Mark Gameng

WA4

1.

a. P(A,C)

P(A = T, C = T) = P(A = T, C = T, B = T) + P(A = T, C = T, B = F) = 0.014 + 0.012 = 0.026
P(A = T, C = F) = P(A = T, C = F, B = T) + P(A = T, C = F, B = F) = 0.126 + 0.048 = 0.174
P(A = F, C = T) = P(A = F, C = T, B = T) + P(A = F, C = T, B = F) = 0.392 + 0.144 = 0.536
P(A = F, C = F) = P(A = F, C = F, B = T) + P(A = F, C = F, B = F) = 0.168 + 0.096 = 0.264

A	C	P(A,C)
T	T	0.026
T	F	0.174
F	T	0.536
F	F	0.264

b. P(C)

P(C = F) = P(C = F, A = T, B = T) + P(C = F, A = T, B = F) + P(C = F, A = F, B = T) + P(C = F, A = F, B = F) = 0.126 + 0.048 + 0.168 + 0.096 = 0.438
P(C = T) = P(C = T, A = T|F, B = T|F) = 0.014 + 0.012 + 0.392 + 0.144 = 0.562

C	P(C)
F	0.438
T	0.562

c. P(A|C)

P(A|C) = P(A, C) / P(C)
P(A = T| C = T) = P(A = T, C = T) / P(C = T) = 0.026 / 0.562 = 0.046
P(A = T| C = F) = P(A = T, C = F) / P(C = F) = 0.174 / 0.438 = 0.397
P(A = F| C = T) = P(A = F, C = T) / P(C = T) = 0.536 / 0.562 = 0.954
P(A = F| C = F) = P(A = F, C = F) / P(C = F) = 0.264 / 0.438 = 0.603

A	C	P(A|C)
T	T	0.046
T	F	0.397
F	T	0.954
F	F	0.603

d. P(A, B | C)

P(A, B | C) = P((A, B), C) / P(C) = P(A, B, C) / P(C)
P(A = T, B = T|C = T) = P(A = T, B = T, C = T) / P(C = T) = 0.014 / 0.562 = 0.025
P(A = T, B = T|C = F) = P(A = T, B = T, C = F) / P(C = F) = 0.126 / 0.438 = 0.288
P(A = T, B = F|C = T) = P(A = T, B = F, C = T) / P(C = T) = 0.012 / 0.562 = 0.021
P(A = T, B = F|C = F) = P(A = T, B = F, C = F) / P(C = F) = 0.048 / 0.438 = 0.110

P(A = F, B = T|C = T) = P(A = F, B = T, C = T) / P(C = T) = 0.392 / 0.562 = 0.698
P(A = F, B = T|C = F) = P(A = F, B = T, C = F) / P(C = F) = 0.168 / 0.438 = 0.384
P(A = F, B = F|C = T) = P(A = F, B = F, C = T) / P(C = T) = 0.144 / 0.562 = 0.256
P(A = F, B = F|C = F) = P(A = F, B = F, C = F) / P(C = F) = 0.096 / 0.438 = 0.219

A	**B**	**C**	**P(A,B | C)**
T	**T**	**T**	**0.025**
T	**T**	**F**	**0.288**
T	**F**	**T**	**0.021**
T	**F**	**F**	**0.110**
F	**T**	**T**	**0.698**
F	**T**	**F**	**0.384**
F	**F**	**T**	**0.256**
F	**F**	**F**	**0.219**

2.

a. **P(X2)P(X3)P(X4)P(X5|X2,X3)P(X6|X3,X4)P(X7|X5)P(X8|X5,X3,X6)P(X9|X7,X5,X8)**
b. 2 * 3 * 4 * 5 * 6 * 7 * 8 * 9 = 362880 – 1 = **362879** independent parameters
c. P(X2) = 2 – 1 = 1, P(X3) = 3 – 1 = 2, P(X4) = 3, P(X5|X2,X3) = (5-1) (2*3) = 4*6 = 24
P(X6|X3,X4) = 5*3*4 = 60, P(X7|X5) = 6*5 =30 , P(X8|X5,X3,X6) = 7*5*3*6 = 630
P(X9|X7,X5,X8) = 8*7*5*8 = 2240
1 + 2 + 3 + 24 + 60 + 30 + 630 + 2240 = **2990** independent parameters required for this network

