CS536, Spring 2022
Practice Final Exam

SOLUTIONS

Name ______________________________

IIT Email ______________________________

Important notes:

- This exam has 11 pages. Make sure you have them all.
- You have 120 minutes to complete the exam. We suggest looking through the questions first to see where to focus your time.
- Write your answers in the space provided. If you need more space for answers or scrap, you can use the back of the page, but clearly mark where your answers are.
- You are permitted to refer to **three** double-sided 8.5" $\times$ 11" sheets of notes, as well as the provided reference material. We will collect your sheets of notes at the end of the exam, so if you want them back, please make sure your full name is clearly written on both pages (or the first one if they're attached). **No other outside aids (including electronics) or notes are permitted.**
- Sign the statement below:
  I have not used any unauthorized resources or received or given help during this exam.

Signed______________________________ Date____________________

1 Multiple Choice (17 points)

1. (Student-submitted Q!) Which of the following is a valid interleaving of the parallel program $[s_1; s_2 \| s3; s_4]$?

   A) $s_3; s_4; s_1; s_2$

   B) $s_2; s_1; s_4; s_3$

   C) $s_2; s_4; s_1; s_3$

   D) None of thse

   A

2. (Student-submitted Q!) Which of the following gives information about s (i.e., is not either true for all s or false for all s)?

   A) $\models \{F\}\ s\ \{T\}$

   B) $\models \{F\}\ s\ \{F\}$

   C) $\models \{T\}\ s\ \{F\}$

   D) $\models \{T\}\ s\ \{T\}$

   C

3. Which of these is **not** a difference between loop bounds in standard Hoare Logic and potential functions in Quantitative Hoare Logic (QHL)?

   A) Loop bounds only need to count iterations of a loop; QHL is interested in tracking resource usage even within one iteration of a loop.

   B) The loop bound needs to decrease at every iteration of the loop; in QHL, the potential function needs to be a loop invariant.

   C) Loop bounds can be an overestimate; potential functions in QHL must be exact.

   D) It makes no sense to talk about loop bounds for loop-free programs; QHL can track potential functions for any program.

   C

4. Which of the following is logically equivalent to $(P \wedge Q) \vee (R \wedge T)$?

   A) $(P \rightarrow Q) \wedge (R \rightarrow T)$

   B) $(P \vee R) \rightarrow ((P \rightarrow Q) \wedge (R \rightarrow T))$

   C) $(P \wedge Q) \rightarrow (R \wedge T)$

   D) $(\neg P \vee \neg Q) \rightarrow (R \wedge T)$

   D

NOTE: *For Multiple Choice questions 5–7, multiple answers are correct. Select* ***ALL*** *of the correct answers.*

5. Which of the following could be a deadlock condition for the program (note: you're looking for possible deadlock conditions, regardless of whether the statements themselves are contradictions).

$[\{x > 0\}\ \textsf{await}\ y \leq 5\ \textsf{then}\ \{s_1\}\{x = 10 \wedge y = 2\} \| \{y = 0\}\ s_2; \{y > 0\}\textsf{await}\ x = 10\ \textsf{then}\ \{s_3\}\{x < 0\}]$

A) $y > 0 \wedge x = 10 \wedge y = 2$

B) $x = 10 \wedge y = 2 \wedge x < 0$

C) $y = 0 \wedge x < 0$

D) $x > 0 \wedge x < 0$

E) $x > 0 \wedge y > 0$

A, E

6. Which of the following are **weakest** preconditions of the program $x := a[i]$ and the postcondition $x > 0$?

A) $0 \leq i < |a|$

B) $0 \leq i < |a| \wedge \forall j \in [0, |a| - 1].a[j] > 0$

C) $0 \leq i < |a| \wedge a[i] > 0$

D) $0 \leq i \leq |a| - 1 \wedge a[i] \geq 1$

E) $0 \leq i < |a| \wedge a[i] > 1$

F) $a[i] > 0 \wedge 0 \leq i < |a|$

C, D, F

7. Which statements could be executed in the program

$$\textsf{branch}\ \{x \geq 0 \rightarrow s_1 \,\Box\, y \geq 0 \rightarrow s_2 \,\Box\, x < 10 \rightarrow s_3 \,\Box\, x < y \rightarrow s_4\}$$

in state $\{x = -1, y = 2\}$?

A) s_1

B) s_2

C) s_3

D) s_4

B, C, D

2 Fill in the Blank(s) (9 points)

1. (3 points) $\forall x.\exists y.x = y^2$ cannot be a contingency(contradiction/tautology/contingency)

   because none of its variables are free.

