CptS 260 - Introduction to Computer Architecture
Spring 2022

Midterm #2
April 1, 2022
Take home exam
(Read the policies carefully before you start)

NAME: ID:

	Total Points	Earned
Problem 1	20	
Problem 2	20	
Problem 3	20	
Problem 4	20	
Problem 5	20	

Policy:

- You are one sheet of notes
- Calculator is allowed
- You are **not allowed** to collaborate with your classmates
- Boolean rules are provided
- Show your work for each question.
- MIPS reference data is provided!

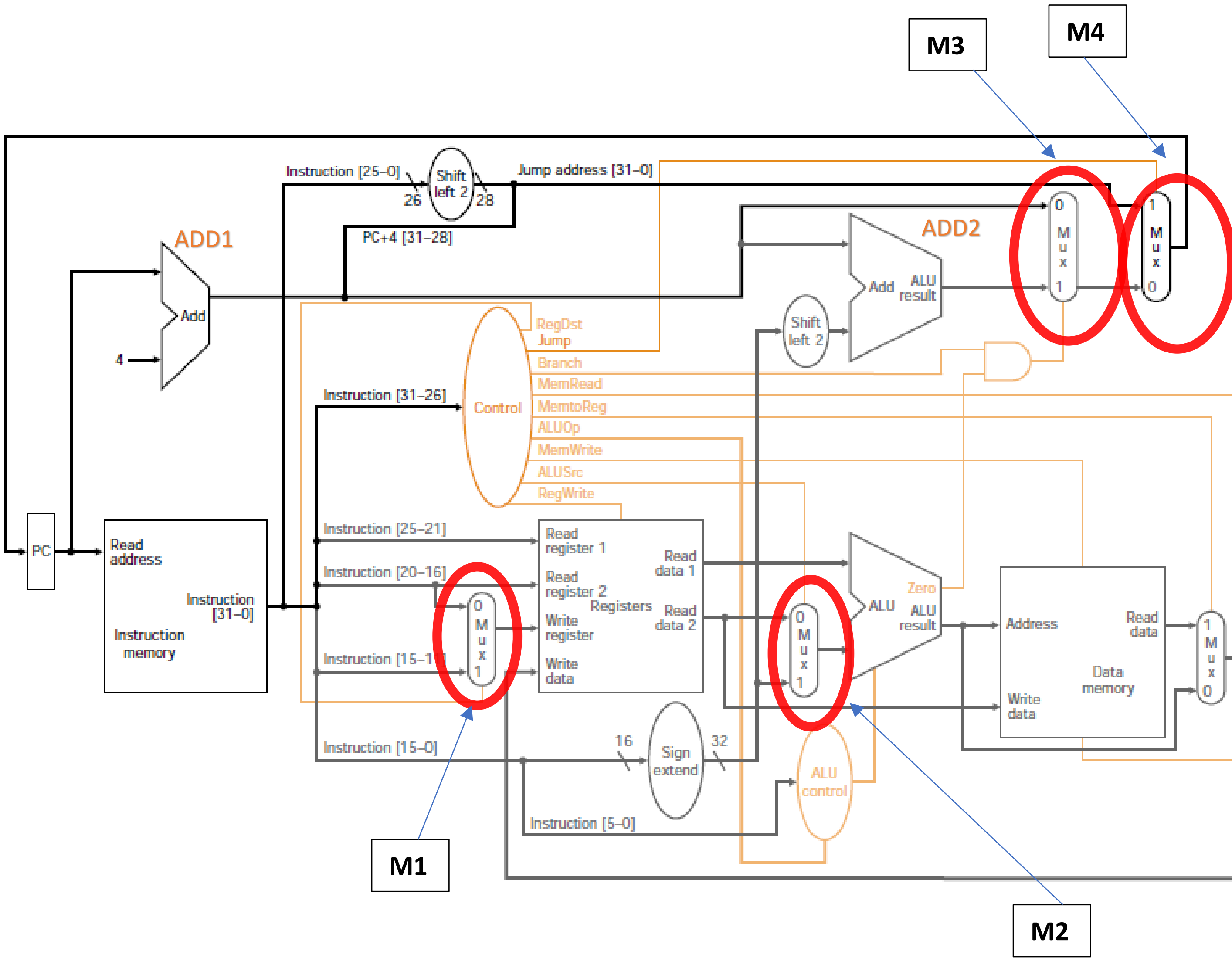
1. **(20 points)** Consider the following Boolean expression:

