Mark Gameng

WA4

* 1. P(A,C)  
     P(A = T, C = T) = P(A = T, C = T, B = T) + P(A = T, C = T, B = F) = 0.014 + 0.012 = 0.026  
     P(A = T, C = F) = P(A = T, C = F, B = T) + P(A = T, C = F, B = F) = 0.126 + 0.048 = 0.174  
     P(A = F, C = T) = P(A = F, C = T, B = T) + P(A = F, C = T, B = F) = 0.392 + 0.144 = 0.536  
     P(A = F, C = F) = P(A = F, C = F, B = T) + P(A = F, C = F, B = F) = 0.168 + 0.096 = 0.264

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| **A** | **C** | **P(A,C)** |
| **T** | **T** | **0.026** |
| **T** | **F** | **0.174** |
| **F** | **T** | **0.536** |
| **F** | **F** | **0.264** |

* 1. P(C)  
     P(C = F) = P(C = F, A = T, B = T) + P(C = F, A = T, B = F) + P(C = F, A = F, B = T) + P(C = F, A = F, B = F) = 0.126 + 0.048 + 0.168 + 0.096 = 0.438  
     P(C = T) = P(C = T, A = T|F, B = T|F) = 0.014 + 0.012 + 0.392 + 0.144 = 0.562

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| **C** | **P(C)** |
| **F** | **0.438** |
| **T** | **0.562** |

* 1. P(A|C)  
     P(A|C) = P(A, C) / P(C)  
     P(A = T| C = T) = P(A = T, C = T) / P(C = T) = 0.026 / 0.562 = 0.046  
     P(A = T| C = F) = P(A = T, C = F) / P(C = F) = 0.174 / 0.438 = 0.397  
     P(A = F| C = T) = P(A = F, C = T) / P(C = T) = 0.536 / 0.562 = 0.954  
     P(A = F| C = F) = P(A = F, C = F) / P(C = F) = 0.264 / 0.438 = 0.603

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| **A** | **C** | **P(A|C)** |
| **T** | **T** | **0.046** |
| **T** | **F** | **0.397** |
| **F** | **T** | **0.954** |
| **F** | **F** | **0.603** |

* 1. P(A, B | C)   
     P(A, B | C) = P((A, B), C) / P(C) = P(A, B, C) / P(C)  
     P(A = T, B = T|C = T) = P(A = T, B = T, C = T) / P(C = T) = 0.014 / 0.562 = 0.025  
     P(A = T, B = T|C = F) = P(A = T, B = T, C = F) / P(C = F) = 0.126 / 0.438 = 0.288  
     P(A = T, B = F|C = T) = P(A = T, B = F, C = T) / P(C = T) = 0.012 / 0.562 = 0.021  
     P(A = T, B = F|C = F) = P(A = T, B = F, C = F) / P(C = F) = 0.048 / 0.438 = 0.110  
     P(A = F, B = T|C = T) = P(A = F, B = T, C = T) / P(C = T) = 0.392 / 0.562 = 0.698  
     P(A = F, B = T|C = F) = P(A = F, B = T, C = F) / P(C = F) = 0.168 / 0.438 = 0.384  
     P(A = F, B = F|C = T) = P(A = F, B = F, C = T) / P(C = T) = 0.144 / 0.562 = 0.256  
     P(A = F, B = F|C = F) = P(A = F, B = F, C = F) / P(C = F) = 0.096 / 0.438 = 0.219

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| **A** | **B** | **C** | **P(A,B | C)** |
| **T** | **T** | **T** | **0.025** |
| **T** | **T** | **F** | **0.288** |
| **T** | **F** | **T** | **0.021** |
| **T** | **F** | **F** | **0.110** |
| **F** | **T** | **T** | **0.698** |
| **F** | **T** | **F** | **0.384** |
| **F** | **F** | **T** | **0.256** |
| **F** | **F** | **F** | **0.219** |

* 1. **P(X2)P(X3)P(X4)P(X5|X2,X3)P(X6|X3,X4)P(X7|X5)P(X8|X5,X3,X6)P(X9|X7,X5,X8)**
  2. 2 \* 3 \* 4 \* 5 \* 6 \* 7 \* 8 \* 9 = 362880 – 1 = **362879** independent parameters
  3. P(X2) = 2 – 1 = 1, P(X3) = 3 – 1 = 2, P(X4) = 3, P(X5|X2,X3) = (5-1) (2\*3) = 4\*6 = 24  
     P(X6|X3,X4) = 5\*3\*4 = 60, P(X7|X5) = 6\*5 =30 , P(X8|X5,X3,X6) = 7\*5\*3\*6 = 630  
     P(X9|X7,X5,X8) = 8\*7\*5\*8 = 2240  
     1 + 2 + 3 + 24 + 60 + 30 + 630 + 2240 = **2990** independent parameters required for this network
  4. P(B)  
     Eliminate D, C, A  
     P(A)P(B|A)P(C|B)P(D|C)  
     Eliminate D

