Additional Elaborations on the Challenge of *k*

Figure S1 shows how the cost-adjusted demand curve is affected while α is fixed and k and Q_0 vary (also see Gilroy et al., 2020). The top-left panel of Figure S1 demonstrates the normalization (all points line up with a slope of -1 ins log-log space to Q_0 of one) when k is set to a common value. This also corresponds with the same α having the same peak expenditure shown in the bottom-left panel of Figure S1. However, when k is not fixed, the $P_{\max}$ values no longer align. The consumption curve no longer has $P_{\max}$ lie on the same line (top-right panel of Figure 2) and thus α no longer corresponds to a common $O_{\max}$ (bottom-right panel of Figure 2). *This is the major challenge with how k is used within and between analyses* (i.e., comparing commodities). If k is fixed within an analysis, then α values are directly interpretable within the HS framework of *EV*, but only within that analysis. If k values are free to vary to improve model fits within an analysis, α values are no longer directly comparable within the HS framework. This also becomes a problem across analyses, as α values across studies and commodities cannot be directly compared if a common k was not used.

References

Gilroy, S. P., Kaplan, B. A., & Reed, D. D. (2020). Interpretation(s) of elasticity in operant demand. *Journal of the Experimental Analysis of Behavior*, *114*(1), 106–115. https://doi.org/10.1002/jeab.610

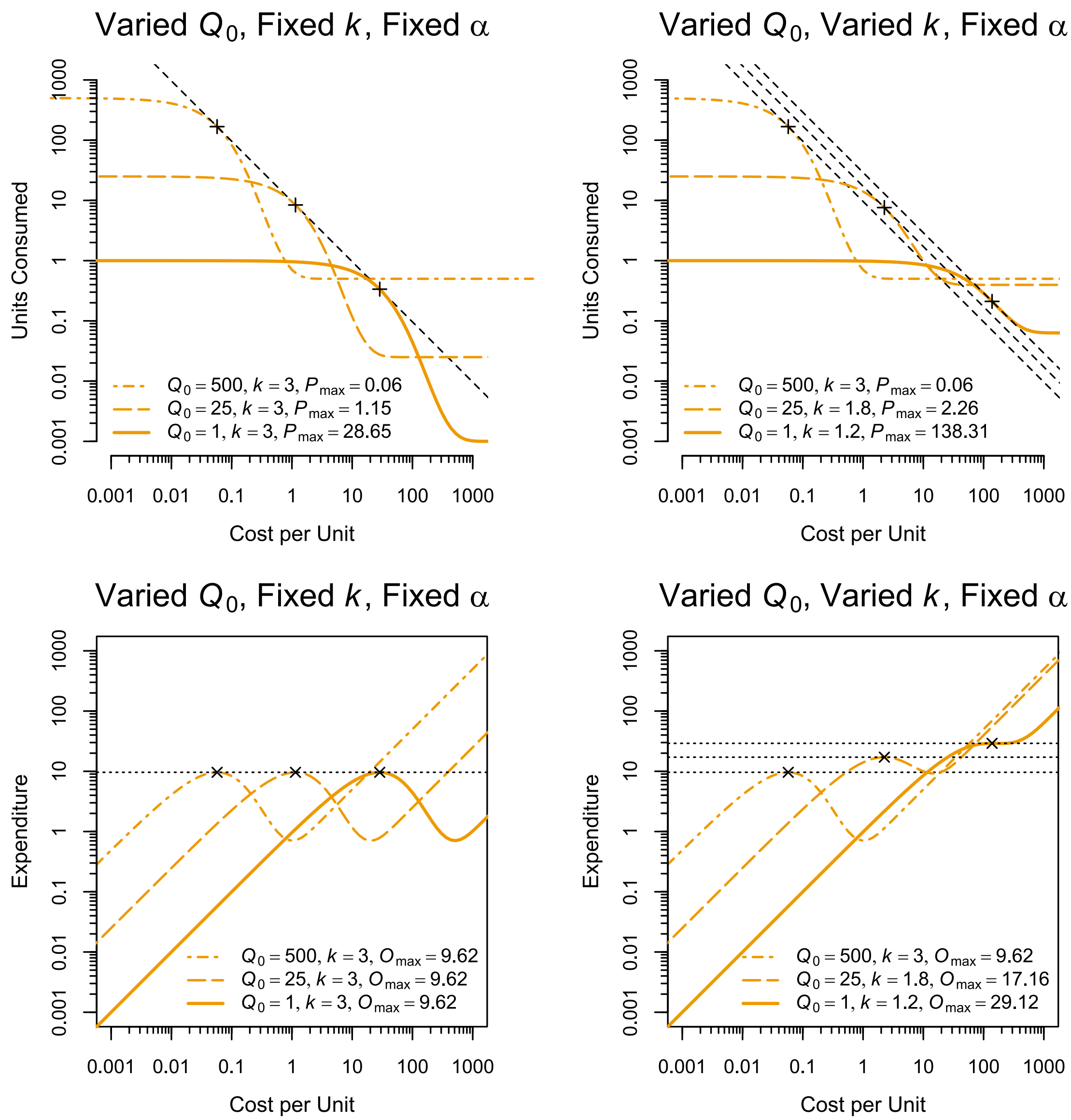

Figure S1. Demonstrations of the holding k constant (left column) and changing k with varying Q_0 values and a fixed α values. Top row are consumption curves, bottom row are expenditure curves (cost times consumption). Values of α are .006 for all panels, with Q_0 values of 500 (dot-dash line), 25 (long-dash line), and 1 (solid line). Short-dashed black lines indicate slope of -1 in log-log space. Dotted black lines indicate peak expenditure (O_{max}). Crosses indicate P_{max} on the consumption curves which was determined using Gilroy et al.'s (2019) solution. Xs indicate peak O_{max} at P_{max} on expenditure curves. Both axes are log-scaled.

