Lab Project

Jacobi Method

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Input

The matrix A

$$\begin{bmatrix} 5 & -1 & 0 & 0 & 0 \\ -1 & 4.5 & 0.2 & 0 & 0 \\ 0 & 0.2 & 1 & -0.4 & 0 \\ 0 & 0 & -0.4 & 3 & 1 \\ 0 & 0 & 0 & 1 & 3 \end{bmatrix}$$

Output

The eigenvalues $u_1, \ldots, u_n$

Jacobi's method for a symmetric matrix A is described by

$$A_1 = A,$$

$$A_2 = P_1 A_1 P_1^t$$

and, in general,

$$A_{i+1} = P_i A_i P_i^t.$$

The matrix A_{i+1} tends to a diagonal matrix, where P_i is a rotation matrix chosen to eliminate a large off-diagonal element in A_i. Suppose $a_{j,k}$ and $a_{k,j}$ are to be set to 0, where $j \neq k$. If $a_{jj} \neq a_{kk}$, then

$$(P_i)_{jj} = (P_i)_{kk} = \sqrt{\frac{1}{2}\left(1 + \frac{b}{\sqrt{c^2 + b^2}}\right)}, \quad (P_i)_{kj} = \frac{c}{2(P_i)_{jj}\sqrt{c^2 + b^2}} = -(P_i)_{jk},$$

where

$$c = 2a_{jk}\operatorname{sgn}(a_{jj} - a_{kk}) \quad \text{and} \quad b = |a_{jj} - a_{kk}|,$$

or if $a_{jj} = a_{kk}$,

$$(P_i)_{jj} = (P_i)_{kk} = \frac{\sqrt{2}}{2}$$

and

$$(P_i)_{kj} = -(P_i)_{jk} = \frac{\sqrt{2}}{2}.$$

Develop an algorithm to implement Jacobi's method by setting $a_{21} = 0$. Then set a_{31}, a_{32}, a_{41}, a_{42}, $a_{43}, \ldots, a_{n,1}, \ldots, a_{n,n-1}$ in turn to zero.This is repeated until a matrix A_k is computed with

$$\sum_{i=1}^{n} \sum_{\substack{j=1 \\ j \neq i}}^{n} |a_{ij}^{(k)}|$$

sufficiently small. The eigenvalues of A can then be approximated by the diagonal entries of A_k.