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CS 458 – Dong Jin

RSA

3.1 – Task 1: Deriving the private key

```
//initialize p, q, e
BN_hex2bn(&p, "F7E75FDC469067FFDC4E847C51F452DF");
BN_hex2bn(&q, "E85CED54AF57E53E092113E62F436F4F");
BN_hex2bn(&e, "0D88C3");
BN_dec2bn(&one, "1");

//n = p * q
BN_mul(n, p, q, ctx);
printBN("n = p * q = ", n);

//phi = (p - 1) * (q - 1)
BN_sub(p, p, one);
BN_sub(q, q, one);
BN_mul(phi, p, q, ctx);
printBN("Phi = (p - 1) * (q - 1) = ", phi);

//d
BN_mod_inverse(d, e, phi, ctx);
printBN("d = ", d);
```

Given **p**, **q**, and **e**, I made a similar program in the lab description and found **n** which is **p * q**. I then calculated the private key **d** by first calculating **phi**.

```
[10/13/20]seed@VM:~/.../RSA$ task1
n = p * q = E103ABD94892E3E74AFD724BF28E78366D9676BCCC70118BD0AA1968DBB143D1
Phi = (p - 1) * (q - 1) = E103ABD94892E3E74AFD724BF28E78348D52298BD687C44DEB3A8
1065A7981A4
d = 3587A24598E5F2A21DB007D89D18CC50ABA5075BA19A33890FE7C28A9B496AEB
```

It was very easy to create the private key. I wonder if there is an easier process than this but also as secure or even better?

3.2 – Task 2: Encrypting a message

We want to encrypt “A top secret!” but first it needs to be turned in to a hexadecimal.

```
[10/13/20]seed@VM:~/.../RSA$ python -c 'print("A top secret!".encode("hex"))'
4120746f702073656372657421
```

Public keys are given so we can encrypt it. Encrypted = message ^e mod n. The private key was also given so we can double check by decrypting and it should give the hex of the message.

```
//initialize var
BN_hex2bn(&n, "DCBFFE3E51F62E09CE7032E2677A78946A849DC4CDDE3A4D0CB81629242FB1A5");
BN_hex2bn(&e, "010001");
BN_hex2bn(&m, "4120746F702073656372657421");
BN_hex2bn(&d, "74D806F9F3A62BAE331FFE3F0A68AFE35B3D2E4794148AACBC26AA381CD7D30D");

//m^e mod n -- encrypting
BN_mod_exp(encrypted, m, e, n, ctx);
printBN("Encrypted message = ", encrypted);

//encryption^d mod n -- decrypting
BN_mod_exp(decrypted, encrypted, d, n, ctx);
printBN("Decrypted message = ", decrypted);
```

```
[10/13/20]seed@VM:~/.../RSA$ task2
Encrypted message = 6FB078DA550B2650832661E14F4F8D2CFAEF475A0DF3A75CACDC5DE5CFC
5FADC
Decrypted message = 4120746F702073656372657421
```

Here we see that “A top secret!” was encrypted and decrypting using the private key **d**, we get the same hex of “A top secret!”. Thus, we have successfully encrypted a message and decrypted it using a private key.

3.3 – Task 3: Decrypting a message

We are given ciphertext, **C**, to decode and using the same values, we can decrypt it.

```
//initialize var
BN_hex2bn(&n, "DCBFFE3E51F62E09CE7032E2677A78946A849DC4CDDE3A4D0CB81629242FB1A5");
BN_hex2bn(&d, "74D806F9F3A62BAE331FFE3F0A68AFE35B3D2E4794148AACBC26AA381CD7D30D");
BN_hex2bn(&encrypted, "8C0F971DF2F3672B28811407E2DABBE1DA0FEBBDFC7DCB67396567EA1E2493F");
/*ciphertext C given*/

//encryption^d mod n -- decrypting
BN_mod_exp(decrypted, encrypted, d, n, ctx);
printBN("Decrypted message = ", decrypted);
```

```
[10/13/20]seed@VM:~/.../RSA$ task3
Decrypted message = 50617373776F72642069732064656573
```

```
[10/13/20]seed@VM:~/.../RSA$ python -c 'print("50617373776F72642069732064656573"
.decode("hex"))'
Password is dees
```

Getting the decrypted hex and then converting it to ASCII, we get that the decrypted ciphertext **C**, is “Password is dees”.