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| --- | --- | --- |
| D | C | P(D|C) |
| T | T | 0.82 |
| T | F | 0.37 |
| F | T | 0.18 |
| F | F | 0.63 |

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| --- | --- |
| C | f(C) |
| T | 0.82+0.18 = 1 |
| F | 0.37+0.63 = 1 |

P(A)P(B|A)P(C|B)f(C)  
Eliminate C

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| --- | --- | --- |
| C | B | P(C|B)f(C) |
| T | T | 0.7\*1 = 0.7 |
| T | F | 0.4\*1 = 0.4 |
| F | T | 0.3\*1 = 0.3 |
| F | F | 0.6\*1 = 0.6 |

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| --- | --- |
| B | f(B) |
| T | 0.7+0.3=1 |
| F | 0.4+0.6=1 |

P(A)P(B|A)f(B)  
Eliminate A

|  |  |  |
| --- | --- | --- |
| B | A | P(A)P(A|B)f(B) |
| T | T | 0.4\*0.1\*1=0.04 |
| T | F | 0.6\*0.8\*1=0.48 |
| F | T | 0.4\*0.9\*1=0.36 |
| F | F | 0.6\*0.2\*1=0.12 |

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| B | f(B) = P(B) |
| T | 0.04+0.48 = 0.52 |
| F | 0.36+0.12 = 0.48 |

P(B = T) = P(B = T | A = T)P(A = T) + P(B = T | A = F)P(A = F) = 0.1\*0.4+0.8\*0.6 = 0.52  
P(B = F) = 0.9\*0.4 + 0.2\*0.6 = 0.48

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| **B** | **P(B)** |
| **T** | **0.52** |
| **F** | **0.48** |

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| --- | --- | --- |
| D | C | P(D|C) |
| T | T | 0.82 |
| T | F | 0.37 |
| F | T | 0.18 |
| F | F | 0.63 |

|  |  |
| --- | --- |
| C | f(C) |
| T | 0.82+0.18 = 1 |
| F | 0.37+0.63 = 1 |

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| --- | --- | --- |
| B | C | P(B|A=T)P(C|B)f(C) |
| T | T | 0.1\*0.7\*1 = 0.07 |
| T | F | 0.1\*0.3\*1 = 0.03 |
| F | T | 0.9\*0.4\*1 = 0.36 |
| F | F | 0.9\*0.6\*1 = 0.54 |

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| --- | --- |
| C | f(C) |
| T | 0.07+0.36=0.43 |
| F | 0.03+0.54=0.57 |

* 1. P(C|A=T)  
     Eliminate D, B  
     P(A)P(B|A)P(C|B)P(D|C)  
     Eliminate D  
       
       
       
       
       
       
     P(A)P(B|A)P(C|B)f(C)  
     Eliminate B  
       
       
       
       
       
       
     P(A=T)f(C)  
     P(C=T|A=T) = 0.43 \* 0.4 = 0.172  
     P(C=F|A=T) = 0.57 \* 0.4 = 0.228  
     0.172 + 0.228 = 0.4 -> (0.172/0.4, 0.228/0.4) = (0.43, 0.57)

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| **C** | **P(C|A=T)** |
| **T** | **0.43** |
| **F** | **0.57** |

* 1. P(A,B | C = T, D = F)  
     = P(A,B,C = T,D = F) / P(C = T, D = F)  
     P(C=T, D = F)  
     Eliminate B, A  
     P(A)P(B|A)P(C|B)P(D|C)  
     Eliminate B

|  |  |  |
| --- | --- | --- |
| B | A | P(B|A)P(C=T|B) |
| T | T | 0.1\*0.7 = 0.07 |
| T | F | 0.8\*0.7 = 0.56 |
| F | T | 0.9\*0.4 = 0.36 |
| F | F | 0.2\*0.4 = 0.08 |

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| A | f(A,C=T) |
| T | 0.07+0.36=0.43 |
| F | 0.56+0.08=0.64 |

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| --- | --- |
| A | P(A)f(A,C=T) |
| T | 0.4\*0.43=0.172 |
| F | 0.6\*0.64=0.384 |

P(A)f(A,C=T)P(D|C)  
Eliminate A

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| --- | --- | --- |
| B | A | P(A,B,C=T,D=F) |
| T | T | 0.4\*0.1\*0.7\*0.18=0.00504 |
| T | F | 0.6\*0.8\*0.7\*0.18=0.06048 |
| F | T | 0.4\*0.9\*0.4\*0.18=0.02592 |
| F | F | 0.6\*0.2\*0.4\*0.18=0.00864 |

f(C=T) = 0.172 + 0.384 = 0.556  
f(C=T)P(D=F|C=T) = 0.556 \* 0.18 = 0.10008 = P(C = T, D = F)  
P(A,B,C=T,D=F)

