

317 Homework #10

1. Build a Turing Machine accepting $(b+c)^+ \# a^+$

* We know that $(b+c)$ and c^+ are $(b+c)^+$ means any number of b or c repeats at least one of them *

* a^+ means any number of a's at least 1 given $(b+c)^+ \# a^+$ that can be build with intermediate symbol # and we combine the two parts into one *

Turing machine $M = \langle Q, \Gamma, B, E, \delta, q_0, F \rangle$

Q - set of states

Γ - Tape symbols

B - Blank symbol \perp

E - input alphabet $E \subseteq \Gamma$

δ - Transition function

q_0 - starting state

$F = \{q\}$ final accepting states

$I = (b+c) \# a^+$

$E = \{a, b+c\} = \{a, b, c, \perp \#^+\}$

↓

Transition function as follows

$\delta(Q \times \Gamma) \rightarrow Q \times \Gamma \times \{L, R, S\}$

$(q_0, b) \rightarrow (q_1, b, R)$ reading at least one b to move

to next state

$(q_0, c) \rightarrow (q_1, c, R)$ all

$(q_1, b) \rightarrow (q_2, b, R)$ / Reading more than one 'c'

$(q_1, c) \rightarrow (q_2, c, R)$

$(q_1, \#) \rightarrow (q_2, \#, R)$ // End of first block (b+c)

$(q_2, b) \rightarrow (q_3, b, R)$
"Reading at least one to move state"

$(q_2, c) \rightarrow (q_3, c, R)$ continue to read

$(q_2, \#) \rightarrow (q_4, \#, R)$ end of tape to start process

2. Build a Turing Machine accepting $\{x \# x^r : x \in \{a, b\}^+\}$

// state q_0 : Read input

// Action: End string

q_0 : Read input

if input = "" then go to q_{Accept}

else go to q_1

// state q_1 : check first character

// Action: Read the first character

q_1 :

Read first character

if first character = "#" then go to q_{Reject}

else go to q_2

// state q_2 : store the first character

// Action: Store the first character

q_2 :

Store the first character Go to q_3

Go to q_3

// state q_3 : move the right

// Action: Move one cell to the right

q_3 :

Move one cell to the right

Go to q_4

// state q_4 : check for "#"

// Action: Read the character

q_4 : Read the character

if character = "#" then go to q_{Reject}

else go to q_5

// state q_5 : check if

character is the same

as the first character

// Action: Compare the

character with the stored

first character

q_5 :

Compare the character with

the stored first character

if character match then

go to q_6 else go to

q_{Reject}

// state q_6 : Move back to left

// Action: move one cell to left

q_6 : Move one cell to the

left

Go to q_7

// state q_7 : Read the

character

// Action: Read the

character

q_7 :

Read the character

if character = "" then go to

q_{Accept}

else go to q_{Reject}

// state q_{Accept} : Accept

// Action: Accept the input

q_{Accept} :

Accept the input

// state q_{Reject} :

Reject

// Action: Reject the

input

q_{Reject} :

Reject the input

3.

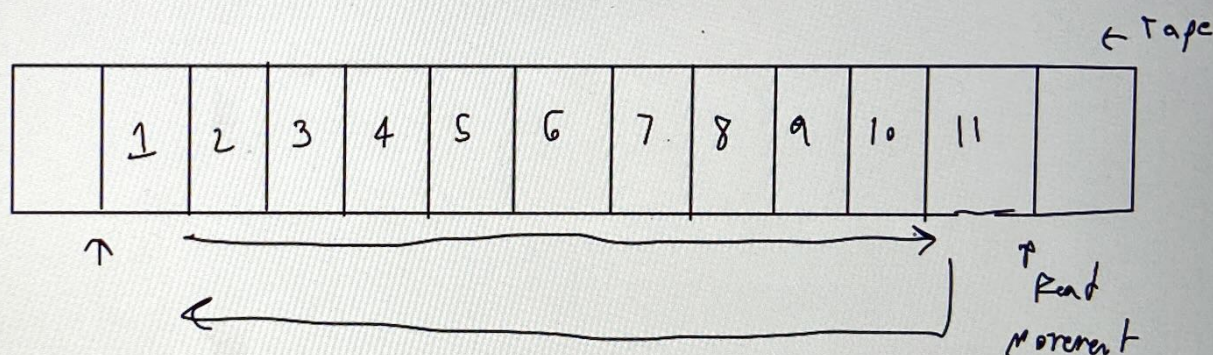
→ Context free language are recognized by PDA

→ PDA use stack, for recognising given string

Now,

Turing machine is a one-turn Turing Machine. Such that during any execution and on any inputs M makes at most one turn on the tape.

Means, if the tape is moving right, later it (May move left but never moves right again).



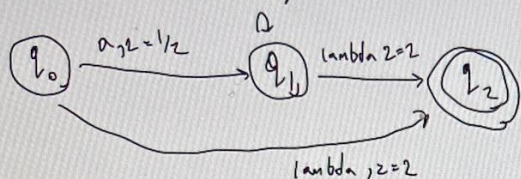
→ This is similar to the stack, because after turning the last element is accessed first.

→ Hence, the tape has become a stack, so this machine acts like PDA.

* The language accepted by one-turn Turing machine are context free *

4.

Let $\Gamma = \{1, 2\}$, $\Sigma = \{a, b\}$



We simulate a 2-PDA on a Turing machine by using the tape as a queue of possible configurations of the 2-PDA (say by simply keeping them in sequence with delimiter $\$$, popping by reading the first configuration and then moving the other left, and pushing by adding configurations at the end). Each configuration in the queue encodes a state of the 2-PDA, the contents of both stacks, and the remaining input. Our TM will then pop a configuration, iterate through the possible 2-PDA transitions from this configuration and for each transition pushing the resulting configuration onto the stack. If the transition popped has no more input and is in an accept state, the TM accepts: if the queue is ever empty, the TM rejects.

The 2-PDA accepts if and only if there is some sequence of valid transitions which brings it to an accept state with no input left.

If such a sequence exists the TM will eventually follow it and accept. Conversely, if no such sequence exists the TM will not accept. Thus this TM accepts if and only if the 2-PDA did not.