MagLev Project Part 3

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The control objectives, linearized equations of motion, and plant transfer functions can all be seen below for the MagLev train.

Control Objectives

$T_r \leq 0.18\,s$ ⋯⋯ Operating Points: $\bar{h} = 10\,mm$

$\%OS < 40\%$ $\qquad \bar{V} = 42.20\,V$

$\qquad \bar{i} = 602.91\,A$

SS error for 5mm step < 1mm

" " 4000N step < 1mm

Assume system behaves like a second order underdamped system.

$$\rightarrow T_r \approx \frac{1.8}{\omega_n} \rightarrow \omega_n = \frac{1.8\,s}{0.18\,s} = 10\ ^{rad}/_{s}$$

$$\%OS = 40\% \rightarrow 0.4 = e^{-\pi\zeta/\sqrt{1-\zeta^2}} \rightarrow \zeta = \frac{-\ln(0.4)}{\sqrt{\pi^2 + \ln(0.4)^2}} = 0.28$$

Linearized EoM:

$$① \approx \bar{V} - R\bar{i} - R(i(t) - \bar{i}) - L\left(\frac{di}{dt}(t) - \frac{d\bar{i}}{dt}\right) - B_0 l \frac{1}{(h_0-\bar{h})^2}\frac{d\bar{h}}{dt}(h(t)-\bar{h}) - B_0 l \frac{1}{h_0-\bar{h}}\left(\frac{dh}{dt}(t) - \frac{d\bar{h}}{dt}\right)$$
$$+ (V(t) - \bar{V})$$

$$② \approx -NB_0 l \frac{1}{h_0-\bar{h}}\bar{i} + mg - NB_0 l \frac{1}{h_0-\bar{h}}(i(t)-\bar{i}) - NB_0 l \frac{1}{(h_0-\bar{h})^2}\bar{i}\,(h(t)-\bar{h}) + m\left(\frac{d^2h}{dt^2}(t) - \frac{d^2\bar{h}}{dt}\right)$$

With $h(t) = \hat{h}(t) + \bar{h}$, $v(t) = \hat{v}(t) + \bar{v}$, $i(t) = \hat{i}(t) + \bar{i}$

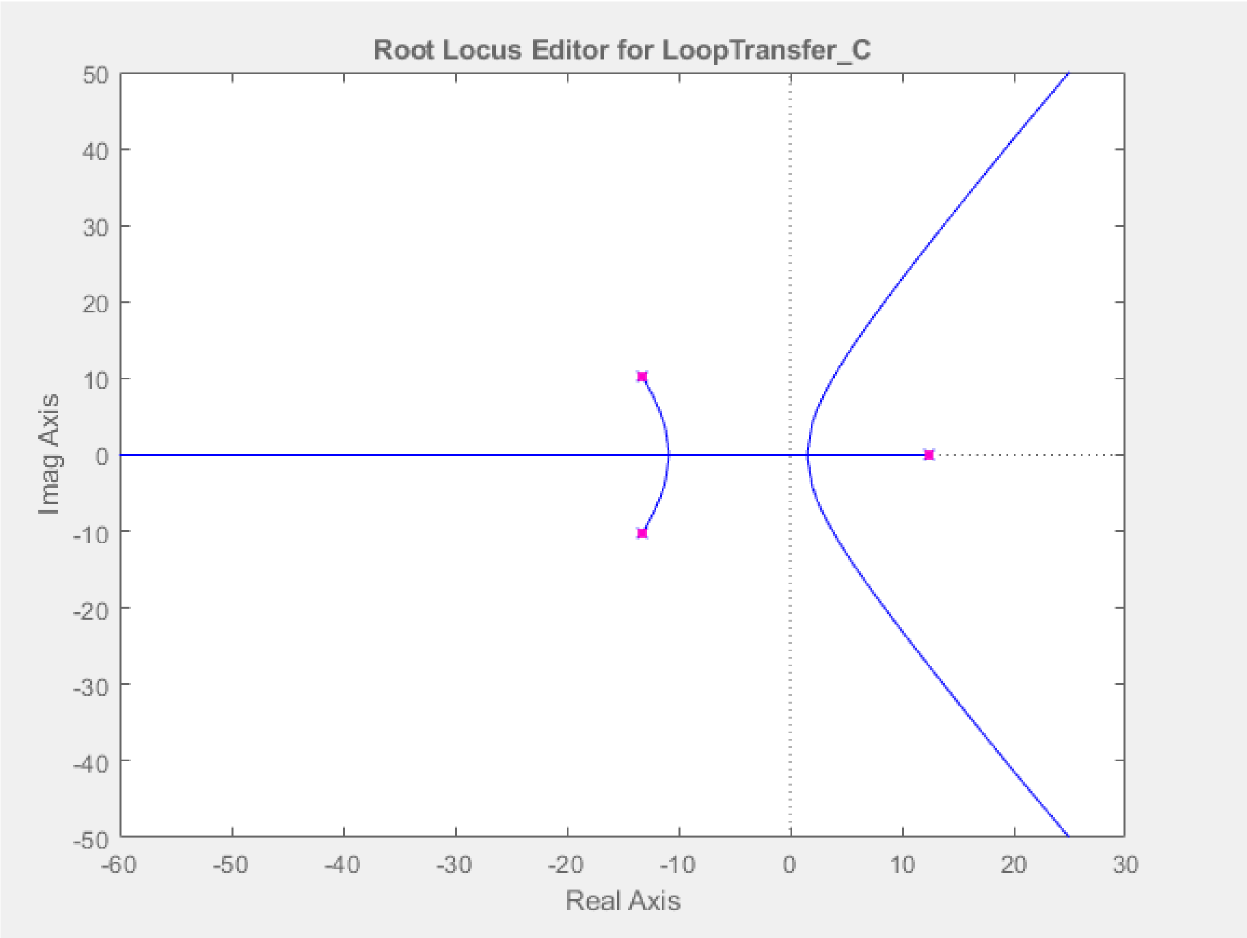
$$① \text{ becomes } -R\hat{i}(t) - L\frac{d\hat{i}}{dt} - B_0 l \frac{1}{h_0-\bar{h}} \cdot \frac{d\hat{h}}{dt}(t) + \hat{v}(t) \approx 0$$