2. (2 points) $\models [\ F\ ]$ while true {skip} $[\ T\ ]$

3. (2 points) If we know $\neg(\exists i \in \mathbb{Z}.P(i))$, we can conclude $\forall i \in \mathbb{Z}.\neg P(i)$ by

   DeMorgan's Laws.

4. (2 points) (Student-submitted Q!) $\models \{y \geq 0 \wedge z > 0\}\ x{:=}y/z\ \{x > 0\}$ (Many possible answers)

3 Proofs (25 points)

Do **either** proof A or proof B. For whichever one you choose:

a) Give appropriate loop invariants **and bounds** for any loops.

b) Write a **full** proof for the program, **including** proving that the loop bound decreases. You may write a full proof outline or a Hilbert-style proof. (We would not recommend using a proof tree.)

c) You **do not** need to worry about runtime errors, just loop termination.

If you start both, tell us which one you want us to grade: ___ Proof A ___ Proof B

Proof A

$\{x \geq 0\}$

$i := \overline{0}$
while $i < 1$ {
  if $i \leq \overline{0} \land x > 0$ {
    $x := x - \overline{1}$
  } else {
    $i := i + \overline{1}$
  }
} $\{x = 0\}$

$\{x \geq 0\}$

$i := \overline{0}$
$\{\textbf{inv } p \equiv x \geq 0 \land (i > 0 \to x = 0)\}$
$\{\textbf{dec } x + 1 - i\}$
while $i < 1$ { $\{p' \equiv p \land i < 1 \land x + 1 - i = t_0\}$
  if $i \leq \overline{0} \land x > 0$ { $\{p' \land i \leq 0 \land x > 0\}$
    $\Rightarrow \{x - 1 \geq 0 \land (i > 0 \to x - 1 = 0) \land x - i < t_0\}$
    $x := x - \overline{1}$ $\{p \land x + 1 - i < t_0\}$
  } else { $\{p' \land (i > 0 \lor x \leq 0)\}$
    $\Rightarrow \{x \geq 0 \land (i + 1 > 0 \to x = 0) \land x - i < t_0\}$
    $i := i + \overline{1}$ $\{p \land x + 1 - i < t_0\}$
  } $\{p \land x + 1 - i < t_0\}$
} $\{x = 0\}$

Proof B

On the real exam, there will be a Proof B of similar difficulty to Proof A. Here, there is not. Sorry.

4 Calculations (20 points)

All parts to this question were submitted by students!

(a) (5 points) Calculate $[x/y]((\forall x.x \geq y) \rightarrow (\exists y.z \leq y))$.

$$
\begin{aligned}
& [x/y]((\forall a.a \geq y) \rightarrow (\exists y.z \leq y)) \\
= \quad & ((\forall a.a \geq x) \rightarrow (\exists y.z \leq y))
\end{aligned}
$$

(b) (5 points) Calculate $M(\mathsf{while}\ e > 0\ \{e := e - \overline{1}; x := x * \overline{2}\}, \{e = 5.x = 1\})$. You do not need to show your work. (*Editor's note: the student who submitted this question probably didn't know it, but this is a program to compute Stefan's age.*)

$$\{e = 0, x = 32\}$$

(c) (5 points) Calculate $wp(\mathsf{if}\ x > 2\ \mathsf{then}\ \{y := \overline{1}\}\ \mathsf{else}\ \{y := \overline{-1}\} y > 0,)$. **Simplify** your answer as much as you can.

$$(x > 2 \to wp(y := \overline{1}, y > 0)) \land (x \le 2 \to wp(y := \overline{-1}, y > 0))$$
$$(x > 2 \to 1 > 0) \land (x \le 2 \to -1 > 0)$$
$$(x > 2 \to T) \land (x \le 2 \to F)$$
$$x > 2$$

(d) (5 points) Calculate $wp(\mathsf{if}\ a[x] \ge 0\ \mathsf{then}\ \{z := sqrt(a[x])\}\ \mathsf{else}\ \{z := sqrt(-a[x])\}, y < z)$. You **do not** need to simplify your answer.

