Claim. If A and B are matrices of the same size, then the following formula must hold:

$$rank(A+B) = rank(A) + rank(B)$$

Proof. Suppose the claim is true. Let

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$
$$B = \begin{bmatrix} -1 & -1 \\ -1 & -1 \end{bmatrix}$$

with $A+B$ resulting in the zero matrix. To find the ranks, put the matrices in reduced row-echelon form:

$$rref(A) = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$$
$$rref(B) = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$$
$$rref(A+B) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Notice that

$$rank(A) = 1$$
$$rank(B) = 1$$
$$rank(A+B) = 0$$

which means

$$rank(A+B) \neq rank(A) + rank(B)$$
$$0 \neq 1 + 1$$
$$0 \neq 2$$

Therefore, the claim is false. $\square$