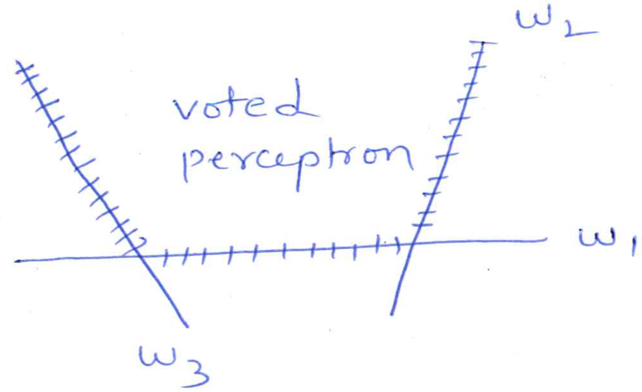
HW #3 Solutions

1\. (a) The decision boundary of voted perceptron is non-linear. You cannot represent the decision function as a linear function. Take a simple two dimensional example, it a piece-wise linear function.



(b) Average perceptron is linear.

2\. Scale learning rate using importance weight h_i $\rightarrow$ You want to more aggressive w.r.t learning for examples which are important.

$$\boxed{w_{new} = w_{old} + \eta_i y_i x_i}$$

where, $\eta_i = h_i * \eta$

3. Balance the unbalanced by either

(a) up-sampling the minority class, i.e, create more copies of +ve examples

(or)

(b) Down-sampling the majority class, i.e, Remove negative examples appropriate

(Alternative)

We can use solution of Q2 to solve Q3.

H+ will be the weight of all +ve examples

H- will be the weight of all -ve examples

Make H+ larger than H- appropriately

Say H+ = 0.9 & H- = 0.1

Q4.

$$SCORE(i) = w \cdot f(x_i)$$

$$SCORE(j) = w \cdot f(x_j)$$

If candidate i is better than j, then

$$SCORE(i) > SCORE(j)$$

$$w \cdot F(x_i) > w \cdot f(x_j)$$

$$w \cdot (f(x_i) - f(x_j)) > 0$$

$$\Rightarrow \quad \text{sign}(w \cdot \underbrace{f(x_i) - f(x_j)}_{x}) > 0$$

If mistake, then update weights

$$w_{new} = w_{old} + \eta \underbrace{(f(x_i) - f(x_j))}_{x}$$

$$y = +1$$

Alternatively,

similar to multi-class classification

$\uparrow$ increase score of i & $\downarrow$ decrease score of j

$$\left.\begin{aligned} w &= w + f(x_i) \\ w &= w - f(x_j) \end{aligned}\right\} \text{same net result}$$

Q5.

(a) $\|x - c_+\| \leq \|x - c_-\|$

Square both sides will make life easier

$$\|x - c_+\|^2 \leq \|x - c_-\|^2$$

$$\cancel{\|x\|^2} - 2x \cdot c_+ + \|c_+\|^2 \leq \cancel{\|x\|^2} - 2x \cdot c_- + \|c_-\|^2$$

bring all the terms to one side.

$$2x \cdot c_+ - 2x \cdot c_- + \|c_-\|^2 - \|c_+\|^2 \geq 0$$

$$\underbrace{2(c_+ - c_-)}_{w} \cdot x + \underbrace{\|c_-\|^2 - \|c_+\|^2}_{b} \geq 0$$

$$w = 2(c_+ - c_-)$$

$$b = \|c_-\|^2 - \|c_+\|^2$$

(b)

$$w = \frac{2\sum_{i=1}^{n+} x_i}{n+} - \frac{2\sum_{j=1}^{n-} x_j}{n-}$$

$$\left.\begin{array}{ll} \alpha = \frac{2}{n+} & \text{for +ve examples} \\ \alpha = \frac{2}{n-} & \text{for -ve examples} \end{array}\right\}$$

(c) All are support vectors