Homework #8

37.

a.

x	25	40	65
$P(x)$	.2	.5	.3

Sampling distribution

$\bar{x}$	25	32.5	40	45	52.5	65
$P\bar{x}$	.04	.1+.1 = .2	.25	.06+.06 = .12	.15+.15 = .30	.09

$E(\bar{x}) = \sum \bar{x} P(\bar{x}) =$

$= \begin{Bmatrix} 25(.04) + 32.5(.2) + 40(.25) + 45(.12) \\ + 52.5(.3) + 65(.09) \end{Bmatrix}$

$= 1 + 6.5 + 10 + 5.4 + 15.75 + 5.85$

$= 44.5$

$\mu = \sum x P(X = x)$

$= 25 \times .2 + 40 \times .5 + 65 \times .3$

$= \underline{44.5}$

x_1	x_2	$p(x_1, x_2)$	$\bar{x} = \frac{x_1 + x_2}{2}$	$s^2 = (x_1 - \bar{x})^2 + (x_2 - \bar{x})^2$
25	25	$.2 \times .2 = .04$	$(25+25)/2 = 25$	$(25-25)^2 + (25-25)^2 = 0$
25	40	$.2 \times .5 = .1$	$(25+40)/2 = 32.5$	$(25-32.5)^2 + (40-32.5)^2 = 112.5$
25	65	$.2 \times .3 = .06$	45	800
40	25	$.5 \times .2 = .1$	32.5	112.5
40	40	$0.5 \times .5 = .25$	40	0
40	65	$.5 \times .3 = .15$	52.5	312.5
65	25	$.3 \times .2 = .06$	45	800
65	40	$.3 \times .5 = .15$	52.5	312.5
65	65	$.3 \times .3 = .09$	65	0

B.

s^2	0	112.5	312.5	800
Ps	0.38 0.04 + .25 + .09	0.20 .10 + .1	.3 .15 + .15	.12 .06 + .06

$E(s^2) = \sum s^2 p(s^2)$

$= 0(.38) + 112.5(.2) + 312.5(.3) + 800(.12)$

$= \underline{212.25}$

$\sigma^2 = \sum x^2 P(X=x) - \{\sum x P(X=x)\}^2$

$= \{(25)^2 \times .2 + (40)^2 \times 0.5 + (65)^2 \times .3\} - (44.5)^2$

$= 2192.5 - (44.5)^2$

$= \underline{212.25}$

So, $E(s^2) = \sigma^2$

Homework #8

38. A

a.

$$P(T_0=0)=P(X_1=0)\,P(X_2=0)$$
$$=.2\times.2$$
$$=\underline{.04}$$

$$P(T_0=1)=P(X_1=0).P(X_2=1)+P(X_1=1).P(X_2=0)$$
$$=.2\times.5+.5\times.2$$
$$=.1+.1=\underline{.2}$$

$$P(T_0=2)=P(X_1=1).P(X_2=1)+P(X_1=0).P(X_2=2)+P(X_1=2).P(X_2=0)$$
$$=.5\times.5+.2\times.3+.3\times.2$$
$$=.25+.06+.06$$
$$=\underline{.37}$$

$$P(T_0=3)=P(X_1=2).P(X_2=1)+P(X_1=1).P(X_2=2)$$
$$=.3\times.5+.5\times.3$$
$$=.15+.15=\underline{.3}$$

$$P(T_0=4)=P(X_1=2).P(X_2=2)$$
$$=.3\times.3$$
$$=\underline{.09}$$

T_0	0	1	2	3	4
$P(T_0)$	.04	.2	.37	.3	.09

B

b.

$$\mu_{T_0}=\sum T_0\cdot P(T_0)$$
$$=(0\times.04)+(1\times.2)+(2\times.37)+(3\times.3)+(4\times.09)$$
$$=.2+.74+.9+.36=\underline{2.2}$$
$$\mu_{T_0}=2\mu$$

