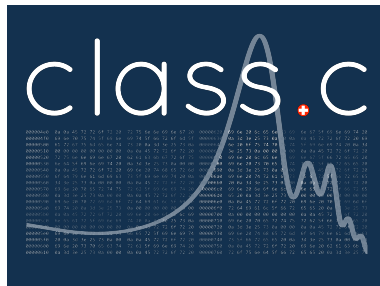


CLASS

Cosmological Linear Anisotropy Solving System



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These slides available at https://github.com/MarkMos/class_lecture

Visit <http://class-code.net/> for more info!

What to expect in these lectures:

- Basics: Why use **class**?
- Usage: Installation
- Usage: Python Interface
- Basics: Existing Species
- Basics: Module Overview
- Theory: What is **class** based upon?
- Coding: Implementing features

We will learn **how to use class** and **which models** can be run with it.

What is an Einstein-Boltzmann solver?

Often just called a *Boltzmann code* for brevity, a typical Boltzmann code will:

- Solve coupled Einstein and Boltzmann equations.
- Generally work at linear level in perturbation theory.
- Compute global (Background+Thermodynamic) quantities *and* perturbations.

$$\underbrace{G_{\mu\nu} = 8\pi T_{\mu\nu}}_{\text{Einstein-equation}} \quad \underbrace{\frac{df}{d\lambda} = C[f]}_{\text{Boltzmann-equation}} \quad (1)$$

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- Emulated non-linear power spectra
- CMB spectral distortions

All computed in a matter of seconds!

Why use a Boltzmann code?

This has several use cases:

- Analysis of CMB experiments
- Analysis of LSS experiments
- Initial conditions for non-linear simulations (N -body, etc.)
- Consistent treatment of background/thermodynamic evolution

All easy to do with `class`!

Fast execution \Rightarrow ideal for use in an MCMC pipeline.

Why use class?

class is:

- Accurate: class & camb cross-check each other
- Versatile: Interfaces with MontePython, Cobaya, Cosmosis, Procoli, CosmoPower, OLÉ, CONNECT, and others!
- Comprehensive: Computes a wide range of cosmological observables for a large selection of models beyond Λ CDM.
- Modular and well-documented: ReadTheDocs page and Doxygen documentation, thoroughly commented source code, easy to modify

All strong arguments to use **class**!

Installing class

Using class

If you have no intention of modifying source code:

```
> pip install classy
```

And the class wrapper will be ready to use in your Python environment.

This is the easiest way to install and ideal if you only plan to call class via the Python wrapper.

Modifying class

If you wish to modify source code:

```
> git clone git@github.com:
    lesgourg/class_public.
    git class
> cd class/
> make clean; make -j
```

The wrapper can be used in your Python environment, and the binary executable can be called from the terminal.

Documentation

`classy` is documented in various places:

- The file `explanatory.ini` lists all input parameters.
(downloadable from GitHub page if `classy` installed via pip)

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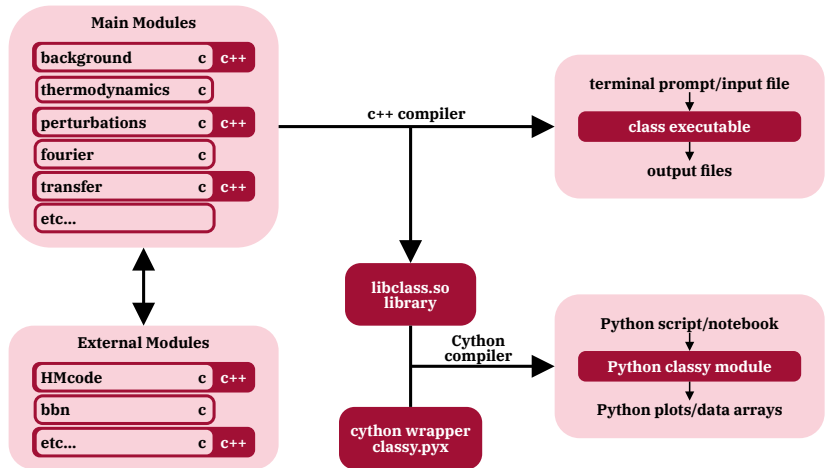
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- Online documentation at https://github.com/lesgourg/class_public/wiki
- Old course notes linked on <http://class-code.net> and <https://schoeneberg.github.io/>

Installing class

Let's take a few minutes to install class and confirm it is working!

- If you have installed classy via pip, try running the script `classy_test.py`
- If you have installed class via git clone and compiling, try also running `./class default.ini`

The code structure



class vs classy

Let's clarify a bit of nomenclature!

