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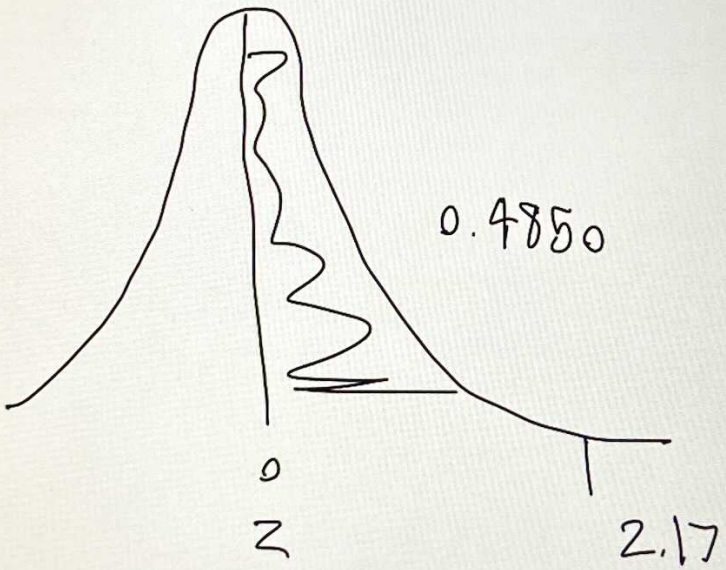
Homework #6

Q28.

a. $P(0 \leq z \leq 2.17) = P(z \leq 2.17) - P(z \leq 0)$

$= \Phi(2.17) - \Phi(0)$

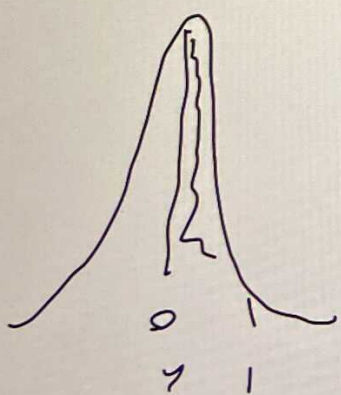
$= .9850 - 0.5000 = \underline{0.4850}$



b. $P(0 \leq z \leq 1) : P(0 \leq z \leq 1) - P(z \leq 0)$

$= \Phi(1) - \Phi(0) = 0.8413 - 0.5000$

$= \underline{0.3413}$



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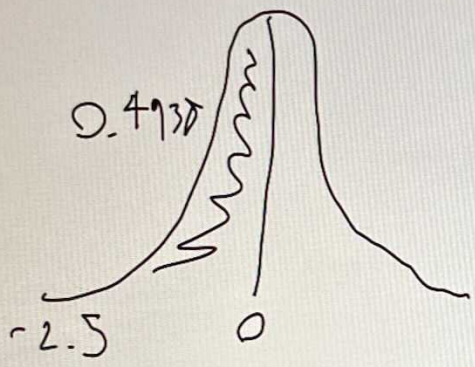
Homework #6

