

$$1. \quad Q = \{q_0, q_1, q_2, q_3, q_4, q_5, q_6\}$$

$$\Sigma = \{a, b\}$$

$$\Gamma = \{A, B\}$$

$$\delta = (q_0, a, Z_0) \rightarrow (q_1, AZ_0)$$

$$(q_1, a, A) \rightarrow (q_1, AA)$$

$$(q_1, b, A) \rightarrow (q_2, bA)$$

$$(q_2, a, B) \rightarrow (q_2, BA)$$

$$(q_2, b, B) \rightarrow (q_3, bB)$$

$$(q_3, a, A) \rightarrow (q_4, aA)$$

$$(q_3, b, A) \rightarrow (q_3, bA)$$

$$(q_4, a, B) \rightarrow (q_4, Ba)$$

$$(q_4, b, B) \rightarrow (q_5, Bb)$$

$$(q_5, a, A) \rightarrow (q_5, aA)$$

$$(q_5, b, A) \rightarrow (q_6, bA)$$

$$(q_6, a, B) \rightarrow (q_6, aB)$$

$$(q_6, b, B) \rightarrow (q_6, bB)$$

$$(q_0, b, Z_0) \rightarrow (q_2, bZ_0)$$

If the string is being derived using $S \rightarrow AB$, then the PDA will need to be in state q_3 and will need to read an A followed by a B . This will transition the PDA to state q_4 .

3. Let L_1 and L_2 be two regular languages. Show that $L_3 = \{xx^r : x \in L_1 \text{ and } x^r \in L_2\}$ is a context free language.

$$L_1 = \{a^n b^n \mid n \geq 0\}$$

$$f: \{a, b\} \Rightarrow \Sigma \quad f: \{a, b\} \Rightarrow \Sigma$$

$$L_2 = f(L_1) = \{uv \mid u, v \in \Sigma^*, |u| = |v|\}$$

$$L_3 = L_1 \cap L_2$$

$$h: \Sigma \cup \{\$ \} \Rightarrow \Sigma^* \text{ as follow. } h(a) = a, h(b) = b$$

$$h(L_3) = \{ab \mid a \in x, b \in x^r\}$$

L_3 is a context free language.

4. Show that $L = \{a^{2^M} b^{3^M} c^{4^M} : M \geq 0\}$ is not context free language.

L is CFL

$$W = aa\ bb\ cccc$$

$$W = \frac{aa}{u} \frac{bb}{v} \frac{bc}{x} \frac{cc}{y} \frac{c}{z}$$

$$W = UV^i x y^i z$$

$$\text{if } i=2 \\ = aa\ bbbbbb\ ccccc$$

$$W \notin L$$

Given language L is not CFL