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| --- | --- | --- |
| **B** | **A** | **P(A,B | C = T, D = F)** |
| **T** | **T** | **0.00504 / 0.10008 = 0.504** |
| **T** | **F** | **0.06048 / 0.10008 = 0.604** |
| **F** | **T** | **0.02592 / 0.10008 = 0.259** |
| **F** | **F** | **0.00864 / 0.10008 = 0.086** |

P(A,B | C = T, D = F) = P(A,B,C = T,D = F) / P(C = T, D = F)

1. S

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| X | Y | P(X)P(Y|X) |
| T | T | 0.4\*0.2=0.08 |
| T | F | 0.4\*0.8=0.32 |
| F | T | 0.6\*0.7=0.42 |
| F | F | 0.6\*0.3=0.18 |

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| --- | --- |
| Y | P(Y) |
| T | 0.08+0.42=0.5 |
| F | 0.32+0.18=0.5 |

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| **Action** | **Expected Utility** |
| **a** | **0.5\*800+0.5\*200=500** |
| **~a** | **0.5\*400+0.5\*1000=700** |

* 1. P(X)P(Y|X)  
       
       
       
       
       
       
       
       
       
       
     **The expected action is ~a, with 700, since it has greater expected utility, the MEU.**
  2. P(Y|Z) = P(X)P(Y|X)P(Z|Y) = P(Y)P(Z|Y)

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| Y | Z | P(Y|Z) |
| T | T | 0.5\*0.9=0.45 |
| T | F | 0.5\*0.1=0.05 |
| F | T | 0.5\*0.2=0.1 |
| F | F | 0.5\*0.8=0.4 |

P(Y|Z=T) = (0.45,0.1) -> (0.82, 0.18)  
a -> 0.82\*800+0.18\*200 = 692  
~a -> 0.82\*400+0.18\*1000 = 508  
**MEU|Z=T -> 692 with a**  
P(Y|Z=F) = (0.05, 0.4) -> (0.11, 0.89)  
a -> 0.11\*800+0.89\*200 = 266  
~a -> 0.11\*400+0.89\*1000 = 934  
**MEU|Z=F -> 934 with ~a**P(Z=T) = 0.45+0.1 = 0.55  
P(Z=F) = 0.05+0.4 = 0.45  
**Value of Information of Z = 0.55 \* 692 + 0.45 \* 934 = 800.9 – 700 = 100.9**

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| --- | --- | --- |
| Y | X | P(X)P(Y|X) |
| T | T | 0.4\*0.2=0.08 |
| F | T | 0.4\*0.8=0.32 |
| F | F | 0.6\*0.3=0.18 |
| T | F | 0.6\*0.7=0.42 |

* 1. X = T -> (0.08, 0.32) -> (0.2, 0.8)  
     a -> 0.2\*800+0.8\*200 = 320  
     ~a -> 0.2\*400+0.8\*1000 = 880  
     **MEU|X=T -> 880 with ~a**  
     X = F -> (0.42, 0.18) -> (0.7, 0.3)  
     a -> 0.7\*800+0.3\*200 = 620  
     ~a -> 0.7\*400+0.3\*1000 = 580  
     **MEU|X=F -> 620 with a  
     Value of Information of X = 0.4 \* 880 + 0.6 \* 620 = 724 – 700 = 24**

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| Y | X | P(Y|X, Z=T) |
| T | T | 0.4\*0.2\*0.9=0.072 |
| T | F | 0.6\*0.7\*0.9=0.378 |
| F | T | 0.4\*0.8\*0.2=0.064 |
| F | F | 0.6\*0.3\*0.2=0.036 |

* 1. P(Y|X,Z=T)=P(X)P(Y|X)P(Y|Z=T)  
       
       
       
       
       
     X=T -> (0.072, 0.064) -> (0.53, 0.47)  
     a -> 0.53\*800+0.47\*200 = 518  
     ~a -> 0.53\*400+0.47\*1000 = 682  
     **MEU|X=T, Z=T -> 682 with ~a**  
     X=F -> (0.378, 0.036) -> (0.91, 0.09)  
     a -> 0.91\*800+0.09\*200 = 746  
     ~a -> 0.91\*400+0.09\*1000 = 454  
     **MEU|X=F, Z=T -> 746 with a**P(X=T|Z=T) = 0.072 + 0.064 = 0.136  
     P(X=F|Z=T) = 0.378 + 0.036 = 0.414  
     P(X|Z=T) -> (0.136, 0.414) -> (0.25, 0.75)  
     **Value of Information of X|Z=T = 0.25\*682+0.75\*746 = 730 – 692 = 38**

1. take = 0\*(1-p)\*100+0\*p\*0+1\*(1-p)\*20+1\*p\*70  
   = (1-p)\*20+p\*70 = 20 – 20p + 70p  
   = 20 + 50p  
   ~take = 1\*(1-p)\*100+1\*p\*0+0\*(1-p)\*20+0\*p\*70  
   = (1-p)\*100  
   = 100 – 100p  
   take = ~take  
   20 + 50p = 100 – 100p  
   150p = 80  
   **p = 80/150 = 8/15**