Section 14.2 – Calculus of Vector-Valued Functions

- Derivative and Tangent Vector:
  Let $\boldsymbol{r}(t) = f(t)\boldsymbol{i} + g(t)\boldsymbol{j} + h(t)\boldsymbol{k}$, where f, g, and h are differentiable functions on (a, b). Then $\boldsymbol{r}$ has a derivative on (a, b) and
  $$\boldsymbol{r}'(t) = f'(t)\boldsymbol{i} + g'(t)\boldsymbol{j} + h'(t)\boldsymbol{k}$$
  Provided $\boldsymbol{r}'(t) \neq 0, \boldsymbol{r}'(t)$ is a tangent vector at the point corresponding to $\boldsymbol{r}(t)$.

- Compute the derivative of the following functions:
  - $\boldsymbol{r}(t) = \langle \cos t, t^3, \ln t \rangle$
  - $\boldsymbol{r}(t) = \langle te^{-t}, 2\sin 3t, (2t+1)^{-1} \rangle$

- Unit Tangent Vector: Let $\boldsymbol{r}(t) = f(t)\boldsymbol{i} + g(t)\boldsymbol{j} + h(t)\boldsymbol{k}$ be a smooth parametrized curve, for $a \le t \le b$. The unit tangent vector for a particular value of t is

$$\boldsymbol{T}(t) = \frac{\boldsymbol{r}'(t)}{|\boldsymbol{r}'(t)|}$$

- Find the unit tangent vector for the following parametrized curve.
  - $\boldsymbol{r}(t) = \langle 2t, 2t, t \rangle$ for $0 \le t \le 1$

- Compute $\boldsymbol{r}''(t)$ for the following:
  - $\boldsymbol{r}(t) = \langle \sqrt{t+4}, \ln(t+1), 2e^{-4t} \rangle$

- <u>Indefinite Integral of a Vector-Valued Function:</u>

$$\int \boldsymbol{r}(t)dt \ = \boldsymbol{R}(t) + \boldsymbol{C}$$

$$\int \langle f(t), g(t), h(t)\rangle dt = \langle F(t), G(t), H(t)\rangle + \langle C_1, C_2, C_3\rangle$$

- Compute

$$\int \langle e^{3t}, \frac{1}{1+t^2}, -\frac{1}{\sqrt{2t}}\rangle \, dt$$

- <u>Definite Integral of a Vector-Valued Function:</u>

$$\int_b^a \boldsymbol{r}(t)dt = \left(\int_b^a f(t)dt\right)\boldsymbol{i} + \left(\int_b^a g(t)dt\right)\boldsymbol{j} + \left(\int_b^a h(t)dt\right)\boldsymbol{k}$$

- Evaluate the following:

$$\int_1^4 (6t^2\boldsymbol{i} + 8t^3\boldsymbol{j} + 9t^2\boldsymbol{k})dt$$

- Find the function **r** that satisfies the following conditions:
    - $\boldsymbol{r}'(t) = \langle e^t, \sin t, \sec^2 t \rangle; \boldsymbol{r}(0) = \langle 2, 2, 2 \rangle$
    - $\boldsymbol{r}'(t) = \langle e^{2t}, 1 - 2e^{-t}, 1 - 2e^t \rangle;\ \boldsymbol{r}(0) = \langle 1,1,1 \rangle$