FINM 25000 **Mark Hendricks**
Summer 2021

Homework #4

Due on Monday, July 26, at 6:00pm CT.

Background Case: **Grantham, Mayo, and Van Otterloo, 2012: Estimating the Equity Risk Premium [9-213-051]**

1 GMO

This section will not be graded, but it will be discussed. It requires no empirical analysis; answer solely based on the material given in the case.

1. GMO's approach.
   - (a) Why does GMO believe they can more easily predict long-run than short-run asset class performance?
   - (b) What predicting variables does the case mention are used by GMO? Does this fit with the goal of long-run forecasts?
   - (c) How has this approach led to contrarian positions?
   - (d) How does this approach raise business risk and managerial career risk?
2. The market environment.
   - (a) We often estimate the market risk premium by looking at a large sample of historic data. What reasons does the case give to be skeptical that the market risk premium will be as high in the future as it has been over the past 50 years?
   - (b) In 2007, GMO forecasts real excess equity returns will be negative. What are the biggest drivers of their pessimistic conditional forecast relative to the unconditional forecast. (See Exhibit 9.)
   - (c) In the 2011 forecast, what components has GMO revised most relative to 2007? Now how does their conditional forecast compare to the unconditional? (See Exhibit 10.)
3. Consider the asset class forecasts in Exhibit 1.
   - (a) Which asset class did GMO estimate to have a negative 10-year return over 2002-2011?
   - (b) Which asset classes substantially outperformed GMO's estimate over that time period?
   - (c) Which asset classes substantially underperformed GMO's estimate over that time period?
4. Fund Performance.
   - (a) In which asset class was GMWAX most heavily allocated throughout the majority of 1997-2011?
   - (b) Comment on the performance of GMWAX versus its benchmark. (No calculation needed; simply comment on the comparison in the exhibits.)

2 Analyzing GMO

This section utilizes data in the file, `gmo_returns.xlsx`.

Examine GMO's performance. For simplicity, just examine total returns.[1]

1. Calculate the mean, volatility, and Sharpe ratio for GMWAX. Do this for three samples:

   - from inception through 2011
   - 2012-present
   - inception - present

   Has the mean, vol, and Sharpe changed much since the case?

2. GMO believes a risk premium is compensation for a security's tendency to lose money at "bad times". For all three samples, analyze extreme scenarios by looking at

   - Min return
   - 5th percentile (VaR-5th)
   - Maximum drawdown

   (a) Does GMWAX have high or low tail-risk as seen by these stats?
   (b) Does that vary much across the two subsamples?

3. For all three samples, regress excess returns of GMWAX on excess returns of SPY.

   (a) Report the estimated alpha, beta, and r-squared.
   (b) Is GMWAX a low-beta strategy? Has that changed since the case?
   (c) Does GMWAX provide alpha? Has that changed across the subsamples?

4. Calculate the rolling, 60-month beta of GMWAX on SPY over the entire sample, (starting 5-years after their shared return history.)

   Does GMWAX show much variation in market beta over time?

[1] Technically, a Sharpe ratio is defined on *excess* returns, but the difference is negligible, and it is common to examine total returns.

3 Forecast Regressions

This section utilizes data in the file, `sp500_fundamentals.xlsx`.

1. Consider the lagged regression, where the regressor, $(X,)$ is a period behind the target, (r^{SPY}).

$$r_t^{SPY} = \alpha^{SPY,\boldsymbol{X}} + \left(\boldsymbol{\beta}^{SPY,\boldsymbol{X}}\right)' \boldsymbol{X}_{t-1} + \epsilon_t^{SPY,\boldsymbol{X}} \tag{1}$$

   Estimate (1) and report the $\mathcal{R}^2$, as well as the OLS estimates for α and β. Do this for...

   - $\boldsymbol{X}$ as a single regressor, the dividend-price ratio.
   - $\boldsymbol{X}$ as a single regressor, the earnings-price ratio.
   - $\boldsymbol{X}$ as three regressors, the dividend-price ratio, the earnings-price ratio, and the 10-year yield.

   For each, report the r-squared.

2. For each of the three regressions, let's try to utilize the resulting forecast in a trading strategy.

   - Build the forecasted SPY returns: $\hat{r}_{t+1}^{\text{SPY}}$. Note that this denotes the forecast made using $\boldsymbol{X}_t$ to forecast the $(t+1)$ return.
   - Set the scale of the investment in SPY equal to 100 times the forecasted value:

$$w_t = 100\,\hat{r}_{t+1}^{\text{SPY}}$$

     We are not taking this scaling too seriously. We just want the strategy to go bigger in periods where the forecast is high and to withdraw in periods where the forecast is low, or even negative. Later, we'll reset the scaling to make sure it is all comparable.

