

Cpt S 317 Homework #10

Please print your name!

1. (1). Show that if both  $L$  and (the complement)  $\bar{L}$  are r.e., then  $L$  is recursive. (2). Show that r.e. languages are not closed under complement.
2. Show that recursive languages are closed under complement.
3. Recall that we use  $M_i$  to denote the “ $i$ -th Turing Machine”. That is, the string encoding  $\langle M_i \rangle$  of  $M_i$  is exactly  $w_i$ , the  $i$ -th word in the dictionary ordering. Show that, for each Turing machine  $M$ , there are infinitely many  $i$  such that  $L(M) = L(M_i)$  (i.e.,  $M$  and  $M_i$  accept the same language.). (If a program is understood as a Turing machine, what does this exercise say about the program?)
4. For a TM  $M$ , a  $L$ -instruction (resp.  $R$ -instruction) is a move in the form of  $\delta(q, a) = (p, b, L)$  (resp.  $\delta(q, a) = (p, b, R)$ ). Show that the following problem is **decidable**:  
**Given a TM  $M$ , whether  $M$  contains the same number of  $L$ -instructions and  $R$ -instructions?**
5. Show that the following problem is also **decidable**:  
**Given a TM  $M$ , whether there exists a TM  $M'$  such that  $M'$  contains the same number of  $L$ -instructions and  $R$ -instructions, and  $L(M') = L(M)$ ?**