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S.E Comps - A 4/8/23 10/10

Laplace transform Tutorial-1

Q.1 $\left(\frac{\sin 2t}{\sqrt{t}}\right)^2$

Q.2 $e^{-t}\int_0^t \frac{\sin t}{t}\,dt$

Q.3 $f(t) = \begin{cases} 5\sin 3\left(t-\frac{\pi}{4}\right) & t > \frac{\pi}{4} \\ 0 & t < \frac{\pi}{4} \end{cases}$

Q.4 If $L[\operatorname{erf}(\sqrt{t})] = \frac{1}{s\sqrt{s+1}}$, find $L\left[te^{3t}\operatorname{erf}(\sqrt{t})\right]$

Q. 1 $f(t) = \left(\dfrac{\sin 2t}{\sqrt{t}}\right)^2$

$= \dfrac{\sin^2 2t}{t}$

using division by t

$L[f(t)] = \int_s^\infty L(\sin^2 2t)\,dt$

$= \int_s^\infty L\left(\dfrac{1 - \cos 4t}{2}\right) dt$

$\cancel{= \int_s^\infty L\left(\frac{1}{2} - \right.}$

$= \dfrac{1}{2}\int_s^\infty L(1 - \cos 4t)\,dt$ ✓

$= \dfrac{1}{2}\int_s^\infty \left(\dfrac{1}{s} - \dfrac{s}{s^2+16}\right) dt$

$= \dfrac{1}{2}\left[\log s - \dfrac{1}{2}\log(s^2+16)\right]_s^\infty$

$= \dfrac{1}{2}\left[\log s - \log(s^2+16)^{1/2}\right]_s^\infty$

$= \dfrac{1}{2}\left(\log \dfrac{s}{\sqrt{s^2+16}}\right)_s^\infty$

$= \dfrac{1}{2}\left(0 - \log \dfrac{s}{\sqrt{s^2+16}}\right)$

$$= \frac{1}{2}\left(-\log\frac{s}{\sqrt{s^2+16}}\right)$$

$$= \frac{1}{2}\log\frac{\sqrt{s^2+16}}{s}$$

$$= \frac{1}{2}\log\frac{\sqrt{s^2+16}}{s}$$

//

Q. 2 $e^{-t} \int_0^t \frac{\sin t}{t} \, dt$

$L(f(t)) = e^{-t} L\left(\frac{\sin t}{t}\right) dt$

$= e^{-t} \int_s^\infty L(\sin t) \, dt$

$= e^{-t} \int_s^\infty \frac{1}{s^2+1} \, dt$

$= e^{-t} \left[\tan^{-1}(s)\right]_s^\infty$

$= e^{-t} \left[\tan^{-1}(\infty) - \tan^{-1}(s)\right]$

$= e^{-t} \left[\frac{\pi}{2} - \tan^{-1} s\right]$

$\cancel{= e^{-t} \cot^{-1}}$

$= e^{-t} \cot^{-1} s$

Q.3 ~~f(t) =~~

$$\therefore L[f(t)] = \int_0^{\frac{\pi}{4}} e^{-st}(0)\,dt + \int_{\frac{\pi}{4}}^{\infty} e^{-st}\, 5\sin 3\left(t-\frac{\pi}{4}\right)$$

$$L = \left[5\int_{\frac{\pi}{4}}^{\infty} e^{-st} \cdot e^{\frac{\pi}{4}t} \sin 3t \right]$$

$$\therefore L = \left[5\int_{\frac{\pi}{4}}^{\infty} e^{\left(\frac{\pi}{4}-s\right)t} \cdot \sin 3t \right]$$

$$= 5\int_{\frac{\pi}{4}}^{\infty} \left[\sin 3t \, \frac{e^{\left(\frac{\pi}{4}-s\right)t}}{\frac{\pi}{4}-s} - \int \cos 3t \cdot \frac{e^{\left(\frac{\pi}{4}-s\right)t}}{\frac{\pi}{4}-s} \right]$$

$$= 5e^{\left(\frac{\pi}{4}-s\right)t} \int_{\frac{\pi}{4}}^{\infty} L[\sin 3t]$$

$$= 5e^{\left(\frac{\pi}{4}-s\right)t} \int_{\frac{\pi}{4}}^{\infty} \frac{3}{s^2+9}$$

$$= 15e^{\left(\frac{\pi}{4}-s\right)t} \left[\frac{1}{9}\tan^{-1}\left(\frac{s}{3}\right)\right]_{\frac{\pi}{4}}^{\infty}$$

$$= \frac{15}{9} e^{\left(\frac{\pi}{4}-s\right)t} \left(\frac{\pi}{2} - \tan^{-1}\frac{\pi}{12}\right)$$

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$= \cancel{\frac{15}{9}}\, e^{(\frac{\pi}{2}-s)t}$

$= \frac{15}{9}\, e^{(\frac{\pi}{2}-s)t} \cdot \cot^{-1} \frac{\pi}{12}$ //

Q.4 $L[\text{erf}(\sqrt{t})] = \dfrac{1}{s\sqrt{s+1}}$

$$\therefore L[e^{3t}\,\text{erf}\sqrt{t}] = \frac{1}{(s-3)\sqrt{(s-3)+1}}$$

$$= \frac{1}{(s-3)\sqrt{s-2}}$$

$$= -\frac{d}{dt}\left[\frac{1}{(s-3)(\sqrt{s-2})}\right]$$

$$= \frac{1}{(s-3)^2(s-2)}\frac{d}{dt}\left[(s-3)\sqrt{s-2}\right]$$

$$= \frac{1}{(s-3)^2(s-2)}\left[(s-3)\cdot\frac{1}{2\sqrt{s-2}} + \sqrt{s-2}\right]$$

$$= \frac{1}{2(s-3)^2(s-2)^{3/2}}\left[(s-3)\cancel{+2\sqrt{s-2}} + 2(s-2)\right]$$

$$= \frac{3s-7}{2(s-3)^2(s-2)^{3/2}}$$

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Tutorial 2 : Inverse Laplace transform