- **class** is the C code that does all the work of solving the EBS. It is used from the command line.
- **classy** is the Python wrapper of **class**, and internally uses it. It is used from a python interpreter.

The two share same input/output, except different naming and format (e.g. **.ini file** vs **python dictionary** as input)

Running class in terminal

Run any input file with extension *.ini:

- Simple first-usage file

```
./class default.ini
```

- Huge reference file containing all possible input parameters with comments

```
./class explanatory.ini
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- Slim file matching Planck 2018 "baseline model" bestfit

```
./class base_2018_plikHM_TTTEEE_lowl_lowE_lensing.ini
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- All input is presented in detail in `explanatory.ini` (apart from precision parameters)
- This is a *reference* file; we advise you to not modify it:
 - either start from a slim file (like `default.ini`),
 - or copy it and reduce it to a shorter and more friendly file,
 - or write your own from scratch with only needed input lines.

class input parameters

The common 'language' for input is as follows

```
parameter = value
```

with the python dictionary equivalent of

```
{'parameter': 'value'}
```

where value is passed as a **python string**.

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where value is passed as a **python string**.

Special cases include

```
option = yes/no  
selection = a,b,c,d
```

and comments are

```
parameter = value #comment behind parameter  
#comment in its own line
```

class vs classy

Let's compare **class** vs **classy** for an execution:

class	classy
.ini file with parameter = value	python dictionary with 'parameter': 'value'
Solving of equations in class	
Output to files Options write files	Output from python functions Options enable functions (otherwise error message)
<code>./class myfile.ini</code>	<code>import classy cosmo = classy.Class() cosmo.set(mydictionary)</code>

Input parameters

If nothing given → **Planck 2013 cosmology** with $\sum m_\nu = 0$

Any parameter **overwrites** the defaults (from **.ini file** or **cosmo.set**)

Precision file **cl_ref.pre** → close to ‘optimal’ precision

Input parameters

Most common/important parameters:

- output = $\underbrace{\text{tCl}}$, $\underbrace{\text{pCl}}$, $\underbrace{\text{lCl}}$,
 Temperature C_ℓ^{TT} Polarization $C_\ell^{TE/EE/BB}$ Lensing $C_\ell^{\phi\phi}$,
 $\underbrace{\text{mPk}}$, $\underbrace{\text{mTk}}$, $\underbrace{\text{vTk}}$,
 Matter power $P(k, z)$ Matter transfer $\delta_i(k, z)$ Velocity transfer $\theta_i(k, z)$,
 $\underbrace{\text{nCl}}$, $\underbrace{\text{sCl}}$,
 Number count C_ℓ^{dd} Galaxy Lensing count C_ℓ^{ss}

Input parameters

Most common/important parameters:

- `l_max_scalars = 2500`
- `P_k_max_1/Mpc = 1`
- `z_pk = 0, 1, 2` for `class`
- `z_max_pk = 10` for `classy`
- `format = CAMB` for initial conditions
- `write_warnings = yes` for `class`

Input parameters

Most common/important parameters:

- `input_verbose = 1`
- `background_verbose = 2`
- `thermodynamics_verbose = 1`
- `perturbations_verbose = 1`
- `fourier_verbose = 1`
- `output_verbose = 1`

Verbosity parameters for `class`

Input parameters

The Λ CDM parameters $\{H_0, A_s, n_s, \Omega_m, \Omega_b h^2, \tau_{\text{reio}}\}$:

- Hubble constant

$H_0=67$

$h=0.67$

$100 \times \theta_s = 1.042$

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$$h = 0.67$$

$$100 \theta_s = 1.042$$

- Primordial amplitude

$$A_s = 2.1 \times 10^{-9}$$

$$\ln 10^{10} A_s = 3.0 \quad = \ln(10^{10} A_s)$$

$$\ln A_s \times 10^{10} = 3.0$$

$$\sigma_8 = 0.825 \quad = \int \frac{k^3 P(k, z=0)}{2\pi^2} T^2(k R_8) d \ln k$$

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$$\Omega_m = 0.3$$

$$\Omega_m h^2 = 0.14 \quad = \Omega_m h^2$$

$$\Omega_{\text{cdm}} = 0.25$$

$$\Omega_{\text{cdm}} h^2 = 0.11 \quad = \Omega_{\text{cdm}} h^2$$

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 $\Omega_m h^2=0.14 \quad = \Omega_m h^2$
 $\Omega_{\text{cdm}}=0.25$
 $\Omega_{\text{cdm}} h^2=0.11 \quad = \Omega_{\text{cdm}} h^2$
- Baryon abundance
 $\Omega_b=0.05$
 $\Omega_b h^2=0.02233 \quad = \Omega_b h^2$
- Reionization time
 $z_{\text{reio}}=7.0$
 $\tau_{\text{reio}}=0.05 \quad \approx \int_0^{z_{\text{reio}}} \sigma_T n_e d\eta$

