CLASS

Cosmological Linear Anisotropy Solving System

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These slides available at https://github.com/MarkMos/class_lecture
Visit http://class-code.net/ for more info!

What to expect in these lectures:

- Basics: Why use `class`?
- Usage: Installation
- Usage: Python Interface
- Basics: Existing Species
- Basics: Module Overview
- Theory: What is `class` based upon?
- Coding: Implementing features

We will learn how to use `class` and which models can be run with it.

École Rodolphe Clédassou
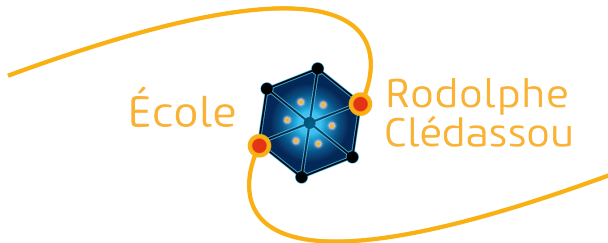
What is an Einstein-Boltzmann solver?

Often just called a *Boltzmann code* for brevity, a typical Boltzmann code will:

- Solve coupled Einstein and Boltzmann equations.
- Generally work at linear level in perturbation theory.
- Compute global (Background+Themodynamic) quantities *and* perturbations.

$$\underbrace{G_{\mu\nu} = 8\pi T_{\mu\nu}}_{\text{Einstein-equation}} \qquad \underbrace{\frac{\mathrm{d}f}{\mathrm{d}\lambda} = C[f]}_{\text{Boltzmann-equation}} \qquad (1)$$

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- CMB spectral distortions

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- Emulated non-linear power spectra
- CMB spectral distortions

All computed in a matter of seconds!

Why use a Boltzmann code?

This has several use cases:

- Analysis of CMB experiments
- Analysis of LSS experiments
- Initial conditions for non-linear simulations (*N*-body, etc.)
- Consistent treatment of background/thermodynamic evolution

All easy to to with `class`!

Fast execution $\Rightarrow$ ideal for use in an MCMC pipeline.

Why use `class`?

`class` is:

- Accurate: `class` & `camb` cross-check each other
- Versatile: Interfaces with `MontePython`, `Cobaya`, `Cosmosis`, `Procoli`, `CosmoPower`, `OLÉ`, `CONNECT`, and others!
- Comprehensive: Computes a wide range of cosmological observables for a large selection of models beyond ΛCDM.
- Modular and well-documented: ReadTheDocs page and Doxygen documentation, thoroughly commented source code, easy to modify

All strong arguments to use `class`!

Installing `class`

Using `class`

If you have no intention of modifying source code:

```
> pip install classy
```

And the `class` wrapper will be ready to use in your Python environment.

This is the easiest way to install and ideal if you only plan to call `class` via the Python wrapper.

Modifying `class`

If you wish to modify source code:

```
> git clone git@github.com:
    lesgourg/class_public.
    git class
> cd class/
> make clean; make -j
```

The wrapper can be used in your Python environment, and the binary executable can be called from the terminal.

Documentation

class is documented in various places:

- The file `explanatory.ini` lists all input parameters.
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  `https://github.com/lesgourg/class_public/wiki`
- Old course notes linked on `http://class-code.net` and
  `https://schoeneberg.github.io/`

Installing `class`

Let's take a few minutes to install class and confirm it is working!

- If you have installed `classy` via `pip`, try running the script `classy_test.py`
- If you have installed `class` via `git clone` and compiling, try also running `./class default.ini`

The code structure
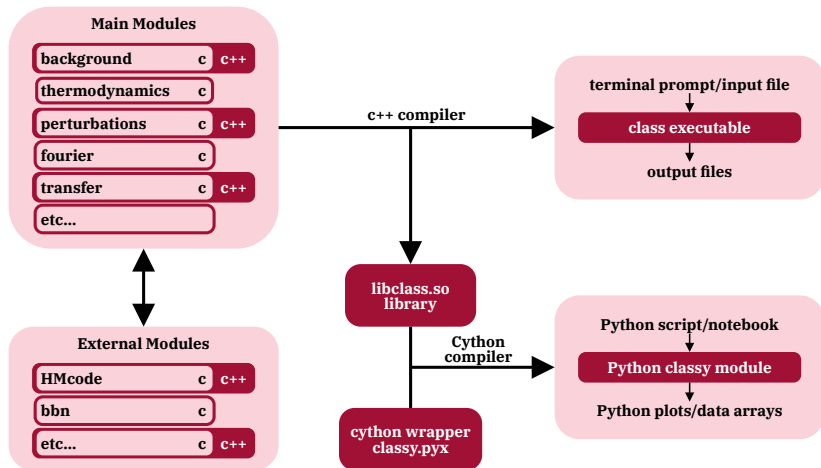
`class` vs `classy`

Let's clarify a bit of nomenclature!

