

## Homework #2

Due on Tuesday, July 14, at 6:00pm CT.

*I know you have a short week. Only Sections 2 and 3 are required to be submitted. We will discuss Sections 1 and 3 in class on Monday.*

Background Case: **Smart Beta Exchange-Traded Funds and Factor Investing**

### 1 The Case

*This section will not be graded, but it will be discussed in class.*

1. Describe the four factors: Market, Size, Value, and Momentum.
  - (a) Are they constructed from portfolios going long stocks or portfolios that go long some stocks and short others?
  - (b) What is the point of figures 1-6?
  - (c) Do Fama and French believe Momentum is a pricing factor?
2. How is a “smart beta” ETF different from a traditional ETF?
3. Is it possible for all investors to invest in the Value factor?
4. How does factor investing differ from traditional diversification?

### 2 Pricing Factors Stats

On Canvas, find the data files used in Homework #2,

- **industry\_equity\_data.xlsx**
- **fama\_french\_data.xlsx**

Note that the FF factor labels are as follows:

- Market: MKT
- Size: SMB (Small Minus Big)
- Value: HML (High Minus Low)
- Momentum: UMD (Up Minus Down)

The Fundamental Theorem of Asset Pricing says that these priced factors are sufficient to construct the Tangency Portfolio. Let's examine both of those implications for the excess returns of the 4 factors,  $\tilde{f}^i$ , as tested on the industry equity excess returns<sup>1</sup>,  $\tilde{r}^i$ .

1. Use the “RF” factor in the Fama-French data to convert both the factor data and the equity data into excess returns.
2. Report the mean excess return for the four factors. Are they all strongly positive? If one of the means is negative, what would that say about the pricing model?
3. Report the correlation matrix of the four factors' excess returns.
4. Use both the equity data,  $\tilde{r}$ , and factor data,  $\tilde{f}$ , to construct the tangency portfolio weights. Report the weights.
5. Is much weight put on the equity data, or is most of the weight on the factors?
6. If the factors were sufficient for pricing, what should we find?

### 3 Testing Pricing Models with Alpha

The Fundamental Theorem of Asset Pricing says that if the four factors are the pricing factors, then

$$\mathbb{E}[\tilde{r}^i] = \beta^{i,\text{mkt}}\mathbb{E}[\tilde{f}^{\text{mkt}}] + \beta^{i,\text{smb}}\mathbb{E}[\tilde{f}^{\text{smb}}] + \beta^{i,\text{hml}}\mathbb{E}[\tilde{f}^{\text{hml}}] + \beta^{i,\text{umd}}\mathbb{E}[\tilde{f}^{\text{umd}}] \quad (1)$$

Let's use regression methods to test whether the selected four pricing factors work.

For each equity security, estimate the following regression to test the 4-factor model:

$$\tilde{r}_t^i = \alpha^i + \beta^{i,\text{mkt}}\tilde{f}_t^{\text{mkt}} + \beta^{i,\text{smb}}\tilde{f}_t^{\text{smb}} + \beta^{i,\text{hml}}\tilde{f}_t^{\text{hml}} + \beta^{i,\text{umd}}\tilde{f}_t^{\text{umd}} + \epsilon_t \quad (2)$$

So you are running that regression 15 times, once for each security,  $\tilde{r}^i$ .

1. For each regression, report the estimated  $\alpha$  and r-squared.  
Note that you already did this in Homework #2, problem 3.3 for Apple and Goldman Sachs. Feel free to re-use that code!
2. Calculate the mean-absolute-error of the estimated alphas, (one for each security,  $\tilde{r}^i$ .)

$$\text{MAE} = \frac{1}{n} \sum_{i=1}^n |\hat{\alpha}^i|$$

If the pricing model worked, should these alpha estimates be large or small? Why? Based on your MAE stat, does this seem to support the pricing model or not?

---

<sup>1</sup>excluding SHV, since we already have a risk-free rate.

## 4 Testing Pricing Models with Cross-Sectional Regression

*This section will not be graded, but it will be discussed in class, and you are expected to learn it.*

Let's further evaluate whether the pricing model in (1) works. Instead of evaluating the alphas from the time-series regressions, we try to directly estimate (1). Run the following cross-sectional regression:

$$\bar{\tilde{r}}_t^i = \theta + \phi^{\text{mkt}} \hat{\beta}^{i,\text{mkt}} + \phi^{\text{smb}} \hat{\beta}^{i,\text{smb}} + \phi^{\text{hml}} \hat{\beta}^{i,\text{hml}} + \phi^{\text{umd}} \hat{\beta}^{i,\text{umd}} + v^i \quad (3)$$

This may seem like a strange regression, but it is the estimated version of (1).

- It is a cross-sectional regression, not a time-series regression. That is, we are running one single regression, with 15 data points, one for each security.
- The left-side variable is the sample average of each equity's excess return,  $\bar{\tilde{r}}^i$ .
- The right-side regressors are the estimated betas (15 sets of them) from the previous (15) time-series regressions.
- The errors, usually denoted  $\epsilon_t$  are now  $v^i$ , (one for each security.)
- This regression will return an intercept, which we are calling  $\theta$ . Usually we call the regression intercept  $\alpha$ , but that would be confusing given that we used  $\alpha$  to denote the intercepts of the time series regressions.
- This regression will return regression "betas" that we are notating  $\phi$ . So in this regression,  $X$  is actually the estimates betas from the time-series regressions in (2) and  $\beta$  is actually  $\phi$ .

Report the r-squared from (3). Also report the estimate for  $\theta$  and for the four  $\phi$ .

3. If the pricing model worked, what would we expect for this r-squared,  $\theta$ , and  $\phi$ ?
4. Is your conclusion that this model seems to work, or not?
5. Would you expect a linear factor pricing model to work better on an industry ETF, (portfolio,) or on a single-name equity?