

**Task 1.1**

- (a)  $[y + x/x](2x + y \geq z) = 2(y + x) + y \geq z = 3y + 2x \geq z$
- (b)  $[z/x](x \geq 0 \rightarrow (\forall x.x * z > y) \wedge x > -1)$   
 $= (x \geq 0)[z/x] \rightarrow (\forall x.x * z > y)[z/x] \wedge (x > -1)[z/x]$   
 $= z \geq 0 \rightarrow (\forall x.x * z > y) \wedge z > -1$
- (c)  $[x/y]\forall x.(y > 0 \rightarrow \exists y.y = x) = \forall x.(x > 0 \rightarrow \exists y.y = x)$
- (d)  $[x + 2/x]\exists x.\forall y.x > y = \exists x.\forall y.x > y$

**Task 2.1**

$\{x \neq y\}$  if  $y > x$  then  $\{t := x; x := y; y := t\}$  else  $\{\text{skip}\}$   $\{x > y\}$

$\{x \neq y\}$

if  $(y > x)\{$

$t := x$   $\{x \neq y \wedge y > x\}$

$x := y$   $\{x \neq y \wedge y > x \wedge t = x\}$

$y := t$   $\{x \neq y \wedge y > x \wedge t = x \wedge x = y\} \Rightarrow \{t = x_0 \wedge x = y_0 \wedge y_0 > x_0\}$

$\{t = x_0 \wedge x = y_0 \wedge y = t \wedge y_0 > x_0\} \Rightarrow \{x = y_0 \wedge y = x_0 \wedge y_0 > x_0\} \Rightarrow \{x > y\}$

$\}$  else  $\{$   $\{x \neq y \wedge y < x\}$

$\text{skip}$   $\{x \neq y \wedge x > y\} \Rightarrow \{x > y\}$

$\}$   $\{x > y \vee x > y\} \Rightarrow \{x > y\}$

**Task 2.2**

1.  $\{x = 0 \wedge 0 = 0\}s := 0\{x = 0 \wedge s = 0\}$  Assign
2.  $\{x < 0 \wedge -1 = -1\}s := -1\{s = x/|x|\}$  Assign
3.  $\{x > 0 \wedge 1 = 1\}s := 1\{s = x/|x|\}$  Assign
4.  $\{T\}$  if  $\dots\{s = x/|x| \vee s = x/|x|\}$  if 2, 3
5.  $\{T\}$  if  $\dots\{s = x/|x|\}$  Weakening 4
6.  $\{T\}$  if  $\dots\{(x = 0 \wedge s = 0) \vee s = x/|x|\}$  if 1, 4

### Task 3.1

- (a)  $wp(x := x + y; n := x * z, n = 0) \equiv z = 0$
- (b)  $wp(\text{if } x = y \text{ then } \{z := 1\} \text{ else } \{z := x/y\}, z = 1)$   
 $\equiv (x = y \rightarrow wp(z := 1, z = 1)) \wedge (x \neq y \rightarrow wp(z := x/y, z = 1))$   
 $\equiv (x = y \rightarrow 1 = 1) \wedge (x \neq y \rightarrow y \neq 0 \wedge x/y = 1)$   
 $\equiv \text{True} \wedge (x \neq y \rightarrow y \neq 0 \wedge x/y = 1)$   
 $\equiv x \neq y \rightarrow y \neq 0 \wedge x/y = 1$
- (c)  $sp(x = 1, \text{if } y > 0 \text{ then } x := x + 1 \text{ else } \{\text{skip}\})$   
 $\equiv sp(x = 1 \wedge y > 0, x := x + 1) \vee sp(x = 1 \wedge \neg(y > 0), \text{skip})$   
 $\equiv ([x_0/x](x = 1 \wedge y > 0) \wedge x = [x_0/x](x + 1)) \vee sp(x = 1 \wedge y \leq 0, \text{skip})$   
 $\equiv ((x_0 = 1 \wedge y > 0) \wedge x = x_0 + 1) \vee (x = 1 \wedge y \leq 0)$   
 $\equiv (y > 0 \wedge x = 2) \vee (x = 1 \wedge y \leq 0)$
- (d)  $sp(x \geq 0, x := 1; \text{if } x > 0 \text{ then } \{x := x - 1\} \text{ else } x := 0)$   
 $\equiv sp(sp(x \geq 0, x := 1), \text{if } x > 0 \text{ then } \{x := x - 1\} \text{ else } x := 0)$   
 $\equiv sp([x_0/x](x \geq 0) \wedge x = [x_0/x]1, \text{if } x > 0 \text{ then } \{x := x - 1\} \text{ else } x := 0)$   
 $\equiv sp(x_0 \geq 0 \wedge x = 1, \text{if } x > 0 \text{ then } \{x := x - 1\} \text{ else } x := 0)$   
 $\equiv sp(x_0 \geq 0 \wedge x = 1 \wedge x > 0, x := x - 1) \vee sp(x_0 \geq 0 \wedge x = 1 \wedge \neg(x > 0), x := 0)$   
 $\equiv ([x_1/x](x_0 \geq 0 \wedge x = 1 \wedge x > 0) \wedge x = [x_1/x](x - 1)) \vee ([x_1/x](x_0 \geq 0 \wedge x = 1 \wedge x \leq 0) \wedge x = [x_1/x]0)$   
 $\equiv (x_0 \geq 0 \wedge x_1 = 1 \wedge x_1 > 0 \wedge x = x_1 - 1) \vee (x_0 \geq 0 \wedge x_1 = 1 \wedge x_1 \leq 0 \wedge x = 0)$   
 $\equiv x_0 \geq 0 \wedge x_1 = 1 \wedge x_1 > 0 \wedge x = x_1 - 1$

### Task 4.1

I spent about 4 hours on this.