Math 273 Comprehensive Exam Review:

This is just a snap shot of the material you should be studying!

Be sure to study your notes, homeworks, and reviews as well to get a full review of the material covered. For the final exam you will also want to look over Exam 1 and Exam 2.

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1. Find the equation of the line that goes through the points $\left(0, \frac{1}{2}, 1\right)$ and $(2, 1, -3)$.
2. Find the equation of the line where the planes $2x + y + 3z = 1$ and $x - y + 4z = 2$ intersect.
3. Find an equation of the plane through $(3, -1, 1), (1, 2, -2)$, and $(2, 1, 0)$.
4. Let $\vec{r_1}(t) = \langle 2 + 3t, 1 - t, 0 \rangle$ and $\vec{r_2}(t) = \langle t, -2t, 1 + t \rangle$. Find an equation for the line which is perpendicular to both lines and which passes through the point $(1, 1, 1)$.
5. Suppose the space curve $\boldsymbol{r}(t) = \langle 3t, \ln(t), \sqrt{t} \rangle$ gives the position of a particle as a function of time t.
    a. Find parametric equations for the line which is tangent to this curve at the point where $t = 1$.
    b. Find the particle's velocity and acceleration functions.
6. Let $\vec{r}(t) = \langle 2\cos(t^2), 2\sin(t^2), t^3 \rangle$ on $-\infty < t < \infty$. For arbitrary $a \geq 0$, find the arc length of $\vec{r}$ on the interval $-a \leq t \leq a$ in terms of a.
7. Suppose an object is launched from $\langle 0, 0, 2 \rangle$ m with initial velocity $\langle 10, 20, 30 \rangle$ m/s. Assuming gravity ($g \approx 10 \, {}^{m}/_{s}$) is the only force acting on the object and that the plane $z = 0$ represents the ground, find the duration of the flight.

8. Let $\overrightarrow{r_1}(t) = \langle 2+3t, 1-t, -1+t \rangle$ and $\overrightarrow{r_2}(t) = \langle 5+t, -2t, t \rangle$. Observing that these lines intersect, find the equation of the plane which contains both lines.
9. Use traces to sketch and identify the surface.
    a. $x = y^2 + 4z^2$
    b. $36x^2 + y^2 + 36z^2 = 36$
    c. $-x^2 + 4y^2 - z^2 = 4$
    d. $y = z^2 - x^2$
    e. $x = z$
    f. $x = y^3$
10. Find the unit tangent vector at the point with the given value of the parameter t.
    a. $\boldsymbol{r}(t) = \cos(t)\,\boldsymbol{i} + 3t\boldsymbol{j} + 2\sin(2t)\,\boldsymbol{k}, \quad t = 0$
    b. $\boldsymbol{r}(t) = \langle t^3 + 3t, t^2 + 1, 3t + 4 \rangle, \quad t = 1$
11. A particle moves with position function

$$\boldsymbol{r}(t) = \langle t\ln(t), t, e^{-t} \rangle$$

    Find the velocity, speed, and acceleration of the particle.
12. A particle starts at the origin with initial velocity $\langle 1, -1, 3 \rangle$. Its acceleration is $\boldsymbol{a}(t) = \langle 6t, 12t^2, -6t \rangle$. Find its position function.
13. Find the length of the curve of $\vec{r}(t) = \langle t, 8\sin t, 8\cos t \rangle$ for $0 \le t \le 4\pi$.
14. Graph several level curves of the following functions:
    a. $z = x^2 + y^2$
    b. $z = \sqrt{y - x^2 - 1}$
    c. $z = e^{-x^2 - 2y^2}$
    d. $z = x^2 - y$

15. Find the following limits:

a. $\lim\limits_{(x,y)\to(2,0)} \frac{(x^2-3xy^2)}{x+y}$

b. $\lim\limits_{(x,y)\to(2,2)} \frac{y^2-4}{xy-2x}$

16. Prove the following limits do not exist by finding two different directions of approach as $(x, y) \to (0, 0)$ leading to different "limits".

a. $\lim\limits_{(x,y)\to(0,0)} \frac{y^2\sin^2(x)}{x^4+y^4}$

b. $\lim\limits_{(x,y)\to(0,0)} \frac{xy^4}{x^2+y^8}$

c. $\lim\limits_{(x,y)\to(0,0)} \frac{x^2ye^y}{x^4+4y^2}$

17. Find all the second partial derivatives of $f(x, y) = 2x^5y^2 + y^3\sin(4x)$.

18.Let $x = t^2, y = t^3$. Define $z = x\sin y$. Find $\frac{dz}{dt}$. You may leave your answer in terms of x, y, and t.

