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VECTOR CALCULUS

Quiz 2

Problem 1.1. Flux of $\vec{F} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ through $x^2 + y^2 + z^2 = 1, z \geq 0$
Solution: For the unit sphere, $\hat{N} = (x, y, z)$, so $\vec{F} \cdot \hat{N} = 1$
$\iint_S \vec{F} \cdot \hat{N}\, dS = \iint_S dS = 2\pi$ (the area of the upper hemisphere)

Problem 1.2. Flux of $\vec{F} = \hat{\imath}$ through $x^2 + 4y^2 + 16z^2 = 1, z \geq 0$
Solution: Change of coordinates: $(x, y, z) \to (x, 2y, 4z)$
The ellipsoid then becomes the unit sphere, and $\vec{F} \cdot \hat{N} = x$
$\iint_S \vec{F} \cdot \hat{N}\, dS = \iint_S x\frac{1}{x}\, dy\, dz = \int_{y^2+z^2 \leq 1, z \geq 0} dy\, dz = \frac{\pi}{2}$
Change back to old coordinates, the flux is $\frac{\pi}{2} \cdot \frac{1}{2} \cdot \frac{1}{4} = \frac{\pi}{16}$

Problem 1.3. Flux of $\vec{F} = 3y\hat{\jmath} - 2\hat{k}$ through $x^2 + 4y^2 + 9z^2 = 1$
Solution: The equation of the ellipsoid can be rewritten as $x^2 + \frac{y^2}{(1/2)^2} + \frac{z^2}{(1/3)^2} = 1$, hence its volume is $\frac{4}{3}\pi abc = \frac{4}{3}\pi(1)(1/2)(1/3) = \frac{2\pi}{9}$
By Divergence Theorem,

$$\begin{aligned}
\iint_S \vec{F} \cdot \hat{N}\, dS &= \iiint_V \vec{\nabla} \cdot \vec{F}\, dV \\
&= \iiint_V 3\, dV \\
&= 3 \cdot \frac{2\pi}{9} = \frac{2\pi}{3}
\end{aligned}$$

Problem 1.4. Flux of $\vec{F} = \langle y, -x, 0 \rangle$ through $x^2 + y^2 + z^2 = 4, z > 1$
Solution: For the unit sphere, $\hat{N} = (x, y, z)$, so $\vec{F} \cdot \hat{N} = 0$
$\iint_S \vec{F} \cdot \hat{N}\, dS = \iint_S 0\, dS = 0$