CptS 355- Programming Language Design

Functional Programming in Haskell
Part-1

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Fall 2023

Washington State University
World Class. Face to Face.

Functional Programming - A Brief History

- Based on *lambda calculus* by *Alonzo Church* (1930s)
- First real language: **Lisp** by *John McCarthy* (1950s)
- Popularized by many, especially *John Backus*
- **ML** developed by *Robin Milner* at Edinburgh (1973)
- **Miranda** and **Haskell** in late 1980s
  - It is named after logician Haskell Curry.

Haskell

- Haskell is:
  - pure functional programming language – there are no side effects
  - lazy evaluation – values are only computed on demand, allowing the implementation of infinite data structures
  - type system – statically typed, no type declarations needed (types are inferred); supports polymorphic types

- Benefits:
  - allows for concise programs -- Haskell makes code easier to understand and maintain
  - much cleaner mathematically
  - strong typing catches many bugs at compile time
  - functional code permits better testing methodologies

Installing Haskell

- Install the Haskell Platform, which includes the GHC compiler.
  - Windows: https://www.haskell.org/platform/
  - Mac and Linux: https://www.haskell.org/downloads/
- Check the instructions posted on Canvas:
  - Assignment 0

Getting started with Haskell

- Create a file called hello.hs with the following contents:

```haskell
main = putStrLn "Hello, world!"
```

- Compile your program to a native executable like this:

```
$ ghc --make hello
[1 of 1] Compiling Main             ( hello.hs, hello.o )
Linking hello ...
$ ./hello
Hello, world!
```

- Or run it in the GHCI interpreter like this:

```
$ ghci hello.hs
GHCi, version 7.0.3: http://www.haskell.org/ghc/  :? for help
...
Ok, one module loaded.
Prelude Main> main
Hello, world!
Prelude Main>
```

Getting started with Haskell

- If you need multiple I/O actions in one expression, you can use a `do` block

```
main = do putStrLn "What is 4 * 5?"
          x <- readLn
          if x == 20
             then putStrLn "You're right!"
             else putStrLn "You're wrong!"
```

- The indentation is significant.

Getting started with Haskell (cont.)

- Or run Haskell interpreter and load the file within Haskell :

```
$ ghci
ghci > :load hello
Ok, one module loaded.
ghci> main
Hello, world!
ghci>

OR
ghci > :l hello
```

- If you've subsequently edited the file with an external editor, use:

```
ghci> :reload
OR
ghci> :r
```

- To change the prompt:

```
ghci> :set prompt "haskell> "
```

- To quit:

```
ghci> :quit
OR
ghci> :q
```

Building Blocks: Functions

- A function has two components:
  - Input: arguments passed to function
  - Output: result of running function
- Functions are first class: treated like any other value
  - Can be passed into other functions
  - Can be returned from other functions
- Combine functions to build new functions

In Haskell every value, expression, and function has a type

- **Some basic types:**
  - `Bool` - either `True` or `False`
  - `Char` - a unicode code point (i.e., a character)
  - `Int` - fixed-size integer
  - `Integer` - an arbitrary-size integer
  - `Double` - an IEEE double-precision floating-point number
  - `String` – which is an alias for [`Char`]
  - `type1` → `type2` - a function from `type1` to `type2`
  - `(type1, type2, ..., typeN)` – a tuple
  - `[type1]` – a list

- You can declare the type of a symbol or expression with `::`

```
y = 1 :: Int
x = (1 :: Integer) + (1 :: Integer) :: Integer
```

  - `::` has lower precedence than any function operators (including +)

Bindings

- Haskell uses the "=" sign to declare bindings:

```
x = 2                           -- Two hyphens introduce a comment
y = 3                           -- ...that continues to end of line.
main = let z = x + y            -- let introduces local bindings
       in print z               -- program will print 5
```

  - Bound names cannot start with upper-case letters
  - Bindings are separated by ";", which is usually auto-inserted by a layout rule