Simplified Exponential with Standardized Data

Figure S2 demonstrates the same standardization as in Figure 2, but with the simplified exponential with normalized decay. Notice the same patterns as HS models.

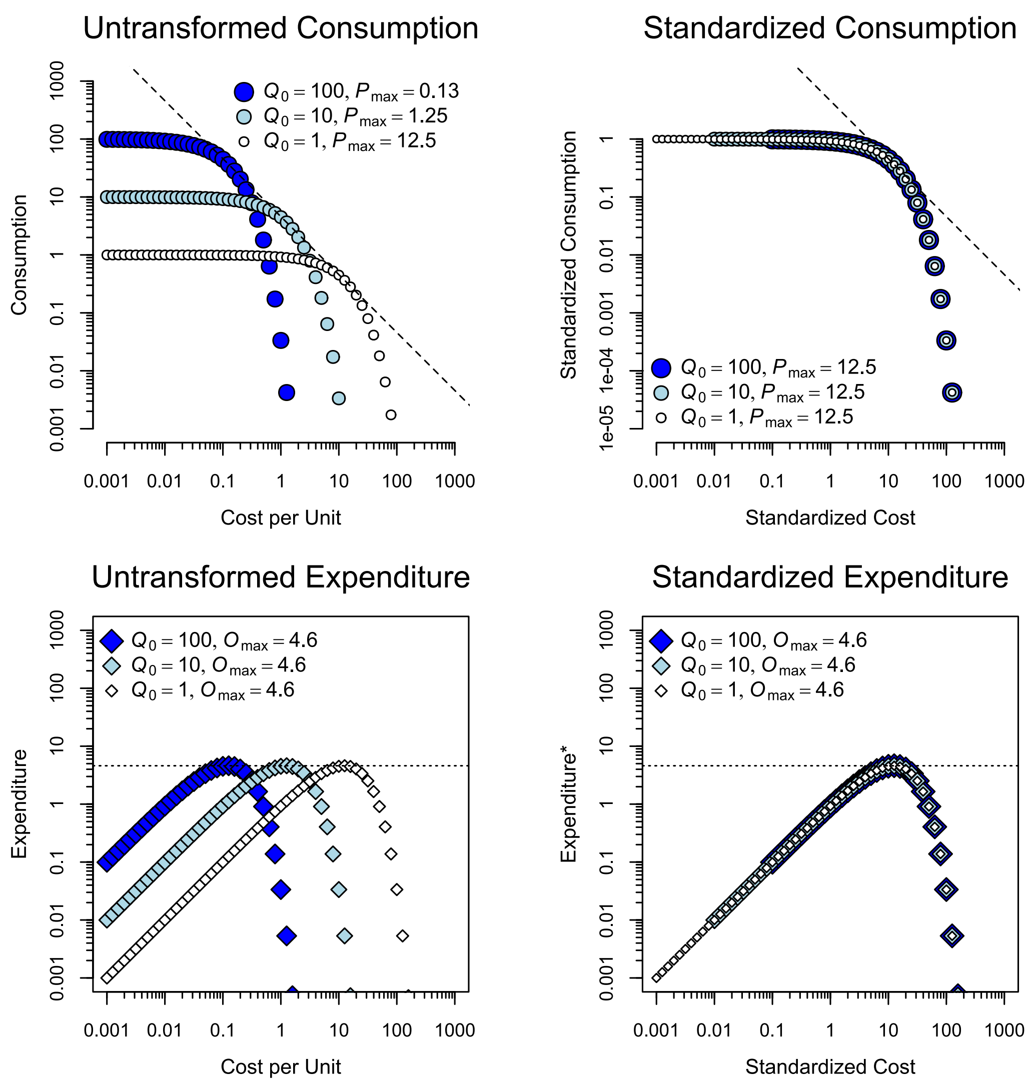

Figure S2. Demonstrations of applying standardization to data and its effect on P_{max} and O_{max} on the simplified exponential model with normalized decay. Untransformed consumption data with the same α and different Q_0 values (top left), standardized consumption and standardized price (top right), untransformed expenditure data (bottom left), and standardized expenditure and standardized cost (bottom right). The standardization consists of dividing consumption/expenditure by the respective Q_0 for a data series (i.e., vertical shift) and multiplying the unit price for the data by its respective Q_0 (i.e., horizontal shift). In the bottom right panel, the y-axis is labeled "Expenditure*" to denote that standardized expenditure is the same as untransformed expenditure when standardized cost and standardized consumption is used to calculate standardized expenditure.

