84.

The centers for disease control and prevention reported in 2012 that 1 in 88 american children had been diagnosed with an autism spectrum disorder (ASD)

a.

$$P = \frac{1}{88} \times 200 = 2.273$$

$$\sqrt{np(1-p)}$$

$$= \sqrt{200 \times \frac{1}{88} - (1 - \frac{1}{88})}$$

$$= \sqrt{2.246901}$$

$$= 1.498967$$

$$= 1.499$$

b.

$P(x \leq 2(\text{mean} - \text{standard deviation})) = P(x \leq 2(0.0114 - 0.1503))$

$$= P(x \geq 2) = 1 - P(x \leq 1)$$

$$= 1 - \left[\frac{e^{-2.273}}{0!} + \frac{e^{-2.273}(2.273)^1}{1!}\right]$$

$$= 1 - [0.103003 + 0.234125]$$

$$= 1 - 0.337128$$

$$= 0.662872$$

$$= 0.663$$

c.

$$352 \times \frac{1}{88} = 4$$

$$P(x < 5) = P(x \leq 4)$$

$[0.018316 + 0.073263 + 0.146525 + 0.195367 + 0.19$

$= .628830$

$= 0.629$

86.

a. what is the probability that one cubic meter of discharge contains at least 8 organisms?

$P(x \geq 8) = 1 - P(x < 7)$

$= 1 - P(x \leq 7)$

$= 1 - \left[\sum^{7} P(x = x)\right]$

$= 1 - \left[\sum^{7} \frac{e^{-10}(10)^x}{x!}\right]$

$= 1 - [= \text{poisson.Dist}(7, 10, \text{True})]$

$1 - 0.2202$

$= {}^{*}0.7798^{*}$

b.

$P(Y > \mu_Y + \sigma_Y) = P(Y > 15 + \sqrt{15})$

$= P(Y > 18.873)$

$\approx 1 - P(Y \leq 18)$

$= 1 - \left[e^{-15} \sum^{3} \frac{(15)^x}{x!}\right]$

$= 1 - [\text{poisson.Dist}(18, 15, \text{true})]$

$= 1 - 0.8195$

$= {}^{*}0.18053^{*}$

c. $e^{-\lambda} = 0.001$

$\ln(e^{-\lambda}) = \ln(0.001) \quad -\ln(0.001)$

$\lambda = 6.9078 = \lambda \approx 7$ discharge is $^{*}7/m^3{}^{*}$

5.

a.

$$\int_0^2 kx^2\,dx = 1$$

$$\rightarrow \left[\frac{kx^3}{3}\right] = 1 \rightarrow \frac{k}{3}\left[x^3\right]_0^2 = 1 \rightarrow \frac{k}{3}(2^3 - 0^3) = 1$$

$$\rightarrow \frac{k}{3}(8) = 1 \rightarrow k = \frac{3}{8} \rightarrow {}^*k = 0.375^*$$

b.

$$p(x<1) = \int_0^1 kx^2\,dx = \int_0^1 \frac{3}{8}x^2\,dx = \frac{3}{8}\int_0^1 x^2\,dx$$

$$= \frac{3}{8}\left[\frac{x^3}{3}\right]_0^1 = \frac{1}{8}(1-0) = {}^*0.125^*$$

c.

$$\frac{3}{8}\int_1^{1.5} x^2\,dx = \frac{3}{8}\left[\frac{x^3}{3}\right]_1^{1.5} = \frac{1}{8}\left[1.5^3 - 1\right] =$$

$${}^*.296875^*$$

d.

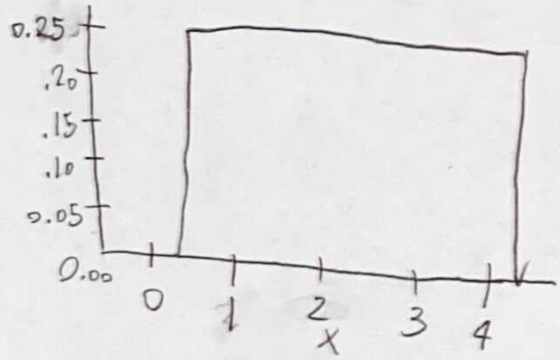
$$p(x \geq 1.5) = \int_{1.5}^2 \frac{3}{8}x^2\,dx = \frac{3}{8}\left[\frac{x^3}{3}\right]_{1.5}^2 = \frac{2^3 - (1.5)^3}{8}$$

$$= {}^*0.578125^*$$

7.

a.

