

For  $A$  to be invertible, its columns must be linearly independent. To check this, consider the following equation

$$A\vec{x} = \vec{0}$$

$$\vec{v}_1x_1 + \vec{v}_2x_2 + \dots + \vec{v}_nx_n = \vec{0}$$

Let  $i = 1$  and  $j = 2$  so that the first two columns of  $A$  are equal. Then, the equation becomes

$$\vec{v}_1(x_1 + x_2) + \dots + \vec{v}_nx_n = \vec{0}$$

Notice that to get  $\vec{0}$  as a solution, any constant such that  $x_1 = -x_2$  will cancel out the first term. Then, the rest of  $\vec{x}$  can be zeros. Since there is an infinite number of solutions rather than only the trivial  $\vec{x} = \vec{0}$ , the columns of  $A$  are linearly dependent. Therefore,  $A$  is not invertible.