Fundamentals of Computing Autumn 2019 Coursework

For return by 27 January 2020 (late submission 10 February 2020)

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1. **(6%)** (a) A set of Boolean formulas is *consistent* if there is an assignment of the truth-values 0, 1 to the propositional variables under which all of the formulas in the set are true. For each of the following sets, state whether it is consistent or inconsistent:

(i) $\{A \vee B,\ A \to C,\ C \wedge \neg B\}$

(ii) $\{\neg A \wedge \neg B,\ \neg C \to A,\ \neg C \vee B\}$

(b) Is the following argument logically correct: if propositions $\neg(A \wedge B)$, $C \to A$ and $C \wedge B$ are true, then C is also true.

2. **(6%)** (a) Construct the truth-table for the Boolean function given by the formula

$$\neg\big((A \to \neg B) \wedge (C \to A)\big).$$

(b) Find a Boolean circuit with AND, OR and NOT gates only that computes the Boolean function above and contains as few gates as possible.

(c) Determine whether the formula in (a) is equivalent to the formula

$$(B \to \neg A) \to (C \wedge \neg A).$$

Show your working.

3. **(3%)** Design a Boolean circuit (with as few gates as possible) for the Boolean function that checks whether a 3-bit unsigned binary number is less than 3 (hint: construct the truth-table for the input bits A_2, A_1, A_0).

4. **(5%)** Formalise the following argument in Boolean logic, and decide whether it is correct or not. Explain your answer.

> If Jones did not meet Smith last night then either Smith was the murderer or Jones is lying. If Smith was not the murderer then Jones did not meet Smith last night and the murder took place after midnight. If the murder took place after midnight then either Smith was the murderer or Jones is lying. Therefore, Smith was the murderer.

(Suggestion: use the propositional variable A for 'Jones did not meet Smith last night', B for 'Smith was the murderer', C for 'Jones is lying', and D for 'The murder took place after midnight'.)

5. **(6%)** Given the machine 32-bit word

1100 0001 1011 0000 0000 0000 0000 0000

find the decimal number represented by this word assuming that it is

(a) a two's complement integer;

(b) an unsigned integer;

(c) a single precision IEEE 754 floating-point number.

6. **(6%)** Find computer representations of the following numbers:

(a) -107 as a two's complement 32-bit binary number;

(b) -107 as an IEEE 754 32-bit floating-point number;

(c) -14.375 as an IEEE 754 32-bit floating-point number.

7. **(6%)** Give an example of a $\mathbb{N} \to \mathbb{N}$ function that is

(a) one-to-one but not onto,

(b) onto but not one-to-one,

(c) both one-to-one and onto (but different from the identical function),

(d) neither one-to-one nor onto.

8. **(6%)** For each of the following functions, determine whether or not it is one-to-one and whether or not it is onto:

(a) $f\colon \mathbb{R} \to \mathbb{R}$ given by $f(x) = (x+1)/(x-2)$;

(b) $f\colon \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ given by $f(m,n) = m + n - 1$;

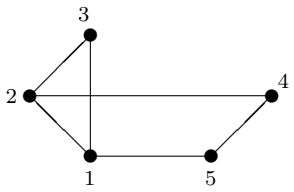
(c) $f\colon \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ given by $f(m,n) = |m| - |n| - 1$.

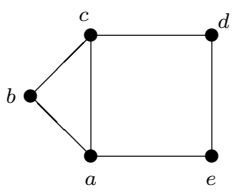
9. **(9%)** (a) Draw an undirected graph represented by the following adjacency matrix:

$$\begin{bmatrix} 1 & 2 & 0 & 1 \\ 2 & 0 & 3 & 0 \\ 0 & 3 & 1 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

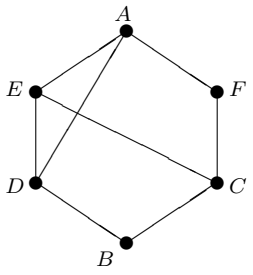
Is the graph simple? Explain your answer.

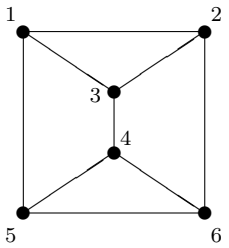
(b) Determine whether the following graphs are isomorphic or not and explain your answer:



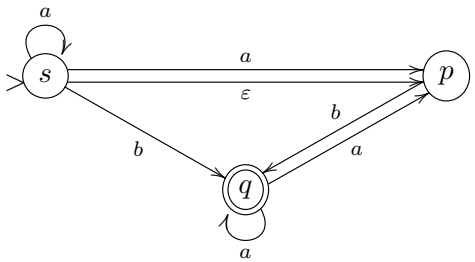


(c) Determine whether the following graphs are isomorphic or not and explain your answer:





10. **(12%)** Consider the following NFA:



(a) Give all the computations of the automaton on the input strings bb, ab, aba, and ε, and determine if the strings are accepted.

(b) Transform the automaton, using the subset construction, into an equivalent deterministic finite automaton and remove the unreachable states. Show your working.

(c) Describe the language of the automaton in English.

(d) Describe the language of the automaton by means of a regular expression.

(e) Describe the language of the automaton by means of a context-free grammar.

11. **(8%)** Design a (deterministic or nondeterministic) finite automaton A such that $L(A)$ consists of all words over the alphabet $\{0, 1\}$ that contain at least two 0's and do not end with 11. Find a regular expression representing the language $L(A)$.

12. **(6%)** Convert the regular language $L\big[\, ab\big((c \cup de)ab^*\big)^* c\,\big]$ to a finite automaton accepting it. (If you do not follow the algorithm discussed in the lectures, explain the intuition behind your construction.)

13. **(8%)**(a) Give a regular expression defining the same language as the following context-free grammar:

$$S \to Sa, \quad K \to aa, \quad S \to Kb, \quad K \to Ka.$$

(b) What is the language over the alphabet $\{a, b\}$ defined by the following context-free grammar with start variable S:

$$S \to aSa, \quad S \to bTb, \qquad T \to bT, \quad T \to b$$

Is this language regular? Give an informal explanation of your answer.

14. **(13%)** Find a context-free grammar generating the language $\{a^m b^n c^k \mid m + k = n\}$ and construct a pushdown automaton for this language.