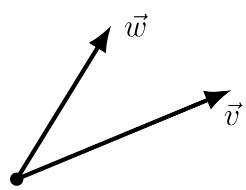
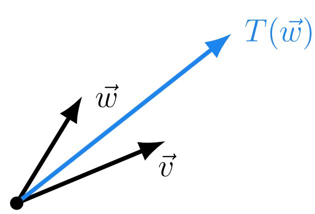
MATH 262 - Homework 7.1

8. Arguing geometrically, find all eigenvectors and eigenvalues of the linear transformation below. Then find an eigenbasis if you can, and thus determine whether the given transformation is diagonalizable.

   The linear transformation with $T(\vec{v}) = \vec{v}$ and $T(\vec{w}) = \vec{v} + \vec{w}$ for the vectors $\vec{v}$ and $\vec{w}$ in $\mathbb{R}^2$ sketched below.

   The definition of an eigenvalue states that it is a λ such that $A\vec{u} = \lambda\vec{u}$. For $T(\vec{w})$, this would mean $\vec{v} + \vec{w}$ has to be a scalar multiple of $\vec{w}$. Thus, $\vec{v}$ would have to be collinear with $\vec{w}$. However, the sketch above shows that they are not collinear. As further proof, the sketch below illustrates $T(\vec{w})$. Observe that it is not collinear with $\vec{w}$.

   Therefore, T is not diagonalizable.