- `class` is the C code that does all the work of solving the EBS. It is used from the command line.
- `classy` is the Python wrapper of `class`, and internally uses it. It is used from a python interpreter.

The two share same input/output, except different naming and format (e.g. `.ini` file vs `python dictionary` as input)

Running `class` in terminal

Run any input file with extension `*.ini`:

- Simple first-usage file

  ```
  ./class default.ini
  ```

- Huge reference file containing all possible input parameters with comments

  ```
  ./class explanatory.ini
  ```

- Slim file matching Planck 2018 "baseline model" bestfit

  ```
  ./class base_2018_plikHM_TTTEEE_lowl_lowE_lensing.ini
  ```

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- All input is presented in detail in `explanatory.ini` (apart from precision parameters)

- This is a *reference* file; we advise you to not modify it:
  - either start from a slim file (like `default.ini`),
  - or copy it and reduce it to a shorter and more friendly file,
  - or write your own from scratch with only needed input lines.

class input parameters

The common 'language' for input is as follows

```
parameter = value
```

with the python dictionary equivalent of

```
{'parameter':'value'}
```

where value is passed as a python string.

`class` input parameters

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```
parameter = value
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with the python dictionary equivalent of

```
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where value is passed as a `python string`.

Special cases include

```
option = yes/no
selection = a,b,c,d
```

and comments are

```
parameter = value #comment behind parameter
#comment in its own line
```

class vs classy

Let's compare `class` vs `classy` for an execution:

class	classy
`.ini` file with `parameter = value`	`python dictionary` with `'parameter':'value'`
Solving of equations in `class`	
Output to files Options write files	Output from python functions Options enable functions (otherwise error message)

`./class myfile.ini`	`import classy`
	`cosmo = classy.Class()`
	`cosmo.set(mydictionary)`

Input parameters

If nothing given → Planck 2013 cosmology with $\sum m_\nu = 0$

Any parameter overwrites the defaults (from `.ini file` or `cosmo.set`)

Precision file `cl_ref.pre` → close to 'optimal' precision

Input parameters

Most common/important parameters:

- output =   tCl   ,   pCl   ,   lCl   ,

  Temperature C_ℓ^{TT} Polarization $C_\ell^{TE/EE/BB}$ Lensing $C_\ell^{\phi\phi}$

      mPk   ,   mTk   ,   vTk   ,

  Matter power $P(k,z)$ Matter transfer $\delta_i(k,z)$ Velocity transfer $\theta_i(k,z)$

      nCl   ,   sCl

  Number count C_ℓ^{dd} Galaxy Lensing count C_ℓ^{ss}

Input parameters

Most common/important parameters:

- `l_max_scalars = 2500`
- `P_k_max_1/Mpc = 1`
- `z_pk = 0, 1, 2` for class
- `z_max_pk = 10` for classy
- `format = CAMB` for initial conditions
- `write_warnings = yes` for class

Input parameters

Most common/important parameters:

- input_verbose = 1
- background_verbose = 2
- thermodynamics_verbose = 1
- perturbations_verbose = 1
- fourier_verbose = 1
- output_verbose = 1

Verbosity parameters for class

Input parameters

The ΛCDM parameters $\{H_0, A_s, n_s, \Omega_m, \Omega_b h^2, \tau_{\text{reio}}\}$:

- Hubble constant
  ```
  H_0=67
  h=0.67
  100*theta_s = 1.042
  ```

Input parameters

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- Hubble constant
  ```
  H_0=67
  h=0.67
  100*theta_s = 1.042
  ```
- Primordial amplitude
  ```
  A_s=2.1e-9
  ln10^{10}A_s = 3.0      =ln(10^10 A_s)
  ln_A_s_1e10 = 3.0
  sigma8 = 0.825       =∫ (k^3 P(k,z=0))/(2π^2) T^2(kR_8) d ln k
  ```

  $$\ln10^{10}A_s = 3.0 \qquad =\ln(10^{10}A_s)$$

  $$\text{sigma8} = 0.825 \qquad =\int \frac{k^3 P(k,z=0)}{2\pi^2} T^2(kR_8)\, \mathrm{d}\ln k$$

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- Primordial tilt
  ```
  n_s=0.96
  ```

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- Primordial tilt
  ```
  n_s=0.96
  ```
- Matter abundance
  ```
  Omega_m=0.3
  omega_m=0.14      = Omega_m h^2
  Omega_cdm=0.25
  Omega_cdm=0.11    = Omega_cdm h^2
  ```
  $\texttt{omega\_m=0.14} \quad = \Omega_m h^2$

  $\texttt{Omega\_cdm=0.11} \quad = \Omega_{\mathrm{cdm}} h^2$

Input parameters

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  $= \int \frac{k^3 P(k,z=0)}{2\pi^2} T^2(kR_8) {\rm d} \ln k$

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- Matter abundance
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  Omega_m=0.3
  omega_m=0.14         = Ω_m h^2
  Omega_cdm=0.25
  Omega_cdm=0.11       = Ω_cdm h^2
  ```