- A binding may declare a function of one or more arguments
  - Function and arguments are separated by spaces (when defining or invoking them)

```
addOne arg = 1 + arg              -- defines function addOne
four = addOne 3                   -- invokes function addOne

add arg1 arg2 = arg1 + arg2       -- defines function add
five = add 2 3                    -- invokes function add
```

- Parentheses can wrap compound expressions, must do so for arguments

```
bad = print add 2 3               -- error! (print should have only 1 argument)
main = print (add 2 3)            -- ok, calls print with 1 argument, 5
```

Functions

- Functions map input values to output values

head body

```
add arg1 arg2 = arg1 + arg2    -- defines function add
five = add 2 3                 -- invokes function add
```

```
exclaim sentence = sentence ++ "!"    -- defines function exclaim
s = exclaim "Hello"                   -- invokes function exclaim
```

Type Signatures (of functions)

- A complete function definition (with type signature) appears as follows:

```
double :: Integer -> Integer     -- type signature
double x = 2*x                   -- function expression
```

- Function type signatures are optional, but it's good form to declare them.
  - They provide documentation for other programmers and help the Haskell system to spot type errors.

- Haskell will infer the types of most things:

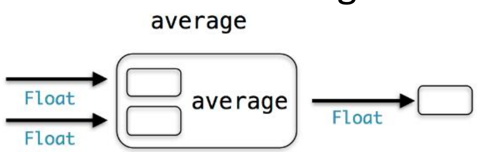
```
double x = 2.0 * ( x :: Double)
:t double

double :: Double -> Double
```

  - `:type` (or `:t`) retrieves the type of a binding, expression, or function.

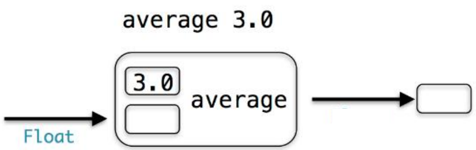
More on Functions

```
average :: Float -> Float -> Float
average a b = (a + b) / 2.0
a = average 3.0 4.0
```

- The type of a function with more than one argument separates the arguments with an arrow (→)
- Function application happens one argument at a time (a.k.a. "currying")
  - You can view a function with two arguments, such as `average`, as a box with two free slots:

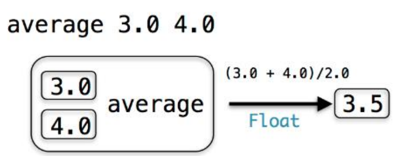


  - Once the function is applied to an argument of type `Float`, the first slot is filled, and it results in a new function which maps a value `b` provided as an argument to `(3.0 + b)/2.0`.



Will return a function of type `Float -> Float`

  - Only when the second argument is provided, and all slots are filled, can the function be fully evaluated and return the result value of type `Float`:



Will return `Float`

- So the type of function "average" is `Float -> (Float -> Float)`

More on Functions

- Function application happens one argument at a time (a.k.a. "*currying*")

```
average :: Float -> Float -> Float
average a b = (a + b) / 2.0
a = average 3.0 4.0
```

  - So `average 3.0 4.0` is equivalent to `(average 3.0) 4.0`
  - `(average 3.0)` takes `4.0` returns `3.5`, so `(average 3.0)` has type `Float -> Float`

- Functions of multiple arguments that can be applied to their arguments one at a time are called *curried* functions
  - after the mathematician *Haskell B. Curry* — the Haskell language was named after him as well).
  - In Haskell, all functions of multiple arguments are curried by default.