Q.1 $L^{-1}\left[\frac{6s-4}{s^2-4s+20}\right]$

Q.2 $L^{-1}\left[\frac{s^2+2s-4}{(s^2+2s+2)(s^2+2s+5)}\right]$

Q.3 $L^{-1}\left[\log\left(\frac{s^2+1}{s(s+1)}\right)\right]$

Q.4 $L^{-1}\left[\frac{2s}{s^4+4}\right]$

Q.1 $L^{-1}\left[\dfrac{6s-4}{s^2-4s+20}\right]$

∴ by completing square method,

$$= 6L^{-1}\left[\frac{s-2/3}{s^2-4s+4+16}\right]$$

$$= 6L^{-1}\left[\frac{s-2/3}{(s-2)^2+16}\right]$$

$$= 6L^{-1}\left[\frac{(s-2)+2-2/3}{(s-2)^2+16}\right]$$

$$= 6e^{2t}L^{-1}\left[\frac{s+4/3}{s^2+4^2}\right] \quad \ldots \{\text{second shifting theorem}\}$$

$$= 6e^{2t}\left[L^{-1}\left(\frac{s}{s^2+4^2}\right)+\frac{4}{3}L^{-1}\left(\frac{1}{s^2+4^2}\right)\right]$$

$$= 6e^{2t}\left[\cos 4t+\frac{4}{3}\frac{\sin 4t}{4}\right]$$

$$= 6e^{2t}\left[\cos 4t+\frac{1}{3}\sin 4t\right]$$

//

Q.2 $L^{-1}\left[\dfrac{s^2+2s-4}{(s^2+2s+2)(s^2+2s+5)}\right]$

by using completing square method,

$$= L^{-1}\left[\frac{s^2+2s+1-\cancel{3}5}{(s^2+2s+1+1)(s^2+2s+1+4)}\right]$$

$$= L^{-1}\left[\frac{(s+1)^2-5}{((s+1)^2+1)((s+1)^2+4)}\right]$$

$$= e^{-t} L^{-1}\left[\frac{s^2-5}{(s^2+1)(s^2+4)}\right]$$

By using Partial fractions,

$$\frac{s^2-5}{(s^2+1)(s^2+4)} = \frac{As+B}{s^2+1} + \frac{Cs+D}{s^2+4}$$

$$s^2-5 = (As+B)(s^2+4) + (Cs+D)(s^2+1)$$

$$= As^3 + 4As + Bs^2 + 4B + Cs^3 + Cs + Ds^2 + D$$

$$s^2-5 = s^3(A+C) + s^2(B+D) + s(4A+C) + 1(4B+D)$$

$\therefore$ by comparing,

$A+C=0$, $B+D=1$, $4A+C=0$, $4B+D=-5$

$\therefore$ Solving above equation, we get

$A = 0, \quad B = -2, \quad C = 0, \quad D = 3$

$$\therefore e^{-t} L^{-1}\left[\frac{s^2 - s}{(s^2+1)(s^2+4)}\right]$$

$$= e^{-t} L^{-1}\left[\frac{-2}{s+1} + \frac{3}{s^2+4}\right]$$

$$= e^{-t}\left[L^{-1}\left(\frac{-2}{s+1}\right) + L^{-1}\left(\frac{3}{s^2+2^2}\right)\right]$$

$$= e^{-t}\left[-2e^{-t} + 3\,\frac{\sin 2t}{2}\right] \text{ //}$$

Q.3 $L^{-1}\left[\log\left(\frac{s^2+1}{s(s+1)}\right)\right]$

= by using formula

$$= -\frac{1}{t} L^{-1} \frac{d}{ds}\left[\log\left(\frac{s^2+1}{s(s+1)}\right)\right]$$

$$= -\frac{1}{t} L^{-1} \frac{d}{ds}\left[\log(s^2+1) - \log[s(s+1)]\right]$$

$$= -\frac{1}{t} L^{-1} \cancel{\frac{d}{ds}}\left[\frac{1}{s^2+1}\cdot 2s - \frac{1}{s(s+1)}\cdot(2s+1)\right]$$

$$= -\frac{1}{t} L^{-1}\left[\frac{2s}{s^2+1^2} - \frac{2s+1}{s(s+1)}\right]$$

$$= -\frac{1}{t}\left[L^{-1}\left(\frac{2s}{s^2+1^2}\right) - \left[L^{-1}\left(\frac{2\cancel{s}}{\cancel{s}(s+1)}\right) + L^{-1}\left(\frac{1}{s(s+1)}\right)\right]\right]$$

$$= -\frac{1}{t}\left[2\cos t - 2e^{-t} - L^{-1}\left(\frac{1}{s^2+s}\right)\right]$$

$\frac{\text{[2]}}{t}$ By using partial fraction,

$$\frac{1}{s(s+1)} = \frac{A}{s} + \frac{B}{s+1}$$

$\therefore 1 = A(s+1) + Bs$

$\therefore 1 = As + A + Bs$

$\therefore 1 = s(A+B) + A$

$\therefore A + B = 0 \qquad A = 1$

$\therefore B = -1$

$$\therefore \frac{1}{s(s+1)} = \frac{1}{s} - \frac{1}{s+1}$$

$$\therefore L^{-1}\left[\log\left(\frac{s^2+1}{s(s+1)}\right)\right]$$

$$= -\frac{1}{t}\left[2\cos t - 2e^{-t} - L^{-1}\left(\frac{1}{s} - \frac{1}{s+1}\right)\right]$$

$$= -\frac{1}{t}\left[2\cos t - 2e^{-t} - 1 + e^{-t}\right]$$

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Q.4 $L^{-1}\left[\dfrac{2s}{s^4+4}\right]$

$= L^{-1}\left[\dfrac{2s}{(s^2)^2+2^2}\right]$

$= L^{-1}\left[\dfrac{2s}{(s^2)^2+2^2+4s^2-4s^2}\right]$

$\cancel{= L^{-1}\left[\dfrac{2s}{(s^2)^2-}\right]}$

$\cancel{= L^{-1}\left[\dfrac{2s}{(s^2+2)^2-4s^2}\right]}$ $= L^{-1}\left[\dfrac{2s}{(s^2+2)^2-4s^2}\right]$

