Performance Attribution and Hedging

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MLP Training: Portfolio Management

Outline

Evaluating Performance

Hedging and Tracking

Risk-adjusted performance

- Asset may have impressive $\mathbb{E}[r]$, but we want to know how this compares to risk.
- Otherwise, a fund manager might obtain high returns by loading excessively on risk.

Factor decomposition of return variation

A Linear Factor Decomposition (LFD) of $\tilde{r}^i$ onto the factor $\mathbf{x}_t$ is given by the regression,

$$\tilde{r}^i_t = \alpha + \boldsymbol{\beta}^{i,\mathbf{x}}\mathbf{x}_t + \epsilon_t$$

- The variation in returns is decomposed into the variation explained by the benchmark, x_t and by the residual, ϵ_t.
- These factors, $\mathbf{x}$, in the LFD should give a high R-squared in the regression if they really explain the variation of returns well.

Elements of the regression

Consider the following elements of the regression:

- Alpha. Expected return beyond what can be explained by the factor.
- Beta. Risk related to the factor. If x moves, how much will our return move?
- Residual. The risk of the return uncorrelated to the factor.

Interpreting alpha

Using alpha as a measure of performance is sensitive to which factors are used in the regression.

- High α will always lead to the question of whether the performance is good, or whether we used a bad model.
- Perhaps alpha is really just some missing beta from the model.
- Still, if this missing beta is not widely known or understood, it may make sense for investors to pay fees to get access to this beta knowledge, even though the fund is passively tracking a model.

Luck or skill

Estimating alpha is statistically imprecise.

- It would be easy to get a large α due to in-sample luck.
- So when faced with a large α, it may be a sign of high performance, or it may just be luck in that sample.

Treynor's Ratio

Treynor's measure is an alternative measure of the risk-reward tradeoff. For the return of asset, i,

$$\text{Treynor Ratio} = \frac{\mathbb{E}\left[\tilde{r}^i\right]}{\beta^{i,m}}$$

Information ratio

The **information ratio** refers to the Sharpe Ratio of the non-factor component of the return: $\alpha + \epsilon_t$.

$$\mathrm{IR} = \frac{\alpha}{\sigma_\epsilon}$$

where σ_ϵ and α come from

$$\tilde{r}^i_t = \alpha + \beta^{i,j}\tilde{r}^j_t + \epsilon_t$$

- α measures the excess return beyond what is explained by the factor, j.
- σ_ϵ measures the non-factor volatility.

Manipulating the benchmark

Investment managers (of hedge funds, mutual funds, etc.) might prefer to be evaluated against a benchmark which does not capture all risks.

- Hedge funds may want to be evaluated against a simple equity benchmark.
- But these funds are likely achieving excess returns by taking on other forms of risk which do not show up in the regression.

Outline

Net exposure

Investor is long \$1 of i and hedges by selling h\$ of j.

- Time t net exposure is,

$$\epsilon_t = r_t^i - h\, r_t^j$$

- This net exposure after hedging is known as **basis**.
- A position in i is perfectly hedged over horizon t if $\epsilon_t = 0$ with probability one.

Why basis?

Why cross-hedging rather than perfectly hedging with i through futures, options, etc.

- Maybe asset i is a non-tradable exposure.
- Maybe asset i is a traded security, but it has no market in futures and shorting i is costly.

Instead, the investor must hedge using asset j.

Basis risk

$$\epsilon_t = r_t^i - h\, r_t^j$$

Basis risk refers to volatility in ϵ_t, denoted as σ_ϵ.

$$\sigma_\epsilon^2 = \sigma_i^2 + h^2\sigma_j^2 - 2h\,\sigma_i\sigma_j\,\rho_{i,j}$$

- Denoted as σ_ϵ^2 because basis is the error in the hedge.
- For $\rho_{i,j} = \pm 1$, the basis risk can be eliminated.

Optimal hedge ratio

The **optimal hedge ratio**, h^*, minimizes basis risk.

$$
\begin{aligned}
h^* &= \arg\min_h \sigma_\epsilon^2 \\
&= \arg\min_h \left\{\sigma_i^2 + h^2\sigma_j^2 - 2h\,\sigma_i\sigma_j\,\rho_{i,j}\right\}
\end{aligned}
$$

Solve by taking derivative,

$$
h^* = \frac{\sigma_i}{\sigma_j}\,\rho_{i,j}
$$

- Higher correlation implies larger hedge ratio, h.
- High relative volatility of i implies larger hedge ratio.
- With negative correlation, must go long the hedging security.