$$② \text{ becomes } -NB_0 l \frac{1}{h_0-\bar{h}}\hat{i}(t) - NB_0 l \frac{1}{(h_0-\bar{h})^2}\bar{i}\,\hat{h}(t) + m\frac{d^2\hat{h}}{dt^2}(t) \approx 0$$

Transfer Function $\frac{\hat{H}(s)}{\hat{V}(s)} = G_2(s)$

$$G_2(s) = \frac{NB_0 l \frac{1}{h_0-\bar{h}}}{NB_0^2 l^2 \frac{1}{(h_0-\bar{h})^2}s + (R+Ls)\left(ms^2 + NB_0 l \frac{1}{(h_0-\bar{h})^2}\bar{i}\right)}$$

A controller needed to be designed to make the system stable and be within the criteria above. Simple P-control is not sufficient to achieve a stable closed loop system which is explained below.



Above is the root locus plot for the plant transfer function. Changing *k* will move the closed loop poles of the function along the root locus lines shown. From the graph, it can be seen that there is currently a closed loop pole in the right half plane, making the system unstable. As *k* is increased, this pole moves closer to the left half plane until it hits the breakaway point at 1.534. A pole from one of the left half plane pole approaches this value from the left at the same time. When *k* is increased further, these poles move along the root locus lines and stay in the right half plane, keeping the system unstable. From this, no value of *k* can be obtained to create a stable system.

The final design for the controller using root locus methods can be seen below.

$$K_{lead-lead-lag}(s) = 1.3945 * 10^7 \frac{(s+15)^2}{(s+103.26)^2} \frac{(s+2)}{(s+0.009)}$$

Design iterations on paper can be seen at the end of the report. Using Excel, a spreadsheet was created to manipulate the poles the root locus was forced through to find different pole locations and gain values for lead zeros of -15. A table of the results is summarized on the next page.

Lead Zero	Forced Location	Lead Pole	Gain	Alpha
-15	-4+12i	28.41	3924.0	5.79
-15	-6+20i	50.37	15554.1	4.59
-15	-10+30i	77.31	54705.9	3.07
-15	-10+5i	15.69	759.2	9.12
-15	-14+20i	103.26	63299.5	4.74
-15	-8+25i	65.20	3924.0	3.73