Infix and Prefix Application of Functions

Infix notation

```
  3 + 4
2.5 * 4.0
```

Prefix notation

```
(+) 3 4
(*) 2.5 4.0
```

Infix notation

```
3.0 `average` 4.0
    3 `add` 4
```

Prefix notation

```
average 3.0 4.0
add 3 4
```

Note that the function name is in *backquotes*

- Binary functions in backquotes are, by default, left associative, i.e.,

```
6.5 `average` 7.2 `average` 6.2
```

is equivalent to:

```
(6.5 `average` 7.2) `average` 6.2
```

Parameterized types

- Types can have parameters sort of the way functions do:

```
myNum :: Num p => p
myNum = 3
```

```
pi :: Floating a => a
pi = 3.141592653589793
```

```
double :: Num a => a -> a
double x = x * 2
```

```
equal :: Eq a => a -> a -> Bool
equal x y = (x == y)
```

```
bigger :: Ord a => a -> a -> Bool
bigger x y = (x > y)
```

- Num, Eq, Ord are all *type classes*; p and a are *type variables*.

```
concat x y = x ++ y
```

```
first a b = a
```

Parameterized types

- Here is an overview of some frequently used type classes, and some overloaded operations on these type classes.

- Typeclass **Show**
  - functions: show :: Show a => a -> String: convert the given value into a string.
  - member types: almost all predefined types, excluding function types.
- Typeclass **Eq**
  - functions: (==), (/=) :: Eq a => a -> a -> Bool: equality and inequality.
  - member types: almost all predefined types, excluding function types.
- Typeclass **Ord**
  - functions: (<), (>), (<=), (>=) :: Ord a => a -> a-> Bool: less than, greater than, less or equal, greater or equal
  - member types: almost all predefined types, excluding function types.
  - all types in Ord are already in Eq, so if you are using both == and < on a value, it is sufficient to require it to be in Ord.
- Typeclass **Num**
  - functions: (+), (-), (*) :: Num a => a -> a -> a: arithmetic operations.
  - member types: Float, Double, Int, Integer
- Typeclass **Integral**
  - functions: div, mod :: Integral a => a -> a -> a: division.
  - member types: Int (fixed precision), Integer (arbitrary precision)
- Typeclass **Fractional**
  - functions: (/) :: Fractional a => a -> a -> a: division.
  - member types: Float, Double

Haskell Tuples

- A tuple combines multiple components into one compound value.
  - The values in a tuple can be of different types.
  - The values in a tuple has a specific order.

```
myTuple :: (Bool, Integer, String)
myTuple = (True, 1, "one")

nestedTuple :: (Bool, (Integer, String), Double)
nestedTuple = (True, (2,"two"), 2.0)
```

- Decomposing values of a pair (a 2-tuple):

```
fst (True, (2,"two")) -- returns the first element : True
snd (True, (2,"two")) -- returns the second element (2,"two")
```

Haskell Tuples – cont.

- Example functions taking tuple as argument:

```
swap :: (Integer, String) -> (String, Integer)
swap (x,y) = (y,x)

swap (2, "two")       -- will return ("two", 2)
swap ("2","two")    -- will give a type error (see the type signature)
```

- Example functions returning tuples:

```
strPair :: Integer -> (Integer, String)
strPair x = (x, show x)

strPair 5  -- will return (5,"5")
```

Haskell Lists

- Haskell lists can be of arbitrary size. They can have values of various types, but all elements must be the same type.

```
tenPrimes :: [Integer]
tenPrimes = [2, 3, 5, 7, 11, 13, 17, 19, 23, 27]
```

- We don't need to explicitly write every single element if our list elements are just a sequence of consecutive numbers — or any type whose values can be enumerated:

```
oneToTwenty :: [Integer]
oneToTwenty = [1..20]
```

```
-- return all positive odd numbers up to maxNumber
oddNumbers :: Integer -> [Integer]
oddNumbers maxNumber = [1,3..maxNumber]

oddNumbers 10 – will return [1, 3, 5, 7, 9]
```

- The difference between tuples and lists can be seen by comparing their types:

```
(1, 2, "green") :: (Integer, Integer, String)
```

```
[1, 2, 3, 4] :: [Integer]
```

Haskell Lists – cont.

