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VECTOR CALCULUS: QUIZ 1

Problem 1.1. *Compute the area of the domain bounded by the ellipse*

$$\vec{r}(t) = (a\cos(t), b\sin(t)), t \in [0, 2\pi)$$

Solution:

$$\begin{aligned}
\iint dA &= \frac{1}{2}\int_C x\,dy - y\,dx \\
&= \frac{1}{2}\int_0^{2\pi} [(a\cos(t))(b\cos(t)) - (b\sin(t))(-a\sin t)]\,dt \\
&= \frac{1}{2}\int_0^{2\pi} (ab\cos^2(t) + ab\sin^2(t))\,dt \\
&= \frac{1}{2}\int_0^{2\pi} ab\,dt \\
&= \pi ab
\end{aligned}$$

Problem 1.2. *Let C be the spiral $\vec{r}(t) = e^{-t}(\cos(t)\hat{\imath} + \sin(t)\hat{\jmath}), t \geq 0$. Compute the line integral of $\vec{F}(x, y) = y\hat{\imath} + x\hat{\jmath}$ along C, from $\vec{r}(0) = (1, 0)$ to $\vec{r}(\infty) = (0, 0)$*

Solution: $\int y\,dx = \int x\,dy = xy$
So $\vec{F} = \vec{\nabla} f$, where $f(x, y) = xy$

$$\begin{aligned}
\int_C \vec{F} \cdot d\vec{r} &= \int_C \left(\frac{\partial f}{\partial x}\,dx + \frac{\partial f}{\partial y}\,dy\right) \\
&= f(0, 0) - f(1, 0) = 0
\end{aligned}$$