Input parameters

One-parameter extensions

- Curvature
 $\Omega_{\text{k}} = 0$
- Dark radiation
 $N_{\text{ur}} = 3.044 \hat{=} N_{\text{eff}}$
- CMB temperature
 $T_{\text{cmb}} = 2.7255$
- Lensing enhancement
 $A_{\text{L}} = 1$
- Primordial running
 $\alpha_{\text{s}} = 0$

Input parameters

Dark Energy options

- Cosmological constant
Omega_Lambda
- Fluid
Omega_fld
- Scalar field
Omega_scf

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Omega_fld
- Scalar field
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Universe is always filled (to be as curved as Ω_k) by putting whatever dark energy is still available (If you want to fill with scalar field, you need $\Omega_{scf} < 0$).

Using the Budget equation

$$\sum \Omega_i = 1 + \Omega_k \quad (2)$$

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Examples:

Omega_m=0.3	⇒	Omega_Lambda -> 0.7
Omega_fld=0.5, Omega_m=0.3	⇒	Omega_Lambda -> 0.2
Omega_Lambda=0, Omega_m=0.3	⇒	Omega_fld -> 0.7
Omega_Lambda=0, Omega_fld=0, Omega_scf=-1, Omega_m=0.3	⇒	Omega_scf -> 0.7
Omega_k=0.2, Omega_m=0.3	⇒	Omega_Lambda -> 0.9

Input parameters

Neutrino mass

- $N_{\text{ncdm}}=1$ (single massive neutrino)
- $m_{\text{ncdm}}=0.06$ (in eV)
- $N_{\text{ur}} = 3.044 - 1.0132 N_{\text{ncdm}} = 2.0308$
(each massive neutrino contributes 1.0132 to N_{eff} [QED corrections, could be optimized in future])
or
- $N_{\text{eff}} = 3.044$ will automatically adjust N_{ur} to achieve the desired N_{eff}

Input parameters

Neutrino mass

- $N_{\text{ncdm}}=3$ (three massive neutrinos)
- $m_{\text{ncdm}}=0, 0.0495, 0.0582$ (inverted hierarchy)
- $N_{\text{ur}} = 3.044 - 1.0132 N_{\text{ncdm}} = 0.0044$
or
- $N_{\text{eff}} = 3.044$ will automatically adjust N_{ur} to achieve the desired N_{eff}

Input parameters

Neutrino mass

- $N_{\text{ncdm}}=1$ (one *degenerate* massive neutrinos)
- $\text{deg_ncdm}=3$ (triply degenerate)
- $m_{\text{ncdm}}=0.02$ (total mass 0.06eV)
- $N_{\text{ur}} = 3.044 - 1.0132 N_{\text{ncdm}} \star \text{deg_ncdm} = 0.0044$
or
- $N_{\text{eff}} = 3.044$ will automatically adjust N_{ur} to achieve the desired N_{eff}

You can get plots

- 1 Manually: using the output files with e.g. gnuplot, IDL, python, Mathematic, GNU Octave...
- 2 Automatically: using **python** and script **CPU.py**, or MATLAB and script **plot_CLASS_output.m**
- 3 Interactively: using **class** as a python module, within a python session or a **Jupyter Notebook**

Running `class` from Python

`class` as a Python module

- based on wrapper located in `python/classy.pyx` (developed initially by B. Audren and extended by many others)
- the compilation produces a python module `classy.py` and installs it on your computer (can be called from anywhere)
- wrapper written in `Cython`, encapsulates most useful `class` variables/functions, contains extra functions (e.g. MontePython-motivated)
- (project: get most of the wrapper generated automatically from C code at compilation - Coming soon!)
- goal: obtain, manipulate and plot the results directly within (i)python scripts or notebooks (recommended)

Existing content already in class

Λ CDM:

- **Baryons** Ω_b or $\omega_b = \Omega_b h^2$

Existing content already in class

Λ CDM:

- **Baryons** Ω_b or $\omega_b = \Omega_b h^2$
- **Cold Dark matter** Ω_{cdm} or $\omega_{\text{cdm}} = \Omega_{\text{cdm}} h^2$ or Ω_m or $\omega_m = \Omega_m h^2$

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- **Hubble** $H_0 = H_0$ or $h = h$ or $100 \theta_s = 100 \theta_s$

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- **Hubble** $H_0 = H_0$ or $h = h$ or $100 \theta_s = 100 \theta_s$
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- **Primordial tilt** $n_s = n_s$