- Haskell lists can be nested or may include composite values (tuples, functions, etc.) :

```
[[1,2,3],[1,2],[3,4],[]]
[(1,"one"),(2,"two"), (3,"three")]
```

```
> :type []
[] :: [a] -- Polymorphic type – a is a type variable
```

- The below won't work.

```
[[1,2,3],["1","2"],[3,4]]
[(1, 2,"one"),(2,"two")]
```

- A String is just a list of Char, so `['a','b','c'] == "abc"`

Haskell List Processing

- The `:` operator appends an item to the head of an already existing list.
  - ": " is pronounced "cons"
  - It takes a value and a list and returns a list where the value is added to the beginning of the list.
  - ":" is right-associative
- Examples:
  - `"blue" : []` ⇒ `["blue"]`
  - `"yellow":["red","green","blue"]` ⇒ `["yellow","red","green","blue"]`
  - `"yellow":"red":"green":"blue":[]` ⇒ `["yellow","red","green","blue"]`
  - `["red", "green", "blue"] : "yellow"` ⇒ *Error!*

- The cons-operator is another example of a *polymorphic* function, as it works on lists of any type.
  - The only restriction is that the element we are adding is of the same type as the

```
(:) :: a -> [a] -> [a]
```

Some basic list functions

- Appending two lists:

```
(++) :: [a] -> [a] -> [a]

[1,2,3] ++ [4,5,6,7] ⇒ [1,2,3,4,5,6,7]
["red","green","blue"] ++ ["yellow"] ⇒ ["red","green","blue“,"yellow"]
```

- Extract the element at a specific index position out of a list

```
(!!) :: [a] -> Int -> a
["zero","one","two","three","four","five","six"] !! 5 ⇒ "five"
"CptS355“ !!  4 ⇒ '3'
```

- Split a list into its first element and the rest

```
head :: [a] -> a
head [0, 1, 2, 3]   ⇒   0
head "mouse"      ⇒   'm'

tail :: [a] -> [a]
tail [0, 1, 2, 3]   ⇒   [1, 2, 3]
tail "mouse"      ⇒   "ouse"
```

Some basic list functions – cont.

- Length of a list:

```
length :: [a] -> Integer
length [0, 1, 2, 3]   ⇒   4
```

- Check if an item is contained in a list

```
elem :: Eq a => a -> [a] -> Bool
elem 3 [0, 1, 2, 3]   ⇒   True
elem 6 [1, 2, 3, 4]   ⇒   False
elem ‘t’ "CptS"       ⇒   True
```

- Add up or multiply the elements of a list

```
maximum :: Ord a => [a] -> a
minimum :: Ord a => [a] -> a
sum :: Num a => [a] -> a
product :: Num a => [a] -> a

maximum [0, 1, 2, 3]   ⇒   3
minimum [0, 1, 2, 3]   ⇒   0
sum [0, 1, 2, 3]   ⇒   6
product [1, 2, 3, 4]   ⇒   24
```

Conditionals : if/else

- Examples of *if/else* statements:

```haskell
max' :: Ord a => a -> a -> a
max' x y = if x >= y then x else y
```

```haskell
signum' :: (Ord a, Num a) => a -> Integer
signum' x = if x < 0 then -1 else if x == 0 then 0 else 1
```

Conditionals: *guards*

- Cascading conditional expressions are difficult to read; *guards* provide an easier syntax:

```
signum :: (Ord a, Num a) => a -> Int
signum x = if x < 0 then -1 else if x == 0 then 0 else 1
```

```
signum :: (Ord a, Num a) => a -> Int
signum x | x < 0 = -1
         | x == 0 = 0
         | x > 0 = 1
```

- The *guards* are checked in the order they are listed
- Usually, the last *guard* should catch all the cases not covered before.
  - We use the special guard `otherwise`, which always evaluates to True

```
signum :: (Ord a, Num a) => a -> Int
signum x | x < 0 = -1
         | x == 0 = 0
         | otherwise = 1
```

