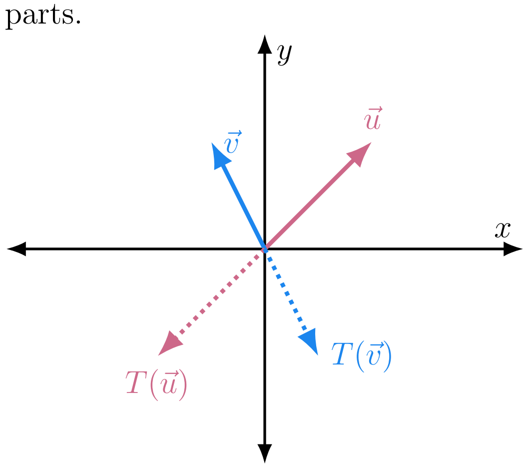
MATH 262 - Homework 7.1

6. Arguing geometrically, find all eigenvectors and eigenvalues of the linear transformation:

   $$\text{Rotation through an angle of } 180^\circ \text{ in } \mathbb{R}^2$$

   Then find an eigenbasis if you can, and thus determine whether the given transformation is diagonalizable.

   Below is a plot of two vectors in $\mathbb{R}^2$ and their transformed counterparts.



   Notice both $T(\vec{u})$ and $T(\vec{v})$ go in the opposite directions of $\vec{u}$ and $\vec{v}$, respectively, but otherwise remain the same. Therefore, each vector's eigenvalue is -1. Thus, the transformation is diagonalizable and the eigenbasis is the basis of $\mathbb{R}^2$ i.e. $\begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}0\\1\end{bmatrix}$. All vectors in $\mathbb{R}^2$ are eigenvectors of this transformation.