MagLev Project Part 2

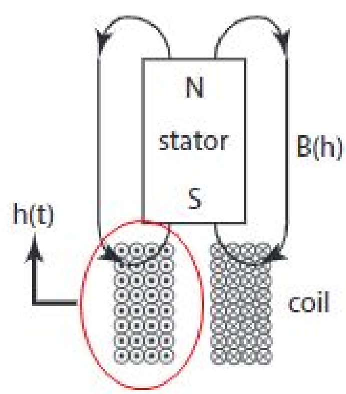
Mark Luke

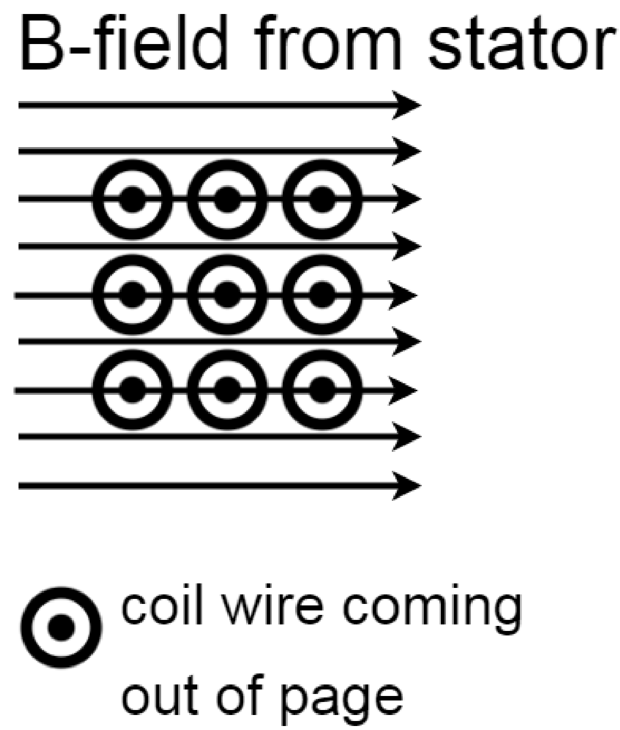
Vihn Peters

ME3360: Dr. Hoelzle

3/23/2018

a) From the figure in the problem statement, the left portion of the coil is experiencing a magnetic field in the right direction from the stator coil, shown below.





Using the right hand rule, pointing fingers in the direction of the B-field and thumb in the direction of the current, the force from the field is in the direction of the palm which should be facing upwards. Therefore, the coil will move up towards the stator.

b) Assuming each coil turn is experiencing the same magnetic field and the magnetic field intensity is given by:

$$B = B_0 * \frac{1}{h_0 - h(t)}$$

The minimum number of stator coils can be calculated using the Bli law with constants B_0 and h_0 shown in the calculation on the next page. Each coil is a length of 20 m with a max current of 1000 A.

$$B = B_0 \cdot \frac{1}{h_0 - h(t)} \qquad B_0 = 0.12\ \mathrm{Wb/m}$$
$$h_0 = 50\,\mathrm{mm} = 0.05\,\mathrm{m}$$

$I_{max} = 1000\ \mathrm{A}$
$L_{max} = 20\,\mathrm{m}$

Find min # stator coil pairs to hold $\bar{T}$ @ 10mm
Use an FOS of 1.5

$F = BI\ell \sin\theta$ - assume $\theta = 90°$ → $\sin\theta = 1$

$$\Rightarrow F_{coil} = B_0 \cdot \frac{1}{h_0 - h(t)} \cdot L_{max} I_{max} = 0.12\,\tfrac{\mathrm{Wb}}{\mathrm{m}} \cdot \frac{1}{(0.05-0.01)\,\mathrm{m}} \cdot 20\,\mathrm{m} \cdot 1000\,\mathrm{A} = 60{,}000\,\mathrm{N}$$

