CptS260: Introduction to Computer Architecture

Boolean Examples

Problem 1:

For each of the following Boolean expressions create the truth table:

a. $(\bar{a} + b \cdot \bar{d}) \cdot (c \cdot b \cdot a + \bar{c} \cdot d)$

b. $\overline{(w + \bar{x})(z\bar{y} + x)}$

a.

a	b	c	d	$(\bar{a} + b \cdot \bar{d})$	$(c \cdot b \cdot a + \bar{c} \cdot d)$	output
0	0	0	0	1	0	0
0	0	0	1	1	1	1
0	0	1	0	1	0	0
0	0	1	1	1	0	0
0	1	0	0	1	0	0
0	1	0	1	1	1	1
0	1	1	0	1	0	0
0	1	1	1	1	0	0
1	0	0	0	0	0	0
1	0	0	1	0	1	0
1	0	1	0	0	0	0
1	0	1	1	0	0	0
1	1	0	0	1	0	0
1	1	0	1	0	1	0
1	1	1	1	0	1	0

b.

w	x	y	z	$(w+\bar{x})$	$(z\bar{y}+z)$	output
0	0	0	0	1	0	0
0	0	0	1	1	1	1
0	0	1	0	1	0	0
0	0	1	1	1	1	1
0	1	0	0	0	0	0
0	1	0	1	0	1	0
0	1	1	0	0	0	0
0	1	1	1	0	1	0
1	0	0	0	1	0	0
1	0	0	1	1	1	1

1	0	1	0	1	0	0
1	0	1	1	1	1	1
1	1	0	0	1	0	0
1	1	0	1	1	1	1
1	1	1	1	1	1	1

Problem 2:
Use DeMorgan's Theorems to simplify the following expressions:

a. $\overline{(\overline{a+d})\cdot(\overline{\bar{b}+c})}$

$$\overline{(\overline{a+d})\cdot(\overline{\bar{b}+c})} = \overline{\overline{(a+d)}} + \overline{\overline{(\bar{b}+c)}} = (a+d)+(\bar{b}+c) = a+\bar{b}+c+d$$

b. $\overline{\bar{a}+d}\cdot\overline{b+\bar{c}}\cdot\overline{\bar{c}+d}$

$$\overline{\bar{a}+d}\cdot\overline{b+\bar{c}}\cdot\overline{\bar{c}+d} = \bar{\bar{a}}\cdot\bar{d}\cdot\bar{b}\cdot\bar{\bar{c}}\cdot\bar{\bar{c}}\cdot\bar{d} = a\cdot\bar{d}\cdot\bar{b}\cdot c\ \cdot c\cdot\bar{d} = a\cdot\bar{d}\cdot\bar{b}\cdot c$$

Problem 3:
Simplify the following Boolean expressions to the minimum number of gates that is possible to implement the given expression. List the minimum number of gates needed and what those gates are:

a. $[\mathrm{A\bar{B}(C+BD)+\bar{A}\bar{B}]C}$
   - $[\mathrm{A\bar{B}(C+BD)+\bar{A}\ \bar{B}]C} = [\mathrm{A\bar{B}C+A\bar{B}BD+\bar{A}\bar{B}}]C = [\mathrm{A\bar{B}C}+0+\mathrm{\bar{A}\bar{B}}]C$
     $= [\mathrm{A\bar{B}C+\bar{A}\ \bar{B}}]C = \mathrm{A\bar{B}CC+\bar{A}\ \bar{B}}C = \mathrm{A\bar{B}C+\bar{A}\ \bar{B}}C = (\mathrm{A+\bar{A}})\mathrm{\bar{B}}C$
     $= \mathrm{\bar{B}}C$

     1 NOT Gate, 1 AND Gate

b. $\mathrm{\bar{A}BC+A\bar{B}\bar{C}+\bar{A}\bar{B}\bar{C}+\ A\bar{B}C+ABC}$
   - $\mathrm{\bar{A}BC+A\bar{B}\bar{C}+\bar{A}\bar{B}\bar{C}+\ A\bar{B}C+ABC = A\bar{B}C+(A+\bar{A})\bar{B}\bar{C}+(A+\bar{A})BC}$

$$= \mathrm{A\bar{B}C+\bar{B}\bar{C}+BC} = (\mathrm{A\bar{B}}+B)C+\mathrm{\bar{B}\bar{C}} = \boxed{(\mathrm{A}+B)C+\mathrm{\bar{B}\bar{C}}}$$

2 AND Gates, 2 NOT Gates, 2 OR Gates