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MAS 3105

Homework 10

(made on mobile)

Problem 1

1. T 2. T 3. T 4. T 5. F 6. T

Problem 2

$\begin{bmatrix}1 & 0\\1 & 0\\1 & 1\end{bmatrix}\begin{bmatrix}a\\b\end{bmatrix}=\begin{bmatrix}0\\1\\1\end{bmatrix}$

Least squares solution:

$$\begin{aligned}\begin{bmatrix}a\\b\end{bmatrix} &= (A^TA)^{-1}A^T\vec{b}\\ &= \begin{bmatrix}3 & 1\\1 & 1\end{bmatrix}^{-1}\begin{bmatrix}1 & 1 & 1\\0 & 0 & 1\end{bmatrix}\begin{bmatrix}0\\1\\1\end{bmatrix}\\ &= \frac{1}{2}\begin{bmatrix}1 & -1\\-1 & 3\end{bmatrix}\begin{bmatrix}1 & 1 & 1\\0 & 0 & 1\end{bmatrix}\begin{bmatrix}0\\1\\1\end{bmatrix}\\ &= \frac{1}{2}\begin{bmatrix}1 & 1 & 0\\-1 & -1 & 2\end{bmatrix}\begin{bmatrix}0\\1\\1\end{bmatrix}\\ &= \frac{1}{2}\begin{bmatrix}1\\1\end{bmatrix}\end{aligned}$$

So $f(x)=\frac{x}{2}+\frac{1}{2}$

Problem 3

$\begin{bmatrix}1 & 3\\2 & 6\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix}=\begin{bmatrix}1\\0\end{bmatrix}\Rightarrow\begin{bmatrix}1 & 3\\1 & 3\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix}=\begin{bmatrix}1\\0\end{bmatrix}$

Least squares solution:

$$\begin{aligned}(A^TA)\begin{bmatrix}x_1\\x_2\end{bmatrix} &= A^T\vec{b}\\ \begin{bmatrix}2 & 6\\6 & 18\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix} &= \begin{bmatrix}1\\3\end{bmatrix}\\ \Rightarrow\begin{bmatrix}x_1\\x_2\end{bmatrix} &= \begin{bmatrix}1/2\\0\end{bmatrix}\end{aligned}$$

Problem 4

$\vec{u}=\begin{bmatrix}2\\0\\0\end{bmatrix}\Rightarrow\hat{u}=\frac{\vec{u}}{||\vec{u}||}=\begin{bmatrix}1\\0\\0\end{bmatrix}$

$\vec{v}=\begin{bmatrix}3\\4\\0\end{bmatrix}\Rightarrow\vec{v_1}=\vec{v}-\frac{\langle\vec{v},\vec{u}\rangle}{\langle\vec{u},\vec{u}\rangle}\vec{u}=\begin{bmatrix}3\\4\\0\end{bmatrix}-\frac{6}{4}\begin{bmatrix}2\\0\\0\end{bmatrix}=\begin{bmatrix}0\\4\\0\end{bmatrix}\Rightarrow\hat{v}=\frac{\vec{v_1}}{||\vec{v_1}||}=\begin{bmatrix}0\\1\\0\end{bmatrix}$

$\vec{w} = \begin{bmatrix} 5 \\ 6 \\ 7 \end{bmatrix} \Rightarrow \vec{w_1} = \vec{w} - \frac{\langle \vec{w}, \vec{u} \rangle}{\langle \vec{u}, \vec{u} \rangle}\vec{u} - \frac{\langle \vec{w}, \vec{v_1} \rangle}{\langle \vec{v_1}, \vec{v_1} \rangle}\vec{v_1} = \begin{bmatrix} 5 \\ 6 \\ 7 \end{bmatrix} - \frac{10}{4}\begin{bmatrix} 2 \\ 0 \\ 0 \end{bmatrix} - \frac{24}{16}\begin{bmatrix} 0 \\ 4 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 7 \end{bmatrix}$

$\Rightarrow \hat{w} = \frac{\vec{w_1}}{||\vec{w_1}||} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

Hence the given basis has an orthonormal basis of span$(\hat{i}, \hat{j}, \hat{k})$

Problem 5

$\vec{v_1} = \begin{bmatrix} 3 \\ 4 \end{bmatrix} \Rightarrow \hat{u}_1 = \frac{\vec{v_1}}{||\vec{v_1}||} = \frac{1}{5}\begin{bmatrix} 3 \\ 4 \end{bmatrix}$

$\vec{v_2} = \begin{bmatrix} 4 \\ 5 \end{bmatrix} \Rightarrow \vec{u_2} = \vec{v_2} - \frac{\langle \vec{v_2}, \vec{v_1} \rangle}{\langle \vec{v_1}, \vec{v_1} \rangle}\vec{v_1} = \begin{bmatrix} 4 \\ 5 \end{bmatrix} - \frac{32}{25}\begin{bmatrix} 3 \\ 4 \end{bmatrix} = \frac{1}{25}\begin{bmatrix} 4 \\ -3 \end{bmatrix} \Rightarrow \hat{u}_2 = \frac{\vec{u_2}}{||\vec{u_2}||} = \frac{1}{5}\begin{bmatrix} 4 \\ -3 \end{bmatrix}$

$Q = \frac{1}{5}\begin{bmatrix} 3 & 4 \\ 4 & -3 \end{bmatrix}$

$R = Q^{-1}A = \frac{1}{5}\begin{bmatrix} 3 & 4 \\ 4 & -3 \end{bmatrix}\begin{bmatrix} 3 & 4 \\ 4 & 5 \end{bmatrix} = \begin{bmatrix} 5 & 32/5 \\ 0 & 1/5 \end{bmatrix}$