

Permutations of a set with repeated elements.

Ex How many distinguishable orderings of letters in Mississippi are there? Steps 1. Choose 4 possitions for S's 2. Choose 4 positions for I's 3. Choose 2 positions for Ps 4. Choose I position for M. (The east step idoes not sh crease the count. So, we have (4) for step 1

 $\binom{7}{4}$ for Step 2, $\binom{3}{2}$ for Step 3. total: $\binom{11}{4}\binom{7}{4}\binom{3}{2} = \frac{11!}{4!7!} \frac{3!}{4!3!} \frac{3!}{2!1!} \frac{11!}{4!4!2!}$





Suppose a collection of Consists of M objects of which M1 are of type 1 and are indistinguishable; M2 are of type 2 and are indistinguishable;

Me are of type &

and are indistinguishable

Suppose that M1+M2+-+Mk = 4

Then the humber of distinguishable

per my tations of the nobjects



$$\binom{n}{m_2}\binom{h-n_1}{n_2}\binom{n-n_2-n_2}{n_3}$$
 $\binom{h-n_1-n_2-\dots-n_k-1}{n_k}$

r-Combinations with repetition allowed.

D: How many ways are there to choose relements without ordering from a set of he elements if repetition is allowed?

Def An E-sepetition combination 1809.

With repetition allowed

(or multiset of Size ">), chosen

from a set of X of n elements

is an unordered selection of

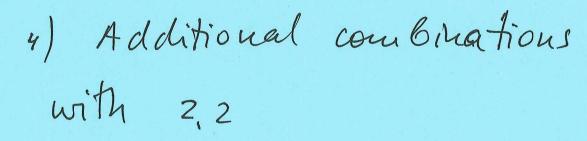
elements taken from X with

repetition allowed.

Ex. 3-combinations from repetitions allowed. $X = \{1, 2, 3, 4\}$

A possible solution:

- 1) combing trous with 1,1
- 2) all additional combrustions with
- 3) additional combinations with 1,3 or 1,4.





- 5) Additional combinations with 2,3 or 2,4
- 6) Additional combinations with 3,3 or 3,4
- 7) an additional combinetion with 4,4.

The complete list



- 1) 41,1,14, 71,1,24, 21,1,34, 11,1,49
- 2) 11,2,29, 11,2,53, {1,2,43
- 3) {1,3,34, { 1,3,4}, { 1,4,4}
- 4/ 523 12,2,29, 52,2,3}, 12,2,49
- 5) 42,3,31, {2,3,4}, {2,4,4}
- 6) {3,3,3}, {3,3,49, {3,4,49
- F) {4,4,4}

thow do we organite such

counts in general?

Think of ellements as categories so now we have four categories.

X - indicates an element chosen teom a given category 1 - separate catégories. A selection is a string such other choices. X | X | X | {1,2,5}]XXI X this means: none from 1 - X (XX | {2,3,3} XX | X | | {1,1,2} 2 from 2 5/2, 04, 43 none from 3 1 from 4 The count is the # of strings of 6: 3 bars and 3 crosses leaves up choices for bars.

(3) is the final count.



Thun The humber of 7-combinotions with repetition allowed that can be selected from a set of h elements is (Ex) Counting triples (i,j,k) with 1 = i = j = k = n (*) Colker an integer 420, And the # of triples i, j, k, not necessa-Tily distinct, and such that

(*) holds.*

Solution Atoiple: N-1 vertical bars and 3 (2088es.

Thus the count is (3+(4-1)) = $= \frac{(h+2)!}{3! (u-1)!} = \frac{(h+2)!}{5! (u-1)!} = \frac{(h(u+1)(u+2)!}{6! (u-1)(u+2)!} = \frac{(h+1)(u+2)!}{3! (u-3)! (u-2)(u-1)} = \frac{(h(u+1)(u+2)!}{3! (u-2)(u-1)!}$

(Ex. 1) How many solutions there to the equation ten 1's in to them. 3 bars separating the categories 10 closses. Count (10+3) = 13! = 10!3! $=\frac{11\cdot 12\cdot 12}{6}=2\cdot 11\cdot 13=286.$ as above, but assuming 2) Same X; ? 1. 1 cross goes noto each

Solution. I cross goes noto tally category. Remaining 6 can be