4. $T(n) = 2^n T(\frac{n}{2}) + n^n$

Master method is not applicable ~~since~~ since a is not constant

5. $T(n) = 16T(\frac{n}{4}) + n$

$a = 16$, $b = 4$, $f(n) = n$

$n^{\log_b a} = n^{\log_4 16} = n^2$

$f(n) < n^{\log_b a}$

$\therefore$ ~~$f(n) = O(n^2)$~~ $\therefore f(n) = O(n^2)$

6. $T(n) = 2T(\frac{n}{2}) + n\log n$

$a = 2$, $b = 2$, $f(n) = n\log n$

$n^{\log_b a} = n^{\log_2 2} = n$

$f(n) > n^{\log_b a}$

$\therefore T(n) = O(n\log n)$

7. $T(n) = 2T(\frac{n}{2}) + \frac{n}{\log n}$

$a = 2$, $b = 2$

$f(n) = \frac{n}{\log n}$

master method is not appliacable because of non-polynomial difference between $f(n)$ and $n^{\log_b a}$

Master method practice

1. $T(n) = 3T\left(\frac{n}{2}\right) + n^2$

$a = 3$, $b = 2$, $f(n) = n^2$

$n^{\log_b a} = n^{\log_2 3} = n^{1.58}$

$\cancel{n^2 > n}$ $\therefore f(n) > n^{\log_b a}$

$f(n) = \Omega(n^{\log_b a + 0.42})$

$\therefore T(n) = \Theta(n^2)$

2. $T(n) = 4T\left(\frac{n}{2}\right) + n^2$

$a = 4$, $b = 2$, $f(n) = n^2$

$n^{\log_a b} = n^{\log_2 4} = n^2$

$F(n) = n^{\log_b a}$

$\therefore T(n) = \Theta(n^2 \log_2 n)$

3. $T(n) = T\left(\frac{n}{2}\right) + 2^n$

$a = 1$, $b = 2$, $f(n) = 2^n$

$n^{\log_b a} = n^{\log_2 1} = n^0 = 1$

$f(n) > n^{\log_b a}$

$f(n) = \Omega(n^{\log_b a + \epsilon})$

$T(n) = O(f(n)) = \cancel{\theta(2^n)}\ \Theta(2^n)$

8. $T(n) = 2T\left(\frac{n}{4}\right) + n^{0.51}$

$a = 2$, $b = 4$, $f(n) = 0.51$

$n^{\log_b a} = n^{\log_4 2} = n^{0.5}$

$f(n) > n^{\log_b a}$

$f(n) = n^{\log_b a + 0.01}$

$\therefore T(n) = \Theta(n^{0.51})$

9. $T(n) = \frac{1}{2} T\left(\frac{n}{2}\right) + \frac{1}{n}$

$a = \frac{1}{2}$ $b = 2$

master method is not applicable because $a < 1$

10. $T(n) = 16T\left(\frac{n}{4}\right) + n!$

$a = 16$, $b = 4$, $f(n) = n!$

$n^{\log_b a} = n^{\log_4 16} = n^2$

$f(n) > n^{\log_b a}$

$T(n) = O(n!)$

11. $T(n) = \sqrt{2}\,T\left(\frac{n}{2}\right) + \log n$

$a = \sqrt{2}$, $b = 2$, $f(n) = \log n$

$n^{\log_b a} = n^{\log_2 \sqrt{2}} = n^{1/2}$

$f(n) < n^{\log_b a}$

$T(n) = O(n^{\log_b a}) = O(\sqrt{n})$

12. $T(n) = 3T\left(\frac{n}{2}\right) + n$

$a = 3$, $b = 2$, $f(n) = n$

$n^{\log_b a} = n^{\log_2 3} = n^{1.58}$

$f(n) < n^{\log_b a}$

$T(n) = O(n^{\log_b a}) = O(n^{1.58})$

13. $T(n) = 3T\left(\frac{n}{3}\right) + \sqrt{n}$

$a = 3$, $b = 2$ $\quad f(n) = \sqrt{n} = n^{1/2}$

$n^{\log_b a} = n^{\log_3 3} = n \quad \therefore f(n) < n^{\log_b a}$

$\therefore T(n) = O(n^{\log_b a}) = O(n)$

14. $T(n) = 4T\left(\frac{n}{2}\right) + cn$

$a = 4,\ b = 2,\ f(n) = cn$

$n^{\log_b a} = n^{\log_2 4} = n^2$

$f(n) < n^{\log_b a}$

$T(n) = O(n^{\log_b a}) = O(n^2)$

15. $T(n) = 3T\left(\frac{n}{4}\right) + n\log n$

$a = 3,\ b = 4,\ f(n) = n\log n$

$n^{\log_b a} = n^{\log_4 3} = n^{0.79}$

$f(n) > n^{\log_b a}$

$T(n) = O(n\log n)$

16. $T(n) = 3T\left(\frac{n}{3}\right) + \frac{n}{2}$

$a = 3,\ b = 3,\ f(n) = \frac{n}{2}$

$n^{\log_b a} = n^{\log_3 3} = n$

$f(n) = n^{\log_b a}$

$\therefore T(n) = \theta(n \log_3 n)$

17. $T(n) = 6T\left(\frac{n}{3}\right) + n^2 \log n$

$a = 6,\ b = 3,\ f(n) = n^2 \log n$

$n^{\log_b a} = n^{\log_3 6}$

$\therefore f(n) > n^{\log_3 6}$

$\therefore T(n) = O(n^2 \log n)$

18. $T(n) = 4T\left(\frac{n}{2}\right) + \frac{n}{\log n}$

$a = 4,\ b = 2,\ f(n) = \frac{n}{\log n}$

$n^{\log_b a} = n^{\log_2 4} = n^2$

$f(n) < n^{\log_b a}$

$T(n) = O(n^2)$

19. $T(n) = 64T\left(\frac{n}{8}\right) - n^2 \log n$

$f(n) = -n^2 \log n$

it cannot be -ve

$\therefore$ master method not applicable

20. $T(n) = 7T\left(\frac{n}{3}\right) + n^2$

$a = 7,\ b = 3,\ f(n) = n^2$

$n^{\log_b a} = n^{\log_3 7} = n^{1.77}$

$f(n) > n^{\log_b a}$

$T(n) = O(n^2)$

Q.21 $T(n) = 4T\left(\frac{n}{2}\right) + \log n$

$a = 4$, $b = 2$, $f(n) = \log n$

~~F(z~~ $n^{\log_b a} = n^{\log_2 4} = n^2$

$f(n) < n^{\log_b a}$

$T(n) = O(n^{\log_b a}) = O(n^2)$

Q.22 $T(n) = T\left(\frac{n}{2}\right) + n(2 - \cos n)$

~~n≥1~~ . $a = 1$, $b = 2$, $f(n) = n(2 - \cos n)$

~~f(n) =~~

$n^{\log_b a} = n^{\log_2 1} = n^0 = 1$

$f(n) > n^{\log_b a}$

$T(n) = O(n(2 - \cos n))$