c. $P(-2.50 \le z \le 0) = P(z \le 0) - P(z \le -2.5)$

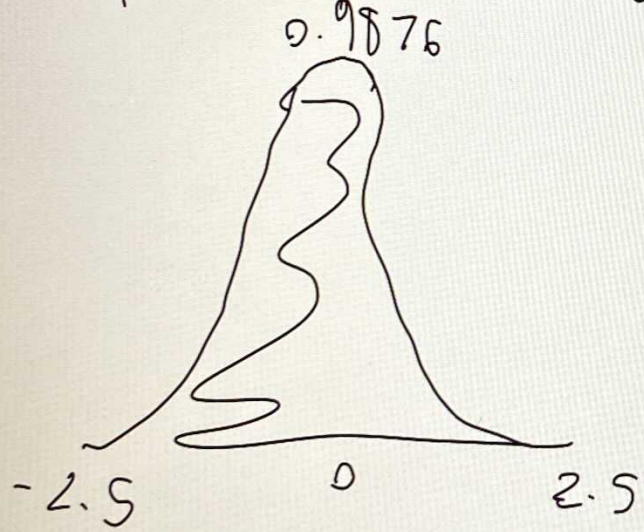


$= \Phi(0) - \Phi(-2.5)$

$\approx .5000 - 0.0062$

≈ 0.4938

d. $P(-2.50 \le z \le 2.50) = P(z \le 2.5) - P(z \le -2.5)$

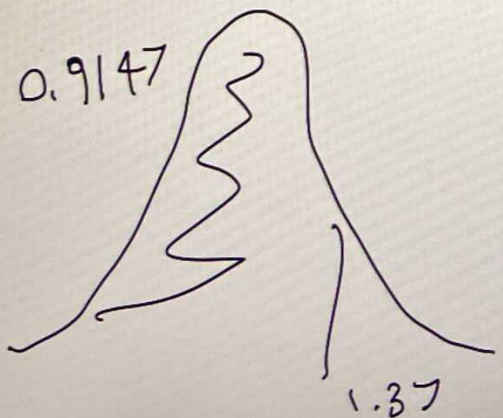


$= \Phi(2.50) - \Phi(-2.5)$

$= .9938 - 0.0062$

$= 0.9876$

e. $P(z \le 1.37) = \Phi(1.37) = 0.9147$



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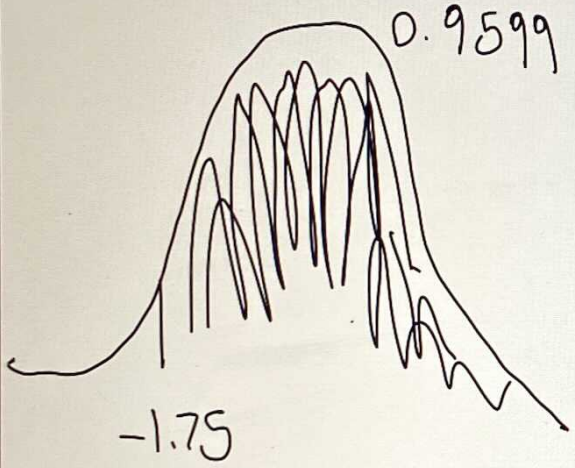
Homework #6

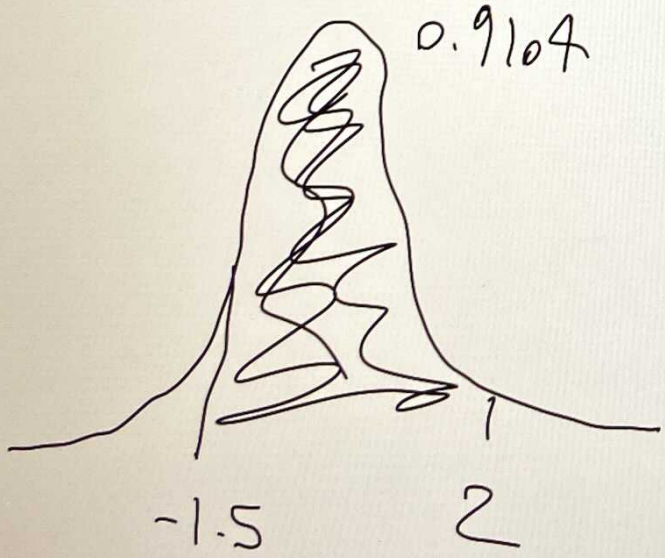
f) $P(-1.75 \leq Z) = 1 - P(Z \leq -1.75)$