- Baryon abundance
  ```
  Omega_b=0.05
  omega_m=0.02233      = Ω_b h^2
  ```

- Reionization time
  ```
  z_reio=7.0
  tau_reio=0.05
  ```
  $\approx \int_0^{z_{\rm reio}} \sigma_T n_e {\rm d}\eta$

Input parameters

One-parameter extensions

- Curvature
  `Omega_k = 0`
- Dark radiation
  `N_ur = 3.044` $\hat{=} N_{\text{eff}}$
- CMB temperature
  `T_cmb = 2.7255`
- Lensing enhancement
  `A_L = 1`
- Primordial running
  `alpha_s = 0`

Input parameters

Dark Energy options

- Cosmological constant
  `Omega_Lambda`
- Fluid
  `Omega_fld`
- Scalar field
  `Omega_scf`

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Universe is always filled (to be as curved as Ω_k) by putting whatever dark energy is still available (If you want to fill with scalar field, you need `Omega_scf<0`).
Using the Budget equation

$$\sum \Omega_i = 1 + \Omega_k \qquad (2)$$

Input parameters

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Examples:

`Omega_m=0.3`	$\Rightarrow$	`Omega_Lambda -> 0.7`
`Omega_fld=0.5,Omega_m=0.3`	$\Rightarrow$	`Omega_Lambda -> 0.2`
`Omega_Lambda=0,Omega_m=0.3`	$\Rightarrow$	`Omega_fld -> 0.7`
`Omega_Lambda=0,Omega_fld=0,` `Omega_scf=-1,Omega_m=0.3`	$\Rightarrow$	`Omega_scf -> 0.7`
`Omega_k=0.2,Omega_m=0.3`	$\Rightarrow$	`Omega_Lambda -> 0.9`

Input parameters

Neutrino mass

- `N_ncdm=1` (single massive neutrino)
- `m_ncdm=0.06` (in eV)
- `N_ur = 3.044 - 1.0132 N_ncdm = 2.0308`
  (each massive neutrino contributes 1.0132 to N_{eff} [QED corrections, could be optimized in future])
  or
- `Neff = 3.044` will automatically adjust `N_ur` to achieve the desired N_{eff}

Input parameters

Neutrino mass

- `N_ncdm=3` (three massive neutrinos)
- `m_ncdm=0,0.0495,0.0582` (inverted hierarchy)
- `N_ur = 3.044 - 1.0132 N_ncdm = 0.0044`
  or
- `Neff = 3.044` will automatically adjust `N_ur` to achieve the desired N_{eff}

Input parameters

Neutrino mass

- `N_ncdm=1` (one *degenerate* massive neutrinos)
- `deg_ncdm=3` (triply degenerate)
- `m_ncdm=0.02` (total mass 0.06eV)
- `N_ur = 3.044 - 1.0132 N_ncdm * deg_ncdm = 0.0044`
  or
- `Neff = 3.044` will automatically adjust `N_ur` to achieve the desired N_{eff}

Plotting

You can get plots

1. Manually: using the output files with e.g. `gnuplot`, `IDL`, `python`, `Mathematic`, `GNU Octave`...

2. Automatically: using `python` and script `CPU.py`, or `MATLAB` and script `plot_CLASS_output.m`

3. Interactively: using `class` as a `python` module, within a `python` session or a `Jupyter Notebook`

Running `class` from Python

`class` as a `Python` module

- based on wrapper located in `python/classy.pyx` (developed initially by B. Audren and extended by many others)
- the compilation produces a python module `classy.py` and installs it on your computer (can be called from anywhere)
- wrapper written in `Cython`, encapsulates most useful `class` variables/functions, contains extra functions (e.g. MontePython-motivated)
- (project: get most of the wrapper generated automatically from C code at compilation - Coming soon!)
- goal: obtain, manipulate and plot the results directly within (i)python scripts or notebooks (recommended)

Existing content already in `class`

ΛCDM:

- Baryons `Omega_b`$= \Omega_b$ or `omega_b`$= \Omega_b h^2$

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ΛCDM:

- Baryons `Omega_b=` Ω_b or `omega_b=` $\Omega_b h^2$
- Cold Dark matter `Omega_cdm=` Ω_{cdm} or `omega_cdm=` $\Omega_{\mathrm{cdm}} h^2$ or `Omega_m=` Ω_{m} or `omega_m=` $\Omega_{\mathrm{m}} h^2$

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ΛCDM:

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- **Hubble** `H0`$= H_0$ or `h`$= h$ or `100*theta_s`$= 100\theta_s$

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ΛCDM:

