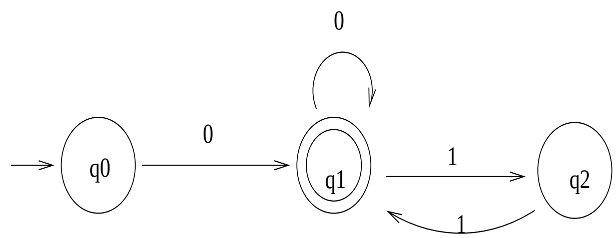
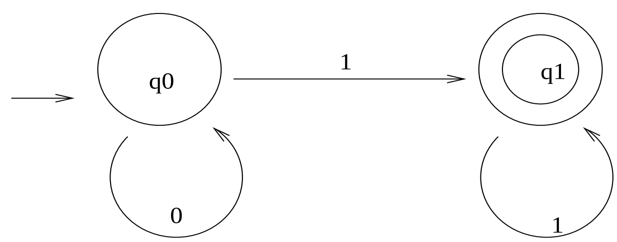
Cpt S 317 Homework #3 Solutions

1. Let L_1 and L_2 be two regular languages. They are specified by the following regular expressions: $L_1 = 0(0+11)^*$ and $L_2 = 0^*11^*$.

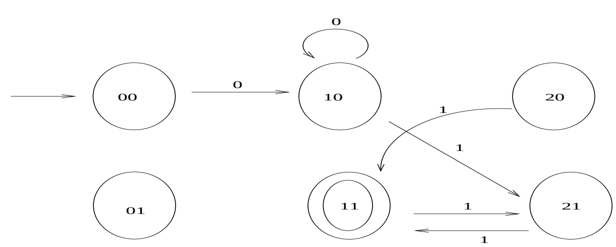
(1). Draw a DFA accepting L_1.



(2). Draw a DFA accepting L_2.



(3). Draw a DFA accepting $L_1 \cap L_2$.



(4). What is the regular expression for $L_1 \cap L_2$?

$00^*11(11)^*$.

2. A natural number can be encoded as a unary string. For instance, 5= the string of *aaaaa*. Therefore, we may treat a set of numbers as a language over a unary alphabet (that contains only one symbol, e.g., a). Write down

the regular expression for the following sets of numbers: (1). all the n such that $n \bmod 3 = 1$. (2). all the n such that $n \bmod 3 = 0$ or $n \bmod 4 = 2$.

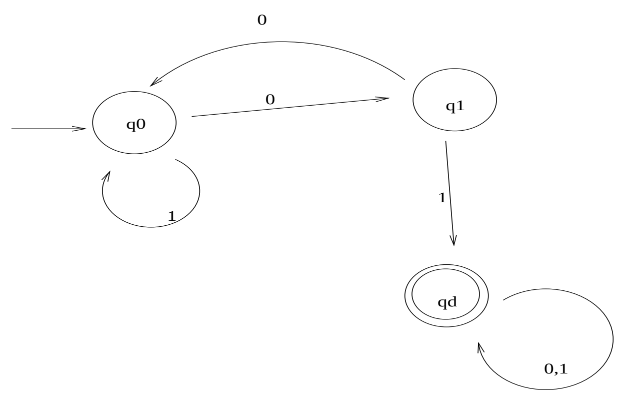
(1). $a(aaa)^*$
(2). $(aaa)^* + aa(aaaa)^*$

3. Show that deterministic FAs are closed under complement. That is, for any deterministic FA M, there is a deterministic FA M' such that $L(M') = \Sigma^* - L(M)$, assuming that both M and M' have the same alphabet.

Assume $M = \langle Q, \Sigma, q_0, A, \delta \rangle$. First we need to make M total. That is, let q_d be a state not in Q. Define $\bar{\delta}$ be a total function such that for each $q \in Q \cup \{q_d\}, a \in \Sigma$, $\bar{\delta}(q, a) = \delta(q, a)$ if $\delta(q, a)$ is defined. Otherwise, $\bar{\delta}(q, a) = q_d$. Now, construct $M' = \langle Q \cup \{q_d\}, \Sigma, q_0', A', \delta' \rangle$ as follows:
$q_0' = q_0$
$A' = Q \cup \{q_d\} - A$
$\delta'(q, a) = \bar{\delta}(q, a)$ for each $q \in Q \cup \{q_d\}, a \in \Sigma$.
Then,
$w \notin L(M)$ iff
$\bar{\delta}^*(q_0, w) \notin A$ iff
$\delta'^*(q_0, w) \notin A$ (since $\delta' = \bar{\delta}$) iff
$\delta'^*(q_0, w) \in A'$ iff
$w \in L(M')$.
Thus, $\Sigma^* - L(M) = L(M')$.

4. According to your proof of Problem 3, draw a deterministic finite automaton that accepts the complement of $(00+1)^*$. And also find a regular expression for the language accepted by M'.

$$L(M') = 1^*0(01^*0)^* + 1^*0(01^*0)^*1(1+0)^*$$



5. Let L be a regular language on Σ and $\Sigma' \subset \Sigma$. The result of dropping symbols in Σ' from a word w is denoted by $w^{-\Sigma'}$. For instance, $aaabacba^{-\{b\}}$ is $aaaaca$. Define $L^{-\Sigma'} = \{w^{-\Sigma'} : w \in L\}$. That is, $L^{-\Sigma'}$ is the result of dropping symbols in Σ' from each word in L. Show that if L is a regular language, then $L^{-\Sigma'}$ is also a regular language. (Hint: use structural induction)

Induction on the definition of regular languages.

Case 1. If $L = \emptyset$, then $L^{-\Sigma'} = \emptyset$ is regular.

Case 2. If $L = \{\Lambda\}$, then $L^{-\Sigma'} = \{\Lambda\}$ is regular.

Case 3. If $L = \{a\}$, $a \in \Sigma$, then $L^{-\Sigma'} = \{\Lambda\}$ if $a \in \Sigma'$, otherwise $L^{-\Sigma'} = \{a\}$. Either case gives $L^{-\Sigma'}$ is regular.

Case 4. If

4(a). $L = L_1 \cup L_2$, then since $L^{-\Sigma'} = L_1^{-\Sigma'} \cup L_2^{-\Sigma'}$, assuming both $L_1^{-\Sigma'}$ and $L_2^{-\Sigma'}$ are regular gives $L^{-\Sigma'}$ is regular.

4(b). $L = L_1L_2$, then since $L^{-\Sigma'} = L_1^{-\Sigma'}L_2^{-\Sigma'}$, assuming both $L_1^{-\Sigma'}$ and $L_2^{-\Sigma'}$ are regular gives $L^{-\Sigma'}$ is regular.

4(c). $L = L_1^*$, then since $L^{-\Sigma'} = (L_1^{-\Sigma'})^*$. So $L^{-\Sigma'}$ is regular if $L_1^{-\Sigma'}$ is.

Thus, $L^{-\Sigma'}$ is regular for any regular language L.