Arithmeticity of Qutrit Clifford+R Gateset

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Mark Deaconu

FoQaCiA workshop 2025, Portugal

Single Qubit Circuits

- **Quantum Protocols** are 2×2 unitary matrices (complex entries)
- **Quantum circuits** implement quantum protocols.
- Circuits are *ideally* comprised of fault tolerant gates

Exact Synthesis

- **Exact synthesis problem**: given a target protocol U_{target}, if possible, compile into a circuit over gate-set G
- For the single qubit gateset Clifford+T there is a **powerful** synthesis algorithm

Recognizable

Computationally Efficient

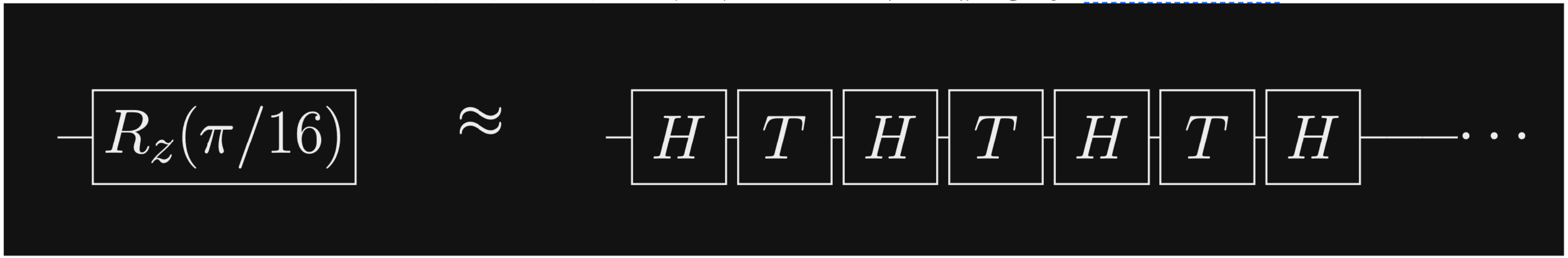
Yields Optimal Cost Circuits

Kliuchnikov, V., Maslov, D., & Mosca, M. (2013). *Fast and efficient exact synthesis of single qubit unitaries generated by Clifford and T gates*. arXiv:1206.5236

Approximate Synthesis

- **Approximate synthesis problem**: given a target unitary U_{target}, compile a circuit over gate-set G such that $||U_{\text{target}} - U_G|| \leq \varepsilon$.
- When this problem is always solvable, the gateset is called **Universal**
- The Clifford+T gate set is universal over 2×2 unitaries… but it's not the only one

Nebe, G., Rains, E. M., & Sloane, N. J. A. (2000). *The invariants of the Clifford groups*. arXiv:math/0001038



Peter Sarnak's Letter

 Feb 2015

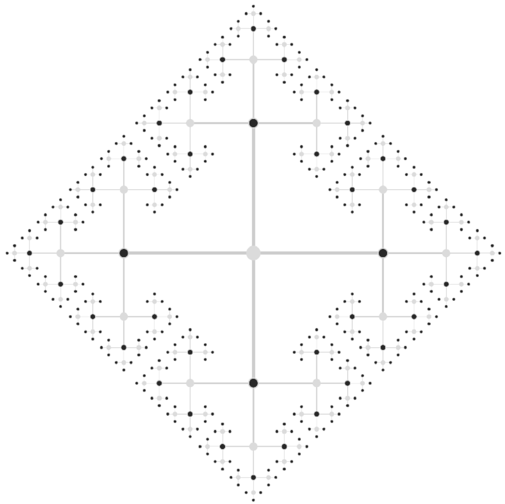
Letter to Scott Aaronson and Andy Pollington on the Solovay-Kitaev Theorem and Golden Gates.

Dear Scott and Andy,

Thanks for pointing me to the papers [K-M-M 1,2], [R-S]. I was not aware of these interesting developments in connection with the Solovay-Kitaev Theorem and the design of universal 1-qubit quantum gates. These papers refer to some others which are also very interesting such as [B-G-S]. All of these results can be understood in a unified way in terms of the arithmetic of quaternion algebras. Doing so clarifies (at least for me) the constructions and it also allows one to prove some fundamental properties for these gates as well as to relate them to some older and more recent developments. I explicate these points below, for myself as much as for the reader.

Tree Structure Relating Quantum Circuits

- Golden gate sets generate **S-arithmetic groups**
- S-arithmetic groups act transitively on a Bruhat-Tits building



(source: https://ariymarkowitz.github.io/Bruhat-Tits-Tree-Visualiser/)

Parzanchevski, O., & Sarnak, P. (2018). *Super-Golden-Gates for PU(2)*. Advances in Mathematics, 327, 869–901

Abramenko, P., & Nebe, G. (2002). *Lattice chain models for affine buildings of classical type*. Mathematische Annalen, 322(3), 537–562

Clifford + R (work in progress)

- Acts on a 4-regular tree
- Has a powerful exact synthesis algorithm related to a p-adic metric
- Collaboration with: Amolak, Nihar, Michele, and Jon

Thank you!

Questions welcome.