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VECTOR CALCULUS
Test 3

Problem 1.1.
$\vec{\nabla} \cdot \vec{A} = \vec{\nabla} \cdot (\vec{\nabla} \times \vec{A}) = 0 \Rightarrow \vec{A}(\vec{r})$ is divergence-free. Hence, from its curl, $\vec{A}(\vec{r})$ can be given by $\vec{A}(\vec{r}) = z[{}^*(\vec{\nabla} \times \vec{A})] = (zx\sin(x)e^{-x^2}, -zy^2\cos(y)e^{-y^2}, 0)$

Problem 1.2.
Because $\vec{A}(\vec{r})$ is divergence-free, a possible solution is $\vec{A}(\vec{r}) = U(y,z)\hat{i}$, where $U(y,z)$ is a scalar potential for ${}^*(\vec{\nabla} \times \vec{A}) = (0, z^2, 1 + 2yz)$
$U = yz^2 + f(z) = z + yz^2 + g(y) \Rightarrow U = z + yz^2$
So $\vec{A}(\vec{r}) = (z + yz^2, 0, 0)$

Problem 1.3. $\vec{E}(\vec{r})$ is in $\mathbb{R}^2$
$\vec{E}(\vec{r}) = \dfrac{2\pi}{S_2(r)}\hat{r} = \dfrac{2\pi}{S_2(1)}\dfrac{\hat{r}}{r^{2-1}} = \dfrac{2\pi}{2\pi}\dfrac{\hat{r}}{r} = \dfrac{\hat{r}}{r}$

Problem 1.4.

$$\begin{aligned} df &= d(xdx \wedge dy) + d(ydy \wedge dz) \\ &= (dx \wedge dx \wedge dy) + (dy \wedge dy \wedge dz) \\ &= 0 \wedge dy + 0 \wedge dz = 0 \end{aligned}$$

Therefore the form f is closed

Problem 1.5.
Since $\vec{\nabla} \times \vec{E} = \vec{0}$, by Gauss' law and divergence theorem, $\vec{\nabla} \cdot \vec{E} = \dfrac{\rho}{C}$
But because $\vec{\nabla} \cdot \vec{E} = \vec{0}$, $\vec{E}$ can only be a constant vector field: $\vec{E} = (C_x, C_y, C_z)$.