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- Hubble `H0`$= H_0$ or `h`$= h$ or `100*theta_s`$= 100\theta_s$
- Reionization `z_reio`$= z_{\rm reio}$ or `tau_reio`$= \tau_{\rm reio}$

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- **Hubble** `H0`$= H_0$ or `h`$= h$ or `100*theta_s`$= 100\theta_s$
- **Reionization** `z_reio`$= z_{\rm reio}$ or `tau_reio`$= \tau_{\rm reio}$
- **Primordial Amplitude** `A_s`$= A_s$ or `ln10^_s`$= \ln(10^{10}A_s)$ or `sigma8`$= \sigma_8$

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- Primordial Amplitude `A_s`$= A_s$ or `ln10^_s`$= \ln(10^{10}A_s)$ or `sigma8`$= \sigma_8$
- Primordial tilt `n_s`$= n_s$

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- Neutrinos m_ν (m_ncdm, N_ncdm, N_ur adjustment see explanatory.ini)

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Standard simple extensions:

- Curvature Ω_k (`Omega_k`)
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- Free-streaming dark radiation N_{eff} (`N_ur`)

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- Free-streaming dark radiation N_{eff} (`N_ur`)
- Helium abundance Y_p (`YHe`)
- Primordial running α_s (`alpha_s`)

Existing content already in `class`

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- Helium abundance Y_p (YHe)
- Primordial running α_s (alpha_s)
- CPL Dark Energy w_0, w_a (Omega_fld, w0_fld, wa_fld, Omega_Lambda=0?)

Existing content already in `class`

Standard simple extensions:

- Curvature Ω_k (`Omega_k`)
- Neutrinos m_ν (`m_ncdm`, `N_ncdm`, `N_ur` adjustment see `explanatory.ini`)
- Free-streaming dark radiation N_{eff} (`N_ur`)
- Helium abundance Y_p (`YHe`)
- Primordial running α_s (`alpha_s`)
- CPL Dark Energy w_0, w_a (`Omega_fld`, `w0_fld`, `wa_fld`, `Omega_Lambda=0?`)
- CMB temperature T_{cmb}

Existing content already in `class`

Dark matter:

- Thermal Warm Dark Matter (`m_ncdm`, `T_ncdm`, `omega_ncdm`)
- Annihilating dark matter
- Decaying dark matter (`Omega_dcdmdr`, `Gamma_dcdm`)
- Non-trivial phase-space distribution (`ncdm` framework), neutrino flavor mixing, neutrino chemical potential
- Interacting with photons, baryons, neutrinos (`idm`)

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- Fluid/Self-interacting (`idr`)
- Viscous (`ceff2_ur`, `cvis_ur`)

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- Fluid/Self-interacting (`idr`)
- Viscous (`ceff2_ur`, `cvis_ur`)

Dark energy:

- CPL, EDE (`fld`) (this `EDE` is not the usual EDE → https://github.com/PoulinV/AxiCLASS and https://github.com/flo1984/TriggerCLASS to be merged)
- Other fluid-like DE (`fld`)
- Quintessence/Scalar field (`scf`)

Existing content already in `class`

Thermal modeling

- Recfast recombination
- Hyrec-2recombination
- Tanh reionization
- Multi-tanh reionization
- Reionization from file
- Energy injection (PBH Evaporation,PBH Accretion,DM Decay,DM Annihilation)

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Spectral distortions

- y and μ distortions
- PCA of intermediate distortions

Existing content already in `class`

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Inflation/Primordial Powerspectrum

- Arbitrary potential $V(\phi)$
- Automatic finding of observable window
- Can impose inflation duration
- Tensor modes included automatically
- Read from file

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- Perturbation theory (very soon)

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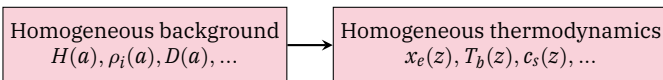- Perturbation theory (very soon)

Modified Gravity
$\rightarrow$ HiCLASS branch (Bellini, Sawicki, Zumalacarregui, http://www.hiclass-code.net)

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http://www.hiclass-code.net)

Extension to second-order perturbation theory
SONG (Fidler, Pettinari, Tram, https://github.com/coccoinomane/song)

Fundamental layout of Einstein-Boltzmann solvers

```
┌─────────────────────────────┐      ┌─────────────────────────────────┐
│  Homogeneous background     │ ──→  │  Homogeneous thermodynamics     │
│  $H(a), \rho_i(a), D(a), ...$ │      │  $x_e(z), T_b(z), c_s(z), ...$  │
└─────────────────────────────┘      └─────────────────────────────────┘
```