Existing content already in class

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- **Curvature** Ω_k (Omega_k)
- **Neutrinos** m_ν (m_ncdm, N_ncdm, **N_ur adjustment** see explanatory.ini)

Existing content already in class

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- **Free-streaming dark radiation** N_{eff} (N_ur)

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- **Helium abundance** Y_p (YHe)

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- **Primordial running** α_s (alpha_s)

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Standard simple extensions:

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- **Free-streaming dark radiation** N_{eff} (N_ur)
- **Helium abundance** Y_p (YHe)
- **Primordial running** α_s (alpha_s)
- **CPL Dark Energy** w_0, w_a (Omega_fld, w0_fld, wa_fld, **Omega_Lambda=0?**)

Existing content already in class

Standard simple extensions:

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- **CPL Dark Energy** w_0, w_a (Omega_fld, w0_fld, wa_fld, **Omega_Lambda=0?**)
- CMB temperature T_{cmb}

Existing content already in class

Dark matter:

- Thermal Warm Dark Matter (m_{ncdm} , T_{ncdm} , ω_{ncdm})
- Annihilating dark matter
- Decaying dark matter ($\Omega_{\text{dcdm} \rightarrow \text{r}}$, Γ_{dcdm})
- Non-trivial phase-space distribution (ncdm framework), neutrino flavor mixing, neutrino chemical potential
- Interacting with photons, baryons, neutrinos (idm)

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- Viscous (ceff2_{ur} , cvis_{ur})

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- Viscous ($c_{\text{eff}2,\text{ur}}$, $c_{\text{vis},\text{ur}}$)

Dark energy:

- CPL, EDE (fld) (this EDE is **not** the usual EDE → <https://github.com/PoulinV/AxiCLASS> and <https://github.com/flo1984/TriggerCLASS> to be merged)
- Other fluid-like DE (fld)
- Quintessence/Scalar field (scf)

Existing content already in class

Thermal modeling

- Recfast recombination
- Hyrec-2recombination
- Tanh reionization
- Multi-tanh reionization
- Reionization from file
- Energy injection (PBH Evaporation,PBH Accretion,DM Decay,DM Annihilation)

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Spectral distortions

- y and μ distortions
- PCA of intermediate distortions

Existing content already in class

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- Arbitrary potential $V(\phi)$
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- Can impose inflation duration
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→ **HiCLASS** branch (Bellini, Sawicki, Zumalacarregui,
<http://www.hiclass-code.net>)

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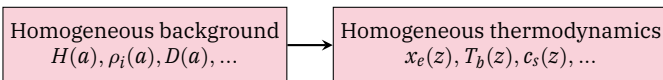
Modified Gravity

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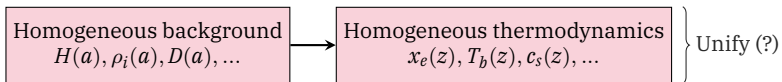
Extension to **second-order perturbation theory**

SONG (Fidler, Pettinari, Tram, <https://github.com/coccoinomane/song>)

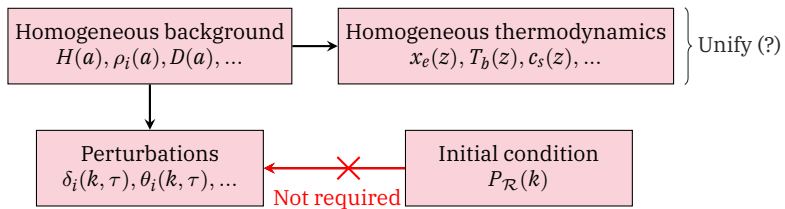
Fundamental layout of Einstein-Boltzmann solvers



