Answer Key

Math 273 – Written Homework #3

Remember to show work or explanation on everything. Writing the answer only will not be accepted. Homework should be submitted on Crowdmark by the due date. Late work is not accepted. You will lose 1% for every minute your assignment is submitted after the deadline.

All graphs must be hand drawn. Any work done using a calculator or computer will receive a zero.

Your work should be unique! You may work together but identical work will receive a zero for the first assignment and then will be reported to the Student Board of Conduct.

No Written Homeworks may be uploaded to Chegg or similar websites. Anyone found uploading homework sets to a tutoring webpage will be given a zero and possibly fail the course.

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1. Find the arc length of the trajectory given on the specified interval.

   $\boldsymbol{r}(t) = \langle -\frac{1}{7}, 4e^{2t} - 1, 3e^{2t} + 3 \rangle \quad 0 \le t \le \ln 2$

$$\vec{r}\,'(t) = \langle 0, 8e^{2t}, 6e^{2t} \rangle$$

$$|\vec{r}\,'(t)| = \sqrt{64e^{4t} + 36e^{4t}} = \sqrt{100e^{4t}} = 10e^{2t}$$

$$L = \int_0^{\ln(2)} 10e^{2t}\,dt = 5e^{2t}\Big|_0^{\ln(2)}$$

$$= 5e^{2\ln(2)} - 5e^0$$

$$= 5e^{\ln(4)} - 5$$

$$= 5(4) - 5 = \boxed{15}$$

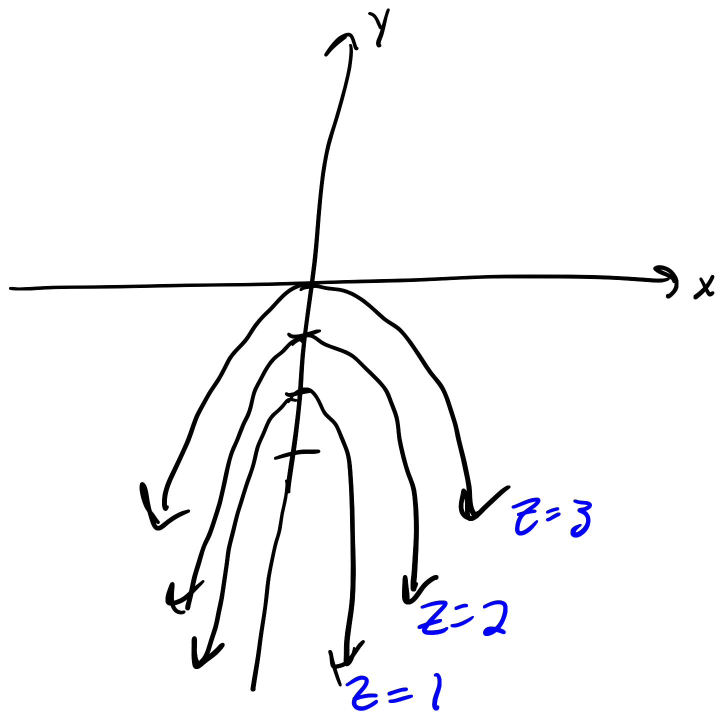
2. Answer the following for the function $f(x, y) = 3 + x^2 + y$

   a. Graph at least 3 level curves for the function. Be sure to label the curves with their z-values.

