

IIT CS536: Science of Programming

Homework 2: State, Expression and Types

Prof. Stefan Muller
TAs: Chaoqi Ma, Zhenghao Zhao

Out: Thursday, Jan. 27
Due: Monday, Feb. 7, 11:59pm CST

Updated Feb. 2

This assignment contains 7 written task(s) for a total of 60 points.

SOLUTIONS

Logistics

Submission Instructions

Please read and follow these instructions carefully.

- Submit your homework on Blackboard under the correct assignment by the deadline (or the extended deadline if taking late days).
- You may submit multiple times, but we will only look at your last submission. Make sure your last submission contains all necessary files.
- Email the instructor and TAs ASAP if
 - You submit before the deadline but then decide to take (more) late days.
 - You accidentally resubmit after the deadline, but did not intend to take late days.

Otherwise, you do not need to let us know if you're using late days; we'll count them based on the date of your last submission.

- Submit your written answers in a single PDF or Word document. Typed answers are preferred (You can use any program as long as you can export a .pdf, .doc or .docx; LaTeX is especially good for typesetting logic and math, and well worth the time to learn it), but *legible* handwritten and scanned answers are acceptable as well.
- Your Blackboard submission should contain only the file with your written answers. Do not compress or put any files in folders.

Collaboration and Academic Honesty

Read the policy on the website and be sure you understand it.

Task 1.1 (Written, 9 points).

For b) please give the value of x.

- What is $\sigma[x \mapsto 4][x \mapsto 8]$?
- What is $\sigma[y \mapsto 9](x)$?
- What is $\sigma[x \mapsto 2][z \mapsto 4]$?

- $\sigma[x \mapsto 4][x \mapsto 8] = \{x = 4, y = 3, b = [4; 5; 6; 7]\}[x \mapsto 8] = \{x = 8, y = 3, b = [4; 5; 6; 7]\}$
- $\sigma[y \mapsto 9](x) = \sigma(x) = 1$
- $\sigma[x \mapsto 2][z \mapsto 4] = \{x = 2, y = 3, b = [4; 5; 6; 7], z = 4\}$

Task 1.2 (Written, 12 points).

- Does $\{x = 9, y = 3\} \models \exists x \in \mathbb{Z} . x < y$?
- Does $\{x = 2, y = 4\} \models \forall x \in \mathbb{Z} . x \leq y^2$?
- Does $\{b = [1; 3; -4; 6]\} \models \forall i \in \mathbb{Z} . 0 \leq i \wedge i < 2 \rightarrow b[i] > 0$?
- Does $\{x = 2, y = 6, b = [1; 6; 8]\} \models (\exists x \in \mathbb{Z} . \exists j \in \mathbb{Z} . b[j] < x \wedge x < y)$?

- a) Yes, $\{x = 9, y = 3\}[x \mapsto 1] \models x . x < y$
- b) No, for example, $x = 17$
- c) Yes
- d) Yes

Task 1.3 (Written, 7 points).

Every element in array b is an integer. Complete the definition of

$$\sigma = \{x = \underline{\hspace{1cm}}, y = \underline{\hspace{1cm}}, z = 8, b = \underline{\hspace{1cm}}\}$$

$$x = 40, \ y = 16, \ b = [2, 5, 6]$$

Task 1.4 (Written, 7 points).

Fill in ____ with “some” or “all” *Updated Feb. 2*: There should be no σ before the \models in each question. We want to know the situation required for each statement to be a tautology.

- a) $\models (\exists x \in \mathbb{Z} . x < 0)$ if for _____ states σ , it is true that $\sigma[x \mapsto n] \models x < 0$, for _____ $n \in \mathbb{Z}$.
- b) $\models (\forall x \in \mathbb{Z} . x < 0)$ if for _____ states σ , it is true that $\sigma[x \mapsto n] \models x < 0$, for _____ $n \in \mathbb{Z}$.
- c) $\models (\exists x \in \mathbb{Z} . \forall y \in \mathbb{Z} . x < y^2)$ if for _____ states σ , it is true that $\sigma[x \mapsto n][y \mapsto m] \models x < y^2$, for _____ $n \in \mathbb{Z}$ and for _____ $m \in \mathbb{Z}$.

- a) all, some
- b) all, all
- c) all, some, all

2 Expression and Types

Task 2.1 (Written, 10 points).

For each of the following expressions, say whether it is legal (allowed by the syntax of the language we defined in class) or illegal (not allowed). b and c are 1-dimensional arrays, d is a 2-dimensional array. x, y, z are integers.

- a) $(x = y ? z : b[z])$
- b) $b[(x < 3 ? y : z + 2)]$
- c) $(x = 2 ? b : c)[y]$
- d) $(x > 3 ? \text{sqrt}(0) : \text{max}(2, 5))$
- e) $d[0] + b[2][1]$

- a) Legal
- b) Legal
- c) Illegal: You can only index into an array, not an expression
- d) Legal
- e) Illegal: Array dimensions are wrong

Task 2.2 (Written, 15 points).

Evaluate each of the following expressions with the state $\sigma = \{x = 5; y = 2; a = [2; 5; 6]\}$.

- a) $\sigma(x + y)$
- b) $\sigma(x * a[y])$
- c) $\sigma(\text{max}(a[x - 4], a[y]))$
- d) $\sigma(x > y ? a[x - 4] : a[y])$
- e) $\sigma(a[\text{size}(a) - y])$

- a) 7
- b) 30
- c) 6
- d) 5
- e) 5

3 One more wrap-up question

Task 3.1 (Written, 0 points).

How long (approximately) did you spend on this homework, in total hours of actual working time? Your honest feedback will help us with future homeworks.