Exam 2 – Part B – Math 273 Spring 2021 – Version #2

Exam Part A [MML]

- Available from 12:00am to 11:59pm on Wednesday, April 7th.
- Once you begin the MML portion, you will have 40 minutes to complete it.
- There are 7 problems worth a total of 30 points on the MML portion of the exam.

Exam Part B [Crowdmark]

- Available from 4:30pm to 6:00pm on Wednesday, April 7th.
- Complete this portion of the exam and have it uploaded **no later than** 6:00pm.
  DO NOT WAIT UNTIL THE LAST MINUTE!
- You will be penalized for late submissions.
- There are 4 problems worth a total of 70 points on the Crowdmark portion of the exam.
- Completing the work for this portion of the exam should take you no more than 60 minutes, although you have a 90-minute period of time. Additional time should be used for downloading/uploading. You should plan on finishing and uploading ***early*** so you can deal with any computer/internet/uploading issues without stress!

Read and follow all directions carefully.

1. Simplify all answers completely but leave them in EXACT form. For example, $\sqrt{3}$ is an EXACT answer, while 1.732 is an APPROXIMATE answer.

2. You are required to crop your images appropriately, rotate the images to be upright, and make sure you are submitting a clear and readable image. If you fail to check the orientation or if your image is not cropped, there will be a penalty of 1 point per violation.

3. When uploading your solutions, attach each problem in the appropriately numbered space. Uploading multiple problems in the space for number 1 means we will not see your solutions for 2, 3, 4, etc. when we are grading.

4. You do not need to print the exam, but a pdf of the entire exam will be attached to the Crowdmark assignment if you wish to print it.

5. You must show all of your own original work/each step. The grader must be able to follow your work, including following your reasoning. You will receive credit only if you show your process and answer questions completely. Correct answers without supporting work will receive 0 points.

6. No typed work will be accepted. All work submitted must be hand-written. Work written on a tablet is acceptable, as long as it is not typed.

7. Clearly indicate your final answer for each question.

8. Label units appropriately.

9. The following list of items **are allowed** for you to utilize during the exam:
   - You may use your textbook.
   - You may use class notes.
   - You may email the instructor.

10. You **<u>are NOT allowed</u>** to use any of the following resources:
    - You cannot use a calculator.
    - You cannot use your cellphone in any other manner than to take images to submit your exam. If you use your phone to submit images on Crowdmark you may still do so, but you should only be on your phone at the end of the exam to submit, not at all beforehand. Anyone on their phone before they are finished with the exam will face consequences for cheating on the exam.
    - You cannot use any internet resource.
    - You cannot use any software package(s) such as Excel, Mathway, Wolfram Alpha, or Matlab
    - **You cannot use any website other than Crowdmark.**
    - **You may not take or post images of any part of the exam other than for the purpose of submitting your work to Crowdmark. It is copyrighted material. No distribution without written permission.**
    - **You cannot receive help from another student or tutors of any kind or persons in any manner (other than the instructor).**
    - You cannot help any other students complete their exam.

11. You must sign and submit under problem number 1 in Crowdmark the attestation that:

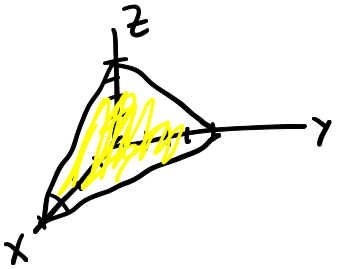
"I have not and will not receive any unauthorized aid during this exam in accordance with the policies outlined in the directions."

1. [0] The remainder of the exam will not be graded unless you sign your name attesting that the following statement is true. This is required as part of your exam.

   "I have not and will not receive any unauthorized aid during this exam in accordance with the policies outlined in the directions."

