CLASS

Cosmological Linear Anisotropy Solving System

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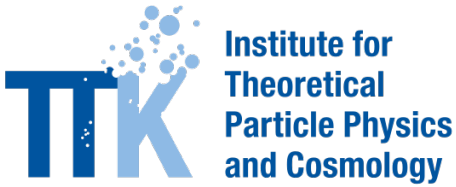
These slides available at https://github.com/MarkMos/class_lecture
Visit http://class-code.net/ for more info!

class in Aachen

What to expect in these lectures:

- Basics: Why use class?
- Usage: Installation
- Usage: Python Interface
- Basics: Existing Species
- Basics: Module Overview
- Theory: What is class based upon?
- Coding: Implementing features

We will learn how to use class and which models can be run with it.



What is an Einstein-Boltzmann solver?

Often just called a *Boltzmann code* for brevity, a typical Boltzmann code will:

- Solve coupled Einstein and Boltzmann equations.
- Generally work at linear level in perturbation theory.
- Compute global (Background+Themodynamic) quantities *and* perturbations.

$$\underbrace{G_{\mu\nu} = 8\pi T_{\mu\nu}}_{\text{Einstein-equation}} \qquad \underbrace{\frac{\mathrm{d}f}{\mathrm{d}\lambda} = C[f]}_{\text{Boltzmann-equation}} \tag{1}$$

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Modern Boltzmann codes offer:

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- CMB spectral distortions

All computed in a matter of seconds!

Why use a Boltzmann code?

This has several use cases:

- Analysis of CMB experiments
- Analysis of LSS experiments
- Initial conditions for non-linear simulations (N-body, etc.)
- Consistent treatment of background/thermodynamic evolution

All easy to to with `class`!

Fast execution $\Rightarrow$ ideal for use in an MCMC pipeline.

Why use `class`?

`class` is:

- Accurate: `class` & `camb` cross-check each other
- Versatile: Interfaces with `MontePython`, `Cobaya`, `Cosmosis`, `Procoli`, `CosmoPower`, `OLÉ`, `CONNECT`, and others!
- Comprehensive: Computes a wide range of cosmological observables for a large selection of models beyond ΛCDM.
- Modular and well-documented: ReadTheDocs page and Doxygen documentation, thoroughly commented source code, easy to modify

All strong arguments to use `class`!

Installing `class`

Using `class`

If you have no intention of modifying source code:

```
> pip install classy
```

And the `class` wrapper will be ready to use in your Python environment.

This is the easiest way to install and ideal if you only plan to call `class` via the Python wrapper.

Modifying `class`

If you wish to modify source code:

```
> git clone git@github.com:
    lesgourg/class_public.
    git class
> cd class/
> make clean; make -j
```

The wrapper can be used in your Python environment, and the binary executable can be called from the terminal.

Documentation

`class` is documented in various places:

- The file `explanatory.ini` lists all input parameters.
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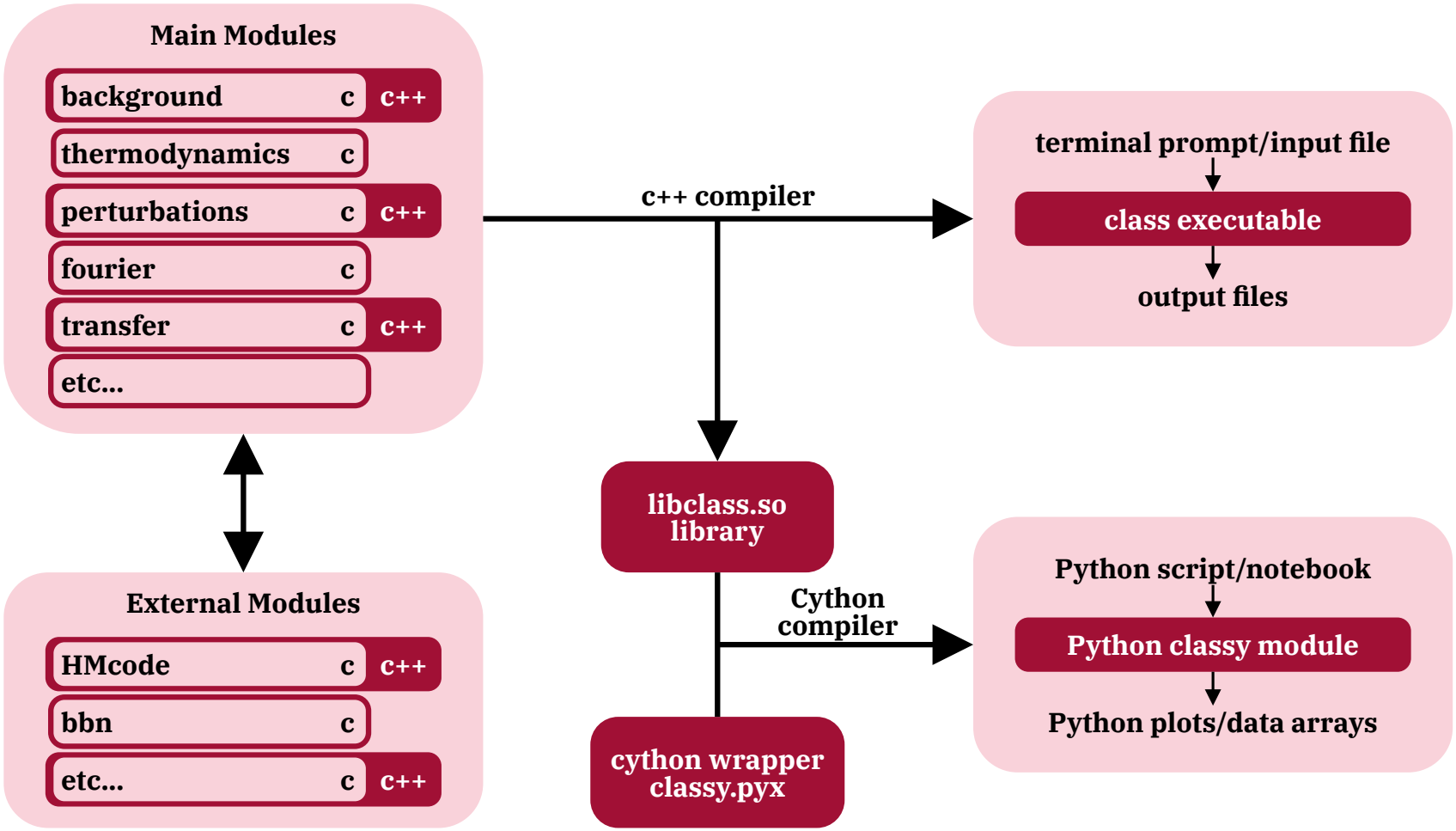
- Online documentation at
  https://github.com/lesgourg/class_public/wiki
- Old course notes linked on http://class-code.net and
  https://schoeneberg.github.io/

Installing `class`

Let's take a few minutes to install class and confirm it is working!

- If you have installed `classy` via `pip`, try running the script `classy_test.py`
- If you have installed `class` via `git clone` and compiling, try also running `./class default.ini`

The code structure



`class` vs `classy`

Let's clarify a bit of nomenclature!

- `class` is the `C` code that does all the work of solving the EBS. It is used from the command line.
- `classy` is the `Python` wrapper of `class`, and internally uses it. It is used from a python interpreter.

The two share same input/output, except different naming and format (e.g. `.ini` file vs `python dictionary` as input)

Running `class` in terminal

Run any input file with extension `*.ini`:

- Simple first-usage file

```
./class default.ini
```

- Huge reference file containing all possible input parameters with comments

```
./class explanatory.ini
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- Slim file matching Planck 2018 ”baseline model” bestfit

```
./class base_2018_plikHM_TTTEEE_lowl_lowE_lensing.ini
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- All input is presented in detail in `explanatory.ini` (apart from precision parameters)
- This is a *reference* file; we advise you to not modify it:
  - either start from a slim file (like `default.ini`),
  - or copy it and reduce it to a shorter and more friendly file,
  - or write your own from scratch with only needed input lines.

`class` input parameters

The common 'language' for input is as follows

```
parameter = value
```

with the python dictionary equivalent of

```
{'parameter':'value'}
```

where value is passed as a `python string`.

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where value is passed as a python string.