3.4 – Task 4: Signing a message

```
[10/13/20]seed@VM:~/.../RSA$ python -c 'print("I owe you $2000.".encode("hex"))'
49206f776520796f752024323030302e
[10/13/20]seed@VM:~/.../RSA$ python -c 'print("I owe you $3000.".encode("hex"))'
49206f776520796f752024333030302e
```

Image above shows two messages with its hex values. Those two will be used to compare signatures. Signature is just RSA function with the private key.

```
//initialize var
BN_hex2bn(&n, "DC8FFE3E51F62E09CE7032E2677A78946A849DC4CDE3A4D0CB81629242FB1A5");
BN_hex2bn(&e, "010001");
BN_hex2bn(&m1, "49206f776520796f752024323030302e"); /* I owe you $2000 */
BN_hex2bn(&m2, "49206f776520796f752024333030302e"); /* I owe you $3000 */
BN_hex2bn(&d, "74D806F9F3A62BAE331FFE3F0A68AFE35B3D2E4794148AACBC26AA381CD7D30D");

//m^e mod n -- signature -- RSA func with private key
BN_mod_exp(m1s, m1, d, n, ctx);
printBN("M1 Signature = ", m1s);
BN_mod_exp(m2s, m2, d, n, ctx);
printBN("M2 Signature = ", m2s);
```

```
M1 Signature = 55A4E7F17F04CCFE2766E1EB32ADDBA890BBE92A6FBE2D785ED6E73CCB35E4CB
M2 Signature = BCC20FB7568E5D48E434C387C06A6025E90D29D848AF9C3EBAC0135D99305822
```

Here we see that even though we only made a slight change to the message **M**, we get very different signatures. I thought it would be quite similar because the hex of the messages are similar except for one byte. That's shows how good the function is and use of exponent and modulus.

3.5 Task 5: Verifying a signature

Given a message with signature and public key, we can verify the signature is legit.

```
//initialize var
BN_hex2bn(&n, "AE1CD4DC432798D933779FBD46C6E1247F0CF1233595113AA51B450F18116115");
BN_hex2bn(&m, "4c61756e63682061206d697373696c652e"); //M = Launch a missile.
BN_hex2bn(&s, "643D6F34902D9C7EC90CB0B2BCA36C47FA37165C0005CAB026C0542CBDB6802F");
BN_hex2bn(&e, "010001");

BN_mod_exp(ver, s, e, n, ctx);
printBN("Verifying = ", ver);
```

```
[10/13/20]seed@VM:~/.../RSA$ task5
Verifying = 4C61756E63682061206D697373696C652E
[10/13/20]seed@VM:~/.../RSA$ python -c 'print("4C61756E63682061206D697373696C652E".decode("hex"))'
Launch a missile.
```

To verify the signature, we do $\text{signature}^e \bmod n$, and we get the message. Cross checking the message, we see that the messages are equal, so the signature is indeed Alices.

Changing the last byte of the signature, this is what happens:

```
[10/13/20]seed@VM:~/.../RSA$ task5
Verifying = 91471927C80DF1E42C154FB4638CE8BC726D3D66C83A4EB6B7BE0203B41AC294
[10/13/20]seed@VM:~/.../RSA$ python -c 'print("91471927C80DF1E42C154FB4638CE8BC726D3D66C83A4EB6B7BE0203B41AC294".decode("hex"))'
91471927C80DF1E42C154FB4638CE8BC726D3D66C83A4EB6B7BE0203B41AC294
```

We can clearly see that the signature is not Alice's or is corrupted because the hex is different and decoding it got something random. This is surprising because we only changed the last byte of the signature, and it completely changed it. I would have thought it would only made a small difference.

Step 1:

I used www.google.com and got the certificate and copied it to c0.pem and c1.pem.