Additional Elaborations on the Logic of the HS-to-SND Conversions

The goal for the conversion function was to find when P_{maxSND} and $EC_{1/e\text{HS}}$ would become equivalent between the two equations, but simple calculations between the two indicated that the two equations were not equal (Equation S1).

$$\frac{1}{\alpha_{SND} \cdot Q_0} \neq \frac{\ln\left(\frac{\log_b(1/e)}{k} + 1\right)}{-\alpha_{HS} \cdot Q_0} \tag{S1}$$

Based on previous attempts to identify an *EV* from the HS model of equations that wash away the influence of *k* described earlier, we assumed a direct relationship between *EC* of 1/*e* for the SND (i.e., P_{maxSND}) and an *EC* of 1/*e* for HS, and when α_{HS} and some factor for a given base, F_b, replaces the α_{SND} in the P_{maxSND}, resulting in Equation S2.

$$\frac{1}{\alpha_{HS} \cdot F_b \cdot Q_0} = \frac{\ln\left(\frac{\log_b(1/e)}{k} + 1\right)}{-\alpha_{HS} \cdot Q_0} \tag{S2}$$

Which after solving for F_b becomes Equation S3.

$$F_b = \frac{-1}{\ln\left(1 - \frac{1}{k\ln(b)}\right)} \tag{S3}$$

Thus, the conversion factor F_b between the SND and HS models is Equation S3. Now, based on this F_b, we can solve for the value required to change α_{HS} to α_{SND} and vice versa. If P_{maxSND} and the lefthand-side of Equation S2 are equal, this results in Equation S4.

$$\frac{1}{\alpha_{SND} \cdot Q_0} = \frac{1}{\alpha_{HS} \cdot F_b \cdot Q_0} \tag{S4}$$

Therefore, conversions between the two decay parameters can be found by solving for the respective α values, which after expanding and simplifying for the SND becomes Equation S5.

$$\alpha_{SND} = \alpha_{HS} \cdot F_b = \frac{-\alpha_{HS}}{\ln\left(1 - \frac{1}{k \cdot \ln(b)}\right)} \tag{S5}$$

Or should the inverse direction be desired, Equation S6 can be used.

$$\alpha_{HS} = \frac{\alpha_{SND}}{F_b} = -\alpha_{SND} \cdot \ln\left(1 - \frac{1}{k \cdot \ln(b)}\right) \tag{S6}$$

To demonstrate this conversion visually, Figure S3 has three sets of consumption and expenditure data. In the left column, α for both the HS and SND equations is set to 0.1. In this case, the cost identified as the *EC* with a proportion of 1/*e* for both models occur at different *C* values. In the center column, when applying Equation S5 to the α_{HS} and the plotting the curves, the *EC* values with a proportion of 1/*e* now occur at the SND P_{max} (i.e., converting α_{HS} to α_{SND}). In the right column Equation S6 is used, resulting in the SND *EC* values (i.e., P_{maxSND}) to line up with the HS *EC* values.

It should be noted that HS version of *EC* finds the *C* value where consumption decreases to some proportion of Q_0 but does not take into consideration $Q_{LowerLimit}$. That is, if *k* is not large enough to set $Q_{LowerLimit}$ below the desired proportion of Q_0 then *EC* will not be solvable. To specifically convert between values as presented in Table 2, these conversions are possible so long as $k > 1/\ln(b)$, as *k* values lower than that do not allow for the curve to reach a proportion of Q_0 that is approximately 0.3678794, or 1/*e*. While this conversion technically works at very low *k* values, if *k* was empirically determined to be around $1/\ln(b)$, a horizontal line might be more appropriate to use rather than an exponential function. What appears to be occurring with the conversion between the two models is that the conversion factor, F_b, allows for the point EC_p would have occurred the HS model should *k* have been infinity. While not shown here, when *k* is low but $P_{\max}$ solvable for the HS model, it is shifted to the right of $P_{\max}$ for the SND model. As k increases, the difference between $P_{\max\mathrm{HS}}$ and $P_{\max\mathrm{SND}}$ decreases.

