FINM 25000 **Mark Hendricks**
Summer 2021

Homework #1

Due on Monday, June 28, at 6:00pm.

The Harvard Management Company and Inflation-indexed Bonds [HBS 9-201-053].

1 HMC's Approach

Section 1 is not graded, and you do not need to submit your answers. But you are encouraged to think about them, and we will discuss them.

1. The HMC framing of the portfolio allocation problem.

   (a) There are thousands of individual risky assets in which HMC can invest. Explain why MV optimization across 1,000 securities is infeasible.

   (b) Rather than optimize across all securities directly, HMC runs a two-stage optimization. First, they build asset class portfolios with each one optimized over the securities of the specific asset class. Second, HMC combines the asset-class portfolios into one total optimized portfolio.

   In order for the two-stage optimization to be a good approximation of the full MV-optimization on all assets, what must be true of the partition of securities into asset classes?

   (c) Should TIPS form a new asset class or be grouped into one of the other 11 classes?

2. Portfolio constraints.

   The case discusses the fact that Harvard places bounds on the portfolio allocation rather than implementing whatever numbers come out of the MV optimization problem.

   (a) How might we adjust the stated optimization problem from the lecture slides to reflect the extra constraints Harvard is using in their bounded solutions given in Exhibits 5 and 6. Just consider how we might rewrite the optimization; don't try to solve this extra-constrained optimization.

   (b) Exhibits 5 shows zero allocation to domestic equities and domestic bonds across the entire computed range of targeted returns, (5.75% to 7.25%). Conceptually, why is the constraint binding in all these cases? What would the unconstrained portfolio want to do with those allocations and why?

   (c) Exhibit 6 changes the constraints, (tightening them in most cases.) How much deterioration do we see in the mean-variance tradeoff that Harvard achieved?

2 Mean-Variance Optimization

This section is graded for a good-faith effort by your group. Submit your write-up- along with your supporting code. Don't just submit code or messy numbers; submit a coherent write-up based on your work.

- The exhibit data is in a spreadsheet posted on Canvas,but you do not need to use it; I provide it only in case you wish to do extra comparisons to the case data.
- For our analysis, we use more current data found in `multi_asset_returns.xlsx`.[1]
- The time-series data gives monthly returns for the 11 asset classes from mid-2009 to mid-2021.
- We will be working with the risky MV frontier for 11 risky asset classes, and we will use the excess-return formulation and frontier. The data you are provided has already subtracted short-term treasury returns, (a measure of the risk-free rate.) So go ahead and use these returns as **excess** returns on risky securities.

1. Summary Statistics
   (a) Calculate and display the mean and volatility of each asset's excess return. (Recall we use volatility to refer to standard deviation.)
   (b) Which assets have the best and worst Sharpe ratios?
2. The MV frontier.
   (a) Compute and display the weights of the tangency portfolios: $\boldsymbol{w}^{\text{tan}}$.
   (b) Compute the mean, volatility, and Sharpe ratio for the tangency portfolio corresponding to $\boldsymbol{w}^{\text{tan}}$.
3. The allocation.
   (a) Compute and display the weights of MV portfolios with target returns of $\mu^p = .0075$.[2]
   (b) What is the mean, volatility, and Sharpe ratio for $\boldsymbol{w}^p$?
   (c) Discuss the allocation. In which assets is the portfolio most long? And short?
   (d) Does this line up with which assets have the strongest Sharpe ratios?
4. Long-short positions.
   (a) Consider an allocation between only domestic and foreign equities. (Drop all other return columns and recompute $\boldsymbol{w}^p$ for $\mu^p = .0075$.)
   (b) Is the portfolio balanced?
   (c) Make an adjustment to $\mu^{\text{foreign equities}}$ of $+0.002$. Recompute $\boldsymbol{w}^p$ for $\mu^p = .0075$ for these two assets.
   How does the allocation among the two assets change?
   (d) Does this two-asset example raise any issues for the 11-asset problem about the statistical precision of the MV solutions?
5. Robustness

[1] The case does not give time-series data, so this data has been compiled outside of the case, and it intends to represent the main asset classes under consideration via liquid investment vehicles. For details on the specific securities/indexes, check the "Info" tab of the data.

[2] This is monthly data, so while this target looks small, it is reasonable.

(a) Recalculate the full allocation, again with the unadjusted $\mu^{\text{foreign equities}}$ and again for $\mu^p = 0.0075$. This time, make one change: in building $\boldsymbol{w}^{\text{tan}}$, do not use $\boldsymbol{\Sigma}$ as given in the formulas in the lecture. Rather, use a diaganolized $\boldsymbol{\Sigma}^D$, which zeroes out all non-diagonal elements of the full covariance matrix, $\boldsymbol{\Sigma}$.

How does the allocation look now?

(b) What does this suggest about the sensitivity of the solution to estimated means and estimated covariances?

(c) HMC deals with this sensitivity by using explicit constraints on the allocation vector. Conceptually, what are the pros/cons of doing that versus modifying the formula with $\boldsymbol{\Sigma}^D$?

6. Out-of-Sample Performance

Let's divide the sample to both compute a portfolio and then check its performance out of sample.

(a) Using only data through the end of 2018, compute $\boldsymbol{w}^p$ for $\mu^p = .0075$, allocating to all 11 assets.

(b) Calculate the portfolio's Sharpe ratio within that sample, through the end of 2018.

(c) Calculate the portfolio's Sharpe ratio based on performance in 2019-2021.

(d) How does this out-of-sample Sharpe compare to the 2009-2018 performance of a portfolio optimized to μ^p using 2009-2018 data?

7. Robust Out-of-Sample Performance

Recalculate $\boldsymbol{w}^p$ on 2009-2018 data using the diaganolized covariance matrix, $\boldsymbol{\Sigma}^D$. What is the performance of this portfolio in 2019-2021? Does it do better out of of sample than the portfolio constructed on 2009-2018 data using the full covariance matrix?