MATH 262 - Homework 7.3

1. Let $A = \begin{bmatrix} 0 & -1 \\ 1 & 2 \end{bmatrix}$.

   Find all (real) eigenvalues for A. Then find a basis of each eigenspace, and diagonalize A, if you can. Do not use technology.

   First, find the eigenvalues for A.

$$\text{tr}(A) = 0 + 2 = 2$$

$$\det(A) = (0 \cdot 2) - (-1 \cdot 1) = 0 - (-1) = 0 + 1 = 1$$

$$\begin{aligned} 0 &= det(A - \lambda I_2) \\ &= \lambda^2 - \text{tr}(A)\lambda + \det(A) \\ &= \lambda^2 - 2\lambda + 1 \\ &= (\lambda - 1)^2 \end{aligned}$$

   A has one eigenvalue $\lambda_1 = 1$ with a multiplicity of 2. Now, use this eigenvalue to find the eigenspace and its basis.

$$E_1 = \ker(A - I_2) = \ker \begin{array}{c} \begin{matrix} -1 & 1 \end{matrix} \\ \begin{bmatrix} -1 & -1 \\ 1 & 1 \end{bmatrix} \end{array} = \text{span}\left\{ \begin{bmatrix} -1 \\ 1 \end{bmatrix} \right\}$$

   Because only one linearly independent eigenvector can be found, it's not possible to construct an eigenbasis for A. Thus, A fails to be diagonalizable.