Special cases include

```
option = yes/no
selection = a,b,c,d
```

and comments are

```
parameter = value #comment behind parameter
#comment in its own line
```

class vs classy

Let's compare class vs classy for an execution:

class	classy
.ini file with parameter = value	python dictionary with 'parameter':'value'
Solving of equations in class	
Output to files Options write files	Output from python functions Options enable functions (otherwise error message)
./class myfile.ini	import classy cosmo = classy.Class() cosmo.set(mydictionary)

Input parameters

If nothing given $\rightarrow$ Planck 2013 cosmology with $\sum m_\nu = 0$

Any parameter overwrites the defaults (from `.ini file` or `cosmo.set`)

Precision file `cl_ref.pre` $\rightarrow$ close to 'optimal' precision

Input parameters

Most common/important parameters:

- `output = tCl , pCl , lCl ,`
  - `tCl`: Temperature C_ℓ^{TT}
  - `pCl`: Polarization $C_\ell^{TE/EE/BB}$
  - `lCl`: Lensing $C_\ell^{\phi\phi}$
  - `mPk , mTk , vTk ,`
    - `mPk`: Matter power $P(k,z)$
    - `mTk`: Matter transfer $\delta_i(k,z)$
    - `vTk`: Velocity transfer $\theta_i(k,z)$
  - `nCl , sCl`
    - `nCl`: Number count C_ℓ^{dd}
    - `sCl`: Galaxy Lensing count C_ℓ^{ss}

Input parameters

Most common/important parameters:

- `l_max_scalars = 2500`
- `P_k_max_1/Mpc = 1`
- `z_pk = 0, 1, 2` for `class`
- `z_max_pk = 10` for `classy`
- `format = CAMB` for initial conditions
- `write_warnings = yes` for `class`

Input parameters

Most common/important parameters:

- `input_verbose = 1`
- `background_verbose = 2`
- `thermodynamics_verbose = 1`
- `perturbations_verbose = 1`
- `fourier_verbose = 1`
- `output_verbose = 1`

Verbosity parameters for `class`

Input parameters

The ΛCDM parameters $\{H_0, A_s, n_s, \Omega_m, \Omega_b h^2, \tau_{\rm reio}\}$:

- Hubble constant

```
H_0=67
h=0.67
100*theta_s = 1.042
```

Input parameters

The ΛCDM parameters $\{H_0, A_s, n_s, \Omega_m, \Omega_b h^2, \tau_{\rm reio}\}$:

- Hubble constant
  `H_0=67`
  `h=0.67`
  `100*theta_s = 1.042`
- Primordial amplitude
  `A_s=2.1e-9`
  `ln10^{10}A_s = 3.0` $=\ln(10^{10} A_s)$
  `ln_A_s_1e10 = 3.0`
  `sigma8 = 0.825` $=\int \frac{k^3 P(k, z=0)}{2\pi^2} T^2(kR_8) \mathrm{d}\ln k$

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- Matter abundance
  `Omega_m=0.3`
  `omega_m=0.14` $= \Omega_m h^2$
  `Omega_cdm=0.25`
  `Omega_cdm=0.11` $= \Omega_{\rm cdm} h^2$

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  `Omega_cdm=0.25`
  `Omega_cdm=0.11` $= \Omega_{\rm cdm} h^2$
- Baryon abundance
  `Omega_b=0.05`
  `omega_m=0.02233` $= \Omega_b h^2$
- Reionization time
  `z_reio=7.0`
  `tau_reio=0.05` $\approx \int_0^{z_{\rm reio}} \sigma_T n_e \mathrm{d}\eta$

Input parameters

One-parameter extensions

- Curvature
  `Omega_k = 0`
- Dark radiation
  `N_ur = 3.044` $\hat{=} N_{\rm eff}$
- CMB temperature
  `T_cmb = 2.7255`
- Lensing enhancement
  `A_L = 1`
- Primordial running
  `alpha_s = 0`

Input parameters

Dark Energy options

- Cosmological constant
  `Omega_Lambda`
- Fluid
  `Omega_fld`
- Scalar field
  `Omega_scf`

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Universe is always filled (to be as curved as Ω_k) by putting whatever dark energy is still available (If you want to fill with scalar field, you need `Omega_scf<0`).
Using the Budget equation

$$\sum \Omega_i = 1 + \Omega_k \tag{2}$$

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Examples:

`Omega_m=0.3`	$\Rightarrow$	`Omega_Lambda -> 0.7`
`Omega_fld=0.5,Omega_m=0.3`	$\Rightarrow$	`Omega_Lambda -> 0.2`
`Omega_Lambda=0,Omega_m=0.3`	$\Rightarrow$	`Omega_fld -> 0.7`
`Omega_Lambda=0,Omega_fld=0, Omega_scf=-1,Omega_m=0.3`	$\Rightarrow$	`Omega_scf -> 0.7`
`Omega_k=0.2,Omega_m=0.3`	$\Rightarrow$	`Omega_Lambda -> 0.9`

Input parameters

Neutrino mass

- `N_ncdm=1` (single massive neutrino)
- `m_ncdm=0.06` (in eV)
- `N_ur = 3.044 - 1.0132 N_ncdm = 2.0308`
  (each massive neutrino contributes 1.0132 to $N_{\rm eff}$ [QED corrections, could be optimized in future])
  or
- `Neff = 3.044` will automatically adjust `N_ur` to achieve the desired $N_{\rm eff}$

Input parameters

Neutrino mass

- `N_ncdm=3` (three massive neutrinos)
- `m_ncdm=0,0.0495,0.0582` (inverted hierarchy)
- `N_ur = 3.044 - 1.0132 N_ncdm = 0.0044`
  or
- `Neff = 3.044` will automatically adjust `N_ur` to achieve the desired $N_{\rm eff}$

Input parameters

Neutrino mass

- `N_ncdm=1` (one *degenerate* massive neutrinos)
- `deg_ncdm=3` (triply degenerate)
- `m_ncdm=0.02` (total mass 0.06eV)
- `N_ur = 3.044 - 1.0132 N_ncdm * deg_ncdm = 0.0044`
  or
- `Neff = 3.044` will automatically adjust `N_ur` to achieve the desired $N_{\rm eff}$

Plotting

You can get plots

1. Manually: using the output files with e.g. `gnuplot`, `IDL`, `python`, `Mathematic`, `GNU Octave`...
2. Automatically: using `python` and script `CPU.py`, or `MATLAB` and script `plot_CLASS_output.m`
3. Interactively: using `class` as a `python` module, within a `python` session or a `Jupyter Notebook`

Running `class` from Python

`class` as a `Python` module

- based on wrapper located in `python/classy.pyx` (developed initially by B. Audren and extended by many others)
- the compilation produces a python module `classy.py` and installs it on your computer (can be called from anywhere)
- wrapper written in `Cython`, encapsulates most useful `class` variables/functions, contains extra functions (e.g. MontePython-motivated)
- (project: get most of the wrapper generated automatically from C code at compilation - Coming soon!)
- goal: obtain, manipulate and plot the results directly within (i)python scripts or notebooks (recommended)

Existing content already in `class`

ΛCDM:

- Baryons `Omega_b`$= \Omega_b$ or `omega_b`$= \Omega_b h^2$

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- Primordial tilt `n_s`$= n_s$

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- Neutrinos m_ν (`m_ncdm`, `N_ncdm`, `N_ur` adjustment see `explanatory.ini`)

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- Free-streaming dark radiation $N_{\rm eff}$ (`N_ur`)

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- Helium abundance Y_p (`YHe`)

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- Helium abundance Y_p (`YHe`)
- Primordial running α_s (`alpha_s`)

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- Helium abundance Y_p (`YHe`)
- Primordial running α_s (`alpha_s`)
- CPL Dark Energy w_0, w_a (`Omega_fld`, `w0_fld`, `wa_fld`, `Omega_Lambda=0?`)

Existing content already in `class`

Standard simple extensions:

- Curvature Ω_k (`Omega_k`)
- Neutrinos m_ν (`m_ncdm`, `N_ncdm`, `N_ur` adjustment see `explanatory.ini`)
- Free-streaming dark radiation $N_{\rm eff}$ (`N_ur`)
- Helium abundance Y_p (`YHe`)
- Primordial running α_s (`alpha_s`)
- CPL Dark Energy w_0, w_a (`Omega_fld`, `w0_fld`, `wa_fld`, `Omega_Lambda=0?`)
- CMB temperature $T_{\rm cmb}$

Existing content already in `class`

Dark matter:

- Thermal Warm Dark Matter (`m_ncdm`, `T_ncdm`, `omega_ncdm`)
- Annihilating dark matter
- Decaying dark matter (`Omega_dcdmdr`, `Gamma_dcdm`)
- Non-trivial phase-space distribution (`ncdm` framework), neutrino flavor mixing, neutrino chemical potential
- Interacting with photons, baryons, neutrinos (`idm`)

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Dark radiation:

- Fluid/Self-interacting (`idr`)
- Viscous (`ceff2_ur`, `cvis_ur`)

