Claim. A system of four linear equations in three unknowns is always inconsistent.

Proof. Suppose the claim is true. A counterexample to that claim is the following system, which has a trivial solution of all unknowns being zero:

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Therefore, the claim is false. □