Homework #8

8.1 #4

Define a relation P on Z as follows: For all m, n ∈ Z, m P n ⟺ m and n have a common prime factor

a. is 15 P 25?

- yes, 15 P 25 because 15 and 25 are both divisible by 5, which is prime.

b. 22 P 27?

- No, 22 $\not P$ 27 because 22 and 27 have no common prime factors

c. is 0 P 5?

- yes, 0 P 5 because 0 and 5 are both divisible by 5, which is prime.

d. is 8 P 8?

- 8 P 8 because 2 divides both 8 and 2 is prime

8.1 #7

Define a relation R on Z as follows: For all integers m and n,
$m R n \leftrightarrow 5 \mid (m2 - n2)$

a. is $1 R (-9)$?

- $1 R (-9) \leftrightarrow 5 \mid (1^2 - (-9)^2)$

$\leftrightarrow 5 \mid (1^2 - 81)$

$\leftrightarrow 5 \mid -(-8\circ)$

yes, $1 R (-9)$

b. $2 R 13 \leftrightarrow 5 \mid (2^2 - 13^2)$

$\leftrightarrow 5 \mid -(4 - 169)$

$\leftrightarrow 5 \mid -(165)$

yes, $2 R 13$

c. is $2 R (-8)$?

- $2 R (-8) \leftrightarrow 5 \mid (2^2 - (-8)^2)$

$\leftrightarrow 5 \mid (4 - 64)$

$\leftrightarrow 5 \mid (-60)$ because $-60 = (-5) . 12$

yes, $2 R (-8)$

d. $(-8) R 2 \leftrightarrow 5 \mid ((-8)^2 - 2^2)$

$\leftrightarrow 5 \mid (64 - 4)$

$\leftrightarrow 5 \mid 60$ because $60 = 5.12$

yes, $(-8) R 2$

Homework 8

8.1 #17

Draw the directed graphs of the relations.

Exersize

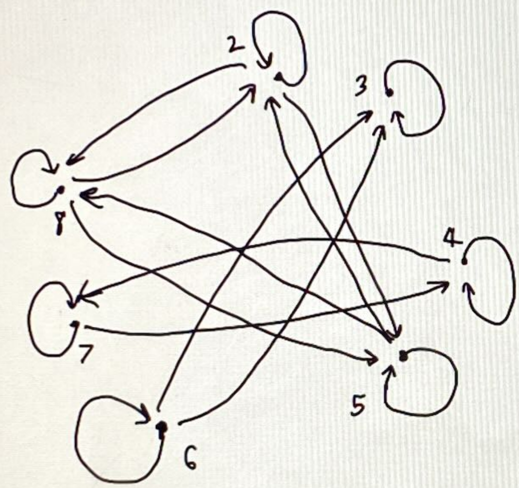
Let $A = \{2,3,4,5,6,7,8\}$ and define a relation T on A as follows: For all $x, y \in A$,

$x\,T\,y \leftrightarrow 3 \mid (x-y)$

By definition of the binary relation of T

$T = \{(2,2),(2,5),(2,8),(3,3),(4,4),(4,7),(5,5),(5,8),(5,2),(6,6),(6,3),(7,7),(7,4),(8,8),(8,5),(8,2)\}$

$x\,T\,y \rightarrow 3 \mid (x-y)$



8.1 #20

$A = \{-1, 1, 2, 4\}$ and $B = \{1, 2\}$

$A \times B = \{(-1,1),(-1,2),(1,1),(1,2),(2,2),(4,1),(4,2)\}$

$R = \{(-1,1)(1,1)(2,2)\}$

Since $xRy \Rightarrow |x| = |y|$ so $|-1| = 1 \Rightarrow (-1,1) \in R$
and $|2| = 2 \Rightarrow (2,2) \in R$

$S = \{(-1,1)(1,1)(2,2)(4,2)\}$

because $x\,S\,y \Rightarrow x-y$ is even so $-1-1 = -2$ is even and $1-1 = 0$ is even, $2-2=0$
$4-2=2$ are even.

