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Homework #5

Section 4.5 #29

a. Use the quotient-remainder theorem with $d=3$ to prove that the square of any integer has the form $3k$ or $3k+1$ for some integer k.

b. Use the mod notation

a. When $d=3$ or n is divided by 3, the possible remainders are 0, 1, and 2. When $d=3$, any integer n can be written in any one of the following three forms for some integer q:

(i) when $r=0$, $n=3q$

(ii) when $r=1$, $n=3q+1$

(iii) when $r=2$, $n=3q+2$

case (i)

$n^2 = (3q)^2$
$= 3(3q^2)$
$= 3k$

$k = 3q^2$ is an integer as product of integers is also an integer

case (ii)

$n^2 = (3q+1)^2$
$= 9q^2 + 6q + 1$
$= 3(3q^2+2q)+1$
$= 3k+1$

$k = 3q^2+2q$ is an integer as sum of product of integers is also an integer

case (iii)

$n^2 = (3q+2)^2$
$= 9q^2 + 12q + 4$
$= 9q^2 + 12q + 3 + 1$
$= 3(3q^2 + 4q + 1) + 1$
$= 3k+1$

* The square of any integer has the form $3k$ or $3k+1$ for some integer k

b.

The square of any integer has the form $3k$ or $3k+1$ for some integer k means that the remainder obtained when the square of any integer is divided by 3 is 0 or 1

So, if n is the positive integer, then $n^2 \bmod 3 = 0$ or 1

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Homework #5

4.7 #4

Use proof by contradiction to show that for every integer m, $7m+4$ is not divisible by 7.

$\Rightarrow \exists\ n \in \mathbb{Z}$ such that

$$7m+4=7n$$

$$\Rightarrow 7m-7n+4=0$$

$$\Rightarrow 7n-7m=4$$

$$\Rightarrow 7(n-m)=4$$

But, $n-m \in \mathbb{Z}$ whereas $\frac{4}{7} \notin \mathbb{Z}$

$\therefore$ $7m+4$ is not divisible by 7, for any $m \in \mathbb{Z}$

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Homework #5

4.9 #7

Either draw a graph with the specified properties or explain why no such graph exists. Graph with four vertices of degrees 1, 1, 1, and 4.

- There is no graph with four vertices of degrees 1, 1, 1, and 4. We prove using contradiction method. If possible, there is a graph G with four vertices of degrees 1,1,1 and 4. Then, the total degree of the graph is $1+1+1+4=7$, 7 is odd. This is in contradiction to every graph whose total degree must be even.

Therefore, there is no such graph.

4.9 # 15

a. $18 + 8 + 20 + 3x = 82$

$\rightarrow 3x = 82 - 43$

$3x = 39$

$x = \frac{39}{3} = 13$

13 people are network friends with 3 other people in network

b. Total number of people in network =

$3 + 1 + 5 + 13 = 22$

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4.10 #16

Use the Euclidean algorithm to hand-calculate the greatest common divisors of each of the pairs of integers in 13-16.

16. 4,131 and 2,431

→ $2431 \overline{)4131}$

2431

1700 remainder

$4131 = 2431 \times 1 + 1700$

$GCD(4131, 2431) = GCD(2431, 1700)$

→ $1700 \overline{)2431}$

1700

731 remainder

$2431 = 1700 \times 1 + 731$, $GCD(2431, 1700) = GCD(1700, 731)$

→ $731 \overline{)1700}$

1462

238 remainder

$1700 = 731 \times 2 + 238$ $GCD(1700, 731) = GCD(731, 238)$

→ $238 \overline{)731}$

714

17 remainder

$731 = 238 \times 3 + 17$ $GCD(731, 238) = GCD(238, 17)$

→ $238 \div 17 = 14 \Rightarrow 17 \overline{)238}$ $= 0$ $238 = 17 \times 14 + 0$ $GCD(238, 17) = GCD(17, 0)$

238

$GCD(4131, 2431) = 17$

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Homework # 5

4.10 #18

Make a trace table to track the action 4.10.2 for the input variables given in 17-19.

18. 5,859 and 1,232

A	5,859						
B	1,232						
r		931	301	28	21	7	0
b	1,232	931	301	28	21	7	0
a	5,859	1231	931	301	28	21	7
GCD							7