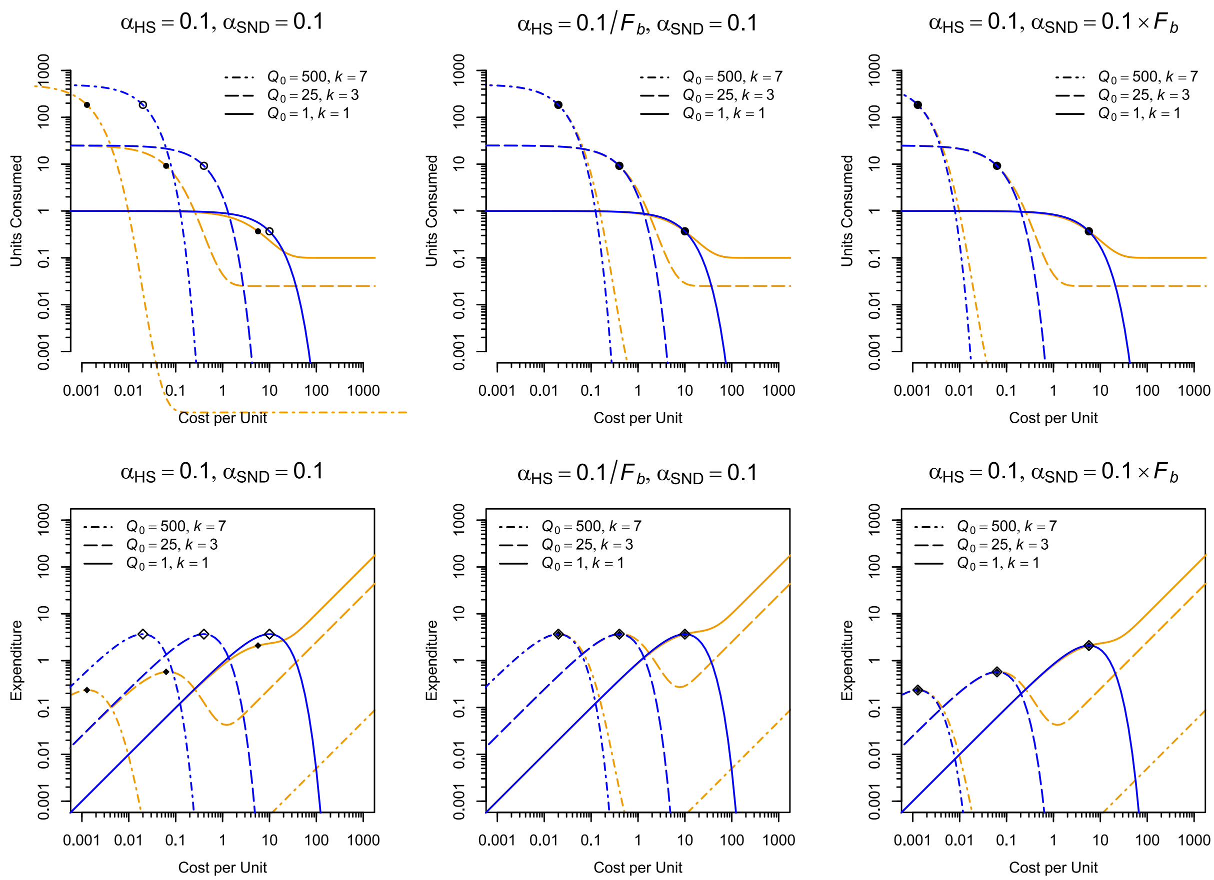

Figure S3. Demonstrations of the conversion between the HS (orange, light grey) and SND (blue, dark grey) models. Top row are consumption curves, bottom row are expenditure curves (cost times consumption). Short-dash lines indicate slope of -1 in log-log space. Q_0 values are 500, 25, and 1 for both equations with corresponding k values of 7, 3, and 1 respectively for the HS equation. Filled circles indicate $EC_{1/e\mathrm{HS}}$ and open circles indicate $P_{\max\mathrm{SND}}$. Left column are the HS and SND equations with α_{HS} and α_{SND} both set to 0.1. Center column has HS model α_{HS} drawn with conversion to α_{SND}. Center column has SND model α_{SND} drawn with conversion to α_{HS}.

Additional Model Fits and Correlations

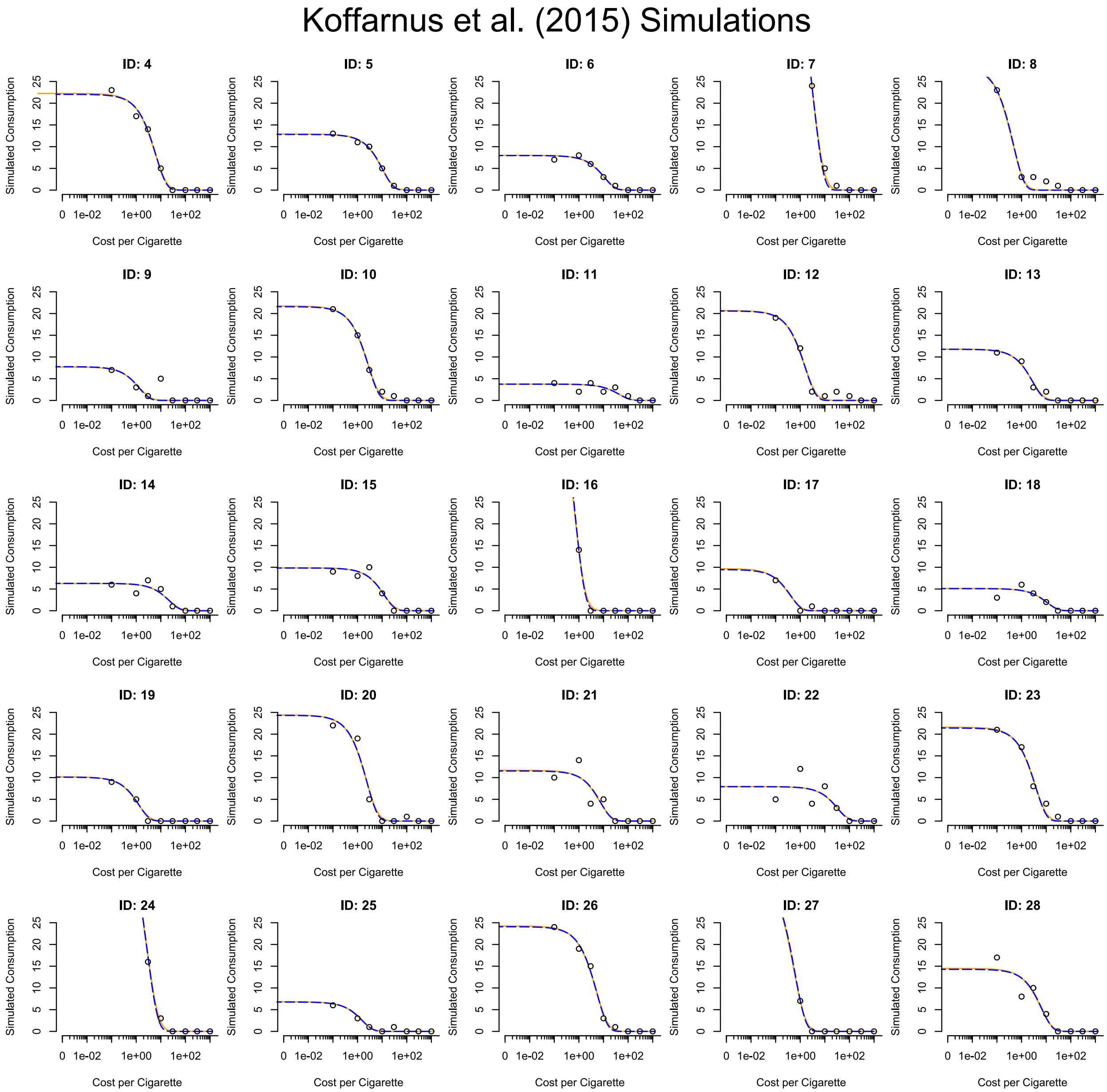

Figure S4. Random effects estimates from the first fourth to 28th simulations from the Koffarnus et al. (2015) data. Dashed blue (dark grey) lines indicate fits from the SND model, whereas solid orange (light grey) lines indicate fits from the EXPD model. Y-axes are in linear scaling whereas the x-axis is log-scaled. Purchasing at zero has been placed at .001 so they can be included on the log scale.

