Section 14.3 – Motion in Space

- Position: $\boldsymbol{r}(t) = \langle x, y, z \rangle$
- Instantaneous Velocity: $\boldsymbol{v}(t) = \boldsymbol{r}'(t) = \langle x'(t), y'(t), z'(t) \rangle$
- Speed: $|\boldsymbol{v}(t)| = \sqrt{x'(t)^2 + y'(t)^2 + z'(t)^2}$
- Acceleration: $\boldsymbol{a}(t) = \boldsymbol{v}'(t) = \boldsymbol{r}''(t)$

- Find the velocity and speed of the object. Then find the acceleration of the object.
  $\boldsymbol{r}(t) = \langle 3 + t, t^2, e^{-t} \rangle$ for $t \geq 0$

- Find the velocity and speed of the object. Then find the acceleration of the object.
  $\boldsymbol{r}(t) = \langle 3\sin t\,, 5\cos t\,, 4\sin t\rangle$ for $0 \leq t \leq 2\pi$

- Uniform straight-line motion: $\boldsymbol{r}(t) = \langle x_0 + at, y_0 + bt, z_0 + ct \rangle$
- Circular motion: $\boldsymbol{r}(t) = \langle A\cos t\,, A\sin t \rangle$ for $0 \le t \le 2\pi$, $\boldsymbol{r}$ and $\boldsymbol{a}$ are parallel but point in opposite directions, and $\boldsymbol{r} \cdot \boldsymbol{v} = \boldsymbol{a} \cdot \boldsymbol{v} = 0$ which means position and acceleration are both orthogonal to the velocity.
- Given an acceleration vector, initial velocity $\langle u_0, v_0 \rangle$, and initial position $\langle x_0, y_0 \rangle$, find the velocity and position vectors for $t \ge 0$.

$$\boldsymbol{a}(t) = \langle 1, 2 \rangle, \;\; \langle u_0, v_0 \rangle = \langle 1, 1 \rangle, \langle x_0, y_0 \rangle = \langle 2, 3 \rangle$$

- Two-Dimensional Motion in a Gravitational Field:
  Consider an object moving in a plane with a horizontal x-axis and a vertical y-axis, subject only to the force of gravity, $\boldsymbol{a}(t) = \langle 0, -g \rangle$. Given the initial velocity $\boldsymbol{v}(0) = \langle u_0, v_0 \rangle$ and the initial position $\boldsymbol{r}(0) = \langle x_0, y_0 \rangle$, the velocity of the object, for $t \ge 0$, is

$$\boldsymbol{v}(t) = \langle x'(t), y'(t) \rangle = \langle u_0, -gt + v_0 \rangle$$

  and the position is

$$\boldsymbol{r}(t) = \langle x(t), y(t) \rangle = \langle u_0 t + x_0, -\frac{1}{2} g t^2 + v_0 t + y_0 \rangle$$

$$g \approx 9.8\, {}^{m}/_{s^2} \;\; or \; 32\, {}^{ft}/_{s^2}$$

- Find the velocity and position vectors for $t \geq 0$.
  A soccer ball has an initial position $\langle x_0, y_0 \rangle = \langle 0, 0 \rangle$ when it is kicked with an initial velocity of $\langle u_0, v_0 \rangle = \langle 30, 6 \rangle \, m/s$.

- Determine the time of flight and range of the object.

- Determine the maximum height of the object.

- Three-dimensional motion:
- Find the velocity and position vectors for $t \geq 0$.

$$\boldsymbol{a}(t) = \langle 1, t, 4t \rangle, \langle u_0, v_0, w_0 \rangle = \langle 20, 0, 0 \rangle, \langle x_0, y_0, z_0 \rangle = \langle 0, 0, 0 \rangle$$

- $\boldsymbol{a}(t) = \langle 0, 0, -g \rangle$. We include any crosswinds, spins, or slices in other force components.
- For the following problems:
    - Find the velocity and position vectors for $t \geq 0$.
    - Determine the time of flight and range of the object.
    - Determine the maximum height of the object.
- A bullet is fired from a rifle 1 m above the ground in a northeast direction. The initial velocity of the bullet is $\langle 200, 200, 0 \rangle$ m/s.

- A baseball is hit 3 feet above home plate with an initial velocity of $\langle 60, 80, 80 \rangle$ ft/s. The spin on the baseball produces a horizontal acceleration of the ball of 10 ft/s^2 in the eastward direction.