*Theorem 21.* Consider a system of $m$ equations in $n$ unknowns. Let $\left[A \mid \vec{b}\right]$ be the augmented matrix associated to this system and let $\left[H \mid \vec{c}\right]$ be the reduced row-echelon form of $\left[A \mid \vec{b}\right]$. The matrices $A$ and $H$ are $m \times n$ matrices and

1. $rank(A) \leq n$ and $rank(A) \leq m$
2. If $rank(A) = m$ then the system has at least one solution.
3. If $rank(A) = n$ then the system has at most one solution.
4. If $rank(A) < n$ then the system has either infinitely many or no solutions.

*Claim.* It is false that there exists a $5 \times 5$ matrix $A$ of rank 4 such that the system $A\vec{x} = \vec{0}$ has only the solution $\vec{x} = \vec{0}$.

*Proof.* Let $n = 5$, the number of columns in the matrix $A$. Notice that for $A$

$$\begin{aligned} rank(A) &< n \\ 4 &< 5 \end{aligned}$$

By Therem 21, the system has either infinitely many or no solutions. Therefore, the claim is true. □