$$F = \bar{A}\,\bar{B}\,\bar{C} + A\,\bar{B}\,\bar{C} + \bar{A}\,B\,\bar{C}$$

Assume that you are allowed to use only these gates: NOT, 2-input AND, 2-input OR to design the digital circuit for '*F*'. Furthermore, assume that the latency of each gate is 2 ns.

a. Draw equivalent digital circuit for non-simplified function '*F*'.
b. What is the overall latency of the circuit in part (a)?
c. Simplify function '*F*' as much as possible.
d. Draw equivalent digital circuit for the simplified function.
e. What is the overall latency of the simplified circuit?

2. Consider the following simple single cycle MIPS architecture and instruction format, which is the same architecture discussed in the class.



Instruction Format for R-Type, I-Type, and Jump instructions

R	opcode	rs	rt	rd	shamt	funct
	31 26	25 21	20 16	15 11	10 6	5 0
I	opcode	rs	rt	immediate		
	31 26	25 21	20 16	15 0		
J	opcode	address				
	31 26	25 0				

a. Assume that the following instruction is in the datapath to execute. The instruction is located in address 1000d (i.e., address 1000 decimal) in the instruction memory. We also know that $t1 = 10d, and $t2 = 24d.

BEQ $t1, $t2, 10

- What is the value of PC after this instruction is executed.
- What is the value of selection pin for each multiplexer in the datapath (0 or 1). The MUXs are labeled as M1, M2, M3, and M4.

b. Specify the value of each control signal listed in the following table during execution of each instruction listed in the first column.

Instruction	RegDst	Branch	MemRead	MemtoReg	MemWrite	AluSrc	RegWrite
add $9, $7, $8							
Beq $1, $2, 10							
sw $4, 0($3)							

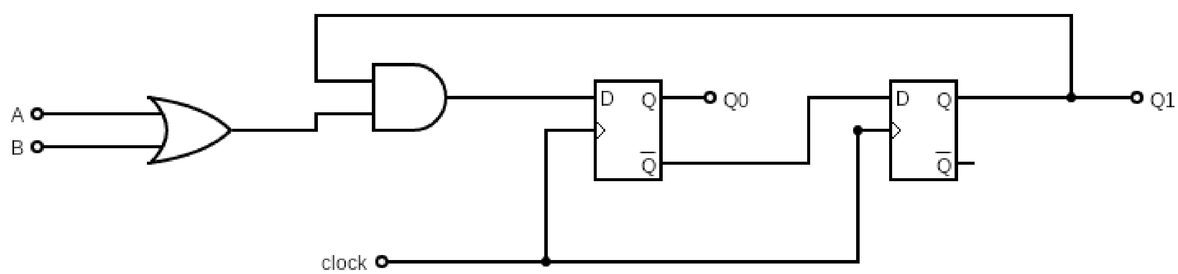
c. Assume instruction “BEQ $t1, $t2, 24d” is located in address 2004d (i.e., address 2004 decimal) in the instruction memory. We also know that $t1 = 15d, and $t2 = 15d. What is the value of PC in decimal after this instruction is executed?

d. Fill out the table below to show the units that are utilized by each instruction?

	Instruction Memory	Register File	Sign-Extend	ALU	Data Memory	PC	ADD1	ADD2
SUB								
OR								
J								
SW								

3. **(20 Points)** The following figure shows a sequential circuit with three D Flip Flops with a common input clock. Note that the inverted output of the first flip-flop ($\bar{Q}_0$) is connected to the input of the second flip-flop (D input of the far right flip flop). Assume that the flip flops are **initialized at '1' and '1', respectively**. That is, $Q_0 = 0$ and $Q_1 = 1$ during Cycle 0. This means the output of this circuit is initially Q_1Q_0 = '11'.

   a. Compute the value of each output signal (Q_1, and Q_0) for 10 cycles using the table below.
   b. Convert the 2-bit binary Q_1Q_0 to its equivalent decimal in the table.
   c. How many unique output states (i.e., Q_1Q_0) does this circuit produce after the initial state?



