Linear Algebra / AI Bootcamp

matrix operations

1.

$$A = \begin{bmatrix} 1 & 5 \\ 8 & 6 \\ -6 & 5 \end{bmatrix} \quad B = \begin{bmatrix} -6 & 7 \\ -1 & 4 \\ 3 & 0 \end{bmatrix}, \text{ find } A + B$$

$$A + B = \begin{bmatrix} -5 & 12 \\ 7 & 10 \\ -3 & 5 \end{bmatrix}$$

2.

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 7 & 9 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 4 & -6 & 0 \\ 3 & 10 & -2 \end{bmatrix}, \text{ find } A - B$$

$$A - B = \begin{bmatrix} -2 & 7 & 1 \\ 4 & -1 & 2 \end{bmatrix}$$

3.

$$A = \begin{pmatrix} 5 & 1 & 1 \\ -1 & 4 & 3 \end{pmatrix} \quad B = \begin{pmatrix} 24 & -6 \\ 6 & -5 \\ 0 & 2 \end{pmatrix}, \text{ find } A^T + B$$

$$A^T + B = \begin{pmatrix} 5 & -1 \\ 1 & 4 \\ 1 & 3 \end{pmatrix} + \begin{pmatrix} 24 & -6 \\ 6 & -5 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} 29 & -7 \\ 7 & -1 \\ 1 & 5 \end{pmatrix}$$

4. $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$ $B = \begin{pmatrix} 1 & -1 \\ 2 & -3 \end{pmatrix}$, find $A^T \cdot B$

$$A^T \cdot B = \begin{pmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{pmatrix} \cdot \begin{pmatrix} 1 & -1 \\ 2 & -3 \end{pmatrix} = \begin{pmatrix} 9 & -13 \\ 12 & -17 \\ 15 & -21 \end{pmatrix}$$

5. $A = \begin{pmatrix} -1 & 2 \\ 4 & 1 \end{pmatrix}$ $B = \begin{pmatrix} 1 & -1 & 1 \\ 2 & -3 & 0 \end{pmatrix}$, find $B \cdot B^T - A$

$$B \cdot B^T = \begin{pmatrix} 1 & -1 & 1 \\ 2 & -3 & 0 \end{pmatrix} \cdot \begin{pmatrix} 1 & 2 \\ -1 & -3 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 3 & 5 \\ 5 & 13 \end{pmatrix}$$

$$B \cdot B^T - A = \begin{pmatrix} 3 & 5 \\ 5 & 13 \end{pmatrix} - \begin{pmatrix} -1 & 2 \\ 4 & 1 \end{pmatrix} = \begin{pmatrix} 4 & 3 \\ 1 & 12 \end{pmatrix}$$

6. $A = \begin{pmatrix} -1 & 3 \\ 0 & 1 \end{pmatrix}$ $B = \begin{pmatrix} 5 & -1 & 4 \\ 1 & -2 & -1 \end{pmatrix}$, find $A^TA + BB^T$

$$A^TA = \begin{pmatrix} -1 & 0 \\ 3 & 1 \end{pmatrix} \cdot \begin{pmatrix} -1 & 3 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & -3 \\ -3 & 10 \end{pmatrix}$$

$$B \cdot B^T = \begin{pmatrix} 5 & -1 & 4 \\ 1 & -2 & -1 \end{pmatrix} \cdot \begin{pmatrix} 5 & 1 \\ -1 & -2 \\ 4 & -1 \end{pmatrix} = \begin{pmatrix} 42 & 3 \\ 3 & 6 \end{pmatrix}$$

$$A^TA + B \cdot B^T = \begin{pmatrix} 1 & -3 \\ -3 & 10 \end{pmatrix} + \begin{pmatrix} 42 & 3 \\ 3 & 6 \end{pmatrix} = \begin{pmatrix} 43 & 0 \\ 0 & 16 \end{pmatrix}$$

7. $A = \begin{pmatrix} 3 & 2 \\ -1 & 1 \end{pmatrix}$, find A^{-1}

$$\begin{pmatrix} 3 & 2 \\ -1 & 1 \end{pmatrix}^{(-1)} = \frac{1}{3 \cdot 1 - 2 \cdot (-1)} \cdot \begin{pmatrix} 1 & -2 \\ 1 & 3 \end{pmatrix} =$$

$$= \begin{pmatrix} \frac{1}{5} & -\frac{2}{5} \\ \frac{1}{5} & \frac{3}{5} \end{pmatrix}$$

8. $A = \begin{pmatrix} -1 & 1 \\ 2 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 3 & 1 & 2 \\ 1 & 1 & 0 \end{pmatrix}$,

$C = \begin{pmatrix} 1 & 0 & 5 \\ 2 & -2 & 1 \end{pmatrix}$, find $(A^{-1}B)^T + C^T$

$$A^{-1} = \frac{1}{-1 \cdot 1 - 1 \cdot 2} \cdot \begin{pmatrix} 1 & -1 \\ -2 & -1 \end{pmatrix} = \begin{pmatrix} -\frac{1}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{1}{3} \end{pmatrix}$$

$$A^{-1} \cdot B = \begin{pmatrix} -\frac{1}{3} & \frac{1}{3} \\ \frac{2}{3} & \frac{1}{3} \end{pmatrix} \cdot \begin{pmatrix} 3 & 1 & 2 \\ 1 & 1 & 0 \end{pmatrix} = \begin{pmatrix} -2/3 & 0 & -2/3 \\ 7/3 & 1 & 4/3 \end{pmatrix}$$

$$(A^{-1} \cdot B)^T = \begin{pmatrix} -2/3 & 7/3 \\ 0 & 1 \\ -2/3 & 4/3 \end{pmatrix}$$

$$(A^{-1} \cdot B)^T + C^T = \begin{pmatrix} -2/3 & 7/3 \\ 0 & 1 \\ -2/3 & 4/3 \end{pmatrix} + \begin{pmatrix} 1 & 2 \\ 0 & -2 \\ 5 & 1 \end{pmatrix} =$$

$$= \begin{pmatrix} 1/3 & 13/3 \\ 0 & -1 \\ 13/3 & 7/3 \end{pmatrix}$$

9. $V = \begin{pmatrix} x \\ y \end{pmatrix}$, $A = (3x \quad 2y^2)$, find A'_V

$$A'_V = \begin{pmatrix} \frac{d}{dx}(3x) & \frac{d}{dx}(2y^2) \\ \frac{d}{dy}(3x) & \frac{d}{dy}(2y^2) \end{pmatrix} = \begin{pmatrix} 3 & 0 \\ 0 & 4y \end{pmatrix}$$

10. $V = \begin{pmatrix} x \\ y \end{pmatrix}$, $A = (5x \quad 2y)$, $B = \begin{pmatrix} 3 \\ y+5 \end{pmatrix}$

find $(AB)'_V$

$$AB = (5x \quad 2y) \cdot \begin{pmatrix} 3 \\ y+5 \end{pmatrix} = 5 \cdot 3x + 2y(y+5) =$$

$$= 15x + 2y^2 + 10y$$

$$(AB)'_V = \begin{pmatrix} \frac{d}{dx}(15x + 2y^2 + 10y) \\ \frac{d}{dy}(15x + 2y^2 + 10y) \end{pmatrix} = \begin{pmatrix} 15 \\ 4y + 10 \end{pmatrix}$$