C

c. $$\sigma^2_{T_0}=\sum(t_0-\mu_{T_0})^2\,p(T_0)$$
$$=\left\{\begin{array}{l}(0-2.2)^2(0.04)+(1-2.2)^2(0.2)+(2-2.2)(.37)\\+(3-2.2)^2(.3)+(4-2.2)^2(0.09\end{array}\right\}$$
$$=0.1936+0.2880+0.0148+0.1920+0.2916$$
$$=\underline{0.98}$$

The variance of T_0 is twice the population variance $\sigma^2=.49$

$$\sigma^2_{T_0}=2\sigma^2$$

Homework #8

38.

d. $T_0 = X_1 + X_2 + X_3 + X_4$

$E(T_0) = E(X_1 + X_2 + X_3 + X_4)$

$= E(X_1) + E(X_2) + E(X_3) + E(X_4)$

$= 1.1 + 1.1 + 1.1 + 1.1$

$= 4.4$

$V(T_0) = V(X_1 + X_2 + X_3 + X_4)$

$= V(X_1) + V(X_2) + V(X_3) + V(X_4)$

$= .49 + .49 + .49 + .49$

$= 1.96$

* The mean and variance of the new random variable T_0 are 4.4 and 1.96 *

e. $p(T_0 = 8)$ and $p(T_0 \geq 7)$

$P(T_0 = 8) = p(T_0 = 4) \times p(T_0 = 4)$

$= .09 \times .09$

$= .0081$

$P(T_0 \geq 7) = p(T_0 = 7) + p(T_0 = 8)$

$p(T_0 = 7) = p(T_0 = 3) \times p(T_0 = 4) + p(T_0 = 4) \times p(T_0 = 3)$

$= .3 \times .09 + .09 \times .3$

$= .027 + .027 = .054$

$P(T_0 \geq 7) = p(T_0 = 7) + p(T_0 = 8)$

$= .054 + .0081 = 0.0621$

Homework #8

52. Mean:

$$E(Y) = E\left(\sum_{i=1}^{4} x_i\right) = \sum_{i=1}^{4} E(x_i) = \sum_{i=1}^{4} 10 = 4 \times 10 = 40$$

Standard deviation:

$$SD(Y) = \sqrt{Var\left(\sum_{i=1}^{4}\right.} = \sqrt{\sum_{i=1}^{4} var(x_i)} = \sqrt{\sum_{i=1}^{4} 1} = \sqrt{4} = 2$$

* The linear combination of the normal distributed random variable has normal distribution, Y is also normal with mean value 40 hours and standard deviation 2 hours *

$$P\{Y > y_{95}\} = 5\%$$

$$P\{Y \le y_{95}\} = 95\%$$

$$P\left\{\frac{Y - E(Y)}{SD(Y)} \le \frac{y_{95} - E(Y)}{SD(Y)}\right\} = .95$$

$$P\left\{Z \le \frac{y_{95} - 40}{2}\right\} = .95$$

$$P\{Z \le 1.645\} = .95$$

$$\frac{y_{95} - 40}{2} = 1.645$$

$$y_{95} = 40 + (1.645)(2) = 40 + 3.29 = 43.29 \text{ hours}$$

53.

a. $E(\bar{x}) = \mu = 50$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{1.2}{\sqrt{9}} = .4$$

$$P(\bar{x} \ge 51) = P\left(\frac{\bar{x} - \mu}{\sigma_x} \ge \frac{51 - 50}{.4}\right) = P(z \ge 2.5) = 1 - P(z \ge 2.5) = 1 - 0.9938 = .0062$$

b. $E(\bar{x}) = \mu = 50$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{1.2}{\sqrt{40}} = 0.1897$$

$$P(\bar{x} \ge 51) = P\left(\frac{\bar{x} - \mu}{\sigma_{\bar{x}}} \ge \frac{51 - 50}{0.1897}\right) = P(z \ge 5.2715) = 1 - P(z \le 5.2715) = 1 - 1 = 0$$