$= L^{-1}\left[\dfrac{2s}{((s^2+2)-2s)((s^2+2)+2s)}\right]$

$= L^{-1}\left[\dfrac{2s}{(s^2-2s+2)(s^2+2s+2)}\right]$

$\cancel{= L^{-1}\left[\dfrac{2s}{((s^2-2s+1)+1)((s^2+2s+1)+1)}\right]}$

$\cancel{= L^{-1}\left[\dfrac{2s}{((s-1)^2+1)((s+1)^2+1)}\right]}$

Using partial fraction,

$$\frac{2s}{(s^2-2s+2)(s^2+2s+2)} = \frac{As+B}{s^2-2s+2} + \frac{Cs+D}{s^2+2s+2}$$

$$\therefore \quad 2s = (As+B)(s^2+2s+2) + (Cs+D)(s^2-2s+2)$$

$$\therefore \quad 2s = As^3 + 2As^2 + 2As + Bs^2 + 2Bs + 2B$$
$$+ Cs^3 - 2Cs^2 + 2Cs + Ds^2 - 2Ds + 2D$$

$$\therefore 2s = s^3(A+C) + s^2(2A+B-2C+D)$$
$$+ s(2A+2B+2C-2D) + 1((2D)+2B)$$

$\therefore$ comparing

$A+C=0 \qquad 2A+B-2C+D=0 \qquad 2A+2B+2C-2D=2 \qquad 2D+2B=0$

$\therefore \; A=0 \;,\; B=\frac{1}{2} \;,\; C=0 \;,\; D=-\frac{1}{2}$

$$\therefore L^{-1}\left[\frac{2s}{(s^2-2s+2)(s^2+2s+2)}\right]$$
$$= L^{-1}\left[\frac{1/2}{s^2-2s+2} - \frac{1/2}{s^2+2s+2}\right]$$

Q.4

$$= \frac{1}{2} L^{-1}\left[\frac{1}{(s^2-2s+1)+1} - \frac{1}{(s^2+2s+1)+1}\right]$$

$$= \frac{1}{2} L^{-1}\left[\frac{1}{(s-1)^2+1} - \frac{1}{(s+1)^2+1}\right]$$

$$= \frac{1}{2}\left[e^{t} L^{-1}\left[\frac{1}{s^2+1}\right] - e^{-t} L^{-1}\left[\frac{1}{s^2+1}\right]\right]$$

$$= \frac{1}{2}\left[e^{t}\sin t - e^{-t}\sin t\right]$$

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Find fourier series Tutorial 3

Q.1 $f(x) = \begin{cases} x & \text{for } 0 \le x \le \pi \\ 2\pi - x & \text{for } \pi \le x \le 2\pi \end{cases}$

deduce

$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots = \frac{\pi^2}{8}$$

Q.2 Find half-orange sine series for

$$f(x) = \begin{cases} 1/4 - x, & \text{for } 0 < x < \frac{1}{2} \\ x - 3/4 & \text{for } 1/2 < x < 1 \end{cases}$$

Q.1 $f(x) = \begin{cases} x & 0 \le x \le \pi \\ 2\pi - x & \pi \le x \le 2\pi \end{cases}$

$2T = 2\pi - 0 = 2\pi$

$\therefore T = \pi$

$$a_0 = \frac{1}{2T}\left[\int_0^{\pi} x\,dx + \int_{\pi}^{2\pi} 2\pi - x\,dx\right]$$

$$= \frac{1}{2T}\left[\left[\frac{x^2}{2}\right]_0^{\pi} + \left[2\pi x - \frac{x^2}{2}\right]_{\pi}^{2\pi}\right]$$

$$a_0 = \frac{1}{2\pi}\left[\frac{\pi^2}{2} + \left[4\pi^2 - \frac{4\pi^2}{2} - 2\pi^2 + \frac{\pi^2}{2}\right]\right]$$

$$= \frac{1}{2\pi}\left[\frac{\pi^2}{2} + \cancel{4\pi^2} - \cancel{2\pi^2} - \cancel{2\pi^2} + \frac{\pi^2}{2}\right]$$

$$= \frac{1}{2\pi}\left[\pi^2\right]$$

$$\boxed{a_0 = \frac{\pi}{2}}$$

$$a_n = \frac{1}{T}\int_0^{2\pi} f(x)\cos\left(\frac{n\pi}{T}\right)x\,dx$$

$$= \frac{1}{\pi}\left[\int_0^{\pi} x\cos nx\,dx + \int_{\pi}^{2\pi}(2\pi - x)\cos nx\,dx\right]$$

$$= \frac{1}{\pi}\left[\left(x\cdot\frac{\sin nx}{n} - (1)\left(-\frac{\cos nx}{n^2}\right)\right)_0^{\pi} + \left[(2\pi - x)\frac{\sin nx}{n} - (-1)\left(-\frac{\cos nx}{n^2}\right)\right]_{\pi}^{2\pi}\right]$$

$$= \frac{1}{\pi}\left[\frac{\cos n\pi}{n^2} - \frac{\cos 0}{n^2} - \left(\frac{\cos 2\pi n}{n^2} - \frac{\cos n\pi}{n^2}\right)\right]$$

$$= \frac{1}{\pi}\left[\frac{1}{n^2}\left(\cos n\pi - 1 - 1 + \cos n\pi\right)\right]$$

$$= \frac{1}{\pi n^2}\left(2(-1)^n - 2\right)$$

$$\cancel{= \frac{2}{\pi n^2}\left((-1)^2 - 2\right)}$$

$$\boxed{a_n = \frac{2}{\pi n^2}\left((-1)^n - 1\right)}$$

$$\therefore -\frac{\pi}{2} = \frac{2}{a} \sum_{n=1}^{\infty} \frac{1}{n^2}\left((-1)^n - 1\right).$$

$$\therefore -\frac{\pi^2}{4} = \frac{1}{1^2}(-2) + 0 + \frac{1}{3^2}(-2) + 0 + \frac{1}{5^2}(-2) + \cdots$$

$$\therefore \frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots$$

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Q.2 $f(x) = \begin{cases} 1/4 - x, & 0 < x < 1/2 \\ x - 3/4 & 1/2 < x < 1 \end{cases}$

$\therefore l = 1 - 0 = 1$

half range sine series

$\therefore a_0 = a_n = 0$

$$b_n = \frac{2}{l}\int_0^l f(x) \sin\left(\frac{n\pi}{l}\right)x \, dx$$

$$\therefore b_n = \frac{2}{1}\left[\int_0^{1/2}\left(\frac{1}{4} - x\right)\sin(n\pi x)\,dx + \int_{1/2}^{1}\left(x - \frac{3}{4}\right)\sin(n\pi x)\,dx\right]$$

$$= 2\left[\cancel{\frac{x}{4} - \frac{x^2}{2}}\right.$$

$$= 2\left[\left[\left(\frac{1}{4} - x\right)\left(-\frac{\cos(n\pi x)}{n\pi}\right) - (-1)\left(-\frac{\sin(n\pi x)}{n^2\pi^2}\right)\right]_0^{1/2}\right.$$