$= 1 - \Phi(-1.75)$

$= 1 - 0.401 = \underline{0.9599}$

g) $P(-1.50 \leq Z \leq 2.00) : P(Z \leq 2.00) - P(Z \leq -1.5)$



$= \Phi(2.00) - \Phi(-1.5)$

$= 0.9772 - 0.0668$

$= \underline{0.9104}$

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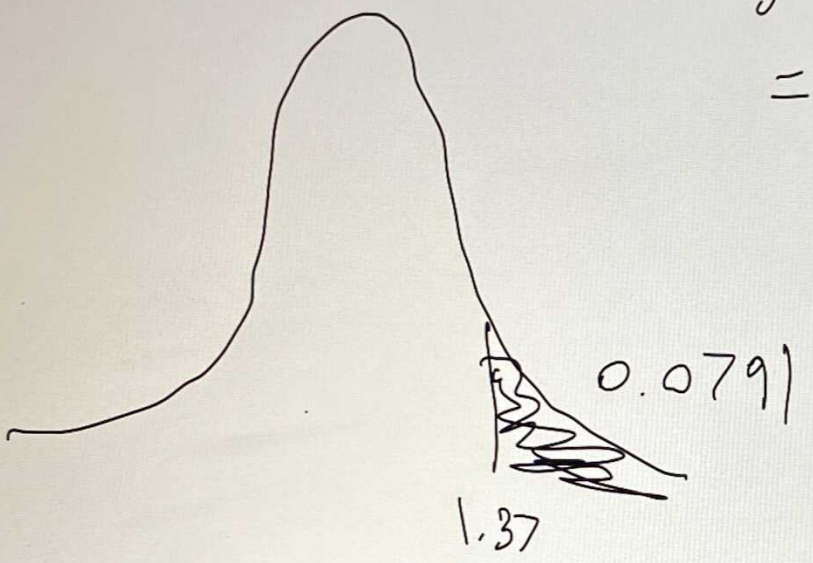
Homework #6

h) $P(1.37 \leq Z \leq 2.50) = \Phi(2.50) - \Phi(1.37)$

$= 0.9938 - 0.9147$

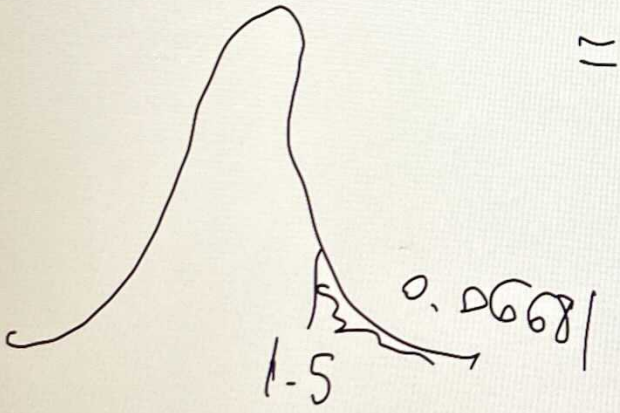
$= 0.0791$



i) $P(1.50 \leq Z) = 1 - P(Z \leq 1.50)$

$= 1 - \Phi(1.50) = 1 - 0.9332$

$= 0.0668$

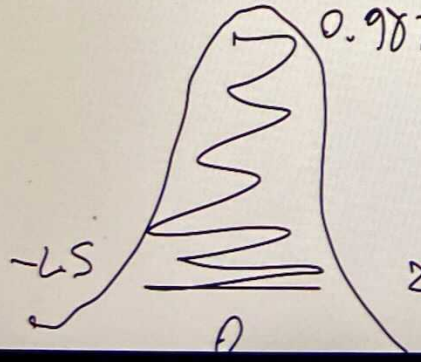


j) $P(|Z| \leq 2.50) = P(-2.50 \leq Z \leq 2.50)$

$= P(Z \leq 2.50) - P(Z \leq -2.50)$

$= \Phi(2.50) - \Phi(-2.50)$

$= 0.9938 - 0.0062 =$ 0.9876



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Homework #6

Q3.