Fundamental layout of Einstein-Boltzmann solvers

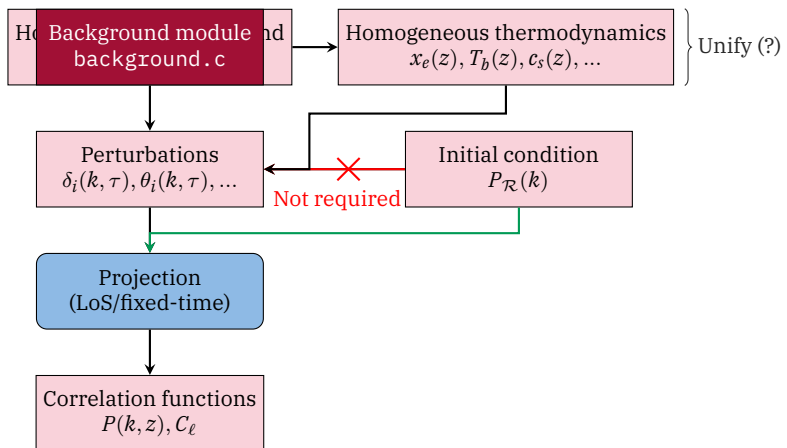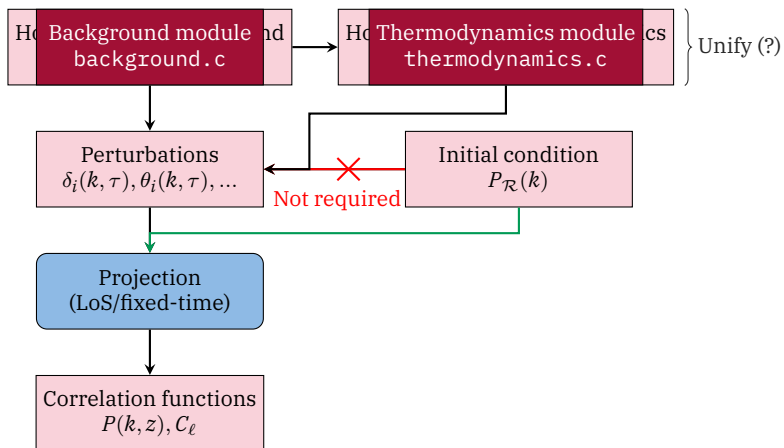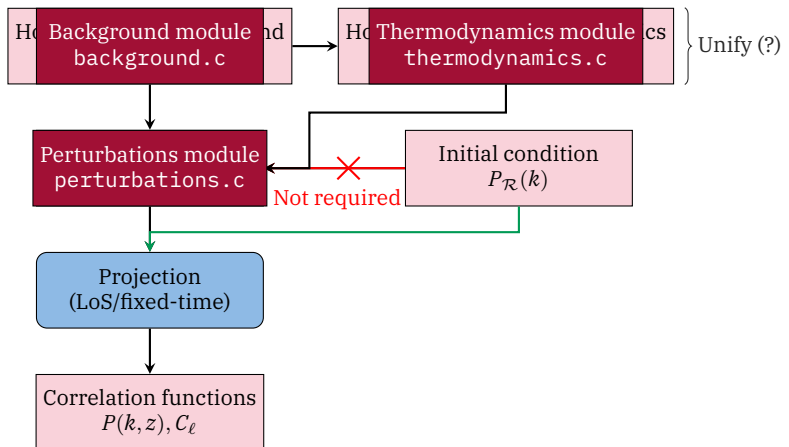| Homogeneous background $H(a), \rho_i(a), D(a), ...$ | → | Homogeneous thermodynamics $x_e(z), T_b(z), c_s(z), ...$ | } Unify (?) |

Fundamental layout of Einstein-Boltzmann solvers

Fundamental layout of Einstein-Boltzmann solvers

Fundamental layout of Einstein-Boltzmann solvers

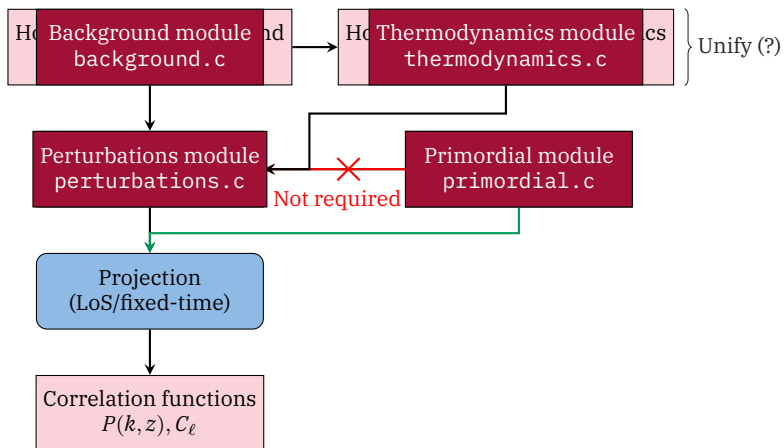# Fundamental layout of Einstein-Boltzmann solvers

