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MAS 3105

Homework 11

Problem 1

1. F 2. T 3. T 4. T 5. F 6. T 7. T 8. T 9. T 10. F

1, 2: $\det(-A) = (-1)^n \det(A)$. If n is even, the given equality does not hold
3: $\det(A \cdot A^T) = \det(A) \cdot \det(A^T) = (\det(A))^2 = 1$
4: Matrix for orthogonal projection: $A(A^T \cdot A)^{-1}A^T$
5: Counterexample: $\det(I_2 + I_2) \neq \det(I_2) + \det(I_2)$
7: True by 6

Problem 2

$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 3 & 4 & 5 \end{bmatrix} \Rightarrow \det(A) = 1 \det \begin{bmatrix} 3 & 4 \\ 4 & 5 \end{bmatrix} - 2 \det \begin{bmatrix} 2 & 4 \\ 3 & 5 \end{bmatrix} + 3 \det \begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix} = 1(-1) - 2(-2) + 3(-1) = 0$

$B = \begin{bmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{bmatrix} \Rightarrow \det(A) = 2 \det \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} - 1 \det \begin{bmatrix} 1 & 1 \\ 0 & 2 \end{bmatrix} + 0 \det \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} = 2(3) - 1(2) = 4$

Problem 3

- A_1 is the result of swapping row II and row III in A. Hence, $\dfrac{\det(A_1)}{\det(A)} = -1$
- A_2 is obtained from A_1 by adding a multiple of row II to row III. Hence, the determinant does not change, and $\dfrac{\det(A_2)}{\det(A)} = -1$
- B is the result of multiplying row III by 2 in A_2. Therefore, $\dfrac{\det(B)}{\det(A)} = \dfrac{\det(B)}{\det(A_2)} \cdot \dfrac{\det(A_2)}{\det(A)} = -2$

So $\dfrac{\det(A)}{\det(B)} = -\dfrac{1}{2}$

Problem 4

$A - \lambda \cdot I_2 = \begin{bmatrix} 4-\lambda & 3 \\ -2 & -1-\lambda \end{bmatrix}$

$A - \lambda \cdot I_2$ is not invertible $\Leftrightarrow (4-\lambda)(-1-\lambda) + 6 = \lambda^2 - 3\lambda + 2 = 0$
Using the provided quadratic equation, we get $\lambda = 1$ or $\lambda = 2$