$$\left.+\left[\left(x - \frac{3}{4}\right)\left(-\frac{\cos n\pi x}{n\pi}\right) - (1)\left(-\frac{\sin n\pi x}{n^2\pi^2}\right)\right]_{1/2}^{1}\right]$$

$$= 2\left\{\left[\left(\frac{1}{4}-\frac{1}{2}\right)\left(-\frac{\cos n\pi/2}{n\pi}\right)-\left(\frac{\sin n\pi/2}{n^2\pi^2}\right)\right.\right.$$

$$\left. + \left(\frac{1}{4}\right)\cdot\frac{1}{n\pi} + 0\right]$$

$$+\left[\left(1-\frac{3}{4}\right)\left(-\frac{\cos n\pi}{n\pi}\right) + \left(\frac{1}{2}-\frac{3}{4}\right)\left(\frac{\cos n\pi/2}{n\pi}\right)\right.$$

$$\left.\left. -\left(\frac{\sin n\pi/2}{n^2\pi^2}\right)\right]\right]$$

$$= 2\left[\cancel{+\frac{1}{4}\frac{\cos n\pi/2}{n\pi}} - \frac{\sin n\pi/2}{n^2\pi^2} + \frac{1}{4}\frac{1}{n\pi} - \frac{1}{4}\frac{\cos n\pi}{n\pi}\right.$$

$$\left. \cancel{-\frac{1}{4}\frac{\cos n\pi/2}{n\pi}} - \frac{\sin n\pi/2}{n^2\pi^2}\right]$$

$$= 2\left[\frac{1}{4}\left[\frac{1}{n\pi} - \frac{\cos n\pi}{n\pi}\right] - \frac{2\sin n\pi/2}{n^2\pi^2}\right]$$

$$= \frac{1}{n\pi}\left[\frac{1}{2}(1-\cos n\pi) - 4\frac{\sin n\pi/2}{n\pi}\right]$$

$$= \frac{1}{n\pi}\left[\frac{1}{2}\left(1-(-1)^n\right) - 4\frac{\sin n\pi/2}{n\pi}\right]$$

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Q.2

~~if n = odd~~

~~$\therefore b_n =$~~

$$f(x) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi}{l}\right)x$$

$$\therefore f(x) = \sum_{n=1}^{\infty} \frac{1}{n\pi}\left[\frac{1}{2}\left(+1-(-1)^n\right) - \frac{4\sin n\pi/2}{n\pi}\right]\sin\left(\frac{n\pi}{l}\right)x$$

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Maths - Tutorial-4 Complex Variable

Q.1 Verify if $f(z) = \begin{cases} \frac{xy^2(x+iy)}{x^2+y^4}, & z \neq 0 \\ 0, & z = 0 \end{cases}$

is analytic or not?

Q.2 Find the value of a and b such that the function

$f(z) = x^2 + ay^2 - 2xy + i(bx^2 - y^2 + 2xy)$ is analytic. Also find $f'(z)$.

Q.3 If $u - v = (x-y)(x^2 + 4xy + y^2)$ and $f(z) = u + iv$ is an analytic function of $z = x + iy$, find $f(z)$ in terms of z.

Q.4 Find analytic function $f(z) = u + iv$ in terms of z whose imaginary part is $\cos x \cosh y$.

Q.1 $f(z) = \dfrac{xy^2(x+iy)}{x^2+y^4}$

$= \dfrac{x^2y^2 + ixy^3}{x^2+y^4}$

$= \dfrac{x^2y^2}{x^2+y^4} + i\dfrac{xy^3}{x^2+y^4}$

$u = \dfrac{x^2y^2}{x^2+y^4} \qquad v = \dfrac{xy^3}{x^2+y^4}$

$u_x = \dfrac{(x^2+y^4)(2xy^2) - (x^2y^2)(2x)}{(x^2+y^4)^2}$

$= \dfrac{\cancel{2x^3y^2} + 2xy^6 - \cancel{2x^3y^2}}{(x^2+y^4)^2}$

$u_x = \dfrac{2xy^6}{(x^2+y^4)^2}$

?

$u_y = \dfrac{(x^2+y^4)(2x^2y) - (x^2y^2)(4x^3)}{(x^2+y^4)^2}$

$= \dfrac{2x^4y + 2x^2y^5 - 4x^2y^5}{(x^2+y^4)^2}$

$u_y = \dfrac{2x^4y - 2x^2y^5}{(x^2+y^4)^2}$

$$V_x = \frac{(x^2+y^4)(y^3) - (xy^3)(2x)}{(x^2+y^4)^2}$$

$$V_x = \frac{x^2y^3 + y^7 - \cancel{4xy^6}\; 2x^2y^3}{(x^2+y^4)^2} = \frac{-x^2y^3 + y^7}{(x^2+y^4)^2}$$

$$V_y = \frac{(x^2+y^4)(3xy^2) - (xy^3)(4y^3)}{(x^2+y^4)^2}$$

$$v_y = \frac{3x^2y^2 + 3xy^6 - 4xy^6}{(x^2+y^4)^2}$$

$$V_y = \frac{3x^2y^6 - xy^6}{(x^2+y^4)^2} = \frac{2xy^6}{(x^2+y^4)^2}$$

$\therefore\ u_x = v_y$

$\therefore$ function is analytic.

