Section 13.5

- Vector-valued function: $\boldsymbol{r}(t) = \langle x(t), y(t), z(t)\rangle$
- Equation of a Line: An equation of the line passing through the point $P_0(x_0, y_0, z_0)$ in the direction of the vector $\boldsymbol{v} = \langle a, b, c\rangle$ is $\boldsymbol{r} = \boldsymbol{r_0} + t\boldsymbol{v}$, or
$$\langle x, y, z\rangle = \langle x_0, y_0, z_0\rangle + t\langle a, b, c\rangle, \qquad -\infty < t < \infty$$
Equivalently, the parametric equations of the line are
$$x = x_0 + at, \qquad y = y_0 + bt, \qquad z = z_0 + ct, \qquad -\infty < t < \infty$$

- Find an equation for the line through $(-3, 2, -1)$ in the direction of the vector $\boldsymbol{v} = \langle 1, -2, 0\rangle$

- Find parametric equations for the line through the points $P\left(0, \frac{1}{2}, 1\right)$ and $Q(2, 1, -3)$.

- Find the line through $(1, 2, 3)$ that is perpendicular to the lines

  $\boldsymbol{r}_1(t) = \langle 3 - 2t, 5 + 8t, 7 - 4t \rangle$ and $\boldsymbol{r}_2(t) = \langle t, t, -t \rangle$

- Plane in $\mathbb{R}^3$: Given a fixed point P_0 and a nonzero vector $\boldsymbol{n}$, the set of points P in $\mathbb{R}^3$ for which $\overrightarrow{P_0P}$ is orthogonal to $\boldsymbol{n}$ is called a plane.
- General Equation of a Plane in $\mathbb{R}^3$: The plane passing through the point $P_0(x_0, y_0, z_0)$ with a normal vector $\boldsymbol{n} = \langle a, b, c\rangle$ is described by the equation
$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$
or
$$ax + by + cz = d$$
where $d = ax_0 + by_0 + cz_0$.

- Find an equation of the plane that passes through the point P_0 with a normal vector $\boldsymbol{n}$.
$$P_0(1, 2, -3); \boldsymbol{n} = \langle -1, 4, -3\rangle$$

- Find an equation of the plane that passes through the points $(2, -1, 4)$, $(1, 1, -1)$, and $(-4, 1, 1)$.

- Find the points at which the following planes intersect the coordinate axes and find equations of the lines where the planes intersect the coordinate planes. Sketch a graph of the plane.
$$x + 3y - 5z - 30 = 0$$

- Parallel and Orthogonal Planes: Two distinct planes are parallel if their respective normal vectors are parallel (scalar multiples of each other). Two planes are orthogonal if their respective normal vectors are orthogonal ($\boldsymbol{u} \cdot \boldsymbol{v} = 0$).

- Find an equation of the plane parallel to the plane $-x + 2y - 4z = 1$ passing through the point $P_0(1, 0, 4)$.

- Find an equation of the line where the planes Q and R intersect.

$$Q: x + 2y - z = 1$$
$$R: x + y + z = 1$$