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- Fluid/Self-interacting (`idr`)
- Viscous (`ceff2_ur`, `cvis_ur`)

Dark energy:

- CPL, EDE (`fld`) (this `EDE` is not the usual EDE → `https://github.com/PoulinV/AxiCLASS` and `https://github.com/flo1984/TriggerCLASS` to be merged)
- Other fluid-like DE (`fld`)
- Quintessence/Scalar field (`scf`)

Existing content already in `class`

Thermal modeling

- Recfast recombination
- Hyrec-2recombination
- Tanh reionization
- Multi-tanh reionization
- Reionization from file
- Energy injection (PBH Evaporation,PBH Accretion,DM Decay,DM Annihilation)

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Spectral distortions

- y and μ distortions
- PCA of intermediate distortions

Existing content already in `class`

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Inflation/Primordial Powerspectrum

- Arbitrary potential $V(\phi)$
- Automatic finding of observable window
- Can impose inflation duration
- Tensor modes included automatically
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Modified Gravity
$\rightarrow$ HiCLASS branch (Bellini, Sawicki, Zumalacarregui, `http://www.hiclass-code.net`)

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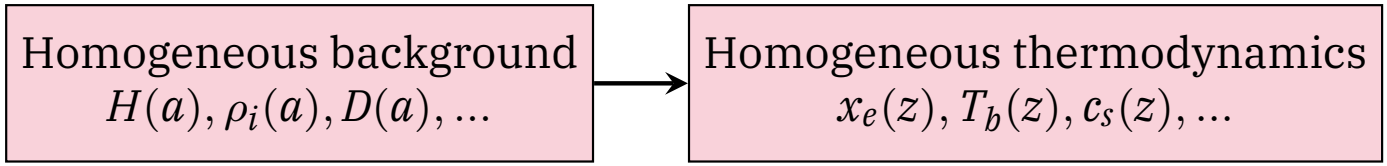
Extension to second-order perturbation theory
SONG (Fidler, Pettinari, Tram, `https://github.com/coccoinomane/song`)

Fundamental layout of Einstein-Boltzmann solvers



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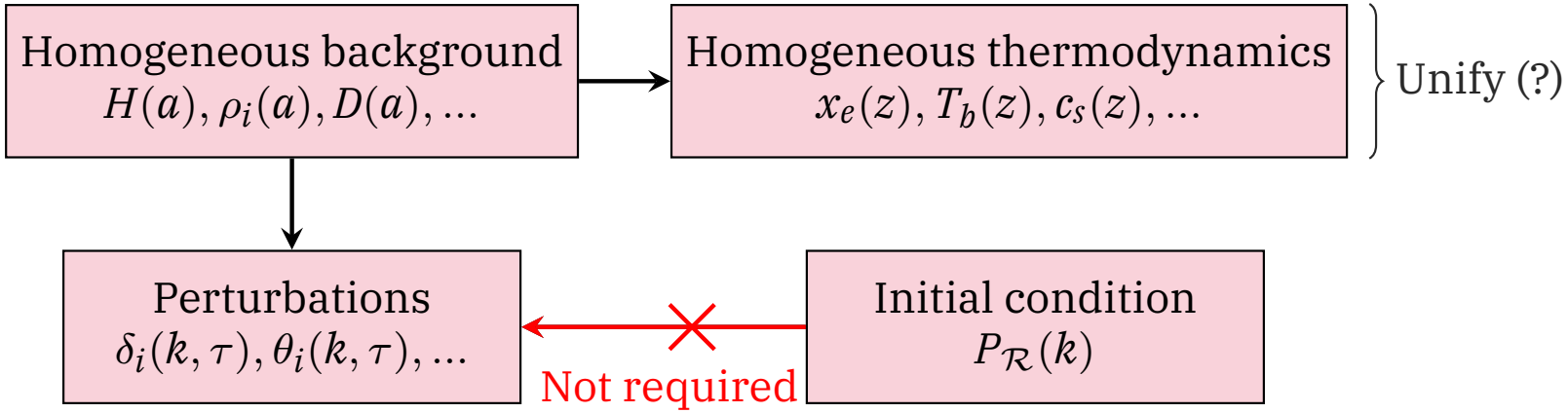

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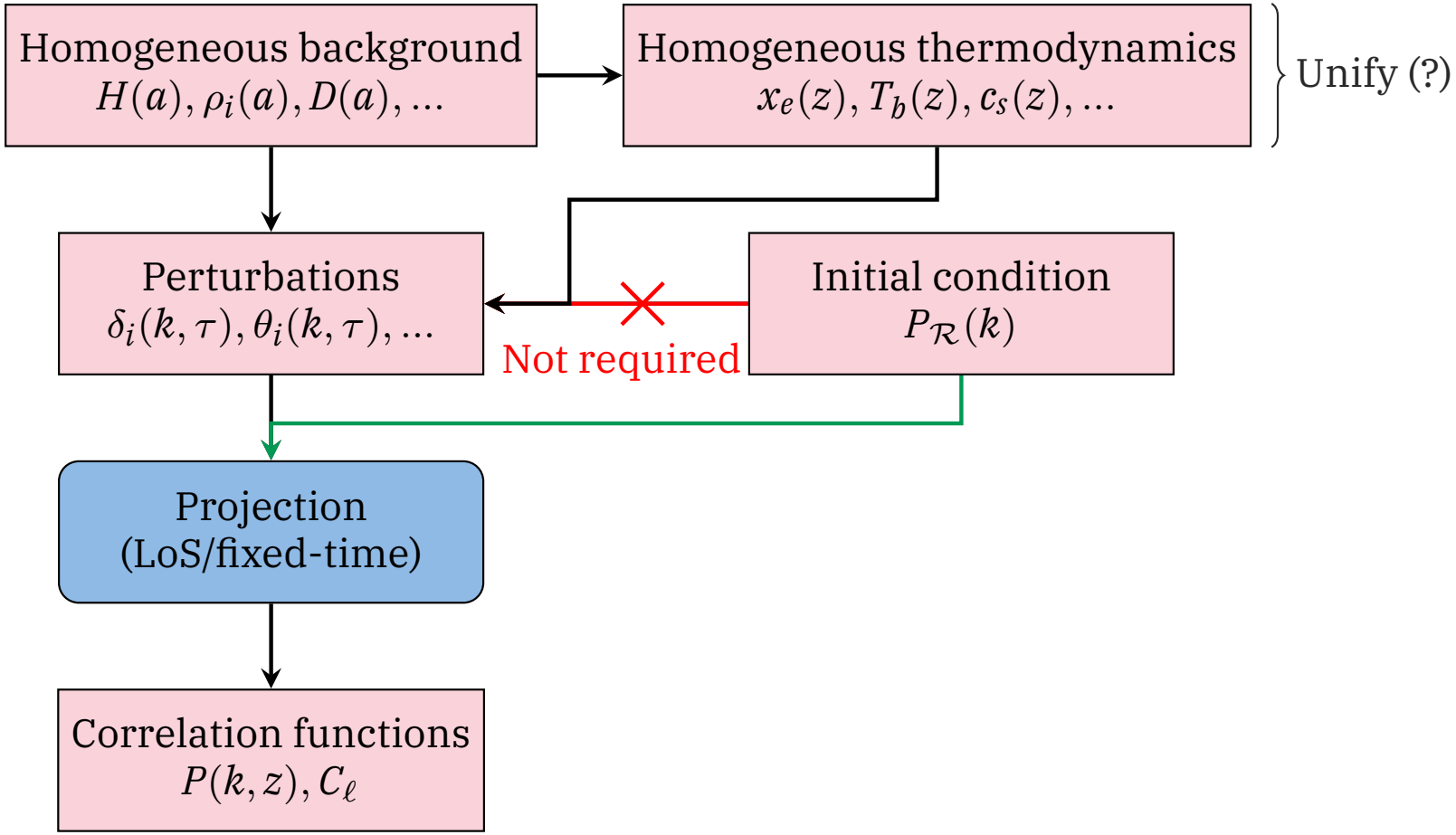