Clock Cycle	A	B	Q_1 Q_0	Decimal ($Q = Q_1Q_0$)
0	0	1	1 1	3
1	1	1		
2	1	0		
3	1	0		
4	0	0		
5	0	0		
6	1	1		
7	0	1		
8	0	1		
9	1	1		
10	0	0		

4. **(20 points)** Provide brief answers to the questions below.

a) What are the main differences between 1) digital systems without states (combinational logic) and 2) synchronous systems with state information?

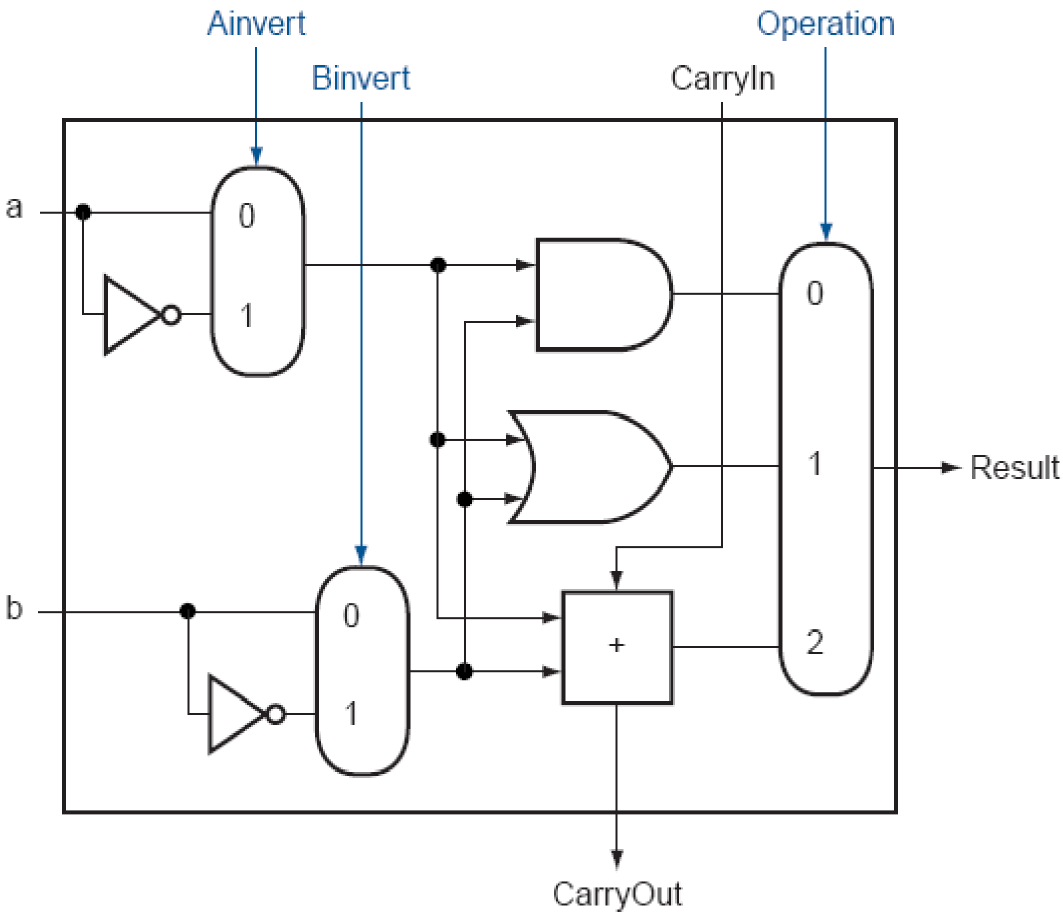
b) Briefly describe the function of an encoder. You may use a 4 to 2 decoder to explain the functionality. What are some situations in which an encoder is used in a digital circuit?

c) Why do all MIPS arithmetic instructions have the form below? What are the potential advantages of using this form?

operation, $r1, $r2, $r3?

d) What are the primary functions of the control unit in the MIPS architecture?

