# Homework #11

## 9.7 #15

$$\sum_{i=2}^{n+1} \binom{i}{2} = \binom{n+1}{r+1}$$

$$\sum_{i=2}^{n+1} \binom{i}{2} = \binom{2}{2} + \binom{3}{2} + \ldots + \binom{n+1}{2} = \binom{n+2}{3}$$

$$\sum_{i=r}^{n} \binom{i}{r} = \binom{r}{r} + \binom{r+1}{r} + \binom{r+2}{r} + \ldots \binom{n}{r}$$

$${}^{n}C_r + {}^{n}C_{r-1} = {}^{n+1}C_r$$

$$s = \binom{r}{r} + \binom{r+1}{r} + \binom{r+2}{r} + \ldots \binom{r+k-1}{r} + \binom{r+k}{r}$$

$$= \binom{r+3}{r+1} + \ldots + \binom{r+k-1}{r} + \binom{r+k}{r}$$

$$= \binom{r+k}{r+1} + \binom{r+k}{r} = \binom{r+k+1}{r+1}$$

$$= \binom{n+1}{r+1}$$

## 9.7 #23

$$(p-2q)^4 = \sum_{k=0}^{4} \binom{4}{k} (p)^{4-k} (-2q)^k$$

$$= \binom{4}{0}(p)^4(-2q)^0 + \binom{4}{1}(p)^3(-2q)^1 + \binom{4}{2}(p)^2(-2q)^2 + \binom{4}{3}(p)^1(-2q)^3 + \binom{4}{4}(-2q)^4$$

$$= p^4 - 8p^3q + 24p^2q^2 - 32pq^3 + 16q^4$$

## 9.7 #37

$$(1+2)^n = \sum_{k=0}^{n} \binom{n}{k} (1)^{n-k} (2)^k$$

$$\binom{n}{0}(1)^n(2)^0 + \binom{n}{1}(1)^{n-1}(2)^1 + \binom{n}{2}(1)^{n-2}(2)^2 + \ldots + \binom{n}{n}(1)^{n-n}(2)^n$$

$$= \binom{n}{0} + \binom{n}{1}2^1 + \binom{n}{2}2^2 + \ldots + \binom{n}{n}(2)^n$$

## 9.8 #13

$$P(n): P(A_1 \cup A_2 \cup \ldots \cup A_n) = \sum_{k=1}^{n} P(A_k)$$

L.H.S of $P(2) = P(A_1 \cup A_2)$

$= P(A_1) + P(A_2) - P(A_1 \cap A_2)$

$= P(A_1) + P(A_2) [\because P(A_1 \cap A_2)$

R.H.S of $P(2) = \sum_{k=1}^{2} P(A_k)$

$= P(A_1) + P(A_2)$

L.H.S of $P(2)$ = R.H.S of $P(2)$

$P(n)$ is true for $n=2$

L.H.S of $P(m+1) = -P(\{A_1 \cup A_2 \cup \ldots \cup A_n\} \cap A_{m+1})$

$= P(A_1 \cup A_2 \cup \ldots \cup A_m) + P(A_{m+1}) - P((\emptyset) \cup (\emptyset) \cup \ldots \cup (\emptyset)$

$= P(A_1 \cup A_2 \cup \ldots \cup A_m) + P(A_{m+1}) - P(\emptyset)$

$=$ R.H.S of $P(m+1) = \sum_{k=1}^{m+1} P(A_k)$

$\therefore$

Thus by the method of mathematical induction,

$P(n): P(A_1 \cup A_2 \cup \ldots \cup A_n) = \sum_{k=1}^{n} P(A_k)$ is

$\forall n \geq 2$

## 9.8 #9

a. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= .4 + .5 - .2 = .7$

b. $P(C) = P(S - (A \cup B))$

$= 1 - P(A \cup B)$

$= 1 - .7 = .3$

C. $P(A^c) = 1 - P(A)$

$= 1 - .4 = .6$

d. $P(A^c \cap B^c) = P(A \cup B)^c$

$= 1 - P(A \cup B)$

$= 1 - .7 = .3$

e. $P(A^c \cup B^c) = P(A \cap B)^c$

$= 1 - P(A \cap B)$

$= 1 - .2 = .8$

f. $P(B^c \cap C) = P((A \cup B)^c)$

$= 1 - P(A \cup B) = 1 - .7 = .3$

# Homework #11

## 9.8 # 14

$E(x) = \sum_{k=1}^{n} x_k \, p(x_k)$

$E(x) = x_1 p(x_1) + x_2 p(x_2) + x_3 p(x_3) + x_4 p(x_4)$

$= 1,999,998 \times \frac{1}{1,500,000} + 20 \times \frac{10000}{1,500,000} + 4 \times \frac{50000}{1,500,000} + (-2) \times \frac{1,439,999}{1,500,000}$