3.

a. P(B)
Eliminate D, C, A
P(A)P(B|A)P(C|B)P(D|C)
Eliminate D

D	C	P(D|C)
T	T	0.82
T	F	0.37
F	T	0.18
F	F	0.63

C	f(C)
T	0.82+0.18 = 1
F	0.37+0.63 = 1

P(A)P(B|A)P(C|B)f(C)
Eliminate C

C	B	P(C|B)f(C)
T	T	0.7*1 = 0.7
T	F	0.4*1 = 0.4
F	T	0.3*1 = 0.3
F	F	0.6*1 = 0.6

B	f(B)
T	0.7+0.3=1
F	0.4+0.6=1

P(A)P(B|A)f(B)
Eliminate A

B	A	P(A)P(A|B)f(B)
T	T	0.4*0.1*1=0.04
T	F	0.6*0.8*1=0.48
F	T	0.4*0.9*1=0.36
F	F	0.6*0.2*1=0.12

B	f(B) = P(B)
T	0.04+0.48 = 0.52
F	0.36+0.12 = 0.48

P(B = T) = P(B = T | A = T)P(A = T) + P(B = T | A = F)P(A = F) = 0.1*0.4+0.8*0.6 = 0.52
P(B = F) = 0.9*0.4 + 0.2*0.6 = 0.48

B	**P(B)**
T	**0.52**
F	**0.48**

b. P(C|A=T)
Eliminate D, B
P(A)P(B|A)P(C|B)P(D|C)
Eliminate D

D	C	P(D|C)
T	T	0.82
T	F	0.37
F	T	0.18
F	F	0.63

C	f(C)
T	0.82+0.18 = 1
F	0.37+0.63 = 1

P(A)P(B|A)P(C|B)f(C)
Eliminate B

B	C	P(B|A=T)P(C|B)f(C)
T	T	0.1*0.7*1 = 0.07
T	F	0.1*0.3*1 = 0.03
F	T	0.9*0.4*1 = 0.36
F	F	0.9*0.6*1 = 0.54

C	f(C)
T	0.07+0.36=0.43
F	0.03+0.54=0.57

P(A=T)f(C)
P(C=T|A=T) = 0.43 * 0.4 = 0.172
P(C=F|A=T) = 0.57 * 0.4 = 0.228
0.172 + 0.228 = 0.4 -> (0.172/0.4, 0.228/0.4) = (0.43, 0.57)

C	**P(C|A=T)**
T	**0.43**
F	**0.57**

c. P(A,B | C = T, D = F)
= P(A,B,C = T,D = F) / P(C = T, D = F)
P(C=T, D = F)
Eliminate B, A
P(A)P(B|A)P(C|B)P(D|C)
Eliminate B

B	A	P(B|A)P(C=T|B)
T	T	0.1*0.7 = 0.07
T	F	0.8*0.7 = 0.56
F	T	0.9*0.4 = 0.36
F	F	0.2*0.4 = 0.08

A	f(A,C=T)
T	0.07+0.36=0.43
F	0.56+0.08=0.64

P(A)f(A,C=T)P(D|C)

Eliminate A

A	P(A)f(A,C=T)
T	0.4*0.43=0.172
F	0.6*0.64=0.384

f(C=T) = 0.172 + 0.384 = 0.556

f(C=T)P(D=F|C=T) = 0.556 * 0.18 = 0.10008 = P(C = T, D = F)

P(A,B,C=T,D=F)

B	A	P(A,B,C=T,D=F)
T	T	0.4*0.1*0.7*0.18=0.00504
T	F	0.6*0.8*0.7*0.18=0.06048
F	T	0.4*0.9*0.4*0.18=0.02592
F	F	0.6*0.2*0.4*0.18=0.00864

P(A,B | C = T, D = F) = P(A,B,C = T,D = F) / P(C = T, D = F)

