

FDTD 1D example with Python

Example task for lectures "Computations in Physics"

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Konstantin Ladutenko

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All the sources are available at
<https://github.com/kostyfisik/fdtd-1d>

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Task 0:

Simulate a system with magnetic mirror boundary condition ($H=0$) on one side and electric mirror ($E=0$) on the other side. The source is a Gaussian profile propagating to boundaries and back, source located exactly in the center of the simulated domain. The successful presentation should provide a sequence of images as a time evolution (or animation) for electric and magnetic fields. The simulation should finish at the moment where the electric field is vanished (all energy is in the magnetic field).

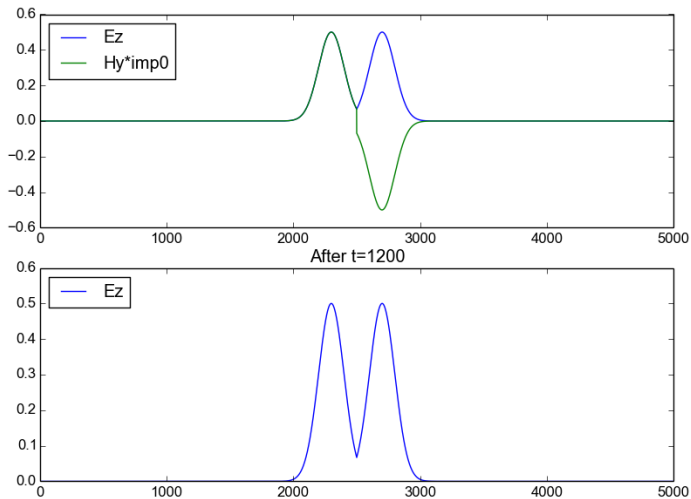


Figure 1: Source produces two Gaussian pulses

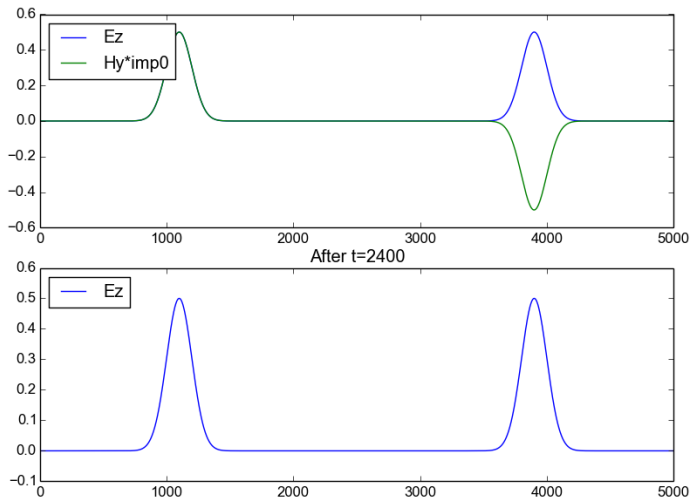


Figure 2: Before reflection

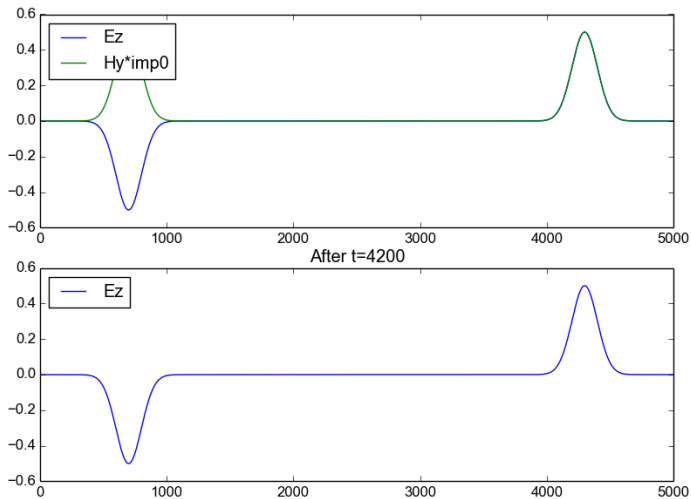


Figure 3: After reflection

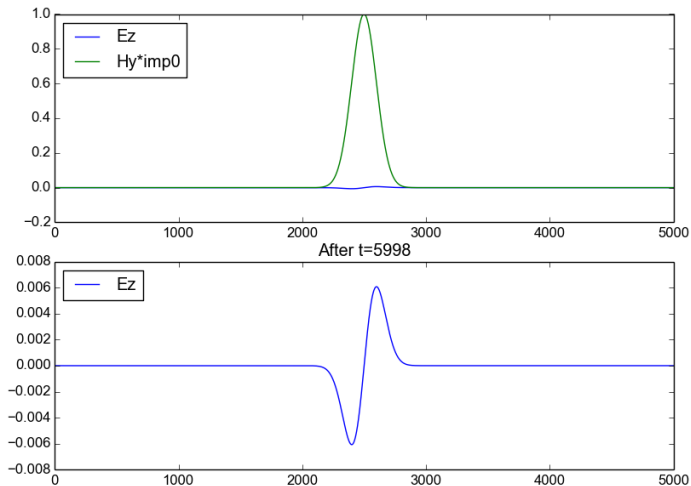


Figure 4: Last step just before vanishing E field

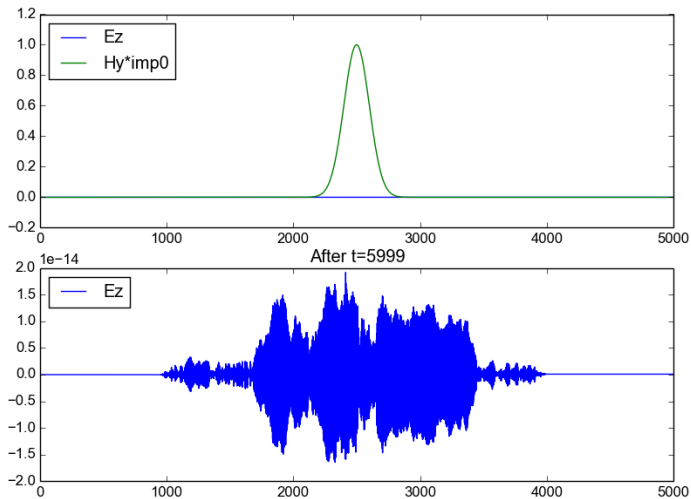


Figure 5: E field vanished. Noise corresponds to numerical accuracy.

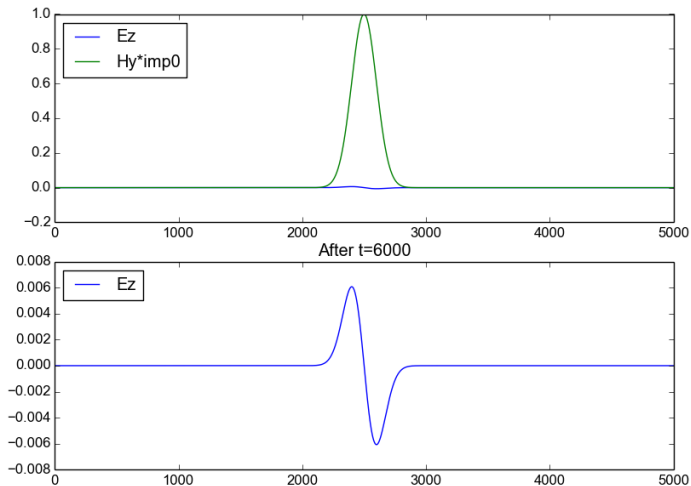


Figure 6: The next time step

Step 1, 2, and 3:

Provide absorbing (Mur ABC) and PML (simplified CPML) boundary condition. Compare ABC with 5, 10, and 20 cell PML.