Q.2 $f(z) = x^2 + ay^2 - 2xy + i(bx^2 - y^2 + 2xy)$

$\therefore u = x^2 + ay^2 - 2xy$

$v = bx^2 - y^2 + 2xy$

$f(z)$ is analytic

$\therefore u_x = v_y$

$\therefore u_y = -v_x$

$\therefore u_x = 2x - 2y \qquad v_x = 2xb + 2y$

$u_y = 2ay - 2x \qquad v_y = -2y + 2x$

$u_y = -v_x$

$\therefore 2ay - 2x = -(2xb + 2y)$

$\therefore -2x + 2ay = -2xb - 2y$

$\therefore$ comparing,

$\cancel{-2x} = \cancel{-2x}b \qquad \cancel{2}ay = -\cancel{2}y$

$\therefore b = 1 \qquad \therefore a = -1$

$\therefore \boxed{\begin{aligned} a &= -1 \\ b &= 1 \end{aligned}}$

$f(z) = x^2 + ay^2 - 2xy + i(bx^2 - y^2 + 2xy)$

$\therefore f(z) = x^2 - y^2 - 2xy + i(x^2 - y^2 + 2xy)$

$\therefore f'(z) = 2x \cancel{2x} - 2y + i(2x + 2y)$

$f'(z) = 2(x - y) + 2i(x + y)$

$f(z) = x^2 + ay^2 - 2xy + i(bx^2 - y^2 + 2xy)$

using M-T and $a = -1, b = 1$

$x = z, y = 0$

$f(z) = z^2 + i(z^2)$

$f(z) = z^2(1+i)$

$\therefore f'(z) = 2z\cdot(1+i)$ //

Q.3 $u - v = (x-y)(x^2+4xy+y^2)$

$= x^3 + 4x^2y + xy^2 - x^2y - 4xy^2 - y^3$

$= x^3 + 3x^2y - 3xy^2 - y^3$

$f(z) = u + iv \quad - ①$

$\therefore if(z) = iu - v \quad - ②$

adding ① and ②

$f(z) + if(z) = u + iv + iu - v$

$\therefore f(z)(1+i) = (u-v) + i(u+v)$

$f(z)(1+i) = U + iV \quad - ③$

Here $U = u - v = x^3 + 3x^2y - 3xy^2 - y^3$

$\therefore U_x = 3x^2 + 6xy - 3y^2$

$U_y = 3x^2 - 6xy - 3y^2$

derivating eq ③

$f'(z)(1+i) = U_x + iV_x$

By C-R equations,

$u_x = v_y \qquad u_y = -v_x$

$\therefore v_x = -u_y$

$\therefore f'(z)(1+i) = U_x - iU_y$

$\therefore f'(z) (1+i) = u_x + i v_y$

$f'(z)(1+i) = (3x^2+6xy-3y^2) + i(3x^2-6xy-3y^2)$.

$\therefore$ by Milne – thompson rule,

$x = z,\ y = 0$

$\therefore f'(z)(1+i) = (3z^2) + i(3z^2)$

$f'(z)\cancel{(1+i)} = 3z^2\cancel{(1+i)}$

$\therefore f'(z) = 3z^2$

$\therefore$ Integrating

$\int f'(z) = \int 3z^2$

$\therefore \boxed{\cancel{f(z) =} z^3 + C}$

Ans. $\therefore \boxed{f(z) = z^3 + C}$

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Q.4 $f(z) = u + iv$

$v = \cos x \cosh y$

$\therefore v_x = (-\sin x)\cosh y$

$v_y = \cos x \sinh y$

$f'(z) = u_x + iv_x$

by C-R relation

$u_x = v_y \quad , \quad u_y = -v_x$

$\therefore f'(z) = v_y + iv_x$

$= \cos x \sinh y + i(-\sin x)\cosh y$

~~$= \cos x \sinh y - i \sin x$~~

$f'(z) = \cos x \sinh y - i \sin x \cosh y$

$\therefore$ by Milne - thompson rule

$x = z, \ y = 0$

$\therefore f'(z) = \cos z \sinh 0 - i \sin z \cosh 0$

$f'(z) = -i \sin z$

$\therefore \int f'(z) = -i \int \sin z$

$\therefore \boxed{f(z) = i\cos z + c}$

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S.E Comps - A

$\frac{10}{10}$ FR. CONCEICAO RODRIGUES COLLEGE OF ENGINEERING

15/9/23 Tutorial 5: Topic - Correlation, Regression and Curve Fitting

Q.1

Price in Rs	100	98	85	92	90	84	88	90	93	95
Sales Units	500	610	700	630	670	800	800	750	700	650

Q.2 Given two regression lines $3x+2y=26$ and $6x+y=31$.

1) Identify type of equation
2) Find $\bar{x}$ and $\bar{y}$
3) Find r

Q.1

Price in Rs (X-92)			Sales Units (Y-670)			Product
x	dx	dx^2	y	dy	dy^2	dx dy
100	8	64	500	-170	28900	-1360
98	6	36	610	-60	3600	-360
85	-7	49	700	30	900	-210
92	0	0	630	-40	1600	0
90	-2	4	670	0	0	0
84	-8	64	800	130	16900	-1040
88	-4	16	800	130	16900	-520
90	-2	4	750	80	6400	-160
93	1	1	700	30	900	30
95	3	9	690	20	400	40

$N = 10$ $\Sigma dx = -5$ $\Sigma dx^2 = 247$ $\Sigma dy = 150$ $\Sigma dy^2 = 76500$ $\Sigma dxdy = -3560$

~~using~~ with product method

$$r = \frac{\Sigma dx\,dy - \frac{(\Sigma dx)(\Sigma dy)}{N}}{\sqrt{\Sigma dx^2 - \frac{(\Sigma dx)^2}{N}}\sqrt{\Sigma dy^2 - \frac{(\Sigma dy)^2}{N}}}$$

$$\therefore r = \frac{-3560 - \frac{(-5) \times 150}{10}}{\sqrt{247 - \frac{25}{10}}\sqrt{76500 - \frac{22500}{100}}}$$

$$= \frac{-3485}{4261}$$

$$\therefore \boxed{r = -0.8179}$$

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without product

x	x^2	y	y^2	xy
100	10000	500	250000	50000
98	9004	610	372100	59780
85	7225	700	490000	59500
92	8464	630	396900	57960
90	8100	670	448900	60300
84	7056	800	640000	67200
88	7744	800	640000	70400
90	8100	750	562500	67500
93	8649	~~690~~ 700	490000	65100
95	9025	690	476100	65550
915	83967	6850	4766500	623290

i) y on x

$\Sigma y = Na + b\Sigma x$

$6850 = 10a + 915b$ — (1)

$\Sigma xy = a\Sigma x + b\Sigma x^2$

$623290 = 915a + 839676b$ — (2)

Solving (1), (2)

$a = 1989.20$

$b = -14.2535$

ii) x on y

$\Sigma x = Na_1 + b_1\Sigma y.$

$\therefore 915 = 10a_1 + 6850b_1$ — (1)

$\Sigma xy = a_1\Sigma y + b_1\Sigma y^2$

$\therefore 623290 = 6850a_1 + 4766500b_1$ — (2)

$\therefore a_1 = 123.65$

$b_1 = -0.0469$

$\therefore b = -14.25 \quad b_1 = -0.0469$

$\therefore r = -\sqrt{b_1 \cdot b}$

$\therefore \boxed{r = -0.8125}$

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Q.2

1) Assume that $3x + 2y = 26$ is y on x

$6x + y = 31$ is x on y

$b_{xy} = \frac{-1}{6} = -0.166$, $b_{yx} = \frac{-3}{2} = -1.5$

$r = -\sqrt{b_{xy} b_{yx}} = -0.4999$

$\therefore -1 \le r \le 1$. $\therefore$ assumption is correct.