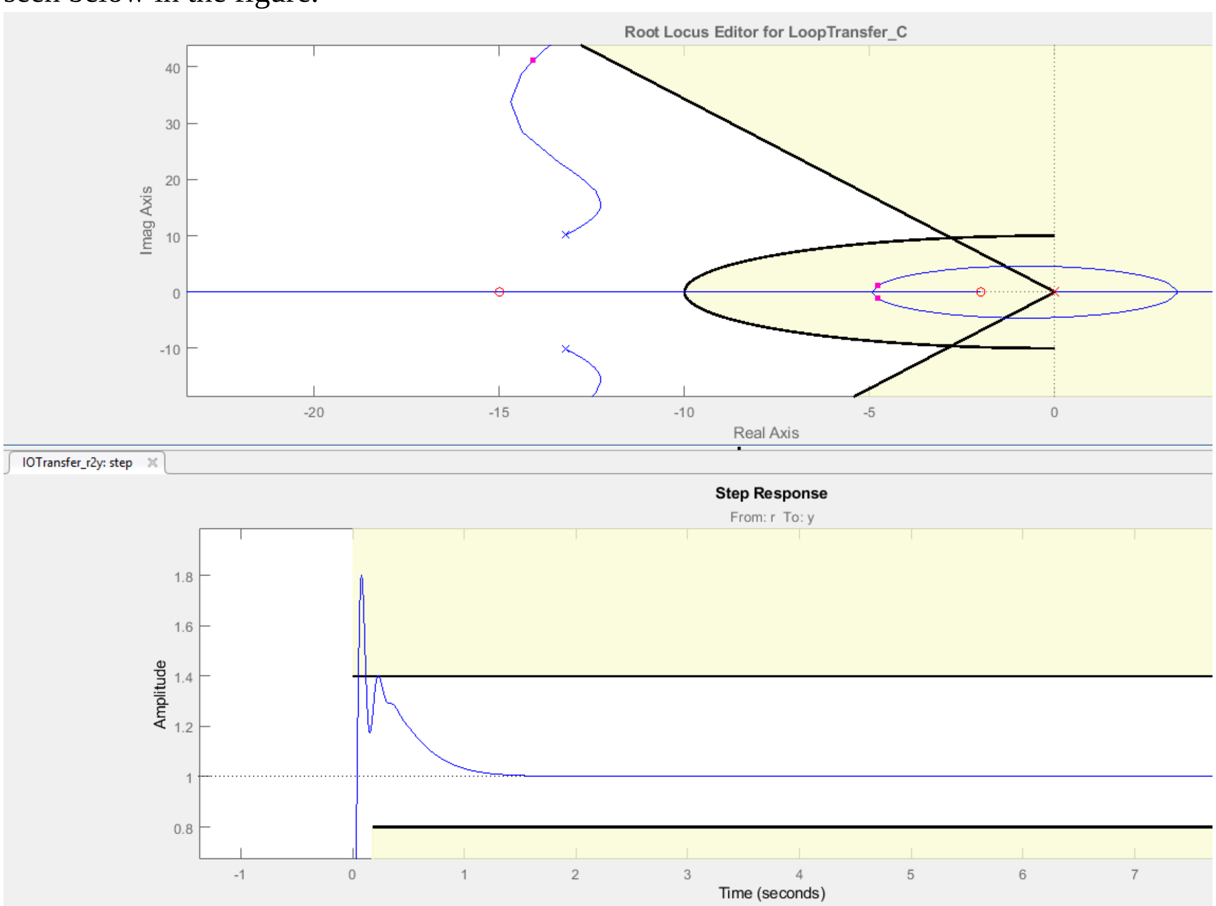
The pole location of 103.26 was selected based on results from MatLab's sisotool. All pole locations were evaluated by using the angle criteria the for root locus which can be seen below.

$$\sum_{i=1}^{m} \angle(s-z_i) - \sum_{i=1}^{n} \angle(s-p_i) = 180^\circ + 360^\circ(a-1)$$

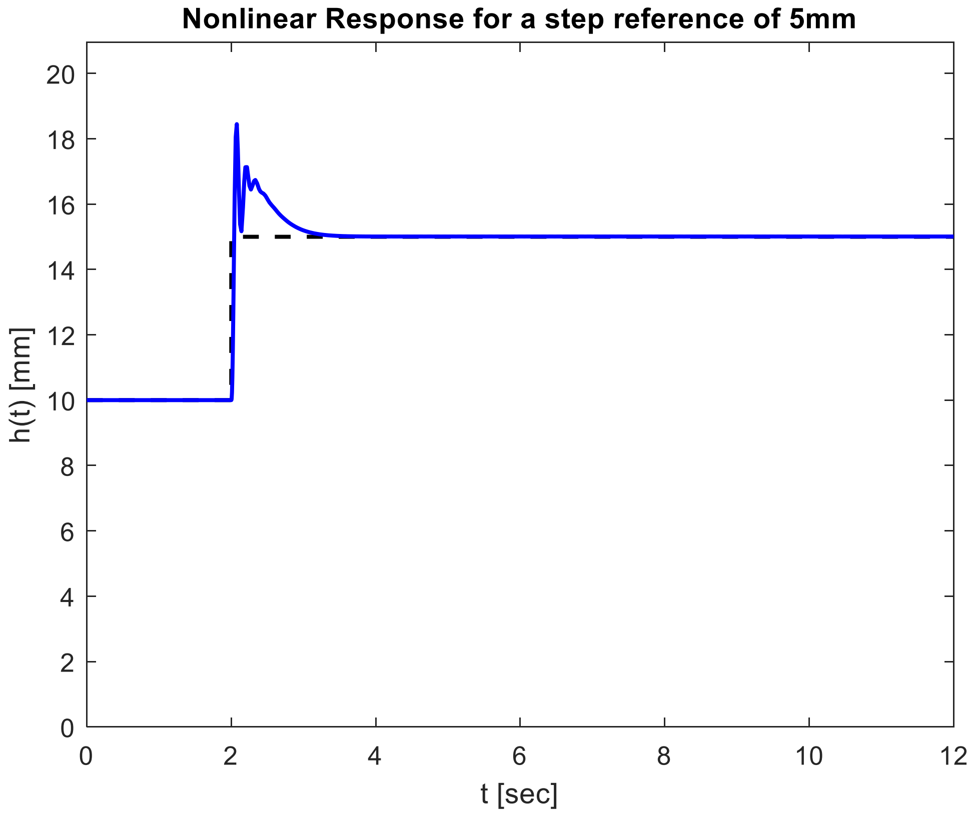
The lag controller was then solved for using the final value theorem for a step reference and type zero system:

$$e_{final} = \lim_{s\to 0} \cancel{s} \frac{\cancel{s^0}^{1} A(s)}{\cancel{s^0}^{1} A(s)+B(s)} \frac{1}{\cancel{s}} = \frac{A(0)}{A(0)+B(0)}$$

Where e_{final} equals -0.2. The chosen pole locations and step response of the linear system can be seen below in the figure.



