Carbon elastics for commissioning

November 20, 2017

1 Kinematics equations

- Carbon target mass, M_C = 12*931.5 MeV
- Beam energy, E_B
- Scattered electron energy and angle, E_e and θ_E
- Scattered carbon energy and angle, E_C and θ_C
- Carbon excitation energy, E_x

Conservation of energy:

$$E_B + M_C = E_e + E_C + E_x \tag{1}$$

Conservation of momentum (assume $E_e = p_E$):

$$\begin{aligned} E_e * \sin(\theta_e) &= p_C * \sin(\theta_C) & (2)\\ E_B &= E_e * \cos(\theta_e) + p_C * \cos(\theta_C) & (3) \end{aligned}$$

Solving for momentum equations for E_C:

$$\begin{aligned} E_e^2 * \sin^2(\theta_e) &= p_C^2 * \sin^2(\theta_C) & (4)\\ (E_B - E_e * \cos(\theta_e))^2 &= p_C^2 * \cos^2(\theta_C) & (5)\\ (E_B - E_e * \cos(\theta_e))^2 + E_e^2 * \sin^2(\theta_e) &= p_C^2 = E_C^2 - M_C^2 & (6)\\ -2E_B E_e \cos(\theta_e) + E_e^2 &= E_C^2 - M_C^2 - E_B^2 & (7) \end{aligned}$$

From energy equation:

$$E_C^2 - M_C^2 - E_B^2 = +2E_B M_C + (E_e + E_x)^2 - 2(E_B + M_C)(E_e + E_x) \tag{8}$$

Combining the two:

$$2E_B M_C + 2E_e E_x + E_x^2 - 2(E_B + M_C)(E_e + E_x) = -2E_B E_e \cos(\theta_e) \tag{9}$$

Solving for E_e :

$$E_e = \frac{E_B M_C + E_x^2/2 - E_x(E_B + M_C)}{[E_B(1 - \cos(\theta_e)) + M_c - E_x]} \tag{10}$$

Solve quadratic equation for the excitation energy:

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\begin{aligned}
E_x &= \frac{1}{2A} * \left[-B \pm \sqrt{B^2 - 4 * A * C)} \right] && (11)\\
A &= \frac{1}{2} && (12)\\
B &= E_e - M_C - E_B && (13)\\
c &= E_e E_B (\cos(\theta_e) - 1) + M_C (E_B - E_e) && (14)
\end{aligned}
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