Hw 1 Solutions.

1. (1) $\{\langle n, p, q\rangle : n = p \cdot q$ and p, q are primes$\}$

where $\langle n, p, q\rangle$ is the string encoding of n, p, q.

(2). $\{\langle n\rangle : \exists p, q.\ n = p \cdot q$ and p, q are primes$\}$

where $\langle n\rangle$ is the string encoding of n

(3). $\{\langle A, w\rangle : A$ accepts $w\}$ where $\langle A, w\rangle$ is the string encoding of A and w.

(4). $\{\langle A\rangle : \exists w.\ A$ accepts $w\}$ where $\langle A\rangle$ is the string encoding of A.

2. (1). Function $2n^3-18n$ is $O(n^3)$

why? $\lim_{n\to\infty} \frac{2n^3-18n}{3n^3} = \frac{2}{3} < 1$.

It is also $O(n^4)$.

why? $\lim_{n\to\infty} \frac{2n^3-18n}{n^4} = 0 < 1$.

It is NOT $O(n^2 \log n)$.

why? Assume it is $O(n^2 \log n)$. Then, $\exists c > 0$,

$$\lim_{n\to\infty} \frac{2n^3-18n}{c \cdot n^2 \log n} = \lim_{n\to\infty} \left(\frac{2n}{c \log n} - \frac{18}{c n \log n}\right)$$

$$= +\infty < 1, \text{ contradictio}$$

(2). Function $3n^2 2^{2n}$ is $2^{O(n)}$.

why? $\lim_{n\to\infty} \frac{3n^2 2^{2n}}{2^{3n}} = \lim_{n\to\infty} \frac{3n^2}{2^n} = 0 < 1$.

3. Graded on the effort that you put into the problems.