MATH 262 - Homework 4.1

14. True or False? Upload your reasoning. If the statement is true give a proof or detailed reason why it is true. If it is false give a counter example.

Claim. If W_1 and W_2 are subspaces of a linear space V, then the intersection $W_1 \cap W_2$ must be a subspace of V as well.

Proof. Let $\vec{v}$ and $\vec{w}$ be vectors such that $\vec{v}, \vec{w} \in W_1 \cap W_2$. By the definition of a set intersection, $\vec{v}, \vec{w} \in W_1$ and $\vec{v}, \vec{w} \in W_2$. By the definition of a linear subspace, $\vec{v} + k\vec{w} \in W_1$ and $\vec{v} + k\vec{w} \in W_2$ for some $k \in \mathbb{R}$. Therefore, by the definition of a set intersection, $\vec{v} + k\vec{w} \in W_1 \cap W_2$.

By the definition of a linear subspace, both W_1 and W_2 contain the neutral element 0 of V. Therefore, by the definition of a set intersection, $W_1 \cap W_2$ also contains the neutral element 0 of V.

It has been shown that $W_1 \cap W_2$ is closed under linear combinations and contains the neutral element 0 of V. Therefore, the definition of a linear subspace is satisified and the claim is true. □

Hint: Start with $k \in \mathbb{R}$ and $\vec{v}, \vec{w} \in W_1 \cap W_2$. Then, are $\vec{v}$ and $\vec{w}$ both in W_2? If so, why is $\vec{v} + k\vec{w} \in W_2$? Can you conclude that $\vec{v} + k\vec{w} \in W_1 \cap W_2$?