Section 14.4 – Length of Curves

- Arc Length for Vector Functions:
  Consider the parametrized curve $\boldsymbol{r}(t) = \langle f(t), g(t), h(t) \rangle$, where f', g' and h' are continuous, and the curve is traversed once for $a \leq t \leq b$. The arc length of the curve between $\left(f(a), g(a), h(a)\right)$ and $\left(f(b), g(b), h(b)\right)$ is
  $$L = \int_a^b \sqrt{f'(t)^2 + g'(t)^2 + h'(t)^2}\,dt = \int_a^b |\boldsymbol{r}'(t)|\,dt$$

- Find the length of the trajectory on the given interval.
  $$\boldsymbol{r}(t) = \langle 2t, -t, 5t \rangle, \quad 0 \leq t \leq 4$$

- Find the length of the trajectory on the given interval.

$$\boldsymbol{r}(t) = \langle t, 8\sin t, 8\cos t\rangle, 0 \le t \le 4\pi$$

- <u>Arc Length as a Function of a Parameter:</u>
  Let $\boldsymbol{r}(t)$ describe a smooth curve, for $t \geq a$. The arc length is given by
  $$s(t) = \int_a^t |\boldsymbol{v}(u)| du\,,$$
  where $|\boldsymbol{v}| = |\boldsymbol{r}'|$. Equivalently, $\frac{ds}{dt} = |\boldsymbol{v}(t)| > 0$. If $|\boldsymbol{v}(t)| = 1$, for all $t \geq a$, then the parameter t corresponds to arc length.

- Determine whether the following curve uses arc length as a parameter. If not, find a description that uses arc length as a parameter.
  $$\boldsymbol{r}(t) = \langle 2\cos t\,, 2\sin t \rangle, \qquad 0 \leq t \leq 2\pi$$