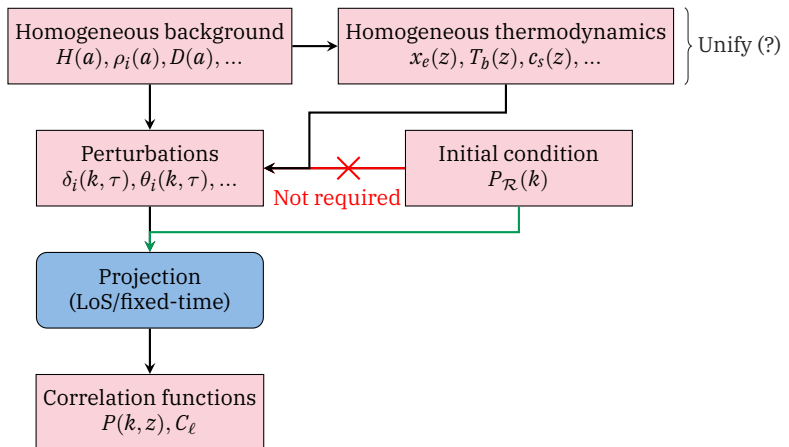
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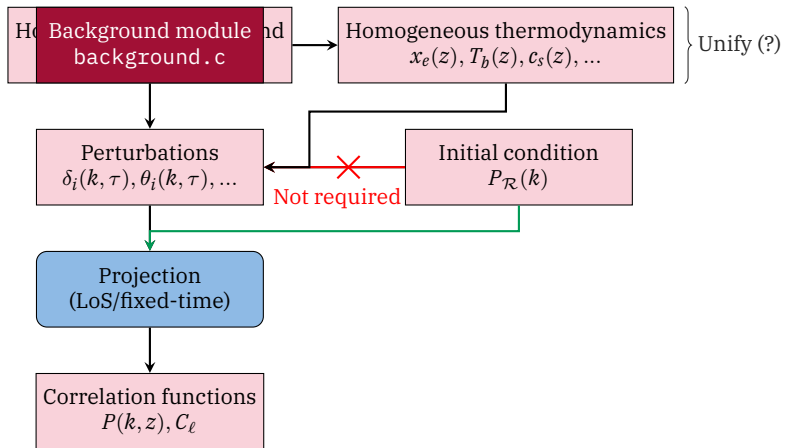
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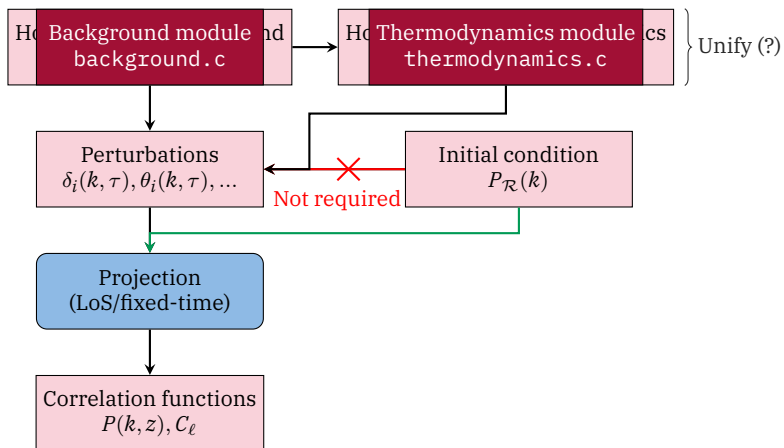
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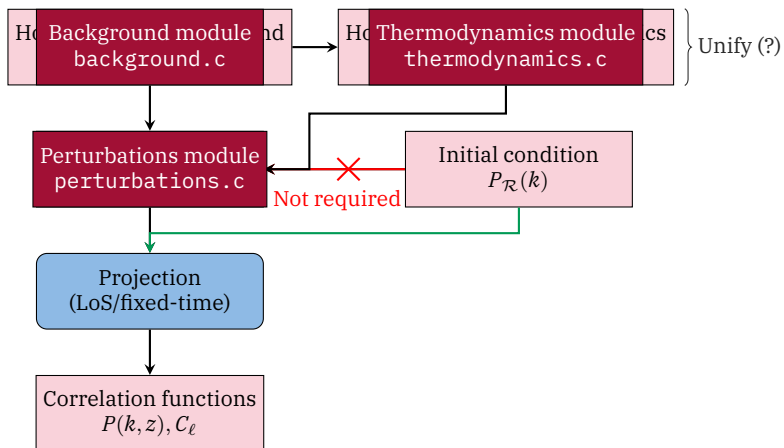
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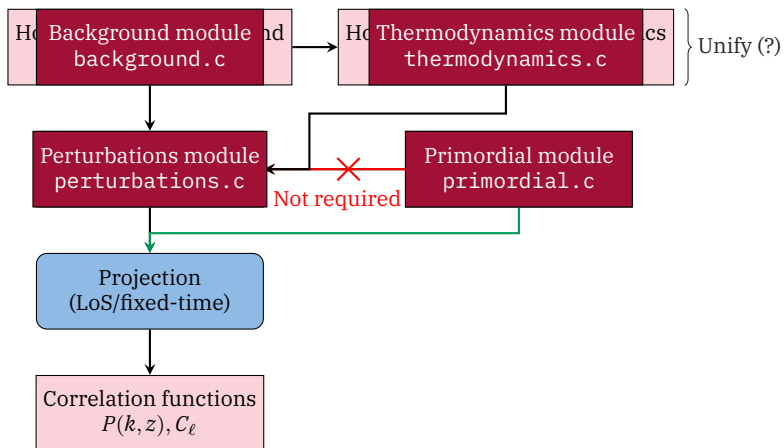
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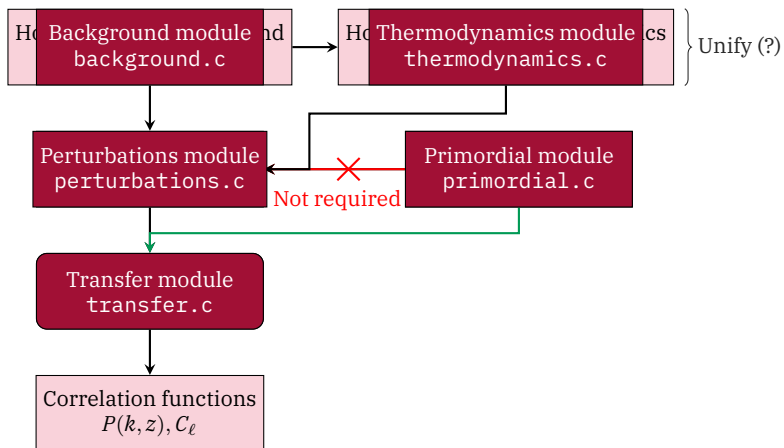