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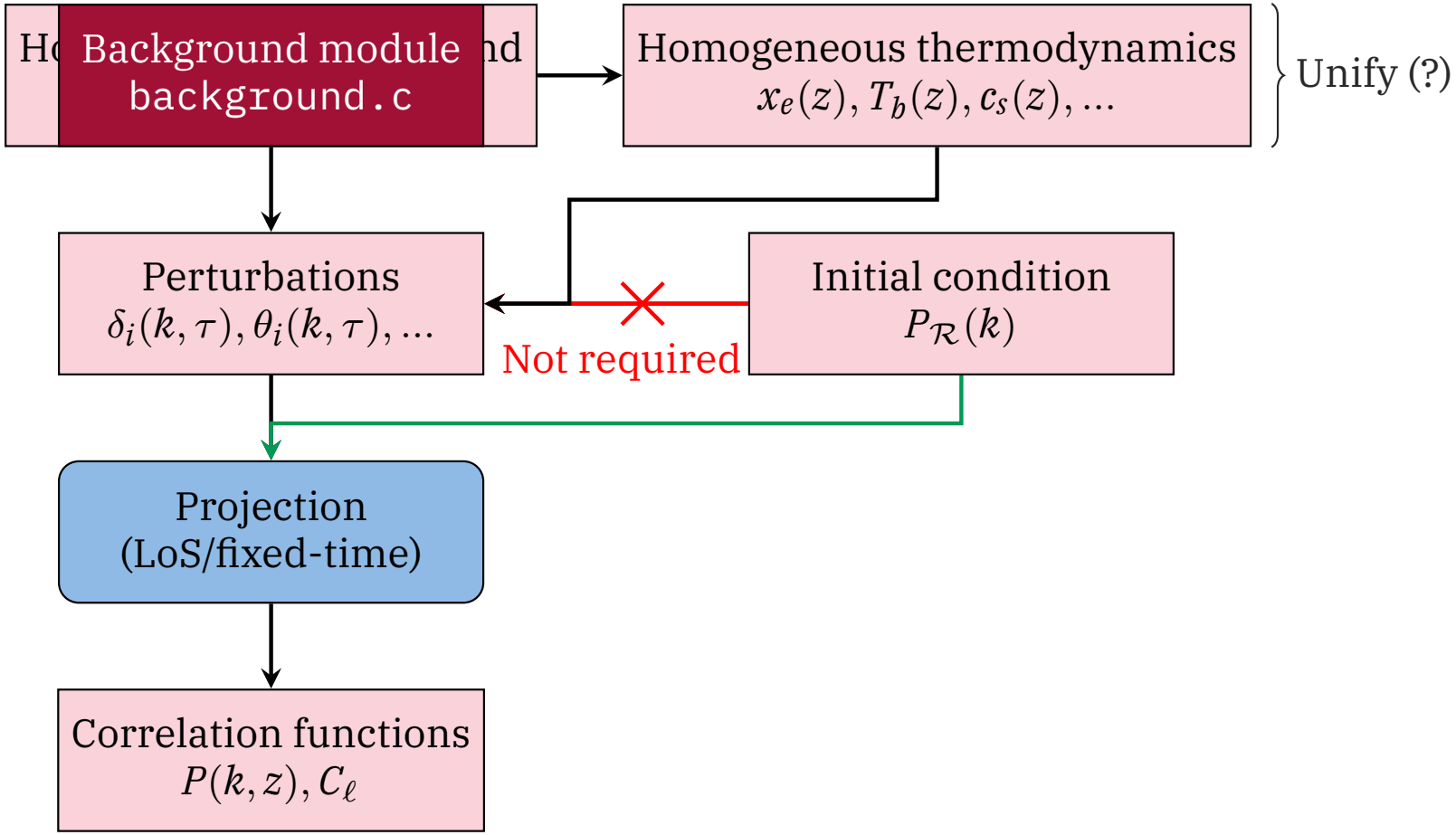

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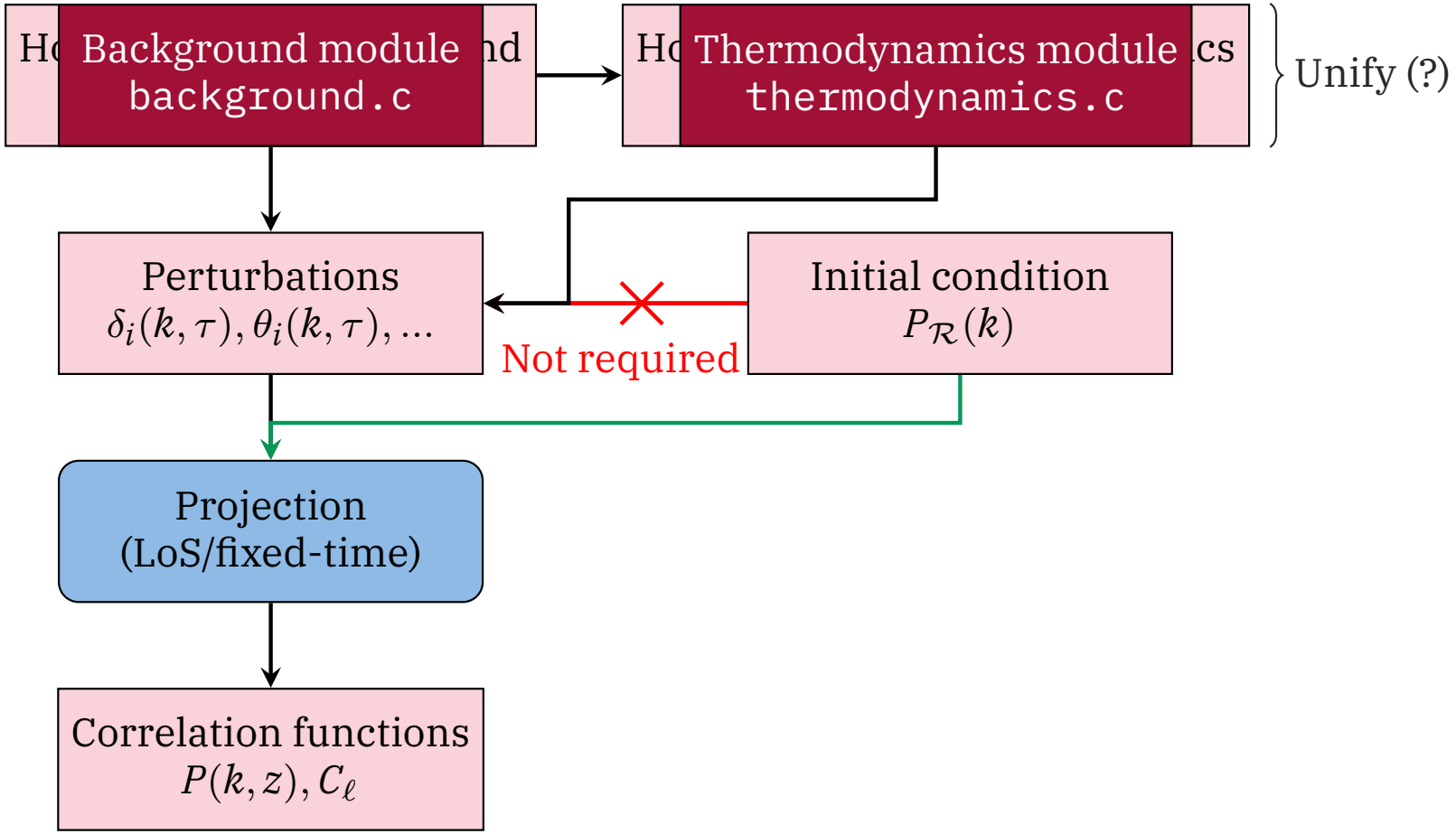