2) To find $\bar{x}$, $\bar{y}$

$3x + 2y = 26$

$6x + y = 31 \times 2$

$(-)$ $\therefore 3x + \cancel{2y} = 26$

$12x + \cancel{2y} = 62$

$9x = 36$

$\therefore x = 4$

$3x + 2y = 26$

$\therefore 12 + 2y = 26$

$\therefore y = 7$

$\therefore \boxed{\bar{x} = 4, \ \bar{y} = 7}$

8) $3x+2y=26$ (Y on X)

$\therefore 2y=-3x+26$

$\therefore Y=-\frac{3}{2}x+13$

$\therefore byx=-\frac{3}{2}$

$6x+y=31$ (X on Y)

$\therefore 6x=-y+31$

$\therefore x=-\frac{1}{6}y+\frac{31}{6}$

$\therefore bxy=\frac{-1}{6}$

$r=\sqrt{byx \cdot bxy}$

$=\sqrt{\left(-\frac{3}{2}\right)\left(-\frac{1}{6}\right)}$

$=\sqrt{\frac{1}{4}}$

$\therefore r=\frac{1}{2}$

$\therefore \boxed{r=0.5}$

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22/9/23

Tutorial 6 - Probability theory

Q.1

$X = x$	0	1	2	3
$p(x)$	0.1	0.3	0.5	0.1

$Y = X^2 + 2X$, Find $E(Y)$ and $Var(Y)$.

Q.2 Given cumulative distribution function (cdf, $F(x)$) of a RV X, find its pdf $f(x)$.

$$F(x) = \begin{cases} 0, & x \le -1 \\ \frac{x+1}{4}, & -1 \le x \le 3 \\ 1, & x \geq 3 \end{cases}$$

Q.3 A RV X has pmf

$$p(x) = \frac{1}{2^x}, \quad x = 1, 2, 3, \dots, \infty.$$

Find MGF of X and hence the mean and variance of X.

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~~Q.4 $[(p \to q) \wedge \sim q] \to p$~~

~~$= [(\sim p \vee q) \wedge$~~

Q.1

$E(x) = \Sigma x \cdot p_i$

$= (0 \times 0.1) + (1 \times 0.3) + (2 \times 0.5) + 3(0.1)$

$= 0.3 + 1 + 0.3$

$E(x) = 1.6$

$E(x^2) = \Sigma x^2 \cdot p_i$

$= (0^2 \times 0.1) + (1^2 \times 0.3) + (2^2 \times 0.5) + (3^2 \times 0.1)$

$= 3.2$

$V(x) = E(x^2) - [E(x)]^2$

$= 3.2 - (1.6)^2$

$V(x) = 0.64$

$E(Y) = E(x^2 + 2x) = E(x^2) + 2E(x)$

$= 3.2 + 2 \times 1.6$

$\therefore \boxed{E(Y) = 6.4}$

$V(Y) = V(x^2 + 2x) = V(x^2) + 4V(x)$

$= 4(E(x))^2 \cdot Var(x) + 4V(x)$

.... by taylor's rule

$= 4 \times (1.6)^2 \times 0.64 + 4 \times 0.64$

$\therefore \boxed{V(Y) = 9.11}$

$$E(y^2) = (0.1 \times (0^2+2\times 0)^2) + (0.3 \times (1^2+2\times 1)^2)$$
$$+ (0.5 \times (2^2+2*2)^2) + (0.1 \times (3^2+2*3)^2)$$

$$= 0 \cdot 2.7 + 30 + 22.5$$

$$\therefore E(y^2) = 55.2$$

$$\therefore V(y) = E(y^2) - [E(y)]^2$$

$$= 55.2 - 6.4^2$$

$$\therefore \boxed{\cancel{V(y) = 48.8}}$$

$$\boxed{V(y) = 14.24}$$

Q.2 $f_x(x) = \frac{d}{dx} F(x)$

$$\therefore f(x) = \begin{cases} \cancel{0, \ x \leq -1} \\ \frac{1}{4}, & -1 \leq x \leq 3 \\ 0, & x \geq 3 \end{cases}$$

The required condition for a PDF is:-

$$\int_{-\infty}^{\infty} f(x)\,dx = 1$$

$$\therefore \int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{-1} f(x)\,dx + \int_{-1}^{3} f(x)\,dx + \int_{3}^{\infty} f(x)\,dx$$

$$= \int_{-\infty}^{-1} 0 \cdot dx + \int_{-1}^{3} \frac{1}{4}\,dx + \int_{3}^{\infty} 0 \cdot dx$$

$$= 0 + \frac{1}{4}[x]_{-1}^{3} + 0$$

$$= \frac{1}{4}(3-(-1))$$

$$\therefore \int_{-\infty}^{\infty} f(x)\,dx = 1$$

$\therefore f(x)$ satisfies condition of pdf.

$$\therefore f(x) = \begin{cases} \frac{1}{4}, & -1 \leq x \leq 3 \\ 0, & x \geq 3 \end{cases}$$

//

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Q-3 $p(x) = \frac{1}{2^x}$; $x = 1, 2, 3, \ldots, \infty$.

i) MGF

$M_x(t) = E[e^{tx}]$

$= \sum_{x=1}^{\infty} e^{tx} \cdot p(x)$

$= \sum_{x=1}^{x=\infty} e^{tx} \cdot \frac{1}{2^x}$

$= \sum_{x=1}^{x=\infty} (e^t)^x \cdot \left(\frac{1}{2}\right)^x = \sum_{x=1}^{x=\infty} \left(\frac{e^t}{2}\right)^x$

$\therefore M_x(t) = \left(\frac{e^t}{2}\right)^1 + \left(\frac{e^t}{2}\right)^2 + \left(\frac{e^t}{2}\right)^3 + \ldots \infty$

$= \frac{e^t}{2}\left[1 + \frac{e^t}{2} + \left(\frac{e^t}{2}\right)^2 + \ldots \infty\right]$

$= \frac{e^t}{2}\left[1 - \frac{e^t}{2}\right]^{-1} \ldots \left\{\frac{1}{1-x} = 1 + x + x^2 + \ldots\right\}$

