Calculus | AI Bootcamp

Diff-exercises

1. $y = \frac{1}{x} + \sqrt{x} - \frac{1}{\sqrt{x}}$

$$y' = -\frac{1}{x^2} + \frac{1}{2\sqrt{x}} - \left(-\frac{1}{2}\cdot x^{-\frac{3}{2}}\right) = -\frac{1}{x^2} + \frac{1}{2\sqrt{x}} + \frac{1}{2x^{3/2}}$$

3. $y = \ln(\sqrt{5x^2-4}) = \frac{1}{2}\ln(5x^2-4)$

$$y' = \frac{1}{2}\cdot\left(\frac{1}{5x^2-4}\cdot 5\cdot 2x\right) = \frac{5x}{5x^2-4}$$

4. $y = \left(e^{x^2+2}\right)^2 = e^{2\cdot(x^2+2)} = e^{2x^2+4}$

$$y' = 4x\cdot e^{2x^2+4}$$

7. $y = \frac{\sqrt{\ln x}}{e^{2x}} = e^{-2x}\cdot\sqrt{\ln(x)}$

$$y' = (e^{-2x})'\cdot\sqrt{\ln x} + e^{-2x}\cdot(\sqrt{\ln x})' =$$

$$= -2e^{-2x}\cdot\sqrt{\ln x} + e^{-2x}\cdot\frac{1}{2\sqrt{\ln(x)}}\cdot\frac{1}{x} =$$

$$= -2e^{-2x}\sqrt{\ln x} + \frac{e^{-2x}}{2x\sqrt{\ln x}} =$$

$$= -\frac{e^{-2x}(4x\ln(x)-1)}{2x\sqrt{\ln(x)}}$$

12. $y = \frac{e^{3x}}{x} + x\ln x$

$$y' = \frac{3e^{3x}\cdot x - e^{3x}}{x^2} + \ln x + \frac{x}{x} =$$

$$= \ln(x) + \frac{3xe^{3x} - e^{3x}}{x^2} + 1$$

14*. $x^3 + x^2y + y^2 - x = 0$

$$\frac{d}{dx}(x^3 + x^2y + y^2 - x) = \frac{d}{dx}(0)$$

$$3x^2 + 2xy + \frac{dy}{dx} \cdot x^2 + 2y\frac{dy}{dx} - 1 = 0$$

$$3x^2 + 2xy + \frac{dy}{dx}(x^2 + 2y) - 1 = 0$$

$$\frac{dy}{dx} = \frac{-3x^2 + 2xy + 1}{x^2 + 2y}$$

30. $y = e^{-x^2} + \ln x^4$

$$y' = e^{-x^2} \cdot (-2x) + \frac{4x^3}{x^4} =$$

$$= \frac{4}{x} - 2xe^{-x^2}$$

32. $y = 5x^3 - 3x + \frac{2}{x} - \sqrt{x} + 3$

$$y' = 15x^2 - 3 - \frac{2}{x^2} - \frac{1}{2\sqrt{x}} =$$

$$= 15x^2 - \frac{2}{x^2} - \frac{1}{2\sqrt{x}} - 3$$

47. $y = \ln\left[e^x \cdot e^{\sqrt{x}}\right] = x + \sqrt{x}$

$$y' = \frac{1}{2\sqrt{x}} + 1$$

86*. $$xy = (x-y)^2 + 1$$

$$\frac{d}{dx}(xy) = \frac{d}{dx}\left((x-y)^2 + 1\right)$$

$$y + x \cdot \frac{dy}{dx} = 2(x-y) \cdot \left(1 - \frac{dy}{dx}\right)$$

$$x\frac{dy}{dx} + y = 2(y-x) \cdot \left(\frac{dy}{dx} - 1\right)$$

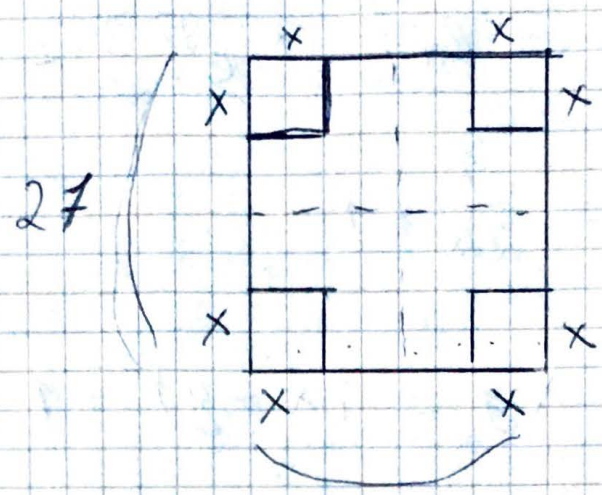
$$x\frac{dy}{dx} + y = 2y\frac{dy}{dx} - 2x\frac{dy}{dx} - 2y + 2x$$

$$x\frac{dy}{dx} + 2x\frac{dy}{dx} - 2y\frac{dy}{dx} = -3y + 2x$$

$$\frac{dy}{dx} = \frac{3y - 2x}{2y - 3x}$$

opt-exercises

1.



$0 \leq x \leq \frac{27}{2} = 13.5$

$V(x) = (27-2x)(27-2x) \cdot x$

$V(x) = (729 - 108x + 4x^2) \cdot x$

$V(x) = 729x - 108x^2 - 4x^3$

$V'(x) = 729 - 216x - 12x^2$

$-12x^2 - 216x + 729 = 0 \quad (\cdot -1)$

$12x^2 + 216x - 729 = 0$

$D = 216^2 - 4 \cdot (-729) \cdot 12 = 81648$

$\sqrt{D} \simeq 285.74$

$x_1 = 2.9$

$x_2 = -20.9 < 0 \Rightarrow$ (can't be

$\Downarrow$

$x = 2.9$ (max. point)

Dimensions: $l = 21.2$ $w = 21.2$ $h = 2.9$

4. $x + y = 120$ $\quad x > 0$, $y > 0$

maximize $x^2 \cdot y$

$f = x^2 \cdot y \qquad y = 120 - x$

$f = x^2(120 - x) = 120x^2 - x^3$

$f' = 240x - 3x^2$

$240x - 3x^2 = 0$

$x(240 - 3x) = 0$

$x = 0$ x can't be 0

$3x = 240$

$x = 80$

$\Rightarrow \quad x = 80, \; y = 40$

12. 480 m of fencing

$480 = 2 \cdot (\ell + h) + h = 2\ell + 3h$

$A = \ell \cdot h \qquad 2\ell = 480 - 3h$

$\ell = \frac{480 - 3h}{2}$

$A = \frac{480 - 3h}{2} \cdot h = \frac{480h - 3h^2}{2}$

$$A' = 240 - 3h$$

$$240 - 3h = 0$$

$$h = 80 \Rightarrow l = 120$$

$$A = 9600 \ m^2$$