$$\begin{aligned}
& wp(\mathsf{if}\ a[x] \ge 0\ \mathsf{then}\ \{z := sqrt(a[x])\}\ \mathsf{else}\ \{z := sqrt(-a[x])\}, y < z) \\
& \land D(\mathsf{if}\ a[x] \ge 0\ \mathsf{then}\ \{z := sqrt(a[x])\}\ \mathsf{else}\ \{z := sqrt(-a[x])\}) \\
= \; & (a[x] \ge 0 \to wlp(z := sqrt(a[x]), y < z)) \land (a[x] < 0 \to wlp(z := sqrt(-a[x]), y < z)) \\
& \land D(a[x] \ge 0) \land (a[x] \ge 0 \to D(sqrt(a[x]))) \land (a[x] < 0 \to D(sqrt(-a[x]))) \\
= \; & (a[x] \ge 0 \to y < \sqrt{a[x]} \land a[x] \ge 0) \land (a[x] < 0 \to y < \sqrt{-a[x]} \land -a[x] \ge 0) \\
= \; & (a[x] \ge 0 \to y < \sqrt{a[x]}) \land (a[x] < 0 \to y < \sqrt{-a[x]})
\end{aligned}$$

5 Logic Proof (15 points)

Prove the following logical implication.

$$((\exists x.P(x)) \rightarrow Q) \land \neg Q \Rightarrow \forall x.\neg P(x)$$

Write a statement and a justification on each line, as you did on HW1. The justifications should be logical laws (See Section A of the reference material) with a reference to the lines you're using. For example, if line 3 has the statement P and line 4 has the statement Q, you could justify $P \land Q$ with "Conjunction(3, 4)". If a justification uses only one line and it's the line immediately before, you can leave it out. You can leave lines left over; there may be more than you need.

1	$((\exists x.P(x)) \rightarrow Q) \land \neg Q$	Assumption
2	$(\neg(\exists x.P(x)) \lor Q) \land \neg Q$	Definition of Conditional
3	$((\forall x.\neg P(x)) \lor Q) \land \neg Q$	DeMorgan
4	$((\forall x.\neg P(x)) \land \neg Q) \lor (Q \land \neg Q)$	Distributivity
5	$((\forall x.\neg P(x)) \land \neg Q) \lor F$	Contradiction
6	$((\forall x.\neg P(x)) \land \neg Q)$	Identity
7	$\forall x.\neg P(x)$	Simplify

6 Parallel Programs (14 points)

Consider the program $[s_1 \| s_2 \| s_3]$ where

$$\begin{aligned} s_1 &= x := c + d \\ s_2 &= y := \overline{1} - d \\ s_3 &= z := z + c \end{aligned}$$

(a) (6 points) Complete the following table to determine the interference of the various threads.

i	j	$Change(i)$	$Vars(j)$	i Interferes w/ j?
1	2	x	y, d	No
1	3	x	z, c	No
2	1	y	x, c, d	No
2	3	y	z, c	No
3	1	z	x, c, d	No
3	2	z	y, d	No

(b) (8 points) Draw an evaluation graph for $M([s_1 \| s_2 \| s_3], \{c = 1, d = 2, z = 5\})$, where (as before):

$$\begin{array}{rcl} s_1 & = & x := c + d \\ s_2 & = & y := \overline{1} - d \\ s_3 & = & z := z + c \end{array}$$

$$\begin{tikzcd}[column sep=tiny]
 & \langle [s_1 \| s_2 \| s_3], \{c = 1, d = 2, z = 5\} \rangle \arrow[dl] \arrow[d] \arrow[dr] & \\
\langle \textsf{skip} \| s_2 \| s_3, \{c = 1, d = 2, z = 5, x = 3\} \rangle \arrow[d] \arrow[dr] & \langle [s_1 \| \textsf{skip} \| s_3], \{c = 1, d = 2, z = 5, y = -1\} \rangle \arrow[dl] \arrow[dr] & \langle [s_1 \| s_2 \| \textsf{skip}], \{c = 1, d = 2, z = 6\} \rangle \arrow[dl] \arrow[d] \\
\langle \textsf{skip} \| \textsf{skip} \| s_3, \{c = 1, d = 2, z = 5, x = 3, y = -1\} \rangle \arrow[dr] & \langle \textsf{skip} \| s_2 \| \textsf{skip}, \{c = 1, d = 2, z = 6, x = 3\} \rangle \arrow[d] & \langle [s_1 \| \textsf{skip} \| \textsf{skip}], \{c = 1, d = 2, z = 6, y = -1\} \rangle \arrow[dl] \\
 & \langle \textsf{skip}, \{c = 1, d = 2, z = 6, y = -1, x = 3\} \rangle &
\end{tikzcd}$$