$= \frac{e^t}{2}\left[\frac{2-e^t}{2}\right]^{-1}$

$= \frac{e^t}{\cancel{2}}\left[\frac{\cancel{2}}{2-e^t}\right]$

$\therefore \boxed{M_x(t) = \frac{e^t}{2-e^t}}$

ii) Mean $= \frac{d}{dt}[M_x(t)]_{t=0}$

$$= \left[\frac{(2-e^t)e^t - e^t(0-e^t)}{(2-e^t)^2}\right]_{t=0}$$

$$= \frac{1 - 1(-1)}{1^2}$$

$$= 2 \qquad \therefore \boxed{\text{Mean} = 2}$$

iii) Variance $= E(x^2) - [E(x)]^2$

$$E(x^2) = \frac{d^2}{dt^2}[M_x t]_{t=0}$$

$$= \frac{d}{dt}\left[\frac{(2-e^t)e^t - e^t(0-e^t)}{(2-e^t)^2}\right]_{t=0}$$

$$= \frac{d}{dt}\left[\frac{2e^t - e^{2t} + e^{2t}}{(2-e^t)^2}\right]_{t=0}$$

$$= \frac{d}{dt}\left[\frac{2e^t}{(2-e^t)^2}\right]_{t=0}$$

$$= \left[\frac{(2-e^t)^2 \cdot 2e^t - 2e^t \cdot (2)(2-e^t)(-e^t)}{(2-e^t)^4}\right]_{t=0}$$

$$= \frac{(2-1)^2(2)(1) + 2(1)(2)(2-1)(1)}{(2-1)^4}$$

$$= \frac{2+4}{1^4}$$

$$= 6$$

$$\therefore V(x) = 6 - 2^2$$

$$\therefore \boxed{\text{Variance} = 2}$$

M-7

Title: Application of Laplace transform

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Project no. 7

Graded

25/10/19

Application of Laplace transform for signal processing

Laplace transform is a mathematical tool that is widely used in signal processing for a variety of applications.

1. System Analysis and control: Laplace transform are used to analyze linear time-variant systems in terms of their transfer functions. This is crucial for control systems engineering, where engineers design controllers to achieve desired system behaviour.

2. Signal representation can be made more simpler by Laplace transform, making it easy to perform mathematical operations on them

3. Stability analysis is crucial to ensure that system remains ~~safe~~ within safe operational bound. Laplace transforms help in analyzing and characterizing the stability of linear time-invariant systems, enabling engineers to design control systems that manage stability.

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4. Convolution and Time-domain analysis:
Convolution is a fundamental operation in signal processing and Laplace transforms make it easier to perform convolution operations in the Laplace domain.

5. Frequency domain analysis:
By transforming signals and system function into laplace domain, signal processing engineers can study the frequency response, resonance, damping, and other characteristics that are crucial for applications like filter design, spectrum analysis, and communication systems.

Applications of Laplace Transform in Heat Conduction and Wave Propagation

In science and engineering understanding heat conduction and wave propagation is essential. Laplace transforms are powerful tools that simplify the mathematical description of these phenomena, making complex problem-solving more accessible

Heat Conduction:

Heat conduction involves the transfer of thermal energy within materials. The heat equation, partial differential equation, is used to describe this process, Laplace transforms are used to convert this equation into a simpler algebraic form in the Laplace domain. This simplification aids in efficiently solving heat conduction problems. For e.g.

consider the non-homogeneous heat equation in 1D in a normalised form:

$$k(x,t) ** [u_t(x,t) - u_{xx}(x,t)] = \sum_{i=1}^{n} f_i(x,t) ** y(x,t)$$

where $k(x,t)$ is a polynomial defined above under initial conditions.

$$u(x,0) = q_1(x) * q_2(x);$$
$$u(0,t) = w_1(t) * w_2(t);$$

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$$u_x(0,t) = \partial/\partial t\,(\omega_1(t) * \omega_2(t))$$

By applying Laplace transform we can reduce it to:

$$u(t,x) = L_s^{-1} L_p^{-1}\left[\frac{Q_1(p)\,Q_2(p) - p\,W_1(s)\,W_2(s)}{(s-p^2)}\,h. \frac{}{s-p^2}\right]$$

$$- L_s^{-1} L_p^{-1}\left[\frac{W_1(s)\,(s\,W_2(s) - W_2(0))}{(s-p^2)}\right] +$$

$$L_s^{-1} L_p^{-1}\left[\frac{\sum_{i=1}^{n} F_i(p,s)\,G(p,s)}{(s-p)^2 \quad R(p,s)}\right]$$

Wave propagation:

Wave propagation, found in various branches of physics and engineering is crucial. for understanding disturbances and oscillations. through mediums. The wave propagation equation, a partial differential equation, governs wave behaviour. Laplace transform simplifies the wave equation by transforming it into the Laplace domain. This simplifications is valuable in solving wave propagation problems, especially when dealing with complex initial or boundary conditions. This technique helps engineers and scientists gain insight into wave phenomena, making it applicable in telecommunication acoustics and seismology.

PROBABILITY AND STATISTICS.

The laplace transform is a powerful mathematical tool that finds extensive applications in various fields, including probability and statistics. It enables the transformation of functions from the time or space domain to the complex frequency domain. In the realm of probability and statistics, the laplace transform is particularly useful for analyzing the behaviour of random variables and probability distributions. Lets explore its application in the context of probability density functions (PDF'S) and cumulative distribution functions (CDF'S) with a specific example and couple of diagrams

EXAMPLE: EXPONENTIAL DISTRIBUTION

Consider a random variable x that follows an exponential distribution with the probability density function given by:
$f(x) = \lambda e^{-\lambda x}$ for $x \geq 0$ and 0 elsewhere where λ is rate parameter

STEP 1: Applying laplace transform
To find the laplace transform of the probability density function $f(x)$, we integrate it with respect to x from 0 to ∞.

$$F(s) = \int_0^\infty f(x)\, e^{-sx}\, dx$$

$$= \int_0^\infty \lambda e^{-\lambda x} e^{-sx}\, dx$$

$$= \int_0^\infty \lambda e^{-(\lambda+s)x}\, dx$$

$$= \frac{\lambda}{\lambda+s}.$$

STEP 2: Analyzing the laplace transform

The laplace transform of the exponential distribution's probability density function is $\lambda/\lambda+s$. This function is known as the moment generating function (MGF) of the exponential distribution. The MGF helps in finding moments of a random variable; making it a crucial tool for statistical analysis.

STEP 3: Visualization

With the help of laplace transform we can include graph of probability density function and laplace transform graph.

CONCLUSION:

The transformation enables the evaluation of statistical properties and moments of the exponential distribution thereby facilitating deeper insights into the behaviour of random variables and probability distributions. The use of laplace transform in probability and statistics offers a valuable framework for analyzing and understanding various probability distributions and their properties.