5. (20 points) Consider the 1-bit ALU architecture below.



a. Can we execute the NAND instruction using the available hardware in the ALU? If yes, what would be the values for all the control signals? If not, what additional hardware is needed to implement NAND?

b. Can we execute the XOR operation using the available hardware in the ALU? If yes, what would be the values for all the control signals? If not, what additional hardware is needed to implement XOR?

Boolean Expression Rules:

$$A + \bar{A} = 1$$
$$A \cdot \bar{A} = 0$$

$$\bar{\bar{A}} = A$$

$$A \cdot (A + B) = A$$
$$A + AB = A$$

$$A + \bar{A}B = A + B$$

$$A + A = A$$

$$A \cdot A = A$$

$$A \cdot 0 = 0$$
$$A + 1 = 1$$
$$A \cdot 1 = A$$
$$A + 0 = A$$

MIPS Reference Data Card ("Green Card") 1. Pull along perforation to separate card 2. Fold bottom side (columns 3 and 4) together

①

MIPS Reference Data

CORE INSTRUCTION SET

NAME, MNEMONIC		FOR-MAT	OPERATION (in Verilog)		OPCODE / FUNCT (Hex)
Add	`add`	R	R[rd] = R[rs] + R[rt]	(1)	$0 / 20_{hex}$
Add Immediate	`addi`	I	R[rt] = R[rs] + SignExtImm	(1,2)	8_{hex}
Add Imm. Unsigned	`addiu`	I	R[rt] = R[rs] + SignExtImm	(2)	9_{hex}
Add Unsigned	`addu`	R	R[rd] = R[rs] + R[rt]		$0 / 21_{hex}$
And	`and`	R	R[rd] = R[rs] & R[rt]		$0 / 24_{hex}$
And Immediate	`andi`	I	R[rt] = R[rs] & ZeroExtImm	(3)	c_{hex}
Branch On Equal	`beq`	I	if(R[rs]==R[rt]) PC=PC+4+BranchAddr	(4)	4_{hex}
Branch On Not Equal	`bne`	I	if(R[rs]!=R[rt]) PC=PC+4+BranchAddr	(4)	5_{hex}
Jump	`j`	J	PC=JumpAddr	(5)	2_{hex}
Jump And Link	`jal`	J	R[31]=PC+8;PC=JumpAddr	(5)	3_{hex}
Jump Register	`jr`	R	PC=R[rs]		$0 / 08_{hex}$
Load Byte Unsigned	`lbu`	I	R[rt]={24'b0,M[R[rs] +SignExtImm](7:0)}	(2)	24_{hex}
Load Halfword Unsigned	`lhu`	I	R[rt]={16'b0,M[R[rs] +SignExtImm](15:0)}	(2)	25_{hex}
Load Linked	`ll`	I	R[rt] = M[R[rs]+SignExtImm]	(2,7)	30_{hex}
Load Upper Imm.	`lui`	I	R[rt] = {imm, 16'b0}		f_{hex}
Load Word	`lw`	I	R[rt] = M[R[rs]+SignExtImm]	(2)	23_{hex}
Nor	`nor`	R	R[rd] = ~ (R[rs] | R[rt])		$0 / 27_{hex}$
Or	`or`	R	R[rd] = R[rs] | R[rt]		$0 / 25_{hex}$
Or Immediate	`ori`	I	R[rt] = R[rs] | ZeroExtImm	(3)	d_{hex}
Set Less Than	`slt`	R	R[rd] = (R[rs] < R[rt]) ? 1 : 0		$0 / 2a_{hex}$
Set Less Than Imm.	`slti`	I	R[rt] = (R[rs] < SignExtImm)? 1 : 0	(2)	a_{hex}
Set Less Than Imm. Unsigned	`sltiu`	I	R[rt] = (R[rs] < SignExtImm) ? 1 : 0	(2,6)	b_{hex}
Set Less Than Unsig.	`sltu`	R	R[rd] = (R[rs] < R[rt]) ? 1 : 0	(6)	$0 / 2b_{hex}$
Shift Left Logical	`sll`	R	R[rd] = R[rt] << shamt		$0 / 00_{hex}$
Shift Right Logical	`srl`	R	R[rd] = R[rt] >>> shamt		$0 / 02_{hex}$
Store Byte	`sb`	I	M[R[rs]+SignExtImm](7:0) = R[rt](7:0)	(2)	28_{hex}
Store Conditional	`sc`	I	M[R[rs]+SignExtImm] = R[rt]; R[rt] = (*atomic*) ? 1 : 0	(2,7)	38_{hex}
Store Halfword	`sh`	I	M[R[rs]+SignExtImm](15:0) = R[rt](15:0)	(2)	29_{hex}
Store Word	`sw`	I	M[R[rs]+SignExtImm] = R[rt]	(2)	$2b_{hex}$
Subtract	`sub`	R	R[rd] = R[rs] - R[rt]	(1)	$0 / 22_{hex}$
Subtract Unsigned	`subu`	R	R[rd] = R[rs] - R[rt]		$0 / 23_{hex}$