Patterns

- In Haskell we can access components of lists (or tuples) directly by using patterns. The context in which the identifier appears tells us the part of the structure it references.
- Examples:

```
x::(Integer,Integer)
x = (1,2)
(h,t) = x            --h will be assigned to 1 and t will be assigned to 2

myList::[Integer]
myList = [1,2,3]
[v1,v2,v3] = myList  --v1, v2, and v3 will be assigned to 1,2, and 3, respectively.
[1,v4,3] = myList    --v4 will be assigned to 2

[1,ys] = myList      -- This won't work. Why?
(x:ys) = myList      --x will be assigned to 1 and ys will be assigned to [2,3]
```

Patterns – cont.

- An underscore (_) may be used as a “wildcard” or “don’t care” symbol. It matches part of a structure without defining a new binding.

```
y:_  = ['c','a','t'] -- y will be assigned to 'c’

_:xs = ['c','a','t'] -- xs will be assigned to ['a','t'] or 'at'
```

- Patterns can be nested too:

```
x :: ((Integer,Double),Integer)
nestedTuple = ((1,3.0),5)

((_,y),_) = nestedTuple -- y will be assigned to 3.0
```

How can I extract the grade for the first class (i.e., “CptS355”) from the below list?

```
courses  = [("CptS355",3), ("CptS322",4), ("CptS360",2), ("CptS321",3) ]
```

Pattern matching in functions : *cons*

- We use pattern matching to decompose lists into their first element and the rest of the list
- Head of a list (*head* in *Prelude*):

```
head':: [a] -> a
head' (x:xs) = x
```

Partial function

This parenthesis is necessary

```
head' :: [a] -> a
head' [] = error "head: empty list"
head' (x:_) = x
```

- Tail of a list (*tail* in *Prelude*):

```
tail' :: [a] -> [a]
tail' (x:xs) = xs
```

Partial function

```
tail' :: [a] -> [a]
tail' [] = error "tail: empty list"
tail' (_:xs) = xs
```

- If your functions don't cover all possible cases, you may get a run-time “Match” exception.

```
partial :: (Ord a, Num a, Num p) => a -> p
partial x | x < 0 = -1
          | x > 0 = 1
```

Pattern matching in functions : *cons*

- Check if a list is empty (*null* in *Prelude*):

```
isNull :: Eq a => [a] -> Bool
isNull x = if x==[] then True else False
```

vs

```
isNull :: [a] -> Bool
isNull [] = True
isNull (x:xs) = False
```

- What is the difference between the above *isNull* definitions? (Note the difference in type)

Haskell is a *pure* functional language

- Unlike variables in imperative languages, Haskell bindings are:
  - *immutable* - can only bind a symbol once in a given scope (We still call bound symbols "variables" though)

```
x = 5
x = 6          -- error, cannot re-bind x
```

  - *order-independent* - order of bindings in source code does not matter
  - *lazy* - definitions of symbols are evaluated only when needed

```
safeDiv x y =
    let q = div x y              -- safe as q never evaluated if y == 0
     in if y == 0 then 0 else q
main = print (safeDiv 1 0)     -- prints 0
```

Role of variable

- In Haskell variables are immutable
- A variable just maps to a value that it is bound to.
- There is no "assignment statement" in Haskell for changing what a variable maps to

- In imperative languages, a variable is a *location* that can hold a value, and which can be changed through an assignment.

  x = x + 1;

How to program without mutable variables?

- In C, we use mutable variables to create loops:

```
long factorial (int n)
{
   long result = 1;
   while (n > 1)
         result *= n--;
   return result;
}
```

- In Haskell, can use recursion to "*re-bind*" argument symbols in new scope

```
factorial n = if n > 1
              then n * factorial (n-1)
              else 1
```

  – Recursion often fills a similar need to mutable variables
  – But the above Haskell factorial is inferior to the C one--why?