   - Calculate the return on this strategy:

$$r_{t+1}^{\text{x}} = w_t r_{t+1}^{\text{SPY}}$$

   You should now have the trading strategy returns, r^{x} for each of the forecasts. For each strategy, estimate

   - mean, volatility, Sharpe,
   - max-drawdown
   - market alpha
   - market beta
   - market Information ratio

4 Out-of-Sample Forecasting

This section utilizes data in the file, `sp500_fundamentals.xlsx`.

Reconsider the problem above, of estimating (1) for $\boldsymbol{x}$. The reported $\mathcal{R}^2$ was the in-sample $\mathcal{R}^2$–it examined how well the forecasts fit in the sample from which the parameters were estimated.
This time, only consider the case where we are regressing on the earnings-price ratio.

Let's consider the out-of-sample r-squared. To do so, we need to do the following:

- Start at $t = 60$.

- Estimate (1) only using data through time t.

- Use the estimated parameters of (1), along with $\boldsymbol{x}_{t+1}$ to calculate the out-of-sample forecast for the following period, $t + 1$.
$$\hat{r}_{t+1}^{SPY} = \hat{\alpha}_t^{SPY,\boldsymbol{x}} + \left(\boldsymbol{\beta}^{SPY,\boldsymbol{x}}\right)' \boldsymbol{x}_t$$

- Calculate the $t + 1$ forecast error,
$$e_{t+1}^{x} = r_{t+1}^{SPY} - \hat{r}_{t+1}^{SPY}$$

- Move to $t = 61$, and loop through the rest of the sample.

You now have the time-series of out-of-sample prediction errors, e^x.

Calculate the time-series of out-of-sample prediction errors e^0, which are based on the null forecast:

$$\begin{aligned}\bar{r}_{t+1}^{SPY} =& \frac{1}{t}\sum_{i=1}^{t} R_i^{SPY}\\ e_{t+1}^{0} =& r_{t+1}^{SPY} - \bar{r}_{t+1}^{SPY}\end{aligned}$$

1. Report the out-of-sample $\mathcal{R}^2$:
$$\mathcal{R}_{OOS}^2 \equiv\ 1 - \frac{\sum_{i=61}^{T}\left(e_i^x\right)^2}{\sum_{i=61}^{T}\left(e_i^0\right)^2}$$

   Note that unlike an in-sample r-squared, the out-of-sample r-squared can be anywhere between $(-\infty, 1]$.

   Did this forecasting strategy produce a positive OOS r-squared?

2. Re-do problem 3.2 using this OOS forecast.

   How much better/worse is the OOS Earnings-Price ratio strategy compared to the in-sample version of 3.2?

5 Time-Series Models of Volatility

This section utilizes data in the file, `sp500_fundamentals.xlsx`.

We will calculate a time-series of volatility estimates using a few different methods. For each, we use σ_t to denote our estimate of the time-t return volatility, as based on data over periods through $t-1$, but not including t itself.

Estimate the following using the SPY return series. We use a common (but biased) version of the usual variance estimator by ignoring μ and dividing by the number of data points, rather than by the degrees of freedom.

- Expanding Series

$$\sigma_t^2 = \frac{1}{t-1}\sum_{\tau=1}^{t-1} \tilde{r}_\tau^2$$

  Begin the calculation at $t = 61$, so that the first estimate is based on 60 data points.

- Rolling Window

$$\sigma_t^2 = \frac{1}{m}\sum_{l=1}^{m} \tilde{r}_{t-l}^2$$

  Use $m = 60$, and begin the calculation at the $t = 61$, (so that the calculation has a full 60 data points.) Consider, (if you have a pandas dataframe named, `df`,) using pandas method `df.rolling(60).std()`

- IGARCH: famously used in *Risk Metrics*, and in this form, simply an Exponentially Weighted Moving Average (EWMA) of squared returns:

$$\sigma_t^2 = \theta\sigma_{t-1}^2 + (1-\theta)\tilde{r}_{t-1}^2$$

  Rather than estimating θ, simply use $\theta = 0.97$, and initialize with $\sigma_1 = 0.15\left(\frac{1}{\sqrt{12}}\right)$.

  This is easy to program with a loop, or you can try figuring out the (poorly documented) package `arch.univariate.EWMAVariance`.

- GARCH(1,1)

$$\sigma_t^2 = \omega + \theta\sigma_{t-1}^2 + \gamma\tilde{r}_{t-1}^2$$

  To estimate GARCH(1,1), try using the ARCH package in Python. The default estimation implementation is fine, (and will account for μ and degrees of freedom.)

For each of these methods,

1. plot σ_t. (Plot the volatility, the square-root of the variance.)
2. report the estimation for November 2008 and January 2018.