Fundamental layout of Einstein-Boltzmann solvers

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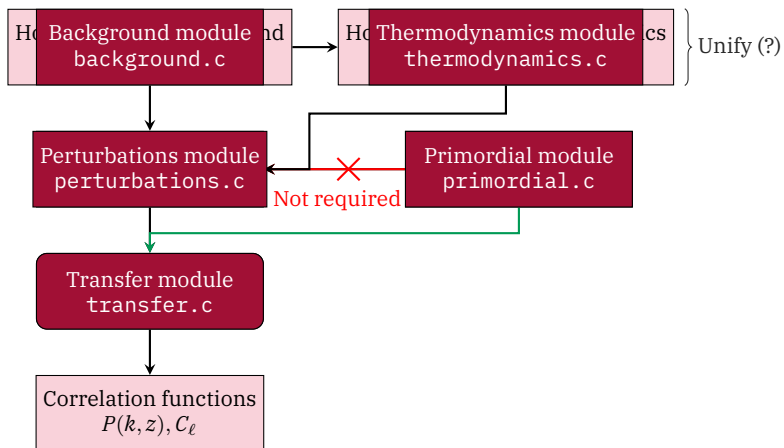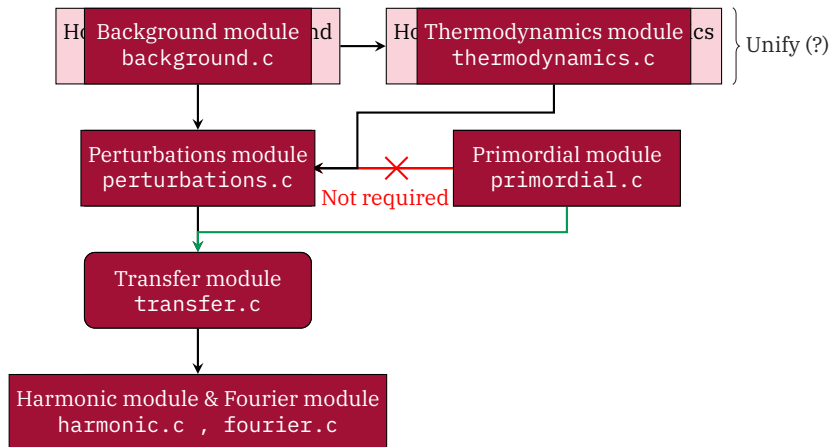
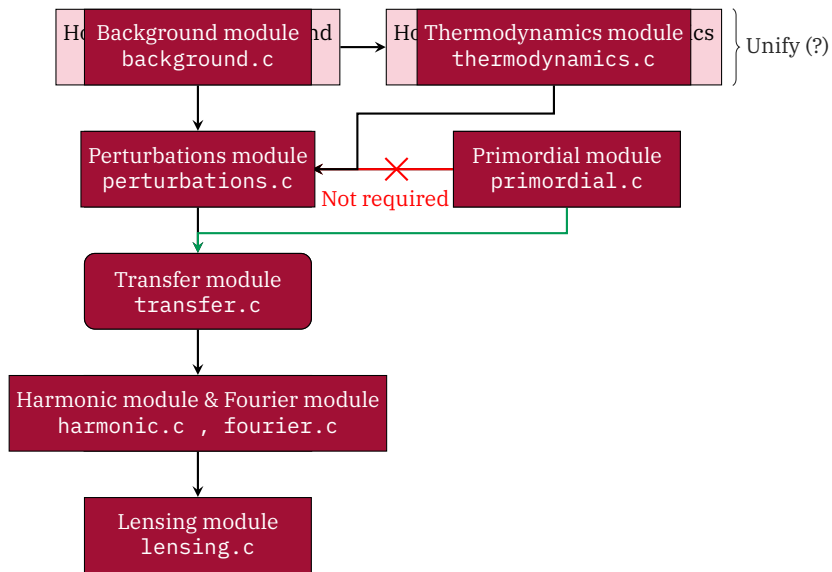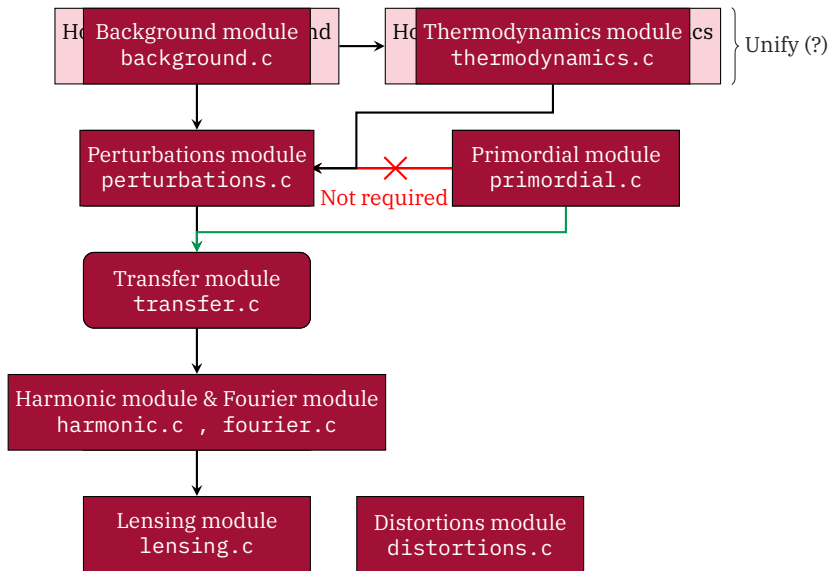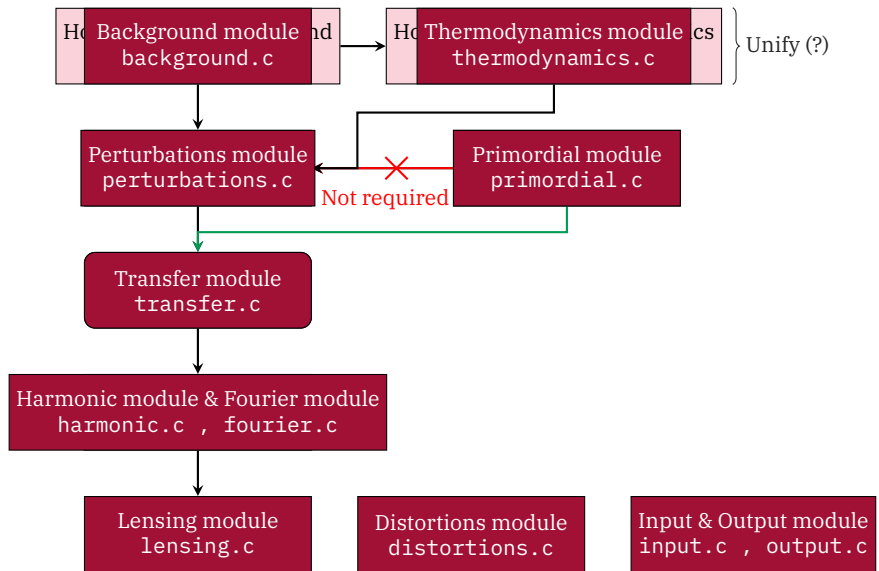# Fundamental layout of Einstein-Boltzmann solvers

