11. Let S be the set of students at your school, let M be the set of movies that have ever been released, and let $V(s, m)$ be "student s has seen movie m." Rewrite each of the following statements without using the symbol $\forall$, the symbol $\exists$, or variables.

b. $\forall s \in S, V(s, \text{Star Wars})$

- $\forall s \in S$: All students at school

$V(s, \text{Star Wars})$: Student s has seen the movie Star Wars

So, the statement at school have seen the movie Star Wars.

12. Let $D = E = \{-2, -1, 0, 1, 2\}$. Write negations for each of the following statements and determine which is true, the given statement or its negation.

c. $\forall x$ in D, $\exists y$ in E such that $xy \geq y$.

- Negation of the statement:

$\exists x$ in D such that $\forall y$ in E, $xy < y$

Here the given statement is true.

For all x in D, take $y = 0$ in E then $xy = x \cdot 0 \geq 0 = y$

$\rightarrow xy \geq y$

---

d. $\exists x$ in D such that $\forall y$ in E, $x \leq y$.

Negation: $\forall x$ in D, $\exists y$ in E such that $x > y$.

$x = -2$, then $\forall y$ in E, $-2 \leq y$

negation is false, if we take -2, cannot find a y in E such that $-2 > y$.

16. $\exists$ a real number u such that $\forall$ real numbers v, $uv = v$

Rewriting in English:

There is a real number whose product with any other real number always equal to the second real number

Negation; $\forall$ real number u, $\exists$ real number v, such that $uv \neq v$.

Rewriting in English:

For every real number, there is a real number such that their product is not equal to second real number.

47. True, There is a triangle x for all circles y, then x is above y.

- To satisfy the given statement there should be a triangle above all the circles
- The domain of all variables is defined in the given Tarski world
- From the given Tarski world we can locate a triangle labeled with "a" as above all the shapes thus as circles.
- Thus the given statements is true for the given Tarski world.

5. For every object x, there is an object y such that if $x \neq y$ then x and y have different colors

- Truth value of the statements is True

Consider any square, circle or tringle in the figure.

For every triangle, there is a square or circle with different color.

For every square, there is a triangle or circle with different color.

For every circle, there is a square or triangle with different color.

So we can say,

"For every object x, there is an object y such that if $x \neq y$ then x and y have different colors."

14. The given statement is invalid by inverse error. Intuitively, a program isn't necessarily correct just because it compiles without any errors. So it is invalid by inverse error.

15. Any sum of two rational numbers is rational.

The sum $r+s$ is rational.

$\therefore$ The numbers r and s are both rational

$$r = \frac{p}{q}, \quad s = \frac{p_1}{q_1} \quad \text{where } p, q, p_1, q_1 \in \mathbb{Z}$$

$$\& \ q, q_1 \neq 0$$

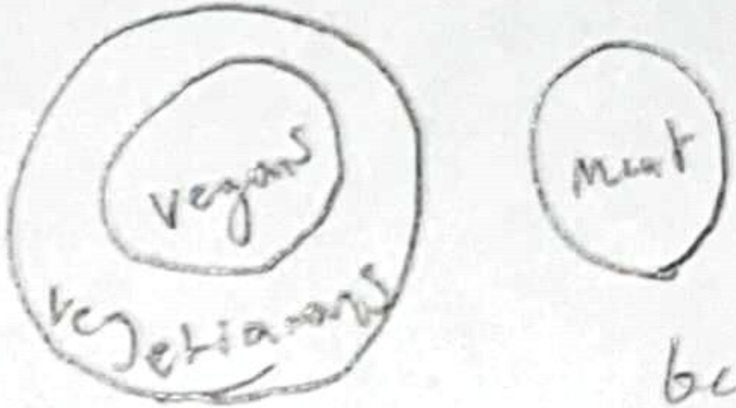
$$r + s = \frac{p}{q} + \frac{p_1}{q_1} = \frac{p_1 q + q_1 p}{q q_1} \in \mathbb{Q}$$

$p_1 q \in \mathbb{Z}$
$q_1 p \in \mathbb{Z}$
$q q_1 \in \mathbb{Z}$

24. No vegetarians eat meat.

All vegans are vegetarian.

