HW7 new solutions — hint.

1.(1). color patterns that sat. the formula are

① all yellow, OR,

② there is a blue, after that, all yellow

③ there are infinitely many blue.

Each can be checked by running SCC on G. For instance, to check ①, we can drop all other nodes with colors not yellow and check that the resulting graph has a looping SCC reachable from the initial node that is also yellow. you can similarly do ② and ③.

1.(2). color patterns that sat. the formula are:

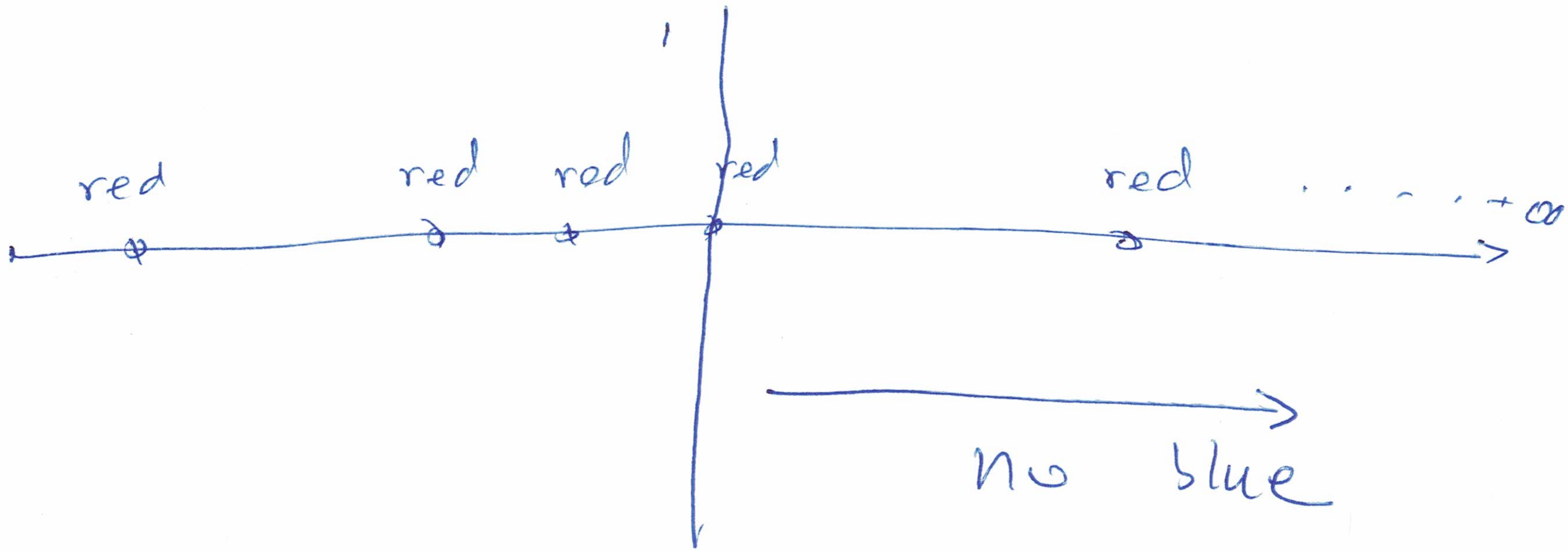
① there are infinitely many yellow, OR

② there are infinitely many blue.

To check ①, you need run SCC on G and check that there is a looping SCC that contains a yellow

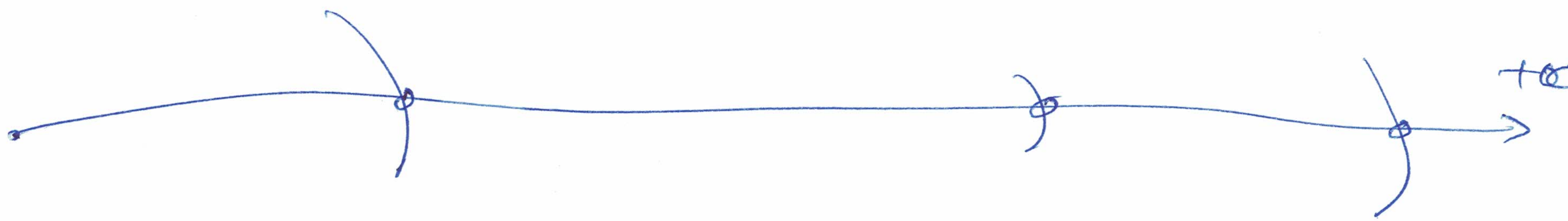
node and the SCC is reachable from the initial node.
you can check ② similarly.

