

MATH 262 - Homework 4.2a

7.

Claim. The following function is a linear transformation.

$$T(f(t)) = t(f'(t)) \text{ from } P_2 \text{ to } P_2$$

Proof. Take some $f(t), g(t) \in P_2$ and $k \in \mathbb{R}$. Show that $T(f(t)) + kT(g(t)) = T(f(t) + kg(t))$.

$$\begin{aligned} T(f(t)) + kT(g(t)) &= t \cdot f'(t) + t \cdot kg'(t) \\ &= t(f'(t) + kg'(t)) \\ &= t\left(\frac{d}{dt}f(t) + \frac{d}{dt}kg(t)\right) \\ &= t \cdot \frac{d}{dt}(f(t) + kg(t)) \\ &= T(f(t) + kg(t)) \end{aligned}$$

$T(f(t))$ has been shown to follow the sum rule and constant-multiple rule. Therefore, it is a linear transformation. \square