The percent overshoot is still around 80 percent, which was as close to the 40% criteria the different iterations of the controller would get. This system was then modelled in Simulink for the nonlinear system, and a similar response was recorded and shown in the figure below.



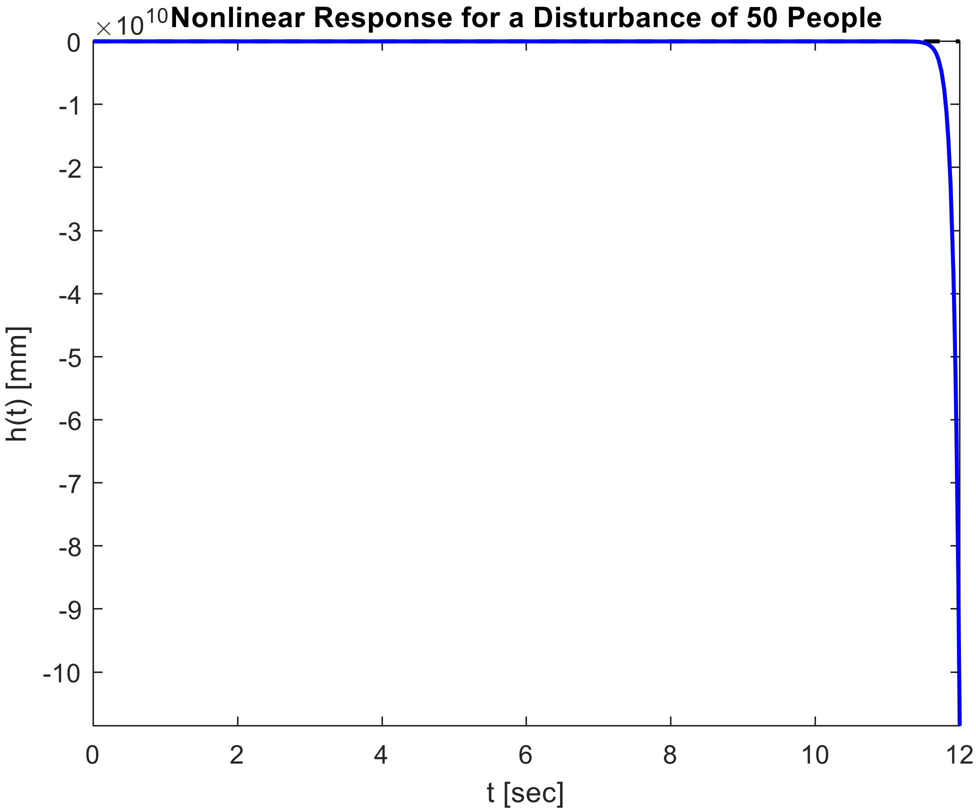
Next, a disturbance is added in the form of 50 passengers stepping onto the train. A transfer function relating the added mass step reference to the height of the train can be seen below.

$$\frac{(Ls+R)}{(Ls+R)\left(ms^2+N_6 B_0 L \frac{1}{(h_0-\bar{h})^T}\right)+\frac{B_0^2 L^2 N}{(h_0-\bar{h})^2}} \cdot M_{pass}\, g \,\frac{1}{s}$$

Work for getting this transfer function can be seen at the end of the report after the design iterations.

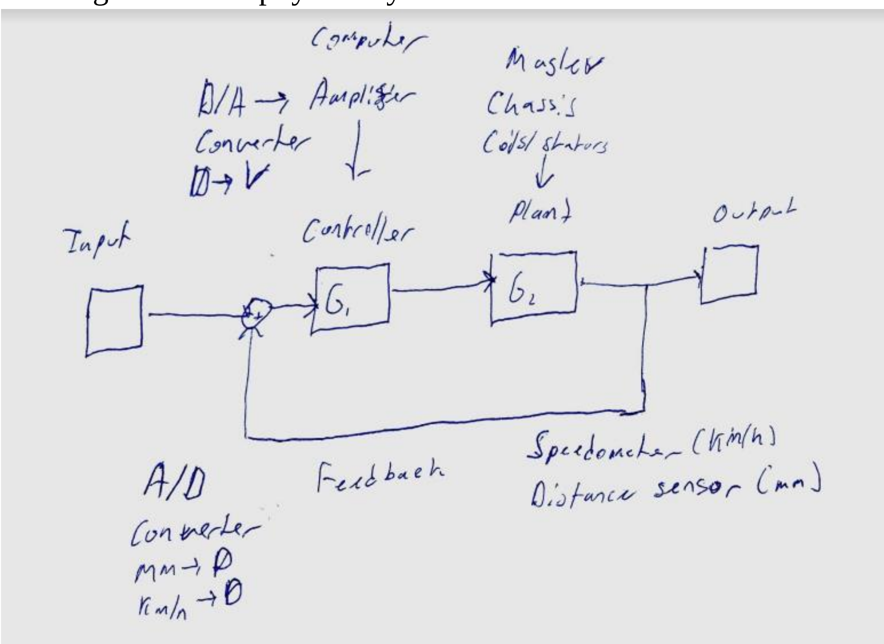
The lag controller design was checked with this new disturbance input. An alpha value of 220.3 was found, which was much greater than 4.74. This new alpha was used in place of the old, keeping the zero of the lag at 2. The new pole came out to be 0.009. This did not affect the transient response much, and the nonlinear response was nearly identical to the one shown above.