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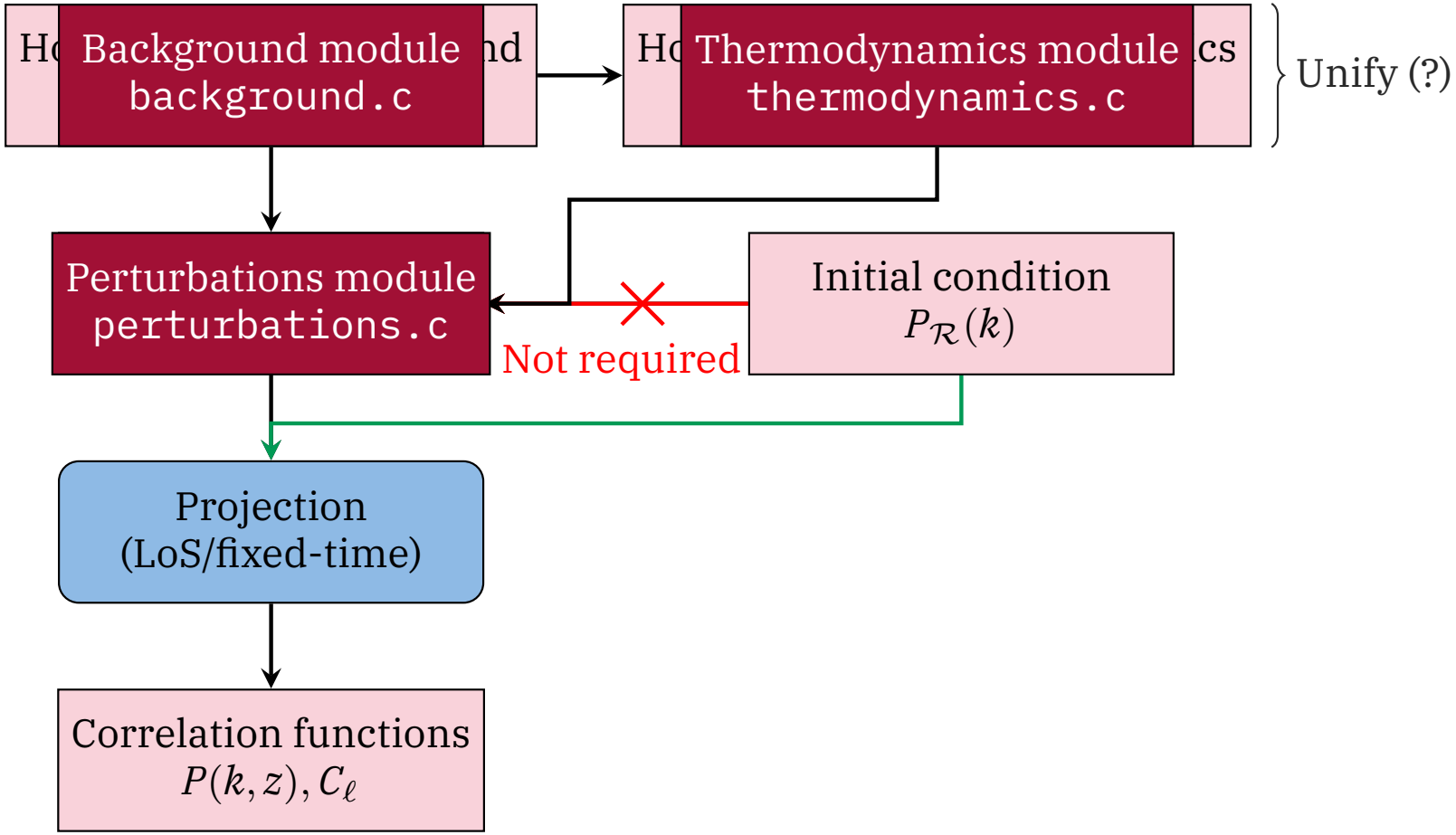

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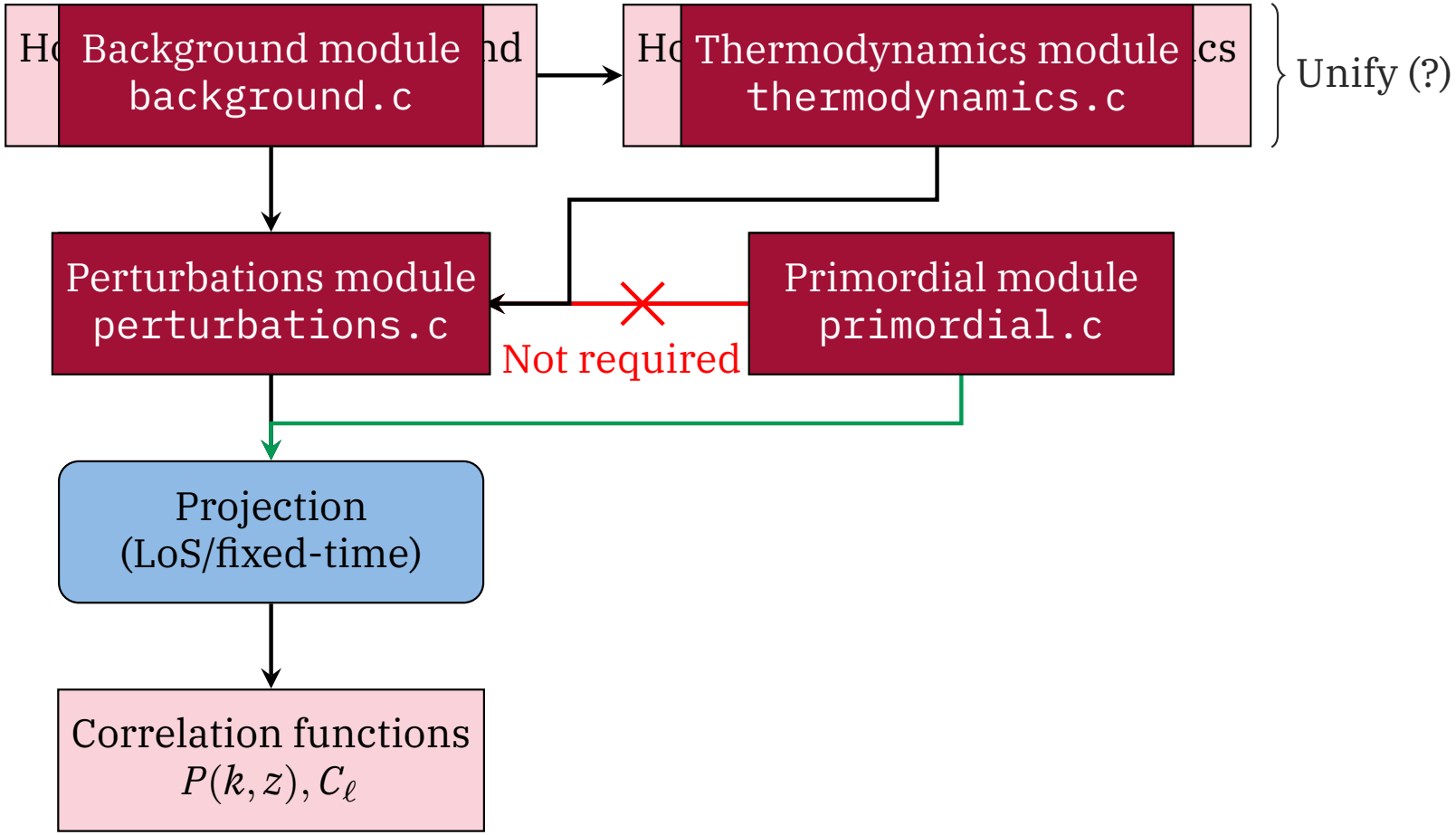

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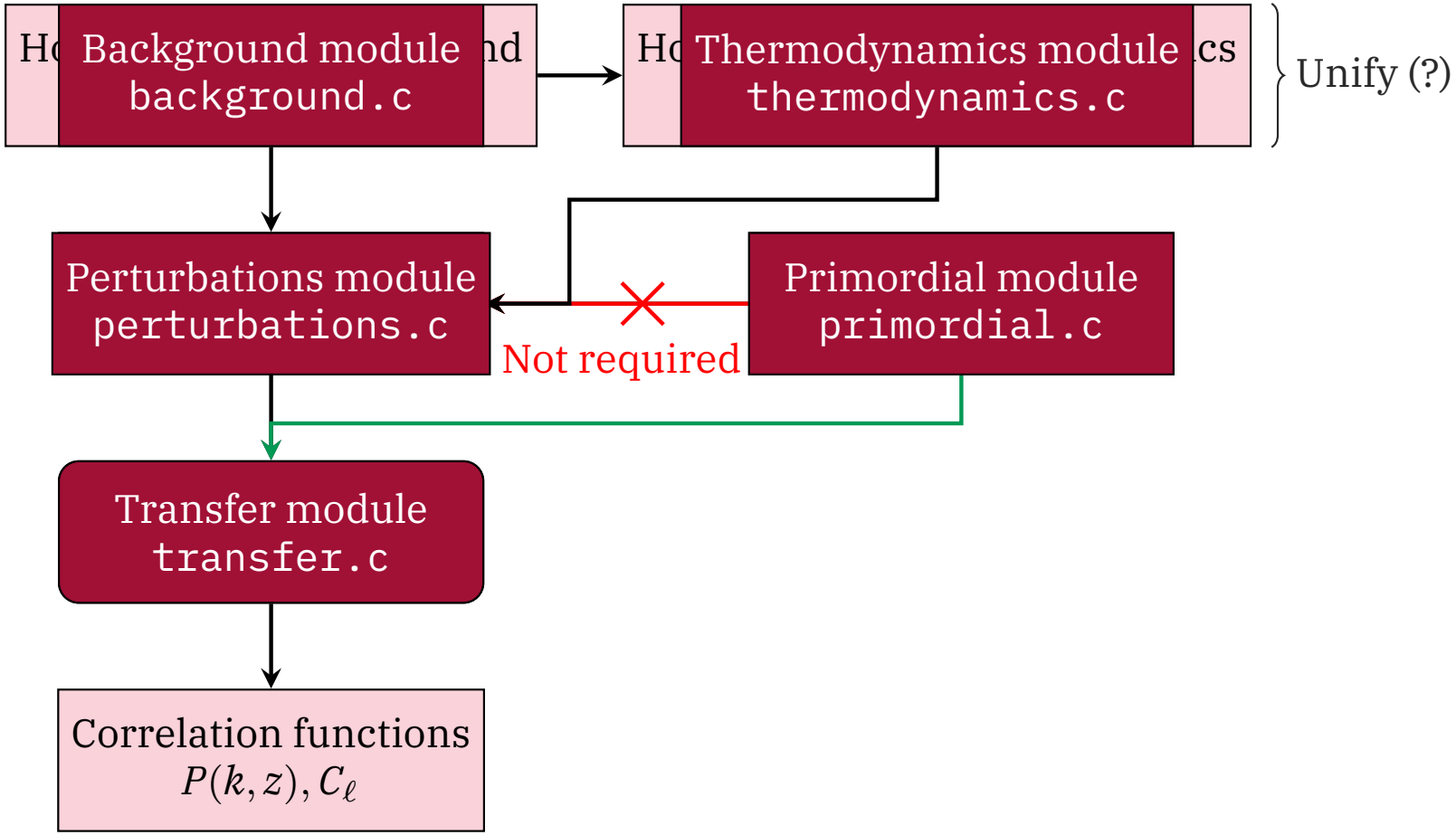

