MATH 262 - Homework 4.2a

2.

Claim. The following function is a linear transformation.

$$T(M) = M + I_2 \text{ from } \mathbb{R}^{2\times 2} \text{ to } \mathbb{R}^{2\times 2}$$

Proof. Let

$$M_1 = M_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$
$$k = 1$$

Assume the claim is true. Thus, $T(M_1) + kT(M_2) = T(M_1 + kM_2)$, $T(M)$ must hold true for any $M_1, M_2 \in \mathbb{R}^{2\times 2}$ and $k \in \mathbb{R}$.

$$\begin{aligned} T(M_1) + kT(M_2) &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} + I_2 + 1\left(\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} + I_2\right) \\ &= 2I_2 \end{aligned}$$

$$\begin{aligned} T(M_1 + kM_2) &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} + 1\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} + I_2 \\ &= I_2 \end{aligned}$$

$T(M_1) + kT(M_2) \neq T(M_1 + kM_2)$, which contradicts the definition of a linear transformation. Therefore, the claim is false. □