Solutions to Homework #8

1. $0^+1^* + 0^*1^+$.

2. $(0+1)^*$

3. $\{0^n 1^{2n} : n \geq 0\}$

4. From Problem 2, $S \to S0|S1|\Lambda$ generates $(0+1)^*$. Also it is easy to see $S \to 10$ generates 10. We just need to concatenate these two grammars – and this is the answer:

$S \to AB$
$A \to A0|A1|\Lambda$
$B \to 10$

5. Note that for each word $0^{3n}1^{2n}0^m1^m$ in the language, the two parts $0^{3n}1^{2n}$ and 0^m1^m are independent since we don't have a dependence relation between m and n. Thus, we first construct a grammar to generate strings $0^{3n}1^{2n}$:

$S \to 000S11|00011$ (note that $n \geq 1$),

and a grammar to generate strings 0^m1^m:

$S \to 0S1|01$.

The answer is the result of concatenating the two grammars:

$S \to AB$
$A \to 000A11|00011$
$B \to 0B1|01$

6. We have to generate a grammar that does the following: whenever a 1 is generated, the grammar generates at least two 0's – in this way, we shall make sure that the words generated are in the form of 0^n1^m with $n \geq 2m$. First, we take the rule

$S \to 00S1$

This rule will give two 0's for each 1 generated. We will be ok if we, at some time, "switch S to a mode that generates 0's only" – this will guarantee that the number of 0's is more than the number of 1's doubled. How to switch? well, look at this:

$S \to A$
$A \to 0A$

The critical part is: whenver S is switched to A, it won't come back to S. So, here is the grammar:

$S \to 00S1$

$S \to A$
$A \to 0A$

7. Similar to Problem 6. The answer is:
$S \to S1|0S1|00S1|1|01$

8. Note that $\{0^n1^m : n \neq 2m\}$ is the union of the two languages in Problem 6 and 7. The answer is:
$S \to D|E$ (D means the S in Problem 6, E means the S in Problem 7. S has two choices: either goes D or goes E – this is "union")
$D \to 00D1$
$D \to A$
$A \to 0A$
(The above three rules are copied from Problem 6 with S replaced by D)
$E \to 00E1$
$E \to B$
$B \to 1B$
(The above three rules are copied from Problem 7 with S replaced by E)

9. Notice that A, B, S are all nullables. Here is the new grammar:
$S \to ASB|SB|AB|AS|A|S|B$ (correspond to $S \to ASB$)
$S \to AB|A|B$ (correspond to $S \to AB$)
$S \to ab$
$A \to AS|A|S|a$
$B \to SB|S|B|A|b$
Then we can get rid of repeated rules and here is the final answer:
$S \to ASB|SB|AB|AS|A|S|B|ab$
$A \to AS|A|S|a$
$B \to SB|S|B|A|b$

10. Notice that $S \Rightarrow_G^* B$, $B \Rightarrow_G^* A$ and $S \Rightarrow_G^* A$. Therefore, the new grammar is:
$S \to SB|ab$
$A \to a|SA$
$S \to a|SA$ (since $S \Rightarrow_G^* A$)
$B \to a|SA$ (since $B \Rightarrow_G^* A$)
$B \to SB|b$
$S \to SB|b$ (since $S \Rightarrow_G^* B$)

11. First add two new nonterminal symbols D_a and D_b:

$D_a \to a$
$D_b \to b$
Then, replace each a by D_a and b by D_b:
$S \to AD_bBD_aS|ASB|AB|D_aD_b$
$A \to D_bB|D_a$
$B \to SBD_b|D_b$
Now we need add a number of rules to make them CNF:
$S \to AE$
$E \to D_bF$
$F \to BG$
$G \to D_aS$
and
$S \to AH$
$H \to SB$
and
$S \to AB|D_aD_b$
$A \to D_bB|D_a$
$B \to SI$
$I \to BD_b$
$B \to D_b$
So, here is the final answer:
$D_a \to a$
$D_b \to b$
$S \to AE|AH|AB|D_aD_b$
$E \to D_bF$
$F \to BG$
$G \to D_aS$
$H \to SB$
$A \to D_bB|D_a$
$B \to SI|D_b$
$I \to BD_b$