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MAS 3105

Homework 6

Problem 1 All 7 statements are True

Problem 2

For a matrix $M = \left(\begin{bmatrix}\vec{v_1} & \vec{v_2} & ... & \vec{v_m}\end{bmatrix}\right)$, $\text{im}(M) = \mathbb{R}^3$ iff $\text{span}(\vec{v_1}, \vec{v_2}, ..., \vec{v_m}) = \mathbb{R}^3$

For A, $\text{span}\left(\begin{bmatrix}1\\2\\0\end{bmatrix}, \begin{bmatrix}0\\1\\1\end{bmatrix}, \begin{bmatrix}1\\-1\\-3\end{bmatrix}\right) \neq \mathbb{R}^3$ because $\begin{bmatrix}1\\-1\\-3\end{bmatrix} = \begin{bmatrix}1\\2\\0\end{bmatrix} - 3\begin{bmatrix}0\\1\\1\end{bmatrix}$

For B, $\text{span}\left(\begin{bmatrix}10086\\2048\\3105\end{bmatrix}, \begin{bmatrix}0\\1\\2\end{bmatrix}\right) \neq \mathbb{R}^3$

For C, $\text{span}\left(\begin{bmatrix}1\\2\\0\end{bmatrix}, \begin{bmatrix}0\\1\\1\end{bmatrix}, \begin{bmatrix}1\\-1\\-3\end{bmatrix}, \begin{bmatrix}1\\3\\1\end{bmatrix}\right) \neq \mathbb{R}^3$ because $\begin{bmatrix}1\\3\\1\end{bmatrix} = \begin{bmatrix}1\\2\\0\end{bmatrix} + \begin{bmatrix}0\\1\\1\end{bmatrix}$

For D, $\text{span}\left(\begin{bmatrix}1\\2\\0\end{bmatrix}, \begin{bmatrix}0\\1\\1\end{bmatrix}, \begin{bmatrix}1\\-1\\-2\end{bmatrix}\right) = \mathbb{R}^3$ because these 3 vectors are linearly independent

Hence D is the only matrix where $\text{im}(D) = \mathbb{R}^3$

Problem 3

1. $\vec{v_1}$ and $\vec{v_2}$ are linearly independent
2. These are not linearly independent: $\vec{v_3} = 675\vec{v_1} + 674\vec{v_2}$
3. These are not linearly independent: $\vec{v_3} = 2\vec{v_1} + \vec{v_2}$
4. These are not linearly independent: $\vec{v_2} = 2\vec{v_1}$

Problem 4

$\vec{v_3} \notin \text{span}(\vec{v_1}, \vec{v_2})$, hence $\text{span}(\vec{v_1}, \vec{v_2}) \neq \text{span}(\vec{v_1}, \vec{v_2}, \vec{v_3})$

Problem 5

$A = \begin{bmatrix}1 & 2 & 3\\2 & 3 & 4\\3 & 5 & 7\end{bmatrix} = \begin{bmatrix}\vec{v_1} & \vec{v_2} & \vec{v_3}\end{bmatrix}$

We have $\vec{v_3} = 2\vec{v_2} - \vec{v_1}$, which implies $\text{im}(A) = \text{span}(\vec{v_1}, \vec{v_2})$

$\begin{bmatrix}1\\2\\3\end{bmatrix} \times \begin{bmatrix}2\\3\\5\end{bmatrix} = \begin{bmatrix}1\\1\\-1\end{bmatrix} \notin \text{span}(\vec{v_1}, \vec{v_2})$, so $\begin{bmatrix}1\\1\\-1\end{bmatrix} \notin \text{im}(A)$