Washington State University
School of Electrical Engineering and Computer Science
Spring 2022

CptS 223 Advanced Data Structures in C++
Homework 3 – Solution
Due: February 2, 2022 (11:59pm pacific time)

1. The following algorithm Histogram1(A,B,N) computes a histogram from integer array A and stores it in array B, both of length N. When completed, B[k] contains the number of k's that appear in A.

```
Histogram1(A,B,N)
1    for k = 1 to N
2         B[k] = 0
3         for i = 1 to N
4              if A[i] == k
5              then B[k] = B[k] + 1
```

   a. Assuming each line of the code has a cost of 1, show the number of times each line of code is executed, compute an exact expression T(N) for the running time of the algorithm, and give an asymptotically tight bound Θ for T(N).

   Solution:

```
Histogram1(A,B,N)                       Times
1    for k = 1 to N                      N+1
2         B[k] = 0                       N
3         for i = 1 to N                 N(N+1)
4              if A[i] == k              N*N
5              then B[k] = B[k] + 1      t1
```

   $T(N) = 2N^2 + 3N + 1 + t1 = \Theta(N^2)$
   The value of t1 depends on the contents of A, but never exceeds N.

   b. Describe the conditions on the inputs for the best-case and worst-case running time for the Histogram1 algorithm.

   Solution: The best-case scenario is when line 5 is never executed (t1 = 0), which occurs when none of the elements of A are between 1 and N. The worst-case scenario is when line 5 is executed a total of N times (t1 = N), which occurs when every element of A is between 1 and N.

2. The following algorithm Histogram2(A,B,N) computes the same result as in Problem 1, but using a different method.

```
Histogram2(A,B,N)
1     for k = 1 to N
2          B[k] = 0
3     for i = 1 to N
4          k = A[i]
5          if (k >= 1) and (k <= N)
6          then B[k] = B[k] + 1
```

   a. Assuming each line of the code has a cost of 1, show the number of times each line of code is executed, compute an exact expression T(N) for the running time of the algorithm, and give an asymptotically tight bound Θ for T(N).

   Solution:

```
Histogram2(A,B,N)                                  Times
1     for k = 1 to N                                N+1
2          B[k] = 0                                 N
3     for i = 1 to N                                N+1
4          k = A[i]                                 N
5          if (k >= 1) and (k <= N)                 N
6          then B[k] = B[k] + 1                     t1
```

   $T(N) = 5N + 2 + t1 = \Theta(N)$
   The value of t1 depends on the contents of A, but never exceeds N.

   b. Describe the conditions on the inputs for the best-case and worst-case running time for the Histogram2 algorithm.

   Solution: Same as in Problem 1. The best-case scenario is when line 6 is never executed ($t1 = 0$), which occurs when none of the elements of A are between 1 and N. The worst-case scenario is when line 6 is executed a total of N times ($t1 = N$), which occurs when every element of A is between 1 and N.

3. Prove that the running time expression $T(N) = 2N+1 = \Theta(N)$. In particular, find constants c_1, c_2 and n_0 such that $c_1N \le 2N+1 \le c_2N$ for all $N \ge n_0$. Show your work.

   Solution:

   For the first inequality (this proves $T(N) = \Omega(N)$):

   $c_1N \le 2N+1$

   $c_1 \le 2+1/N$

   $c_1 \le 3$ for $N \ge 1$

   For the second inequality (this proves $T(N) = O(N)$):

   $2N+1 \le c_2N$

   $2+1/N \le c_2$

   $3 \le c_2$ for $N \ge 1$

   Thus, we have found constants $c_1 = 3$, $c_2 = 3$, $n_0 = 1$ that satisfy the inequality. Note that there are many other solutions, e.g., $c_1 = 2.5$, $c_2 = 2.5$, $n_0 = 2$.

4. Prove that the running time expression $T(N) = N^2 = O(2^N)$. In particular, find constants c_2 and n_0 such that $N^2 \le c_2 2^N$ for all $N \ge n_0$. Show your work.

   Solution:

   $N^2 \le c_2 2^N$

   $c_2 \ge N^2/2^N$

   For $N \ge 4$, $N^2/2^N \le 1$, thus $c_2 = 1$ and $n_0 = 4$ satisfy the inequality.