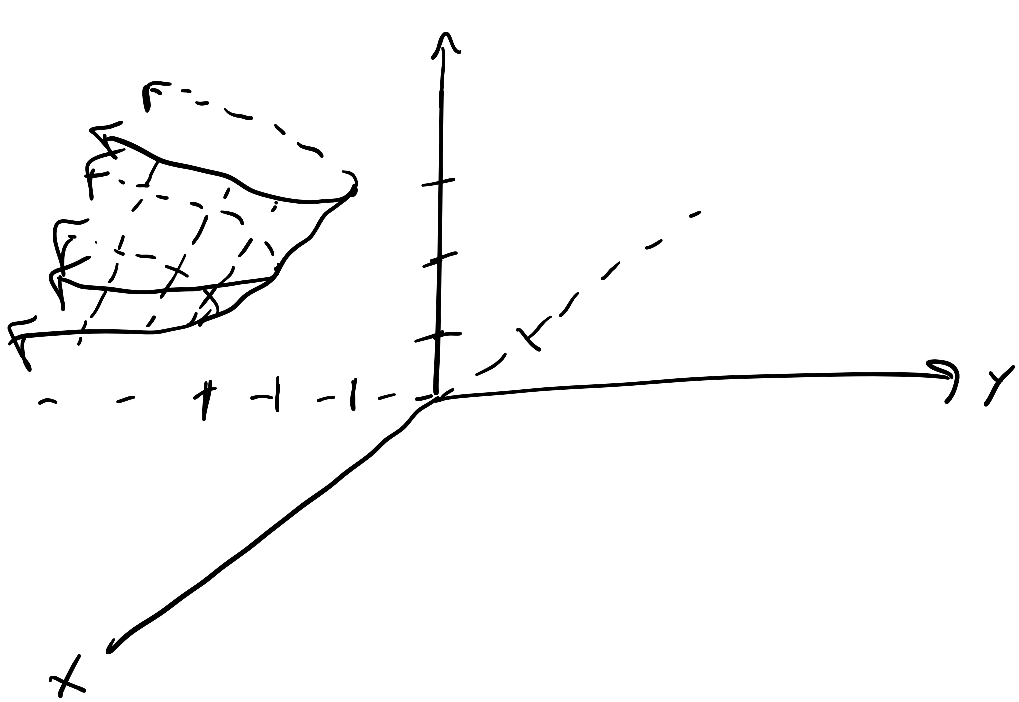
   b. Graph the function in $\mathbb{R}^3$.

a) $y = -x^2 - 3 + z$

z	$y = -x^2 - 3 + z$
3	$y = -x^2$
2	$y = -x^2 - 1$
1	$y = -x^2 - 2$



b)



3. Evaluate the following limit:

$$\lim_{(x,y)\to(-2,3)} \frac{xy - x + 2y - 2}{x^2 - 3x - 10}$$

Sidework:

$$\frac{xy - x + 2y - 2}{x^2 - 3x - 10} = \frac{x(y-1) + 2(y-1)}{(x+2)(x-5)}$$

$$= \frac{\cancel{(x+2)}(y-1)}{\cancel{(x+2)}(x-5)}$$

$$= \frac{y-1}{x-5}$$

$$= \lim_{(x,y)\to(-2,3)} \left(\frac{y-1}{x-5}\right)$$

$$= \frac{3-1}{-2-5} = \boxed{-\frac{2}{7}}$$

4. Use two paths to prove that the following limit does not exist.

$$\lim_{(x,y)\to(0,0)} \frac{5xy}{6x^2 + y^2}$$

Let $y=0$:

$$\lim_{x\to 0} \frac{0}{6x^2} = 0 = L_1$$

Let $y=x$:

$$\lim_{x\to 0} \frac{5x^2}{6x^2+x^2} = \lim_{x\to 0} \frac{5x^2}{7x^2} = \lim_{x\to 0} \frac{5}{7} = \frac{5}{7} = L_2$$

Since $L_1 \neq L_2 \Rightarrow \lim_{(x,y)\to(0,0)} \frac{5xy}{6x^2+y^2} = \text{DNE}$

5. Find the first partial derivatives of the following function:

$$g(x,y) = x^2 \sin(3xy) - e^{7x}y + xy^5 - 2x + y$$

$$g_x = 2x\sin(3xy) + 3x^2y\cos(3xy) - 7e^{7x}y + y^5 - 2$$

$$\frac{\partial g}{\partial y} = 3x^3\cos(3xy) - e^{7x} + 5xy^4 + 1$$