Task 1.1

(a) $\sigma[x \mapsto 4][x \mapsto 8] = \{x = 1, y = 3, b = [4; 5; 6; 7]\}[x \mapsto 4][x \mapsto 8]$
$= \{x = 4, y = 3, b = [4; 5; 6; 7]\}[x \mapsto 8]$
$= \{x = 8, y = 3, b = [4; 5; 6; 7]\}$

(b) $\sigma[y \mapsto 9](x) = \{x = 1, y = 3, b = [4; 5; 6; 7]\}[y \mapsto 9](x)$
$= \{x = 1, y = 9, b = [4; 5; 6; 7]\}(x)$
$= 1$

(c) $\sigma[x \mapsto 2][z \mapsto 4] = \{x = 1, y = 3, b = [4; 5; 6; 7]\}[x \mapsto 2][z \mapsto 4]$
$= \{x = 2, y = 3, b = [4; 5; 6; 7]\}[z \mapsto 4]$
$= \{x = 2, y = 3, b = [4; 5; 6; 7], z = 4\}$

Task 1.2

(a) Yes because $\{y = 3, x = 2\} \models x < y$ holds.

(b) No because $\{x = 17, y = 4\} \not\models x \leq y^2$

(c) Yes because $b[0] > 0$ and $b[1] > 0$

(d) Yes because with $j = 0$ then $\{x = 2, y = 6, b = [1; 6; 8]\} \models (\exists x \in \mathbb{Z}.\exists j \in \mathbb{Z}.b[j] < x \wedge x < y)$

Task 1.3

$x = 2y + z$, $y = 2z$, $z = b[0] + b[2]$, and $4 < b[1] < b[2] < 7$ and $z = 8$
We can get $b[1], b[2] = 5, 6$ from $4 < b[1] < b[2] < 7$
Using $z = 8$ with all the other variables we got, we can get $b[0] = z - b[2] = 8 - 6 = 2$, $y = 2z = 2 * 8 = 16$, $x = 2y + z = 2 * 16 + 8 = 40$
Thus,
$\sigma = \{x = 40, y = 16, z = 8, b = [2, 5, 6]\}$

Task 1.4

(a) $\models (\exists x \in \mathbb{Z}.x < 0)$ if for **all** states σ, it is true that $\sigma[x \mapsto n] \models x < 0$, for **some** $n \in \mathbb{Z}$

(b) $\models (\forall x \in \mathbb{Z}.x < 0)$ if for **all** states σ, it is true that $\sigma[x \mapsto n] \models x < 0$, for **all** $n \in \mathbb{Z}$

(c) $\models (\exists x \in \mathbb{Z}.\forall y \in \mathbb{Z}.x < y^2)$ if for **all** states σ, it is true that $\sigma[x \mapsto n][y \mapsto m] \models x < y^2$, for **some** $n \in \mathbb{Z}$, and for **all** $m \in \mathbb{Z}$

Task 2.1

(a) Legal

(b) Legal

(c) Illegal because b and c are not expressions. $(e?e:e)$. Can instead do $(x = 2?b[y] : b[y])$

(d) Legal

(e) Illegal. d is 2-dimensional while b is 1-dimensional.

Task 2.2

(a) $\sigma(x + y) = \sigma(x) + \sigma(y) = 5 + 2 = 7$

(b) $\sigma(x * a[y]) = \sigma(x) * \sigma(a[y]) = \sigma(x) * \sigma(a)[\sigma(y)] = 5 * a[2] = 5 * 6 = 30$

(c) $\sigma(max(a[x-4], a[y])) = max(\sigma(a[x-4]), \sigma(a[y])]) = max(a[1], a[2]) = max(5, 6) = 6$

(d) $\sigma(x > y?a[x-4] : a[y]) = \sigma(x > y)?\sigma(a[x-4]) : \sigma(a[y]) = 5 > 2?a[1] : a[2] = True?5 : 6 = 5$

(e) $\sigma(a[size(a) - y]) = \sigma(a[3-2])] = \sigma(a[1]) = 5$

Task 3.1

I spent about 2 hours on this.