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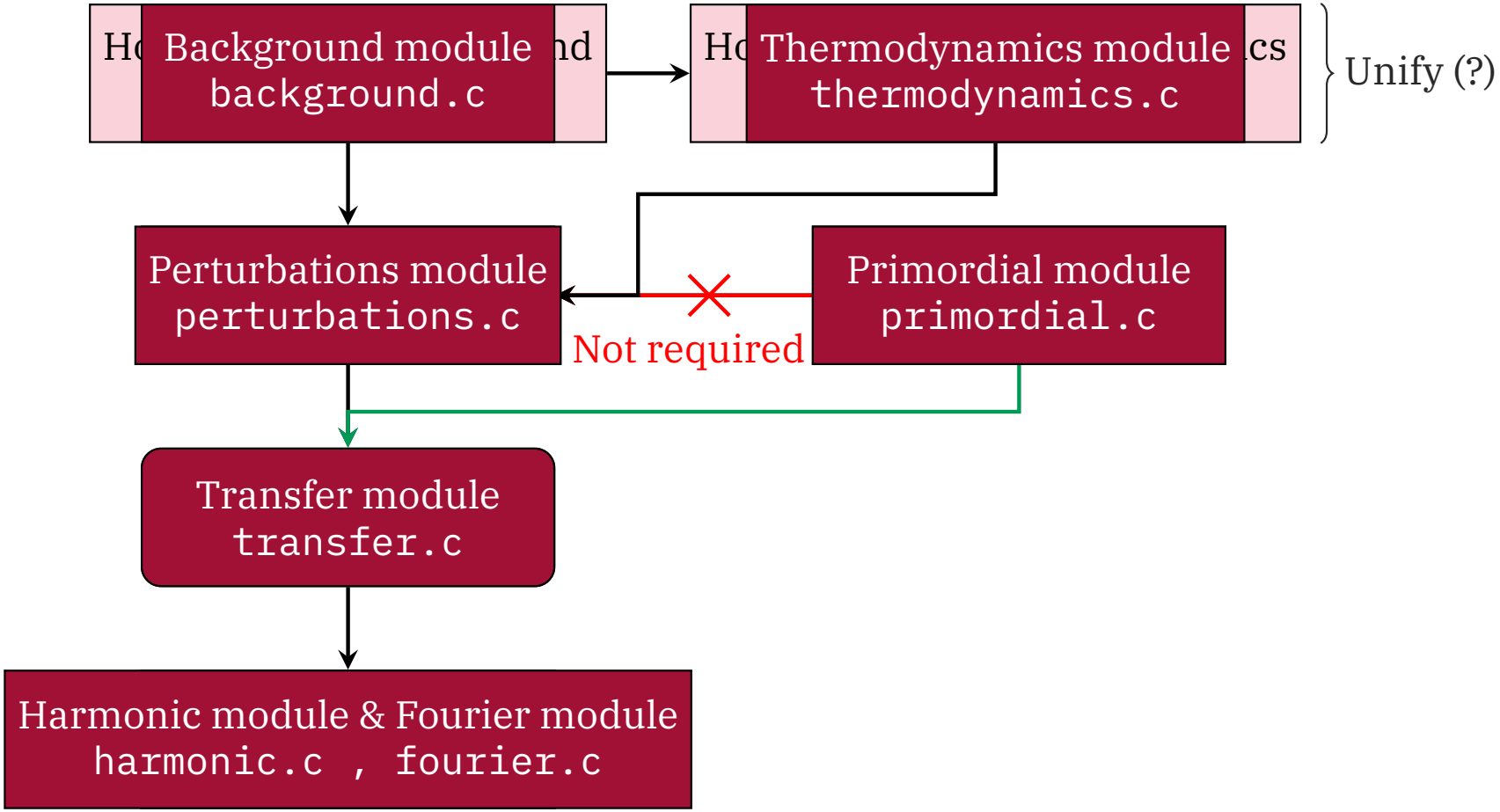

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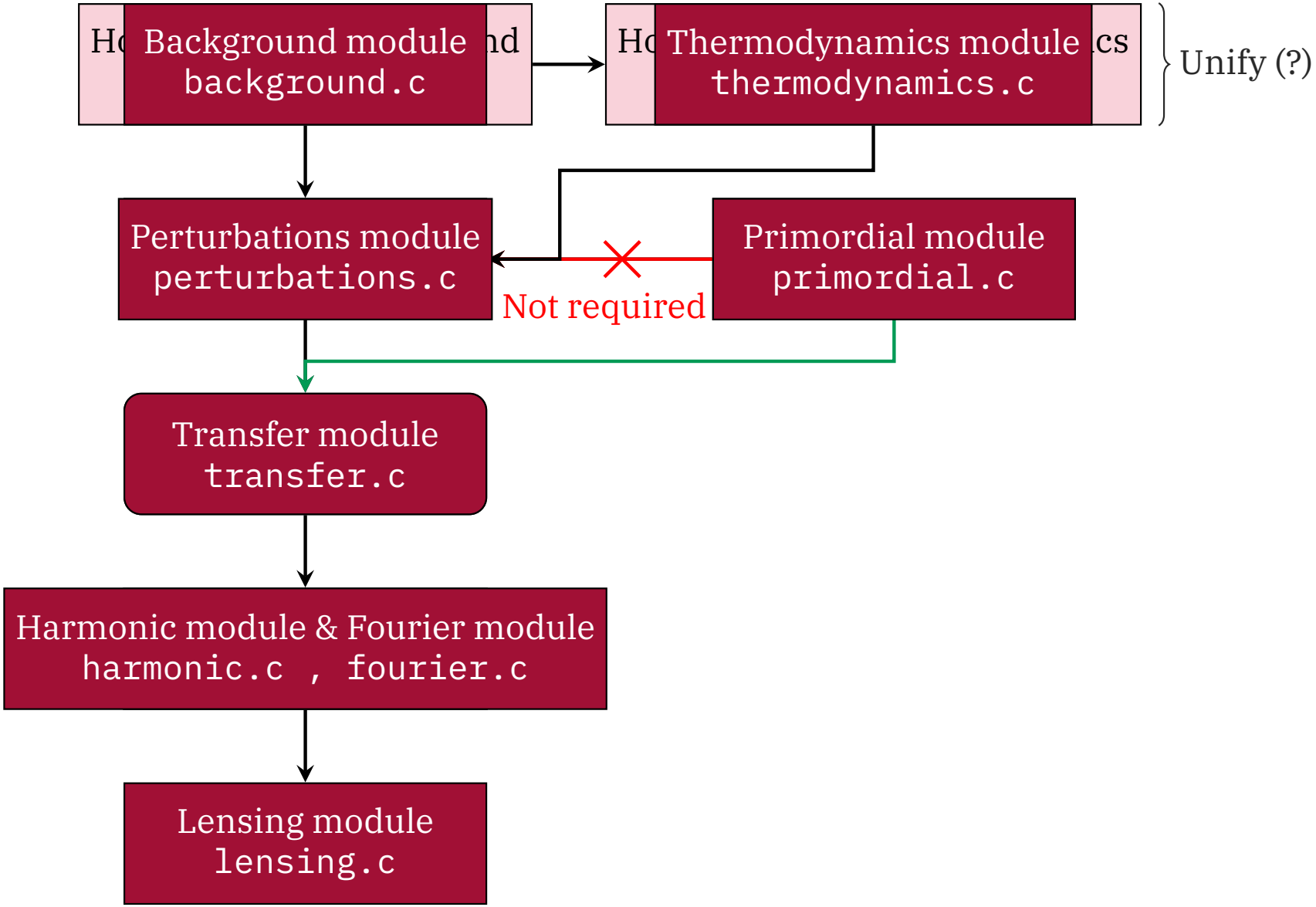