Recursive Functions in Haskell

- Add first n natural numbers:

```haskell
natSum :: (Num a, Ord a) => a -> a
natSum n | n == 0 = 0
         | n > 0 = n + natSum (n - 1)
         | otherwise = error "natSum: Input value is negative!"
```

- Add first n natural numbers (alternative):

```haskell
natSum :: (Num a, Ord a) => a -> a
natSum 0 = 0
natSum n | n > 0 = n + natSum (n - 1)
         | otherwise = error "natSum: Input value is negative!"
```

Recursive Functions in Haskell

- Length of a list:

```
length' :: [a] -> Int
length' [] = 0
length' (x:xs) = 1 + (length xs)

n = length [1,2,2,3]
```

Recursive Functions in Haskell

- Last element of a list:

```
last' :: [a] -> a
last' []     = error "last': Input list is empty."
last' [x]    = x
last' (x:xs) = (last xs)

last' [1,2,3,4]
```

- Caution!
  - Patterns are checked in order and order matters. The first matching pattern is evaluated. In the function , if you have specified the last pattern before the middle, it would not work.
  - If you use the cons (:) patterns, you need to use parenthesis around it.
    - For example: `last' x:xs = (last xs)` will give an error.

Recursive Functions in Haskell

- n^{th} element: Return the nth element in a list. Assume (n>0)

```
nthElement [] n = error "nthElement': The input list is too short."
nthElement (x:xs) 1 = x
nthElement (x:xs) n = (nthElement xs (n-1))
```

How recursion works?

```
copyList [] = []
copyList (x:xs)  = x : (copyList xs)
```

copyList[]	[]
copyList[3]	3:(result of recursive call)
copyList[2,3]	2:(result of recursive call)
copyList[1,2,3]	1:(result of recursive call)

Recursive Functions in Haskell

- Mapping: applying an operation to every element of a list:

– Compute the square of each element in the list:

  `allSquares [x1, x2,… , xn] = [x1 * x1, x2 * x2,… , xn * xn]`

```
allSquares :: Num a => [a] -> [a]
allSquares [] = []
allSquares (x : xs) = (x * x) : (allSquares xs)
```

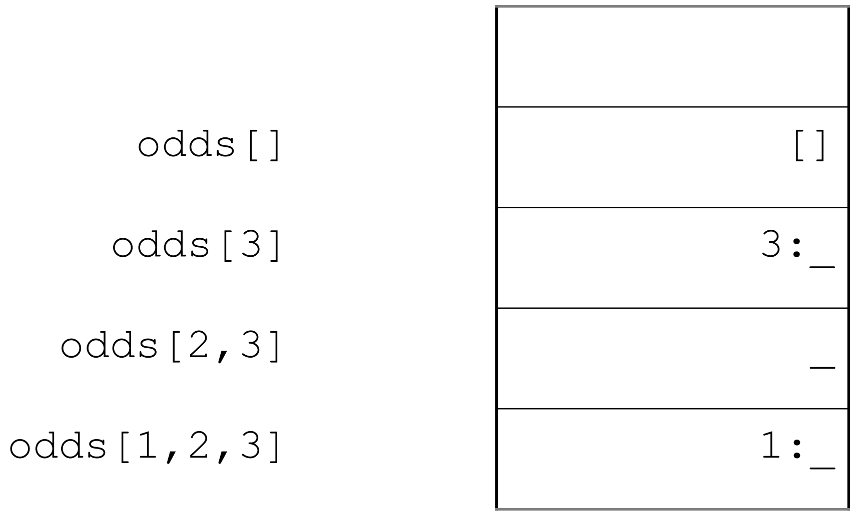
– Make all letters in a string uppercase:

  `strToUpper “Cpts355” = “CPTS355”`

```
import Data.Char

strToUpper :: String -> String
strToUpper [] = []
strToUpper (chr : xs) = (toUpper chr) : (strToUpper xs)
```