Basis as a regression residual

From the previous slide, we can write

$$r_t^i = \beta^{i,j} r_t^j + \epsilon_t$$

where

$$\beta^{i,j} = h^*$$

- The optimal hedge ratio, h^*, is simply a regression beta!
- Optimized basis risk is simply the regression residual variance.
- (Thus the notation of using ϵ to denote basis.)

Hedging returns

These results also apply to hedging with multiple assets:

$$r_t^i = \beta^{i,1} r_t^1 + \beta^{i,2} r_t^2 + \ldots + \beta^{i,k} r_t^k + \epsilon_t$$

- Basis is then the net return exposure.
- Optimal hedge ratios are given by the betas in the return regression above.

Hedging excess returns

These results also apply to hedging excess returns.

$$\tilde{r}_t^i = \beta^{i,1}\tilde{r}_t^1 + \beta^{i,2}\tilde{r}_t^2 + \ldots + \beta^{i,k}\tilde{r}_t^k + \epsilon_t$$

- Optimal hedge ratios are given by the regression beta.
- Intercept is portion of mean returns which can not be replicated by the hedge strategy.

Include an intercept?

In regression for optimal hedge ratio, should we include a constant, (alpha?) Depends on our purpose...

- Do we want to explain the total return (including the mean) or simply the excess-mean return?
- In short samples, mean returns may be estimated inaccurately, (whether in r^i or $\tilde{r}^i$,) so we may want to include α (eliminate means) to focus on explaining variation.

Investment with hedging a factor

- Suppose a hedge fund wants to trade on information regarding a certain asset return, r^i.
- But does not want the trade to be subject to the overall market factor, r^m.
- More generally, imagine anyone that wants to trade on the performance of return r^i **relative** to another factor r^j.

This idea of trading on specific information while hedging out broader market movements is the origination of the term, hedge funds.

Building the market-hedged position

A hedge fund would first run the regression

$$\tilde{r}^i_t = \alpha + \beta^{i,m} \tilde{r}^m_t + \epsilon_t$$

- Then the hedge-fund can go long $\tilde{r}^i$, while shorting $\beta^{i,m}$ times the overall market.
- The fund is then holding

$$\tilde{r}^i_t - \beta^{i,m} \tilde{r}^m_t = \alpha + \epsilon_t$$

Properties of the market-hedged position

$$\tilde{r}^i_t - \beta^{i,m}\tilde{r}^m_t = \alpha + \epsilon_t$$

- This hedged position has mean excess return α; volatility σ_ϵ.
- Compared to simply going long $\tilde{r}^i$, the strategy is no longer subject to the volatility coming from $\beta^{i,m}\tilde{r}^m_t$.
- This allows the hedge fund to minimize the variance of the hedged position.

Hedging vs Tracking

- We have considered the case where an investor wants to completely hedge out some factor, r^j.
- This optimal hedging allows the investor to just trade on the portion of r^i uncorrelated with r^j: $\alpha + \epsilon$.
- Now consider a **tracking portfolio**, r^i, which tracks a factor, r^j, rather than hedging it out.

Tracking portfolios

Regress

$$\tilde{r}^i_t = \alpha + \beta \tilde{r}^j_t + \epsilon_t$$

- ϵ is known as the **tracking error** of $\tilde{r}^i$ relative to $\tilde{r}^j$.
- R-squared measures how well j tracks i.
- The Information Ratio, α/σ_ϵ, measures the tradeoff between obtaining extra mean return α at the cost of taking on tracking error ϵ from the target portfolio.

Of course, this is just another way of looking at the hedging problem.

Tracking funds and hedged funds

For broad market factors, mutual funds are often tracking some factor while hedge funds are trying to hedge it out.

$$\tilde{r}_t^i = \underbrace{\beta \tilde{r}_t^j}_{\text{mutual fund position}} + \underbrace{\alpha + \epsilon_t}_{\text{hedge fund position}}$$

for factors such as the overall market index, industry indexes, value/growth indexes, etc.

- This is not exactly true.
- Hedge funds retain some factor exposure, β, while mutual funds deviate from their benchmark.

Mutual funds benchmarks

Most mutual funds explicitly state that they track some type of benchmark. r^j may equal...

- Market index
- Value index
- Growth index
- Small stock index
- Large stock index
- Foreign equities
- AAA Corporate bonds

Along with many others...