(1) May cause overflow exception
(2) SignExtImm = { 16{immediate[15]}, immediate }
(3) ZeroExtImm = { 16{1b'0}, immediate }
(4) BranchAddr = { 14{immediate[15]}, immediate, 2'b0 }
(5) JumpAddr = { PC+4[31:28], address, 2'b0 }
(6) Operands considered unsigned numbers (vs. 2's comp.)
(7) Atomic test&set pair; R[rt] = 1 if pair atomic, 0 if not atomic

BASIC INSTRUCTION FORMATS

R	opcode	rs	rt	rd	shamt	funct
	31 26	25 21	20 16	15 11	10 6	5 0
I	opcode	rs	rt	immediate		
	31 26	25 21	20 16	15 0		
J	opcode	address				
	31 26	25 0				

②

ARITHMETIC CORE INSTRUCTION SET

NAME, MNEMONIC		FOR-MAT	OPERATION		OPCODE / FMT /FT / FUNCT (Hex)
Branch On FP True	`bc1t`	FI	if(FPcond)PC=PC+4+BranchAddr	(4)	11/8/1/--
Branch On FP False	`bc1f`	FI	if(!FPcond)PC=PC+4+BranchAddr	(4)	11/8/0/--
Divide	`div`	R	Lo=R[rs]/R[rt]; Hi=R[rs]%R[rt]		0/--/--/1a
Divide Unsigned	`divu`	R	Lo=R[rs]/R[rt]; Hi=R[rs]%R[rt]	(6)	0/--/--/1b
FP Add Single	`add.s`	FR	F[fd]= F[fs] + F[ft]		11/10/--/0
FP Add Double	`add.d`	FR	{F[fd],F[fd+1]} = {F[fs],F[fs+1]} + {F[ft],F[ft+1]}		11/11/--/0
FP Compare Single	`c.`*x*`.s`*	FR	FPcond = (F[fs] *op* F[ft]) ? 1 : 0		11/10/--/*y*
FP Compare Double	`c.`*x*`.d`*	FR	FPcond = ({F[fs],F[fs+1]} *op* {F[ft],F[ft+1]}) ? 1 : 0		11/11/--/*y*

* (*x* is `eq`, `lt`, or `le`) (*op* is ==, <, or <=) (*y* is 32, 3c, or 3e)