Result:

For free space Mur ABC is almost perfect. For host media with $\epsilon = 5$ the performance is in between 5 and 10 cell CPML, 20 cell CPML is the best.

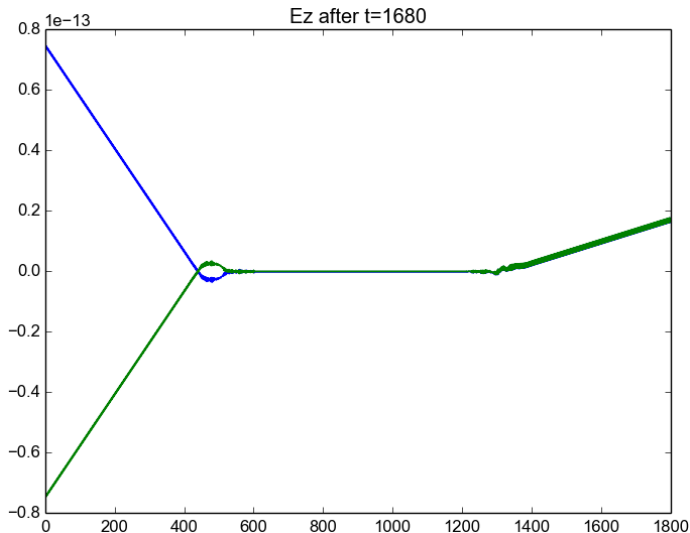


Figure 7: Just after reflection of Gaussian pulses using Mur boundary condition. Almost perfect for free space.

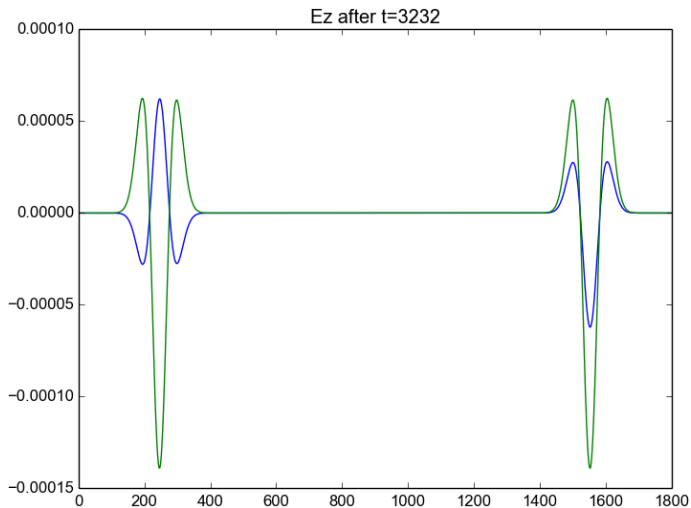


Figure 8: For the host media with $\epsilon = 5$ there is some reflection.

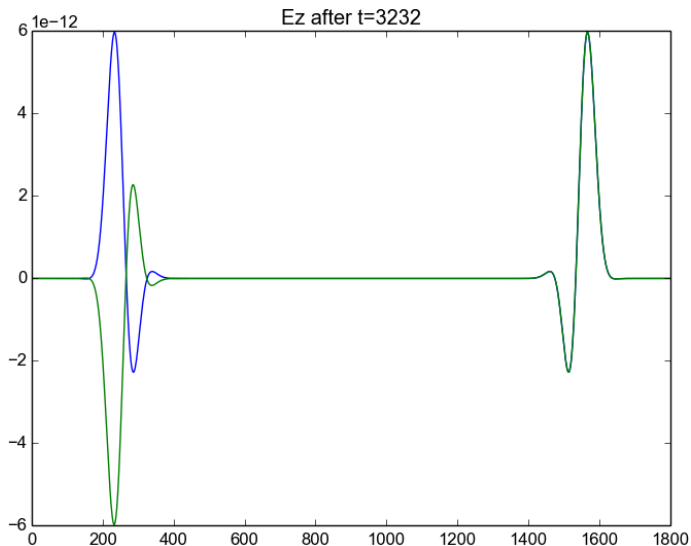


Figure 9: For free space CPML performs very well too.