How recursion works?

```
odds [] = []
odds (x:xs)  | ((x `mod` 2) == 1) = x: (odds xs)
             |  otherwise = (odds xs)
```



"_" **represents** `result of recursive call`

Recursive Functions in Haskell

- Filtering: removing elements from a list

  – Filter out the values smaller than a given value.

```
filterSmaller [] v = []
filterSmaller (x:xs) v | (x >= v) = x:(filterSmaller xs v)
                       | otherwise = (filterSmaller xs v)
```

  – Extract digits:

    `extractDigits “CptS355” = “355”`

```
import Data.Char

extractDigits :: String -> String
extractDigits [] = []
extractDigits (chr : xs) | isDigit chr = chr : (extractDigits xs)
                         | otherwise = extractDigits xs
```

How recursion works?

```
addup []     = 0
addup (x:xs) = x + (addup xs)
```

addup[]	0
addup[3]	3 + _
addup[2,3]	2 + _
addup[1,2,3]	1 + _

"_" represents `result of recursive call`

Recursive Functions in Haskell

- Reductions: combining the elements of a list.

  – Add-up all the elements of a list:

```
addup :: Num p => [p] -> p
addup []      = 0
addup (x:xs) = x + (addup xs)

sum1 = addup [1,2,3,4,5]             -- evaluates to 15
sum2 = addup [1.0,2.0,3.0,4.0,5.0] -- evaluates to 15.0
```

  – Multiply the elements in a list:

```
mul :: Num p => [p] -> p
mul []      = 1
mul (x:xs) = x * (mul xs)

p1 = mul [1,2,3,4,5]     -- evaluates to 120
```

Recursive Functions in Haskell

- Reductions: combining the elements of a list.

  – List is a recursive structure: all lists are constructed from `[]` with `cons(:)` operator.

  ```
  [x1, x2,… , xn] = (x1 : (x2 : ⋯ : (xn : [])⋯)
  ```

  – When we combine the elements of a list, we simple replace the cons `cons(:)` with another operator. For example:

  ```
  mul (x1 : (x2 : ⋯ : (xn : [])) = (x1 * (x2 * ⋯ * (xn * 1))

  mul (3:(5:(6:[])))  ⇒  3 * mul (5:(6:[]))
                      ⇒  3 * (5 * mul (6:[]))
                      ⇒  3 * (5 * (6 * mul []))
                      ⇒  3 * (5 * (6 * 1))
                      ⇒  90
  ```

  – If we generalize this:

  ```
  op (x1 : (x2 : ⋯ : (xn : [])) = (x1 `op` (x2 `op` ⋯ `op` (xn `op` base))
  ```

Recursive Functions in Haskell

- Reductions: combining the elements of a list.
  - Minimum/maximum value in a list:

```
minList :: [Int] -> Int
minList []     = maxBound
minList (x:xs) = x `min` (minList xs)

m = minList [2,6,1,4,3]     -- evaluates to 1
```

  - `maxBound` is *Prelude* constant for the maximum `Int` value.

Recursive Functions in Haskell

- Reverse:
  - We can express reverse as a reduction as well

```
reverse [x1, x2,… , xn] ⇒ [xn,… , x2, x1]
reverse (x1 : xs)  ⇒  (reverse xs) ++ [x1])
                   ⇒  ((([xn] ++ ..) ++ [x2]) ++ [x1])
```

Not actual Haskell code

  - Consider the function snoc:

```
snoc x xs = xs ++ [x]
```

```
reverse (x1 : (x2 : ⋯ : (xn : [])))  ⇒
      (x1 `snoc` (x2 `snoc` ⋯ `snoc` (xn `snoc` [])))
```

  - Now we can implement `reverse` using `snoc`: (we call it `reverse'` since `reverse` is already defined in Prelude)

```
reverse' :: [a] -> [a]
reverse' [] = []
reverse' (x:xs) = x `snoc` (reverse' xs)
```

OR

```
reverse' :: [a] -> [a]
reverse' [] = []
reverse' (x:xs) = (reverse' xs) ++ [x]
```

Recursive Functions in Haskell

- Reverse:
  - What is the time complexity of `reverse'`?

```
snoc x xs = xs ++ [x]
```

```
reverse' :: [a] -> [a]
reverse' [] = []
reverse' (x:xs) = x `snoc` (reverse' xs)
```

  - Later we will give a more efficient definition of `reverse`.

