Section 15.3 – Partial Derivatives

- Notation:
  If $z = f(x, y)$, we write
  $$f_x(x,y) = f_x = \frac{\partial f}{\partial x} = \frac{\partial}{\partial x} f(x,y) = \frac{\partial z}{\partial x} = D_x f$$
  $$f_y(x,y) = f_y = \frac{\partial f}{\partial y} = \frac{\partial}{\partial y} f(x,y) = \frac{\partial z}{\partial y} = D_y f$$
- Rule for finding partial derivatives:
  1. To find f_x regard y as a constant and differentiate $f(x, y)$ with respect to x.
  2. To find f_y regard x as a constant and differentiate $f(x, y)$ with respect to y.

- **Example:**
  Find the first partial derivative of the following:
  $$f(x,y) = x^6 y^3 - 9x^2 y^2 - x + y^2$$

- **Example:**
  Find the first partial derivative of the following:
  $$F(p,q) = \sqrt{p^2 + pq + q^2}$$

- **Example:**
  Find the first partial derivative of the following:
  $$f(x, y, z) = x^2 \sin(yz) + e^{xy}$$

- **Example:**
  Find the first partial derivative of the following:
  $$F(w, x, y, z) = w\sqrt{x + 2y + 3z}$$

- Second Partial Derivatives:
- Notation:

$$(f_x)_x = f_{xx} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial^2 f}{\partial x^2}$$

$$(f_x)_y = f_{xy} = \frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial^2 f}{\partial y \partial x}$$

etc…

- Equality of Mixed Partial Derivatives:
  Assume that f is defined on an open set D of $\mathbb{R}^2$, and f_{xy} and f_{yx} are continuous throughout D. Then $f_{xy} = f_{yx}$ at all points of D.
- **Example:**
  Find all the second partial derivatives:

$$f(x, y) = \sqrt{x^2 + y^2}$$

- Compute the first partial derivatives of the following function:

$$f(x, y) = \ln(1 + e^{-xy})$$

- Conditions for Differentiability:

  Suppose the function f has partial derivatives f_x and f_y defined on an open set containing (a, b), with f_x and f_y continuous at (a, b). Then f is differentiable at (a, b).

- Differentiability Implies Continuity:

  If a function f is differentiable at (a, b), then it is continuous at (a, b).