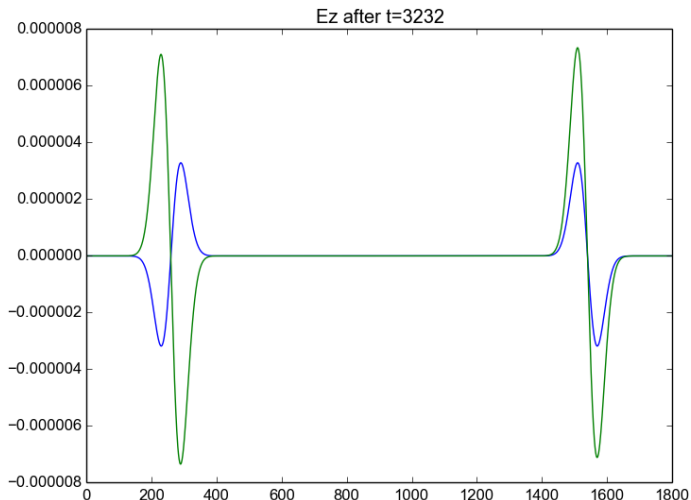


Figure 10: For $\epsilon = 5$ 10 cells CPML has one order of magnitude less reflection.

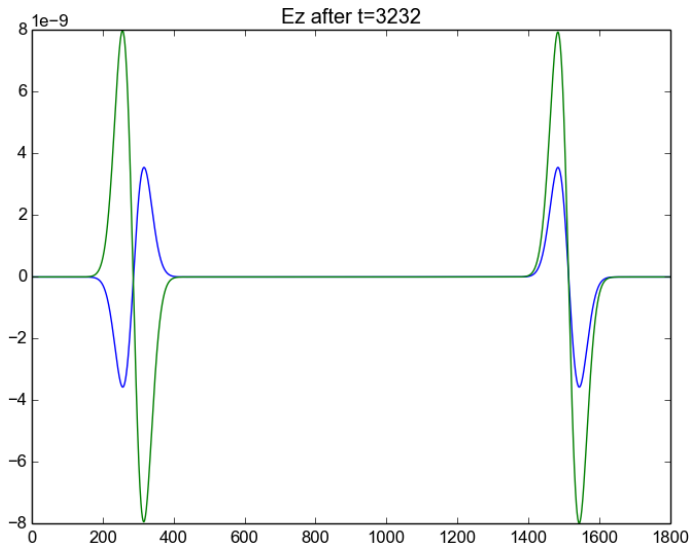


Figure 11: 20 cells CPML reduces reflection for three orders of magnitude compared to 10 cell.

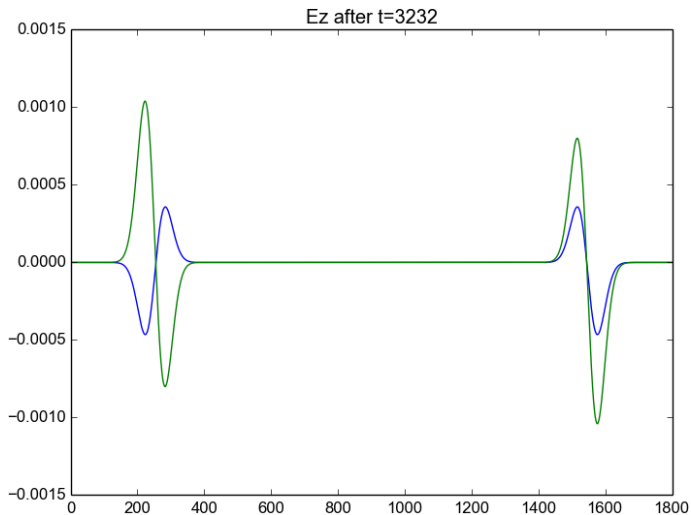


Figure 12: 5 cells PML performs rather poor.