= The expected loss per ticket is \$0.4

## 9.8 # 22

$p(x_2) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

$p(x_3) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$

$p(x_4) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{16}$

$p(x_5 = TTTT) = \frac{1}{2^4} = \frac{1}{16}$

$E(x) = \sum_{k=1}^{n} x_k \, p(x_k)$

$E(x) = x_1 p(x_1) + x_2 p(x_2) + x_3 p(x_3) + x_4 p(x_4) + x_5 p(x_5)$

$= (1)\frac{1}{2} + (2)\frac{1}{4} + (3)\frac{1}{8} + (4)\frac{1}{16} + (4)\frac{1}{16}$

$= \frac{15}{8} = 1.875$, expected number of tosses is 1.875

## 9.9 # 14

$P(B_1) = 4\% = .04$

$P(B_2) = 96\% = .96$

$P(A \mid B_2) = 3\% = .03$

$P(\bar{A} \mid B_1) = 2\% = .02$

$P(A \mid B_1) = 98\% = .98$

$P(\bar{A} \mid B_2) = 97\% = .97$

a. $P(B_1 \mid A) = \frac{P(A \mid B_1) \times P(B_1)}{P(A \mid B_1) \times P(B_1) + P(A \mid B_2) \times P(B_2)}$

$= \frac{98\% \times 4\%}{98\% \times 4\% + 96\% \times 3\%}$

$= 57.6\%$

b. $P(B_2 \mid \bar{A}) = \frac{P(\bar{A} \mid B_2) \times P(B_2)}{P(\bar{A} \mid B_2) \times P(B_2) + P(\bar{A} \mid B_1) \times P(B_1)}$

$= \frac{97\% \times 96\%}{2\% \times 4\% + 97\% \times 96\%} = 99.9\%$

## 9.9 # 5

$P(A \mid B^c) = \frac{P(A \cap B^c)}{P(B^c)}$

$= \frac{P(A - B)}{P(B^c)} = \frac{P(A) - P(A \cap B)}{P(B^c)}$

$= \frac{P(A) - P(A \cap B)}{1 - P(B)} \qquad P(A \mid B) = \frac{P(A \cap B)}{P(B)}$

$\rightarrow P(A \cap B) = P(A \mid B) \times P(B)$

$\therefore P(A \mid B^c) = \frac{P(A) - P(A \mid B) \times P(B)}{1 - P(B)}$

c. $P(W_1) = \frac{3}{10}$

$P(M_2 \mid W_1) = \frac{7}{9}$

$P(M_1) = \frac{7}{10}$

$P(W_2 \mid M_1) = \frac{3}{9}$

$P(W_1 \cap M_2) = \frac{21}{90}$

$P(M_1 \cap M_2) = \frac{21}{90}$

$\therefore P(W_1 \cap M_2) + P(M_1 \cap W_2) = \frac{21}{90} + \frac{21}{90} = \frac{42}{90} = \frac{7}{15}$

## 9.9 # 8

a. $P(W_1) = \frac{C(3,1)}{C(10,1)} = \frac{3}{10}$

$P(W_2 \mid W_1) = \frac{C(2,1)}{C(9,1)} = \frac{2}{9}$

$\therefore P(W_1 \cap W_2) = P(W_1) \times P(W_2 \mid W_1)$

$= \frac{3}{10} \times \frac{2}{9} = \frac{1}{15}$

b. $P(M_1) = \frac{C(7,1)}{C(10,1)} = \frac{7}{10}$

$P(M_2 \mid M_1) = \frac{C(6,1)}{C(9,1)} = \frac{6}{9}$

$P(M_1 \cap M_2) = P(M_1) \times P(M_2 \mid M_1) = \frac{7}{10} \times \frac{6}{9} = \frac{7}{15}$