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54. $\mu = 2.65 \quad \sigma = .85$

a.

$$\sigma_{\bar{x}} = \frac{.85}{\sqrt{25}} = \frac{0.85}{5} = .17$$

$$P(\bar{x} \le 3) = P\left(z \le \frac{3 - 2.65}{.17}\right) = P(z \le 2.06) = \Phi(2.06) = .9803$$

$$P(2.65 \le \bar{x} \le 3) = P(\bar{x} \le 3) - P(\bar{x} \le 2.65)$$

$$= P\left(z \le \frac{3 - 2.65}{0.17}\right) - P\left(z \le \frac{2.65 - 2.65}{0.17}\right)$$

$$= \Phi(2.06) - \Phi(0)$$

$$= 0.9803 - .5 = 0.4803$$

b.

$$P(\bar{x} \le 3.0) = P\left(z \le \frac{3 - 2.65}{\frac{0.85}{\sqrt{n}}}\right) = .99$$

$$\frac{3 - 2.65}{\frac{0.85}{\sqrt{n}}} = 2.33$$

$$\frac{0.85}{\sqrt{n}} = 0.1502$$

$$\sqrt{n} = 5.6585$$

$$n = 32.03$$

$n = 33$ * Required sample size is 33 *

Homework #8

59.

a. $E(Y) = E(x_1 + x_2 + x_3)$

$= E(x_1) + E(x_2) + E(x_3)$

$= \mu_1 + \mu_2 + \mu_3$

$= 60 + 60 + 60 = \underline{180}$

$V(Y) = V(x_1 + x_2 + x_3)$

$= V(x_1) + V(x_2) + V(x_3)$

$= \sigma_1^2 + \sigma_2^2 + \sigma_3^2$

$= 15 + 15 + 15$

$= 45$

$\sigma_Y = \sqrt{V(Y)}$

$= \sqrt{45} = \underline{6.7082}$

$P(Y \le 200) = P\left(\frac{Y - \mu_Y}{\sigma_Y} \le \frac{200 - 180}{6.7082}\right)$

$= P\left(Z \le \frac{20}{6.7082}\right)$

$= P(Z \le 2.98) = 0.9986$

$P(x_1 + x_2 + x_3 \le 200) = \underline{.9986}$

$P(150 \le Y \le 200) = P\left(\frac{150 - \mu_Y}{\sigma_Y} \le \frac{Y - \mu_Y}{\sigma_Y} \le \frac{200 - \mu}{\sigma}\right)$

$= P\left(\frac{150 - 180}{6.7082} \le Z \le \frac{200 - 180}{6.7082}\right)$

$= P(-4.47 \le Z \le 2.98)$

$= \Phi(2.98) - \Phi(-4.47)$

$= .9986 - .0000$

$= \underline{0.9986}$

$\underline{P(150 \le x_1 + x_2 + x_3 \le 200) = 0.9986}$

b. $\bar{x} = \frac{x_1 + x_2 + x_3}{3}$

$E(\bar{x}) = \frac{E(x_1) + E(x_2) + E(x_3)}{3}$

$= \frac{\mu_1 + \mu_2 + \mu_3}{3}$

$= \frac{60 + 60 + 60}{3}$

$= 60$

$\bar{x}$ is $\mu_{\bar{x}} = \underline{60}$

$V(\bar{x}) = V\left(\frac{x_1 + x_2 + x_3}{3}\right)$

$= \frac{1}{9}[V(x_1) + V(x_2) + V(x_3)]$

$= \frac{1}{9}(15 + 15 + 15) = 5$

$\sigma_x = \sqrt{V(\bar{x})} = \sqrt{5} = 2.2361$

$P(\bar{x} \ge 55) = 1 - P(\bar{x} < 55)$

$= 1 - P\left(\frac{\bar{x} - \mu_{\bar{x}}}{\sigma_x} < \frac{55 - 60}{2.2361}\right)$