Recursive Functions in Haskell

- **Append**:
  - We append the first list to the second.

`append [3] [4,5]`	`3:(append [] [4,5])`
`append [2,3] [4,5]`	`2:(append [3] [4,5])`
`append [1,2,3] [4,5]`	`1:(append [2,3] [4,5])`

Recursive Functions in Haskell

- **Append**:
  - We append the first list to the second.
  - Do we need to capture the case where the second list is empty?

```
append :: [a] -> [a] -> [a]
append [] list = list
append (x:xs) list = x:(append xs list)
```

  - If patterns overlap and if some patterns are redundant, you may get a warning from the compiler. For example:

```
append :: [a] -> [a] -> [a]
append [] list = list
append (x:xs) list = x:(append xs list)
append list [] = list
```

redundant pattern

let expression and *where* clause

- All the assignment statements in a Haskell module are "top-level" assignments.
- However, often one needs to make assignments inside other code in order to avoid repeated computation of values or simply to make the implementation clearer.
- Haskell offers two alternatives:
  - ***let*** … ***in*** statement , and
  - ***where*** clause

let expression

- Syntax:
  - Each ***bi*** is any *binding* and ***e*** is any *expression*

```
let  b1 = …
     b2 = …
     …
     bn = …
  in  e
```

- Type-checking: Type-check each ***bi*** and ***e*** in a static environment that includes the previous bindings.
  - Type of whole let-expression is the type of ***e***.
- Evaluation: Evaluate each ***bi*** and ***e*** in a dynamic environment that includes the previous bindings.
  - Result of whole let-expression is result of evaluating ***e***.

➢ A let-expression is ***just an expression***, so we can use it ***anywhere*** an expression can go

let expression and *where* clause

- Consider the `reverse` function we defined before. **`reverse`** calls **`snoc`**:

```
snoc x xs = xs ++ [x]

reverse' :: [a] -> [a]
reverse' [] = []
reverse' (x:xs) = x `snoc` (reverse' xs)
```

- **`let ... in`** introduces a variable/function before it can be used;

```
reverse' :: [a] -> [a]
reverse' []     = []
reverse' (x:xs) = let
                   snoc x xs = xs ++ [x]
                  in x `snoc` (reverse' xs)
```

- whereas **where** assigns a value to a variable after it has been used.

```
reverse' :: [a] -> [a]
reverse' [] = []
reverse' (x:xs) = x `snoc` (reverse' xs)
                  where snoc x xs = xs ++ [x]
```

let expression and *where* clause

- So far we have used both constructs interchangeably. However, there is one significant difference that is important to us.
  - A variable bound with ***let*** has a so called *scope*. That is, it is only “visible” after the ***in*** in the context of a computational block.
  - A variable bound with ***where*** is “visible” anywhere in the body of a function preceding the declaration.
  - Example:
    - **let ... in**

```
basic n = if even n then
                let two = 2
                in two * n
          else
                two * two * n
```

Will yield an out of scope error for **two**.

- **where**

```
basic n = if even n then two * n
          else two * two * n
               where two = 2
```

How recursion works?

```
copyList2 []  buf = reverse buf
copyList2 (x:xs) buf =  copyList2 xs (x:buf)
```

copyList2 [1,2,3] []	buf =[]
copyList2 [2,3] [1]	buf =[1]
copyList2 [3] [2,1]	buf =[2,1]
copyList2 [] [3,2,1]	buf =[3,2,1]