# Computational Advertising*

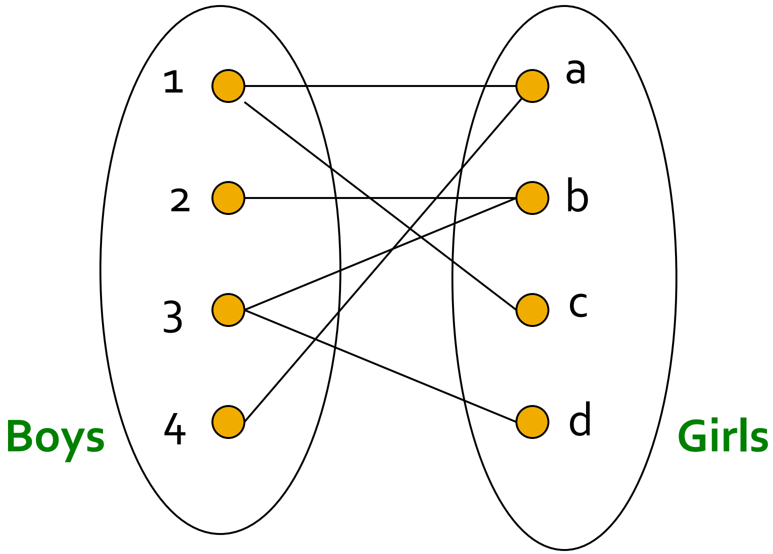
* Slides based on Jeff Ullman, Anand Rajaraman, and Jure Leskovec

# Online vs. Offline Algorithms

- **Classic model of algorithms**
  - You get to see the entire input, then compute some function of it
  - In this context, “offline algorithm”
- **Online Algorithms**
  - You get to see the input one piece at a time, and need to make irrevocable decisions along the way
  - **Similar to the data stream model**

# Aside: Online Bipartite Matching

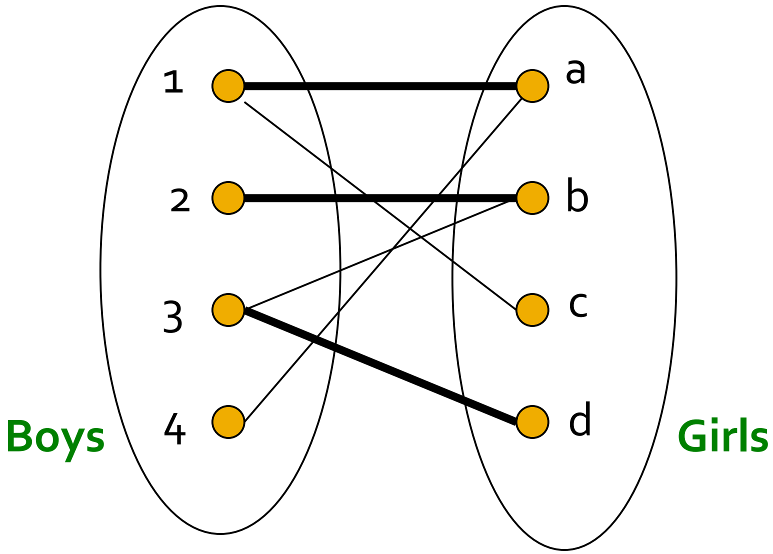
# Example: Bipartite Matching




**Nodes: Boys and Girls; Edges: Preferences**

**Goal: Match boys to girls so that maximum number of preferences is satisfied**

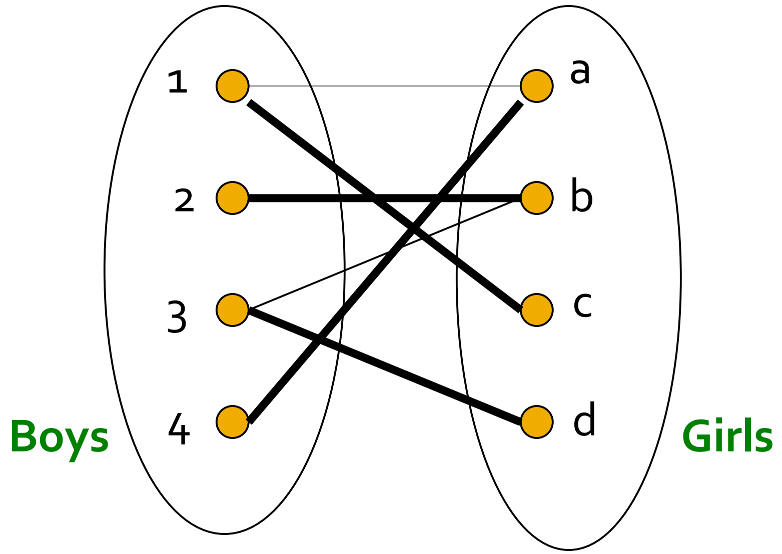
# Example: Bipartite Matching




**M = {(1,a),(2,b),(3,d)}** is a **matching**

**Cardinality of matching = |M| = 3**

# Example: Bipartite Matching




**M = {(1,c),(2,b),(3,d),(4,a)}** is a **perfect matching**

**Perfect matching** … all vertices of the graph are matched
**Maximum matching** … a matching that contains the largest possible number of matches