$= 1 - P(Z < -2.24)$

$= 1 - 0.0125 = \underline{.9875}$

$\underline{P(\bar{x} \ge 55) \text{ is } .9875}$

$P(58 \le \bar{x} \le 62) =$

$P\left(\frac{58 - 60}{2.2361} \le \frac{\bar{x} - \mu_{\bar{x}}}{\sigma_{\bar{x}}} \le \frac{62 - 60}{2.2361}\right)$

$= P(-0.89 \le Z < 0.89)$

$= \Phi(.89) - \Phi(-.89)$

$= .8133 - .1867$

$= \underline{.6266}$

$\underline{P(58 \le \bar{x} \le 62) \text{ is } .6266}$

Homework #8

c. $E(T) = E(x_1 - 0.5x_2 - 0.5x_3)$

$= E(x) - 0.5E(x_2) - 0.5E(x_3)$

$= \mu_1 - 0.5\mu_2 - 0.5\mu_3$

$= 60 - 0.5\times 60 - 0.5\times 60$

$= 0$

$V(T) = V(x_1 - 0.5x_2 - 0.5x_3)$

$= V(x_1) + .25V(x_2) + .25V(x_3)$

$= \sigma_1^2 + .25\sigma_2^2 + .25\sigma_3^2$

$= 15 + .25(15) + 0.25(15)$

$= 22.5$

$\sigma_T = \sqrt{V(T)} = \sqrt{22.5} = 4.7434$

$P(-10 \le T \le 5) = P\left(\frac{-10-0}{4.7434} \le \frac{T-\mu_T}{\sigma_T} \le \frac{5-0}{4.7434}\right)$

$= P(-2.11 \le z \le 1.05)$

$= \Phi(1.05) - \Phi(-2.11)$

$= .8531 - 0.0174$

$= 0.8357$

$P(-10 \le (x_1 - 0.5x_2 - 0.5x_3) \le 5)$ is $\underline{0.8357}$

d. $\mu_1 = 40, \mu_2 = 50, \mu_3 = 60, \sigma_1^2 = 10, \sigma_2^2 = 12, \sigma_3^2 = 14$

$E(T_0) = E(x_1 + x_2 + x_3)$

$= E(x_1) + E(x_2) + E(x_3)$

$= \mu_1 + \mu_2 + \mu_3 = 40 + 50 + 60 = \underline{150}$

$V(T_0) = V(x_1 + x_2 + x_3)$

$= V(x_1) + V(x_2) + V(x_3)$

$= \sigma_1^2 + \sigma_2^2 + \sigma_3^2$

$= 10 + 12 + 14 = \underline{36}$

$\sigma_{T_0} = \sqrt{V(T_0)} = \sqrt{36} = 6$

$P(T_0 \le 160) = P\left(\frac{T_0 - \mu_{T_0}}{\sigma_{T_0}} \le \frac{160-150}{6}\right)$

$= P\left(z \le \frac{10}{6}\right)$

$= P(z \le 1.67)$

$= \underline{.9525}$

$P(x_1 + x_2 + x_3 \le 160) = .9525$

$P(x_1 + x_2 \ge 2x_3) =$

$P(x_1 + x_2 - 2x_3 \ge 0)$

$= P(k \ge 0)$

$= 1 - P(k < 0)$

$= 1 - P\left(\frac{k-\mu_k}{\sigma_k} < \frac{0-(-30)}{8.8318}\right)$

$= 1 - P(z < 3.39)$

$= 1 - 0.9997$

$= \underline{.0003}$

$\underline{P(x_1 + x_2 \ge 2x_3) \text{ is } .0003}$

$E(k) = \mu_1 + \mu_2 - 2\mu_3$

$= 40 + 50 - 2\times 60$

$= 90 - 120 = -30$

$V(k) = V(x_1 + x_2 - 2x_3)$

$= V(x_1) + V(x_2) + V(-2x_3)$

$= V(x_1) + V(x_2) + (-2)^2 V(x_3)$

$= 10 + 12 + (4\times 14)$

$= \underline{78}$