2.



we drop all blue nodes from G and use G' to denote the resulting graph. Then run SCC on G' and check that the initial node in G can reach a looping SCC in G' that contains a red node. Note that the aforementioned "reach" is within the original graph G.

3.



$\longleftarrow$ $\#_{red} = \#_{blue}$ $\longrightarrow$

It suffices to check that, in G, there is a node v such that

① initial can reach v through a walk with $\#_{red} = \#_{blue}$, and

② v can reach back to v through a walk with $\#_{red} = \#_{blue}$.

Hence, we need only demonstrate an alg to ~~check~~: solve the following problem:

Given: G and two nodes u_1, u_2

Question: Is there a walk from u_1 to u_2 sat. $\#_{red} = \#_{blue}$?

This is a very hard problem and we can have a few ways to do it (e.g., using integer linear programming, etc.)

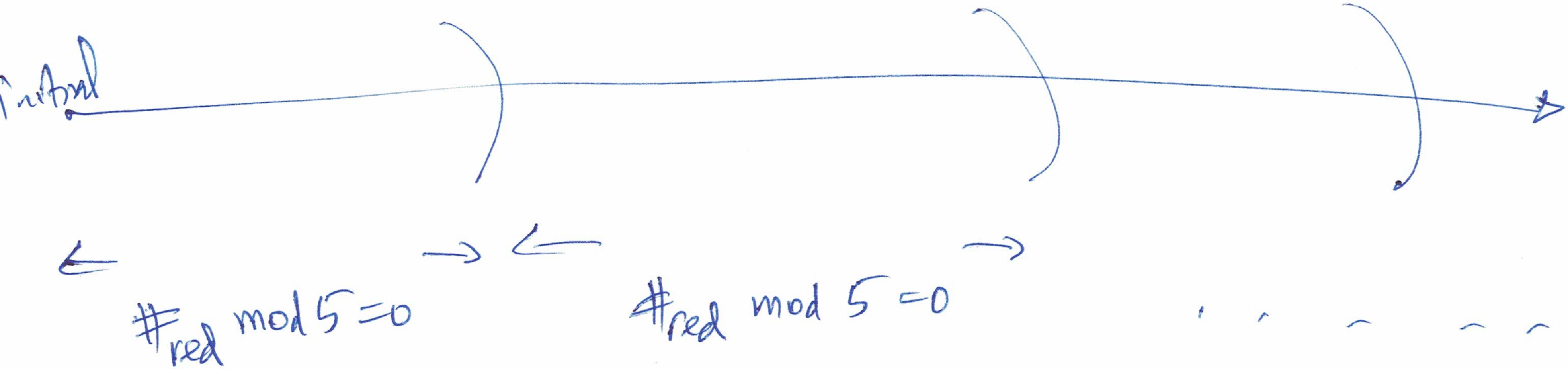
Here, you may use PDA that was in CptS317.

Define

$$\mathcal{L} = \{ w : w \text{ is a walk from } u_1 \text{ to } u_2 \text{ sat. } \#_{red} = \#_{blue} \}.$$

Clearly, $\mathcal{L}$ is context-free. You need only check whether "$\mathcal{L} = \emptyset$?" which has an alg to solve (e.g., using emptiness testing alg for context-free languages or using emptiness testing algo for PDA's.)

4.



It suffices to check that there is a node v sat.

① initial node can reach v through a walk sat. $\#_{red} \bmod 5 = 0$; and,

② v can reach v through a walk sat $\#_{red} \bmod 5 = 0$.

Hence, we only need show an alg to solve the following problem:

Given: G and two nodes u_1 and u_2

Question: Is there a walk from u_1 to u_2 sat. $\#_{red} \bmod 5 = 0$?

There are many ways to solve the problem. For instance, through a careful control using DFS but this DFS-approach would be a mess if you are not careful since such a walk may not be simple (i.e., may contain cycles.) Here we use cpts 317 to show a unified way to solve it. Define

$\mathcal{L} = \{w : w \text{ is a walk from } u_1 \text{ to } u_2 \text{ s.t. } \#_{red} \bmod 5 = 0\}$.

Clearly, $\mathcal{L}$ is regular. (Is it clear?)
Hence, you many construct a ~~DF~~ FA M from G to accept the $\mathcal{L}$ and using DFS, to check whether there is a walk in M from M's initial to M's final state. Notice that such a checking is equivalent to "$\mathcal{L} \neq \phi$."