# Matching Algorithm

- **Problem: Find a maximum matching for a given bipartite graph**
  - A perfect one if it exists

- There is a polynomial-time offline algorithm based on augmenting paths (Hopcroft & Karp 1973, see http://en.wikipedia.org/wiki/Hopcroft-Karp_algorithm)

- **But what if we do not know the entire graph upfront?**

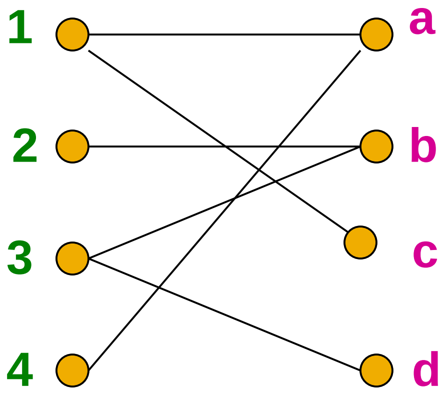
# Online Graph Matching Problem

- **Problem: Find a maximum matching for a given bipartite graph**
  - A perfect one if it exists

- There is a polynomial-time offline algorithm based on augmenting paths (Hopcroft & Karp 1973, see http://en.wikipedia.org/wiki/Hopcroft-Karp_algorithm)

- **But what if we do not know the entire graph upfront?**

# Online Graph Matching: Example




**(1,a)**
**(2,b)**
**(3,d)**

# Greedy Algorithm

- **Greedy algorithm for the online graph matching problem:**
  - Pair the new girl with **any** eligible boy
    - If there is none, do not pair girl

- **How good is the algorithm?**

# Competitive Ratio

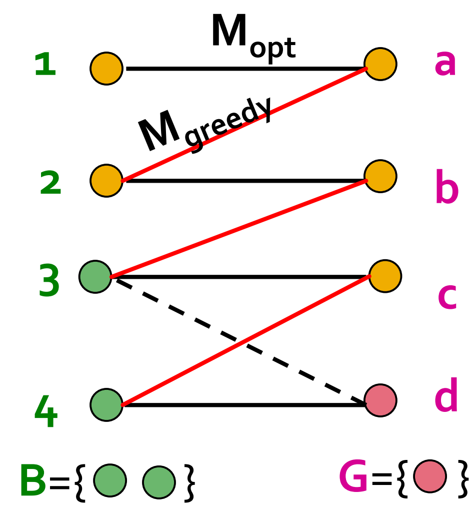
- For input ***I***, suppose greedy produces matching $\boldsymbol{M_{greedy}}$ while an optimal matching is $\boldsymbol{M_{opt}}$

**Competitive ratio =**

$$\min_{\text{all possible inputs } I} \left( |M_{greedy}| / |M_{opt}| \right)$$

**(what is greedy's <u>worst</u> performance <u>over all possible</u> inputs *I*)**

# Analyzing the Greedy Algorithm

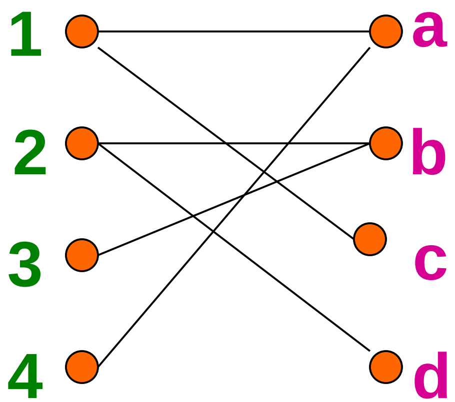



- Consider a case: $M_{greedy} \neq M_{opt}$
- Consider the set ***G*** of girls matched in $M_{opt}$ but not in $M_{greedy}$
- Then every boy ***B*** adjacent to girls in ***G*** is already matched in $M_{greedy}$:
  - If there would exist such non-matched (by $M_{greedy}$) boy adjacent to a non-matched girl then greedy would have matched them
- Since boys ***B*** are already matched in $M_{greedy}$ then **(1)** $|M_{greedy}| \geq |B|$