NAME, MNEMONIC		FOR-MAT	OPERATION		OPCODE / FMT /FT / FUNCT (Hex)
FP Divide Single	`div.s`	FR	F[fd] = F[fs] / F[ft]		11/10/--/3
FP Divide Double	`div.d`	FR	{F[fd],F[fd+1]} = {F[fs],F[fs+1]} / {F[ft],F[ft+1]}		11/11/--/3
FP Multiply Single	`mul.s`	FR	F[fd] = F[fs] * F[ft]		11/10/--/2
FP Multiply Double	`mul.d`	FR	{F[fd],F[fd+1]} = {F[fs],F[fs+1]} * {F[ft],F[ft+1]}		11/11/--/2
FP Subtract Single	`sub.s`	FR	F[fd]=F[fs] - F[ft]		11/10/--/1
FP Subtract Double	`sub.d`	FR	{F[fd],F[fd+1]} = {F[fs],F[fs+1]} - {F[ft],F[ft+1]}		11/11/--/1
Load FP Single	`lwc1`	I	F[rt]=M[R[rs]+SignExtImm]	(2)	31/--/--/--
Load FP Double	`ldc1`	I	F[rt]=M[R[rs]+SignExtImm]; F[rt+1]=M[R[rs]+SignExtImm+4]	(2)	35/--/--/--
Move From Hi	`mfhi`	R	R[rd] = Hi		0 /--/--/10
Move From Lo	`mflo`	R	R[rd] = Lo		0 /--/--/12
Move From Control	`mfc0`	R	R[rd] = CR[rs]		10 /0/--/0
Multiply	`mult`	R	{Hi,Lo} = R[rs] * R[rt]		0/--/--/18
Multiply Unsigned	`multu`	R	{Hi,Lo} = R[rs] * R[rt]	(6)	0/--/--/19
Shift Right Arith.	`sra`	R	R[rd] = R[rt] >> shamt		0/--/--/3
Store FP Single	`swc1`	I	M[R[rs]+SignExtImm] = F[rt]	(2)	39/--/--/--
Store FP Double	`sdc1`	I	M[R[rs]+SignExtImm] = F[rt]; M[R[rs]+SignExtImm+4] = F[rt+1]	(2)	3d/--/--/--

FLOATING-POINT INSTRUCTION FORMATS

FR	opcode	fmt	ft	fs	fd	funct
	31 26	25 21	20 16	15 11	10 6	5 0
FI	opcode	fmt	ft	immediate		
	31 26	25 21	20 16	15 0		

PSEUDOINSTRUCTION SET

NAME	MNEMONIC	OPERATION
Branch Less Than	`blt`	if(R[rs]<R[rt]) PC = Label
Branch Greater Than	`bgt`	if(R[rs]>R[rt]) PC = Label
Branch Less Than or Equal	`ble`	if(R[rs]<=R[rt]) PC = Label
Branch Greater Than or Equal	`bge`	if(R[rs]>=R[rt]) PC = Label
Load Immediate	`li`	R[rd] = immediate
Move	`move`	R[rd] = R[rs]

REGISTER NAME, NUMBER, USE, CALL CONVENTION

NAME	NUMBER	USE	PRESERVED ACROSS A CALL?
$zero	0	The Constant Value 0	N.A.
$at	1	Assembler Temporary	No
$v0-$v1	2-3	Values for Function Results and Expression Evaluation	No
$a0-$a3	4-7	Arguments	No
$t0-$t7	8-15	Temporaries	No
$s0-$s7	16-23	Saved Temporaries	Yes
$t8-$t9	24-25	Temporaries	No
$k0-$k1	26-27	Reserved for OS Kernel	No
$gp	28	Global Pointer	Yes
$sp	29	Stack Pointer	Yes
$fp	30	Frame Pointer	Yes
$ra	31	Return Address	Yes