[illegible]

Step 2:

Can extract the value of **n** and **e** using the following commands, **openssl x509 -in c1.pem -noout -modulus**, and **openssl x509 -in c1.pem -text -noout**

```
[10/13/20]seed@VM:~/.../RSA$ openssl x509 -in cl.pem -noout -modulus
Modulus=D018CF45D48BCDD39CE440EF7EB4DD69211BC9CF3C8E4C75B90F3119843D9E3C29EF500D
10936F058089CF2AA0B9124B0E213D9F581624FE309F0B747755931DA0B74DE1F9D2270F651AC0CC3
B222940F346B981049E70B9D8339DD20C61C2DEFD01186165E723820A82312FFD28427D42FE7446A
5B4DD75066B0AF9E426305FBE01CC46361AF9F6A33FF6297BD48D9D37C1467DC75DC2E69E8F86D78
69D0B71005B8F13C23B24FD1A3374F823E0EC6B198A16C6E3CDA4C0DBBB3A4596038883BAD1DB9
C68CA5731BFBCD9A4ABBCDD3C61D7931598EE81BD8FE264472040064ED7AC97E8B9C05912A14925
23E4ED70342C5B4637CF9A33D83D1C686D24AC07
```

Modulus:

00:d0:18:cf:45:d4:8b:cd:d3:9c:e4:40:ef:7e:b4:
d2:69:21:1b:c9:cf:3c:8e:4c:75:b9:0f:31:19:84:
3d:9e:3c:2b:9f:50:0d:10:93:6f:05:80:80:9f:2a:
a0:bd:12:4b:02:e1:3d:9f:58:16:24:fe:30:9f:0b:
74:77:55:93:1d:4b:f7:4d:e1:92:82:10:f6:51:ac:
0c:3b:22:94:0f:34:6b:98:10:49:e7:0b:9d:83:
39:dd:20:c6:1c:2d:ef:d1:18:61:65:e7:23:83:20:
a8:23:12:ff:d2:24:7f:d4:2f:e7:44:6a:5b:4d:d7:
50:66:b0:af:9e:42:63:05:fb:e0:1c:63:61:af:
9f:6a:33:f7:62:97:bd:48:d9:3d:7c:14:67:dc:75:
dc:2e:69:e8:f8:6d:78:69:d0:b7:10:05:b8:f1:31:
c2:3b:24:fd:1a:33:74:f8:23:e0:ec:6b:19:8a:16:
c6:c3:cd:a4:cd:0b:db:b3:a4:59:60:38:88:3b:ad:
1d:b9:c6:8c:a7:53:1b:fc:bc:d9:a4:ab:bc:dd:3c:
61:d7:93:15:98:ee:81:bd:8f:e2:64:47:20:40:06:
4e:d7:ac:97:e8:b9:c0:59:12:a1:49:25:23:e4:ed:
70:34:2c:a5:b4:63:7c:f9:a3:3d:83:d1:cd:6d:24:
ac:07

Exponent: 65537 (0x10001)

From those, we now have the **n** and **e**.

Step 3:

To get the signature, I did `openssl x509 -in c0.pem -text -noout`

```
Signature Algorithm: sha256WithRSAEncryption
7f:39:e3:6e:6b:cf:d4:62:9b:ca:3e:d1:21:3e:4b:ff:1e:9d:
90:f0:42:5a:9e:ae:ea:da:22:d5:a9:38:72:1a:87:37:b9:a4:
26:46:b3:71:bf:83:1a:84:80:93:3e:c3:59:fb:eb:56:de:8a:
ae:e8:7b:56:af:77:67:dd:20:f4:d9:e9:e5:98:8c:be:41:dd:
e7:e2:4c:56:a5:6a:dd:21:0a:d3:1f:81:77:ff:dc:5b:bd:fa:
e3:3b:2a:9c:23:fd:ec:38:35:92:f3:d9:cf:f4:6a:92:ad:38:
8b:d6:a3:48:63:71:92:23:23:92:41:46:e2:49:4e:b4:1b:8c:
5c:e9:a3:a0:83:f0:6b:cd:62:5d:3d:6d:a5:5f:13:a9:5d:ee:
42:1d:05:27:e5:99:5e:b6:0f:df:71:d2:14:68:04:5c:ee:15:
26:ae:9c:02:1d:68:e5:5e:33:fb:72:8e:17:54:a2:e6:95:40:
cc:81:e3:11:b4:c1:f5:13:f6:58:ae:d7:43:65:64:9b:27:ea:
72:71:09:60:21:27:2c:30:11:79:09:24:19:03:26:25:53:b5:
e2:31:e1:49:17:0a:31:37:3f:1d:73:ca:8a:18:c4:ed:c3:e6:
84:49:44:7a:11:f3:5a:42:4b:b1:8c:18:65:c2:06:38:ff:ac:
86:9c:f5:af
```