The disturbance was input into Simulink and run with the new controller. The output figure can be seen on the next page.



As soon as the disturbance is input into the system, it exponentially decreases and the system is no longer stable. A solution to this has not yet been found.

A feedback block diagram for the physical system can be seen below.



Hardware in the form of the controller and A2D / D2A converters would likely need to be purchased as well as sensors for the train.

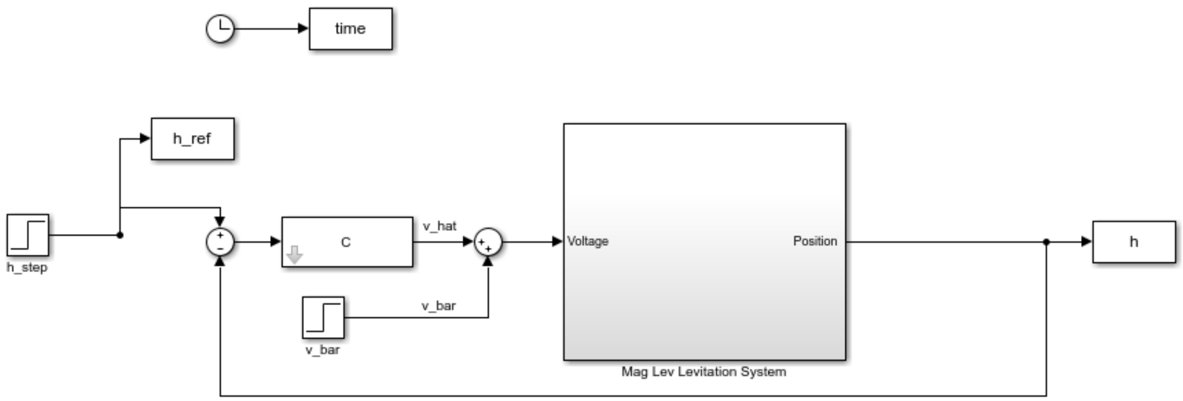
MatLab Code

```
%% ME 3360 Project Parts 2 and 3 Solution

clear
%% Constants
m = 5.9e4;              % kg
l = 20;                 % m
B0 = 0.12;              % Wb/m
h0 = 50e-3;             % m
bar_h = 10e-3;          % m
i_limit = 1000;          % A
g = 9.81;               % m/sec^2
FS = 1.5;               % Factor of safety
R = 0.07;               % ohms
L = 0.005;              % H
t_f = 12;               % s

%% Problem 2b

N_min = (m*g*(h0-bar_h))/(B0*l*i_limit);
N = ceil(N_min*FS)+mod(ceil(N_min*FS),2);      % stator/coil units for
subsequent analyses

%% Problem 2d
bar_i = m*g*(h0-bar_h)/(N*B0*l);
bar_v = R*bar_i;

%% Problem 2f

G_2 = tf([N*B0*l/(h0-bar_h)],([0 0 N*B0^2*l^2/(h0-bar_h)^2 0]+conv([L R],[m 0
-N*B0*l*bar_i/(h0-bar_h)^2])));
%figure(10)
%pzmap(G_2)
%grid on
%%
sisotool(G_2)

%% Part 3
load('Part_3_WITH_C_Controller_ALPHA2.mat')    %Brings in C from RL

G_22 = tf([L R], ([0 0 N*B0^2*l^2/(h0-bar_h)^2 0]+conv([L R],[m 0 -
N*B0*l*bar_i/(h0-bar_h)^2])));
m_pass = 50*80;     %kg
%%
%pzmap(C)
%% Add controller design here

sim part_3_sim_w_train

figure(100)
plot(time, h_ref*1000, 'k--', time, h*1000, 'b-', 'LineWidth', 1.5)
xlabel ('t [sec]')
ylabel ('h(t) [mm]')
title('Nonlinear Response for a Disturbance of 50 People')
```

