Using Dither in Digital Control Loops

"Dither", as a signal or image processing technique, is a method of extending dynamic range by first perturbing (dithering) then averaging. The technique was first developed to enhance the performance of RADAR target algorithms and is now applied to a wide range of applications including navigation systems and consumer audio CD recordings.

Perhaps the most common example of a dithering technique is the synthesis of an artificial color on a computer screen by grouping available colors at adjacent pixels. When viewed by the user, the spatial averaging effect of the eye generates a color that is not available on the computer's color palette.

In Cryo-con's temperature controllers, dither is used to extend the dynamic range of a temperature control loop by outputting available power levels in a controlled sequence so that the average power is somewhere between the levels available in the controller's hardware. Here, the averaging function is performed by the system dynamics.

Control Accuracy

Major error sources in a digital control loop are: the input quantizer (ADC), the Digital Signal Processing mathematical operations and the output quantizer (DAC).

Cryo-con controllers use a 24-bit Analog-to-Digital converter. This is the best available with modern components and it establishes the measurement resolution of the controller. If all other functions were perfect, this ADC would also establish the accuracy of the control loop.

In order to preserve accuracy, the mathematical operations in a digital control loop must be performed to a much higher resolution than the input ADC. Therefore, Cryo-con controllers all use 32-bit floating-point arithmetic.

Finally, a high precision loop output value reaches the output quantizer, which is usually a 16- or 18- bit Digital-to-Analog converter. Since this DAC has much less resolution than the earlier stages, it generally establishes the accuracy of the accuracy of the entire loop. A loop output value has been generated to a very high precision, but the DAC throws away most of this precision to fit its available output levels.

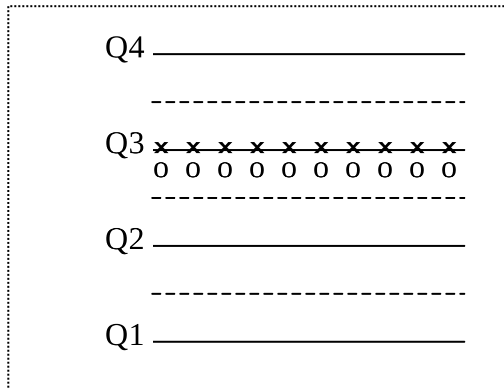
Like the color synthesis example above, signal dithering can be applied to the digital control loop so that the average output value converges to the high precision value computed before output quantization. The result is much greater control accuracy.

Conventional Control Loop Output

The diagram to the right shows the conventional method of generating an analog output from a digital control loop. Here, a high precision loop output value is computed, then the value is truncated or rounded to fit the precision of the output DAC. Precision above the resolution of the DAC is lost.

In this example, the output DAC has four quantization levels labeled Q1 through Q4. Dashed lines show the mid-points between adjacent levels.

Here, the desired high-precision control loop output (o) is between levels Q2 and Q3. For simplicity, ten output intervals of a DC level are shown.

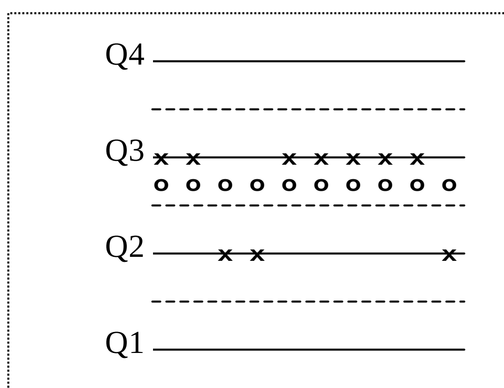


Using an arithmetic 'rounding' scheme, if the desired output is above the mid-point between two quantization levels, the DAC output will be at the higher level. If the value is below the mid-point, the DAC will output the lower level. Therefore, the DAC output (x) for the input shown will simply be Q3.

As can be seen, the average value of the DAC output is equal to the nearest quantization level. In this example, the output (Q3) is slightly higher than the value required to accurately control at the selected set point. Therefore, the control loop will integrate downwards until the DAC output jumps down to Q2. This process of jumping between Q2 and Q3 will continue, establishing an oscillation with an amplitude of one quantization level and a frequency related to the system's closed-loop time constant.

The Dither Algorithm

The signal dithering algorithm used in Cryo-con's digital control loop first generates a dither signal that is a random number within the range of ±0.5 of a quantization level. This is then added to the loop output value just before placing it in the DAC.



If the sum of the desired output plus the dither value is above the midpoint between Q2 and Q3, the DAC will output Q3. If it is below the midpoint, the DAC will output Q2. Therefore, the DAC output to toggles randomly between Q2 and Q3, but the number of times at one level vs. the other is weighted by how close the desired output is to the nearest quantization level.

In this example, the desired output is 25% of the distance from Q3 to Q2. Therefore, 75% of the DAC output samples will be Q3 and the remaining at Q2.

Most importantly, the average value of the DAC output converges to the desired output loop value.

Using this dither technique, the control loop output accuracy will improve as the number of averages increases; up to the limits imposed by the other elements of the control loop.

Fortunately, the number of samples averaged in a given system is proportional to its closed-loop bandwidth, which can be controlled by adjusting PID parameters.

How much improvement does dither provide?

Dither causes the average value of the control loop output to converge to the actual desired output. How close depends on the number of averages accumulated within the closed-loop system.

The accuracy of an estimate of average value for a fixed number of samples is given by the Chi-squared distribution. The 'degrees of freedom' used by this function is the number of samples accumulated. However, this distribution is fairly close to the square root of N.

Using the Cryo-con Model 34, the loop output rate is 10 samples per second. Therefore, if the process being controlled has a time constant on the order of 1.6 seconds, a total of 16 samples will be averaged resulting in a factor of four improvement in control accuracy. This is equivalent to adding two bits to the output DAC.

Since the Model 34 uses a 16-bit output DAC, a 1.6 second closed-loop time constant will result in the equivalent of an 18-bit DAC. Note that 1.6 seconds is an extremely short time constant for a cryogenic temperature process.

Further Reading

1. "Introduction to Signal Processing", Sophocles J. Orfanidis, August 1995. Prentice Hall, ISBN: 0-13-209172-0 http://www.ece.rutgers.edu/~orfanidi/intro2sp/

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