1. Find all the critical points of the following function. Then use the second derivative test to classify them as either a local minimum, local maximum, or a saddle point.

$$f(x, y) = y^4 + 4y^2(x-2) + 8(x-1)^2$$

$f_x = 4y^2 + 16(x-1) = 0 \rightarrow 16x = -4y^2 + 16 \rightarrow x = -\frac{1}{4}y^2 + 1$

$f_y = 4y^3 + 8y(x-2) = 0 \rightarrow 4y^3 + 8y(-\frac{1}{4}y^2 + 1 - 2) = 0$

$4y^3 - 2y^3 - 8y = 0$

$2y^3 - 8y = 0$

$2y(y^2 - 4) = 0$

$y = 0$ | $y = 2$ | $y = -2$

$x = 1$ | $x = 0$ | $x = 0$

critical points: $(1, 0)$ $(0, 2)$ $(0, -2)$

$D(x, y) = (16)(12y^2 + 8(x-2)) - (8y)^2$

$D(1, 0) = (16)(-8) < 0 \rightarrow$ $(1, 0)$ is a saddle point

$D(0, 2) = (16)(48 - 16) - (16)^2 = 496 > 0$

$f_{xx} > 0 \rightarrow$ $(0, 2)$ is a local min.

$D(0, -2) = (16)(48 - 16) - (-16)^2 > 0$

$f_{xx} > 0 \rightarrow$ $(0, -2)$ is a local min.

2. Evaluate the iterated integral:

$$\iint_D 8x^3 e^{x^2 y}\, dA\,; D = \{0 \le x \le 3, 0 \le y \le 1\}$$

$$\int_0^3 \int_0^1 8x^3 e^{x^2 y}\, dy\, dx$$

$$= \int_0^3 8x\, e^{x^2 y}\Big|_0^1 \, dx$$

$$= \int_0^3 \left(8x e^{x^2} - 8x\right) dx$$

$$u = x^2$$

$$du = 2x\, dx$$

$$= \int_0^9 4e^u\, du - \int_0^3 8x\, dx$$

$$= 4e^u\Big|_0^9 - 4x^2\Big|_0^3$$

$$= 4e^9 - 4 - 36 = 4e^9 - 40$$

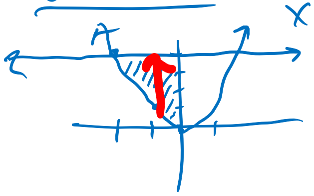
3. Answer the following questions:

a. Evaluate the integral:

$$\int_{-2}^{0}\int_{x^2}^{4} xy\,dy\,dx$$

Bounds: $Y = X^2$ $Y = 4$

$X = -2$ $X = 0$



$$\int_{-2}^{0} \frac{1}{2}xy^2 \Big|_{x^2}^{4} dx$$

$$= \int_{-2}^{0} \left(8x - \frac{1}{2}x^5\right) dx$$

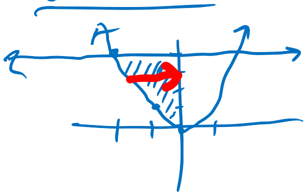
$$= \left(4x^2 - \frac{1}{12}x^6\right)\Big|_{-2}^{0}$$

$$= -16 + \frac{16}{3} = \boxed{\frac{-32}{3}}$$

b. Change the order of integration, then evaluate the integral again. (Hint: you should get the same answer).

Bounds: $X = -\sqrt{Y}$ $X = 0$

$Y = 0$ $Y = 4$



$$\int_{0}^{4}\int_{-\sqrt{y}}^{0} xy\,dx\,dy$$

$$\int_{0}^{4} \frac{1}{2}x^2 y \Big|_{-\sqrt{y}}^{0} dy$$

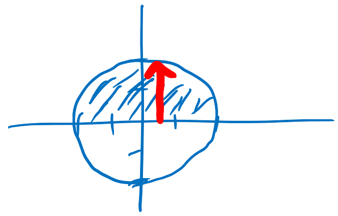
$$\int_{0}^{4} -\frac{1}{2}y^2\,dy$$

$$= -\frac{1}{6}y^3 \Big|_{0}^{4} = \frac{-64}{6} = \boxed{\frac{-32}{3}}$$

4. Convert the following integral to polar coordinates, then evaluate the integral.

$$\int_{-2}^{2}\int_{0}^{\sqrt{4-x^2}} e^{-x^2-y^2}\,dy\,dx$$

Bounds:



$$\int_0^{\pi}\int_0^{2} r e^{-r^2}\,dr\,d\theta \qquad u=-r^2,\quad du=-2r\,dr$$

$$\left(\int_0^{\pi} d\theta\right)\left(\int_0^{-4} -\frac{1}{2}e^{u}\,du\right)$$

$$=\left(\theta\Big|_0^{\pi}\right)\left(-\frac{1}{2}e^{u}\Big|_0^{-4}\right)$$

$$=(\pi)\left(-\frac{1}{2}e^{-4}+\frac{1}{2}\right)$$

$$=\boxed{\frac{\pi}{2}\left(1-e^{-4}\right)}$$