## Section 14.3 – Motion in Space

• Position: 
$$r(t) = \langle x, y, z \rangle$$

• Instantaneous Velocity: 
$$v(t) = r'(t) = \langle x'(t), y'(t), z'(t) \rangle$$

• Speed: 
$$|v(t)| = \sqrt{x'(t)^2 + y'(t)^2 + z'(t)^2}$$

• Acceleration: 
$$a(t) = v'(t) = r''(t)$$

• Find the velocity and speed of the object. Then find the acceleration of the object.

$$r(t) = \langle 3 + t, t^2, e^{-t} \rangle$$
 for  $t \ge 0$ 

• Find the velocity and speed of the object. Then find the acceleration of the object.

 $r(t) = \langle 3 \sin t, 5 \cos t, 4 \sin t \rangle$  for  $0 \le t \le 2\pi$ 

- Uniform straight-line motion:  $r(t) = \langle x_0 + at, y_0 + bt, z_0 + ct \rangle$
- <u>Circular motion</u>:  $r(t) = \langle A \cos t, A \sin t \rangle$  for  $0 \le t \le 2\pi$ , r and a are parallel but point in opposite directions, and  $r \cdot v = a \cdot v = 0$  which means position and acceleration are both orthogonal to the velocity.
- Given an acceleration vector, initial velocity  $\langle u_0, v_0 \rangle$ , and initial position  $\langle x_0, y_0 \rangle$ , find the velocity and position vectors for  $t \ge 0$ .

$$a(t) = \langle 1, 2 \rangle, \ \langle u_0, v_0 \rangle = \langle 1, 1 \rangle, \langle x_0, y_0 \rangle = \langle 2, 3 \rangle$$

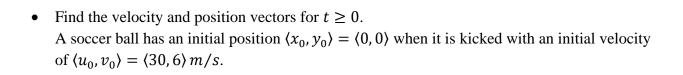
## • Two-Dimensional Motion in a Gravitational Field:

Consider an object moving in a plane with a horizontal x-axis and a vertical y-axis, subject only to the force of gravity,  $\mathbf{a}(t) = \langle 0, -g \rangle$ . Given the initial velocity  $\mathbf{v}(0) = \langle u_0, v_0 \rangle$  and the initial position  $\mathbf{r}(0) = \langle x_0, y_0 \rangle$ , the velocity of the object, for  $t \geq 0$ , is

$$v(t) = \langle x'(t), y'(t) \rangle = \langle u_0, -gt + v_0 \rangle$$

and the position is

$$r(t) = \langle x(t), y(t) \rangle = \langle u_0 t + x_0, -\frac{1}{2}gt^2 + v_0 t + y_0 \rangle$$
  
 $g \approx 9.8 \frac{m}{S^2} \text{ or } 32 \frac{ft}{S^2}$ 



• Determine the time of flight and range of the object.

• Determine the maximum height of the object.

- <u>Three-dimensional motion</u>:
- Find the velocity and position vectors for  $t \ge 0$ .

$$\boldsymbol{a}(t) = \langle 1, t, 4t \rangle, \langle u_0, v_0, w_0 \rangle = \langle 20, 0, 0 \rangle, \langle x_0, y_0, z_0 \rangle = \langle 0, 0, 0 \rangle$$

- $a(t) = \langle 0, 0, -g \rangle$ . We include any crosswinds, spins, or slices in other force components.
- For the following problems:
  - Find the velocity and position vectors for  $t \ge 0$ .
  - o Determine the time of flight and range of the object.
  - o Determine the maximum height of the object.
- A bullet is fired from a rifle 1 m above the ground in a northeast direction. The initial velocity of the bullet is (200, 200, 0) m/s.

•	A baseball is hit 3 feet above home plate with an initial velocity of $(60, 80, 80)$ ft/s. The spin on the baseball produces a horizontal acceleration of the ball of $10$ ft/ $s^2$ in the eastward direction.