

MATH 262 - Homework 7.3

3. Let $A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$.

Find all (real) eigenvalues for A . Then find a basis of each eigenspace, and diagonalize A , if you can. Do not use technology.

Since A is a triangular matrix, its eigenvalues are its diagonal entries. It only has one eigenvalue $\lambda_1 = 1$ with a multiplicity of 3. Now, use this eigenvalue to find the basis of the eigenspace.

$$E_1 = \ker(A - I_2) = \ker \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \text{span} \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \right\}$$

Because only one linearly independent eigenvector can be found, it's not possible to construct an eigenbasis for A . Thus, A fails to be diagonalizable.