B	**A**	**P(A,B | C = T, D = F)**
T	**T**	**0.00504 / 0.10008 = 0.504**
T	**F**	**0.06048 / 0.10008 = 0.604**
F	**T**	**0.02592 / 0.10008 = 0.259**
F	**F**	**0.00864 / 0.10008 = 0.086**

4. S

   a. P(X)P(Y|X)

X	Y	P(X)P(Y|X)
T	T	0.4*0.2=0.08
T	F	0.4*0.8=0.32
F	T	0.6*0.7=0.42
F	F	0.6*0.3=0.18

Y	P(Y)
T	0.08+0.42=0.5
F	0.32+0.18=0.5

Action	**Expected Utility**
a	**0.5*800+0.5*200=500**
~a	**0.5*400+0.5*1000=700**

The expected action is ~a, with 700, since it has greater expected utility, the MEU.

   b. P(Y|Z) = P(X)P(Y|X)P(Z|Y) = P(Y)P(Z|Y)

Y	Z	P(Y|Z)
T	T	0.5*0.9=0.45
T	F	0.5*0.1=0.05
F	T	0.5*0.2=0.1
F	F	0.5*0.8=0.4

P(Y|Z=T) = (0.45,0.1) -> (0.82, 0.18)
a -> 0.82*800+0.18*200 = 692
~a -> 0.82*400+0.18*1000 = 508
MEU|Z=T -> 692 with a
P(Y|Z=F) = (0.05, 0.4) -> (0.11, 0.89)
a -> 0.11*800+0.89*200 = 266
~a -> 0.11*400+0.89*1000 = 934
MEU|Z=F -> 934 with ~a
P(Z=T) = 0.45+0.1 = 0.55
P(Z=F) = 0.05+0.4 = 0.45
Value of Information of Z = 0.55 * 692 + 0.45 * 934 = 800.9 – 700 = 100.9

c.

Y	X	P(X)P(Y|X)
T	T	0.4*0.2=0.08
F	T	0.4*0.8=0.32
F	F	0.6*0.3=0.18
T	F	0.6*0.7=0.42

X = T -> (0.08, 0.32) -> (0.2, 0.8)
a -> 0.2*800+0.8*200 = 320
~a -> 0.2*400+0.8*1000 = 880
MEU|X=T -> 880 with ~a
X = F -> (0.42, 0.18) -> (0.7, 0.3)
a -> 0.7*800+0.3*200 = 620
~a -> 0.7*400+0.3*1000 = 580
MEU|X=F -> 620 with a
Value of Information of X = 0.4 * 880 + 0.6 * 620 = 724 – 700 = 24

d. P(Y|X,Z=T)=P(X)P(Y|X)P(Y|Z=T)

Y	X	P(Y|X, Z=T)
T	T	0.4*0.2*0.9=0.072
T	F	0.6*0.7*0.9=0.378
F	T	0.4*0.8*0.2=0.064
F	F	0.6*0.3*0.2=0.036

X=T -> (0.072, 0.064) -> (0.53, 0.47)
a -> 0.53*800+0.47*200 = 518
~a -> 0.53*400+0.47*1000 = 682
MEU|X=T, Z=T -> 682 with ~a

X=F -> (0.378, 0.036) -> (0.91, 0.09)
a -> 0.91*800+0.09*200 = 746
~a -> 0.91*400+0.09*1000 = 454
MEU|X=F, Z=T -> 746 with a
P(X=T|Z=T) = 0.072 + 0.064 = 0.136
P(X=F|Z=T) = 0.378 + 0.036 = 0.414
P(X|Z=T) -> (0.136, 0.414) -> (0.25, 0.75)
Value of Information of X|Z=T = 0.25*682+0.75*746 = 730 – 692 = 38

5.

take = 0*(1-p)*100+0*p*0+1*(1-p)*20+1*p*70
= (1-p)*20+p*70 = 20 – 20p + 70p
= 20 + 50p
~take = 1*(1-p)*100+1*p*0+0*(1-p)*20+0*p*70
= (1-p)*100
= 100 – 100p
take = ~take
20 + 50p = 100 – 100p
150p = 80
p = 80/150 = 8/15