# Analyzing the Greedy Algorithm

- **Summary so far:**
  - Girls $G$ matched in $M_{opt}$ but not in $M_{greedy}$
  - **(1)** $|M_{greedy}| \geq |B|$
- There are at least $|G|$ such boys ($|G| \leq |B|$) otherwise the optimal algorithm couldn't have matched all girls in $G$
  - **So:** $|G| \leq |\mathbf{B}| \leq |M_{greedy}|$
- By definition of $G$ also: $|\mathbf{M}_{\mathbf{opt}}| \leq |\mathbf{M}_{\mathbf{greedy}}| + |\mathbf{G}|$
  - Worst case is when $|G| = |\mathbf{B}| = |M_{greedy}|$
- $|M_{opt}| \leq 2|M_{greedy}|$ **then** $|M_{greedy}|/|M_{opt}| \geq 1/2$

# Worst-Case Scenario




(1,a)
(2,b)

# Back to Main Topic:
# Computational Advertising

# History of Web Advertising

- **Banner ads (1995-2001)**
  - Initial form of web advertising
  - Popular websites charged *X*$ for every 1,000 "impressions" of the ad
    - Called "**CPM**" rate (Cost per thousand impressions)
    - Modeled similar to TV, magazine ads
  - From **untargeted** to **demographically targeted**
  - **Low click-through rates**
    - Low ROI for advertisers

# Performance-based Advertising

- **Introduced by Overture around 2000**
  - Advertisers **bid** on **search keywords**
  - When someone searches for that keyword, the **highest bidder's ad is shown**
  - Advertiser is charged only if the ad is clicked on

- Similar model adopted by Google with some changes around 2002
  - Called **Adwords**

# Ads vs. Search Results

**Web** Results **1** - **10** of about **2,230,000** for **geico**. (**0.04** secc

**GEICO** Car Insurance. Get an auto insurance quote and save today ...
**GEICO** auto insurance, online car insurance quote, motorcycle insurance quote, online insurance sales and service from a leading insurance company.
www.**geico**.com/ - 21k - Sep 22, 2005 - Cached - Similar pages
Auto Insurance - Buy Auto Insurance
Contact Us - Make a Payment
More results from www.geico.com »

**Geico**, Google Settle Trademark Dispute
The case was resolved out of court, so advertisers are still left without legal guidance on use of trademarks within ads or as keywords.
www.clickz.com/news/article.php/3547356 - 44k - Cached - Similar pages

Google and **GEICO** settle AdWords dispute | The Register
Google and car insurance firm **GEICO** have settled a trade mark dispute over **...** Car insurance firm **GEICO** sued both Google and Yahoo! subsidiary Overture in **...**
www.theregister.co.uk/2005/09/09/google_**geico**_settlement/ - 21k - Cached - Similar pages

**GEICO** v. Google
**...** involving a lawsuit filed by Government Employees Insurance Company (**GEICO**). **GEICO** has filed suit against two major Internet search engine operators, **...**
www.consumeraffairs.com/news04/**geico**_google.html - 19k - Cached - Similar pages

Sponsored Links

Great Car Insurance Rates
Simplify Buying Insurance at Safeco
See Your Rate with an Instant Quote
www.Safeco.com

Free Insurance Quotes
Fill out one simple form to get multiple quotes from local agents.
www.HometownQuotes.com

5 Free Quotes. 1 Form.
Get 5 Free Quotes In Minutes!
You Have Nothing To Lose. It's Free
sayyessoftware.com/Insurance
Missouri

# Web 2.0

- **Performance-based advertising works!**
  - Multi-billion-dollar industry
- **Interesting problem:**
  **What ads to show for a given query?**
  - (Our focus)
- **If I am an advertiser, which search terms should I bid on and how much should I bid?**
  - (We won't cover this question)

# AdWords Problem

- **Given:**
  - **1.** A set of bids by advertisers for search queries
  - **2.** A click-through rate for each advertiser-query pair
  - **3.** A budget for each advertiser (say for 1 month)
  - **4.** A limit on the number of ads to be displayed with each search query

# AdWords Problem

- **Respond to each search query with a set of advertisers such that:**
  - **1.** The size of the set is no larger than the limit on the number of ads per query
  - **2.** Each advertiser has bid on the search query
  - **3.** Each advertiser has enough budget left to pay for the ad if it is clicked upon

# AdWords Problem

- A stream of queries arrives at the search engine: $\boldsymbol{q_1, q_2, \ldots}$
- Several advertisers bid on each query
- When query $\boldsymbol{q_i}$ arrives, search engine must pick a subset of advertisers whose ads are shown