Task 4:

Compare against Fresnel equations, find the limits of FDTD applicability.

Transmission equation:

$$T_{Fresnel} = 1 - \left| \frac{n_1 - n_2}{n_1 + n_2} \right|^2$$

http://en.wikipedia.org/wiki/Fresnel_equations

Results:

$$n_1 = 2.828427, n_2 = 1.414214$$

$$T_{Fresnel} = 0.888889$$

$$\text{FDTD ratio} = 0.888761 \text{ Error} = 0.014358\%$$

$$n_1 = 1.732051, n_2 = 2.236068$$

$$T_{Fresnel} = 0.983867$$

$$\text{FDTD ratio} = 0.983915 \text{ Error} = 0.004879\%$$

$$n_1 = 1.000000, n_2 = 3.000000$$

$$T_{Fresnel} = 0.750000$$

$$\text{FDTD ratio} = 0.750060 \text{ Error} = 0.008024\%$$

Task 5 and 6:

Compare against single dielectric slab (e.g. <http://www.ece.rutgers.edu/orfanidi/ewa/ch05.pdf>), you should provide simulation of reflection-less cases of a quarter-wavelength and half-wavelength slab width cases.

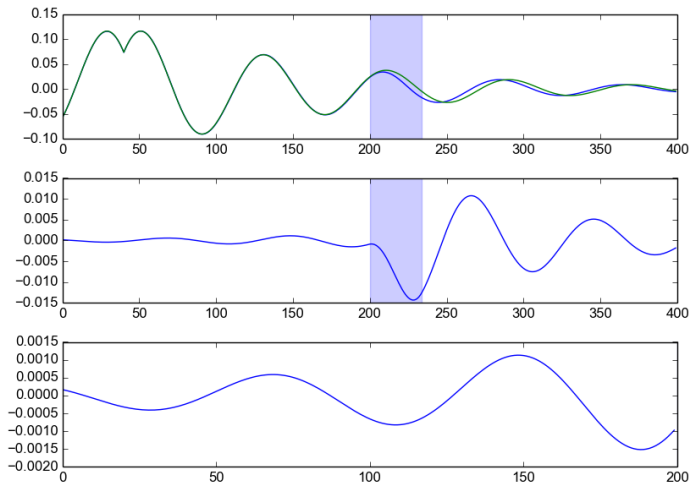


Figure 13: Half-wavelength slab: switching periodic source on. First plane: green curve - wave in free space, blue curve - with a slab. Other two are the difference of these curves. Last plane: left part of the difference, it is the reflection only. During switching on there is some reflection.

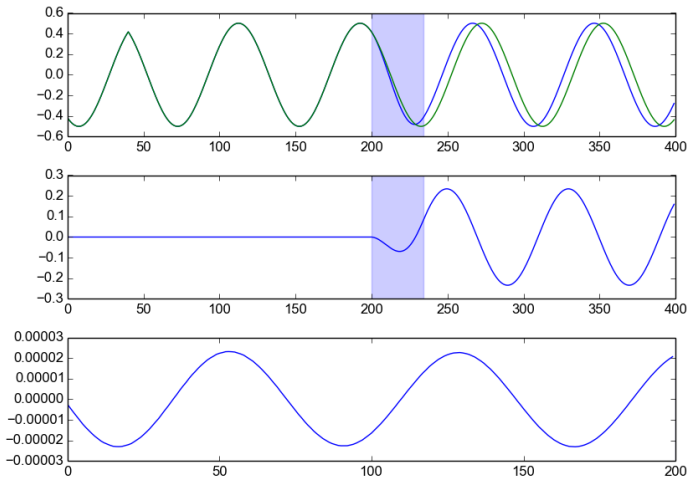


Figure 14: Half-wavelength slab: in steady state the reflection is reduced 100 times more!