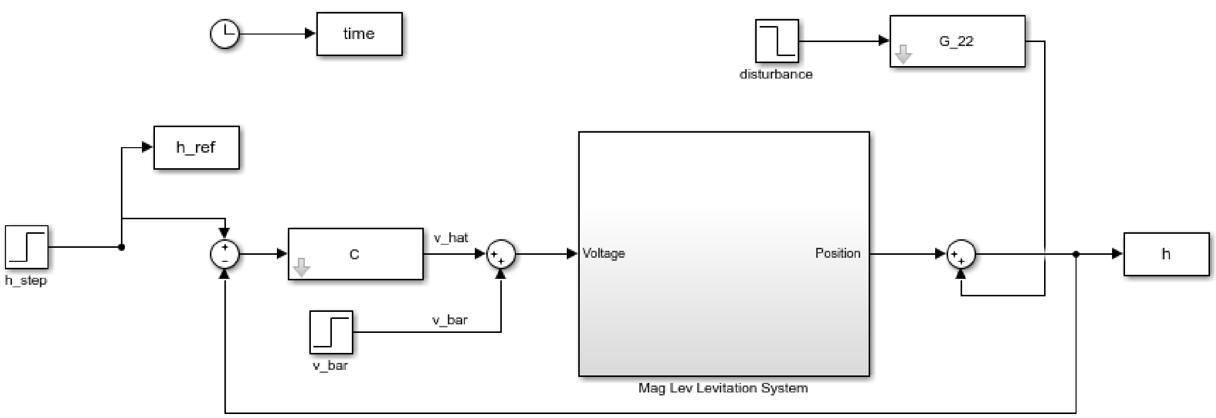
```
ylim ([min([0 min(h)]) max(h)+.0025]*1000)
```

Simulink Diagram

No Disturbance



With Disturbance



Design Iterations

Force asymptote intersection through -50

$$\sigma_a = \frac{\Sigma z - \Sigma p}{m-n} = \frac{(-15-15)-(-13.2-13.2+12.39+2p_c)}{-3} = -50$$

$$-150 = -30 + 14 + 2p_c = -16 + 2p_c$$

$2p_c = 166 \Rightarrow \boxed{p_c = 83}$ – In sisotool – does not move RL enough to get poles in required criteria.

Try $\sigma_a = -100 \rightarrow$

$\Rightarrow 300 = -16 + 2p_c \Rightarrow p_c = 158$

Try $\sigma_a = -150$

$\rightarrow 450 = -16 + 2p_c \rightarrow p_c = 233$ ✓

To find $K \rightarrow$ force RL through $s = -43.5 \pm 113.5i$

$$K = \frac{\sqrt{30.3^2+103^2}\sqrt{30.3^2+124^2}\sqrt{55.9^2+113.5^2}\left(\sqrt{189.5^2+113.5^2}\right)^2}{3.2542\left(\sqrt{28.5^2+113.5^2}\right)^2} = 1.8984 \times 10^6$$

3.0611×10^6

$$\boxed{\rightarrow K_{lead}(s) = 1.8984 \times 10^6 \frac{(s+15)^2}{(s+233)^2}}$$

Now: $K_{lag}(s) = \alpha \dfrac{Ts+1}{\alpha Ts+1} \rightarrow G(s) = \alpha \dfrac{Ts+1}{\alpha Ts+1} K \dfrac{(s+15)^2}{(s+233)^2} \cdot \dfrac{(3.2542)}{(s+12.39)(s^2+26.39s+277.1)}$ $\leftarrow B$ $\leftarrow A$

Want: $e_{final} = -0.2 = \dfrac{A(0)}{A(0)+B(0)}$

$$\rightarrow \frac{(\alpha T\cancel{s}+1)(\cancel{s}+233)^2(\cancel{s}-12.39)(\cancel{s^2}+\cancel{26.39s}+277.1)}{(\alpha T\cancel{s}+1)(\cancel{s}+233)^2(\cancel{s}-12.39)(\cancel{s^2}+\cancel{26.39s}+277.1)+(\alpha)(T\cancel{s}+1)(K)(\cancel{s}+15)^2(3.2542)} = \frac{-186388741}{-186388741 + \alpha K \cdot 732.2} = -0.2$$

$$\rightarrow \alpha = \frac{-186388741 + (0.2)(-186388741)}{-0.2(732.2)(1.8984 \times 10^6)} = 0.805$$

Lead Lead Design

$$k_{lead} = k\frac{(s+z_c)(s+z_c)}{(s+p_c)(s+p_c)} = k\frac{(s+z_c)^2}{(s+p_c)^2} \rightarrow \text{Say } z_c = -15$$

Force RL to leave $-13.2+10.1i$ at $180°$

$$\angle(s-p_j) = \sum_{i=1}^{m}\left(\angle(p_j - z_i)\right) - \sum_{i=1, i\neq j}^{n}\angle(p_j - p_i) - \cancel{180°} = 180°$$

$$\sum_{i=1}^{m}\angle(p_j - z_i) = \angle(-13.2+10.1i+15) + \angle(-13.2+10.1i+15) =$$
$$= 159.8°$$

$$\sum_{i=1, i\neq j}^{n}\angle(p_j - p_i) = \angle(-13.2+10.1i-12.4) + \angle(-13.2+10.1i+p_c) + \angle(-13.2+10.1i+p_c)$$
$$= 158.5° + 2\tan^{-1}\left(\frac{10.1}{p_c - 13.2}\right)$$

$$\Rightarrow 159.8° = 158.5° + 2\tan^{-1}\left(\frac{10.1}{p_c-13.2}\right) \Rightarrow p_c = 903.5 \text{ - not reasonable}$$

