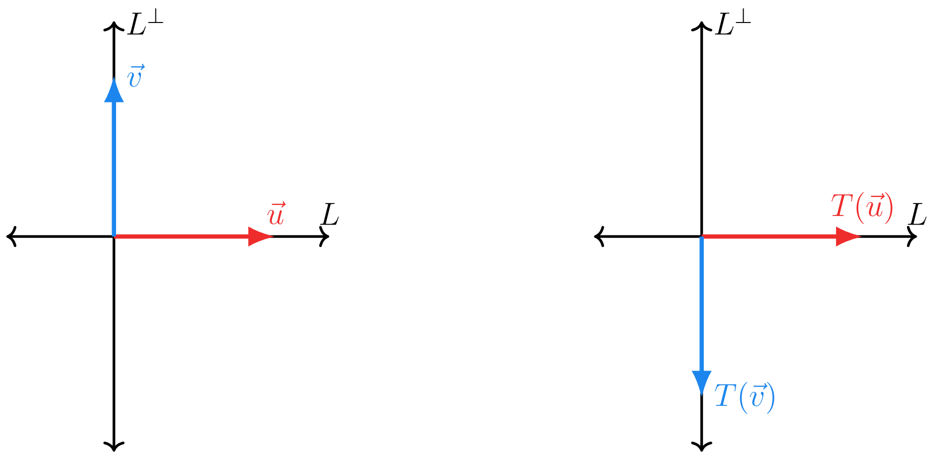
MATH 262 - Homework 7.1

4. Arguing geometrically, find all eigenvectors and eigenvalues of the linear transformation:

   Reflection about a line L in $\mathbb{R}^2$

   Then find an eigenbasis if you can, and thus determine whether the given transformation is diagonalizable.

   Below on the left is a plot of a vector $\vec{u}$ that is on L and a vector $\vec{v}$ that is on $L^\perp$, the line perpendicular to L. On the right is a plot of these vectors reflected about L.



   Notice $T(\vec{u})$ remains the same, so it's eigenvalue is 1. $T(\vec{v})$ remains on $L^\perp$, but is going in the opposite direction, so it's eigenvalue is -1. Therefore, the transformation is diagonalizable and

$$\text{eigenbasis} = \{\, \vec{u}, \vec{v} \,\} \text{ where } \vec{u} \in L, \vec{v} \in L^\perp$$