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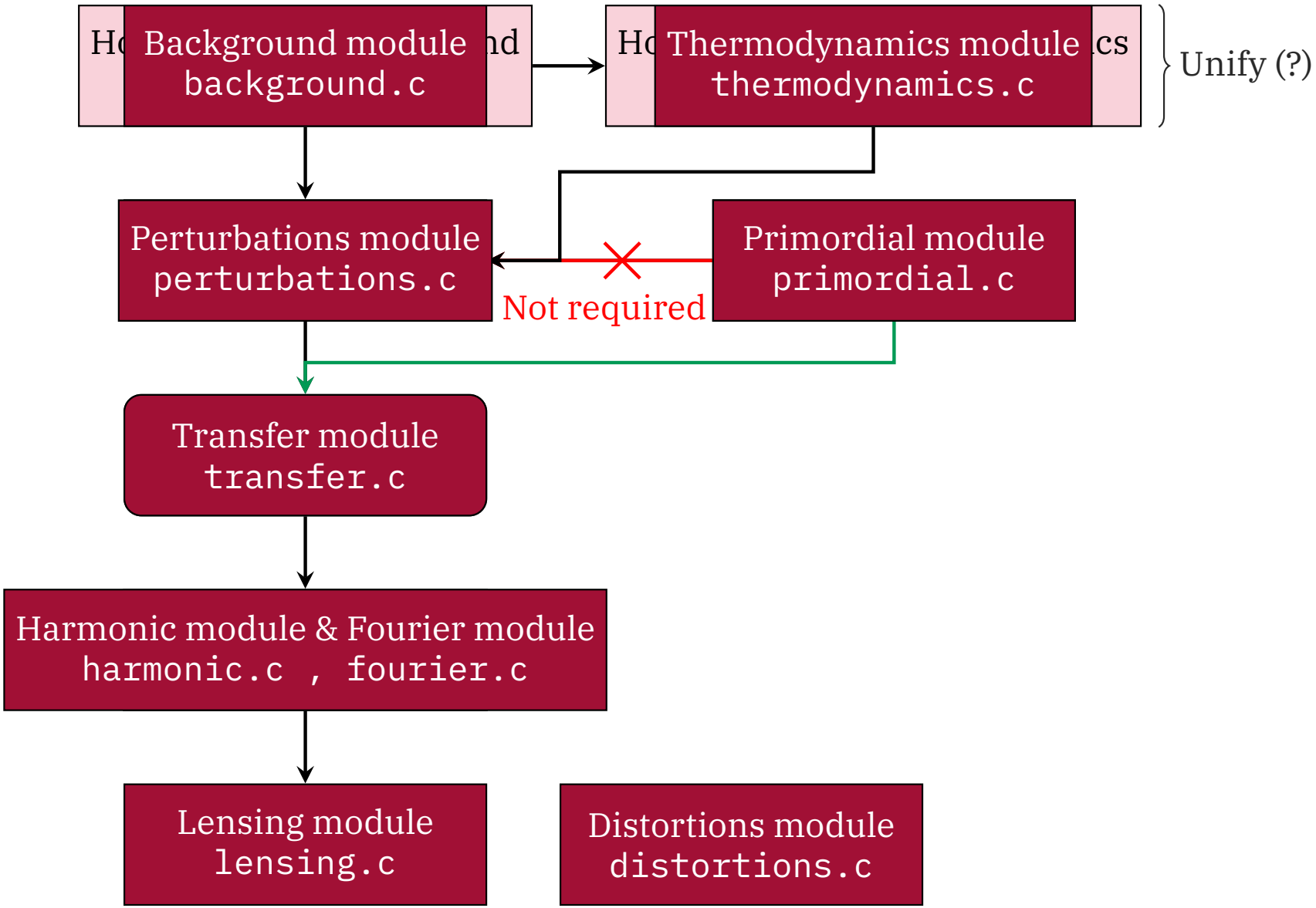

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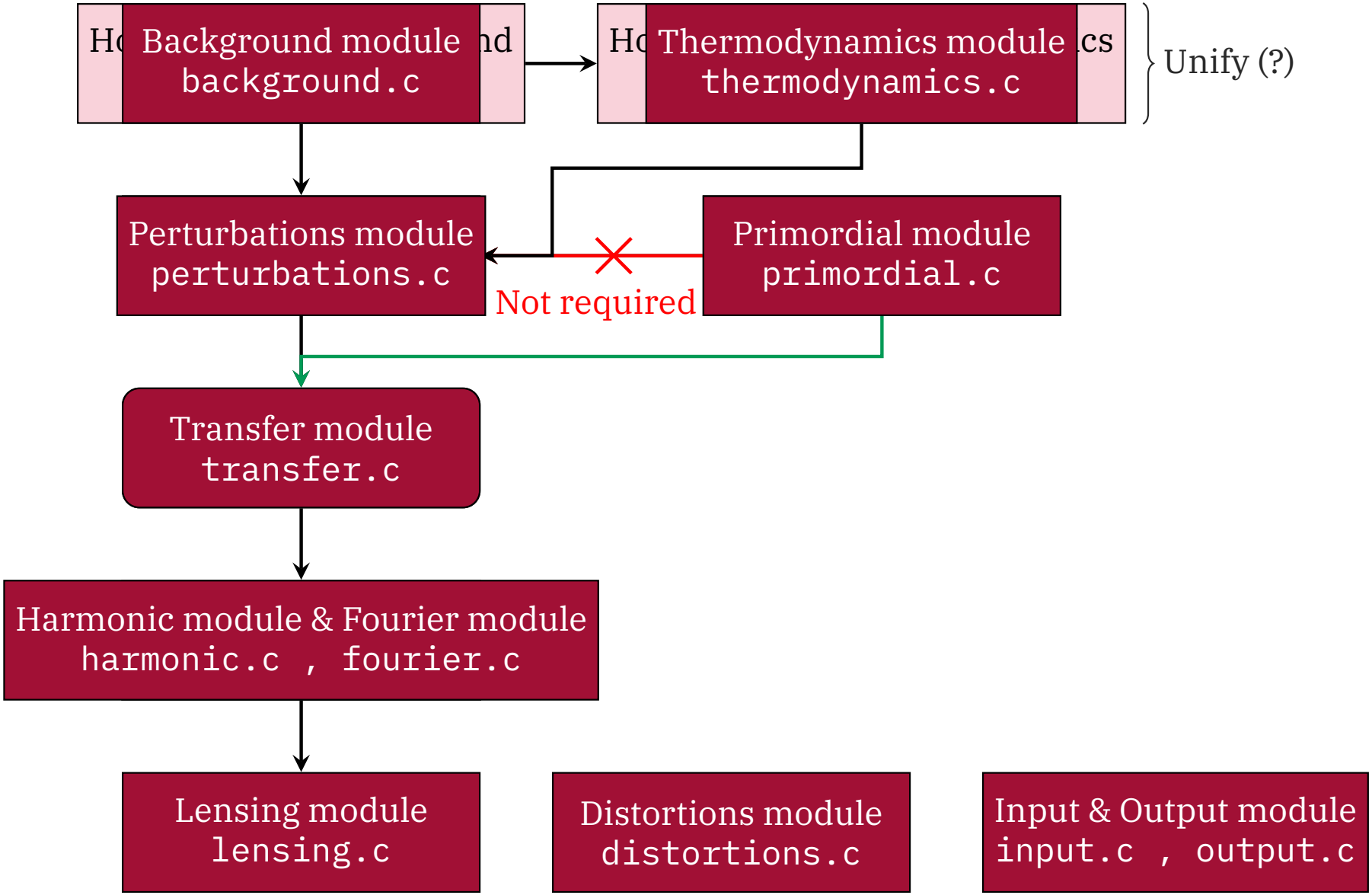

Essentials 1: Input

Read in input files, take care of *shooting*.

```
h = 0.7
#H0 = 70
Omega_m = 0.3
#omega_m = 0.14
sigma8=0.8
```

Special care for equivalent/unknown parameters

Input management in `class`

Terminal	Python wrapper
file `xxx.ini`	
↓	
`input_init(...)`	
(parser)	`.set(...)`
↓	↓
`struct file_content fc;`	(all parameter names/values stored as arrays of strings)
↓	
`input_read_from_file(...)`	
↓	
`input_read_parameters(...)`	
(assign all default values + interprete input + update some parameters)	
↓	
Only *relevant* parameters get stored in the structures of each module	

Input management in `class`

For indirect parameters, use shooting method

Repeated calls of `input_read_parameters(...)`, `class` executions, from `input_read_from_file(...)` until shooting target is met.