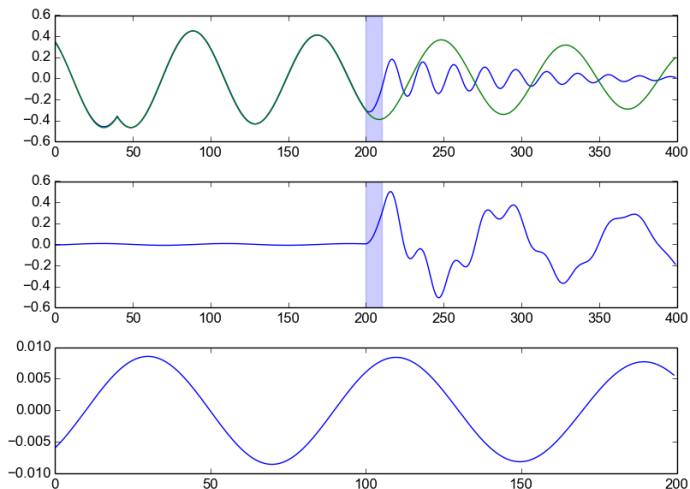


Figure 15: **Quarter-wavelength slab:** switching on.

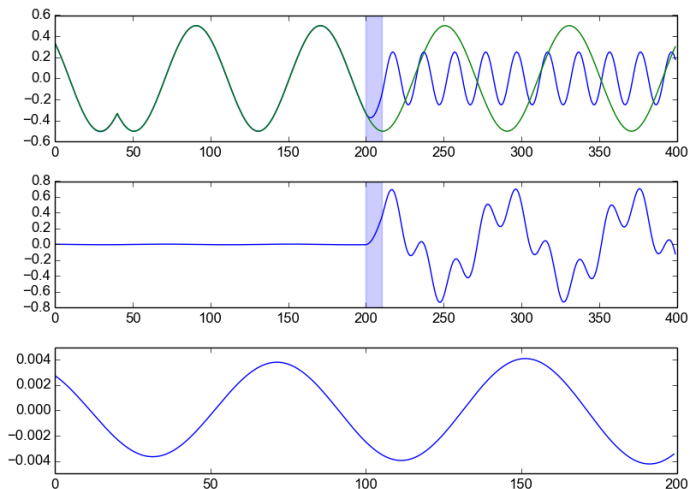


Figure 16: **Quarter-wavelength slab: steady state**

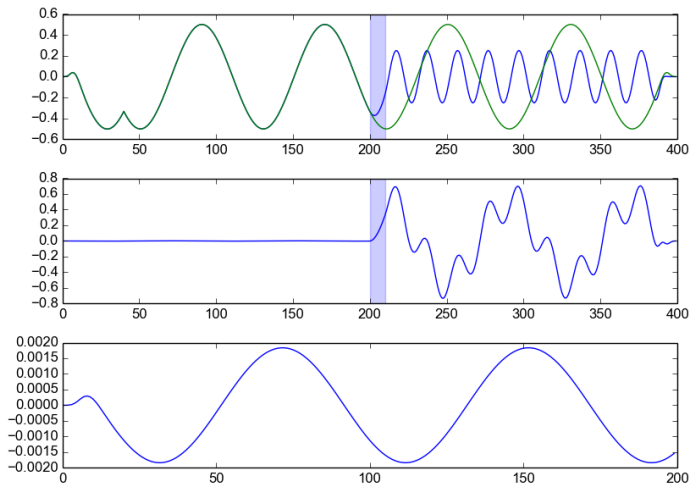


Figure 17: Quarter-wavelength slab: steady state using CPML boundary condition instead of Mur ABC. The reflection was reduced two times just due to change of computational method.