1. Find the following derivatives. $\frac{\partial z}{\partial s}$ and z_t, where $z = e^{2x} + e^{y^3} - (xy - y)^2$, $x = s^2 - 5t$, $y = \cos(st)$. Leave your answer in terms of $x, y, s,$ and t.

$$\frac{\partial z}{\partial s} = \left(2e^{2x} - 2y(xy - y)\right)(2s) + \left(3y^2 e^{y^3} - 2(x-1)(xy-y)\right)(-t\sin(st))$$

$$\frac{\partial z}{\partial t} = z_t = \left(2e^{2x} - 2y(xy-y)\right)(-5) + \left(3y^2 e^{y^3} - 2(x-1)(xy-y)\right)(-s\sin(st))$$

2. Use implicit differentiation to find $\frac{dy}{dx}$ for $ye^{xy} = x + 2$.

$$\frac{dy}{dx} = \frac{-F_x}{F_y} \qquad \underbrace{ye^{xy} - x}_{F} = 2$$

$$\frac{dy}{dx} = \frac{-(y^2e^{xy} - 1)}{e^{xy} + xye^{xy}}$$

3. Compute the directional derivative of the following function at the given point P in the direction of the given vector.

$$f(x, y) = -2x^3 - xy + y^2, P(-2, 4), \vec{v} = \langle 2, 1\rangle$$

$\nabla f = \langle -6x^2 - y, -x + 2y \rangle$

$\nabla f(-2,4) = \langle -28, 10 \rangle$

$\vec{u} = \dfrac{\langle 2, 1\rangle}{\sqrt{2^2+1^2}} = \left\langle \dfrac{2}{\sqrt5}, \dfrac{1}{\sqrt5} \right\rangle$

$D_{\vec{u}} f(-2,4) = \langle -28, 10 \rangle \cdot \left\langle \dfrac{2}{\sqrt5}, \dfrac{1}{\sqrt5} \right\rangle$

$= \dfrac{-56}{\sqrt5} + \dfrac{10}{\sqrt5} = \boxed{\dfrac{-46}{\sqrt5}}$

4. In your own words, briefly explain the difference between a directional derivative and the partial derivatives with respect to x or y.

A directional derivative is able to find a rate of change in any direction whereas the partials only find the rate of change in the direction of the x or y axes.

5. Find the equation of the plane tangent to the given surface at the given point.

$$\underbrace{2x^2 - 5y^3 + yz}_{F} = 0, P(1, -1, 2)$$

$$\nabla F = \langle 4x, -15y^2 + z, y \rangle$$

$$\nabla F(1,-1,2) = \langle 4, -13, -1 \rangle$$

tangent plane:

$$4(x-1) - 13(y+1) - (z-2) = 0$$