4.1 #7

$2^n - 1$

$= 2^7 - 1$

$= 128 - 1$

$= 127$

and 127 is a prime number

4.1 #9

There is a perfect square that can be written as a sum of two other perfect squares.

1)

$25 = 5^2$

$16 = 4^2$

$9 = 3^2$

$25 = 16 + 9$

2)

$36 = 6^2$

$64 = 8^2$

$100 = 10^2$

$100 = 36 + 64$

the statement is true

#15

$$-a^n = (-a)^n$$

$$-a^n = (-1)^n a^n$$

if n is an odd integer

$-a^n = -a^n$ is true

if n is an even integer

$-a^n \neq a^n$

the property is false

#31

If k is any odd integer and m is any even integer, then, k^2+m^2 is odd.

$k=2p+1$, $m=2q$

$$k^2+m^2=(2p+1)^2+(2q)^2=(4p^2+4p+1)+4q^2=2(2p^2+2p+2q^2)+1,$$

where $2p^2+2p+2q^2$ is an integer. So, k^2+m^2 is odd.

4.2 , #5

The numbers in 1-7 are all rational. Write each number as a ratio of two integers.

$.56565656\ldots$

let $x = .56565656$ ①

then $100x = 56.565656$ ②

subtract 1 from 2

$99x = 56$

$x = \frac{56}{99} \rightarrow = .565656$

4.2 # 18 If r and s are any two rational numbers, then $\frac{r+s}{2}$ is rational

1) $\frac{\frac{a}{b} + \frac{c}{d}}{2} = \frac{ab + bc}{2bd}$ $\frac{R+S}{2}$ is a rational number

$R < S$

$R < \frac{R+S}{2} < S$ $R = \frac{R+R}{2} < \frac{R+S}{2} < \frac{S+S}{2} = S$

$R = \frac{A}{B}$, $B = \frac{C}{D}$. $A, B \neq 0$. $C, D \neq 0$

$\frac{R+S}{2} = \frac{\frac{A}{D} + \frac{C}{D}}{2}$ $AD + CD$ $2BD \neq 0$

$\frac{R+S}{2}$ is rational number.

4.2 # 36

proof: Let rational numbers produce a rational number $r = \frac{1}{4}$ and $s = \frac{1}{2}$ be given. $r + s = \frac{1}{4} + \frac{1}{2} = \frac{3}{4}$, which is a rational. This is what was to be shown.

- The given "proof" only works for the given instances of r and s. A proof should be universal, that is, it should hold for all possible values of r and s.

4.3 # 18) Show that the following statement is false: For all integers a and b, if $3 \mid (a+b)$ then $3 \mid (a-b)$

For

$$a = 7 \text{ and } b = 2$$

$$a + b = 7 + 2$$

$$= 9$$

Clearly, 3 divides 9, but $a - b = 5$ is not divisible by 3. Thus, if $3 \mid a + b$ then $3 \mid a - b$ may not be true.

Therefore, the statement false.

4.3 # 38

a) $a^3 = p_1^{3e_1} \; p_2^{3e_2} \; p_3^{3e_3} \cdots p_k^{3e_k}$

b) $2^4 \cdot 3^5 \cdot 7 \cdot 11^2 \cdot k$ will be perfect cube if

$k = 2^2 \cdot 3 \cdot 7^2 \cdot 11$

Now perfect cube is

$$(2^2 \cdot 3^2 \cdot 7 \cdot 11)^3$$

4.4 # #28 case 1

$n = 3q$ for some integer q

$n(n+1)(n+2) = 3q(3q+1)(3q+2) - 3$

$n(n+1)(n+2)$ is divisable by 3

Case 2

$n = 3q+2$ for some integer q

$n(n+1)(n+2) = (3q+\cancel{2}^{1})(3q+\cancel{3}^{2})(3q+\cancel{4}^{1})$

$\underline{m} = (3\underline{q+1})(3\underline{q+2})(q+1)$

case 3

$m = (3q+2)(3q+4)(\cancel{3}q+1)$

Therefore. the product of any three consecutive integers is divisible by 2

4.4 #29

$$n^2 = (3q)^2$$
$$= 9q^2$$
$$= 3(3q^2)$$
$$= 3k \qquad k = 3q^2$$

$$n^2 = (3q+1)^2$$
$$= (3q+1)(3q+1)$$
$$= 9q^2 + 3q + 3q + 1$$
$$= 9q^2 + 6q + 1$$
$$= 3[3q^2 + 2q] + 1$$
$$= 3k+1 \qquad k = 3q^2 + 2q$$

$$n^2 = (3q+2)^2$$
$$= (3q+2)(3q+2)$$
$$= 9q^2 + 6q + 6q + 4$$
$$= 9q^2 + 12q + 4$$
$$= 9q^2 + 12q + 4$$
$$= 9q^2 + 12q + 3 + 1$$
$$= 3[3q^2 + 4q + 1] + 1$$
$$= 3k+1 \qquad [k = 3q^2 + 4q + 1]$$