   Signature: _____

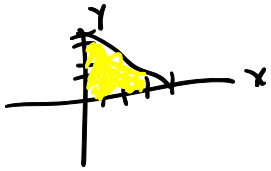
2. *(20 points)* Find the volume of the region which lies below the plane $x + y + z = 4$ and above the xy-plane in the first octant by using a double or triple integral.



xy-plane: $z = 0$

$x + y = 4$

$y = 4 - x$



$$V = \int_0^4 \int_0^{4-x} \int_0^{4-x-y} (1)\, dz\, dy\, dx$$

$$= \int_0^4 \int_0^{4-x} z \Big|_0^{4-x-y} dy\, dx$$

$$= \int_0^4 \int_0^{4-x} (4-x-y)\, dy\, dx$$

$$= \int_0^4 \left(4y - xy - \frac{1}{2}y^2\right)\Big|_0^{4-x} dx$$

$$= \int_0^4 \left(16 - 4x - 4x + x^2 - \frac{1}{2}(16 - 8x + x^2)\right) dx$$

$$= \int_0^4 \left(16 - 8x + x^2 - 8 + 4x - \frac{1}{2}x^2\right) dx$$

$$= \int_0^4 \left(8 - 4x + \frac{1}{2}x^2\right) dx$$

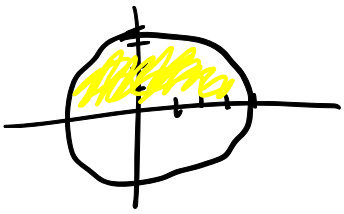
$$= \left(8x - 2x^2 + \frac{1}{6}x^3\right)\Big|_0^4$$

$$= 32 - 32 + \frac{64}{6} = \boxed{\frac{32}{3}}$$

3. *(15 points)* SET UP the following integral in cylindrical coordinates. Do not integrate.

$$\int_0^4 \int_{-\sqrt{16-y^2}}^{\sqrt{16-y^2}} \int_{\sqrt{x^2+y^2}}^{3y} x \, dz \, dx \, dy$$

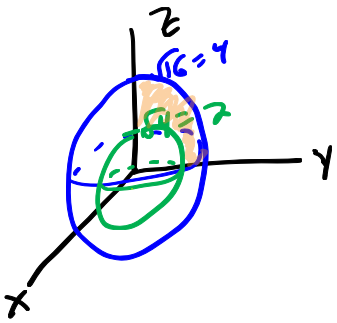
XY-Plane:



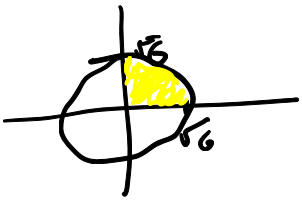
$$\int_0^{\pi} \int_0^{4} \int_{r}^{3r\sin(\theta)} r^2\cos(\theta)\, dz\, dr\, d\theta$$

4. *(15 points)* SET UP the following integral in spherical coordinates. Do not integrate.

$$\int_0^{\sqrt{6}} \int_0^{\sqrt{6-y^2}} \int_{\sqrt{4-x^2-y^2}}^{\sqrt{16-x^2-y^2}} z \, dz \, dx \, dy$$



XY-Plane:



$$\int_0^{\pi/2} \int_0^{\frac{\pi}{2}} \int_2^{4} \rho^3\cos(\phi)\sin(\phi)\, d\rho\, d\phi\, d\theta$$

5. *(20 points)* Find critical points of $f(x,y)$ and classify them as local maximums, local minimums, or saddle points.

$$f(x,y) = y^2(x-5) - x^2$$

$f_x = y^2 - 2x = 0$

$f_y = 2y(x-5) = 0 \rightarrow y = 0 \quad x = 5$

for $y=0$: $0 - 2x = 0 \rightarrow x = 0$

for $x=5$: $y^2 - 10 = 0 \rightarrow y^2 = 10 \rightarrow y = \pm\sqrt{10}$

Critical points: $(0,0), (5,\sqrt{10}), (5,-\sqrt{10})$

$D(x,y) = (-2)(2x-10) - (2y)^2$

$D(0,0) = (-2)(-10) - 0 = +$ → $(0,0)$ is a local max

$f_{xx} = -2 < 0$

$D(5,\sqrt{10}) = (-2)(0) - (2\sqrt{10})^2 = -$ → $(5,\sqrt{10})$ is a saddle point

$D(5,-\sqrt{10}) = (-2)(0) - (-2\sqrt{10})^2 = -$ → $(5,-\sqrt{10})$ is a saddle point