OPCODES, BASE CONVERSION, ASCII SYMBOLS

③

MIPS opcode (31:26)	(1) MIPS funct (5:0)	(2) MIPS funct (5:0)	Binary	Deci-mal	Hexa-deci-mal	ASCII Char-acter	Deci-mal	Hexa-deci-mal	ASCII Char-acter
(1)	sll	add.*f*	00 0000	0	0	NUL	64	40	@
		sub.*f*	00 0001	1	1	SOH	65	41	A
j	srl	mul.*f*	00 0010	2	2	STX	66	42	B
jal	sra	div.*f*	00 0011	3	3	ETX	67	43	C
beq	sllv	sqrt.*f*	00 0100	4	4	EOT	68	44	D
bne		abs.*f*	00 0101	5	5	ENQ	69	45	E
blez	srlv	mov.*f*	00 0110	6	6	ACK	70	46	F
bgtz	srav	neg.*f*	00 0111	7	7	BEL	71	47	G
addi	jr		00 1000	8	8	BS	72	48	H
addiu	jalr		00 1001	9	9	HT	73	49	I
slti	movz		00 1010	10	a	LF	74	4a	J
sltiu	movn		00 1011	11	b	VT	75	4b	K
andi	syscall	round.w.*f*	00 1100	12	c	FF	76	4c	L
ori	break	trunc.w.*f*	00 1101	13	d	CR	77	4d	M
xori		ceil.w.*f*	00 1110	14	e	SO	78	4e	N
lui	sync	floor.w.*f*	00 1111	15	f	SI	79	4f	O
	mfhi		01 0000	16	10	DLE	80	50	P
(2)	mthi		01 0001	17	11	DC1	81	51	Q
	mflo	movz.*f*	01 0010	18	12	DC2	82	52	R
	mtlo	movn.*f*	01 0011	19	13	DC3	83	53	S
			01 0100	20	14	DC4	84	54	T
			01 0101	21	15	NAK	85	55	U
			01 0110	22	16	SYN	86	56	V
			01 0111	23	17	ETB	87	57	W
	mult		01 1000	24	18	CAN	88	58	X
	multu		01 1001	25	19	EM	89	59	Y
	div		01 1010	26	1a	SUB	90	5a	Z
	divu		01 1011	27	1b	ESC	91	5b	[
			01 1100	28	1c	FS	92	5c	\
			01 1101	29	1d	GS	93	5d	]
			01 1110	30	1e	RS	94	5e	^
			01 1111	31	1f	US	95	5f	_
lb	add	cvt.s.*f*	10 0000	32	20	Space	96	60	‘
lh	addu	cvt.d.*f*	10 0001	33	21	!	97	61	a
lwl	sub		10 0010	34	22	"	98	62	b
lw	subu		10 0011	35	23	#	99	63	c
lbu	and	cvt.w.*f*	10 0100	36	24	$	100	64	d
lhu	or		10 0101	37	25	%	101	65	e
lwr	xor		10 0110	38	26	&	102	66	f
	nor		10 0111	39	27	’	103	67	g
sb			10 1000	40	28	(	104	68	h
sh			10 1001	41	29	)	105	69	i
swl	slt		10 1010	42	2a	*	106	6a	j
sw	sltu		10 1011	43	2b	+	107	6b	k
			10 1100	44	2c	,	108	6c	l
			10 1101	45	2d	-	109	6d	m
swr			10 1110	46	2e	.	110	6e	n
cache			10 1111	47	2f	/	111	6f	o
ll	tge	c.f.*f*	11 0000	48	30	0	112	70	p
lwc1	tgeu	c.un.*f*	11 0001	49	31	1	113	71	q
lwc2	tlt	c.eq.*f*	11 0010	50	32	2	114	72	r
pref	tltu	c.ueq.*f*	11 0011	51	33	3	115	73	s
	teq	c.olt.*f*	11 0100	52	34	4	116	74	t
ldc1		c.ult.*f*	11 0101	53	35	5	117	75	u
ldc2	tne	c.ole.*f*	11 0110	54	36	6	118	76	v
		c.ule.*f*	11 0111	55	37	7	119	77	w
sc		c.sf.*f*	11 1000	56	38	8	120	78	x
swc1		c.ngle.*f*	11 1001	57	39	9	121	79	y
swc2		c.seq.*f*	11 1010	58	3a	:	122	7a	z
		c.ngl.*f*	11 1011	59	3b	;	123	7b	{
		c.lt.*f*	11 1100	60	3c	<	124	7c	|
sdc1		c.nge.*f*	11 1101	61	3d	=	125	7d	}
sdc2		c.le.*f*	11 1110	62	3e	>	126	7e	~
		c.ngt.*f*	11 1111	63	3f	?	127	7f	DEL