∴ No vegans eat meat



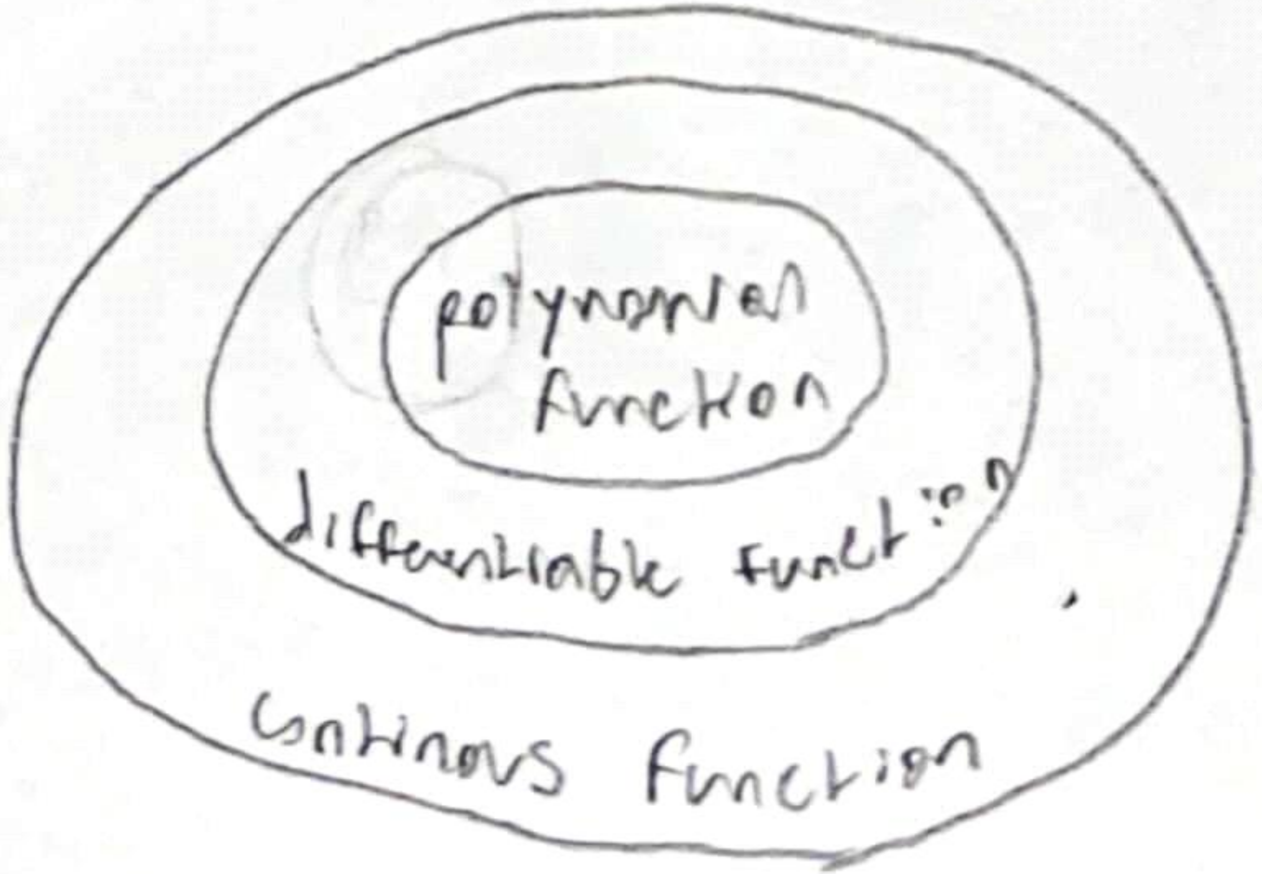
No vegans eat meat - true or false

because in the above diagram all vegans are included in vegetarians and No vegetarian eat meat it means the vegans are also not eat meat. So the given conclusion is "valid".

26 All polynomial functions are differentiable.

All differentiable functions are continuous.

$\therefore$ All polynomial functions are continuous.



The argument is valid

#7 There is an integer $n > 5$ such that $2^n - 1$ is prime

- Let $n = 7$

  then

$$2^n - 1 = 2^7 - 1 = 127$$

which is a prime number. Therefore, there exists $n > 5$ such that $2^n - 1$ is prime.

9 There is a perfect square that can be written as a sum of two other perfect squares

$25 = 5^2$

$16 = 4^2$

$9 = 3^2$

*The statement is true

this implies $25 = 16 + 9$

this 25 is a perfect square that can be written as a sum of two other squares.

15. $-a^n = (-a)^n$

$-a^n = (-a)^n$

$= (-1)^n \cdot a^n$

$(-1)^1$ or $(-1)^3$ or $(-1)^7$ then we will get $-a^n = -a^n$

$-a^n \neq (-a)^n$; if "n" is even

above property is only true for some integers

31. Since k is an odd integer then k can be written as $k = 2p+1$ for some integer and m is an even integer, then m can be written as $m = 2q$ for some integer q

True

Now, $k^2 + m^2 = (2p+1)^2 + (2q)^2$

$= 4p^2 + 4p + 1 + 4q^2 = 2(2p^2 + 2q^2) + w$

which is of the form, twice of some integer plus one ie. odd numbers