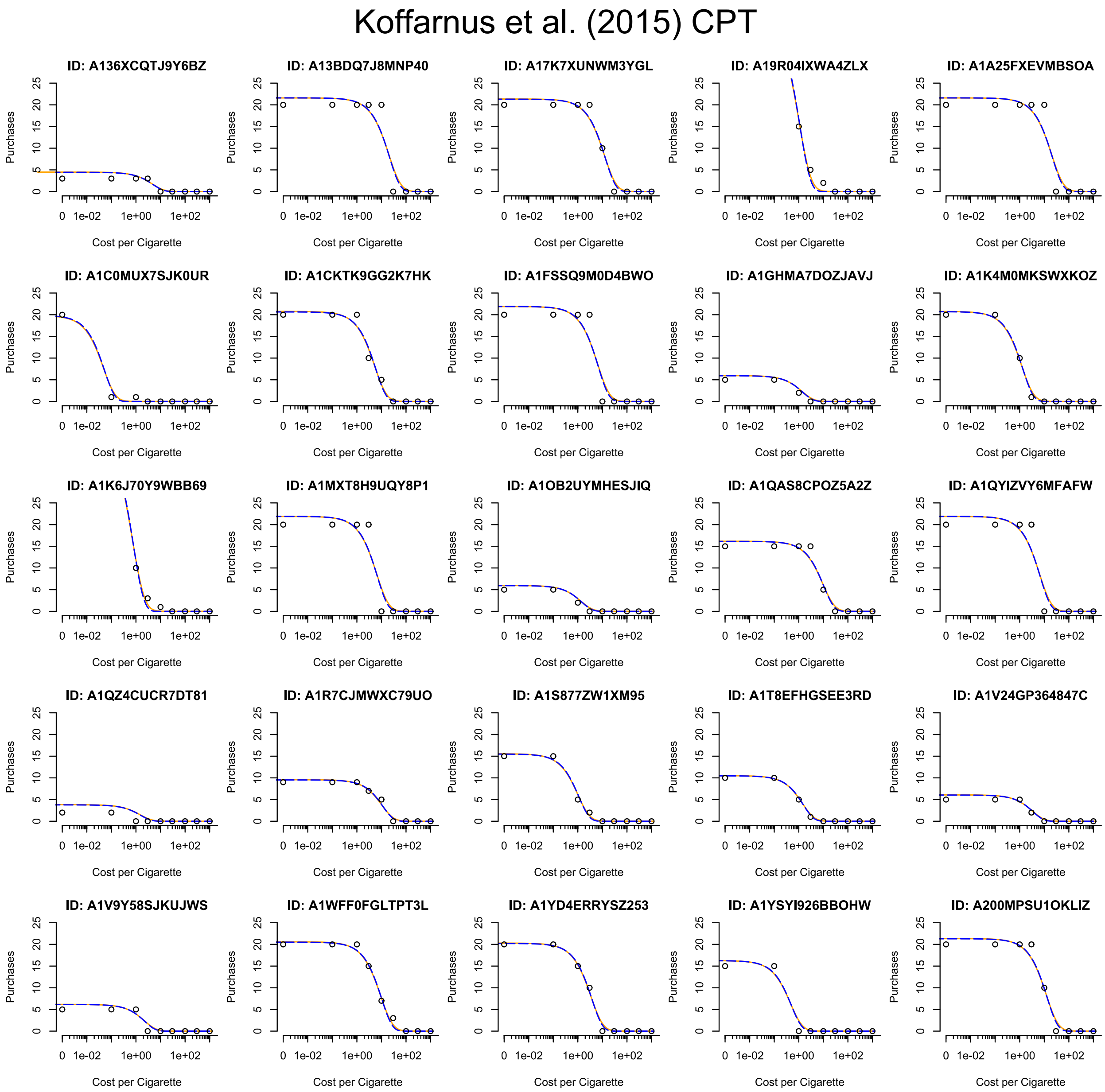

Figure S5. Random effects estimates from the first fourth to 28th participants from the Koffarnus et al. (2015) data. Dashed blue (dark grey) lines indicate fits from the SND model, whereas solid orange (light grey) lines indicate fits from the EXPD model. Y-axes are in linear scaling whereas the x-axis is log-scaled. Purchasing at zero has been placed at .001 so they can be included on the log scale.

