Lab Project

Runge-Kutta-Verner method

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Input
endpoints:
a b
Initial value
α
Tolerance
TOL
Maximum step size
hmax
Minimum step size
hmin

Output
t, w, h

The Runge-Kutta-Verner method (see [Ve]) is based on the formulas

$$w_{i+1} = w_i + \frac{13}{160}k_1 + \frac{2375}{5984}k_3 + \frac{5}{16}k_4 + \frac{12}{85}k_5 + \frac{3}{44}k_6 \quad \text{and}$$

$$\tilde{w}_{i+1} = w_i + \frac{3}{40}k_1 + \frac{875}{2244}k_3 + \frac{23}{72}k_4 + \frac{264}{1955}k_5 + \frac{125}{11592}k_7 + \frac{43}{616}k_8,$$

where

$$k_1 = hf(t_i, w_i),$$

$$k_2 = hf\left(t_i + \frac{h}{6}, w_i + \frac{1}{6}k_1\right),$$

$$k_3 = hf\left(t_i + \frac{4h}{15}, w_i + \frac{4}{75}k_1 + \frac{16}{75}k_2\right),$$

$$k_4 = hf\left(t_i + \frac{2h}{3}, w_i + \frac{5}{6}k_1 - \frac{8}{3}k_2 + \frac{5}{2}k_3\right),$$

$$k_5 = hf\left(t_i + \frac{5h}{6}, w_i - \frac{165}{64}k_1 + \frac{55}{6}k_2 - \frac{425}{64}k_3 + \frac{85}{96}k_4\right),$$

$$k_6 = hf\left(t_i + h, w_i + \frac{12}{5}k_1 - 8k_2 + \frac{4015}{612}k_3 - \frac{11}{36}k_4 + \frac{88}{255}k_5\right),$$

$$k_7 = hf\left(t_i + \frac{h}{15}, w_i - \frac{8263}{15000}k_1 + \frac{124}{75}k_2 - \frac{643}{680}k_3 - \frac{81}{250}k_4 + \frac{2484}{10625}k_5\right),$$

$$k_8 = hf\left(t_i + h, w_i + \frac{3501}{1720}k_1 - \frac{300}{43}k_2 + \frac{297275}{52632}k_3 - \frac{319}{2322}k_4 + \frac{24068}{84065}k_5 + \frac{3850}{26703}k_7\right).$$

The sixth-order method $\tilde{w}_{i+1}$ is used to estimate the error in the fifth-order method w_{i+1}. Construct an algorithm similar to the Runge-Kutta-Fehlberg Algorithm

solve

a. $y' = y/t - (y/t)^2, \quad 1 \le t \le 4, \quad y(1) = 1;$ actual solution $y(t) = t/(1 + \ln t)$.

$TOL = 10^{-6}$, $hmax = 0.5$, and $hmin = 0.05$

Lab Project

SOR Method

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Input

The number

n

The augmented matrix

TOL

the maximum number of iterations

N

The initial value

$x0_i$

The real

w

Output

The solution $x_1, \ldots, x_n$

Use the SOR method to solve the linear system $A\mathbf{x} = \mathbf{b}$ to within 10^{-5} in the l_∞ norm, where the entries of A are

$$a_{i,j} = \begin{cases} 2i, & \text{when } j = i \text{ and } i = 1, 2, \ldots, 80, \\ 0.5i, & \text{when } \begin{cases} j = i + 2 \text{ and } i = 1, 2, \ldots, 78, \\ j = i - 2 \text{ and } i = 3, 4, \ldots, 80, \end{cases} \\ 0.25i, & \text{when } \begin{cases} j = i + 4 \text{ and } i = 1, 2, \ldots, 76, \\ j = i - 4 \text{ and } i = 5, 6, \ldots, 80, \end{cases} \\ 0, & \text{otherwise,} \end{cases}$$

and those of $\mathbf{b}$ are $b_i = \pi$, for each $i = 1, 2, \ldots, 80$.