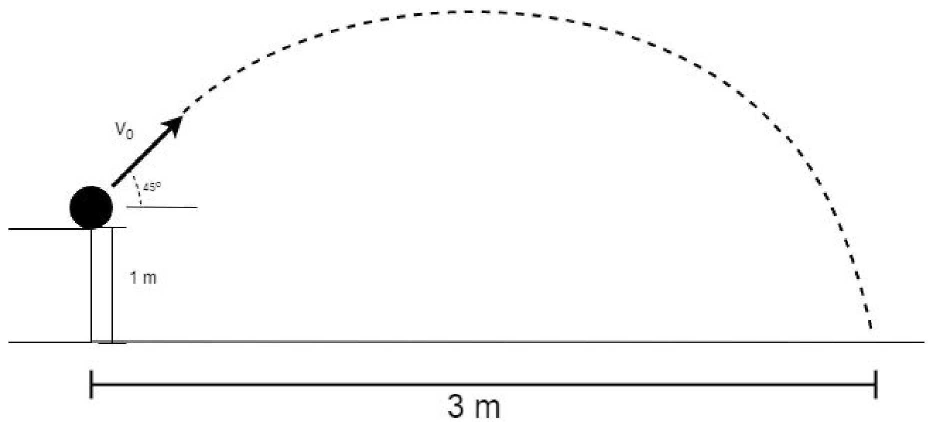
Spring Design:

In order to select a spring to use for the launcher, the projectile motion of the ball needed to be analyzed at the angle of launch for max distance (45^{o}). Projectile motion for an initial launch height of one meter and max distance of three meters was determined.



The equations to solve for initial velocity are below.

$$x = V_0 sin\,(45)\,t$$

$$y = V_0 sin(45)t + \tfrac{1}{2}gt^2$$

Where *x* and *y* are horizontal and vertical distance in meters, V_0 is initial velocity in meters per second, *t* is time in seconds, and *g* is acceleration due to gravity in meters per second squared. Solving the equations yielded an initial velocity of 4.7 m/s to achieve the desired motion.

Next, several springs from McMaster Carr were selected based on their rate. Springs between 5 and 8 inches long, with outer diameters smaller than that of a baseball and fairly small rates were selected. In order to calculate the distance and force needed to propel the ball at 4.7 m/s, the energy equation and Hooke's law for springs were used.

$$\sum E_i = \sum E_f \rightarrow \frac{1}{2}kx^2 = \frac{1}{2}mv^2$$

$$F = kx$$

Where E_i and E_f are the initial and final energies of the spring, k is spring rate, x is the distance the spring is deflected, m is the mass of the baseball and ball cup, v is desired velocity, and F is the force required to compress the spring. The final potential energy of the ball is neglected as it adds a negligible energy value. Equation # was first evaluated for x, then equation # for F. A summary of the results can be seen below in the table.

Table #: Summary of selected springs with deflection and force required for each.

McMaster Carr Spring Number	Spring Rate (lb/in)	Energy Required (J)	Spring Deflection (in)	Force Needed (lb)
9657K469	22.25	22.38	2.98	66.38
96485K135	38.2	22.38	2.28	86.98
96485K156	32.3	22.38	2.48	79.98

Each of these springs were then checked for failure. Due to the need to be operated for a total of 2440 hours per year, it was conservatively estimated that the spring would be deflected 10^6 times per year, and fatigue strength would be the limiting failure mechanism. The equations for max compression spring shear stress can be seen below.

$$\tau_w = K_w \frac{8FD}{\pi d^3}$$

$$K_w = \frac{4c-1}{4c-4} + \frac{0.615}{c}$$

$$c = \frac{D}{d}$$

Where τ_w is max shear stress, K_w is the Wahl factor, and c is the spring index. The fatigue failure limits were then calculated using the equations below.

$$\tau_f = 0.4 S_{ut}$$

$$S_{ut} = B d^a$$

Where τ_f is the shear stress fatigue limit, S_{ut} is the ultimate tensile strength, and B and a are empirically found coefficients for the spring materials from Table 14.1 in the textbook [1]. The factor of safety for each of the springs was then found using the equation below.

$$n = \frac{\tau_f}{\tau_w}$$

Table # below summarizes the results of the fatigue analysis.

Table #: Summary of selected springs for fatigue analysis.

McMaster Carr Spring Number	Max Shear Stress (ksi)	Fatigue Shear Stress Limit (ksi)	Factor of Safety
9657K469	110.53	103.52	0.94
96485K135	63.40	79.46	1.25
96485K156	52.39	78.37	1.50

The analysis assumed each spring was unpeened and has a much higher life cycle than needed giving a conservative factor of safety. From each of these springs, spring 94685K156 was chosen as it give the largest factor of safety for this application, and is greater than the design requirement of 1.25.