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Example:
How would you code the input parameter θ_s?
Use approximate formula $\rightarrow$ inflexible, inaccurate

Input management in class

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Example:
How would you code the input parameter θ_s?
Use approximate formula $\rightarrow$ inflexible, inaccurate

Try out a few values and narrow down (Example: User wants $100\theta_s = 1.04325$)

h	$100\theta_s$
0.7	1.0522492086422521
0.65	1.0270326366580724
0.68215616173	1.0437999980620178
0.68110138476	1.0432819283581667
0.6810337942	1.0432499363679562
0.6810365 0871	1.0432499921072458
0.68103652701	1.0432500079710365
...	...

Input management in class

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0.6810337942	1.0432499363679562
0.681036500871	1.0432499921072458
0.68103652701	1.0432500079710365
...	...

In practice, use more sophisticated Ridder's method / Newton's method

Input management in class

For shooting parameters, establish mapping between *target parameter*, *unknown parameter* and *level*. Currently:

target parameter	unknown parameter	level
$100 \times \theta_s$	h	thermodynamics
$\Omega_{\rm dcdm}$	$\rho^{\rm ini}_{\rm dcdm}$	background
~~σ_8~~	~~A_s~~	~~spectra~~
...	...	...

... plus a few others (alternative parametrizations of decaying CDM, quintessence parameters).

This is what is used e.g. in models of early dark energy!

If you need to add such parameters: see how it is done e.g. for `100*theta_s` and replicate the structure!

Special exception $\tau_{\rm reio} \leftrightarrow z_{\rm reio}$ only concerns reionization and is done independently in `thermodynamics.c`
New Special exception: $\sigma_8 \leftrightarrow A_s$ can be very simply analytically re-scaled (multiplicative property), therefore done independently in `input.c`

Input management in class

Budget equation:

$$\sum_X \Omega_X = 1 + \Omega_k$$

To avoid over-constraining the input, one of the last three (`Omega_Lambda`, `Omega_fld`, `Omega_scf`) must be left unspecified and `class` will assign it using budget equation.

Possibly more advanced in the future

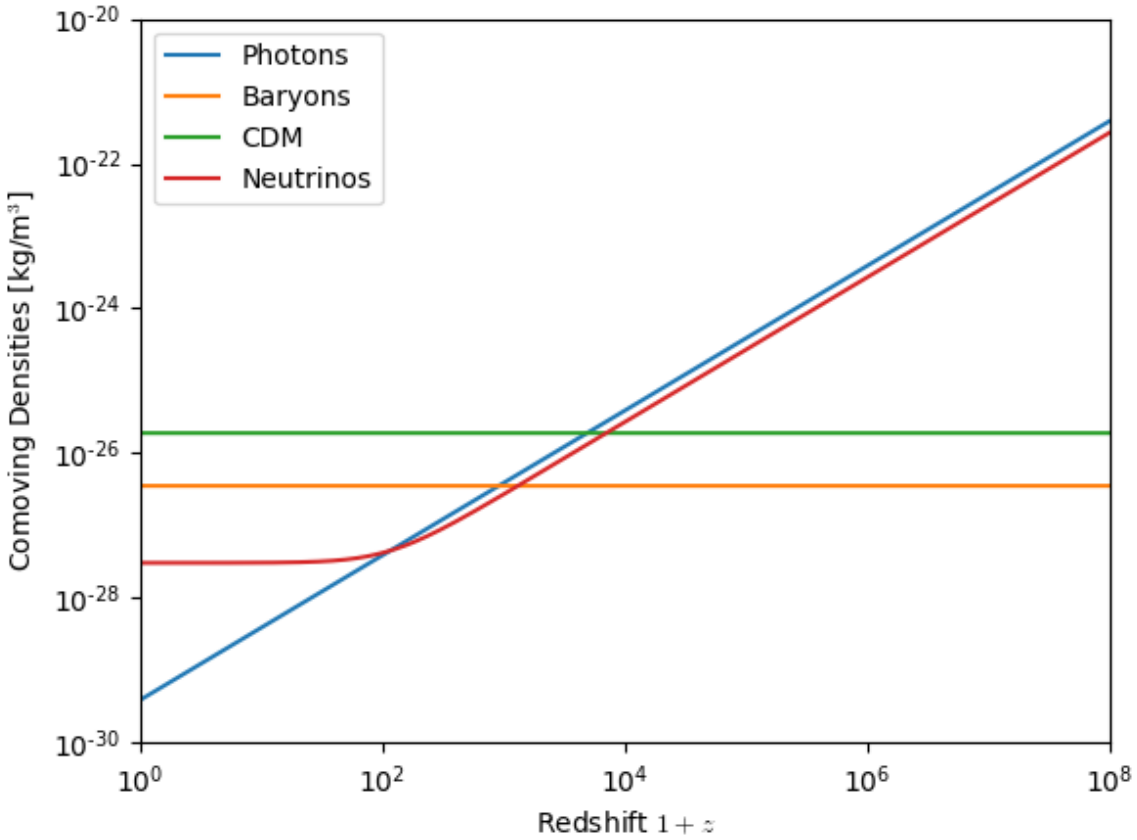
- default: `Omega_Lambda` is automatically adjusted
- if you pass `Omega_Lambda`, `Omega_fld` is automatically adjusted
- if you pass `Omega_Lambda` and `Omega_fld`: `Omega_scf` is automatically adjusted (if you allow, by setting to -1)

This allows whatever combination.
E.g. to get Λ plus a DE fluid:
`Omega_Lambda=0.2`, `Omega_scf=0` or `Omega_fld=0.3`, `Omega_scf=0`

Helpful output by setting background verbose >= 2

Essentials 2: Background

Get all background quantities as function of a scale factor a.



This also gives mapping $a \leftrightarrow z \leftrightarrow t \leftrightarrow$ conf.time

The Background Module

Let's formalize problem!

Three types of parameters:

- $\{A\}$ are analytical functions of scale factor and $\{B\}$ quantities.
- $\{B\}$ need to be integrated over, and are used to compute $\{A\}$
- $\{C\}$ also need to be integrated over, but are not used to compute $\{A\}$.

The Background Module

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ΛCDM and many simple extensions:

- $\{A\} = \{\rho_i(a), p_i(a), H(a), ..., \}$ with e.g. $H(a) = \left(\sum_X \rho_x(a) - \frac{K}{a^2}\right)^{1/2}$
- $\{B\} = \{\}$ (eliminated since `v3.0`)
- $\{C\} = \{t, \tau, r_s, D, f\}$ with e.g. $\frac{dt}{da} = 1/H(a)$, $\frac{dr_s}{da} = c_s(a)/(a \cdot H(a))$

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Example of DE/DM/DR fluid:

- $\{A\} = \{\rho_i(a), p_i(a), H(a), ..., w_{\rm fld}(a)\}$
- $\{B\} = \{\rho_{\rm fld}\}$ with $\frac{d\rho_{\rm fld}}{da} = -3(1 + w_{\rm fld}(a))\rho_{\rm fld}$

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Exemple of extended cosmology with quintessence ϕ:

- $\{A\} = \{\rho_i, p_i, H, ..., V(\phi), \rho_\phi(\phi, \phi')\}$ with e.g. $\rho_\phi(\phi, \phi') = \frac{1}{2}(\phi')^2 + V(\phi)$
- $\{B\} = \{\phi, \phi'\}$ with $\frac{d\phi}{da} = \phi'/[aH(a)]$, $\frac{d\phi'}{da} = -2\phi' - aV(\phi)/H(a)$

The Background Module

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Also Cold Dark Matter decaying into Dark Radiation...

- $\{A\} = \{\rho_i, p_i, H, ...\}$
- $\{B\} = \{\rho_{\rm dcdm}, \rho_{\rm dr}\}$ with $\frac{d\rho_{\rm dcdm}}{da} = -3\rho_{\rm dcdm} - \Gamma(a)/H(a) \cdot \rho_{\rm dcdm}$

The Background Module

Small details:

- Quantities as $D_A(z), D_L(z), r_s, t_{\text{age}}$ can be derived after all A,B,C are computed
- Takes care of NCDM integration of phase-space distribution
- Useful checks & output
- $\rightarrow$ Budget equation output at verbosity level 2

Background exercise

Let's end this first lecture with a hands-on exercise.
Download the jupyter notebook Exercise_background_to_fill.ipynb and follow the steps to plot the properties computed by the background module.