$\rightarrow$ Force to $90°$

$$\Rightarrow 159.8° = 158.5° + 2\tan^{-1}\left(\frac{10.1}{p_c-13.2}\right) + 270 \Rightarrow p_c =$$

Lead Controller Design

$K_{Lead} = k \frac{s + z_c}{s + p_c}$ → Set $z_c = -15$ rad/s $= \omega_n$ as a starting point

Try to force RL through $s = -7 \pm 12.6i$ - within criteria for control objective.

Current poles at $s = -13.20 \pm 10.15i$, 12.39

$\theta_1 = \angle(s + 13.20 + 10.15i) = \angle(6.20 + 2.45i) = 21.56°$ - pole

$\theta_2 = \angle(s + 13.20 - 10.15i) = \angle(6.20 + 22.75i) = 74.76°$ - pole

$\theta_3 = \angle(s - 12.39) = \angle(-19.39 + 12.6i) = 146.98° = 147.0°$ - pole

$\theta_4 = \angle(s + 15) = \angle(8 + 12.6i) = 76.61°$ - Zero x2 =

$\theta_5 = \angle(s + p_c) = \angle(-7 + 12.6i + p_c) = \tan^{-1}\left(\frac{12.6}{p_c - 7}\right)$ - pole

$\sum \angle(s - z_i) - \sum \angle(s - p_i) = 180° = \theta_4 - (\theta_1 + \theta_2 + \theta_3 + \theta_5)$

$\Rightarrow 2\theta_5 = \theta_4 - \theta_1 - \theta_2 - \theta_3 - 180 = 2(76.61°) - 21.56° - 74.76° - 147.0 - 180° = 13.29°$

$\Rightarrow \tan(13.29°) = \frac{12.6}{p_c - 7} \Rightarrow \boxed{p_c = 60.34}$

$$K_{lead} = k \frac{s + 10}{s + 60.34} \rightarrow k = \frac{1}{|G(s)|} = \frac{\sqrt{6.2^2 + 2.45^2}\sqrt{6.2^2 + 22.75^2}\sqrt{19.39^2 + 12.6^2}\sqrt{53.34^2 + 12.6^2}}{3.2542\ \sqrt{13^2 + 12.6^2}}$$

$k = 4726.7 \approx 4727$

$$\boxed{K_{lead} = 4727 \frac{s + 10}{s + 60.34}}$$

Lead Lead Design

$K_{lead}(s) = k\frac{(s+z_c)^2}{(s+p_c)^2}$ Force $z_c = -15$, RL through $s = -8 \pm 25i$

Plant poles: $-13.2 \pm 10.15i$, 12.4

$\theta_1 = \angle(s+15) = \angle(7+25i) = 74.36°$

$\theta_2 = \theta_1 = 74.36°$

$\theta_3 = \angle(s+p_c) = \angle(p_c - 8 + 25i) = \tan^{-1}\left(\frac{25}{p_c - 8}\right)$

$\theta_4 = \theta_3 =$

$\theta_5 = \angle(s + 13.2 + 10.15i) = \angle(5.2 + 14.85i) = 70.70°$

$\theta_6 = \angle(s + 13.2 - 10.15i) = \angle(5.2 + 36.15i) = 81.81°$

$\theta_7 = \angle(s - 12.4) = \angle(-20.4 + 25i) = 129.21$

$\sum\angle(s-z_c) - \sum\angle(s-p_c) = 180 = 2\theta_1 - (2\theta_3 + \theta_5 + \theta_6 + \theta_7) = 180$

$$\theta_3 = \frac{2\theta_1 - \theta_5 - \theta_6 - \theta_7 - 180}{2} = \frac{2(74.36°) - 70.70° - 81.81° - 129.21° - 180°}{2} = -156.5°$$

$$\rightarrow p_c = \frac{25}{\tan(-156.5)} + 8 = 65.5$$

$$K = \frac{1}{|G(i\omega)|} = \frac{\left(\sqrt{57.5^2+25^2}\right)^2\sqrt{5.2^2+14.85^2}\sqrt{5.2^2+36.15^2}\sqrt{20.4^2+25^2}}{3.2542\left(\sqrt{7^2+25^2}\right)^2} = \boxed{33234}$$

$$K_{lead}(s) = 33234\frac{(s+15)^2}{(s+65.5)^2}$$

Lead Lead Design

$K_{lead}(s) = k\,\frac{(s+z_c)^2}{(s+p_c)^2}$ Set $z_c = -15$, force RL through $-4 \pm 12i$