(1) opcode(31:26) == 0

(2) opcode(31:26) == 17_{ten} (11_{hex}); if fmt(25:21)==16_{ten} (10_{hex}) *f* = s (single);
if fmt(25:21)==17_{ten} (11_{hex}) *f* = d (double)

④

IEEE 754 FLOATING-POINT STANDARD

$$(-1)^S \times (1 + \text{Fraction}) \times 2^{(\text{Exponent - Bias})}$$

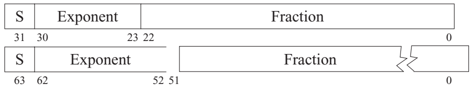
where Single Precision Bias = 127,
Double Precision Bias = 1023.

IEEE 754 Symbols

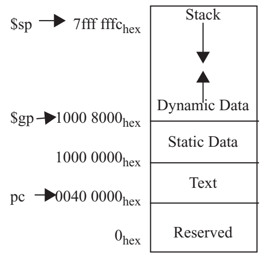
Exponent	Fraction	Object
0	0	± 0
0	≠0	± Denorm
1 to MAX - 1	anything	± Fl. Pt. Num.
MAX	0	±∞
MAX	≠0	NaN

S.P. MAX = 255, D.P. MAX = 2047

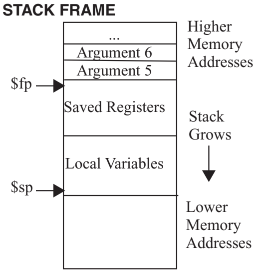
IEEE Single Precision and Double Precision Formats:



MEMORY ALLOCATION



STACK FRAME

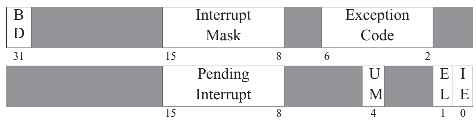


DATA ALIGNMENT

Double Word							
Word				Word			
Halfword		Halfword		Halfword		Halfword	
Byte	Byte	Byte	Byte	Byte	Byte	Byte	Byte
0	1	2	3	4	5	6	7

Value of three least significant bits of byte address (Big Endian)

EXCEPTION CONTROL REGISTERS: CAUSE AND STATUS



BD = Branch Delay, UM = User Mode, EL = Exception Level, IE =Interrupt Enable

EXCEPTION CODES

Number	Name	Cause of Exception	Number	Name	Cause of Exception
0	Int	Interrupt (hardware)	9	Bp	Breakpoint Exception
4	AdEL	Address Error Exception (load or instruction fetch)	10	RI	Reserved Instruction Exception
5	AdES	Address Error Exception (store)	11	CpU	Coprocessor Unimplemented
6	IBE	Bus Error on Instruction Fetch	12	Ov	Arithmetic Overflow Exception
7	DBE	Bus Error on Load or Store	13	Tr	Trap
8	Sys	Syscall Exception	15	FPE	Floating Point Exception

SIZE PREFIXES (10^x for Disk, Communication; 2^x for Memory)

SIZE	PRE-FIX	SIZE	PRE-FIX	SIZE	PRE-FIX	SIZE	PRE-FIX
10^3, 2^{10}	Kilo-	10^{15}, 2^{50}	Peta-	10^{-3}	milli-	10^{-15}	femto-
10^6, 2^{20}	Mega-	10^{18}, 2^{60}	Exa-	10^{-6}	micro-	10^{-18}	atto-
10^9, 2^{30}	Giga-	10^{21}, 2^{70}	Zetta-	10^{-9}	nano-	10^{-21}	zepto-
10^{12}, 2^{40}	Tera-	10^{24}, 2^{80}	Yotta-	10^{-12}	pico-	10^{-24}	yocto-

The symbol for each prefix is just its first letter, except μ is used for micro.

MIPS Reference Data Card (“Green Card”) 1. Pull along perforation to separate card 2. Fold bottom side (columns 3 and 4) together