CIRCUIT ANALYSIS WITH LAPLACE TRANSFORM

The Laplace transform can be used to solve different circuit problems. To solve different circuit problems, first the differential equations of circuits are written. Then, they are solved by using Laplace transform. Also, the circuit itself may be converted into s-domain using Laplace transform and then the algebraic equations can be written and solved.

The electrical circuits can have three circuit elements:- (a) Resistor (R) (b) Inductor (L) and capacitor (C). The analysis of these elements using Laplace transform is given below:

PURE RESISTIVE CIRCUIT

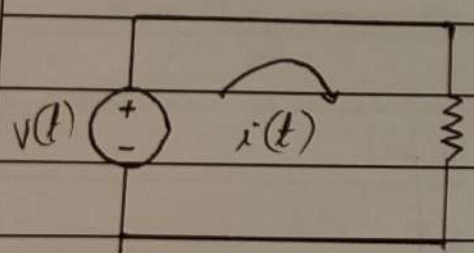

Figure 1

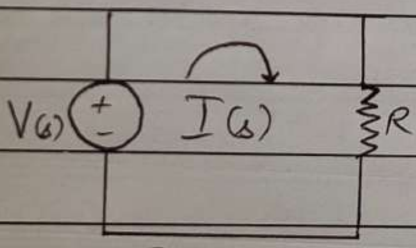

Figure 2

By applying KVL in this circuit, we can write:

$$\boxed{v(t) = R\,i(t)}$$

Therefore, the Laplace transform of this equation is given by,

$$\boxed{V(s) = R\,I(s)}$$

Here, it is noted that the resistance R ~~in~~ in t-domain remains R in s-domain. The Laplace transformed version is shown in figure 2.

PURE INDUCTIVE CIRCUIT

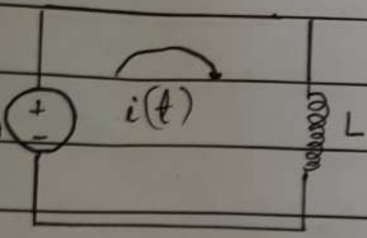

Figure 3

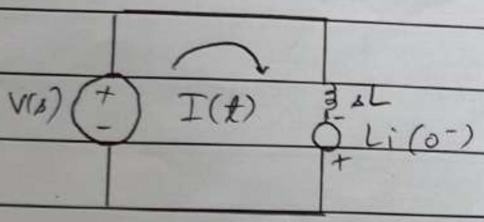

Figure 4

The voltage equation for the pure inductive element is given by:

$$\boxed{v(t) = L\frac{di(t)}{dt}}$$

Taking Laplace transform of the above equation on both sides:

$$V(s) = [sI(s) - i(0^-)]\,L$$

$$\Rightarrow \boxed{V(s) = sLI(s) - Li(0^-)}$$

where $i(0^-)$ is the initial current flowing through inductor.

→ The inductance (L) of the element in time domain becomes (sL) in s-domain.

→ Also, the circuit ~~element~~ current I(s) is opposed by the voltage $Li(0^-)$.

→ The Laplace transformed version of the pure inductive circuit is shown in figure-4.

Pure Capacitive Circuit

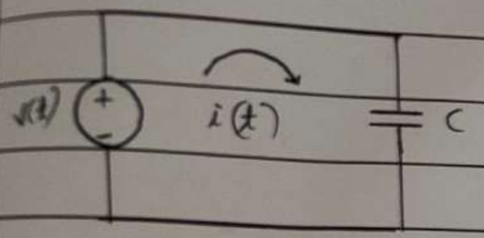

Figure 5

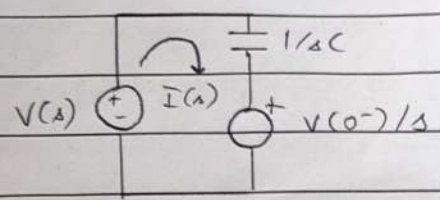

Figure 6

An electric circuit consisting of pure capacitive element is shown above. The current flowing through the capacitor is:

$$\boxed{i(t) = C\frac{dv(t)}{dt}}$$

Taking Laplace transform,

$$I(s) = C[sV(s) - v(0^-)] = sCV(s) - Cv(0^-)$$

$$\therefore \boxed{I(s) = sCV(s) - q(0^-)}$$

Also,

$$V(s) = \frac{1}{sC}I(s) + \frac{1}{sC}q(0^-) \Rightarrow \boxed{V(s) = \frac{I(s)}{sC} + \frac{v(0^-)}{s}}$$

Where $v(0^-)$ is initial voltage across the capacitor. Capacitance (C) in time domain becomes $\left(\frac{1}{sC}\right)$ in s-domain. Voltage $\left(\frac{v(0^-)}{s}\right)$ is introduced in s-domain which opposes the flow of circuit current. The Laplace transformed version of the pure-capacitive is shown in figure 6.

Application of Laplace Transform in Control System

Control System play a vital role in various engineering application, from regulating temperature in Industrial processes to ensuring the Stability & performance of aircraft

Laplace Transforms are a fundamental math tool widely used in Control systems engineering. They provide a powerful means to analyze, design & understand the behaviour of Control System in the following domain :-

Some key application of Laplace Transform in Control System are :-

1. Transfer function Analysis

→ One of the Central concepts in Control system in the transfer function, which relates the inputs & output of a given system. The Laplace Transform is Instrumental in determining transfer functions. By applying Laplace Transform to the governing differential Equation of a system, engineers can obtain a transfer function that satisfies the analysis of the system behavior

2. Stability Analysis

→ Analyzing the stability of control system is a critical step in its design. Laplace Transform allow engineers to determine the stability of a system by examining the poles (eigenvalues) of the transfer function. The location of these poles in the complex plane provide insights into the system stability characteristics.

3. Frequency Response Analysis

→ Understanding how a control system responds to different frequencies in essential for optimizing its performance. Laplace transform enable engineers by converting time-domain differential equation into Laplace ~~s~~ domain

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function that satisfies the analysis of the system behaviour

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3. Frequency Response Analysis

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ii. Controller Design

1) Designing Controllers to regulate system behaviour is a fundamental aspect of Control system engineering. Laplace transform are used to create transfer function for both system & the Controller. Engineers can then combine these transfers function to analyze the overall system response

⇒ Laplace transform are a fundamental math tool in Control System, enabling analysis design & optimisation of Control System.