Plant poles: $s = -13.2 \pm 10.15i,\ 12.4$

$\theta_1 = \angle(s+15) = \angle(11+12i) = 47.5^\circ$

$\theta_2 = \theta_1 = 47.5^\circ$

$\theta_3 = \angle(s+p_c) = \tan^{-1}\left(\frac{12}{p_c-4}\right)$

$\theta_4 = \theta_3$

$\theta_5 = \angle(s+13.2+10.15i) = \angle(9.2+1.85i) = 11.37^\circ$

$\theta_6 = \angle(s+13.2-10.15i) = \angle(9.2+22.15i) = 67.44^\circ$

$\theta_7 = \angle(s-12.4) = \angle(16.4+12i) = 143.81^\circ$

$$\theta_3 = \frac{2\theta_1 - \theta_5 - \theta_6 - \theta_7 - 180}{2} = -153.81^\circ \rightarrow p_c = \frac{12}{\tan(153.81)} + 4 = 28.4$$

Final Iteration

$$K_{lead}(s) = 1.2986 \times 10^5\,\frac{(s+15)^2(s+2)}{(s+103)^2(s+0.422)}$$

Lag Design and finding G22

Lag Design

$G_{lag}(s) = \alpha \cdot \frac{Ts+1}{\alpha Ts+1}$ need $e_{final} < -0.2$ for step ref

$$e_{final} = \frac{A(0)}{A(0)+B(0)} = -0.2$$

$A(0) = (\alpha T\cancel{s}+1)(\cancel{s}+65.5)^2(\cancel{s}-12.4)(\cancel{s}^2+2\cancel{6.34s}+1177.1)$

$= -14.7415 \times 10^6$

$B(0) = \alpha(T\cancel{s}+1)(K)(\cancel{s}+15)^2(3.2542) = 24.33 \times 10^6 \alpha$

$$\rightarrow \frac{-14.7415 \times 10^6}{-14.7415 \times 10^6 + 24.33 \times 10^6 \alpha} = -0.2 \rightarrow \alpha = \frac{-14.7415 \times 10^6 - (0.2)(+14.7415 \times 10^6)}{(0.2)(24.33 \times 10^6)} = 3.635$$

EoM:

$$-R\hat{i}(t) - L\frac{d\hat{i}}{dt} - B_0 l \frac{1}{h_0 - \bar{h}} \frac{d\hat{h}}{dt} + \hat{V}(t) \approx 0$$

$$-NB_0 l \frac{1}{h_0-\bar{h}} \hat{i}(t) - NB_0 l \left(\frac{1}{h_0-\bar{h}}\right)^2 \bar{i}\, \hat{h}(t) + m\frac{d^2\hat{h}}{dt^2}(t) + m_{pass}\, g\, u(t) \approx 0$$

$$\begin{bmatrix} Ls+R & B_0 l \frac{1}{h_0-\bar{h}}(s) \\ -NB_0 l\left(\frac{1}{h_0-\bar{h}}\right) & -N_0 B_0 \left(\frac{1}{h_0-\bar{h}}\right)^2 \bar{i} + ms^2 \end{bmatrix} \begin{bmatrix} \hat{I} \\ \hat{H} \end{bmatrix} = \begin{bmatrix} \hat{V}(s) \\ -m_{pass}\, g \frac{1}{s} \end{bmatrix}$$ disturbance

Cramer's Rule $\rightarrow$

$$\hat{H} = \frac{\det \begin{bmatrix} Ls+R & \hat{V}(s) \\ \frac{-BLN}{h-\bar{h}_0} & -m_{pass}\, g \frac{1}{s} \end{bmatrix}}{\det \begin{bmatrix} Ls+R & B_0 l \frac{1}{h_0-\bar{h}} s \\ \frac{-BLN}{h-h_0} & -N_0 B_0 L \left(\frac{1}{(h_0-\bar{h})}\right)^2 \bar{i} + ms^2 \end{bmatrix}}$$

G_{22}

$$\hat{H} = \frac{\frac{BLN}{h_0-\bar{h}}}{(Ls+R)\left(ms^2 + N_0 B_0 L\left(\frac{1}{h_0-\bar{h}}\right)^2 \bar{i}\right) + \frac{B_0^2 L^2 N}{(h_0-\bar{h})^2}} \cdot \hat{V}(s) - \boxed{\frac{(Ls+R)}{(Ls+R)\left(ms^2 + N_0 B_0 L\left(\frac{1}{h_0-\bar{h}}\right)^{\bar{i}}\right) + \frac{B_0^2 L^2 N}{(h_0-\bar{h})^2}}} \cdot m_{pass}\, g \frac{1}{s}$$

α For $\dfrac{G_{22}}{1+G_{21}G_{11}}$ $\leftarrow B(0) = R * 15^2 * 6330 = 996975\alpha$

$\leftarrow A(0) = -36607774$

$$\frac{A(0)}{B(0)+A(0)} = -0.2 \rightarrow \frac{A(0)+(0.2)A(0)}{(0.2)(996975)} = \alpha = 220.3 \leftarrow \text{Use this } \alpha!$$