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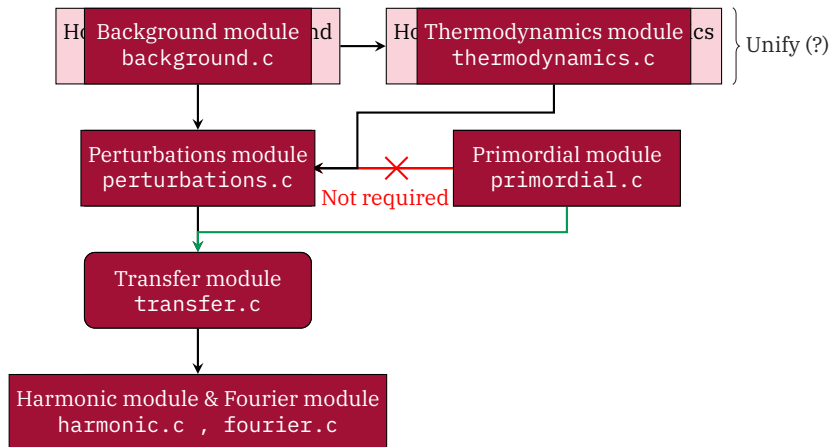
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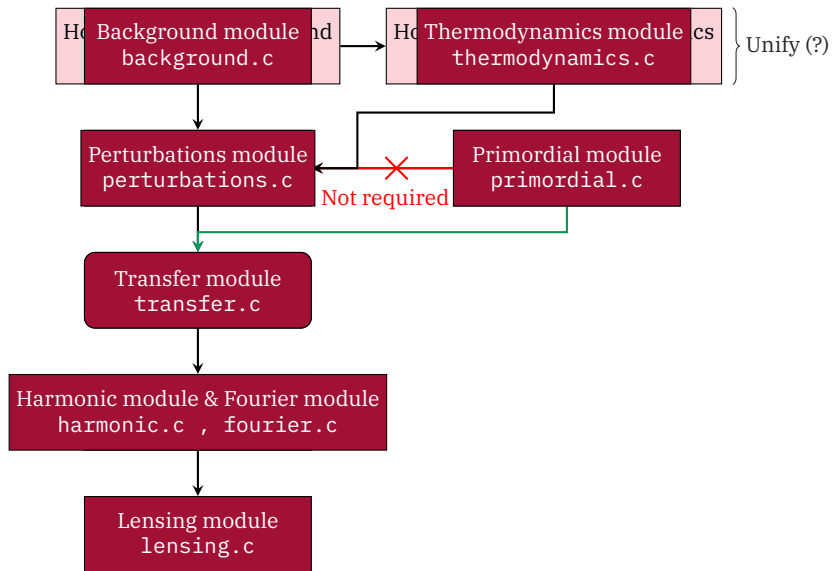
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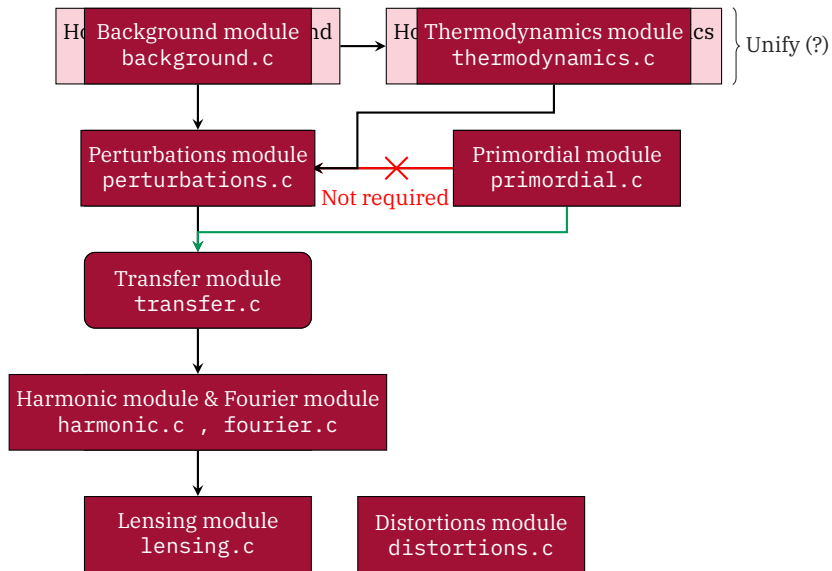
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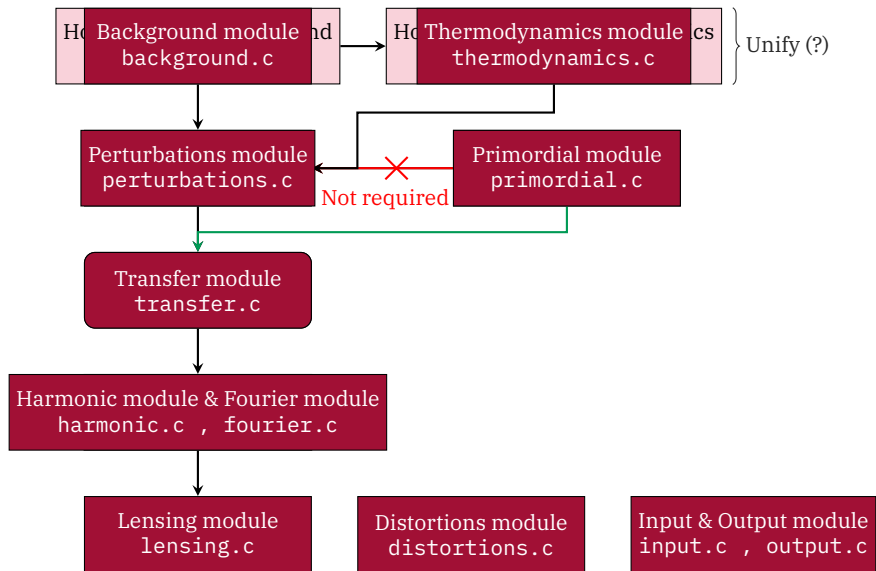
Fundamental layout of Einstein-Boltzmann solvers



