Homework 13

10.2 question #7

- Since the adjacency matrix contains a 0 for all entries in the ith row and ith column, $a_{ii} = 0$ for all integers i and the adjacency matrix contains only 0's in the main diagonal. $a_{ii} = 0$ for all integers i then implies that the vertex v_i is not connected to any other vertex of the graph and so the graph is not completely connected. This being the case, the graph has no loops.

10.2 question #8

- $\begin{bmatrix} 2 & -1 \end{bmatrix}\begin{bmatrix} 1 \\ 3 \end{bmatrix}$ • $\begin{bmatrix} 4 & -1 & 7 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$

a)

Let $A = \begin{bmatrix} 2 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$

$$\begin{bmatrix} 2 & -1 \end{bmatrix}\begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} c_{11} \end{bmatrix}$$

$$c_{11} = 2 \cdot 1 + (-1) \cdot 3 = 2 - 3 = -1$$

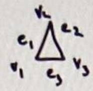
$$AB = \begin{bmatrix} -1 \end{bmatrix}$$

b. $A = \begin{bmatrix} 4 & -1 & 7 \end{bmatrix}$ $B = \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix} = \begin{bmatrix} c_{11} \end{bmatrix}$

$$c_{11} = 4 \cdot 1 + (-1) \cdot 2 + 7 \cdot 0 = 4 - 2 = 2$$

$$AB = \begin{bmatrix} 2 \end{bmatrix}$$

10.2 question #21



$$K_3 = A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$

$$A^3 = AA^2$$

$$= \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}\begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 3 & 3 \\ 3 & 2 & 3 \\ 3 & 3 & 2 \end{bmatrix}$$

* The main diagonal of A^n are equal and all the entries that do not lie along the main diagonal are $n = 1, 2, 3$

$$A^k = \begin{bmatrix} a & b & b \\ b & a & b \\ b & b & a \end{bmatrix}$$

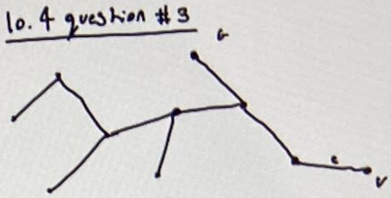
$$A^{k+1} = AA^k$$

$$= \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}\begin{bmatrix} a & b & b \\ b & a & b \\ b & b & a \end{bmatrix}$$

$$= \begin{bmatrix} 2b & a+b & a+b \\ a+b & 2b & a+b \\ a+b & a+b & 2a \end{bmatrix}$$

* all entries along main diagonal of A^n are equal to $2b$ and all entries that do not lie along the main diagonal are equal to $a+b$ for $n = k+1$
* Mathematical induction, all the entries along the main diagonal of A^n are equal and all the entries that do not lie along the main diagonal are equa for all $n \geq 1$.

10.4 question #3



* The number of edges in graph G is equal to, $n_1 - 1 + n_2 - 1 + 1 = n_1 + n_2 - 1 = n - 1$
* a tree with n vertices has a total degree of $2(n-1)$
* The total degree of a tree with n vertices is $2n - 2$

10.4 question #5

(1) All the vertices of T have an edge to the root

- the number of vertices is $k > 2$, excluding the root vertex, we follow that there are $k - 1 \geq 2$ vertices having edges to the root

* there are $k - 1 \geq 2$ vertices with degree 1 in the graph

(2) * while only one leaf is present in the tree, if more internal vertices are there, then the chance is a cycle which leads to a contradiction

- when $n > 1$ internal vertices are present in a tree, then there must be at least n leaves or children in the tree
- the number of vertices of degree 1 is at least $n > 1$ or ≥ 2.

10.2 question #23

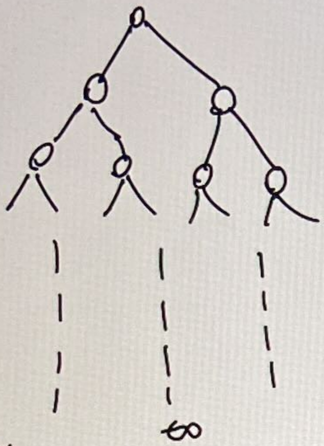
a) if G is a disconnected subgraph, both v and w are in the connected subgraph of G

- if G is a connected graph

- if every pair of adjacent vertices is covered by length 1, then all the n vertices are covered by $n-1$ length. The length of the walk is $n-1$
- we can say that the length of the walk between distinct vertices of a connected graph with n vertices is less than or equal to $n-1$.

- we confirm that there is a walk of length $n-1$ between vertices in a graph of n vertices.

10.4 question 6



* A simple complete binary tree would be an example of infinite tree if it extends to infinity as there would be no vertex with degree 1

10.4 question 26

* The graph is not a tree but connected then it has to have a circuit and if an edge is removed from the graph then the graph is still connected. There will be finitely many circuits and therefore at the end of the process there is a subgraph with n vertices, connected with no circuits and hence this must be a tree which is a contradiction. The graph cannot be connected.

10.4 question 23

- A graph with 9 vertices, the total number of possible edges are 9-1 = 8
- a connected graph having nine vertices cannot be a tree and a connected graph which is a tree cannot have a non trivial circuit
- The connected graph which is not a tree has a non-trivial circuit.

10.4 question 24

- Since the graph is connected in which vertex v is connected to the graph with an edge e having degree 1, it can be removed easily without changing the connectedness of the graph.
- when the vertex v with edge e is removed from the graph, all the vertices in the subgraph G is connected.
- The graph G is connected which is obtained by removing vertex v from connected graph G.