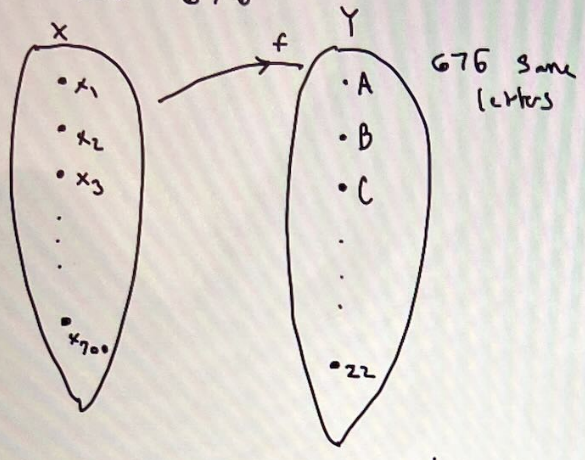
Home Work #10

Section 9.4 #4

consider a group of 700 people

$26 \times 26 = 676$



There are 700 pigeons and 676 pigeonholes, so if 677 pair of initals are used, and then there must be at least 2 people that map to one pigeon.

Section 9.4 # 28

$n = 500 \quad k = 17$ in $\left\lceil \frac{n}{k} \right\rceil$

$$\left\lceil \frac{n}{k} \right\rceil = \left\lceil \frac{500}{17} \right\rceil$$

$$= \lceil 29.42 \rceil$$

$$= 30$$

Yes, there is at least 1 day when the programmer wrote 30 or more lines of program code.

Section 9.5 # 7

a. $$\binom{13}{7} = \frac{13!}{(13-7) \times 7!}$$

$$= \frac{13!}{6! \times 7!}$$

$$= \frac{13 \times 12 \times 11 \times 10 \times 9 \times 8 \times 7!}{6 \times 5 \times 4 \times 3 \times 2 \times 1 \times 7!}$$

$$= \frac{13 \times 11 \times 10 \times 9 \times 8}{5 \times 4 \times 3} = \underline{1716}$$

b. $\binom{7}{4}\binom{6}{3}$

$$\binom{7}{4}\binom{6}{3} = \frac{7!}{4!\,(7-4)!} \times \frac{6!}{(6-3) \times 3!}$$

$$= \frac{7!}{4! \times 3!} \times \frac{6!}{3! \times 3!}$$

$$= \frac{7 \times 6 \times 5 \times 4!}{4! \times 3!} \times \frac{6 \times 5 \times 4 \times 3!}{3! \times 3!} = 35 \times 20 = \underline{700}$$

$$= \binom{13}{7} - \binom{7}{7}$$

$$= \binom{13}{7} - 1 = 1716 - 1 = \underline{1715}$$

$\therefore$ Total number of required selection is:

$$= \binom{7}{3}\binom{6}{4} + \binom{7}{2}\binom{6}{5} + \binom{7}{1}\binom{6}{6} = \underline{658}$$

c. $$\binom{11}{7} = \binom{2}{1}\binom{11}{6} + \binom{11}{7} = \underline{1,254}$$

d. $$\binom{2}{2} \times \binom{11}{5}$$

$\therefore$ Total possibilities are,

$$\binom{2}{2}\binom{11}{5} + \binom{11}{7} = \underline{792}$$

Homework #10

9.5 Section #16

a. $\binom{40}{5} = \frac{40!}{5! \times 35!} = 658{,}008$

b. (total) − (samples which do not contain a defective board)

$= \binom{40}{5} - \binom{37}{5}$

$= 222{,}111$

c. $\therefore N(E) = 222{,}111$

$\therefore N(S) = \binom{40}{5}$

$\therefore p(E) = \frac{N(E)}{N(S)}$

$= \frac{222{,}111}{\binom{40}{5}}$

$= 0.3376$

$= 33.76\%$

9.6 Section #4

a. $\binom{n+m-1}{n}$

$\binom{30+8-1}{30} = \binom{37}{30}$

$= 10295472$

$\therefore$ The total number of inventory of 30 batteries which could be distributed among eight types

10,295,472

b. Total Selections $= \binom{26+8-1}{26}$

$\binom{33}{26} = 4{,}272{,}048$

9.6 Selection 14

$x_1 = a - 10$

$a = x_1 + 10$

$x_2 = b - 10$

$b = x_2 + 10$

$x_3 = c - 10$

$c = x_3 + 10$

$x_4 = d - 10$

$d = x_4 + 10$

$x_5 = e - 10$

$e = x_5 + 10$

$a + b + c + d + e = 500$

$(x_1 + 10) + (x_2 + 10) + (x_3 + 10) + (x_4 + 10) + (x_5 + 10) = 500$

$x_1 + x_2 + x_3 + x_4 + x_5 + 50 = 50$

$x_1 + x_2 + x_3 + x_4 + x_5 = 500 - 50$

$= 450$

$\binom{450 + 4}{450} = \binom{454}{450}$

$= \frac{454!}{450!(454-450)!}$

$= \frac{454!}{450! \times 4!}$

$= \frac{454 \times 453 \times 452 \times 451}{4 \times 3 \times 2 \times 1}$

$= 1{,}746{,}858{,}751$