Fundamental layout of Einstein-Boltzmann solvers



Fundamental layout of Einstein-Boltzmann solvers



Essentials 1: Input

Read in input files, take care of *shooting*.

```
h = 0.7  
#H0 = 70  
Omega_m = 0.3  
#omega_m = 0.14  
sigma8=0.8
```

Special care for equivalent/unknown parameters

Input management in `class`

For indirect parameters, use `shooting method`

Repeated calls of `input_read_parameters(...)`, `class` executions, from `input_read_from_file(...)` until shooting target is met.

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Use approximate formula \rightarrow inflexible, inaccurate

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Try out a few values and narrow down (Example: User wants $100\theta_s = 1.04325$)

h	$100\theta_s$
0.7	1.0522492086422521
0.65	1.0270326366580724
0.68215616173	1.0437999980620178
0.68110138476	1.0432819283581667
0.68103637942	1.0432499363679562
0.68103650871	1.0432499921072458
0.68103652701	1.0432500079710365
...	...

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0.68103650871	1.0432499921072458
0.68103652701	1.0432500079710365
...	...

In practice, use more sophisticated Ridder's method / Newton's method

Input management in **class**

For **shooting** parameters, establish mapping between *target parameter*, *unknown parameter* and *level*. Currently:

target parameter	unknown parameter	level
$100 \times \theta_s$	h	thermodynamics
Ω_{dcdm}	$\rho_{\text{dcdm}}^{\text{ini}}$	background
σ_8	A_s	spectra
...

... plus a few others (alternative parametrizations of decaying CDM, quintessence parameters).

This is what is used e.g. in models of early dark energy!

If you need to add such parameters: see how it is done e.g. for `100*theta_s` and replicate the structure!