19. Let $x = t\cos s\,, y = s\sin t$. Define $f(x, y) = xy - 2x + 3y$. Find $\frac{\partial f}{\partial s}$ and $\frac{\partial f}{\partial t}$. You may leave your answer in terms of x, y, s, and t.

20. Given that $w = xy^2z^3 + \sqrt{yz}, x = t^2, y = \ln(t) + t, z = \frac{1}{t^2}$, use the chain rule to find $\frac{dw}{dt}$.

21. Using implicit differentiation, find $\frac{\partial z}{\partial x}$ for $xy + xz + yz = 3$.

22.Compute the directional derivative of $f(x, y) = 3x^2 + y^3$ at the point $P(1, -2)$ in the direction of $\vec{v} = \langle 1, 2\rangle$.

23.Compute the directional derivative of $f(x, y, z) = ze^{-x^2-y^2}$ at the point $(1, 0, 2)$ in the direction of

$\boldsymbol{v} = \langle 1, -1, 3\rangle$. Is this the maximum rate of change? If not, find the maximum rate of change and its direction.

24. Find the unit vector that gives the direction of steepest ascent and the rate of steepest ascent at P. Then find the unit vector that gives the direction of steepest descent and the rate of steepest descent at P.

$f(x, y) = x^2 + 4xy - y^2$; $P(-1, 2)$.

25. Find an equation of the plane tangent to $xy + y^2 - 2z = 4$ at the point $(1, 2, 1)$.

26. Find an equation of the plane tangent to $z = x^2e^{x-y}$ at the point $(1, 1, 1)$.

27. Find an equation for the tangent plane to the surface $z\sin(4y - 3z) - \frac{yz^2}{x} = 6$ at the point $P\left(-1, \frac{3}{2}, 2\right)$.

28. Find the linear approximation to $f(x, y) = -x^2 + 2y^2$ at $(3, -1)$. Then use it to approximate the value of $f(3.1, -1.04)$.

29. The function $f(x, y) = y(x - 2)^2 + y^2 - y$, has three critical points. Two are saddle points at $(3, 0)$ and $(1, 0)$. Find the other critical point, and classify it as a local max, local min, or saddle point.

30. Find the critical points of the following functions. Then use the second derivative test to classify them as either a local min, local max, or saddle point.

   a. $f(x, y) = 4 + 2x^2 + 3y^2$
   b. $f(x, y) = xy(1 - x - y)$
   c. $f(x, y) = (x^2 + y^2)e^{y^2 - x^2}$

31. Compute the following integrals:

   a. $\int_0^2 \int_1^2 (y + xy^{-2})\,dy\,dx$
   b. $\iint_D y\,dA$, D is bounded by $y = x - 2, x = y^2$.

c. Find the volume of the given solid bounded by the cylinder $y^2 + z^2 = 4$ and the planes $x = 2y, x = 0, z = 0$ in the first octant.

d. $\int_0^{\sqrt{\pi}} \int_0^{x} \int_0^{xz} x^2 \sin y \, dy \, dz \, dx$

e. Evaluate $\iiint_E \sqrt{x^2 + y^2} dV$, where E is the region that lies inside the cylinder $x^2 + y^2 = 16$ and between the planes $z = -5$ and $z = 4$.

f. Find the volume of the solid that lies within both the cylinder $x^2 + y^2 = 1$ and the sphere $x^2 + y^2 + z^2 = 4$.

g. $\int_{-2}^{2} \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} \int_{\sqrt{x^2+y^2}}^{2} xz \, dz \, dx \, dy$

h. Evaluate $\iiint_E (x^2 + y^2) dV$ where E lies between the spheres $x^2 + y^2 + z^2 = 4$ and $x^2 + y^2 + z^2 = 9$.

i. $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{2-x^2-y^2}} xy \, dz \, dy \, dx$

32. A) Evaluate the triple integral $\int_0^1 \int_0^{1-x} \int_0^{2-x^2} 1 \, dz \, dy \, dx$.

B) Give a complete description and/or sketch of the region whose volume is represented by the integral in part (A).