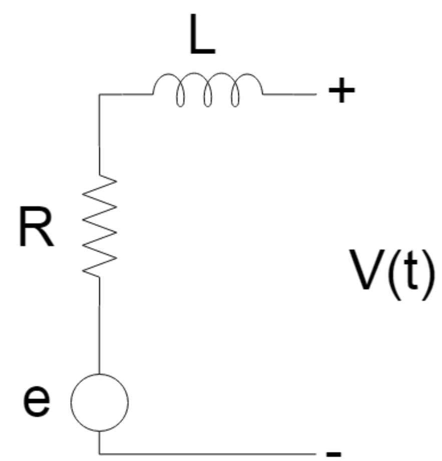
$$F_{train} = mg = (5.9\times10^4\,\mathrm{kg})(9.81\,\mathrm{m/s^2}) = 578790\,\mathrm{N}$$

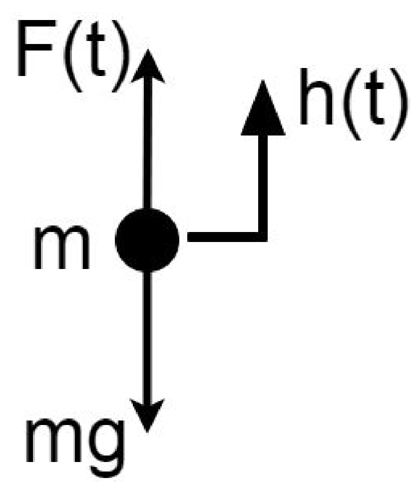
$$\Rightarrow N_{pairs} = \frac{F_{train}}{F_{coil}} = \frac{578790\,\mathrm{N}}{60{,}000\,\mathrm{N}} = 9.65 \Rightarrow \text{round up to } \underbrace{\boxed{10}}_{\text{FOS}} \times 1.5 = \boxed{15}$$

Minimum 15 pairs to make train levitate.

There would need to be a minimum of 15 current pairs with the safety factor included.

c) Assuming the coil acts like an inductor and resistor and generates a back emf *e*, the coil can be viewed as the circuit below, with the free body diagram of the train next to it.





The equations of motion were found and shown below, assuming the Bli and Blu laws can be used for the system and no damping occurs.

From Kirchoff's Voltage Law,

$$V(t) - L\frac{di}{dt} - Ri(t) - e = 0 \quad \text{where } e = Bl \cdot u, \; u = \frac{dh}{dt}$$

$$L\frac{di}{dt} + Ri(t) + Bl\frac{dh}{dt} = V(t) \quad ①$$

From FBD

$$m\frac{d^2h}{dt^2} = F(t) - mg \quad \text{where } F(t) = Bli(t)$$

$$m\frac{d^2h}{dt^2} + mg - Bli(t) = 0 \quad ②$$

EoM:

$$L\frac{di}{dt} + Ri(t) + Bl\frac{dh}{dt} = V(t) \quad ①$$

$$m\frac{d^2h}{dt} + mg - Bli(t) = 0 \quad ②$$

d) Steady state voltage and current were calculated and can be seen below.

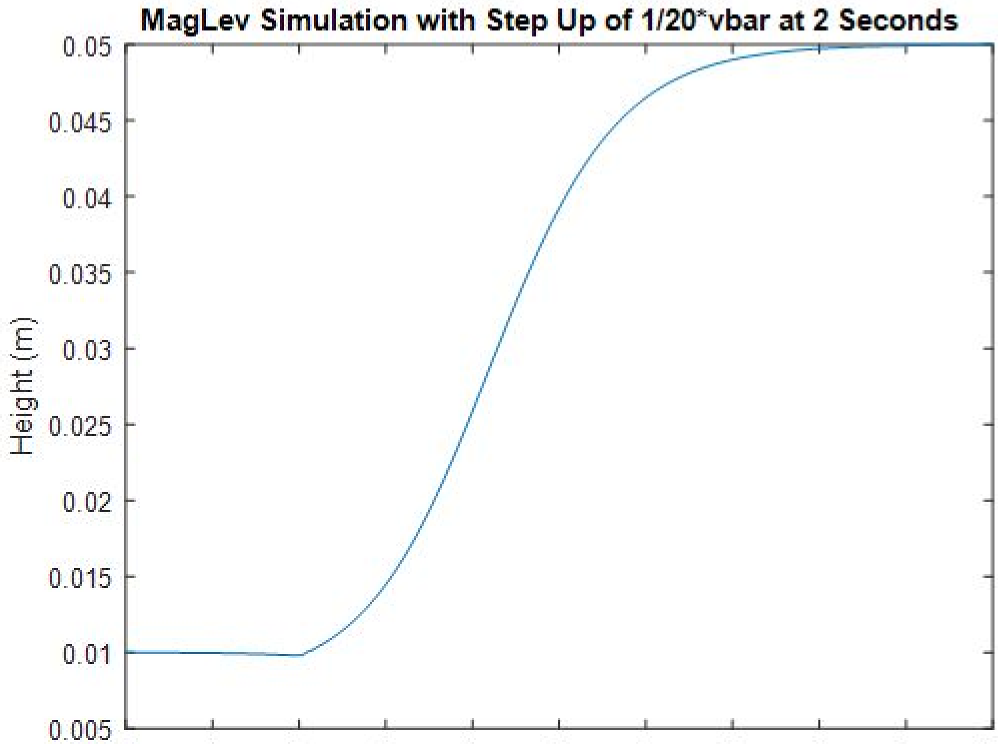
Steady State Analysis: $\bar{h} = 0.01m$

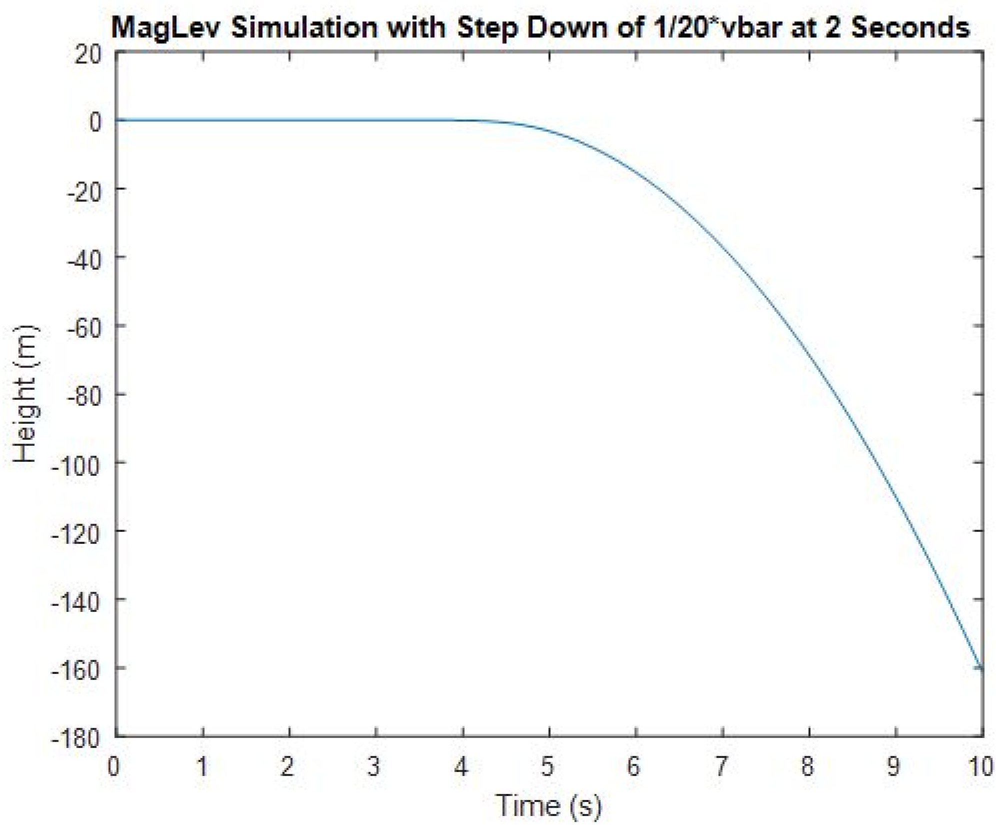
$$① \; \cancel{L\frac{di}{dt}} + Ri(t) + \cancel{Bl\frac{dh}{dt}} = V(t) \Rightarrow R\bar{i} = \bar{V} = (0.07\Omega)(643.1A) = \boxed{45V}$$

$$② \; \cancel{m\frac{d^2h}{dt}} + mg - Bli(t) = 0 \Rightarrow mg = Bl\bar{i} \Rightarrow \bar{i} = \frac{mg}{Bl} = \frac{(5.9\times10^4 \, kg)(9.81 \, m/s^2)}{(0.12 \, Wb/m^2)\left(\frac{1}{0.05-0.01}\right)m \cdot 20m} \cdot \frac{1}{15 \, coils}$$

$$\bar{i} = 643.1 \, A$$

e) The equations of motion from part C were modelled in simulink. A step up and step down response were modelled and plotted below.





The step up response shows the train rising until it hits the stator coil. The step down causes the train to fall uncontrollably to the ground. The code and simulink model can be seen at the end of the report.

e) The linearized equations of motion and calculation of the transfer function can be seen below.

① $L\dot{i} + Ri + BL\dot{h} = V \qquad B = B_0 \cdot \frac{1}{h_0 - h}$, L_c = coil length

$\Rightarrow L\dot{i} + Ri + B_0 \cdot \frac{L_c}{h_0 - h} \cdot \dot{h} - V = 0 = f(\dot{i}, i, h, \dot{h}, V)$

Linearize $f(\dot{i}, i, h, \dot{h}, V)$ assuming $\dot{\bar{i}} = 0$, $\bar{i} = 643.1A$, $\bar{h} = 0.01m$, $\dot{\bar{h}} = 0$, $\bar{V} = 45V$

Steady State: $f(\text{average Values}) = R\bar{i} - \bar{V} = 0$

$$\frac{f(\dot{i}, i, h, \dot{h}, V)}{\partial \dot{i}} \cdot (\dot{i} - \dot{\bar{i}}) = L\dot{i}$$

$$\frac{f(\dot{i}, i, h, \dot{h}, V)}{\partial i} (i - \dot{\bar{i}}) = R(i - \dot{\bar{i}})$$

$$\frac{f(\dot{i}, i, h, \dot{h}, V)}{\partial h} (h - \bar{h}) = \frac{-B_0 \cdot L_c \cdot \cancel{\dot{\bar{h}}}}{(h_0 - \bar{h})^2} \cdot (h - \bar{h}) = 0$$

$$\frac{f(\dot{i}, i, h, \dot{h}, V)}{\partial \dot{h}} (\dot{h} - \dot{\bar{h}}) = \frac{BL_c}{h_0 - \bar{h}} \cdot \dot{h}$$

$$\frac{f(\dot{i}, i, h, \dot{h}, V)}{\partial V} (V - \bar{V}) = -(V - \bar{V}) = \bar{V} - V$$

Combine: $L\dot{i} + Ri - R\dot{\bar{i}} + \frac{BL_c}{h_0 - \bar{h}} \cdot \dot{h} + \bar{V} - V = 0$

$\Rightarrow L\dot{i} + Ri + \frac{BL_c}{h_0 - \bar{h}} \dot{h} = V + \cancel{R\bar{i} - \bar{V}}$ + Laplace transform

$$\boxed{LsI(s) + RI(s) + \frac{BL_c}{h_0 - \bar{h}} \cdot sH(s) = V(s) \quad \text{① Linearized EoM 1}}$$

(2) $m\ddot{h} + mg - Bli = 0 \qquad B = B_0 \frac{L_c}{h_0 - h}$

$\Rightarrow m\ddot{h} + mg - B_0 \frac{L_c}{h_0 - h} i = 0 = f(\ddot{h}, h, i)$ assume $\bar{\ddot{h}} = 0,\ \bar{h} = 0.01m,\ \bar{i} = 643.1A$

Steady State: $mg - B_0 \frac{L_c}{h_0 - \bar{h}} \cdot \bar{i} = 0$

$$\frac{f(\ddot{h}, h, i)}{\partial \ddot{h}} (\ddot{h} - \bar{\ddot{h}}) = m(\ddot{h} - \bar{\ddot{h}}) = m\hat{\ddot{h}}$$

$$\frac{f(\ddot{h}, h, i)}{\partial h} (h - \bar{h}) = \frac{B_0 L_c \bar{i}}{(h_0 - \bar{h})^2} \cdot \underbrace{(h - \bar{h})}_{\hat{h}}$$

$$\frac{f(\ddot{h}, h, i)}{\partial i} (i - \bar{i}) = \frac{-B_0 L_c}{h_0 - \bar{h}} \cdot \underbrace{(i - \bar{i})}_{\hat{i}}$$

combine: $m\hat{\ddot{h}} + \frac{B_0 L_c \bar{i}}{(h_0 - \bar{h})^2} \hat{h} - \frac{B_0 L_c}{h_0 - \bar{h}} \cdot \hat{i} = 0$

LT $\rightarrow ms^2 H(s) + \frac{B_0 L_c \bar{i}}{(h_0 - \bar{h})^2} H(s) - \frac{B_0 L_c}{h_0 - \bar{h}} I(s) = 0$ (2)

Linearized equations can be expressed in matrix form below

$$\begin{bmatrix} Ls+R & \frac{BL_c}{h_0-\bar{h}}s \\ \frac{-BL_c}{h_0-\bar{h}} & ms^2+\frac{BL_c\bar{i}}{(h_0-\bar{h})^2} \end{bmatrix} \begin{bmatrix} I(s) \\ H(s) \end{bmatrix} = \begin{bmatrix} V(s) \\ 0 \end{bmatrix}$$

Using Kramers rule we can solve for $H(s)$

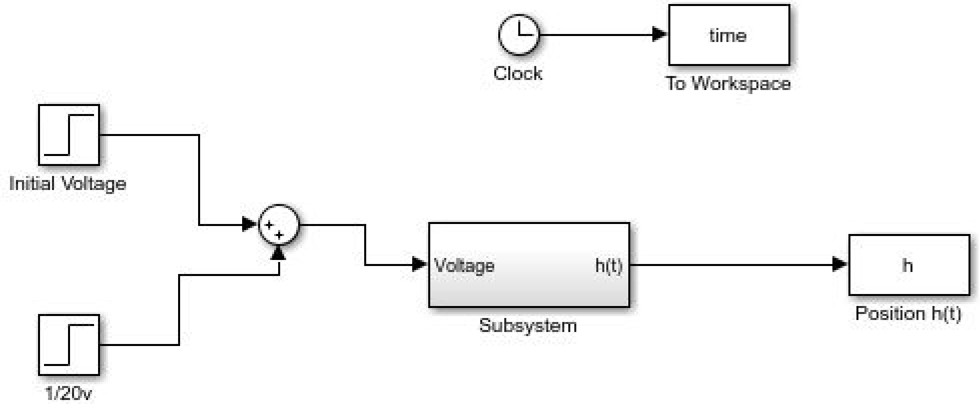
$$H(s) = \frac{\det\begin{bmatrix} Ls+R & V(s) \\ \frac{-BL_c}{h_0-\bar{h}} & 0 \end{bmatrix}}{\det\begin{bmatrix} Ls+R & \frac{BL_c}{h_0-\bar{h}}s \\ \frac{-BL_c}{h_0-\bar{h}} & ms^2+\frac{BL_c\bar{i}}{(h_0-\bar{h})^2} \end{bmatrix}} = \frac{V(s)\frac{BL_c}{h_0-\bar{h}}}{(Ls+R)\left(ms^2+\frac{BL_c\bar{i}}{(h_0-\bar{h})^2}\right)+\frac{B^2L_c^2}{(h_0-\bar{h})^2}s}$$

$$\boxed{\Rightarrow \frac{H(s)}{V(s)} = \frac{\frac{BL_c}{h_0-\bar{h}}}{(Ls+R)\left(ms^2+\frac{BL_c\bar{i}}{(h_0-\bar{h})^2}\right)+\frac{B^2L_c^2}{(h_0-\bar{h})^2}s}}$$

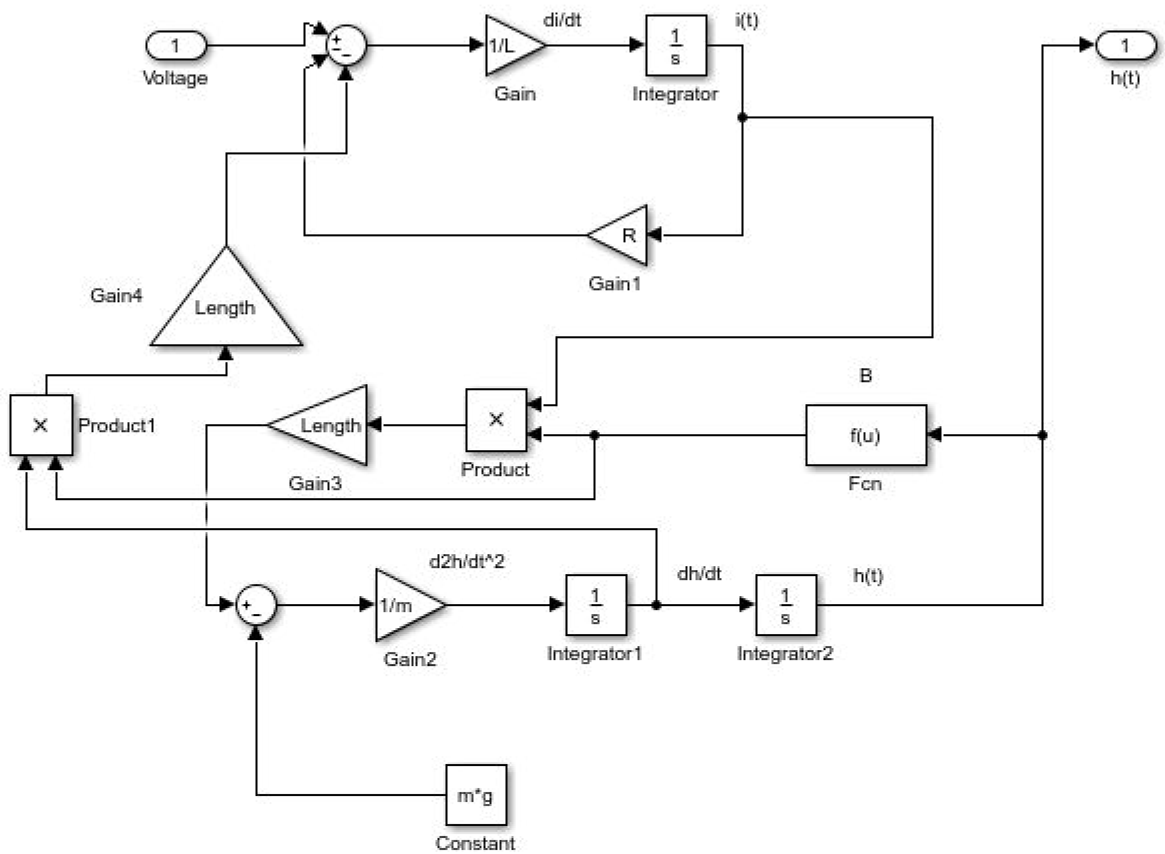
The final transfer function ends up being a third order system with 3 poles and no zeros.

The MatLab code and simulink portion of the project can be found on the following pages.

```
%% ME3360 Project Part 2 %%
clear
close all


L = 0.005;      %H
R = 0.07;       %Ohm
m = 5.9*10^4;   %kg
g = 9.81;       %m/s2
h0 = 0.05;      %m
b0 = 0.12;      %Wb/m2
Length = 20;    %m

vbar = 45;      %V

sim('ME3360_Part_2E')

plot(time, h)
title('MagLev Simulation with Step Down of 1/20*vbar at 2 Seconds')
xlabel('Time (s)')
ylabel('Height (m)')
```


time
Clock
To Workspace
Initial Voltage
Voltage
h(t)
Subsystem
h
Position h(t)
1/20v

Subsystem:


1
Voltage
1/L
di/dt
Gain
1
s
Integrator
i(t)
1
h(t)
R
Gain1
Gain4
Length
B
×
Product1
Length
Gain3
×
Product
f(u)
Fcn
d2h/dt^2
1/m
Gain2
1
s
Integrator1
dh/dt
1
s
Integrator2
h(t)
m*g
Constant