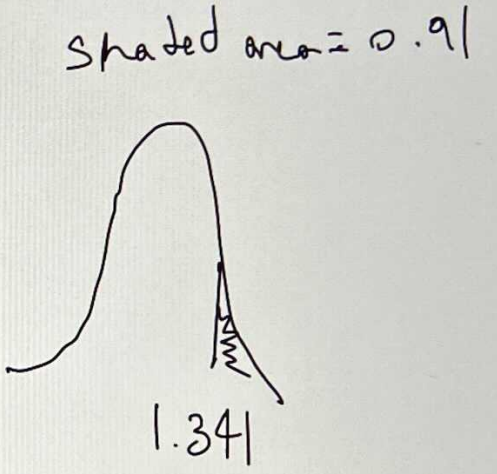
a.

$N(\mu, \sigma^2)$

$P(z < P_{91}) = 0.91$

$P(z < 1.34) = 0.91$

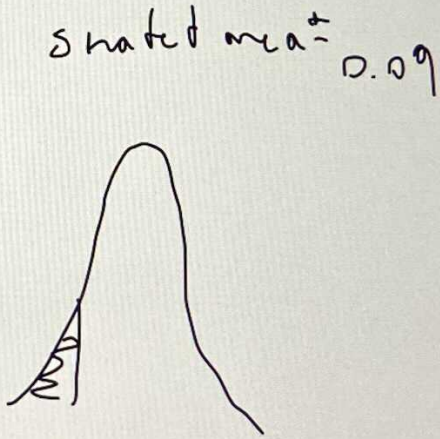
$P_{91} = 1.34$



b. $P(z < P_9) = 0.0900$

$P(z < -1.34) = 0.09$

$P_9 = -1.34$

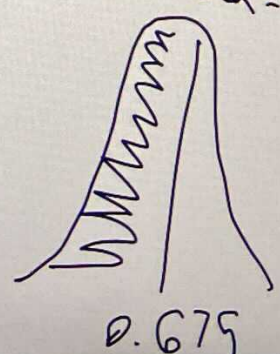


C. $P(z < P_{75}) = 0.7500$

$P(z \leq 0.675) = 0.7500$

$\frac{0.67 + 0.68}{2} = 0.675$

$P_{75} = 0.675$



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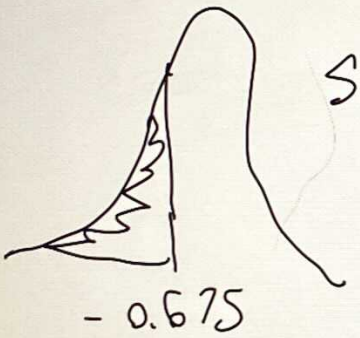
Homework #6

Q30

d) $P(z < P_{25}) = 0.2500$

$$\frac{-0.67 + (-0.68)}{2} = -0.675$$

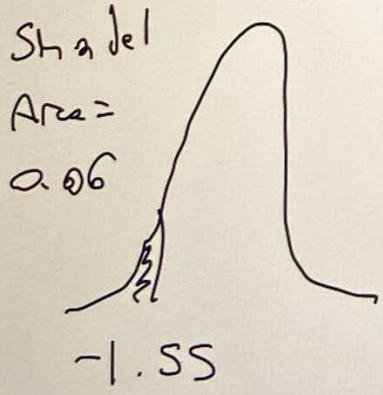
$P(z \leq -0.675) = 0.2500$ $\quad P_{25} = -0.675$



Shaded area = 0.25

25 percentile

Q30 e) $P(z < P_6) = 0.0600$



$P(z < -1.55) = 0.06$

$P_6 = -1.55$

10/16 Homework #6

Q 31

a. $\alpha = 0.0055$

$Z\alpha = Z_{0.0055} = 2.543$

The value of $Z\alpha$ is 2.543

b. $\alpha = 0.09$

$Z\alpha = Z_{0.09} = 1.341$

The value of $Z\alpha$ is 1.341

c. $\alpha = 0.663$

$Z\alpha = Z_{0.663} = 0.421$

The value of $Z\alpha$ is 0.421

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Homework #6

Q36

less than 1500: $P(x < 1500)$

a.