Inputting those values in a text file and removing the spaces and colons, we get the signature.

```
[10/13/20]seed@VM:~/.../RSA$ cat sig.txt | tr -d '[:space:]'
7f39e36e6bcfd4629bca3ed1213e4bff1e9d90f0425a9eaeada22d5a938721a8737b9a42646b371
bf831a8480933ec359fbefb56de8aaee87b56af7767dd20f4d9e9e5988cbe41dde7e24c56a56add21
0ad31f8177ffdc5bbdfae33b2a9c23fdec383592f3d9cff46a92ad388bd6a3486371922323924146
e2494eb41b8c5ce9a3a083f06bcd625d3d6da55f13a95dee423d0527e5995eb60fdf71d21468045c
ee1526ae9c021d68e55e33fb728e1754a2e69540cc81e311b4c1f513f658aed74365649b27ea7271
096021272c30117909241903262553b5e231e149170a31373f1d73ca8a18c4edc3e68449447a11f3
5a424bb18c1865c20638ffac869cf5af[10/13/20]seed@VM:~/.../RSA$
```

Step 4:

To get the body of the certificate, without the signature block, and put it in a file:

```
[10/13/20]seed@VM:~/.../RSA$ openssl asn1parse -i -in c0.pem -strparse 4 -out c0
body.bin -noout
```

Calculating the hash of the body of the certificate:

```
[10/13/20]seed@VM:~/.../RSA$ sha256sum c0_body.bin
fe5623d9b8e79ffa12b3b6471ae96c3ebc8ee5b6144305320c8c7de06443269a  c0_body.bin
```

This will be used to verify the certificate, similar to task 5.

Step 5:

Using a similar program in task 5, but using the **n**, **e**, and **signature** above, we can verify the signature:

```
//initialize var
BN_hex2bn(&n,
"D018CF45D48BCDD39CE440EF7EB4DD69211BC9CF3C8E4C75B90F3119843D9E3C29EF500D10936F0580809F2AA01
BN_hex2bn(&s,
"7f39e36e6bcfd4629bca3ed1213e4bff1e9d90f0425a9eaeada22d5a938721a8737b9a42646b371bf831a8480
BN_hex2bn(&e, "010001");

// decrypting
BN_mod_exp(body, s, e, n, ctx);
printf("Body = ", body);
```


Running the program results in:

```
[10/13/20]seed@VM:~/.../RSA$ task6
Body = 01FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFF
FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFF
4 FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFF
FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFF
5 FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFF
FFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFFF
FFFFFFFFFFFFFFFF003031300D060960864801650304020105000420FE5623D9B8E79FFA12B3B6471A
E96C3EBC8EE5B6144305320C8C7DE06443269A
```

Comparing with the hash in step 5:

```
[10/13/20]seed@VM:~/.../RSA$ sha256sum c0_body.bin
fe5623d9b8e79ffa12b3b6471ae96c3ebc8ee5b6144305320c8c7de06443269a  c0_body.bin
```

The hash looks different, but at the end, they are the same. Thus, we have verified that the signature is valid. I wonder why we get all those irrelevant? hex values in front.

Final Thoughts

This was very mind opening and quite surprising at certain points. In task 5, I knew changing just one byte would make a difference, but never expected it to be that big of a difference. It went from being a legible message to being gibberish due to one byte being off in the signature. It just goes to show how secure RSA. RSA is secure and easy to implement, however I wonder in a security perspective, would it better to have a cipher that is as secure but harder to implement?