$R \cup S = \{(-1,1)(1,1)(2,2)(4,2)\} = S$

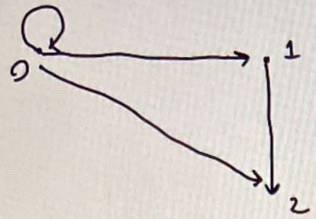
$R \cup S = S$, since $R \cup S$ means the ordered pairs, which belong to R or S

$R \cup S = \{(-1,1)(1,1)(2,2)\} = R \quad \therefore R \cup S = R$ ← Since $R \cap S$ means the ordered pairs, which belongs to both R&S

Homework #8

8.2 #5

(a)



because of $(0,0)$, there is a loop along 0 itself. There are arrows from 0 to 1 and 0 to 2 because of $(0,1)$ and $(0,2)$. There is an arrow from 1 to 2 because of $(1,2)$

(b) R_5 is not reflexive

Since there are no loop along 1 and 2 itself. That is it $1 \in A$, but $(1,1) \notin R_5$ Similarly $2 \in A$, but $(2,2) \notin R_5$

(c) R_5 is not Symmetric

because $(0,1) \in R_5$, but $(1,0) \in R_5$ similarly for remains also. There are no anti loops for any elements.

(d) R_5 is transitive

Since $(0,1) \in R_5$ and $(1,2) \in R_5 \Rightarrow (0,2) \in R_5$

8.2 #14

- O is not reflective

O is reflective $\Leftrightarrow$ for all integers m and mOm by the definition of O, for all integers m, $m - m = 0$, any number. which is false, since 0 is an even number.

- O is Symetric

by the definition of O, for all integers m and n, $m-n$ is odd $\Rightarrow m-n=(2k+1)$ by the definition of odd

$\Rightarrow n-m=-(2k+1)$ number by algebra $-(2k+1)$ is also odd, which implies that $mOn \Rightarrow nOm$ for all $m, n \in \mathbb{Z}$, so O is Symetric.

- O is transitive

by definition of O, for all m, n and $p \in \mathbb{Z}$ $m-n$ is odd. So let $m-n=(2k+1)$ by the definition of odd where $k, l \in \mathbb{Z}$ now, $m-p = m-n+n-p$

$= (m-n)+(n-p)$

$= 2k+1+2l+1$

$= 2k+2l+2$

$= 2(k+l+1)$

$= 2t$ where $t = k+l+1$ an integer

$m-p$ is an odd number which is false. So mOn, nOp but $m \not{O} p$

Homework #8

Section 8.2 # 47

$A = \{0,1,2,3\}$ and $(0,0) \in R$

$0 \, R_S \, 0$

R_S is not irreflexive

$\forall 0,1,2,3 \in A$ $(0,1) \in R_S$ but $(1,0) \notin R_S$
$(0,2) \in R_S$ but $(2,0) \notin R_S$ and $(1,2) \in R_S$
but $(2,1) \notin R_S$

R_S is asymmetric

$\forall 0,1,2 \in A$ $(0,1) \in R_S, (1,2) \in R_S$ $(0,2) \in R_S$

R_S is not intransitive.

Section 8.3 #12

$[-4] = \{m \in A : mR(-4)\}$
$= \{m \in A : 5 \mid m^2 - (-4)^2\}$
$= \{m \in A : 5 \mid m^2 - 16\}$

$5 \mid (-4)^2 - 16, 5 \mid (-1)^2 - 16, 5 \mid 1^2 - 16$
and $5 \mid 4^2 - 16$ follows that the equivalence class is $\{-4,-1,1,4\}$

$n = -3$

$[-3] = \{m \in A : mR(-3)\}$
$= \{m \in A : 5 \mid m^2 - (-3)^2\}$
$= \{m \in A : 5 \mid m^2 - 9\}$

