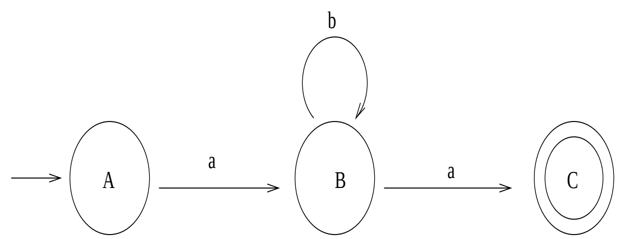
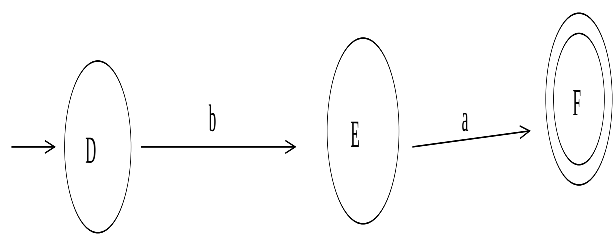
Solutions to Homework #6

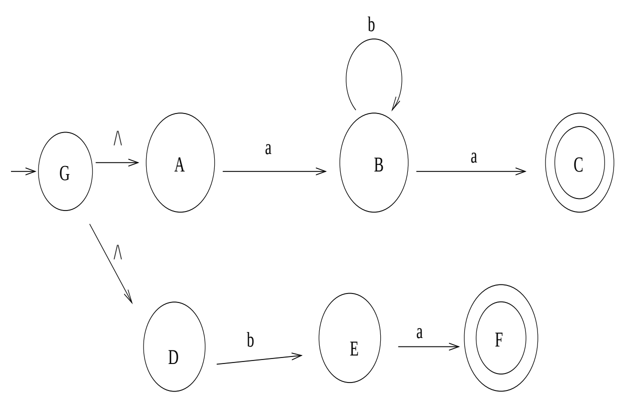
1. First draw M_1 to accept ab^*a.



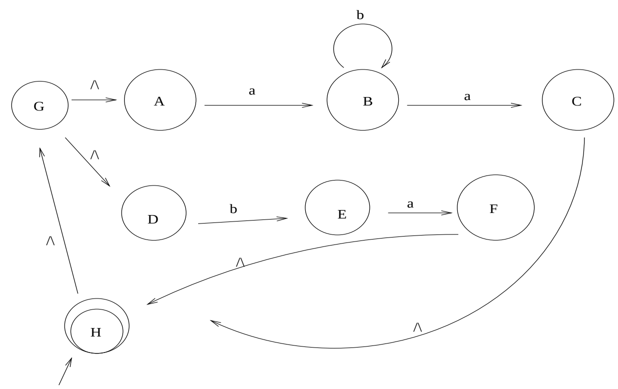
Then draw M_2 to accept ba.



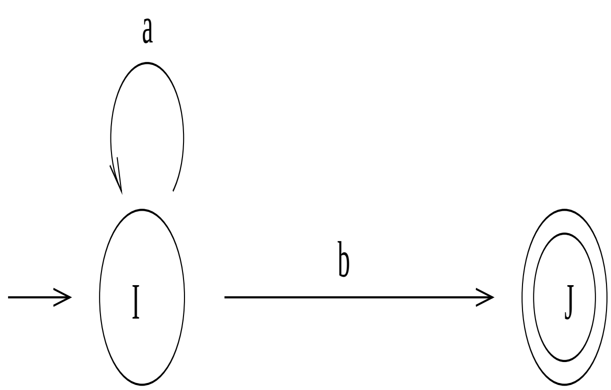
Then draw M_3 to accept $(ab^*a + ab)$.



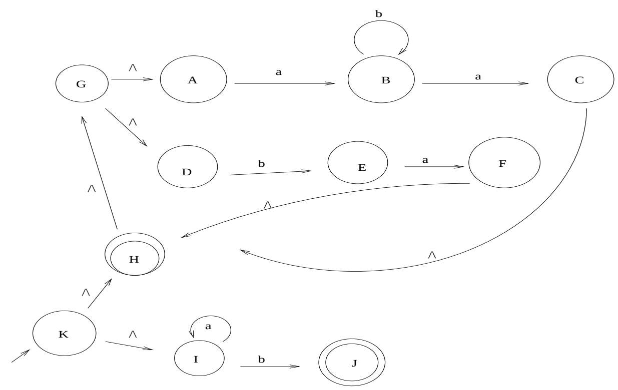
Then draw M_4 to accept $(ab^*a + ab)^*$.



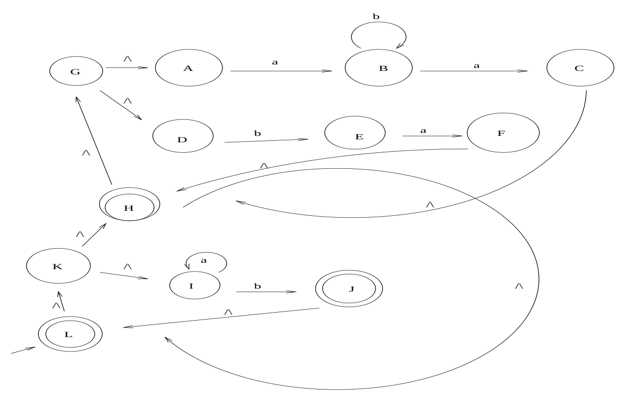
Then draw M_5 to accept a^*b.



Then draw M_6 to accept $(ab^*a+ab)^*+a^*b$.



Then draw M_7 to accept $((ab^*a+ab)^*+a^*b)^*$.
M_7 is the answer.



2. According to the proof, we need to calculate r_{ij}^k as follows:
(1). r_{ij}^0:
$r_{11}^0 = a + \Lambda$
$r_{12}^0 = b$
$r_{21}^0 = b$
$r_{22}^0 = c + \Lambda$
(2). r_{ij}^1:
$r_{11}^1 = a^*$
$r_{12}^1 = r_{11}^0(r_{11}^0)^*r_{12}^0 + r_{12}^0 = a^*b$
$r_{21}^1 = ba^*$

$r_{22}^1 = r_{21}^0(r_{11}^0)^* r_{12}^0 + r_{22}^0 = b(a+\Lambda)^* b + c + \Lambda = ba^*b + c + \Lambda$
The answer is $L(M) =$
$r_{12}^2 = r_{12}^1(r_{22}^1)^* r_{22}^1 + r_{12}^1 = a^*b(ba^*b + c + \Lambda)^*(ba^*b + c + \Lambda) + a^*b$

3. Suppose $L = \{0^n 1^m : n \geq 1, m \geq 1, n \leq m\}$ is regular. Let n be the integer in the pumping lemma. Pick $z = 0^n 1^n \in L$. It can be verified that $|z| \geq n$. From the lemma, there are u, v, w such that $z = uvw$ and $|uv| \leq n$ and $|v| \geq 1$. From the lemma, $uvvw$ (take $i = 2$ in the lemma) should be L. But this is impossible, since $uvvw = 0^{n+|v|}1^n \notin L$ (since $n + |v| > n$). A contradiction. Thus, L is not regular.

4. Suppose $L = \{xx^R x : x \in (a+b)^*\}$ is regular. Let n be the integer in the pumping lemma. Pick $z = a^n b^n b^n a^n a^n b^n \in L$. It can be verified that $|z| \geq n$. From the lemma, there are u, v, w such that $z = uvw$ and $|uv| \leq n$ and $|v| \geq 1$. From the lemma, $uvvw$ (take $i = 2$ in the lemma) should be L. But this is impossible, since $uvvw = a^{n+|v|} b^n b^n a^n a^n b^n \notin L$. A contradiction. Thus, L is not regular.

5. (1). $L = \{0^m 1^n 0^{m+n} : m \geq 1, n \geq 1\}$ is not regular. Suppose L is regular. Let n be the integer in the pumping lemma. Pick $z = 0^n 1^n 0^{2n}$ in L. It can be verified that $|z| \geq n$. From the lemma, there are u, v, w such that $z = uvw$ and $|uv| \leq n$ and $|v| \geq 1$. From the lemma, $uvvw$ (take $i = 2$ in the lemma) should be L. But this is impossible, since $uvvw = 0^{n+|v|}1^n 0^{2n} \notin L$ (since $n + |v| + n \neq 2n$). A contradiction. Thus, L is not regular.

(2). $\{0^m 0^n 0^{m+n} : m \geq 1, n \geq 1\}$ is regular, since it is $0000(00)^*$.

(3). $\{xwx^R : x \in (0+1)^*, w \in (0+1)^*\}$ is regular, since it is $(0+1)^*$.

(4). $\{0^n 1^m : n \geq 1, m \geq 1, n > m\}$ is not regular, since it is the complement of the L in Problem 3. (We know the complement of a regular language is regular – Homework #1 Problem 5).