- Goal: **Maximize search engine's revenues**
  - Simple solution: Instead of raw bids, use the "**expected revenue per click**" (i.e., **Bid*CTR**)
- **Clearly we need an online algorithm!**

# The AdWords Innovation

| Advertiser | Bid | CTR | Bid * CTR |
|---|---|---|---|
| A | $1.00 | 1% | 1 cent |
| B | $0.75 | 2% | 1.5 cents |
| C | $0.50 | 2.5% | 1.125 cents |
| | | Click through rate | Expected revenue |

# The AdWords Innovation

| Advertiser | Bid | CTR | Bid * CTR |
|---|---|---|---|
| B | $0.75 | 2% | 1.5 cents |
| C | $0.50 | 2.5% | 1.125 cents |
| A | $1.00 | 1% | 1 cent |

# Complications: Budget

- Two complications:
  - Budget
  - CTR of an ad is unknown

- Each advertiser has a limited budget
  - Search engine guarantees that the advertiser will not be charged more than their daily budget

# Complications: CTR

- **CTR: Each ad has a different likelihood of being clicked**
  - **Advertiser 1** bids \$2, click probability = 0.1
  - **Advertiser 2** bids \$1, click probability = 0.5
  - **Clickthrough rate (CTR)** is measured **historically**
    - **Very hard problem: Exploration vs. exploitation**
      **Exploit:** Should we keep showing an ad for which we have good estimates of click-through rate
      **or**
      **Explore:** Shall we show a brand new ad to get a better sense of its click-through rate

# Greedy Algorithm

- **Our setting: Simplified environment**
  - There is **1** ad shown for each query
  - All advertisers have the same budget ***B***
  - All ads are equally likely to be clicked
  - Value of each ad is the same (=**1**)

- **Simplest algorithm is greedy:**
  - For a query pick any advertiser who has bid **1** for that query
  - **Competitive ratio of greedy is 1/2**

# Bad Scenario for Greedy

- **Two advertisers A and B**
  - ***A*** bids on query ***x***, ***B*** bids on ***x*** and ***y***
  - Both have budgets of **$4**
- **Query stream:** ***x x x x y y y y***
  - Worst case greedy choice: ***B B B B*** **_ _ _ _**
  - Optimal: **A A A A B B B B**
  - **Competitive ratio = ½**
- **This is the worst case!**
  - **Note:** Greedy algorithm is deterministic – it always resolves draws in the same way

# BALANCE Algorithm

- **BALANCE** Algorithm by Mehta, Saberi, Vazirani, and Vazirani
  - **For each query, pick the advertiser with the largest unspent budget**
    - Break ties arbitrarily (**but in a deterministic way**)

# Example: BALANCE Algorithm

- **Two advertisers A and B**
  - **A** bids on query ***x***, **B** bids on ***x*** and ***y***
  - Both have budgets of **$4**

- **Query stream:** ***x x x x y y y y***

- **BALANCE choice:** **A B A B B B _ _**
  - Optimal: **A A A A B B B B**

- **In general:** For **BALANCE** on **2** advertisers
  **Competitive ratio = ¾**

# General Version of the Problem

- **Arbitrary bids and arbitrary budgets!**
- Consider we have 1 query $\boldsymbol{q}$, advertiser $\boldsymbol{i}$
  - Bid = $\boldsymbol{x_i}$
  - Budget = $\boldsymbol{b_i}$
- **In a general setting BALANCE can be terrible**
  - Consider two advertisers $\boldsymbol{A_1}$ and $\boldsymbol{A_2}$
  - $\boldsymbol{A_1}$: $\boldsymbol{x_1}$ **= 1**, $\boldsymbol{b_1}$ **= 110**
  - $\boldsymbol{A_2}$: $\boldsymbol{x_2}$ **= 10**, $\boldsymbol{b_2}$ **= 100**
  - Consider we see **10** instances of **q**
  - BALANCE always selects $\boldsymbol{A_1}$ and earns **10**
  - Optimal earns **100**

# Generalized BALANCE Algorithm

- **Arbitrary bids:** consider query $q$, bidder $i$
  - Bid = $x_i$
  - Budget = $b_i$
  - Amount spent so far = $m_i$
  - Fraction of budget left over $f_i = 1-m_i/b_i$
  - Define $\psi_i(q) = x_i(1-e^{-f_i})$

- Allocate query $q$ to bidder $i$ with largest value of $\psi_i(q)$

- **Same competitive ratio (1-1/e)**