Chapter 3.1

Simple Harmonic Motion

- mass on a swinging pendulum
- Forces: Gravity and tension
- parallel (x-axis) Forces and perpendicular (y-axis)
- perpendicular: $F_\theta = -mg\sin\theta$
  g - gravity
- Newton's Second Law

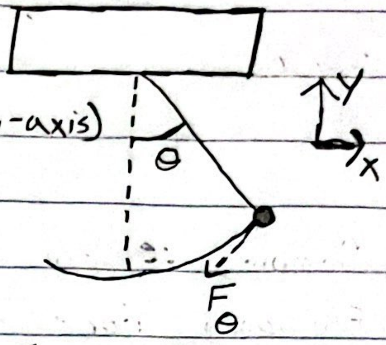

Figure 3.1

$$\Sigma F = ma = F_\theta = m\frac{d^2 s}{dt^2}$$

Displacement: $s = \ell\theta$, ℓ = length

$\sin\theta \approx \theta$

$$\frac{d^2\theta}{dt^2} = -\frac{g}{\ell}\theta$$

- the Oscillations are sinusodal with time without decaying
- Ω = angular frequency / Function of $(\ell, g)_{m, A}$

$$\frac{d\omega}{dt} = -\frac{g}{\ell}\theta \quad , \quad \frac{d\theta}{dt} = \omega \qquad \omega = \text{angular velocity}$$

$$\omega[i+1] = \omega[i] - \frac{g}{\ell}\theta[i]\Delta t \qquad \theta[i+1] = \theta[i] + \omega[i]\,dt$$

- Energy Remains Constant
- Euler Method Cannot Perform correctly

Total Energy: $E = \frac{1}{2}m\ell^2\omega^2 + mg\ell(1-\cos\theta)$

kinetic Energy

gravitational potential Energy

- Euler Method Cannot Conserve Energy
- little Energy transer in projectile motion
- Euler-Cromer does Conserve Energy
- new value of ω is used for new value of θ

Chapter 3.2

Pendulum But More Interesting

Damping ⇒ Friction, air resistance

Frictional Force - $-q(d\theta/dt)$

opposes ~~mo~~ motion

So, damped is

$$\frac{d^2\theta}{dt^2} = -\frac{g}{\ell}\theta - q\frac{d\theta}{dt}$$

Underdamped ⇒ small friction   Overdamped ⇒ large

- Driving Force is applied by sinusodal with time

External driving Force ⇒ $F_D \sin(\Omega_D t)$

- Reasonance - When driving Force matches motion frequency, Amplitude gets larger

No Mechanical energy is conserved + pendulum execute a periodic motion

- depends on Amplitude

Scanned with CamScanner

3.3 Chaos in the Driven Nonlinear Pendulum

- Don't assume θ is small
- Include friction, $-q(d\theta/dt)$
- Sinusodal driving Force, $F_D \sin(\Omega_D t)$

This is a non linear, damped, driven pendulum

$$\frac{d^2\theta}{dt^2} = -\frac{g}{\ell}\sin\theta - q\left(\frac{d\theta}{dt}\right) + F_D \sin(\Omega_D t)$$

"Physical Pendulum"

$\omega[i+1] = \omega[i] - [(g/\ell)\sin\theta[i] - q\omega[i] + F_D \sin(\Omega_D t[i])\,\Delta t$

$\theta[i+1] = \theta[i] + \omega[i+1]\,\Delta t$

$t[i+1] = t[i] + \Delta t$

If $\theta[i+1] \neq$ range $(-\pi, \pi)$ keeps it in a single range.
 then $\pm\ 2\pi$

Pendulum can swing full circles
No driving Force, pendulum quickly comes to rest
After initial motion has angular frequency $\sim \Omega_D$,
 the driving Force keeps in motion
As Driving Force increases, graph is chaotic

Scanned with CamScanner