<u>Section 13.3 & 13.4 Review</u>

- <u>Dot Product</u>: Given two vectors $\boldsymbol{u} = \langle u_1, u_2 \rangle$ and $\boldsymbol{v} = \langle v_1, v_2 \rangle$ in two or three dimensions, their **dot product** is
$$\boldsymbol{u} \cdot \boldsymbol{v} = |\boldsymbol{u}||\boldsymbol{v}| \cos\theta$$
$$\boldsymbol{u} \cdot \boldsymbol{v} = u_1 v_1 + u_2 v_2$$
where θ is the angle between $\boldsymbol{u}$ and $\boldsymbol{v}$ with $0 \le \theta \le \pi$.

- <u>Orthogonal Vectors</u>: two vectors $\boldsymbol{u}$ and $\boldsymbol{v}$ are orthogonal if and only if $\boldsymbol{u} \cdot \boldsymbol{v} = 0$.

- $\cos\theta = \frac{\boldsymbol{u} \cdot \boldsymbol{v}}{|\boldsymbol{u}||\boldsymbol{v}|}$

- Compute the dot product of the vectors $\boldsymbol{u}$ and $\boldsymbol{v}$, and find the angle between the vectors.

    - $\boldsymbol{u} = \boldsymbol{i} + \boldsymbol{j}$ and $\boldsymbol{v} = \boldsymbol{i} - \boldsymbol{j}$

    - $\boldsymbol{u} = \langle 3, 3, -3 \rangle$ and $\boldsymbol{v} = \langle 1, -1, 2 \rangle$

- Projection of $\boldsymbol{u}$ onto $\boldsymbol{v}$:

$$proj_v\boldsymbol{u} = |\boldsymbol{u}|\cos\theta\left(\frac{\boldsymbol{v}}{|\boldsymbol{v}|}\right) = scal_v\boldsymbol{u}\left(\frac{\boldsymbol{v}}{|\boldsymbol{v}|}\right)$$

- Scalar component of $\boldsymbol{u}$ in the direction of $\boldsymbol{v}$:

$$scal_v\boldsymbol{u} = |\boldsymbol{u}|\cos\theta = \frac{\boldsymbol{u}\cdot\boldsymbol{v}}{|\boldsymbol{v}|}$$

- For the given vectors $\boldsymbol{u}$ and $\boldsymbol{v}$, calculate $proj_v\boldsymbol{u}$ and $scal_v\boldsymbol{u}$.
  - $\boldsymbol{u} = \langle -1, 4\rangle$ and $\boldsymbol{v} = \langle -4, 2\rangle$

- <u>Cross Product</u>: Given two nonzero vectors $\boldsymbol{u}$ and $\boldsymbol{v}$ in $\mathbb{R}^3$, the **cross product** $\boldsymbol{u} \times \boldsymbol{v}$ is a vector with magnitude
$$|\boldsymbol{u} \times \boldsymbol{v}| = |\boldsymbol{u}||\boldsymbol{v}| \sin \theta$$
where $0 \leq \theta \leq \pi$ is the angle between $\boldsymbol{u}$ and $\boldsymbol{v}$.
- <u>Evaluating the Cross Product</u>: Let $\boldsymbol{u} = \langle u_1, u_2, u_3 \rangle$ and $\boldsymbol{v} = \langle v_1, v_2, v_3 \rangle$. Then
$$\boldsymbol{u} \times \boldsymbol{v} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ u_1 & u_2 & u_3 \\ v_1 & v_2 & v_3 \end{vmatrix} = \begin{vmatrix} u_2 & u_3 \\ v_2 & v_3 \end{vmatrix} \boldsymbol{i} - \begin{vmatrix} u_1 & u_3 \\ v_1 & v_3 \end{vmatrix} \boldsymbol{j} + \begin{vmatrix} u_1 & u_2 \\ v_1 & v_2 \end{vmatrix} \boldsymbol{k}$$
- Sketch the following vectors. Then compute $|\boldsymbol{u} \times \boldsymbol{v}|$ and show the cross product on your sketch.
  - $\boldsymbol{u} = \langle 0, -2, 0 \rangle$ and $\boldsymbol{v} = \langle 0, 1, 0 \rangle$
- Find a vector normal to the given vectors.
  - $\langle 0, 1, 2 \rangle$ and $\langle -2, 0, 3 \rangle$