Special exception $\tau_{\text{reio}} \leftrightarrow z_{\text{reio}}$ only concerns reionization and is done independently in `thermodynamics.c`

New Special exception: $\sigma_8 \leftrightarrow A_s$ can be very simply analytically re-scaled (multiplicative property), therefore done independently in `input.c`

Input management in `class`

Budget equation:

$$\sum_X \Omega_X = 1 + \Omega_k$$

To avoid over-constraining the input, one of the last three (`Omega_Lambda`, `Omega_fld`, `Omega_scf`) must be left unspecified and `class` will assign it using budget equation.

Possibly more advanced in the future

- default: `Omega_Lambda` is automatically adjusted
- if you pass `Omega_Lambda`, `Omega_fld` is automatically adjusted
- if you pass `Omega_Lambda` and `Omega_fld`: `Omega_scf` is automatically adjusted (if you allow, by setting to -1)

This allows whatever combination.

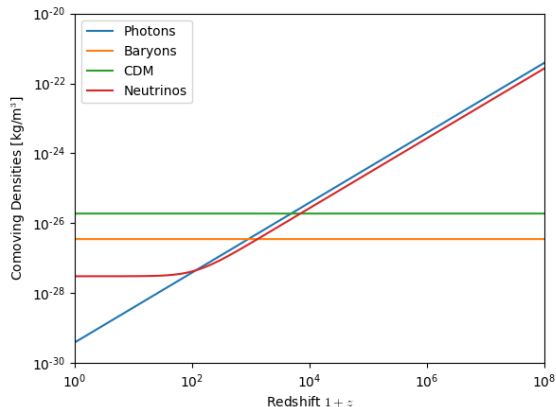
E.g. to get Λ plus a DE fluid:

`Omega_Lambda=0.2, Omega_scf=0` or `Omega_fld=0.3, Omega_scf=0`

Helpful output by setting background verbose `>= 2`

Essentials 2: Background

Get all background quantities as function of a scale factor a .



This also gives mapping $a \leftrightarrow z \leftrightarrow t \leftrightarrow \text{conf.time}$

The Background Module

Let's formalize problem!

Three types of parameters:

- $\{A\}$ are analytical functions of scale factor and $\{B\}$ quantities.
- $\{B\}$ need to be integrated over, and are used to compute $\{A\}$
- $\{C\}$ also need to be integrated over, but are not used to compute $\{A\}$.

The Background Module

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Λ CDM and many simple extensions:

- $\{A\} = \{\rho_i(a), p_i(a), H(a), \dots\}$ with e.g. $H(a) = \left(\sum_X \rho_X(a) - \frac{K}{a^2}\right)^{1/2}$
- $\{B\} = \{\}$ (eliminated since v3.0)
- $\{C\} = \{t, \tau, r_s, D, f\}$ with e.g. $\frac{dt}{da} = 1/H(a)$, $\frac{dr_s}{da} = c_s(a)/(a \cdot H(a))$

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Example of DE/DM/DR fluid:

- $\{A\} = \{\rho_i(a), p_i(a), H(a), \dots, w_{\text{fld}}(a)\}$
- $\{B\} = \{\rho_{\text{fld}}\}$ with $\frac{d\rho_{\text{fld}}}{da} = -3(1 + w_{\text{fld}}(a))\rho_{\text{fld}}$

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Exemple of extended cosmology with quintessence ϕ :

- $\{A\} = \{\rho_i, p_i, H, \dots, V(\phi), \rho_\phi(\phi, \phi')\}$ with e.g. $\rho_\phi(\phi, \phi') = \frac{1}{2}(\phi')^2 + V(\phi)$
- $\{B\} = \{\phi, \phi'\}$ with $\frac{d\phi}{da} = \phi' / [aH(a)]$, $\frac{d\phi'}{da} = -2\phi' - aV(\phi)/H(a)$

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Also Cold Dark Matter decaying into Dark Radiation...

- $\{A\} = \{\rho_i, p_i, H, \dots\}$
- $\{B\} = \{\rho_{\text{cdm}}, \rho_{\text{dr}}\}$ with $\frac{d\rho_{\text{cdm}}}{da} = -3\rho_{\text{cdm}} - \Gamma(a)/H(a) \cdot \rho_{\text{cdm}}$

The Background Module

Small details:

- Quantities as $D_A(z)$, $D_L(z)$, r_s , t_{age} can be derived after all A,B,C are computed
- Takes care of NCDM integration of phase-space distribution
- Useful checks & output
- → Budget equation output at verbosity level 2

Background exercise

Let's end this first lecture with a hands-on exercise.

Download the jupyter notebook [Exercise_background_to_fill.ipynb](#) and follow the steps to plot the properties computed by the background module.