order if and only if its type has more than one arrow." Is this correct? Give a brief argument.

This is true because by definition, a higher order function in Haskell either takes another function as a parameter or returns another function as a result. To do this, the type must have more than one arrow.

For example:

`f_squared f x = f (f x)` has type of $(t \rightarrow t) \rightarrow t \rightarrow t$ which means that it takes in a function as an argument and returns a result. This particular function just applies the function twice. As you can see, it has more than one arrow.

Problem 5

Let $f :: (a \rightarrow a \rightarrow a) \rightarrow a \rightarrow a \rightarrow a$

- Rewrite `f * (2 3)` so that it has no syntax errors and yield 6 if `f h x y = h x y`
`f (*) 2 3` which yields 6.
- Write the definition of a function $g :: ((a,a) \rightarrow a, (a,a) \rightarrow a)$ so that g is an uncurried version of f . Calling your function on `*`, `2`, and `3` should yield 6.
`g (h, (x,y)) = h (x,y)` has a type of $((a,a) \rightarrow a, (a,a) \rightarrow a) \rightarrow a$ but you have to call the function with `uncurry (*)`, `(2,3)` such that `g(uncurry (*), (2,3))` yields 6.
 If you want to just call the function with `*,2,3`, you can do
`g (h, (x,y)) = h x y` which yields 6 if you call it with just `(*)`, `(2,3)`, however it no longer is the same type as specified in the problem description.

Problem 6

Let $f1 = \text{filter } (\lambda x \rightarrow x > 0)$ and $f2 = \text{filter } (\lambda x \rightarrow x < 0)$, and let $\text{nbrFilter } g \ x = \text{length } (\text{filter } g \ x)$.

- (a) Rewrite $f1 \ (f2 \ [-5..15])$ so that it uses function composition to apply just one function to the list.
 $(f1 \ . \ f2) \ [-5..15]$. The infix dot combines the two functions together to one function. So, it only applies one function to the list rather than two.
- (b) Rewrite the nbrFilter function definition to have the form:
 $\text{nbrFilter } g = \text{function composition involving } \text{length} \text{ and } \text{filter} \dots$ and leaving out x .
 $\text{nbrFilter } g = \text{length} \cdot (\text{filter } g)$

Problem 7

- (a) Rewrite $f \ g \ x \ y = g \ x \ (y \ x)$ three ways, first $f \ g \ x =$ unnamed lambda function, then $f \ g =$ unnamed lambda function, and finally $f =$ unnamed lambda function.
 $f \ g \ x = \lambda y \rightarrow g \ x \ (y \ x)$
 $f \ g = \lambda x \ y \rightarrow g \ x \ (y \ x)$
 $f = \lambda g \ x \ y \rightarrow g \ x \ (y \ x)$
- (b) Briefly, how does $\text{var} = \text{lambda function}$ relate to first-class function in Haskell?
That is one way Haskell supports first-class functions. First-class functions means functions are treated like any variable. You can use functions as expressions and can have variables whose values are functions. For example, $\text{var} = \text{lambda function}$

Problem 8

Let's re-implement the foldl function in multiple ways. Your foldl only needs to work on lists.

- (a) Write a definition for foldl using conditional expressions:
 $\text{foldl1a } f \ a \ x = \text{if } x == [] \text{ then etc.}$
 $\text{:}\{\$
 $\text{foldl1a } f \ a \ x = \text{let } \text{len} = \text{length } x \text{ in}$
 $\text{if } \text{len} == 0 \text{ then } a$
 $\text{else if } \text{len} == 1 \text{ then } (f) \ a \ (\text{head } x)$
 $\text{else foldl1a } f \ a \ (\text{h:t}) = \text{let } \text{temp} = (f) \ a \ h \text{ in foldl1a } f \ \text{temp } t$
 $\text{:}\}$

- (b) Rewrite the definition using function definition by cases: *foldl2*...

:{
foldl2 f a [] = a
foldl2 f a (h:t) = let temp = (f) a h in foldl2 f temp t
:}

- (c) Rewrite the definition using a case expression: *foldl3 f a x = case x ...*

foldl3 f a x = case length x of
0 -> a
1 -> (f) a (head x)
_ -> let x' = init(reverse x)
let temp = (f) a (head x)
in foldl3 f temp x'