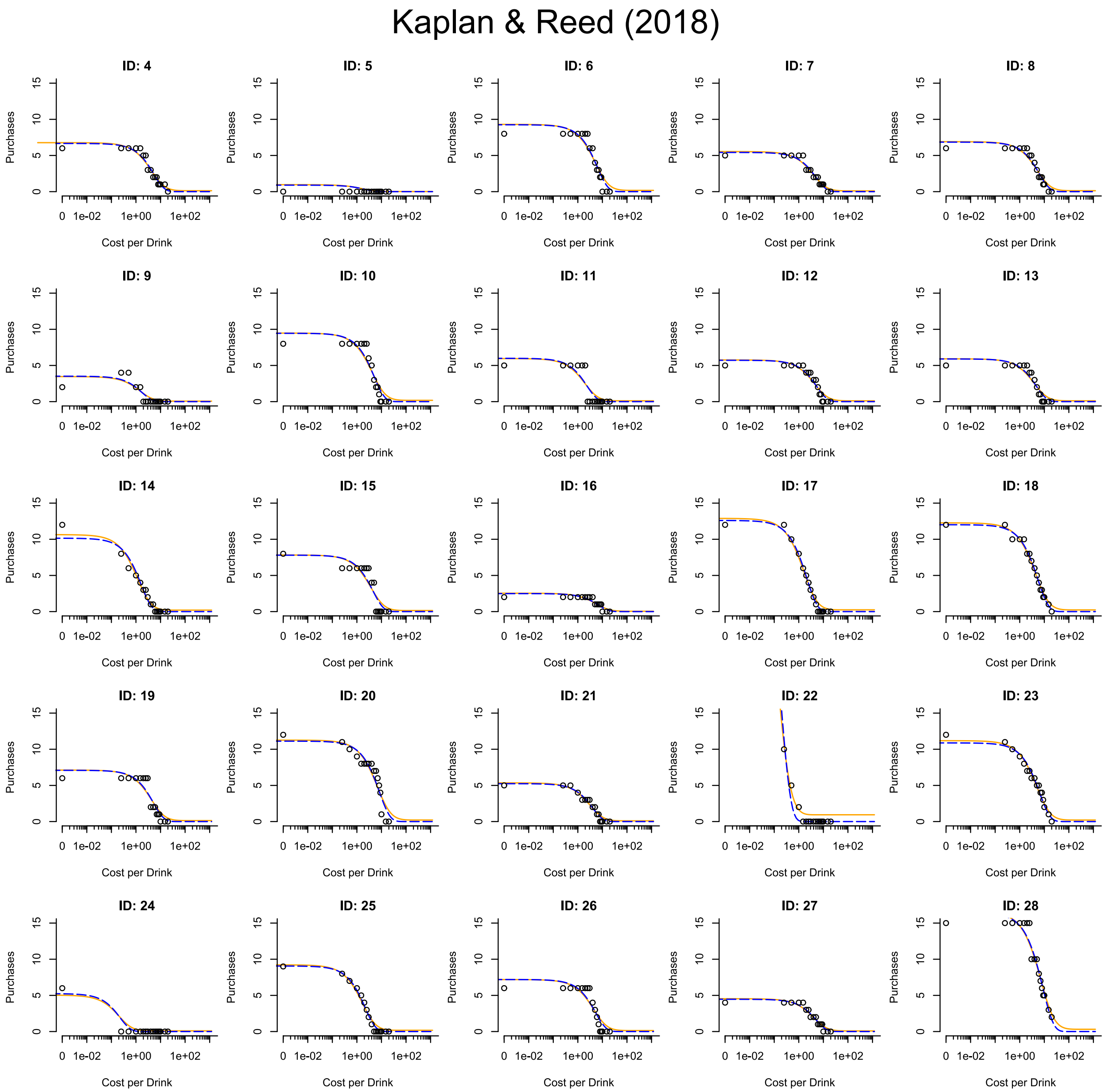

Figure S6. Random effects estimates from the first fourth to 28th participants from the Kaplan & Reed (2018) data. Dashed blue (dark grey) lines indicate fits from the SND model, whereas solid orange (light grey) lines indicate fits from the EXPD model. Y-axes are in linear scaling whereas the x-axis is log-scaled. Purchasing at zero has been placed at .001 so they can be included on the log scale.

