MATH 262 - Homework 4.2a

1.

Claim. The following function is a linear transformation.

$$T(M) = M \begin{bmatrix} 1 & 3 \\ 3 & 9 \end{bmatrix} \text{ from } \mathbb{R}^{2\times 2} \text{ to } \mathbb{R}^{2\times 2}$$

Proof. Take some $M_1, M_2 \in \mathbb{R}^{2\times 2}$ and $k \in \mathbb{R}$.

$$\begin{aligned} T(M_1) + kT(M_2) &= M_1 \begin{bmatrix} 1 & 3 \\ 3 & 9 \end{bmatrix} + kM_2 \begin{bmatrix} 1 & 3 \\ 3 & 9 \end{bmatrix} \\ &= (M_1 + kM_2) \begin{bmatrix} 1 & 3 \\ 3 & 9 \end{bmatrix} \\ &= T(M_1 + kM_2) \end{aligned}$$

Notice that the transformation matrix can be trivially factored out of the initial equation. By showing, $T(M_1) + kT(M_2) = T(M_1 + kM_2)$, $T(M)$ has been shown to follow the sum rule and constant-multiple rule. Therefore, it is a linear transformation. □