As $5 \mid (-3)^2 - 9, 5 \mid (-2)^2 - 9, 5 \mid 2^2 - 9$ follows that the equivalence class is $\{-3,-2,2,3\}$

$[-2] = \{m \in A : mR(-2)\}$
$= \{m \in A : 5 \mid m^2 - (-2)^2\}$
$= \{m \in A : 5 \mid m^2 - 4\}$

As $5 \mid (-3)^2 - 4, 5 \mid (-2)^2 - 4, 5 \mid 2^2$, and $5 \mid 3^2 - 4$
follows that the equivalence class is $\{-3,-2,2,3\}$

$[-1] = \{m \in A : mR(-1)\}$
$= \{m \in A : 5 \mid m^2 - (-1)^2\}$
$= \{m \in A : 5 \mid m^2 - 1\}$

As $5 \mid (-4)^2 - 1, 5 \mid (-1)^2 - 1, 5 \mid 1^2 - 1$, and $5 \mid 4^2 - 1$
follows that the equivalence class is $\{-4,-1,1,4\}$

$[0] = \{m \in A : mR0\}$
$= \{m \in A : 5 \mid m^2 - 0\}$
$= \{m \in A : 5 \mid m^2\}$

As $5 \mid 0^2 - 0$ follows that the equivalence class is $\{0\}$

Therefore, the distinct equivalence classes of R are
$\{-4,-1,1,4\}, \{-3,-2,2,3\}, \{0\}$

Homework #8

Section 8.3 #27

Reflexive:

For $m \in \mathbb{Z}$, $m^2 - m^2 = 0$

As $4 | 0$, it follows that $4 | (m^2 - m^2) \Rightarrow m R m$

therefore, R is reflexive

symmetric:

For $m, n \in \mathbb{Z}$, let mRn then by the definition of relation, $4 | (m^2 - n^2)$

Rewrite the expression $4 | (m^2 - n^2)$ as:

$4 | (m^2 - n^2) \Rightarrow 4 | -(n^2 - m^2)$

it follows that $4 | (n^2 - m^2)$

Thus $n R m$

Therefore, R is symmetric

Transitive:

For $m, n, p \in \mathbb{Z}$, let mRn and nRp

Then by the definition of relation $4 | (n^2 - n^2)$ and $4 | (n^2 - p^2)$

Combine the expression $4 | (m^2 - n^2)$ and $4 | (n^2 - p^2)$

$4 | (m^2 - n^2) + (n^2 - p^2) \Rightarrow 4 | (m^2 - p^2)$

it follows that mRp

therefore, R is transitive

As R being reflexive, symmetric and transitive, so R is an equivalence relation.

$[0] = \{m : m \text{ is even}\}$

$[1] = \{m : m \text{ is even}\}$

$[2] = \{m : m \text{ is even}\}$

$[3] = \{m : m \text{ is odd}\}$

$[0] \cup [2]$ and $[1] \cup [3]$

* Therefore, there are two distinct equivalence classes for the relation R

Section 8.5 #6

To verify anti-symmetricity

As rRs, from the definition, we have that

r is an ancestor of s or $r = s$

sRr

s is an ancestor of r or $s = r$

if rRs and sRr, then $r = s$, since by definition and by the property of equality

To verify transitivity

rRs

r is an ancestor of s or $r = s$

sRt

s is an ancestor of t or $s = t$

Since R is reflexive, anti symmetric, and transitive, R is a partial order relation.

Section 8.5 #7

$2R4$ 2 is prime factor of 4

$4R2$ 4 prime factor of 2

$2 \neq 4$

$2R4$ and $4R2$ but $2 \neq 4$

$rRs \Rightarrow$ every prime factor of r is a prime factor of s

$sRt \Rightarrow$ every prime factor of s is a prime factor of t

rRt is transitive

R is not antisymmetric it is not a partial order relation