$$f(x; 0.20, 4.25) = \begin{cases} \frac{1}{4.05}, & 0.20 \le x \le 4.25 \\ 0, & \text{otherwise} \end{cases}$$



b. $$P(x>3) = \int_3^{4.25} \frac{1}{4.05}\,dx$$

$$= \frac{1}{4.05}(x)\Big|_3^{4.25}$$

$$= \frac{4.25-3}{4.05} \approx 0.3086$$

c.

$$P(|x-\mu| \le 1) = P(|x-2.225| \le 1)$$

$$= P(-1+2.225 \le x \le 1+2.225)$$

$$= P(1.225 \le x \le 3.225)$$

$$= \int_{1.225}^{3.225} f(x)\,dx$$

$$\frac{3.225-1.225}{4.05} \approx 0.4938$$

d. $$\int_a^{a+1} \frac{1}{4.05}\,dx \to \frac{1}{4.05}(x)\Big|_a^{a+1} \to \frac{a+1-a}{4.05} = \frac{1}{4.05} \approx 0.2469$$

11.

a. $P(x \le 1) = F(1) = \frac{1^2}{4} = \boxed{0.25}$

b. $P(0.5 \le x \le 1) = F(1) - F(0.5) = \frac{1^2}{4} - \frac{0.5^2}{4} =$

0.1875

c. $P(x > 1.5) = 1 - F(1.5) = 1 - \frac{1.5^2}{4} = 0.4375$

d. $F(\mu) = 0.5 \quad \frac{\mu^2}{4} = 0.5, \; \mu = \sqrt{2}, \; \mu = 1.4142$

e. $f(x) = \frac{d}{dx}\left(\frac{x^2}{4}\right) = \frac{2x}{4} = 0.5x$

$$F(x) = \begin{cases} 0.5x & 0 < x < 2 \\ 0 & \text{otherwise} \end{cases}$$

f. $\int_0^2 x \cdot (0.5x)\,dx = 0.5\left(\frac{x^3}{3}\right)\Big|_{x=0}^{x=2}$

$= \frac{4}{3} = 1.33$

g. $\int_0^2 x^2 \cdot (0.5x)\,dx$

$= 0.5\left(\frac{x^4}{4}\right)\Big|_{x=0}^{x=2}$

$V(x) = E(x^2) - [E(x)]^2$

$= 2 - \left(\frac{4}{3}\right)^2$

$= \frac{2}{9} = .222$

$\sigma x = \sqrt{V(x)} = \sqrt{.222}$

$= .471$

h)

$E[h(x)] = \int h(x) \cdot f(x)\,dx$

$\int_0^2 x^2 (0.5x)\,dx$

$= 0.5\left[\frac{x^4}{4}\right]\Big|_{x=0}^{x=2}$

$= 2$

22.

a.

$$F(x) = \int_1^x 2\left(1-\frac{1}{x^2}\right)dx = 2\left[\int_1^x 1\,dx - \int_1^x \left(\frac{1}{x^2}\right)dx\right]$$

$$= 2\left(x+\frac{1}{x}\right)\Big|_1^x = 2\left(\left(x+\frac{1}{x}\right)-\left(1+\frac{1}{1}\right)\right)$$

$$= 2\left(x+\frac{1}{x}-2\right)$$

$$F(x) = \begin{cases} 2(x+\frac{1}{x}-2 & 1 \le x \le 2 \\ 1 & x < 2 \\ 0 & x < 1 \end{cases}$$

b

$$2x_p^2 - 4x_p + 2 = x_p p$$

$$2x_p^2 - x_p(p+4) + 2 = 0$$

$$x_p = \frac{1}{4}\left[4 \mp p + \sqrt{p^2+8p}\right]$$

$$x_p = \frac{1}{4}\left[4+p+\sqrt{p^2+8p}\right]$$

$$\tilde{\mu} = \frac{1}{4}\left[4+0.5\right] + \sqrt{0.5^2+8(0.5)}$$

$$\tilde{\mu} = 1.64$$

c.

$$2\int_1^2 x\left(1-\frac{1}{x^2}\right)dx = 2\left[\frac{x^2}{2} - \ln(x)\right]_1^2 = 1.614$$

D. $2\int_1^{1.5} (1.5-x)(1-\frac{1}{x^2})$ variance $= 2\int_1^2 x^2\left(x-\frac{1}{x^2}\right)dx - (1.614)^2$

$$= 2\left[\int_1^{1.5}\left(1.5-x-\frac{1.5}{x^2}+\frac{1}{x}\right)dx\right.$$

$$= 2\left[\frac{x^3}{3} - x\right]_1^2 - 2.605$$

$$= 2\left[1.5x - \frac{x^2}{2} + \frac{1.5}{x} + \ln(x)\right]_1^{1.5}$$

$$= 0.0626$$

$$= 0.061$$