Fundamental layout of Einstein-Boltzmann solvers

Fundamental layout of Einstein-Boltzmann solvers

Fundamental layout of Einstein-Boltzmann solvers

Essential steps in Einstein-Boltzmann solver

Read in input files, take care of *shooting*.

```
h = 0.7
#H0 = 70
Omega_m = 0.3
#omega_m = 0.14
sigma8=0.8
```

Special care for equivalent/unknown parameters

Input management in `class`

Terminal	Python wrapper

file `xxx.ini`

↓

input_init(...)
(parser)

.set(...)

↓   ↓

struct file_content fc; (all parameter names/values stored as arrays of strings)

↓

input_read_from_file(...)

↓

input_read_parameters(...)

(assign all default values + interpret input + update some parameters)

↓

Only *relevant* parameters get stored in the structures of each module

Input management in `class`

For indirect parameters, use shooting method

Repeated calls of `input_read_parameters(...)`, `class` executions, from `input_read_from_file(...)` until shooting target is met.

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Use approximate formula $\rightarrow$ inflexible, inaccurate

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Example:
How would you code the input parameter θ_s?
Use approximate formula $\rightarrow$ inflexible, inaccurate

Try out a few values and narrow down (Example: User wants $100\theta_s = 1.04325$)

h	$100\theta_s$
0.7	1.0522492086422521
0.65	1.0270326366580724
0.68215616173	1.0437999980620178
0.68110138476	1.0432819283581667
0.68103637942	1.0432499363679562
0.68103650871	1.0432499921072458
0.68103652701	1.0432500079710365
...	...

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0.68103637942	1.0432499363679562
0.68103650871	1.0432499921072458
0.68103652701	1.043250079710365
...	...

In practice, use more sophisticated Ridder's method / Newton's method

Input management in `class`

For shooting parameters, establish mapping between *target parameter*, *unknown parameter* and *level*. Currently:

target parameter	unknown parameter	level
$100 \times \theta_s$	h	thermodynamics
Ω_{dcdm}	$\rho_{\mathrm{dcdm}}^{\mathrm{ini}}$	background
~~σ_8~~	~~A_s~~	~~spectra~~
...	...	...

... plus a few others (alternative parametrizations of decaying CDM, quintessence parameters).

This is what is used e.g. in models of early dark energy!