$$z = \frac{x-\mu}{\sigma} = \frac{1500-1050}{150} = \frac{450}{150} = 3 \quad z=3$$

$$P(x<1500) = P(x<3) = 0.99865$$

$$z = \frac{x-\mu}{\sigma} = \frac{1000-1050}{150} = \frac{-50}{150} = -0.33$$

$$P(x \geq 1000) = P(z \geq -0.33) = 1 - P(z < -0.33)$$

$$= 1 - 0.37070 = 0.6293$$

$$\therefore P(x \geq 1000) = 0.6293$$

b.

between 1000 μm & 1500 μm

$$z = \frac{x-\mu}{\sigma} = \frac{1000-1050}{150} = -0.33$$

$$z = \frac{x-\mu}{\sigma} = \frac{1500-1050}{150} = 3$$

$$P(1000 < x < 1500) = P(-0.33 < z < 3) = P(z<3) - P(z<-.33)$$

$$= 0.99865 - 0.37070 = 0.62795$$

$$\therefore P(1000 < x < 1500) = 0.62795$$

10/16

Homework #6

Q36 C. Smallest 2%

$$P(X < x_0) = 0.02$$

$$= P\left(\frac{x-\mu}{\sigma} < \frac{x_0-\mu}{\sigma}\right) = 0.02$$

$$= P\left(z < \frac{x_0-\mu}{\sigma}\right) = 0.02$$

$$\frac{x_0-\mu}{\sigma} = 0.02$$

$$-2.0537$$

$$\frac{x_0-1050}{150} = -2.0537 \qquad x_0 - 1050 = -2.0537 * 150$$

$$x_0 - 1050 = -308.055$$

$$x_0 = -308.055 + 1050$$

$$x_0 = 741.945$$

$$\therefore x_0 = 741.945$$

---

d. $\therefore P[x > 1800] = 0.0359$

$1 - P[x < 0] = 1 - 5_{C_0}(0.0359)^0(1-0.0359)^5$

$= 1 - .83293 = 0.16707$ $P[x \geq 1] = 0.16707$

10/16

Homework #6

Q40

a. $P[\text{At most } 40] = P[X \le 40] =$

$$P\left[\frac{X-\mu}{\sigma} \le \frac{40-43}{4.5}\right]$$

$$= P[Z \le -0.67]$$

$$= \Phi(-0.67) = 0.2514$$

$P(\text{greater than } 60) = P[X > 60]$

$$= P\left[\frac{X-\mu}{\sigma} > \frac{60-43}{4.5}\right]$$

$$= P[Z > 3.78]$$

$$= \phi(-3.78) = 0.0001$$

b. $P[X > c] = 75\% = 0.75$

$$= P\left[\frac{X-\mu}{\sigma} > \frac{c-\mu}{\sigma}\right] = .75$$

$$= 1 - \phi\left(\frac{c-\mu}{\sigma}\right) = .75$$

$$= \Phi\left(\frac{c-\mu}{\sigma}\right) = .25$$

$$= \left(\frac{c-43}{4.5}\right) = \phi^{-1}(0.25) = -0.674$$

$c = 43 + 4.5 \times (-0.674) = 39.967$

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Homework #6

Q 44

a. $P(\mu - 1.5\sigma \le x \le \mu + 1.5\sigma) = P(-1.5 \le z \le 1.5)$

$= P(z \le 1.5) - P(z \le 1.5)$

$= 0.9332 - 0.0668 = 0.8664$

b. $P(x < \mu - 2.5\sigma \text{ or } x > \mu + 2.5\sigma) = 1 - P(\mu - 2.5\sigma \le x \le \mu + 2.5\sigma)$

$= 1 - P(-2.5 \le z \le 2.5)$

$= 1 - [P(z \le 2.5) - P(z \le -2.5)]$

$= 1 - [0.9938 - 0.0062]$

$= 0.0124$

c. $P(\mu - 2\sigma \le x \le \mu + 2\sigma \text{ or } \mu - \sigma \le x \le \mu + \sigma) =$

$\left[\begin{matrix} P(\mu - 2\sigma \le x \le \mu + 2\sigma) - \\ P(\mu - \sigma \le x \le \mu + \sigma) \end{matrix} \right] = P(-2 \le z \le 2) - P(-1 \le z \le 1)$