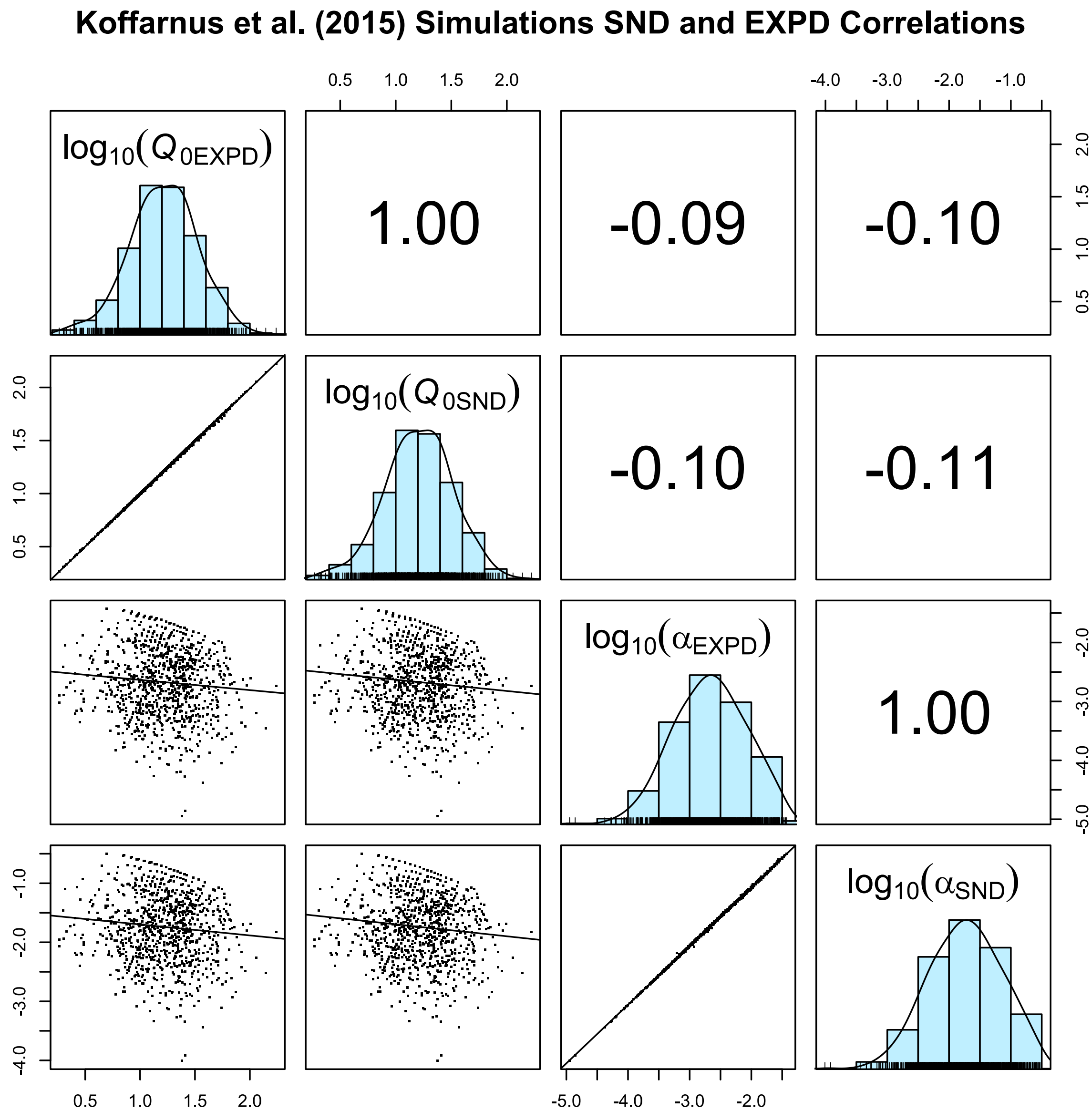

Figure S7. Comparisons between parameters estimated from the EXPD and SND from the Koffarnus et al (2015) CPT data. Parameters estimated are $\log_{10}$ transformed. This was done for each to help approximate normality. Scatterplots are on the bottom of the diagonal, Pearson correlations are on the top of the diagonal, with distributions as histograms on the diagonal.