If you need to add such parameters: see how it is done e.g. for `100*theta_s` and replicate the structure!

Special exception $\tau_{\mathrm{reio}} \leftrightarrow z_{\mathrm{reio}}$ only concerns reionization and is done independently in `thermodynamics.c`
New Special exception: $\sigma_8 \leftrightarrow A_s$ can be very simply analytically re-scaled (multiplicative property), therefore done independently in `input.c`

Input management in `class`

Budget equation:

$$\sum_X \Omega_X = 1 + \Omega_k$$

To avoid over-constraining the input, one of the last three (`Omega_Lambda`, `Omega_fld`, `Omega_scf`) must be left unspecified and `class` will assign it using budget equation.

Possibly more advanced in the future

- default: `Omega_Lambda` is automatically adjusted
- if you pass `Omega_Lambda`, `Omega_fld` is automatically adjusted
- if you pass `Omega_Lambda` and `Omega_fld`: `Omega_scf` is automatically adjusted (if you allow, by setting to -1)
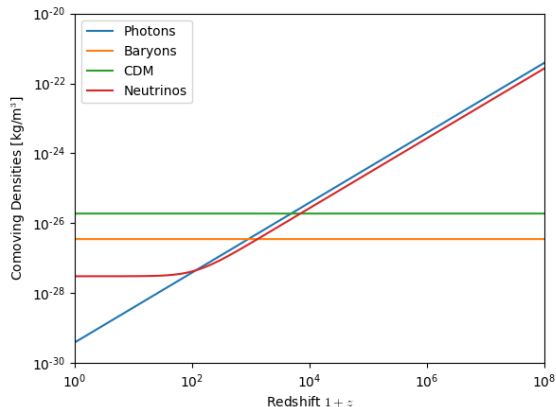
This allows whatever combination.

E.g. to get Λ plus a DE fluid:

`Omega_Lambda=0.2, Omega_scf=0` or `Omega_fld=0.3, Omega_scf=0`

Helpful output by setting background verbose >= 2

Essentials 2: Background

Get all background quantities as function of a scale factor a.

This also gives mapping $a \leftrightarrow z \leftrightarrow t \leftrightarrow$ conf.time

The Background Module

Let's formalize problem!

Three types of parameters:

- $\{A\}$ are analytical functions of scale factor and $\{B\}$ quantities.
- $\{B\}$ need to be integrated over, and are used to compute $\{A\}$
- $\{C\}$ also need to be integrated over, but are not used to compute $\{A\}$.

The Background Module

Let's formalize problem!

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ΛCDM and many simple extensions:

- $\{A\} = \{\rho_i(a), p_i(a), H(a), ..., \}$ with e.g. $H(a) = \left(\sum_X \rho_x(a) - \frac{K}{a^2}\right)^{1/2}$
- $\{B\} = \{\}$ (eliminated since v3.0)
- $\{C\} = \{t, \tau, r_s, D, f\}$ with e.g. $\frac{dt}{da} = 1/H(a)$, $\frac{dr_s}{da} = c_s(a)/(a \cdot H(a))$

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Example of DE/DM/DR fluid:

- $\{A\} = \{\rho_i(a), p_i(a), H(a), ..., w_{\text{fld}}(a)\}$
- $\{B\} = \{\rho_{\text{fld}}\}$ with $\frac{d\rho_{\text{fld}}}{da} = -3(1 + w_{\text{fld}}(a))\rho_{\text{fld}}$

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Example of extended cosmology with quintessence ϕ:

- $\{A\} = \{\rho_i, p_i, H, ..., V(\phi), \rho_\phi(\phi, \phi')\}$ with e.g. $\rho_\phi(\phi, \phi') = \frac{1}{2}(\phi')^2 + V(\phi)$
- $\{B\} = \{\phi, \phi'\}$ with $\frac{d\phi}{da} = \phi'/[aH(a)]$, $\frac{d\phi'}{da} = -2\phi' - aV(\phi)/H(a)$

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Also Cold Dark Matter decaying into Dark Radiation...

- $\{A\} = \{\rho_i, p_i, H, ...\}$
- $\{B\} = \{\rho_{\text{dcdm}}, \rho_{\text{dr}}\}$ with $\frac{d\rho_{\text{dcdm}}}{da} = -3\rho_{\text{dcdm}} - \Gamma(a)/H(a) \cdot \rho_{\text{dcdm}}$

The Background Module

Small details:

- Quantities as $D_A(z), D_L(z), r_s, t_{age}$ can be derived after all A,B,C are computed
- Takes care of NCDM integration of phase-space distribution
- Useful checks & output
- $\rightarrow$ Budget equation output at verbosity level 2

Background exercise

Let's end this first lecture with a hands-on exercise.
Download the jupyter notebook Exercise_background_to_fill.ipynb and follow the
steps to plot the properties computed by the background module.