1.6 Applications of Linear Systems

Balancing Chemical Equations

Ex. When propane gas burns, the propane C_3H_8 combines with oxygen O_2 to form carbon dioxide CO_2 and water H_2O.

$$(x_1)\,C_3H_8 + (x_2)\,O_2 \longrightarrow (x_3)\,CO_2 + (x_4)\,H_2O$$

As atoms are neither created nor destroyed in a chemical reaction, our goal is to determine $x_1, \ldots, x_4$ so that there are equal numbers of C, H, and O atoms on each side of the reaction. We encode each molecule as a vector:

$$C_3H_8 \longrightarrow \begin{bmatrix} 3 \\ 8 \\ 0 \end{bmatrix}, \qquad CO_2 \longrightarrow \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix},$$

$$O_2 \longrightarrow \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}, \qquad H_2O \longrightarrow \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}.$$

we can thus set up a vector equation

$$x_1\begin{bmatrix}3\\8\\0\end{bmatrix} + x_2\begin{bmatrix}0\\0\\2\end{bmatrix} = x_3\begin{bmatrix}1\\0\\2\end{bmatrix} + x_4\begin{bmatrix}0\\2\\1\end{bmatrix}.$$

$$\Rightarrow x_1\begin{bmatrix}3\\8\\0\end{bmatrix} + x_2\begin{bmatrix}0\\0\\2\end{bmatrix} + x_3\begin{bmatrix}-1\\0\\-2\end{bmatrix} + x_4\begin{bmatrix}0\\-2\\-1\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}.$$

Setting up the corresponding augmented matrix and row reducing gives

$$\vec{x} = \begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix} = x_4\begin{bmatrix}1\\5\\3\\4\end{bmatrix}.$$

Checking our solution we see that

$$C_3H_8 + 5O_2 \longrightarrow 3CO_2 + 4H_2O$$

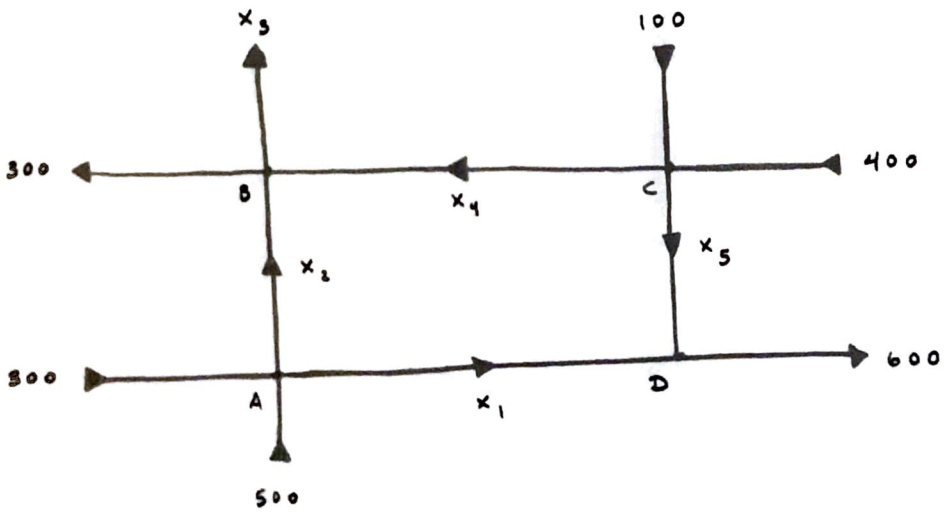
is balanced.

Network Flow

A network consists of a set of points called junctions with lines or arcs called branches connecting some or all of the junctions. The direction of flow in each branch is indicated by an arrow and quantified by a value

or variable. The basic assumption of network flow is that the total flow into the network equals the total flow out of the network and that the total flow into a junction equals the total flow out of the junction.

Ex. consider the traffic flow network



Determine the general flow pattern for the network.

we have

$x_1 + x_2 = 800$, $x_4 + x_5 = 500$,

$x_3 + 300 = x_2 + x_4$, $600 = x_1 + x_5$,

$x_3 + 900 = 1300$

Solving this system gives

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 600 - x_5 \\ 200 + x_5 \\ 400 \\ 500 - x_5 \\ x_5 \end{bmatrix}$$

Often we require $x_i \geq 0$ for each i which would imply here that $x_5 \leq 500$.