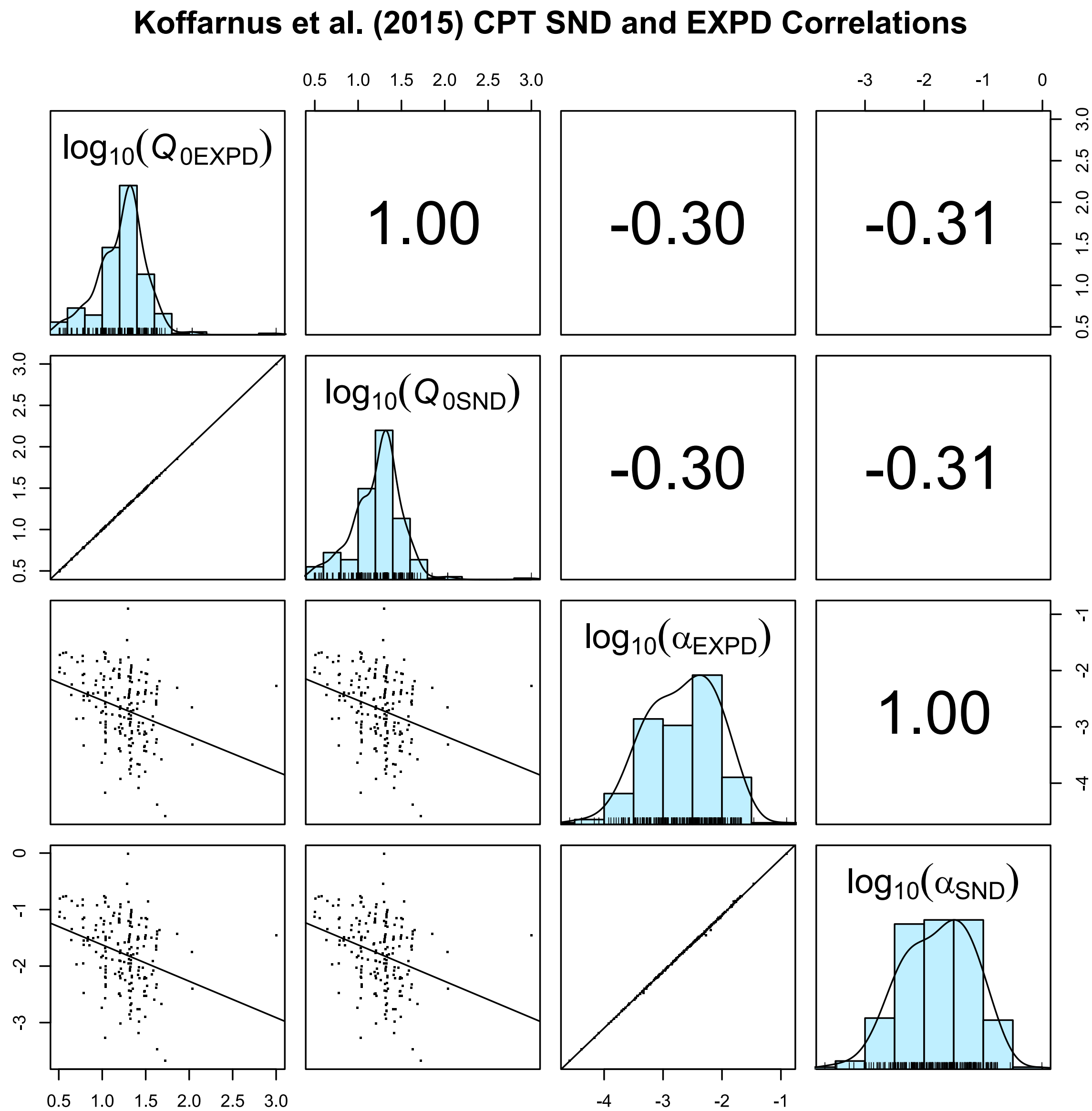

Figure S8. Comparisons between parameters estimated from the EXPD and SND from the Koffarnus et al (2015) simulated data. Parameters estimated are $\log_{10}$ transformed. This was done for each to help approximate normality. Scatterplots are on the bottom of the diagonal, Pearson correlations are on the top of the diagonal, with distributions as histograms on the diagonal.

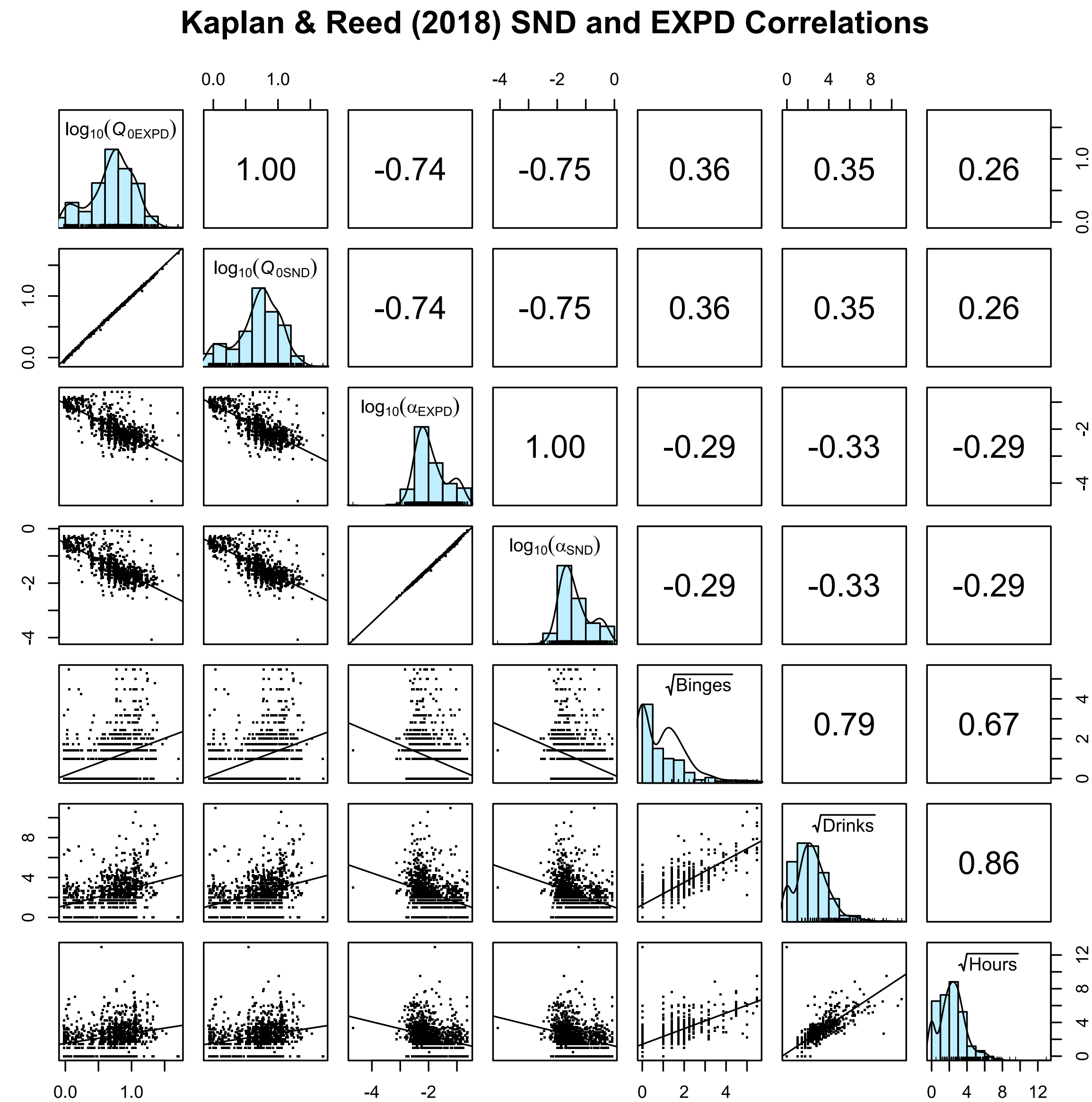

Figure S9. Comparisons between parameters estimated from the EXPD and SND, and their relationship to alcohol use variables from the Kaplan and Reed (2018) data. Parameters estimated are $\log_{10}$ transformed, whereas alcohol use variables are square root transformed. This was done for each to help approximate normality. Scatterplots are on the bottom of the diagonal, Pearson correlations are on the top of the diagonal, with distributions as histograms on the diagonal.