$= \left[\begin{matrix} P(z \le 2) - P(z \le 2) - \\ P(z \le 1) - P(z \le -1) \end{matrix} \right] = [0.9772 - 0.0228] - [0.8413 - 0.1586]$

$= 0.9544 - 0.6827 = 0.2717$

Ch 16
Q 46

Homework #6

b. $P(70 - c \le X \le 70 + c) = 0.95$

$$P(70 - c \le X \le 70 + c) = 0.95$$

$$P\left(\frac{70 - c - 70}{3} \le \frac{X - \mu}{\sigma} \le \frac{70 + c - 70}{3}\right) = 0.95$$

$$P\left(\frac{-c}{3} \le Z \le \frac{c}{3}\right) = 0.95$$

$$2\Phi\left(\frac{c}{3}\right) - 1 = 0.95$$

$$2\Phi\left(\frac{c}{3}\right) = .95 + 1 = 1.95 = \frac{1.95}{2} = .975$$

That is, $\Phi(a) = 1.96$

$$\frac{c}{3} = 1.96 \quad c = 3(1.96)$$

$$= \underline{5.88}$$

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a. $P(Z < 1.67) - P(Z < -1)$

$$= 0.9525 - 0.1587 = \underline{0.7938}$$

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Q 48

Homework #6

a. $P(-1.72 \leq z \leq -0.55) = \Phi(0.55 \leq z \leq 1.72)$

$= P(z < 1.72) - P(z < 0.55)$

$= 0.9573 - 0.7088 = 0.2485$

b. $P(-1.72 \leq z \leq 0.55)$

$P(-1.72 < z < 0.55) = \Phi(z \leq 0.55) - \Phi(z \geq -1.72)$

$= P(z \leq 0.55) - (1 - P(z < -1.72))$

$= 0.7088 - (1 - 0.9573)$

$= 0.7088 - 0.0427 = 0.661$

※ it is not necessary to have a table of $\Phi(z)$ for negative z. Using symmetric property of normal distribution, all probilities of z can be found.

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Q60

Homework #6

Q. $P(X \leq 100) = \int_0^{100} \lambda e^{-\lambda x} dx$

$= (1 - e^{-100\lambda})$

$1 - e^{-100(0.01327)}$

$= 0.7347$

* The probability of the distance at most 100 m is 0.7347

Q.2 $P(X \leq 200) = \int_0^{200} \lambda e^{-\lambda x} dx$

$= 1 - e^{-200\lambda}$

$= 1 - e^{-200(0.01327)}$

$= .9296$

* The probability of distance lies between 100m to 200m is 0.1949

Q.3 $P(100 \leq X \leq 200) = \int_{100}^{200} \lambda e^{-\lambda x} dx$

$= (e^{-100\lambda} - e^{-200\lambda})$

$= e^{-100(0.01327)} - e^{-200(0.01327)} =$

0.1949

* The probability of distance lies between 100m to 200m is 0.1949

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Q 6a

Homework #6

6. $\mu = \frac{1}{0.01327} = 75.3579$

$\sigma = \mu = 75.3579$

$P(x - \mu > 2\sigma) = P(x > \mu + 2\sigma) = 1 - P(x < \mu + 2\sigma)$

$= 1 - F_x(\mu + 2\sigma)$

$= 1 - F_x\left(\frac{1}{0.01327} + \frac{2}{0.01327}\right)$

$= 1 - F_x(226.07) = e^{-0.01327 \times 226.07}$

$= 0.04978$

C. $P(x \leq M_d) = 0.5$

$F_x(M_d) = 0.5 \quad e^{-\lambda \times M_d} = 0.5$

$-\lambda \times M_d = \ln(0.5)$

$M_d = -\left(\frac{\ln(0.5)}{\lambda}\right) = M_d = -\left(\frac{\ln(0.5)}{0.01327}\right)$

$= 52.2341 m$