33. Evaluate the triple integral $\iiint_E z \, dV$, where E is the region above $z = \sqrt{x^2 + y^2}$ and below $x^2 + y^2 + z^2 = 9$.

34. Find the volume of the solid that lies above the disc $x^2 + y^2 \leq 1$ and below the surface $z = 1 - (x^2 + y^2)^{\frac{3}{2}}$.

35. Sketch the following vector fields:

a. $\boldsymbol{F} = \langle -2y, 2x \rangle$

b. $\boldsymbol{F} = \langle y - x, x \rangle$

c. $\boldsymbol{F} = \langle 2x, -y \rangle$

36. Compute $\int_C \boldsymbol{F} \cdot d\boldsymbol{r}$, where:

a. $\boldsymbol{F}(x, y) = \langle 2y, 1 - x \rangle$ and C is the piece of the curve $y = x^2$ from $(-1, 1)$ to $(2, 4)$.

b. $\boldsymbol{F}(x, y) = \langle x^4 - y, y^3 - x \rangle$ and C is the piece of the unit circle from $(0, 1)$ to $(-1, 0)$.

c. $\boldsymbol{F}(x, y) = \langle x - y, 2y - x \rangle$ and C is given by $\boldsymbol{r}(t) = \langle t, 5 - t^2 \rangle, 0 \leq t \leq 2$.

37. Let S be the piece of the surface $z = 4 - x^2 - y$ which lies in the first octant, let E be the solid region bounded by S and the coordinate planes, and let $\boldsymbol{F} = \langle x^2, -y, z \rangle$.

a. Sketch S

b. Set up iterated integrals (limits and all) for calculating the volume of E.

c. Find the net upward flux of $\boldsymbol{F}$ through S. That is, find $\iint_S \boldsymbol{F} \cdot d\boldsymbol{S}$, using upward $d\boldsymbol{S}$.

38. Find $\iint_S xy \, dS$, where S is the surface $z = x + 2y^2, 0 \leq x \leq 1, 0 \leq y \leq 1$.

39. Determine whether each field is conservative. If it is, find a potential function ϕ such that $\boldsymbol{F} = \nabla \phi$.

a. $\boldsymbol{F}(x, y, z) = \langle 2xz, 2x - y, x^2 + z \rangle$

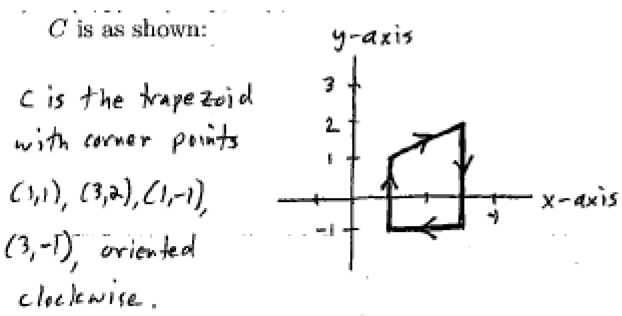
b. $\boldsymbol{F}(x, y, z) = \langle 2xz, -y, x^2 + z \rangle$

40. Let $\boldsymbol{F}(x, y, z) = \langle 2xz, 2x - y, x^2 + z \rangle$, let D be the cube $[0, 1]x[0, 1]x[0, 1]$ in $\mathbb{R}^3$, let S be the entire closed surface of D, and let C be the square boundary of the top side of the cube, oriented counterclockwise from above.

a. Use Stokes' Theorem to find $\oint_C \boldsymbol{F} \cdot d\boldsymbol{r}$.

b. Use the Divergence Theorem to find $\iint_S \boldsymbol{F} \cdot d\boldsymbol{S}$.

41. For each given field and curve, use Green's Theorem to help you compute $\oint_C \boldsymbol{F} \cdot d\boldsymbol{r}$.



a. $\boldsymbol{F}(x, y) = \langle -xy, x^2 \rangle$

b. $\boldsymbol{F}(x, y) = \langle \sin(x) - y^2, x + e^{-y} \rangle$

C is as shown: